
Singular Value Thresholding for Matrix Completion

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Abstract

1 Introduction

2 Different Algorithms

2.1 Baseline Algorithm

2.2 Eigentaste

2.3 SMC

2.4 Singular Value Thresholding

Singular Value Thresholding (SVT) [1] is an algorithm proposed for *nuclear norm minimization* of a matrix X from a few known entries $M_{ij}, (i, j) \in \Omega$. Formally, SVT addresses the optimization problem

$$\begin{aligned} & \underset{X}{\text{minimize}} && \|X\|_* \\ & \text{subject to} && \mathcal{P}_\Omega(X) = \mathcal{P}_\Omega(M), \end{aligned}$$

where $\|\cdot\|_*$ is the *nuclear norm*, or the sum of the singular values and $\mathcal{P}_\Omega(\cdot)$ makes zero all entries $(i, j) \notin \Omega$. This can be thought of as a convex relaxation to the rank minimization problem, and the two are formally equivalent under some conditions. The rank minimization problem is, however, highly non-convex and therefore not a suitable candidate for black-box optimization algorithms.

Singular Value Thresholding works by iteratively constructing X using a low-rank, low-singular value approximation to an auxiliary sparse matrix Y . Y is then adjusted to ensure the resulting approximation in the subsequent step has matching entries $X_{ij} = M_{ij}$. Each iteration consists of the inductive steps

$$\begin{cases} X^k = \text{shrink}(Y^{k-1}, \tau) \\ Y^k = Y^{k-1} + \delta_k \mathcal{P}_\Omega(M - X^k), \end{cases}$$

where $\text{shrink}(\cdot, \cdot)$ is the *singular value shrinkage operator*. Given a singular value decomposition $X = U\Sigma V^T$, $\Sigma = \text{diag}(\{\sigma_i\}_{1 \leq i \leq r})$, we can write this as

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$$\text{shrink}(X, \tau) = U \Sigma_{\tau} V^T, \quad \Sigma_{\tau} = \text{diag}(\{(\sigma_i - \tau)_+\}).$$

These two operations, when repeated, approach a low-nuclear norm solution by repeatedly shrinking the singular values of X . This algorithm has shown success in recovering accurate low-rank solutions when the source of M is also low-rank, even though it does not optimize this objective directly. The original authors discuss its theoretical guarantees in detail, but we choose to omit them in this discussion.

In practice, this system has a number of hyperparameters that must be carefully tuned to guarantee convergence. The shrinkage value τ must be set fairly high in order for the algorithm to converge quickly, but not too high that it dwarfs the true singular values. The stepsizes δ_k are similarly sensitive. These can be set dynamically as well, though we choose to maintain a fixed stepsize throughout. We compute the decomposition of Y^K in batches, which introduces a new batch size parameter l . Also important is the initialization of Y^0 , for which the authors provide helpful strategies. Finally, we use the relative error $\|\mathcal{P}_{\Omega}(X^k - M)\|_F / \|\mathcal{P}_{\Omega}(M)\|$ as a stopping criterion; we terminate when this drops below a small ϵ .

3 Results and Discussion

3.1 Jester Dataset

4 Conclusions

References

- [1] Jian-Feng Cai, Emmanuel J Candès, and Zuowei Shen. A singular value thresholding algorithm for matrix completion. *SIAM Journal on Optimization*, 20(4):1956–1982, 2010.