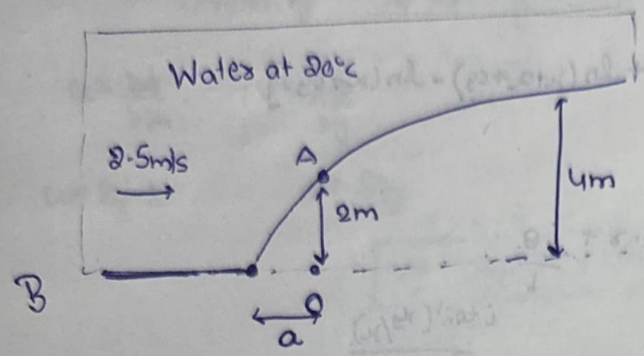
Tutorial - 9

Q2.



$P_B = 130 kPa$

$V_B = 2.5 m/s$

By Bernoulli's eqⁿ:

$$P_B + \frac{1}{2}\rho V_B^2 + 0 = P_A + \frac{1}{2}\rho V_A^2 + \rho g(2)$$

for 2D half rankine body,

$$W(z) = Uz + \frac{m}{2\pi}\ln z$$

$$\frac{dW}{dz} = U + \frac{m}{2\pi z}$$

$$\frac{dW}{dz} = \frac{m}{2\pi z} + U$$

$$\boxed{V = \frac{m}{2\pi z} + U}$$

$z_A = 0 + 2i$

$$V = \frac{m}{2\pi r} e^{-i\theta} + U$$

$$= \frac{m}{2\pi r}(\cos\theta - i\sin\theta) + U$$

$$V = \left(U + \frac{m\cos\theta}{2\pi r}\right) - i\frac{m\sin\theta}{2\pi r}$$

$$= u - iv$$

$$V^2 = u^2 + v^2$$

$$V^2 = \left(U + \frac{m\cos\theta}{2\pi r}\right)^2 + \left(\frac{m\sin\theta}{2\pi r}\right)^2$$

at A, $r=2,\ \theta=\pi/2$

$$V_A^2 = U^2 + \left(\frac{m}{2\pi(2)}\right)^2$$

$$\boxed{V_A^2 = U^2 + \frac{m^2}{16\pi^2}}$$

& $\boxed{\pi a = 4}$

$\psi = -\frac{m\theta}{2\pi} - U r \sin\theta$

at stag. point, $\theta=\pi$

$-\frac{m}{2\pi}\theta - Uy$

$-\frac{m}{2} = -Uy$

$y = \frac{m}{2U} = \pi a$

$a = \frac{m}{2\pi U}$

from stagnan point.

$$\frac{dW}{dz} = U + \frac{m}{2\pi z} = 0$$

$$z = -\frac{m}{2\pi U}$$

$\therefore\ \boxed{a = m/2\pi U}$

$a = 4/\pi$

$\therefore\ \frac{m}{2\pi U} = \frac{4}{\pi}$

$\boxed{m = 8U}$

$$\therefore\ V_A^2 = U^2 + \frac{64U^2}{16\pi^2} = U^2 + \frac{4U^2}{\pi^2}$$

$$\boxed{V_A^2 = U^2\left[1 + \frac{4}{\pi^2}\right]}$$

$$V_A^2 = 2.5^2\left[1 + \frac{4}{\pi^2}\right]$$

$$\boxed{V_A = 2.963\ m/s}$$

Now.

$$130\times10^3 + \frac{1}{2}\times10^3\times(2.5)^2 = P_A + \frac{1}{2}\times10^3\times(2.963)^2 + 10^3\times9.81\times2$$

$$P_A = 130\times10^3 + 3.125\times10^3 - 4.3897\times10^3 - 19.62\times10^3$$

$$P_A = 109.18\times10^3\ Pa$$

$$\boxed{P_A = 109.2\ kPa}$$

Q3.

Crude model: Sink + Free vortex.

Sink $\Rightarrow -\frac{q}{2\pi}\ln z$, vortex $= -i\frac{k}{2\pi}\ln z$

$$w(z) = -\frac{q}{2\pi}\ln z - i\frac{k}{2\pi}\ln z$$

$z = re^{i\theta}$

$$w = -\frac{q}{2\pi}(\ln r + i\theta) - i\left(\frac{k}{2\pi}(\ln r + i\theta)\right)$$

$$= -\frac{q}{2\pi}\ln r - \frac{iq\theta}{2\pi} - \frac{ik}{2\pi}\ln r - i^2\frac{k\theta}{2\pi}$$

$$= -\frac{q}{2\pi}\ln r + \frac{k\theta}{2\pi} - i\left[\frac{q\theta}{2\pi} + \frac{k}{2\pi}\ln r\right]$$

$$w = \phi + i\psi$$

$$\boxed{\phi = -\frac{q}{2\pi}\ln r + \frac{k\theta}{2\pi}}$$

$$\boxed{\psi = \frac{q\theta}{2\pi} + \frac{k}{2\pi}\ln r}$$

$(u_r - iu_\theta)e^{-i\theta}$

$$\frac{dw}{dz} = -\frac{q}{2\pi z} - \frac{ik}{2\pi z}$$

$$V = -\frac{q}{2\pi r}e^{-i\theta} - \frac{ik}{2\pi r}e^{-i\theta}$$

$$= \left(-\frac{q}{2\pi r} - \frac{ik}{2\pi r}\right)(\cos\theta - i\sin\theta)$$

$$= -\frac{q}{2\pi r}\cos\theta + i\frac{q}{2\pi r}\sin\theta - i\frac{k\cos\theta}{2\pi r} - \frac{k\sin\theta}{2\pi r}$$

$$V = \left(-\frac{q}{2\pi r}\cos\theta - \frac{k}{2\pi r}\sin\theta\right) + i\left(\frac{q}{2\pi r}\sin\theta - \frac{k\cos\theta}{2\pi r}\right)$$

$$V = u + iv$$

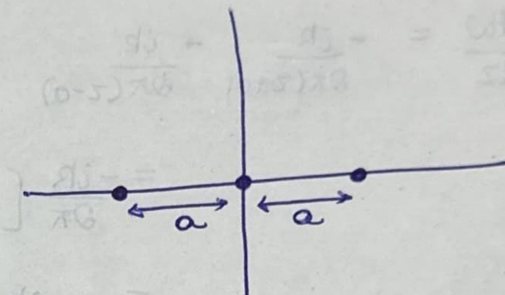
$$u = \frac{-q}{2\pi r}\cos\theta - \frac{k}{2\pi r}\sin\theta$$

$$v = \frac{q}{2\pi r}\sin\theta - \frac{k}{2\pi r}\cos\theta$$

Q4.

First vortex → k $(-a, 0)$

2nd vortex → k $(a, 0)$



$$w(z) = \frac{-ik}{2\pi}\ln(z-(-a)) - \frac{ik}{2\pi}\ln(z-a)$$

$$= \frac{-ik}{2\pi}\ln(z+a) - \frac{ik}{2\pi}\ln(z-a)$$

$$\ln(x+iy) = \ln r + i\theta$$

$$i\tan^{-1}(y/x)$$

$$= \frac{-ik}{2\pi}\ln(x+a+iy) - \frac{ik}{2\pi}\ln(x-a+iy)$$

$$= \frac{-ik}{2\pi}\ln(z+a) - \frac{ik}{2\pi}\ln(z-a)$$

$$= \frac{-ik}{2\pi}\left[\ln(z+a) + \ln(z-a)\right]$$

$\ln re^{i\theta}$

$= \ln r + i\theta$

$= i\tan^{-1}(y/x)$

$= \ln($

$\tan^{-1}\left(\frac{y}{x+a}\right)$

$\ln((x+a)+iy) = \ln\sqrt{(x+a)^2+y^2} + i\tan^{-1}(y/x+a)$

$$= \frac{-ik}{2\pi}\left[\ln\sqrt{(x+a)^2+y^2} + i\tan^{-1}\left(\frac{y}{x+a}\right) + \ln\sqrt{(x-a)^2+y^2} + i\tan^{-1}\left(\frac{y}{x-a}\right)\right]$$

$$= \frac{k}{2\pi}\left[\tan^{-1}\left(\frac{y}{x+a}\right) + \tan^{-1}\left(\frac{y}{x-a}\right) - i\left[\ln\left(\sqrt{(x+a)^2+y^2}\right) + \ln\left(\sqrt{(x-a)^2+y^2}\right)\right]\right]$$

$$= \frac{-k}{2\pi}\left[\right.$$

$$\phi = \frac{k}{2\pi}\left[\tan^{-1}\left(\frac{y}{x+a}\right) + \tan^{-1}\left(\frac{y}{x-a}\right)\right]$$

$$\psi = \frac{-k}{2\pi}\left[\ln\left(\sqrt{(x+a)^2+y^2}\right) + \ln\left(\sqrt{(x-a)^2+y^2}\right)\right]$$

$$w(z) = \frac{-ik}{2\pi}\ln(z+a) - \frac{ik}{2\pi}\ln(z-a)$$

$$\frac{dw}{dz} = \frac{-ik}{2\pi(z+a)} - \frac{ik}{2\pi(z-a)} = \frac{-ik}{2\pi}\left[\frac{1}{z+a} + \frac{1}{z-a}\right]$$

$$= \frac{-ik}{2\pi}\left[\frac{z-\cancel{a}+z+\cancel{a}}{z^2-a^2}\right]$$

$$= \frac{-ik}{\pi}\left[\frac{z}{z^2-a^2}\right]$$

$$= \frac{-ik}{\pi}\left[\frac{x+iy}{x^2-y^2-a^2}\right] = -i\left[\frac{x}{x^2-y^2-a^2}\right]\frac{k}{\pi} + \frac{y}{\pi}\left[\frac{k}{x^2-y^2-a^2}\right]$$

$$\boxed{u = \frac{ky}{\pi(x^2-y^2-a^2)}, \quad v = \frac{xk}{\pi(x^2-y^2-a^2)}}$$

Along yaxis, $\psi(0,y) = \frac{-k}{2\pi}\left[\cancel{\tan^{-1}(a)}\ \ln\left(\sqrt{a^2+y^2}\right) + \ln\left(\sqrt{a^2+y^2}\right)\right]$

$$= \frac{-k}{2\pi}\left[\ln(y^2+a^2)\right]$$

$$u(0,y) = \frac{\partial\psi}{\partial y} = \frac{-k}{\cancel{2}\pi}\cdot\frac{\cancel{2}y}{(y^2+a^2)} = \frac{-ky}{\pi(y^2+a^2)}$$

By bernoulli's eqⁿ

$$P(y) + \frac{1}{2}\rho v(y)^2 = c$$

$$P(y) = P_0 - \frac{1}{2}\rho\left[\frac{k^2 y^2}{\pi^2 (y^2+a^2)^2}\right]$$