TUTORIAL-8

1. Along surface of cylinder $r = a$

$$\therefore u = U\left[1-\left(\frac{a}{a}\right)^2\right]\cos\theta\,\hat{r} - U\left[1+\left(\frac{a}{a}\right)^2\right]\sin\theta\,\hat{\theta}$$

$$= -2U\sin\theta\,\hat{\theta}$$

Apply Bernoulli b/w a pnt on surface of the cylinder and a far away point (neglect elevation head)

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p + \frac{1}{2}\rho V^2$$

$$\Rightarrow p_\infty + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho 4U^2\sin^2\theta$$

$$\Rightarrow p = p_\infty + \frac{1}{2}\rho U^2[1-4\sin^2\theta]$$

Stagnation point $\Rightarrow$ velocity $= 0$

$$\therefore u = 0 \Rightarrow u_r = u_\theta = 0$$

$$\Rightarrow U\left[1-\left(\frac{a}{r}\right)^2\right]\cos\theta = 0 \;\;\&\;\; U\left[1+\left(\frac{a}{r}\right)^2\right]\sin\theta = 0$$

$$\Rightarrow r = a \quad \text{or} \quad \cos\theta = 0 \;\&\; \sin\theta = 0$$

$$\Rightarrow \boxed{r = a;\ \theta = 0^\circ, 180^\circ} \Rightarrow \text{Stagnation point}$$

Points where static pressure on the surface = stream pressure.

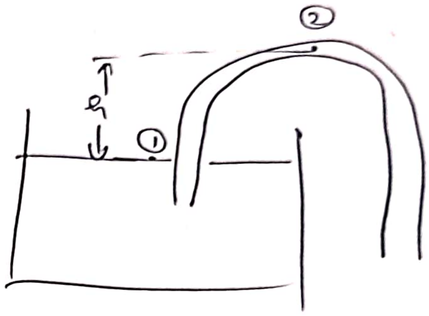
i.e: $P = P_\infty$

$p/\rho_\infty = p/\rho_\infty + \frac{1}{2}\rho U^2[1 - 4\sin^2\theta]$

$\Rightarrow \sin^2\theta = \frac{1}{4} \Rightarrow \sin^2\theta = \pm\frac{1}{2}$

$\Rightarrow \theta = 30°, 150°, 210°, 330° \; - -$

2. $A_2 V_2 = Q$



$\Rightarrow \frac{\pi}{4}(25\times 10^{-3})^2 V_2 = 5\times 10^{-3}$

$\Rightarrow \frac{\pi}{4}\times \cancel{6.25}^{125}\times 10^{-\cancel{4}} \times V_2 = \cancel{5}\times 10^{-3}$

$\Rightarrow V_2 = 10.2\ m/s$

$p_2 = p_{sat}\ @\ 20°C = 2.33\times 10^3$

$\therefore$ Applying Bernoulli eqn

$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$

$\Rightarrow 101325 = 2.33\times 10^3 + \frac{1}{2}\times 1000\times(10.2)^2 + 1000\times 9.8\times h.$

$\Rightarrow \boxed{h = 4.8\ m}$

3. Apply Bernoulli b/w pts ① & ②

$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$

$\Rightarrow p_{atm} + 689\times 10^3 + \frac{1}{2}\times\rho v_1^2 = p_{atm} + \frac{1}{2}\rho\left(v_1\cdot\frac{A_1}{A_2}\right)^2 + \rho g z_2$

$\Rightarrow 689\times 10^3 - 1000\times 9.8\times\left(\frac{75}{2}\times 10^{-3}\right) = \frac{1}{2}\times 1000(v_1^2)\left(\left(\frac{75^2}{25^2}\right)^2 - 1\right)$

$\Rightarrow V_1 = 4.15\ m/s$

$\therefore \varphi = A_1 V_1 = 4.15 \times \frac{3.14}{4}(75 \times 10^{-3})^2 = 0.01832 \, m^3/s$

$= 66 \, m^3/hr$

4. Apply Bernoulli's eqn.

$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$

$\Rightarrow p_{atm} + \frac{1}{2} \times 1000 \times ((6.3)^2 - v_2^2) = p_{atm} + 1000 \times 9.8 \times (1.55)$

$\Rightarrow V_2 = 3.05 \, m/s$

Stagnation pressure @ 2 $= p + \frac{1}{2}\rho v_2^2$

$= p_{atm} + \frac{1}{2}\rho v^2$

$= 101325 + \frac{1}{2} \times 1000 \times (3.05)^2$

$= 105.976 \, kPa$