

EE544: RF Systems and hardware
Project #1 (Systems)
Due: April 8, 2014

- This is the first of two projects in this semester. This is a Matlab-based project where as the next project is Cadence-based
- Groups of 2~3 students each are to be formed. DEN students who can't pair with others due to geographical obstacles may work individually.
- This is a simulation assignment, which every Communications Systems Engineer (Not necessarily every RF engineer) should be able to accomplish.
- All questions on this project should be addressed to me (either via e-mail. Aaron will be responsible for the next project (Cadence).
- Deliverable include: A "single, Typed" report of the entire group including summary (No theory), simulation results, graphs comparing simulation results and theoretical results, Matlab code (M-files) and conclusions.
- For DEN, deliver the project like you do the Homeworks, i.e. send it to denhw@usc.edu. Do NOT e-mail me the projects or the M-files

Project Statement:

In this project, we shall attempt to simulate the performance (as measured by the BER: Bit Error Rate and SER: Symbol Error Rate) of two types of Digital Modulation techniques, namely MPSK: Multi-Level Phase Shift Keying and M-QAM: Multi-Level Quadrature Amplitude Modulation. The channel is modeled as an AWGN: Additive White Gaussian Noise channel and the detector is a coherent detector with and without phase errors. Review class notes and the Tutorial on Digital Modulation Techniques posted in the Tutorial section of our site to gain more understanding on these techniques. The Theoretical results (The one that you will compare your simulation results against) are listed at the end of this brief.

Phase Modulated Carrier:

The general mathematical model for an MPSK signal is given below:

$$S_m(t) = A g_T(t) \cos(2\pi f_c t + \frac{2\pi m}{M}) \quad , m = 0, 1, \dots, M-1 \quad \text{and } 0 \leq t \leq T$$

Where $g_T(t)$ is the pulse shaping at the transmitter, A is the signal amplitude, f_c is the carrier frequency and T is the symbol duration. For the purposes of this project, assume rectangular pulse shaping, i.e. assume

$$g_T(t) = \sqrt{\frac{2}{T}} \quad 0 \leq t \leq T$$

Hence you can represent the modulated carrier by

$$S_m(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi f_c t + \frac{2\pi m}{M})$$

Where E_s is the Symbol Energy. Note that every k bits ($k = \log_2 M$) is mapped to a single symbol. The Bit Energy E_b is therefore related to the Symbol Energy by $E_b = E_s/k$

You can view the above-modulated signal as the sum of the I-channel and the Q-Phase components as follows:

$$S_m(t) = [\sqrt{E_s} \cos \frac{2\pi n}{M}] \psi_1(t) + [\sqrt{E_s} \sin \frac{2\pi n}{M}] \psi_2(t)$$

Where $\psi_1(t)$ and $\psi_2(t)$ are two "normalized" orthogonal functions given

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$\psi_2(t) = -\sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

So geometrically, each signal is represented in a 2-dimensional vector space (we referred to this in class as the "Signal Constellation") with components s_{mi} and s_{mq} where

$$s_{mi} = \sqrt{E_s} \cos \frac{2\pi n}{M}$$

$$s_{mq} = \sqrt{E_s} \sin \frac{2\pi n}{M}$$

The channel is disturbed by AWGN represented as (again to independent noise component, The I and the Q-channels)

$$n(t) = n_i(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)$$

The receiver will "correlate" the received signal $r(t) = S_m(t) + n(t)$ with "exact" replica of $\psi_1(t)$ and $\psi_2(t)$, i.e. we have a "coherent" receiver and generate a "vector" $r = (r_i, r_q)$ as given below:

$$(\sqrt{E_s} \cos \frac{2\pi m}{M} + n_i \quad \sqrt{E_s} \sin \frac{2\pi m}{M} + n_q)$$

The "optimum" detector will then project (i.e. perform a "dot" product) of the received vector with each transmitted vector (There are M of them) and select the transmitted vector that corresponds to the "Largest" projection (That means that the received vector is "closer" to this transmitted vector than any others)

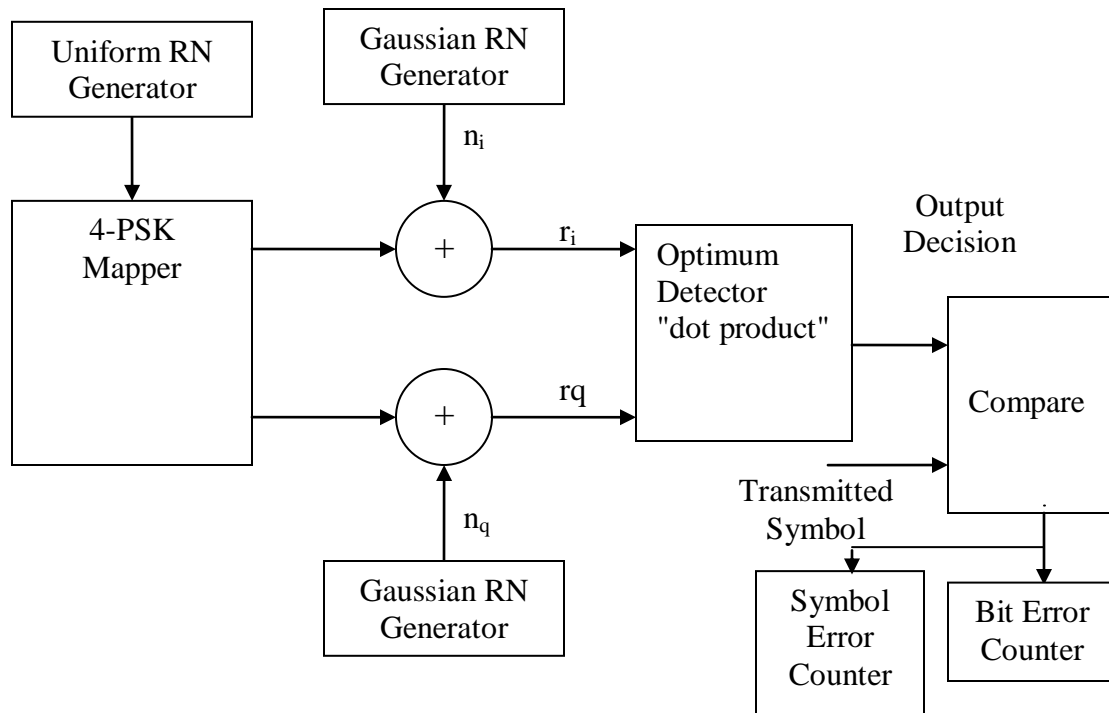
Note: Because all transmitted vectors have the same Energy, you could also calculate the "Phase" of the received vector ($\tan^{-1}(r_q/r_i)$) and select the transmitted vector whose phase is closest to the received phase.

Question #1: M-ary Phase Shift Keying

It is desired to simulate an M-ary PSK system for $M = 2$ (BPSK), $M=4$ (QPSK), $M=8$ (8-PSK) and $M=16$ (16-PSK) and compare your simulation results with the theoretical results. The simulation model for $M = 4$ is shown below. The detector will correlate the received vector with each of the M ($M=4$ in this case) transmitted signal vectors (i.e. perform the dot product $\mathbf{r} \cdot \mathbf{s}$ and selects the largest). These vectors are two-dimensional signal constellations we discussed in class). Start by generating uniformly distributed random numbers from 0 to 1 and divide that range into 4 equal intervals (0-0.25, 0.25-0.5, 0.5-0.75 and 0.75-1). Map those into 00, 01, 11 and 10 (Gray Coding). These pairs are used to select the 4-different phases of the transmitted signal. Here again, you need to generate two statistically independent, zero mean Gaussian noise (The In-phase and Quadrature phase components) with variance σ^2 (note $N_0/2 = \sigma^2$, where $N_0/2$ is the two-sided PSD of the AWGN). You can normalize E_s (The symbol energy) to 1 and control E_s/N_0 by varying the Noise Variance σ^2 . Remember $E_s = kE_b$, where E_b is the Bit Energy (so when $M = 2$ (BPSK), $E_s = E_b = 1$, for $M = 4$, $E_s = 1$ and $E_b = 1/2$ and so on..). Perform the simulation for at least 250,000 symbols at different values of E_b/N_0 . Sketch the symbol and the bit error probabilities (in separate graphs) vs. E_b/N_0 (not E_s/N_0) for $M = 2, 4$ and 8 (on the same

graph). Your graphs should adhere to the standard BER curves discussed in class (The "Niagara-Fall" curves of BER vs. E_b/N_o , see Figure 1-15b, page 24, Steer's book). Compare your results with the Theoretical results given at the end of this document. Provide plots (Signal constellations) on the effect of the noise variance on the constellation signal points (as discussed in class, see Figure 1-28c, page 38).

Next it is desired to investigate the effects of the non-idealities in the I and the Q channels at the receiver. Assume there are 1% and 5% (in amplitude) and 1° and 5° offsets (in phase) from those at the transmitter side. Repeat the above simulations for all permutations of amplitude and phase and compare the results with the ideal case.



Part 2: M-ary Quadrature Amplitude Modulation

In QAM, both the amplitude and the phase of the carrier are going to change according to the information signal. The general "model" for the modulated carrier is

$$S_{mn}(t) = A_m(t)g_T(t)\cos(2\pi f_c t + \varphi_n(t)) \quad \text{where } m = 1, 2, \dots, M_1 \text{ and } n = 1, 2, \dots, M_2$$

For example if $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$ then we say that we have $M = M_1 M_2$ signal points in the constellation each representing $k_1 + k_2$ bits of information where $k_1 + k_2 = \log_2(M_1 M_2)$

In our model, each of the Quadrature carriers $\psi_1(t)$ and $\psi_2(t)$ are modulated by independent sequence of information bits and the transmitted is given by:

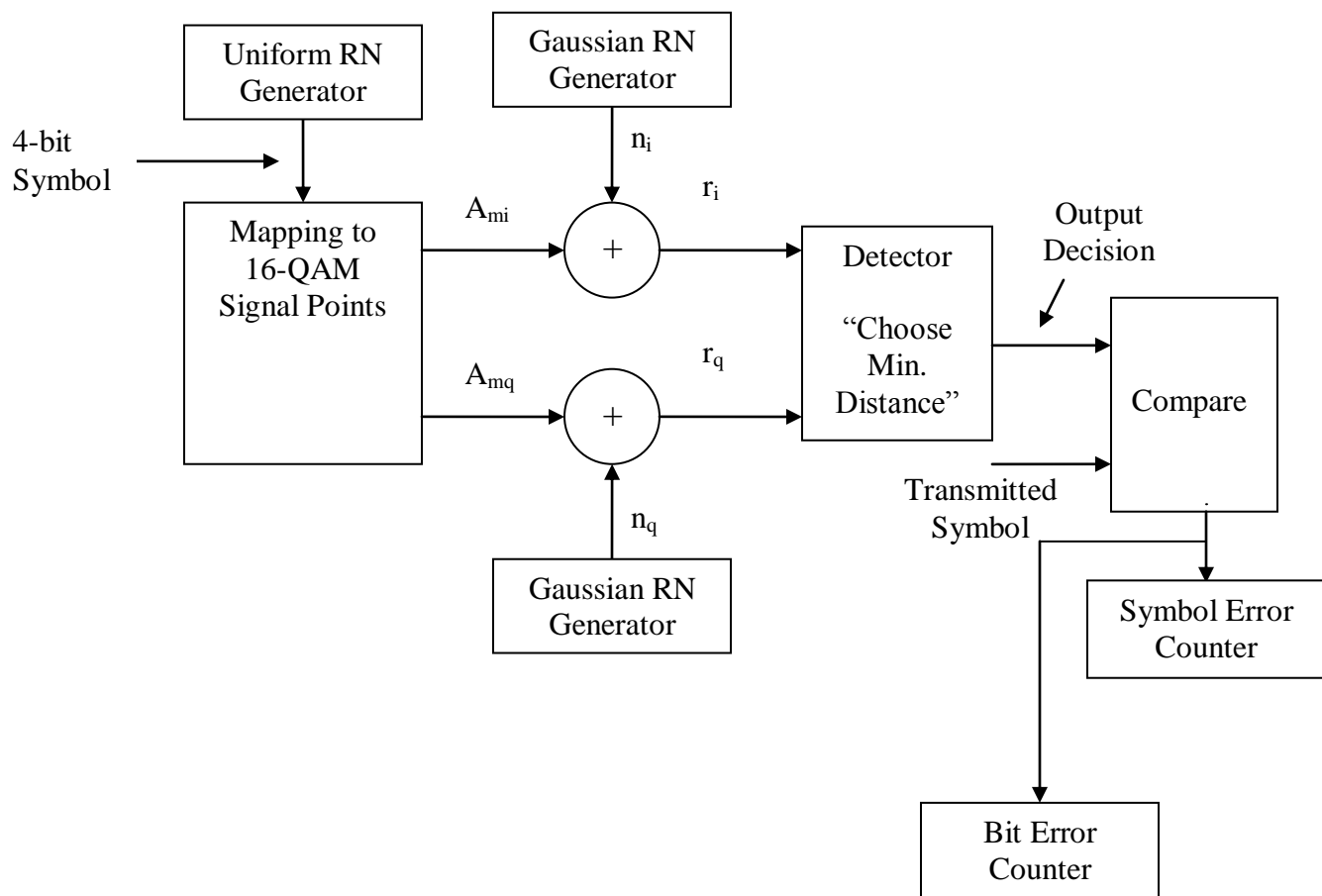
$$S_m(t) = A_{mi}\psi_1(t) + A_{mq}\psi_2(t) \quad \text{where } m = 1, 2, \dots, M$$

A_{mi} and A_{mq} are the set of amplitude levels that are obtained by mapping k -bits sequences into signal amplitudes. That means that each signal is represented by a vector

$$s_m = (\sqrt{E_s} A_{mi} \quad \sqrt{E_s} A_{mq}) \quad m = 1, 2, \dots, M$$

Question #2: M-ary Quadrature Amplitude Modulation

The purpose of this part is to perform Simulation of 16-QAM modulation scheme that uses rectangular single grid (See Figure 1.29b). The sixteen points are $(\pm 1, \pm 1)$, $(\pm 1, \pm 3)$, $(\pm 3, \pm 1)$ and $(\pm 3, \pm 3)$. The random number generator is used to generate a sequence of information bits corresponding to the 16 possible 4-bit combination b_1, b_2, b_3, b_4 . These 16-combinations are mapped to the 16 signal vectors, which have coordinates of $\{A_{mi}, A_{mq}\}$. The detector will compute the distance between the received vector $\mathbf{r} = (r_i, r_q)$ and each of the transmitted vectors and choose the signal vector that is closest to \mathbf{r} . Simulate the Symbol and Bit Error Probabilities vs. E_b/N_0 (not E_s) for at least 250,000 symbols (remember each symbol represent 4-bits) and compare with the theoretical results. Calculate the EVM (Error Vector Magnitude) from your simulation (see class notes and page 39 of Steer). The simulation model is shown below:



Theoretical Results:

The following are:

- The Probability of Bit Error in BPSK (which is almost the same as that of QPSK)
- The Probability of "Symbol" error for MPSK, where M is the number of symbols
- An upper bound on the Probability of Symbol Error for M-QAM. E_{av} represents the "Average Symbol (not Bit) Energy". The Average Bit Energy is of course = $E_{av}/\log_2 M$

$$P_b(\varepsilon) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

$$P_s(\varepsilon) \cong 2Q\left(\sqrt{\frac{2(\log_2 M)E_b}{N_o}} \sin \frac{\pi}{M}\right)$$

$$P_s(\varepsilon) \leq 1 - [1 - 2Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_o}}\right)]^2$$