Uniform Estimators - Problem 5

February 5, 2019

0.0.1 Problem 5

As a first step to the problem, let us try to simulate the data and estimate parameter value L using both Method of moment and Maximum Likelihood estimate.

```
In [1]: #Importing the Library
        import numpy as np
```

```
In [2]: #Assigning values to L = 10 and n = 100
        L = 10
        n = 100
```

Now, lets us calcualte Method of moment estimate for uniform distribution is equal to 2 times the mean of the sample. Let us estimate the value of L for 1000 simulations.

```
In [3]: #MOM Estimate for 1000 simulations
        L_MOM = [2 * np.random.uniform(low = 0, high = L, size = n ).mean()
                 for i in range(1000)]
```

Similarly, lets us calcualte MLE estimate for uniform distribution i.e is equal to maximum of the sample. Let us estimate the value of L for 1000 simulations.

```
In [4]: #MLE Estimate for 1000 Simulations
        L_MLE = [max(np.random.uniform(low = 0, high = L, size = n ))
                 for i in range(1000)]
```

Now lets us calculate the Mean Square error for the estimate which is given by the formula

$$MSE(\hat{L}) = E[(\hat{L} - L)^2]$$

```
In [5]: #Calculating MSE for MOM Estimate
        MSE_L_MOM = sum([(i - L)**2 for i in L_MOM])/1000
        MSE_L_MOM
```

```
Out[5]: 0.3213226434629274
```

```
In [6]: #Calculating MSE for MLE Estimate
        MSE_L_MLE = sum([(i - L)**2 for i in L_MLE])/1000
        MSE_L_MLE
```

```
Out[6]: 0.020194909920636585
```

From the previous questions, we know that theoretical MSE for the MOM Estimate is -

$$\begin{aligned} MSE_{MOM} &= bias^2_{MOM} + var_{MOM} \\ &= L^2/3n \end{aligned}$$

Now let us put the value of n and L,

```
In [7]: #Calculating Theoretical MSE of MOM
        MSE_MOM_Th = (L**2)/(3*n)
        MSE_MOM_Th
```

```
Out[7]: 0.3333333333333333
```

Similarly, we know that theoretical MSE for the MLE Estimate is given by

$$\begin{aligned} MSE_{MLE} &= bias^2_{MLE} + var_{MLE} \\ &= \frac{2L^2}{(n+1)(n+2)} \end{aligned}$$

```
In [8]: #Calculating Theoretical MSE of MLE
        MSE_MLE_Th = (2*(L**2))/((n+1)*(n+2))
        MSE_MLE_Th
```

```
Out[8]: 0.01941370607649
```

Conclusions As, we can see the Theoretical MSE are close to calculated MSE as estimated. Also, we can see that MSE of MLE estimate is lower than that of MSE of MOM which is as expected.