Regression and Error

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In [7]:
```
import numpy as np
import seaborn as sns
import math
```

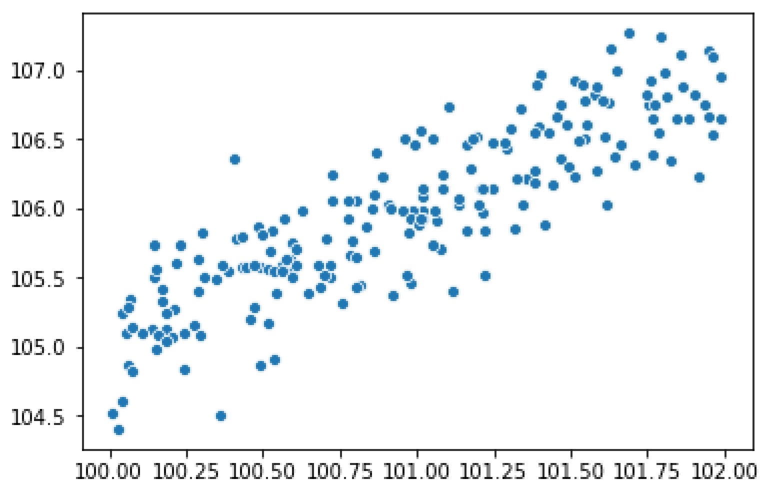
In [8]:
```
m = 200
w = 1
b = 5
sd = math.sqrt(0.1)
```

In [9]:
```
x = list(np.random.uniform(low=100,high =102, size=m))
```

In [10]:
```
y = list(map(lambda i : w*i + b + np.random.normal(0,sd,1), x))
```

In [11]:
```
sns.scatterplot(x,y)
```

Out[11]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x1ca120449e8>
```



In [17]:
```
def check_parameters(w,b,m,sd) :
    w_org = []
    b_org = []
    w_dash = []
    b_dash = []
    for i in range(1000):
        x = list(np.random.uniform(low=100,high =102, size=m))
        x_dash = list(map(lambda j : j - 101, x))
        y_temp = [((w*j) + b) for j in x]
        e = list(np.random.normal(0, sd, size = m))
        y = [a + b for a, b in zip(y_temp, e)]
        w_temp = np.cov(x,y)[0][1]/ np.var(x)
        w_org.append(w_temp)
        b_org.append(np.mean(y) - (w_temp * np.mean(x)))
        w_dash_temp = np.cov(x_dash,y)[0][1]/ np.var(x_dash)
        w_dash.append(w_dash_temp)
        b_dash.append(np.mean(y) - (w_dash_temp * np.mean(x_dash)))
    print("The expected value of w is : ", np.mean(w_org))
    print("The variance of w is : ", np.var(w_org))
    print()
    print("The expected value of b is : ", np.mean(b_org))
    print("The variance of b is : ", np.var(b_org))
    print()
    print("The expected value of w_dash is : ", np.mean(w_dash))
    print("The variance of w_dash is : ", np.var(w_dash))
    print()
    print("The expected value of b_dash is : ", np.mean(b_dash))
    print("The variance of b_dash is : ", np.var(b_dash))
```

In [18]:
```
check_parameters(w,b,m,sd)
```

```
The expected value of w is :  1.0035367302287947
The variance of w is :  0.0014555703102013702

The expected value of b is :  4.643040453497059
The variance of b is :  14.851229282449953

The expected value of w_dash is :  1.0035367302287947
The variance of w_dash is :  0.0014555703102013733

The expected value of b_dash is :  106.00025020660533
The variance of b_dash is :  0.00047436339741031163
```

As expected from our analysis, we can see that variance of b for re-centered data is smaller than the original one. These results are inline with our conclusion of problem 4.

Also from problem 3, the limiting expression for variance are following

$$Var(\hat{w}) \approx \frac{\sigma^2}{m}\frac{1}{Var(x)}$$
$$Var(\hat{b}) \approx \frac{\sigma^2}{m}\frac{E[x^2]}{Var(x)}$$

Let us know manually calculate the variance for w and b -

In [27]:
```
x = list(np.random.uniform(low=100,high =102, size=m))
x_dash = list(map(lambda j : j - 101, x))
y_temp = [((w*j) + b) for j in x]
e = list(np.random.normal(0, sd, size = m))
y = [a + b for a, b in zip(y_temp, e)]
```

In [33]:
```
Var_w = sd**2*(1 / m) * (1 /np.var(x))
print("The limiting variance of w is ", Var_w)
```

```
The limiting variance of w is  0.0015402727974052977
```

In [34]:
```
Var_w_dash = sd**2*(1 / m) * (1 /np.var(x_dash))
print("The limiting variance of w_dash is ", Var_w_dash)
```

```
The limiting variance of w_dash is  0.001540272797405298
```

In [35]:
```
Var_b = sd**2*(1 / m) * np.mean([i**2 for i in x])*(1 /np.var(x))
print("The limiting variance of b is ", Var_b)
```

```
The limiting variance of b is  15.739453951978458
```

In [36]:
```
Var_b_dash = sd**2*(1 / m) * np.mean([i**2 for i in x_dash])*(1 /np.var(x_dash))
print("The limiting variance of b_dash is ", Var_b_dash)
```

```
The limiting variance of b_dash is  0.0005112748689193981
```

As we can see, these values are almost same as what we got from the simulation.