A1 Advanced Heuristic

My heuristic for the A* algorithm is the Manhattan distance from the 2x2 piece to the goal + Manhattan distance from the 2x2 piece to the closest blank square – 1.

Let d = the Manhattan distance from the 2x2 piece to the goal

Let x = Manhattan distance from the 2x2 piece to the closest blank square

$h(n) = d + x - 1$

where n is a state.

Admissible

It is admissible since it is a lower bound on the cost of the optimal solution. Note that if the closest blank square is adjacent to the 2x2 piece this is just the Manhattan heuristic (known to be admissible). In the case where it is not adjacent to a blank square, the 2x2 piece cannot move from its position. Hence, it will take some cost to get the blank squares near to the 2x2 piece to then move it. The fastest possible way to get the blank space adjacent to the 2x2 piece is the Manhattan distance of the piece to the closest blank square. So, this is a lower bound of the cost of the blank space to get to the 2x2 piece. Therefore, it is a lower bound on the overall cost of the optimal solution.

Domination

It dominates the Manhattan heuristic since it is always going to be greater than or equal to the Manhattan heuristic and in at least one state it is strictly greater than the Manhattan heuristic. Note that the Manhattan distance from the 2x2 piece to the closest blank square is at least one since the blank square cannot be in the same square as the 2x2 piece.

$x >= 1$

$x + d >= d + 1$

$x + d - 1 >= d$

$h(n) >= d$

Hence, my heuristic h(n) is greater than or equal to the Manhattan heuristic (since d is the Manhattan heuristic). In the case where there is no blank square adjacent to the 2x2 tile then the Manhattan distance from the 2x2 piece to the closest blank square is at least 2 (or in other words, greater than 1).

$x > 1$

$x + d > d + 1$

$x + d - 1 > d$

$h(n) > d$

Therefore, in this scenario, my heuristic is strictly greater than the Manhattan heuristic which shows that it dominates the Manhattan heuristic.