Final Project

Particle Filtering

Particle filtering is a sequential Monte-Carlo (MC) method that seeks to predict a hidden state variable ($\mathbf{x}$) from a series of observations ($\mathbf{y}$). $p(\mathbf{x}_0)$ is the initial state of the distribution, the transition equation is $p(\mathbf{x}_t|\mathbf{x}_{t-1})$, and $p(\mathbf{y}_t|\mathbf{x}_t)$ is the marginal distribution of the observation. Using Bayes' theorem, we derive an expression for $p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})$, the marginal distribution of the hidden state variable from the observations:

$$p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) = \frac{p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})}{\int p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})d\mathbf{x}_t}$$

$$p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) \propto p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})$$

We can also compute $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ recursively via the marginal distribution:

$$p(\mathbf{x}_t|\mathbf{y}_{1:t}) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1})p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})d\mathbf{x}_{t-1}$$

To find the expected value of $E[f(x_t)]$:

$$E[f(\mathbf{x}_t)] = \int f(\mathbf{x}_{0:t})p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})d\mathbf{x}_{0:t}$$

Do we need intermediate steps here?

$$E[f(\mathbf{x}_t)] = \frac{\int f(\mathbf{x}_{0:t})p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})d\mathbf{x}_{0:t}}{\int p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})d\mathbf{x}_{0:t}}$$

To evalute this integral, we introduce $w(x_{0:t})$, the importance weight. The importance weight is equal to:

$$w(x_{0:t}) = \frac{p(x_{0:t}|y_{1:t})}{\pi(x_{0:t}|y_{1:t})}$$

the importance sampling factor. The importance sampling factor relies on the probability

Dataset

Bibliography

$$x$$