Multiwindow Non-harmonic Analysis of Gravitational Waves

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Abstract

- We don't have a good method for making the time – frequency analysis of the signals like that of the Gravitational waves wherein the signal will have low frequency for a long time and then it shoots up to a high value in a short span of time, which makes it difficult for us to analyse in the cost of both frequency and time domain.
- So, we will have to find a method where in we are changing the windows where the window length must be less for the lower frequency and larger window length for the higher frequency values, by which we can make a better analysis by better resolution of both the frequency and the time.
- Also, if there is less influence of the analysis methods or the transforms on the window length it would be better.
- By all these considerations we are proposing a new method for the analysis of the same type of waves as the gravitational waves.
- There are many methods which can be used but all of these gives significant error as we can change

Important concepts and short discussion about the existing method

- Here mainly we have three methods from which the analysis was made which are the short time Fourier transform, Fourier synchro squeezed transform, and the wavelet transform.
- All these methods have a significant effect on the length of the window length which made it difficult for us to study or make analysis of these kind of waves most.

1. The short time Fourier transform (STFT)

    - This method involved taking a finite window length when you calculated the FFT of the signal for window length points and then we found the dominating frequency and plotted for each window length.
    - As mentioned above here the problem is the length of the analytical window can significantly influence the analytical position of STFT which will make it difficult to analyse signals whose periods are not constant making the analytical precision less.
    - If the length of the window is short the frequency resolution is low where the time resolution decreases in the case of long window length and hence making it not suitable for the analysis of waves such as the gravitational waves or the waves which has similar characteristics to that of the gravitational waves.

2. The wavelet transforms

    - In wavelet transform it is like that of the previous method STFT where in there was a FFT which is nothing but the sum of the cosines or sines but here in this method the wavelet transform has the eigenfunctions nothing but the localised wavelet.

- In this method we consider small wavelet and then we multiply it with the signal to form different which gives us more accurate frequency values than the STFT.
- In this transform also it depends significantly on the analytical window length which makes it difficult for analysis in this method, but we observe that this method is better than that of the STFT as it is giving a better analysis or less error than that of the previous case.

3. The non-harmonic analysis

- This method is better when compared to the previous two methods as the influence of the analytical window length is less on this transformation.
- In the non-harmonic analysis, we start by calculating the FFT for some fixed window lengths and then subtracted from its samples and then square it which is nothing but the cost function which we want to be minimised.
- Hence, we required a method to calculate the minimum of this function which is done with the help of the 'steepest descent method' and the 'newtons method' by taking some coefficient called the weighing coefficient.
- In the method of steepest descent method, we will find the values of frequency, amplitude, phase such that the function is minimum as this function is a function of three variables we use the Jacobian matrix to calculate the required values of frequency amplitude and phase for which the cost function is minimum.

4. Octave division method

- this is the method which we were going to use for making different window lens in the upcoming method called the multiwindow nonharmonic analysis method.
- In this method we will divide the analytical window length in such a way that the time and the frequency resolution will get better when we do the analysis for this multi window.
- The method uses the function

$$f_k = f_0\left(\frac{1}{\frac{n}{m}+1}\right)^k$$

Where the values of m and n are some constants which could be taken by us on how we want to divide them analytical window length to.

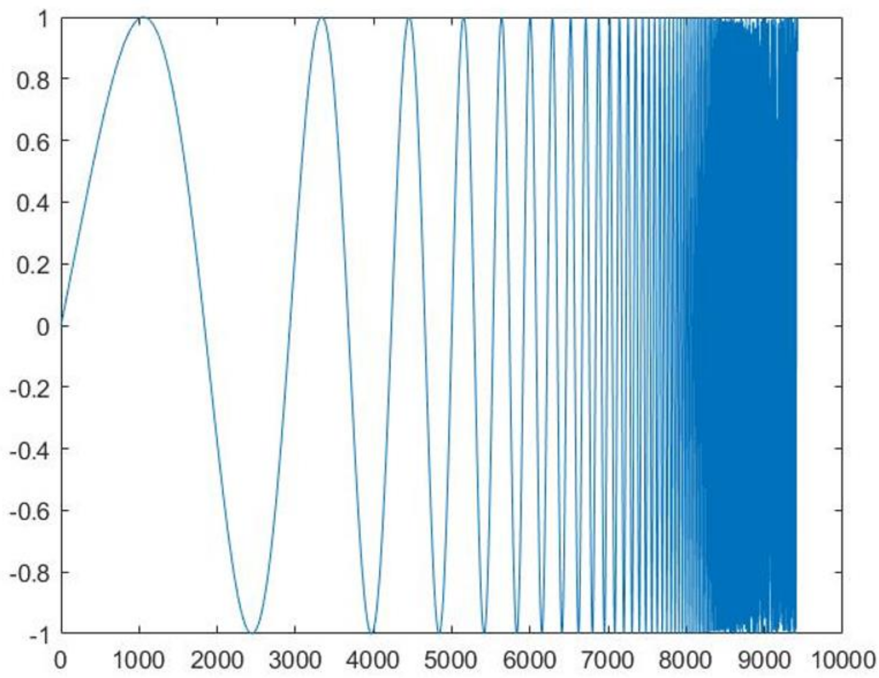
Proposed method: Multiwindow Non-Harmonic Analysis

- In this method firstly we divide the analytical windows based on the above-mentioned octave division method wherein we get the larger window length for higher frequencies and the smaller window length for the lower frequencies.
- We know that the non-harmonic analysis itself does not get influenced much by the change in the analytical window length hence this method will be more acceptable or in this method we will get less error as compared to the original signal which would make a better approximation of the original wave.
- now we will calculate the FFT for all these different window lengths and then subtract from its samples and square it

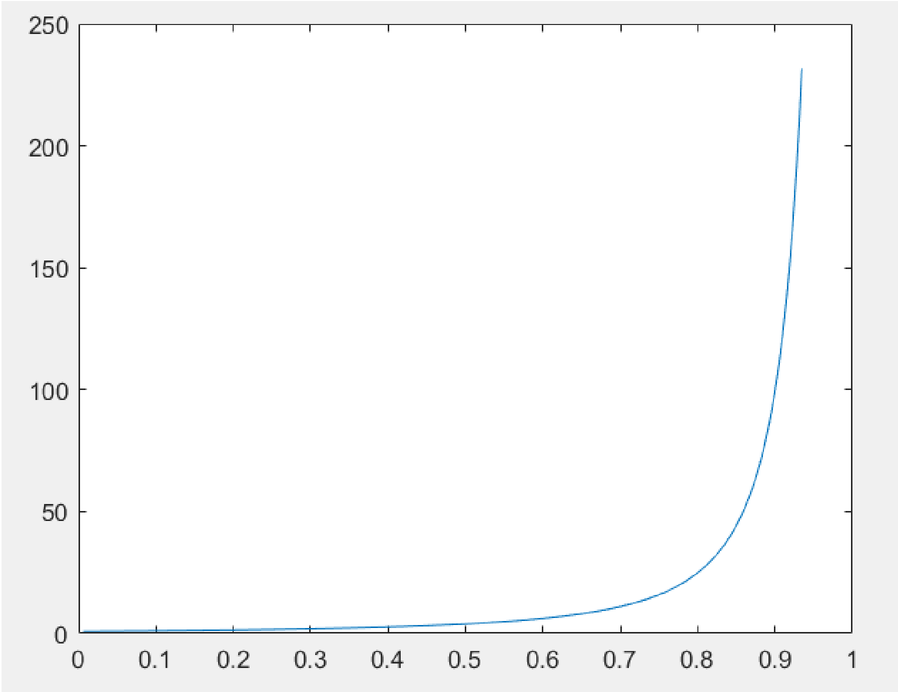
which is nothing but the cost function here which we want to be minimised.

- Hence, we again use the method to calculate the minimum of this cost function which is done with the help of the steepest descent method along with the newtons method by taking the similar coefficient which was introduced as earlier called weighing coefficient.
- In the mentioned steepest descent method, we will find the values of the frequency amplitude and phase such that the cost function is minimum as here we know that the cost function is a function of three variables that is the amplitude frequency and the phase.

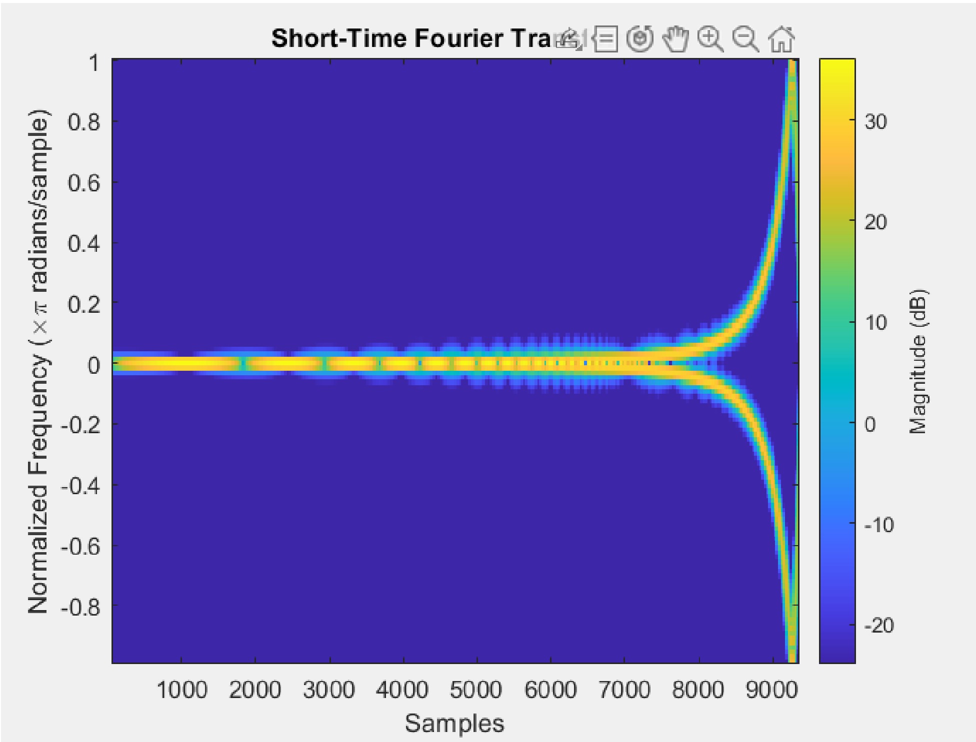
Simulation and observations



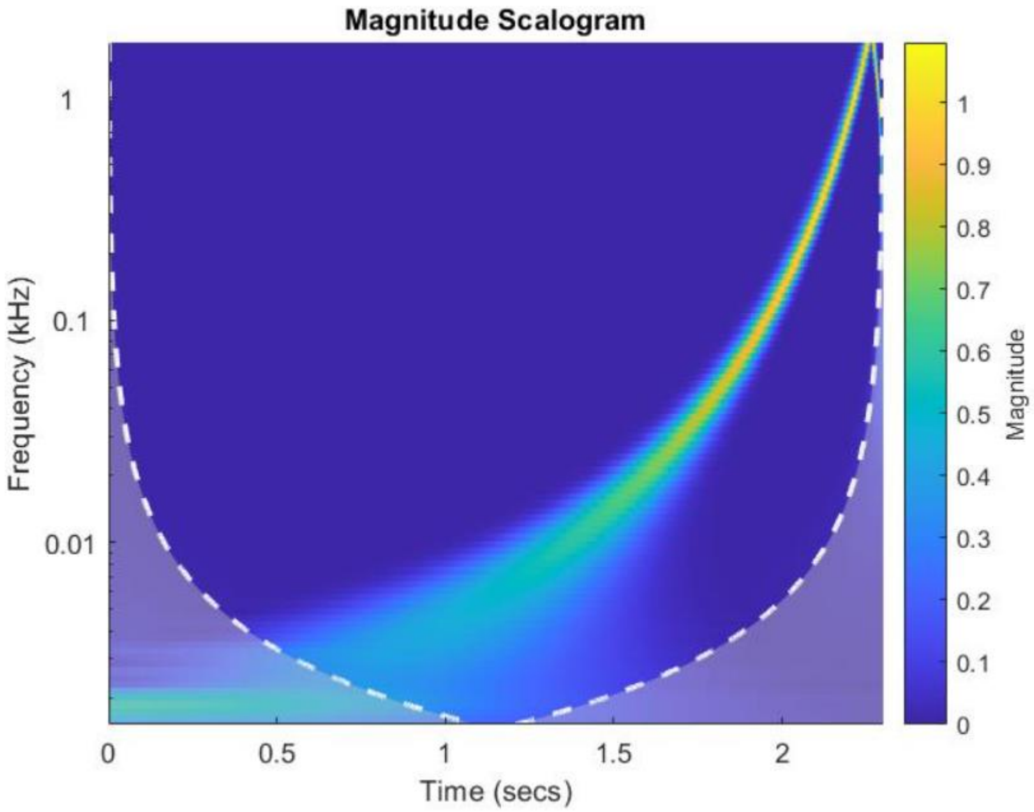
This is the original time domain signal which we used to simulate the GW waves with properties of it.



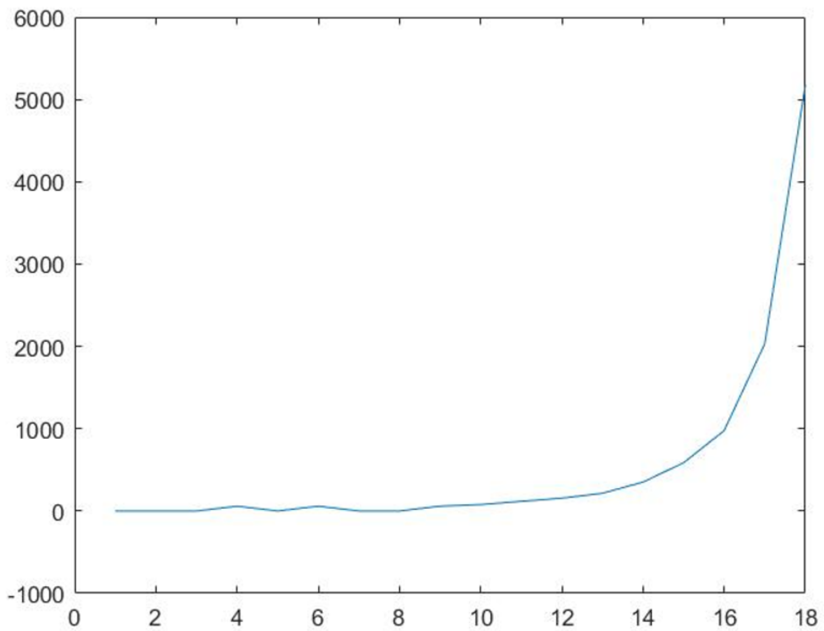
The frequency of the simulated GW signal



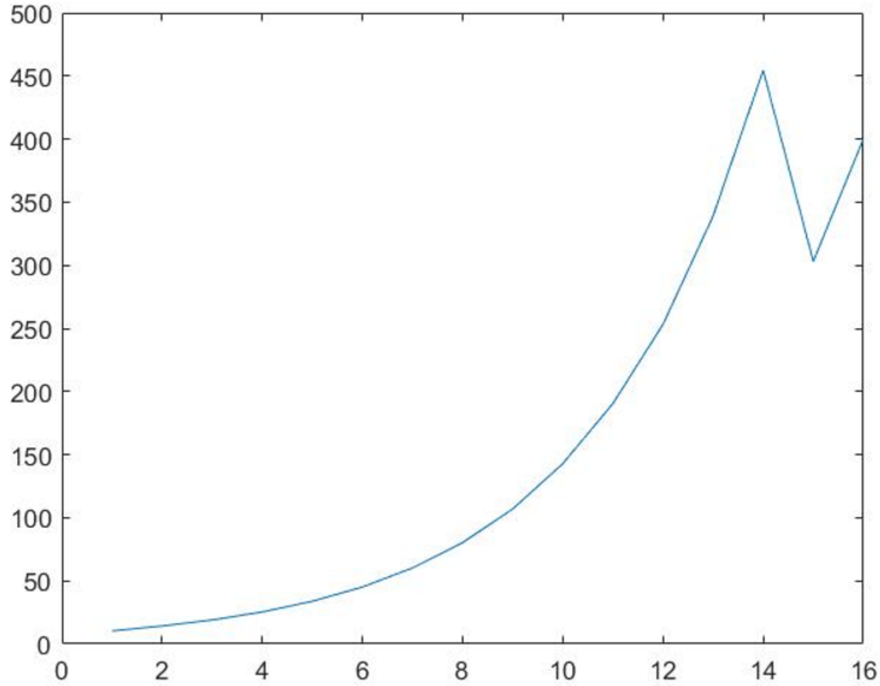
The STFT plot for the simulated signal



The wavelet transform of the given simulation



512 window length NHA



The multi-window NHA with n/m = 1/3

Conclusion

We can observe that the multiwindow NHA method as far better than the STFT, wavelet transform and then NHA as the plotted graph is most approximate to the original simulated wave more than any of the above-mentioned methods, but here we did not consider any noise, the presence of the noise may be unfit for this method to be used.

References

[1] Abrie Oberholster, Stephan Heyns ,' A study of the non-harmonic Fourier analysis technique, 2008

'[2] Stanford University, Mathematical Definition of the STFT (stanford.edu)

*[3] Shawhin Talebi, '' **The Wavelet Transform, Jan 2020***