AM5820
Wind Tunnel and Numerical Experiments

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Free and Forced vortices

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A. Free Vortex

Objective

The aim of this experiment is to create a free vortex and plot its surface profile.

Introduction

In fluid dynamics vortex is the rotation of fluid elements around a common center. Mostly the fluid flows in a spinning motion about an imaginary axis, straight or curve where these motion patterns are called vortical flows. Concentric streamlines in a vortex flow are shown in below figure.

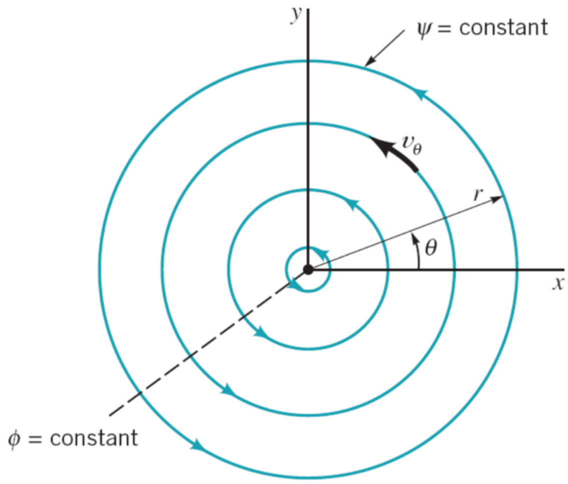


Fig. The streamline pattern for a vortex

The fluid pressure in a vortex is lowest in the center and rises progressively with distance from the center in accordance with Bernoulli's Principle. The core of every vortex can be considered to contain a vortex line, and every particle in the vortex can be considered to be circulating around the vortex line. Vortex lines can start and end at the boundary of the fluid or form closed loops. They cannot start or end in the fluid.

Vortices readily deflect and attach themselves to a solid surface. Two or more vortices that are approximately parallel and circulating in the same direction will merge to form a single vortex. The circulation of the merged vortex will equal the sum of the circulations of the constituent vortices.

Vortices contain a lot of energy in the circular motion of the fluid. In an ideal fluid this energy can never be dissipated, and the vortex would persist forever. However, real fluids exhibit viscosity and this dissipates energy very slowly from the core of the vortex. It is only through dissipation of a vortex due to viscosity that a vortex line can end in the fluid, rather than at the boundary of the fluid.

In a free vortex, the medium spirals towards the center. It is formed when fluid flows out of a vessel through a central hole in a base of a tank in which the degree of the rotation being dependent on initial disturbance. In the free vortex flow, the fluid mass rotates without any external force. The rotation cause by either by internal action or due to some rotation imported previously.

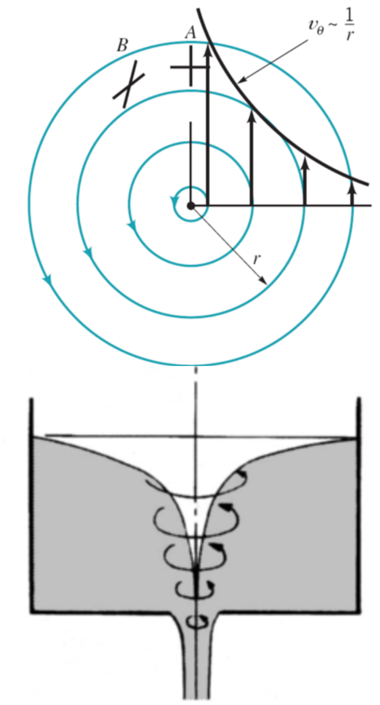


Fig. Motion of fluid element from A to B in free vortex

Practical Applications

In an industry and in a real world, the applications of the vortex flow can be seen in a various area such as turbine design, natural phenomenon and in creating safety against natural disaster. Perhaps the most common being a tornado or a whirlpool. A tornado is formed by high winds whirling around an area of extremely low pressure and characterized by a funnel shaped cloud. Whirlpools can occur where tides flowing in different directions meet or at the base of waterfalls where the effect is a spiraling or swirling of the water again producing a funnel shape.

Equipment description

The apparatus consists of a hydraulic bench & a cylindrical container. There is a pump & rotameter to control the discharge to the container. There are four inlets to the container. Two water inlets are perpendicular & two are tangential. There is a control valve with which we can control the direction of flow to the container whether it is perpendicular or tangential. Also, there is an outlet valve for adjusting the flow. There is an orifice at the center of the container which allow the outflow.

There is a measuring bridge kept at the top of the container with gauge needles and the pitot tube in it. The gauge needles are used for measuring depth of the vortex from the datum. The flow velocity is measured using the hydraulic head formed in the pitot tube.

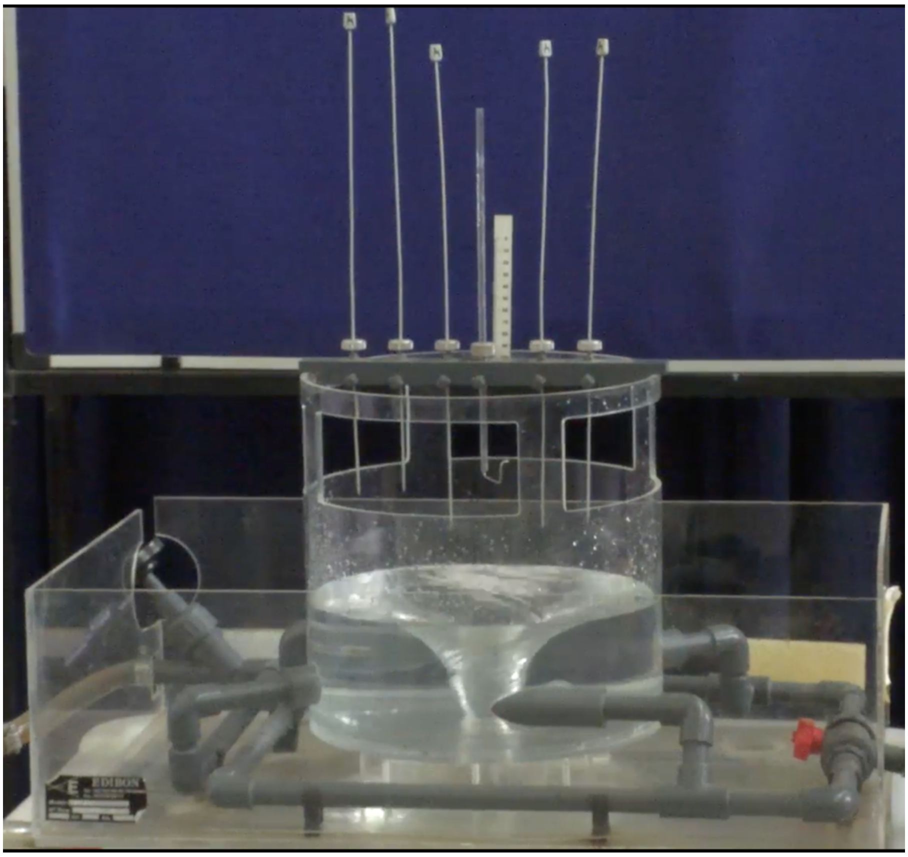


Fig. Experimental setup for Free vortex

Experimental procedure

Normal Inlet

The outlet valve is closed, and the inlet valve is adjusted to let water flow into the container by normal inlet. The water will flow out through the orifice. Allow the water to be filled into the container up to a certain height. It will take some time for the stable

vortex to form. After a stable vortex is formed all the gauge needles should be adjusted in such a way that they touch the free surface of the vortex.
Now, take out the bridge from the container & measure the depth of each needle from the datum using a ruler. These are the y-coordinates of the profile of the vortex. Also, measure the x-coordinate of each needle from the center of the bridge. The plot of these x-coordinate vs y-coordinate will give the experimental profile of the free vortex.

Place the bridge on the container & immerse the pitot tube into the flow. Let stable hydraulic head to be formed in the pitot tube. Now, by keeping the top of the pitot tube closed, take out the bridge & measure the hydraulic head using a ruler. Using this velocity, we can calculate the theoretical profile of the vortex. We need to compare the theoretical & experimental profiles of the vortex. Also, note the discharge rate at which stable vortex is formed for normal inlet.

Tangential Inlet

Now, drain the container via the orifice and turn the inlet valve towards tangential flow. After a stable vortex is formed with the flow coming from a tangential inlet, repeat the same set of measurements as done for the normal inlet flow and plot the profile of the free vortex formed with the tangential inlet. Also, note the discharge rate at which stable vortex is formed for tangential inlet.

We have to observe the difference in the vortex profiles as well as the discharge rate at which the stable vortex is formed by normal & tangential inlet. Also, compare the time it is taking for the formation of a free vortex in the two cases.

Raw data & Results

Normal Inlet

$Discharge\ rate = 1000 litres/hr$

$Hydraulic\ head\ from\ pitot\ tube, H = 6.6cm$

$Pitot\ tube\ arm\ length, R = 1.8cm$

$Reference\ height, h_o = -12.6cm$

$Tangential\ velocity, q = \sqrt{2gH} = 1.13795m/s$

$$\Rightarrow k = qR = 1.13795 \times 1.8 \times 10^{-2} m^2/s$$

$$\Longrightarrow k = 0.02048 m^2/s$$

$X_{exp}(cm)$	$Y_{exp}(cm)$	$k^2/2gr^2$ (cm)	$Y_{theo}(cm)$
3	-18.2	2.375299581	-14.9753
7	-14.4	0.436279515	-13.03628
11	-12.8	0.176675175	-12.776675
-5	-14.6	0.855107849	-13.455108
-9	-13.2	0.263922176	-12.863922

<u>Tangential Inlet</u>

$Discharge\ rate = 600 litres/hr$

$Hydraulic\ head\ from\ pitot\ tube, H = 8.0cm$

$Pitot\ tube\ arm\ length, R = 1.8cm$

$Reference\ height, h_o = -9.8cm$

$Tangential\ velocity, q = \sqrt{2gH} = 1.25284 m/s$

$$\Longrightarrow k = qR = 1.25284 \times 1.8 \times 10^{-2} m^2/s$$

$$\Longrightarrow k = 0.022551 m^2/s$$

$X_{exp}(cm)$	$Y_{exp}(cm)$	$k^2/2gr^2$ (cm)	$Y_{theo}(cm)$
3	-15.2	2.84219481	-12.642195
7	-11.3	0.52203578	-10.322036
11	-10	0.21140292	-10.011403
-5	-13.6	1.02319013	-10.82319
-9	-10.5	0.31579942	-10.115799

Sample Calculation

Each particle moves in a circular path with speed varying inversely as the distance from the center.

The tangential velocity is given by,

$$q = \frac{k}{r}$$

In a steady state condition Bernoulli's equation can be applied

$$\frac{p}{\rho g} + \frac{q^2}{2g} + z = constant$$

Let $h_o$be the height of the vortex when $r \to \infty$ and h be the height at a particular radius (r).

At the free surface $p = 0$

Hence

$$\boxed{h = h_o - \frac{k^2}{2gr^2}}$$

And $q = \sqrt{2gH}$ where H is the hydraulic head

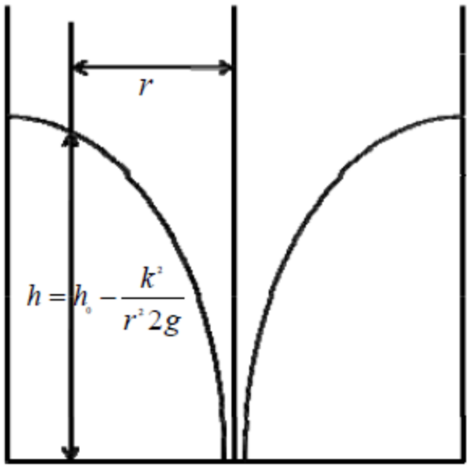
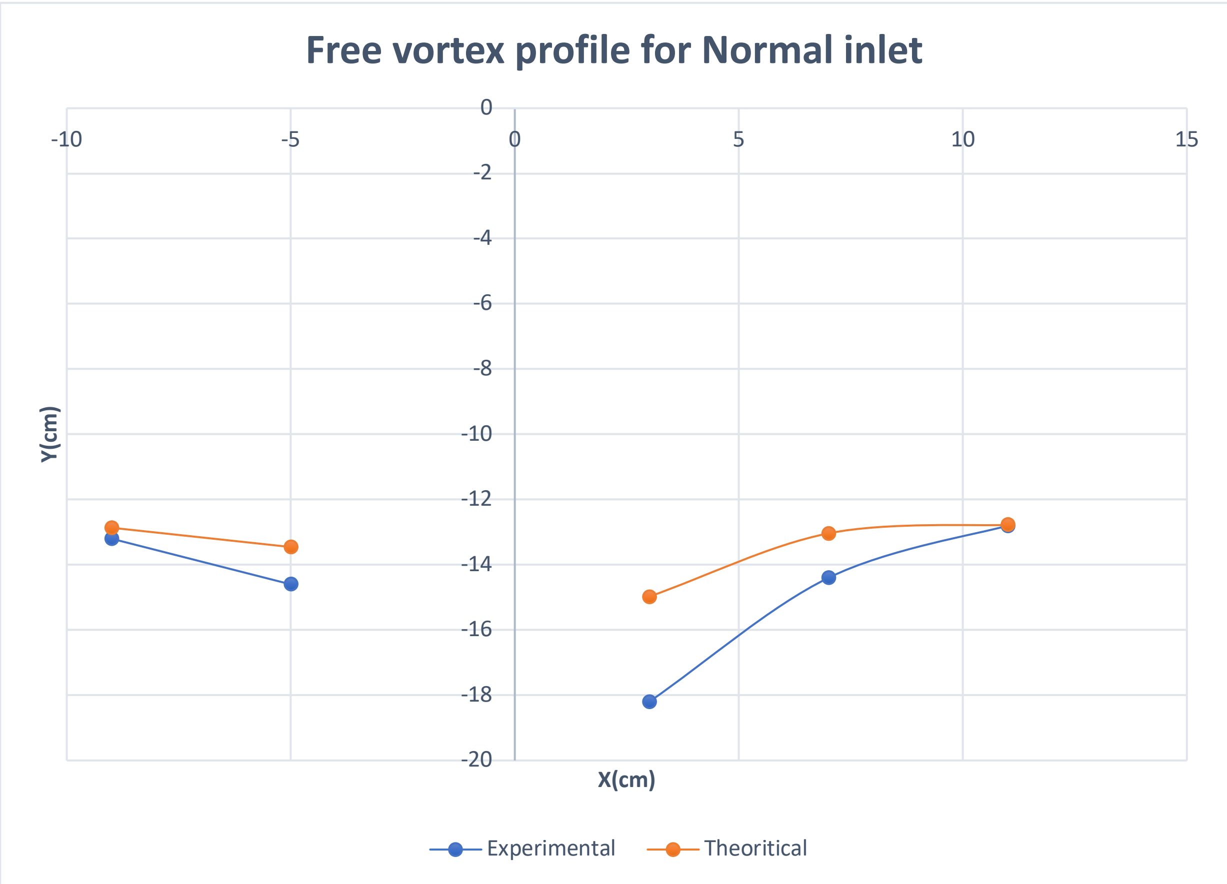


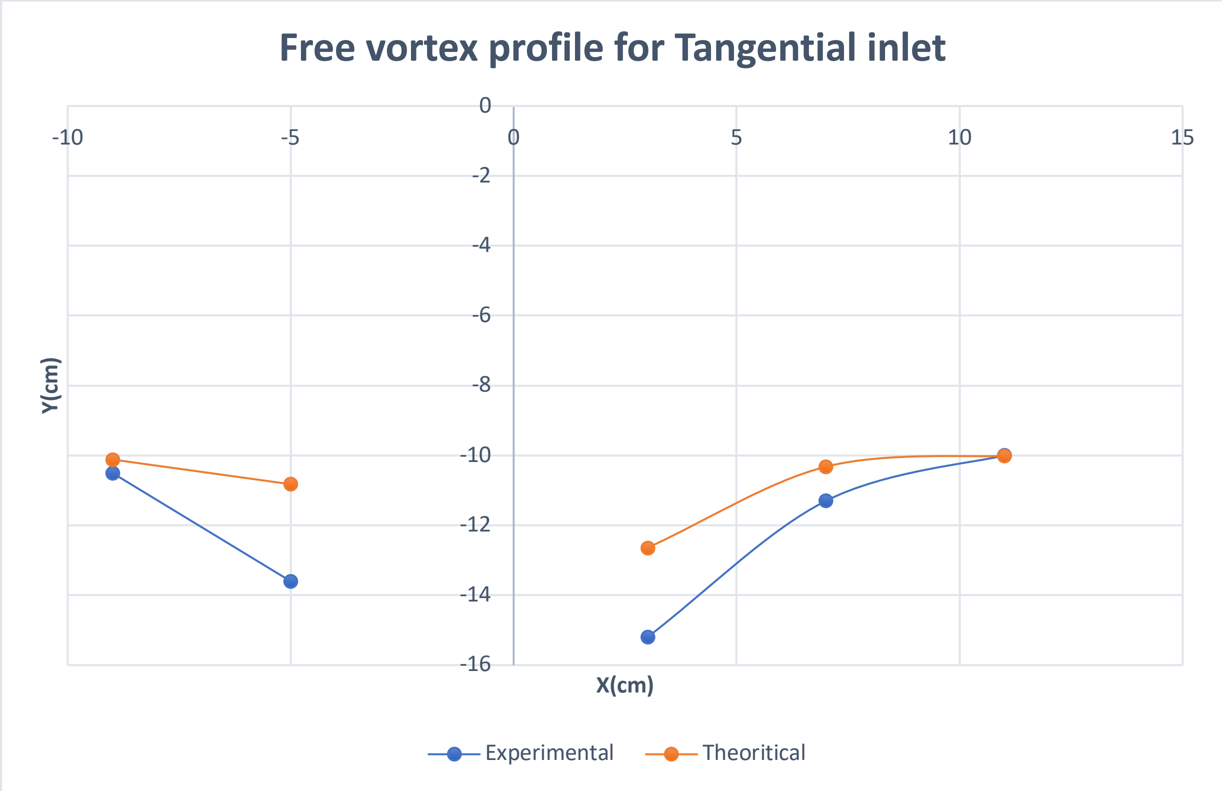
Fig. Free vortex profile

Plot

Normal Inlet



<u>Tangential Inlet</u>



Conclusion

We saw that due to viscous friction and gravitational effects, the fluid tends to organize into a collection of twisted vortices. It acts like downdraft irrotational drain as a surface depression was formed above the drain orifice. This type of vortex flow can move, twist, stretch and interact in complex ways like smoke rings.

Remarks

a) What are the possible sources of error?

Ans:

- The ruler by which the readings are taken might have a zero error of itself, which was not considered
- If the container is not properly levelled, then the readings for the gauge needles won't be accurate

b) Compare and comment on the calculated and measured surface profiles.

Ans: For both the normal & tangential inlet, there is difference between the experimental & theoretical surface profiles, which may be caused due to the errors while conducting the experiment.

c) Discuss the influence of orifice size, discharge flow and the inlet direction on the formation of vortex.

Ans:

- It is the presence of the orifice in the container that is creating the free vortex. Varying the size of the orifice creates changes in the flow rate, thereby changing the rotational speed and size of the vortex profile.
- For a given size of orifice, if we change the discharge rate then it will change the rotational speed and vortex size.
- For tangential inlet, a steady vortex is formed at a lower flow rate as compared to normal inlet. Also, the velocity of vortex was higher for tangential inlet than normal inlet, which can be inferred from the pitot tube readings

B. Forced Vortex

Objective

The aim of this experiment is to create a forced vortex and plot its surface profile.

Introduction

The fluid circles around a center in a forced vortex. During the forced vortex motion, the fluid mass is made to rotate by external source of power which exerts a constant torque on the fluid mass and causes it to rotate with a constant angular velocity. It can be considered as the phenomenon of rigid body rotation. The speed of the forced vortex is zero at the center and keeps increasing proportional to the distance measured from the center.

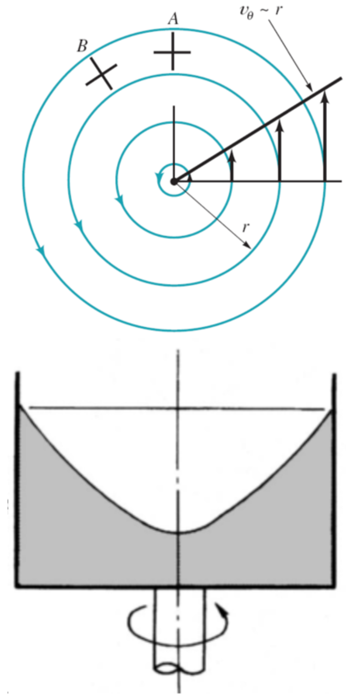


Fig. Motion of fluid element from A to B in forced vortex

Both free and forced vortex exhibit minimum pressure minimum at the center, however free vortex has a much lower minimum pressure compared to forced vortex.

Equipment description

The apparatus consists of a cylindrical container attached to a platform which rotates with a motor. The speed of the motor can be controlled using a variac. A gauge needle is attached to a bridge, which is mounted at the top of the container. This needle can be moved horizontally & vertically using pins. It is used to take readings of x-coordinate & y-coordinate of the vortex profile similar to free vortex experiment. A tachometer is used to measure the RPM of rotation.

Fig. Experimental setup for Forced vortex

Experimental procedure

Switch on the power supply to the motor so that the container with water inside it starts rotating. Using the variac adjust the speed of rotation till vortex is formed. Now, wait for some time so that a stable vortex is formed. After a stable vortex is formed, we have to plot the profile of the forced vortex similar to the free vortex experiment. Adjust the needle using horizontal and vertical scale such that it touches the free surface of the vortex. Now, record the x-coordinate & y-coordinate. Similarly, adjust the needle and take at least five readings along the radius of the vortex so that we get sufficient points to plot the experimental profile.

In forced vortex, all water particles are rotate with same angular velocity therefore we can measure the angular velocity of fluid mass by measuring RPM of any point on the rotating system using a tachometer. If we press side button of the tachometer, we will get a laser beam. This laser beam should be directed towards the white marking on the rotating rod & we will get the corresponding RPM (N).

Using, $\omega = \frac{2\pi N}{60} rad/s$ we can find the angular velocity in rad/s units.

By using $v = r\omega$ relation, we can get the velocity at any radial point. This will give us the theoretical profile of the forced vortex. We need to compare the theoretical & experimental profiles of the vortex.

Raw data & Results

$X_o = 14.9cm$

$h_o = -12.6cm$

$RPM\ (N) = 195$

We know,

$$\omega = \frac{2\pi N}{60} rad/s$$

$$\Rightarrow \omega = \frac{2\pi \times 195}{60} rad/s$$

$$\Rightarrow \omega = 20.4204 rad/s$$

$X_{exp}(cm)$	$Y_{exp}(cm)$	$r = (X_{exp} - X_o)$ (cm)	$\omega^2 r^2/2g\ (cm)$	$Y_{theo}(cm)$
9.5	-6	-5.4	6.19750672	-6.4024933
11.3	-9.6	-3.6	2.75444743	-9.8455526
14.9	-12.6	0	0	-12.6
15.4	-6.8	0.5	0.05313363	-12.546866
16.8	-4.4	1.9	0.76724963	-11.83275

Sample Calculation

In a forced vortex, the fluid rotates as a solid body with $v_\theta = r\omega \;\; and \;\; v_r = 0$

Navier-stokes equation (r-component) is given by

$$\frac{v_\theta^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$\Rightarrow \frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r}$$

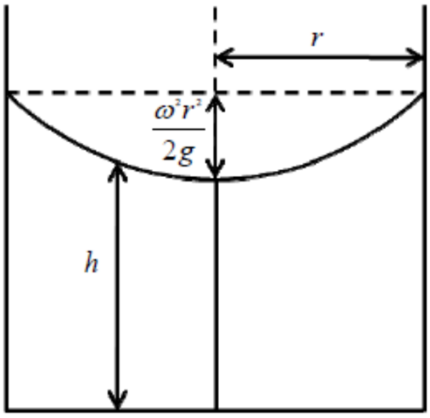


Fig. Forced vortex profile

Navier-stokes equation (z-component) is given by

$$\frac{\partial p}{\partial r} = -\rho g$$

Substituting above two expressions into below differential and integrating, we get

$$\int dp = \int \frac{\partial p}{\partial r} dr + \int \frac{\partial p}{\partial h} dh$$

$$\int dp = \int \frac{\rho v_\theta^2}{r} dr + \int (-\rho g) dh$$

Using $v_\theta = r\omega$ in above expression, we get

$$\int dp = \int \frac{\rho (r\omega)^2}{r} dr - \int \rho g dh$$

$$\int dp = \int \rho \omega^2 r dr - \int \rho g dh$$

$$p = \rho \frac{\omega^2 r^2}{2} - \rho g h + C$$

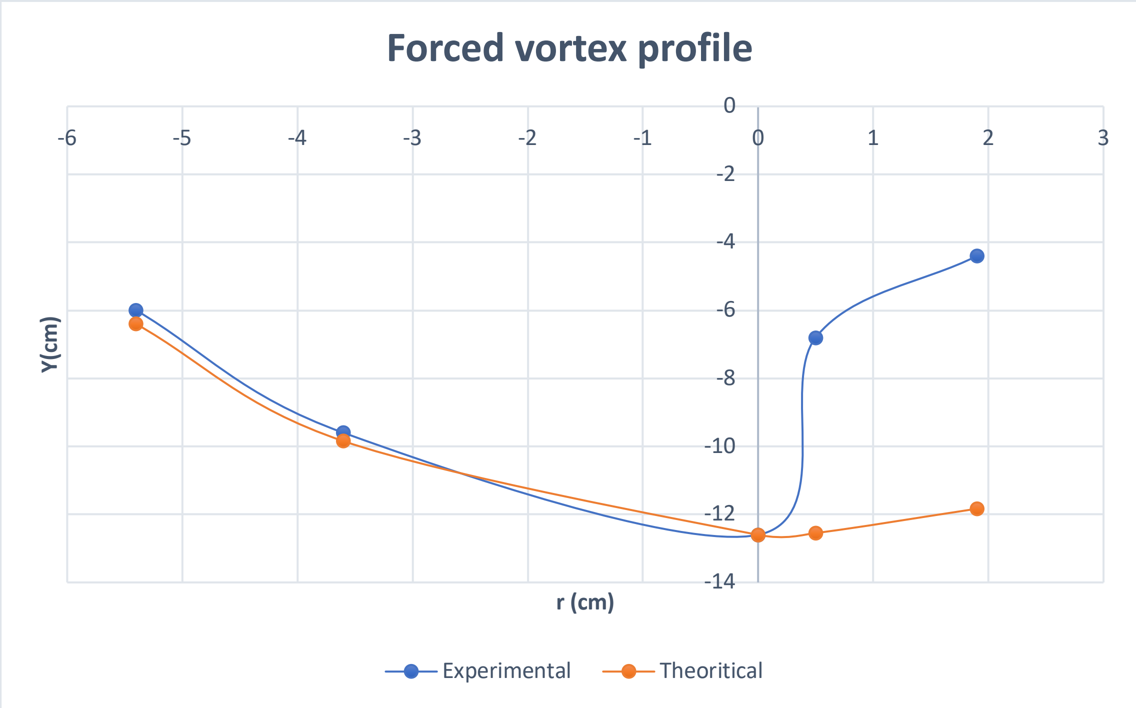
Boundary condition $r = 0, \;\; h = h_o, \;\; p = p_o$

$$p - p_o = -\rho g (h - h_o) + \rho \frac{\omega^2 r^2}{2}$$

When two points are there at free surface $p = p_o$

$$\boxed{h = h_o + \frac{\omega^2 r^2}{2g}}$$

Plot



Conclusion

Due to the application of mechanical power to the fluid flow, we noticed that fluid shifts to vertical upward draft resulting in flywheel vortex which is forced vortex flow by forming paraboloid revolution. Forced vortex flow tends to be rotational flow as fluid under influence of supplied energy constantly rotated

Remarks

a) What are the possible sources of error?

Ans:

- The scales attached to the gauge needle might have a zero error of itself, which was not considered.
- The tachometer would also have some zero error, which was not considered

b) Compare and comment on the calculated and measured surface profiles.

Ans: It can be seen from the plots that the surface profiles found experimentally, almost follows theoretical profile for forced vortex.

c) What is the effect of rotational speed on the formation of vortex?

Ans: Forced vortex can be considered as the phenomenon of rigid body rotation, as each fluid particle rotates with same angular velocity. It can be inferred from this formula

$$h = h_o + \frac{\omega^2 r^2}{2g}$$

that vortex will have higher depth for greater values of rotational speed