AM5820
Wind Tunnel and Numerical Experiments

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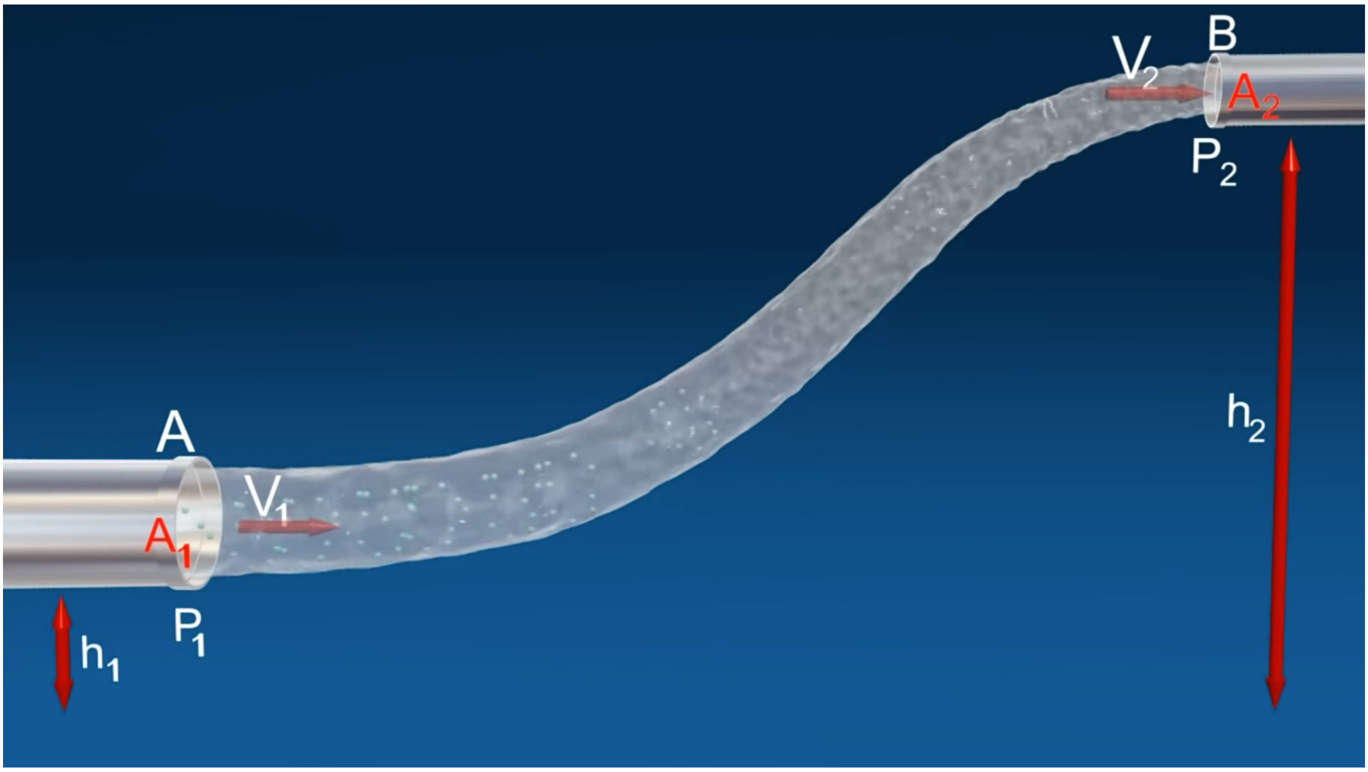
Verification of Bernoulli's theorem



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Objective

The objective of this experiment is:

- To demonstrate the variation of the pressure along a converging-diverging pipe section
- To validate Bernoulli's assumptions and theorem by experimentally proving that the sum of the terms in the Bernoulli equation along a streamline always remains constant

Introduction

The Bernoulli's theorem (developed by Daniel Bernoulli) is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible. The equation is obtained when the Euler's equation is integrated along the streamline for a constant density (incompressible) fluid. The constant of integration (called the Bernoulli's constant) varies from one streamline to another but remains constant along a streamline in steady, frictionless, incompressible flow.

Bernoulli's equation is based on the law of conservation of energy & it states that the "sum of the kinetic energy, the pressure energy and potential energy of the fluid at any point remains constant" provided the flow is steady, irrotational, and frictionless and the fluid used is incompressible. This is however, on the assumption that energy is neither added to nor taken away by some external agency. The key approximation in the derivation of Bernoulli's equation is that viscous effects are negligibly small compared to inertial, gravitational, and pressure effects.

Mathematically, the equation is expressed as

$$P + \frac{1}{2}\rho V^2 + \rho g Z = constant$$

An alternative, but equivalent form of above equation is obtained by dividing each term by the specific weight (ρg)

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = constant$$

Each term in above expression has the units of energy per weight or length and represents: -

$$\frac{P}{\rho g}: \ Pressure\ head$$

$$\frac{V^2}{2g}: Kinetic\ head$$

$$Z: Potential\ head$$

Here,

$P\ is\ pressure\ in\ N/m^2$

$\rho\ is\ density\ in\ N/m^3$

$V\ is\ velocity\ in\ m/s$

$g\ is\ acceleartion\ due\ to\ gravity\ in\ m/s^2$

$Z\ is\ elevation\ in\ m$

Practical Applications

When two boats or buses move side by side parallelly in the same direction, the water (or air) in the region between them moves faster than that on the remote sides. Consequently, in accordance with Bernoulli's principle the pressure between them is reduced and hence due to pressure difference they are pulled towards each other creating the so-called attraction.

Working of aero planes is also based on Bernoulli's principle. Due to their specific shape of wings when the aero plane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise.

Blowing of roofs by windstorms is in accordance with Bernoulli's principle. Magnus effect, which plays as important role in tennis, cricket, and soccer, etc. is based on Bernoulli's principle. Venturi meter used for measuring the rate of flow of liquid through pipes is also based on Bernoulli's theorem. The action of carburetor, paint-gun, scent-spray, or insect-sprayer in an atomizer is also based on Bernoulli's principle.

Equipment description

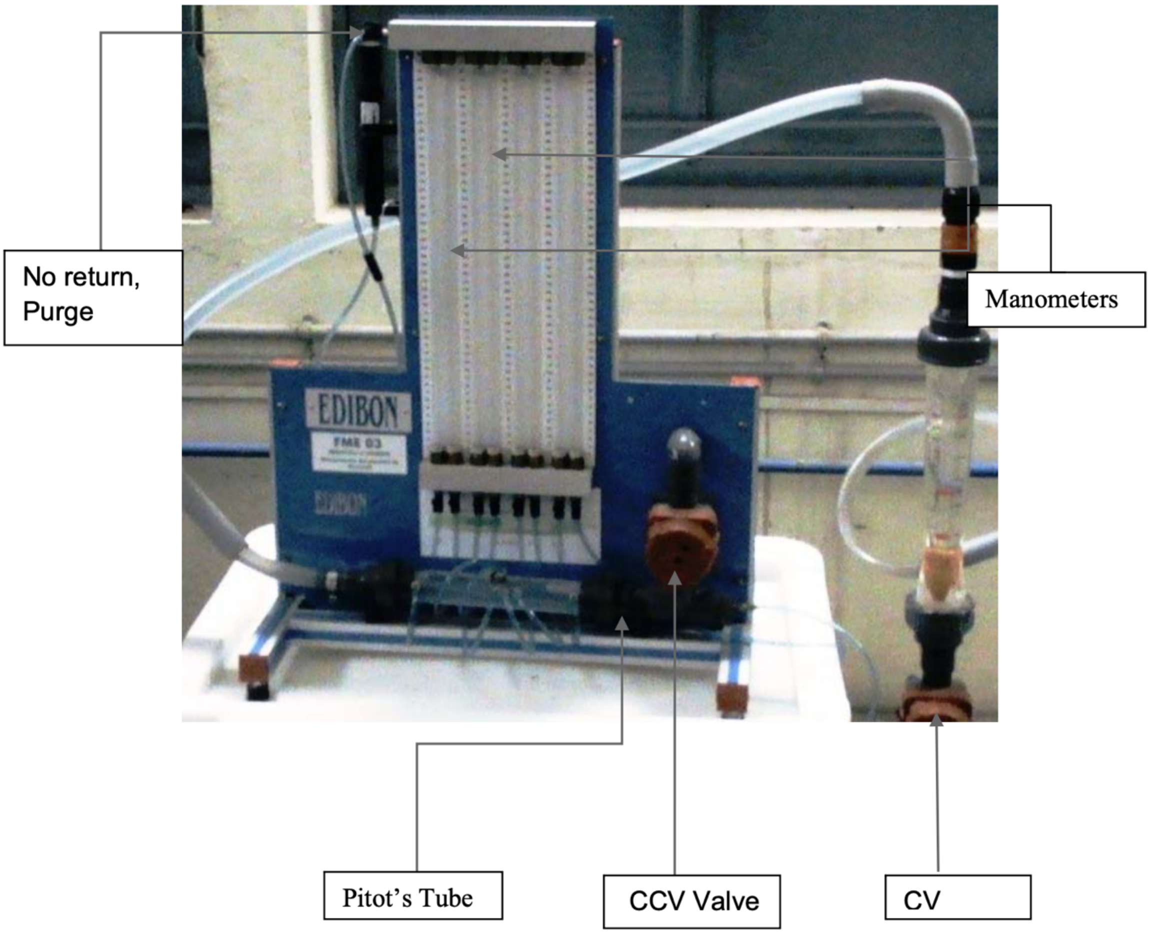


Fig. Apparatus for verification of Bernoulli's theorem

The apparatus is mainly composed of eight manometer tubes and a circular section with a truncated cone attached to inside it. Seven tubes are attached to the static ports along the circular section & eighth tube is attached to the pitot tube whose position can be adjusted inside the circular section by rotating the screw. At the tip of the pitot tube there is a stagnation port where fluid stagnates, and its pressure can be measured. This eighth tube is connected to the last manometer tube to measure the total pressure. Remaining seven tubes are connected to the first seven manometer tubes. The axis can be taken as the datum, and hence the potential energy is same throughout. The flow rate and the pressure in the equipment can be modified by adjusting the control valves and by using the supply valve in the hydraulic bench.

Experimental procedure

There are two parts of the experiment which are described as follows: -

Filling up of the manometer tubes

Before starting check that both the control valves are closed. After switching on the power supply, open the outlet control valve completely. Then open the input control valve. Water will start filling in the manometer tubes. Wait for some time so that it settles without any air bubbles. As, the air bubbles are removed slowly close the outlet control valve & then close the input control valve. Then drain some water by opening the output control valve to get the same initial height in all the manometer tubes. Also, open the surge valve. If the water level is not decreasing, we can use the double action pump. You will get very small water level in the manometer tubes, though the eight tube might take some time to match other tubes. Once, all tubes have dropped to same height close the output control valve.

Actual verification of the Bernoulli's theorem

After closing the surge valve, open the input control valve to give small initial flow rate. The water level will increase in the manometer tubes. Now, open the output control valve till we get a nice pressure variation among the manometer tubes. Adjust the input control valve to give a particular value of flow rate. By adjusting the input & output control valve we can obtain pressure variation in the manometer tubes for a particular flow rate. Since, the ports are connected at different area of cross-section, there is a good velocity variation & hence there is difference in pressure heads in the manometers.

To take the readings, adjust the screw and move the pitot tube to port 1. Measure the static and total pressure from the 1^{st} and the 8^{th} manometer tubes respectively. To take reading of total pressure wait for some time, as it takes time for the total pressure to settle down. Now, using the screw again move the pitot tube to the remaining 6 locations and repeat the process for each point.

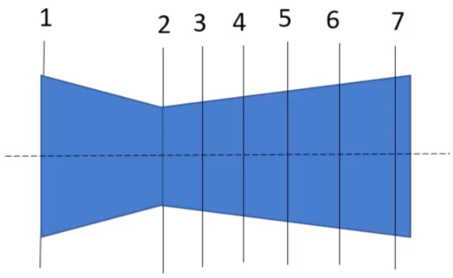
Raw data & Results

$Flow\ rate, Q = 600\ L/hr$

Section	Horizontal distance along pipe (cm)	Area, A (m^2)	Velocity, $v = \frac{Q}{a}$ (m/s)	Dynamic pressure head $v^2/2g$, (m)	Static pressure head, h_s (m)	Total pressure head calculated, $h_s + v^2/2g$, (m)	Total pressure head measured, h_{pt} (m)
1	0	2.173E-04	0.767	0.030	0.348	0.378	0.367
2	2	7.930E-05	2.102	0.225	0.050	0.275	0.247
3	4	8.430E-05	1.977	0.199	0.048	0.247	0.230
4	6	1.373E-04	1.214	0.075	0.130	0.205	0.225
5	8	1.403E-04	1.188	0.072	0.140	0.212	0.206
6	10	2.656E-04	0.628	0.020	0.170	0.190	0.194
7	12	3.717E-04	0.448	0.010	0.187	0.197	0.191

Sample Calculation

Bernoulli's theorem between any two sections can be written as follows



$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

We can take $Z_1 = Z_2$, because there is no change in height with respect to datum as axis

Therefore,

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g}$$

For a real flow, there will be viscous losses (H_l) which can be calculated using below equation

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + H_l$$

For verification of Bernoulli's Theorem:

$$Velocity, v = \frac{Q}{A}$$

We know continuity equation for incompressible flow,

$$A_1 V_1 = A_2 V_2$$

Therefore, velocity will vary with variation in area

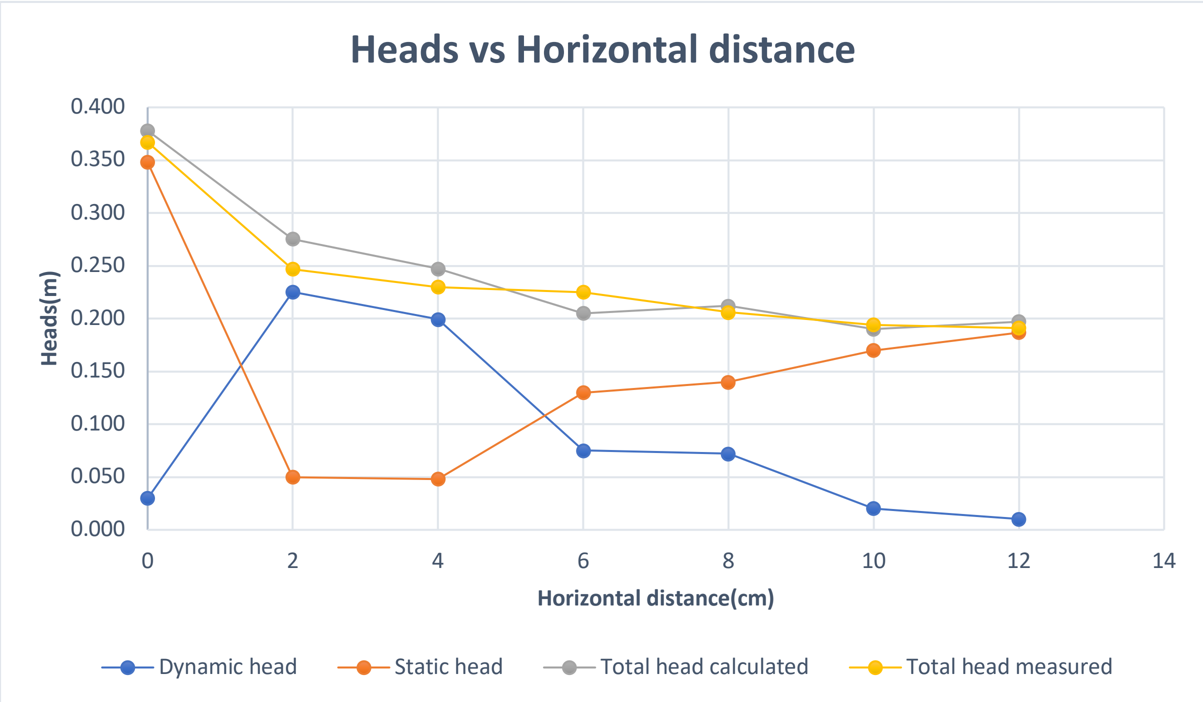
$$\boxed{Dynamic\ head = \frac{v^2}{2g}}$$

$$\boxed{Pressure\ head, h_s = \frac{P}{\rho g}}$$

$$Total\ head\ calculated = \ Pressure\ head + Dynamic\ head$$

$$\Rightarrow \boxed{Total\ head\ calculated = \ h_s + \frac{v^2}{2g}}$$

Plot



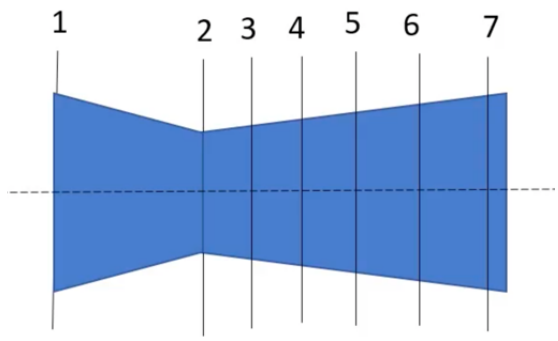
Conclusion

It can be seen from the above plot that with the increase in area of cross-section the velocity is decreasing, hence the dynamic head is decreasing. The opposite happens for the static head. As, we can see that total head calculated is closely varying with the total head measured.

Remarks

a) Which pressure heads, among the static and the dynamics pressures heads, is larger in your experiments? Why is this so? Is the opposite case from what you observe possible? When will both pressure heads be equal?

Ans: With the decrease in cross-section area (from section 1 to 2), dynamic head increases & with increase in cross-section area (from section 2 to 7) dynamic head decrease. This behavior can be explained by continuity equation, as follows for an incompressible flow assumption:



$$A_1V_1 = A_2V_2$$

It can be seen from the plot we got that for some part of the section, dynamic head is larger and for the other part static head is larger.

Yes, the opposite case is possible if we reverse the direction of the section we used in the experiment.

Also, in our plot static head & dynamic head are equal at two sections. As, I have explained earlier this happened because

$$Static\ head + Dynamic\ head = Total\ head$$

With the decrease in static head dynamic head increases as cross-section decreases & opposite happens as cross-section increases.

b) Is the sum of static and dynamic pressure heads constant? If not, why?

Ans: The sum of static and dynamic pressure heads is equal to the total head. And it can be seen from the plot we got that total head is not constant but decreasing along the section. The reason this is happening is due to the viscous losses.

c) Estimate the frictional head losses between the first tap and the last tap from the variation of the sum of static and dynamic pressure heads along the pipe axis.

Ans: We know,

$$Static\ head + Dynamic\ head = Total\ head$$

From the calculated data we can write that,

$$Total\ head\ at\ first\ tap, h_1 = 0.378m$$

$$Total\ head\ at\ last\ tap, h_2 = 0.197m$$

So,

$$Frictional\ head\ loss, H_l = h_1 - h_2$$

$$\Longrightarrow H_l = 0.378 - 0.197m$$

$$\Longrightarrow \boxed{H_l = 0.181m}$$

d) From your variation of static pressure head along the pipe, comment on whether flow can occur from a region of lower static pressure to a region of higher static pressure. If yes, why? If no, why not?

Ans: We know,

$$Static\ head + Dynamic\ head = Total\ head$$

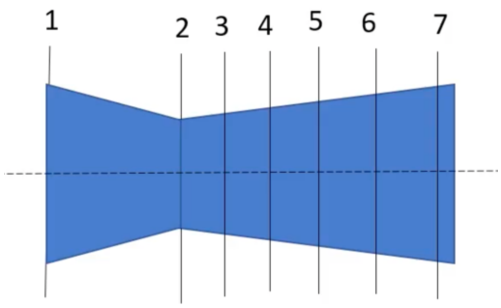
Total head will decrease along the flow. But static head can increase along the flow if the decrease in dynamic head is significant enough that the sum of static and dynamic head decreases. Hence, we can say that flow can occur from lower static pressure to higher static pressure.

e) Discuss the validity of Bernoulli's equation when the flow converges and diverges along the duct. Discuss the results referring to the energy loss and how the components of Bernoulli's equation vary along the length of the test section. Indicate the points of maximum velocity and minimum pressure.

Ans: When the flow diverges, frictional losses are usually significant due to the increased possibility of fluid separation from the walls. As, Bernoulli's equation assumes inviscid flow (no frictional losses), therefore it is invalid in this case.

When flow converges, velocity increases according to continuity equation. At very high velocities flow might not be compressible as Mach number > 0.3 is possible. As, Bernoulli's equation is valid only for incompressible flow therefore, it is invalid for this case

Total head is decreasing along the flow, due to energy losses.



Along the length of test section dynamic head decreases & static head increases. As can be seen from the plot we got, section 2 has maximum dynamic head & hence maximum velocity. Also, section 2 & 3 has minimum static head & hence minimum pressure.