AM5820
Wind Tunnel and Numerical Experiments

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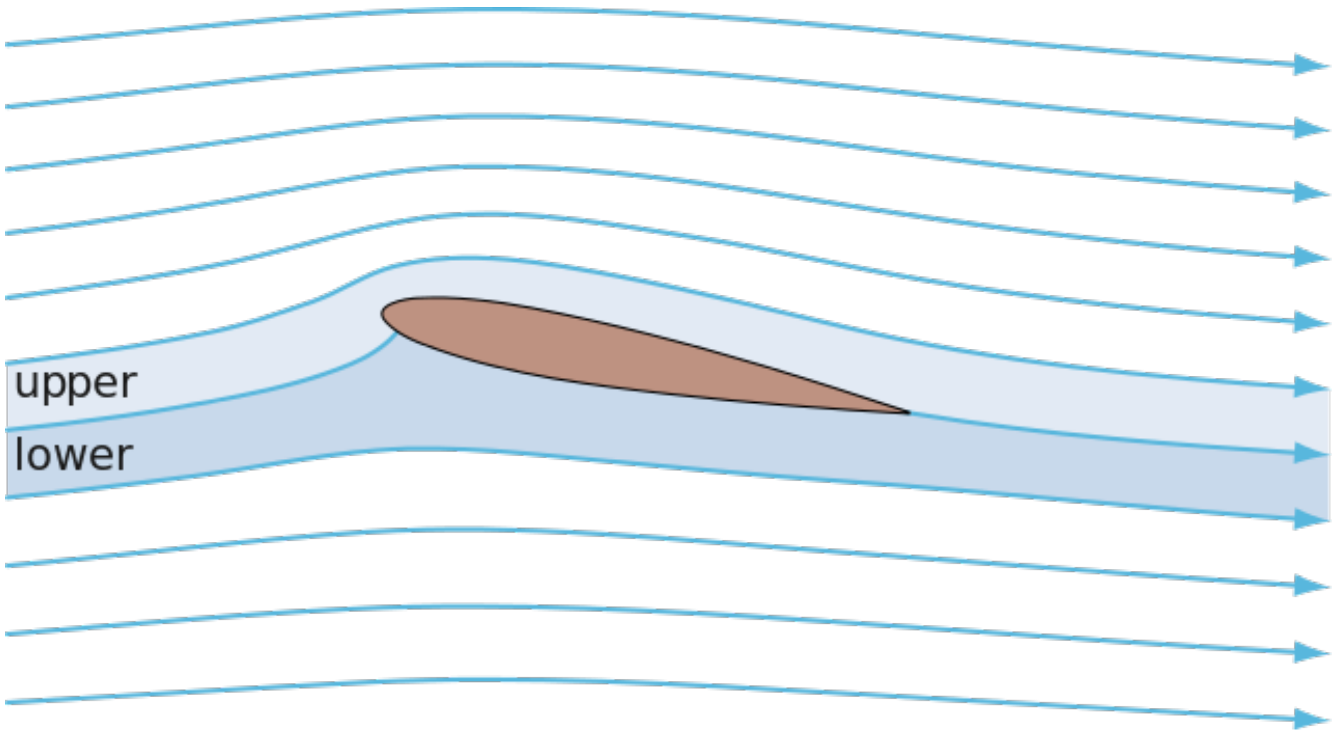
<u>Visualization of potential flows</u>



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Objective

The aim of the experiment is to visualize 2D laminar flow over different models and determine the velocity based on the equations of the potential flow. Qualitative streamline pattern is to be recorded for the following shapes:

1) Cylinder
2) Airfoil
3) Square section

Introduction

When the velocity field of a fluid flow can be represented as the gradient of a scalar quantity called the 'velocity potential', such a flow is known as potential flow. For potential flows,

$$\bar{v} = \bar{\nabla}\varphi \qquad (1)$$

where, φ is the velocity potential

Irrotational flows occur when the vorticity

$$\bar{\omega} = \bar{\nabla} \times \bar{v} = 0 \qquad (2)$$

Eq. (2) shows that irrotationality imposes strict conditions on velocity gradients, viz.

$$\frac{\delta w}{\delta y} = \frac{\delta v}{\delta z}, \qquad \frac{\delta u}{\delta z} = \frac{\delta w}{\delta x}, \qquad \frac{\delta v}{\delta x} = \frac{\delta u}{\delta y} \qquad (3)$$

where u, v, w are the velocity components in the x, y, z direction. It is easy to see that the velocity components of eq. (1) satisfy eq. (3). Hence if any flow is irrotational, the restrictions imposed by the irrotationality condition ensure that the velocity field can be represented by the gradient of a scalar velocity potential.

If the flow is incompressible,

$$\bar{\nabla}.\bar{v} = 0 \qquad (4)$$

$$\nabla^2\varphi = 0 \qquad (5)$$

and the velocity field can be obtained by solving the Laplace equation. For two dimensional flows,

$$\nabla^2\psi = 0$$

where ψ is the stream function defined as,

$$u = \frac{\partial\psi}{\partial y}, \qquad v = \frac{\partial\psi}{\partial x}$$

the stream function also satisfies the Laplace equation. Therefore, when the flow is irrotational the velocity field can be obtained by solving a linear second order elliptic

PDE in one variable φ (or ψ) rather than solving the formidable N-S equations in three velocity components.

Such solenoidal and irrotational velocity vector fields, obtained as the solution of Laplace equation, have some unique properties. Some of these are: the velocity field is independent of temporal changes, it is possible to superpose two solutions to obtain a third solution, values of normal component of velocity at the boundaries uniquely determine the value of velocity everywhere and pressure distribution has no role in deciding the velocity distribution. Since the velocity distribution is determined only by the normal velocity at the boundaries the solutions of Laplace equation allow non-zero tangential velocities at the boundaries. Hence, the no slip condition is not satisfied, and these flows are rarely seen in practice. Often, at high Re, the flow at some distance away from the rigid boundaries where the viscous effects are negligible can be represented as potential flows; a thin viscous 'Boundary Layer' within which the velocity changes from the no slip velocity to the potential flow velocity, occurs at the boundary.

However, it is not true that only inviscid flows are potential flows. Irrotational, and hence potential flows, can occur even when viscous forces are important. It is easy to see why this is so, by writing the N.S equation as

$$\rho\frac{\partial\bar{v}}{\partial t}+\bar{\nabla}\left(p+\frac{1}{2}\rho v^2+\rho gz\right)=\rho\bar{v}\times\bar{\omega}-\mu\bar{\nabla}\times\bar{\omega} \qquad (6)$$

using the vector identities

$$(\bar{v}.\bar{\nabla})\bar{v}=\frac{\bar{\nabla}v^2}{2}-\bar{v}\times\bar{\omega}, \qquad \bar{\nabla}^2\bar{v}=\bar{\nabla}(\bar{\nabla}.\bar{v})-\bar{\nabla}\times\omega \qquad (7)$$

Eq. (6) shows that whatever be the value of viscosity, the right-hand side vanishes for irrotational flows and the flow obeys the inviscid Euler equations, the pressure being related to velocity by the unsteady Bernoulli equation,

$$\rho\left(\frac{\partial\varphi}{\partial t}\right)+p+\frac{1}{2}\rho v^2+\rho gz=constant \qquad (8)$$

Practical Applications

This potential flow lines can be used to explain the lifting of an aircraft using Bernoulli's equation. Hele-Shaw cell is defined as Stokes flow between two parallel flat plates separated by an infinitesimally small gap, named after Henry Selby Hele-Shaw, who studied the problem in 1898.Various problems in fluid mechanics can be approximated to Hele-Shaw flows and thus the research of these flows is of importance. Approximation to Hele-Shaw flow is specifically important to micro-flows. This is due to manufacturing techniques, which creates shallow planar configurations, and the typically low Reynolds numbers of micro-flows. The governing equation of Hele-Shaw flows is identical to that of the inviscid potential flow and to the flow of fluid through a porous medium (Darcy's law). It thus permits visualization of this kind of flow in two dimensions.

Equipment description

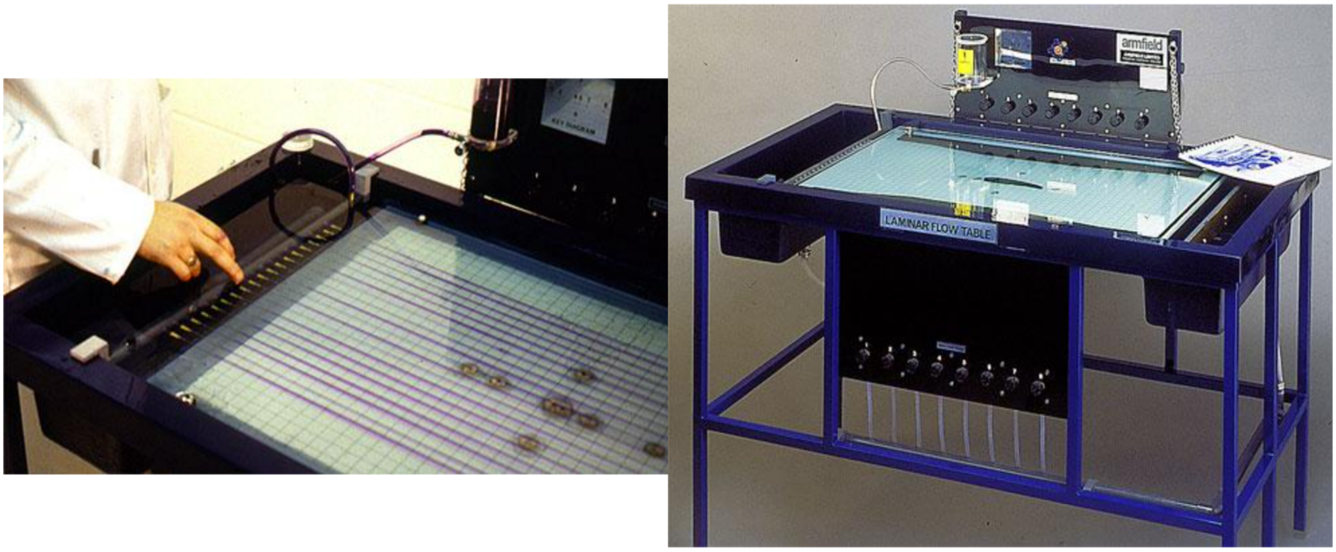


Fig. Laminar flow table

The experimental setup consists of two sections of a tank, between which the flow is regulated. At the outlet of the pump, a flowmeter is installed to regulate the inlet flow. The laminar flow table shown in figure is used to simulate the ideal fluid flows (Potential flows) of combination sources, sinks and uniform flow. Two glass sheets are placed on top of the tank surface to create the test section. The separation between the two glass sheets is approximately 2 mm. Water is made to flow between the two sections of the tank, through the test section. The uniform flow streamlines are simulated by injecting a liquid die which is contained in a separate reservoir, through needle injectors (hypodermic needles) between the glass sheets. The die will flow like a line (represent a streamline) and the streamlines get disturbed due to the presence of either 2D bodies or source & sink flows, there by the streamlines of the complete flow field is visualized.

Experimental procedure

Fill the reservoirs with water & let the flow field is establish properly without bubbles and unwanted particles. Now, place the cylinder model in on the lower plate & close the upper plate on the lower plate. Insert all the injectors inside the gap between two plates. Then the die is injected through the needle injector. The die should form straight lines, where there are no disturbances. If the color lines are not straight, then the flow table should be adjusted till the color lines are straight.

From theory we know that,

$$Velocity\ in\ x\ direction, u = \frac{\Delta\psi}{\Delta y} \qquad (9)$$

Since $\Delta\psi$ between any two streamlines is constant.
Therefore, if Δy is smaller then u will be larger & if Δy is larger the u will be smaller.

Now, replace the cylinder with the aero foil or the square section and repeat the process.
For the aero foil we can observe that Δy above the aero foil is less than Δy below the aero foil. So, velocity above the aero foil is more than velocity below the aero foil by relation (9). Using Bernoulli's equation, we can say that pressure on the top will be lower than pressure at the bottom & this makes the aero foil to lift.

Raw data & Results

Cylinder

$Q = 600\ liters/hr$

$Distance\ along\ the\ object, x(cm)$	$\Delta y\ (cm)$ $Above\ the\ object$	$u(m/s)$ $Above\ the\ object$	$\Delta y\ (cm)$ $Below\ the\ object$	$u(m/s)$ $Below\ the\ object$
0	1.5	0.1852	1	0.2778
1	1	0.2778	0.5	0.5556
2	1.5	0.1852	1.2	0.2315
3	2.4	0.11575	2	0.1389

Airfoil

$Q = 800\ liters/hr$

$Distance\ along\ the\ object, x(cm)$	$\Delta y\ (cm)$ $Above\ the\ object$	$u(m/s)$ $Above\ the\ object$	$\Delta y\ (cm)$ $Below\ the\ object$	$u(m/s)$ $Below\ the\ object$
0	1.6	0.2315	2.4	0.154333333
1	1	0.3704	2.2	0.168363636
2	0.8	0.463	2	0.1852
3	0.8	0.463	2	0.1852
4	1	0.3704	2	0.1852
5	1	0.3704	1.8	0.205777778
6	1.2	0.308666667	1.8	0.205777778
7	1.2	0.308666667	1.8	0.205777778
8	1.2	0.308666667	1.6	0.2315
9	1.4	0.264571429	1.4	0.264571429

Square section

$Q = 800\ liters/hr$

$Distance\ along\ the\ object, x(cm)$	$\Delta y\ (cm)$ $Above\ the\ object$	$u(m/s)$ $Above\ the\ object$	$\Delta y\ (cm)$ $Below\ the\ object$	$u(m/s)$ $Below\ the\ object$
0	1.6	0.2315	1.6	0.2315
1	1	0.3704	1	0.3704
2	0.8	0.463	0.8	0.463
3	1.4	0.264571429	1.2	0.308666667
4	1.8	0.205777778	1	0.3704

Sample Calculation

We know,

$$U_\infty = \frac{Q}{A} \qquad (10)$$

where,

Cross sectional area, $A = 2mm \times 600mm$

Free stream velocity, U_∞

Flow rate, Q

At the start of the streamline, $\Delta y = 2cm$ which is the gap between two injection needles.

<u>Now, for $Q = 600\, ltr/hr$</u>

$$u = \frac{600\, ltr/hr}{1200\, mm^2}$$

$$\Longrightarrow u = \frac{600 \times 10^{-3}}{3600 \times 1200 \times 10^{-6}} m/s$$

$$\Longrightarrow u = \frac{1000}{6 \times 1200} m/s$$

$$\Longrightarrow u = U_\infty = 0.1389 m/s$$

We know from equation (9),

$$\Delta\psi = u \times \Delta y$$

$$\Longrightarrow \Delta\psi = 0.1389 m/s \times 2\mathrm{cm}$$

$$\Longrightarrow \Delta\psi = 0.1389 \times 0.02\, m^2/s$$

$$\Longrightarrow \boxed{\Delta\psi = 0.002778\, m^2/s}$$

<u>And for $Q = 800\, ltr/hr$</u>

$$u = \frac{800\, ltr/hr}{1200\, mm^2}$$

$$\Longrightarrow u = \frac{800 \times 10^{-3}}{3600 \times 1200 \times 10^{-6}} m/s$$

$$\Longrightarrow u = \frac{8000}{36 \times 1200} m/s$$

$$\Longrightarrow u = U_\infty = 0.1852 m/s$$

We know from equation (9),

$$\Delta\psi = u \times \Delta y$$

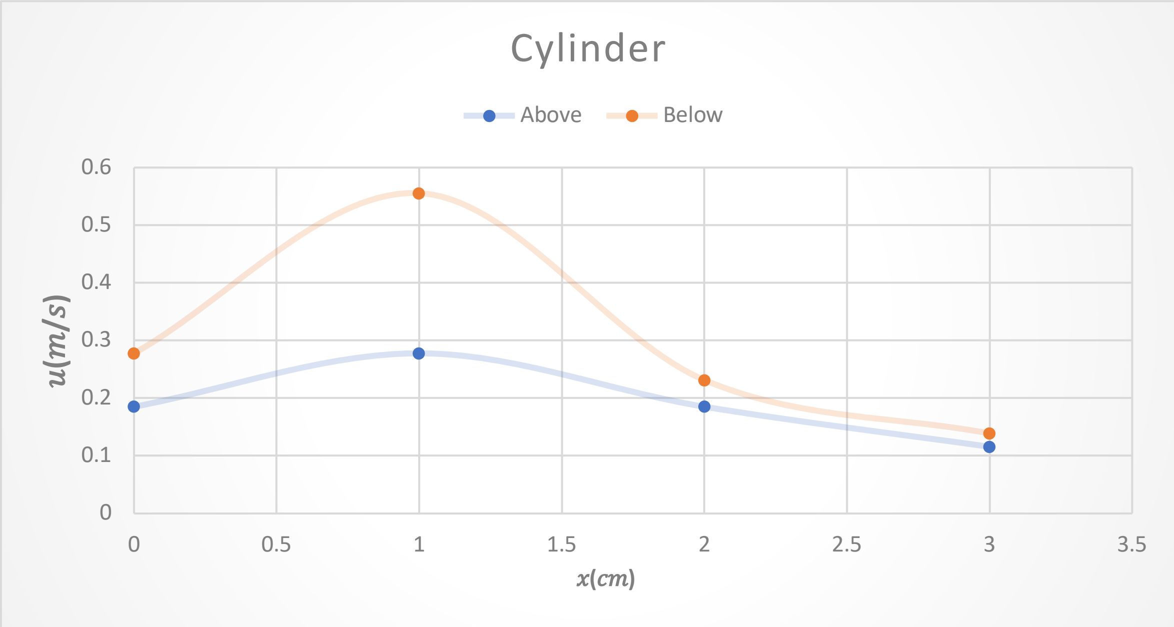
$$\Longrightarrow \Delta\psi = 0.1852 m/s \times 2\mathrm{cm}$$

$$\Longrightarrow \Delta\psi = 0.1852 \times 0.02\, m^2/s$$

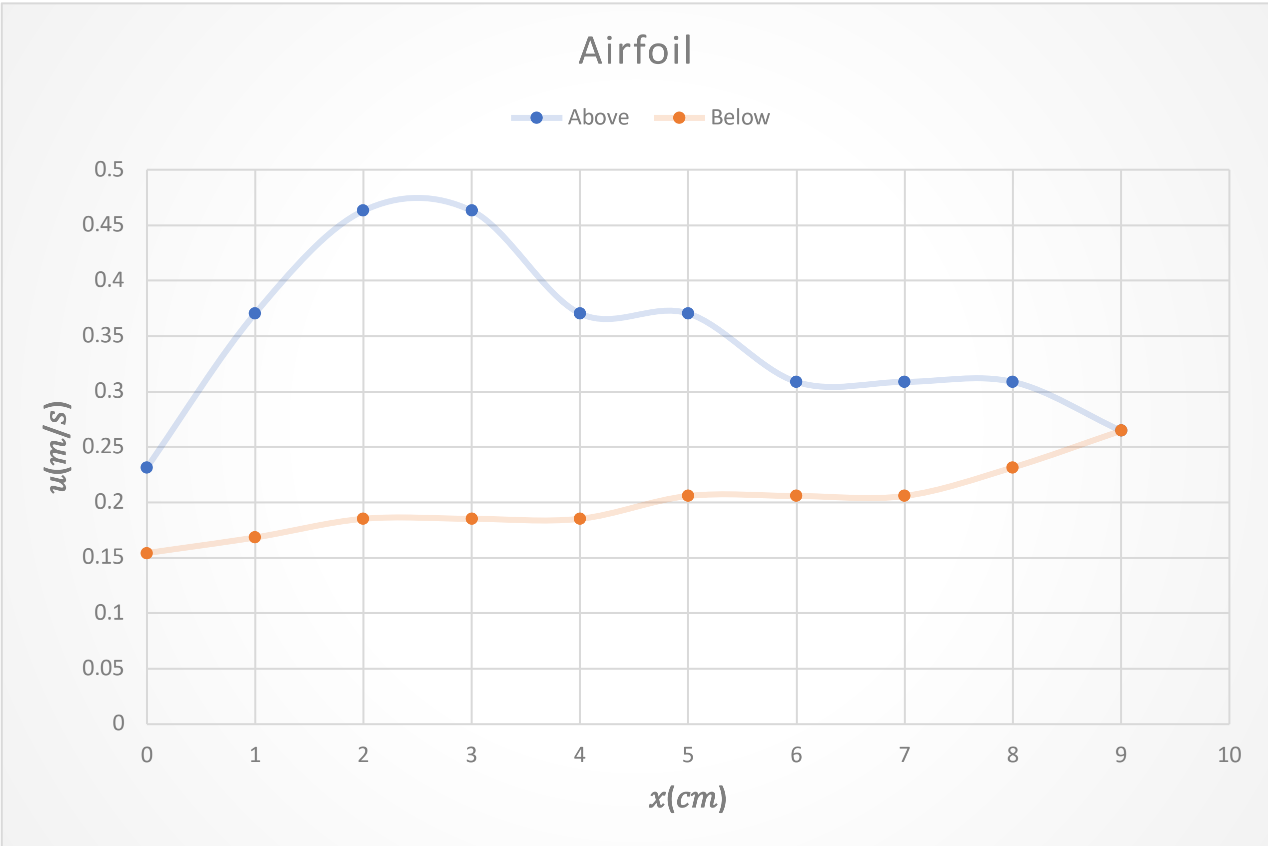
$$\Longrightarrow \boxed{\Delta\psi = 0.003704\, m^2/s}$$

Plot

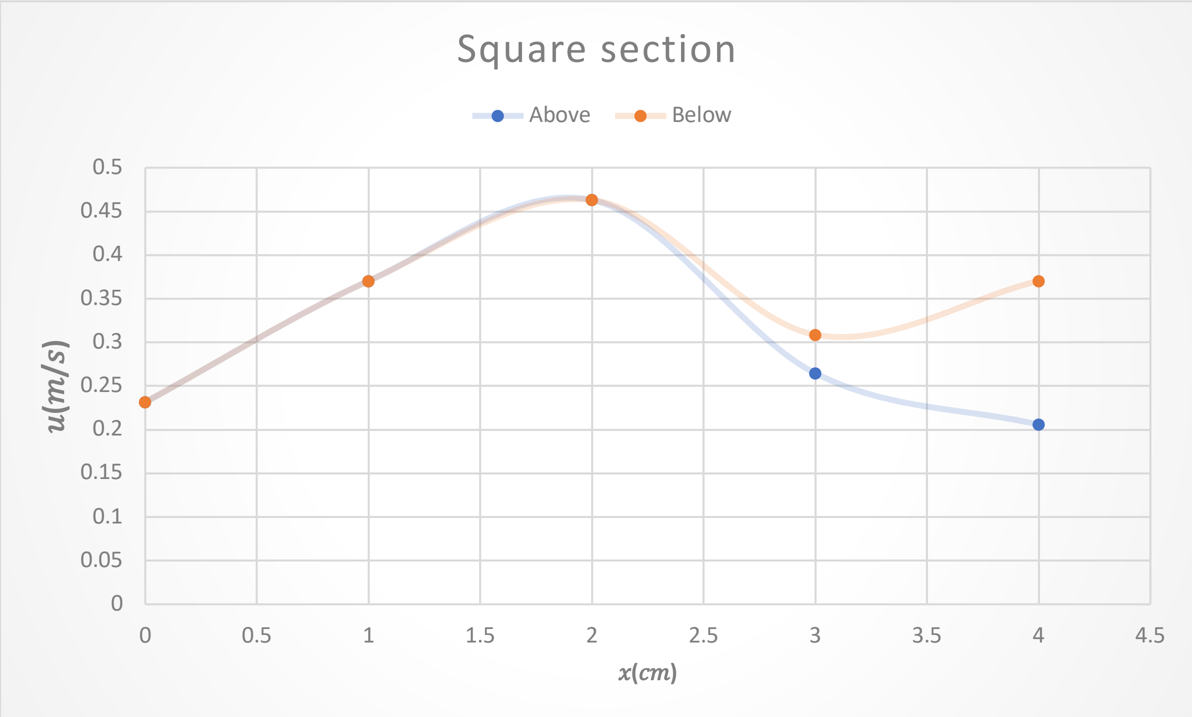
__Cylinder__



__Airfoil__



Square section



Conclusion

It can be seen from above plots that x-direction velocity is almost similar above and below the cylindrical section. Behavior is same for square section as well.

But for airfoil the behavior is significantly different above and below the object. The velocity above the airfoil is higher than below it. This variation is in accordance to the shape of the object, as cylinder & square section are symmetric whereas airfoil is not

Remarks

a) You injected dye. Did you observe streamlines or streak lines? Justify your answer.

Ans: A streak line is the locus of fluid particles that have passed sequentially through a prescribed point in the flow. Whereas streamline is a trajectory of the particles which are under steady flow. It is a curve that is everywhere tangent to the instantaneous local velocity vector.

As, in the experiment lines are formed by continuous introduction of dye from the injectors which are at a fixed location. Therefore, we observed streak lines in the flow.

b) What does the difference between the stream function give you? Which Instrument will you use for this?

Ans: The difference between the stream functions over a pair of streamlines is equal to the volumetric flow rate per unit width between a pair of streamlines.

It is measured using a rotameter

c) When you kept an aero foil at some angle of attack, in which region, the Δy was higher. Why?

Ans: To create lift, the pressure head below the airfoil must be greater than above it. According to Bernoulli's equation the dynamic head above the airfoil must be greater than below it. Or we can say, velocity above the airfoil must be greater than velocity below it. This can only happen if, Δy is higher below the airfoil compared to above it. Hence, for non-zero angle of attack Δy will be higher below the airfoil.

d) What parameter was constant throughout the flow domain?

Ans: The difference between the stream functions, $\Delta\psi$ is constant throughout the flow domain for a given flow rate.