Ques1-

```
Void func ( int n )
{   int j = 1 , i = 0 ;
    while ( i < n )
    {
        i + = j ;
        j + + ;
    }
}
```

for $j = 1$ $\quad i = 1;$

$j = 2$ $\quad i = 1+2;$

$j = 3$ $\quad i = 1+2+3;$

$\Big]\; m$

$\because \quad 1+2+3+ \cdots < n$

$1+2+3+m < n$

$\frac{m(m+1)}{2} < n \longrightarrow m \tilde{\approx} \sqrt{n}$

by summation Method

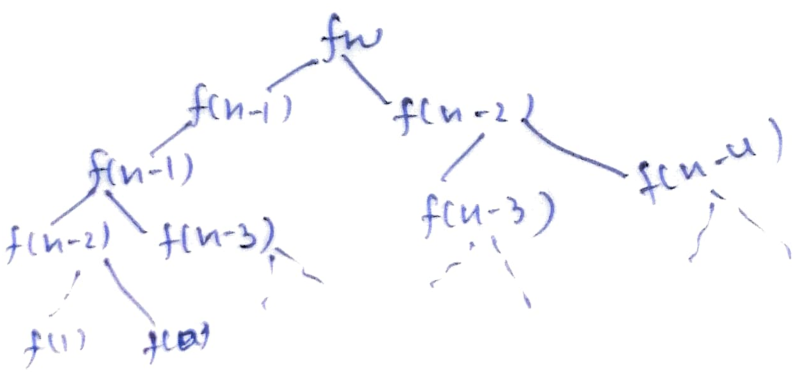
$\sum_{i=1}^{m} 1 \Rightarrow 1+1+ + \cdots \sqrt{n}$ times

$\therefore \quad T(n) = \sqrt{n}$

Ques2- for fibonacci series -

$f(n) = f(n-1) + f(n-2)$

$f(0) = 0 \qquad f(1) = 1$



$\because$ at every function call we get two function calls for n levels :

we have => 2×2 ..... n times

$\therefore T(n) = 2^n$

Maximum Space Considering recursion Stack

no of calls max = n

for each call we have space Complexity O(1)

$\therefore T(n) = O(n)$

Ques3 - (a.) n logn :-

quick Sort

```
Void func [int arr[], int l, int h)
{   if (l<h)
    {
        int pi = partition(a, l, h);
        func (arr, l, pi-1);
        func (arr, pi+1, h);
    }
}
int partition (int arr[], int l, int h)
    int pi = arr [h];
    int i =(l-1);
  for (int j = l ; j<=h; j++)
   {
     if (arr [i] < pi)
   {
       i++;
     swap (arr [i], arr[j] ];
   }
}
  swap ( arr [i+1], arr[h] );
  return (i+1);
}
```

(b)

n^3 :
Multiplication of two Square Matrix

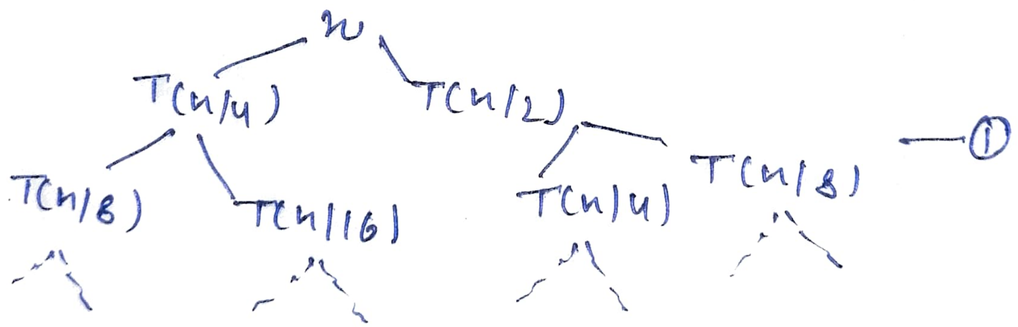
```
for (i=0; i<n; i++)
 {
     for (j=0; j<c ; j++)
   {
     for (k=0 ; k < c1 ; k++)
   {
     res[i][j] += a[i][k] *b[k][j];
```

(c)

log (logn)

```
for ( i=2; i<n ; i= i*i )
  {
      c++;
  }
```

Ques4- $T(n) = T(n/4) + T(n/2) + c*n^2$



At level $0 \rightarrow cn^2$

$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{c\,5n^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

⋮

max level $= \frac{n}{2^k} = 1$

$\Rightarrow k = \log_2 n$

$\because T(n) - [cn^2 + (\frac{5}{16})n^2 + (\frac{5}{16})^2 + \cdots + (5/N)^{\log^n} n^2)$

$T(n) = cn^2 [1 + \frac{5}{16} + (\frac{5}{16})^2 + \cdots (\frac{5}{16})^{\log n}]$

$T(n) = cn^2 \times 1 \times \left[\frac{1 - (\frac{5}{16})^{\log n}}{1 - \frac{5}{16}} \right]$

$= cn^2 \times \frac{11}{5} \left[1 - (\frac{5}{16})^{\log n} \right]$

$\because T(n) = O(n^2 c) \Rightarrow O(cn^2)$

Ques5-

```
int fun (int n)
{
    for (i=j; i<=n ; i++)
    {
        for (j=1; j<n ; j+=i)
        {

        }
    }
}
```

for	i	j
	1	1
	2	1+3+5
	3	1+4+7
	4	1+5+9
	⋮	
	n	

$\sum_{i=1}^{n} \frac{(n-1)}{i}$

$\because T(n) = \frac{(n-1)}{2} + \frac{(n-1)}{2} + \frac{(n-1)}{n}$

$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] - 1 \times n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right]$

$= n \log n - \log n$

$\therefore T(n) = O(n \log n)$

for (i=2; i<=n; i = pow (i, k)]

{ O(1)

}

for → i

2^1

2^k

2^{k^2}

2^{k^3}

⋮

2^{k^m}

where $2^{k^m} <= n$

$k^m = \log n$

$m \log k \quad \log_2 n$

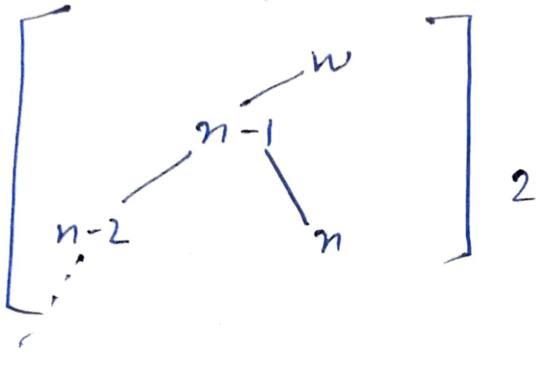
$\because \sum_{i=1}^{m} 1$

$\Rightarrow 1 + + + \cdots m$ times

$\Rightarrow T(n) = O[\log k \log n]$

Ques7 - Given Algo divides array in 99 % & 1 % part

$\therefore \; T(n) = T(n-1) + O(1)$



'n' work is done at each level for merging.

$T(n) = [T(n-1) + T(n-2) + \cdots T(1) + O(1)) * n$

$= n * n$

$T(n) = O(n^2)$

lowest height $= 2$

highest height $= n$

$\therefore$ diff $= n-2 \quad \because (n>1)$

Ans- Considering for large values of 'n'

(a) $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n <$ $\log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

(b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n$ $< n\log n < 2n < 4n < \log(n!) < n^2 < n < 2^{\log 2n} < 5n$

(c) $96 < \log_8 n < \log 2n < 5n < n\log_6 n < n\log_2 n < \log(n!)$ $< 8n^2 < 7n^3 < n! < 8^{2n}$