Ques1 Sol -

$T(n) = 3T(n/2) + n^2$

$a = 3, \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant & f(n) is the function

$\therefore$ Master's theorem is applicable

$c = \log_1 a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $< n^2$

$\therefore$ Case 3 is applicable

$T(n) = O(n^2)$

Q2 soln :-

$T(n) = 4T(n/2) + n^2$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 \qquad n^c = f(n)$

Case 2 $\qquad T(n) = O(n^2 \log n)$

Q3 soln :-

$T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0 \qquad n^c = n^0 = 1$

$f(n) > n^c$

$\therefore$ Case 3

$T(n) = O(2^n)$

Ques4 soln :-

$T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^n$

$\because$ a is not constant, its value depends on n

$\therefore$ Master's theorem not applicable.

Ques5 soln:-

$T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

Case 1

$T(n) = \theta(n^2)$

Ques6 soln:-

$T(n) = 2T(n/2) + n\log n$

$a = 2, \quad b = 2, \quad f(n) = n\log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

$f(n) > n^c$

Case 3 is applied

$T(n) = \theta(n\log n)$

Q7 soln:-

$T(n) = 2T(n/2) + n/\log n$

$a = 2, \quad b = 2 \quad f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n$

non polynomial diff b/w n^c & $f(n)$

$\therefore$ masters theorem not applicable.

Ques8 soln:-

$T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\because f(n) > n^c$

Case 3 is applied

$T(n) = \theta(n^{0.51})$

Q9 Soln:-

$$T(n) = 0.5\,T(n/2) + \frac{1}{n}$$

$a < 1$ ∴ Master's theorem applicable

Q10 Soln:-

$$T(n) = 16T(n/4) + n!$$

$a = 16, \quad b = 4, \quad f(n) = n!$

$c = \log_b a$

$= \log_4 16 = 2$

$n^c = n^2$

$f(n) > n^c$

Case 3

$$T(n) = O(n!)$$

Q11 Soln:-

$$T(n) = 4T(n/2) + \log n$$

$a = 4, \quad b = 2, \quad f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

∴ Case 1 is applied

$$T(n) = O(n^2)$$

Q12 Soln:-

$$T(n) = \sqrt{n}\,T(n/2) + \log n$$

a is not constant, therefore master's theorem not applicable.

Q13 Soln:-

$T(n) = 3T(n/2) + n$

$a = 3, \; b = 2 \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

Case 1

$\therefore T(n) = O(n^{1.58})$

Q14 Soln:-

$T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \; b = 3, \quad f(n) = \sqrt{n}$

$c = \log_b a = 1$

$n^c = n > \sqrt{n}$

Case 1 is applied $T(n) = O(n)$

Q15 Soln:-

$T(n) = 4T(n/2) + cn$

$a = 4, \; b = 2, \; f(n) = c.2$

$n^c = n^2 > f(n)$

$\therefore$ case 1 is applied

$T(n) = O(n^2)$

Q16 Soln:-

$T(n) = 3T(n/4) + n \log n$

$a = 3, \; b = 4, \; f(n) = n \log n$

$c = \log_b a = \log_a 3 = 0.78$

$n^c = n^{0.78} < f(n)$

$\therefore$ case 3 is applied $T(n) = O(\log n)$

Q17 Soln! –

$T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3 \quad f(n) = n/2$

$c = \log_b a = 1$

$n^c = n > f(n)$

Case1 ! $T(n) = \Theta(n)$

Q18 Soln! –

$T(n) = 6T(n/n) + n^2 \log n$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

Case3 is applied $\therefore T(n) = \Theta(n^2 \log n)$

Q19 Soln! –

$T(n) = 4T(n/2) + n \log n$

$c = \log_2 4 = 2$

$n^c = n^2 \quad f(n) = n \log 2$

$n^c > f(n)$

$\therefore$ case1 is applied $T(n) = \Theta(n^2)$

Q20 Soln! –

$T(n) = 64T(n/8) + n^2 \log n$

$a = 64, b = 8 \quad f(n) = n^2 \log n$

$c = \log_b a = 2$

$n^c = n^2 < f(n)$

Case3 is applied $T(n) = \Theta(n^2 \log n)$

Q21 Soln :-

$$T(n) = 7T(n/3) + n^2$$

$a = 7$, $b = 3$, $f(n) = n^2$

$$C = \log_b a = \log_3 7 = 1.77$$

$$n^C = n^{1.77} < f(n)$$

Case 3 is applied $T(n) = O(n^2)$

Q22 Soln :-

$$T(n) = T(n/2) + n(2 - \cos n)$$

$\because$ f(n) is not regular function

$\therefore$ Master's theorem can't applied.