Ques1 Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the Input tends toward a particular value or a limiting value.

Eg - In bubble sort, when the Input array is already sorted, the time taken by algorithm is linear i.e the best case (Ω notation) (Omega notation)

But when the Input array is in reverse condition. the algorithm takes the maximum time to sort the element i.e the worst case (Big - O notation)

when the Input array is neither sorted nor in reverse order, then it takes average time (Θ - notation) (Theta notation)

Q2-

$$\sum_{i=1}^{n} 1+1+\dots k \text{ time}$$

$$\therefore \; 2^k >= n$$

$$2^k = n$$

taking log both side

$$k \log 2 = \log n$$

$$k = \frac{\log n}{\log 2} \qquad \left[\log_b(x) = \frac{\log_a(x)}{\log_a(b)}\right]$$

$$k = \log_2 n$$

$$O(\log n)$$

Ques 3 -

$$T(n) = \begin{cases} 3T(n-1) & n>0 \\ 1 & n=0 \end{cases}$$

$T(n) = 3T(n-1)$ — 1

Let $n = n-1$

Putting n in eq (1)

$T(n-1) = 3T(n-2)$ — 2

Putting 2 in 1

$T(n) = 3^2 T(n-2)$ — 3

Let $n = n-2$

putting n in eq 1

$T(n) = 3T(n-2)$ — 4

putting 4 in 3

$T(n) = 3^3 T(n-3)$

$T(n) = 3^k T(n-k)$

Let $n-k = 0$

$n = k$

$T(n) = 3^n T(0)$

$= O(3^n)$

Ques4 -

$$T(n) = \begin{cases} 2T(n-1) - 1 & n > 0 \\ 1 & n = 0 \end{cases}$$

$T(n) = 2T(n-1) - 1$ ——— ①

Let $n = n-1$ in eq ①

$T(n-1) = 2T(n-2) - 1$ ——— ②

Put this value in eq ①

$T(n) = 2[2T(n-2) - 1] - 1$

$T(n) = 4T(n-2) - 2 - 2$ ——— ③

Let $n = n-2$

$T(n-2) = 2T(n-3) - 1$ ——— ④

Put this value in eq ③

$T(n) = 4[2T(n-3) - 1] - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \dots 2^0$

Put $n - k = 0$

$n = k$

$T(n) = 2^n T(0) - 2^{n-1} \dots 2^0$

$= 2^n - [2^{n-1} + 2^{n-2} + \dots 2^0]$

$\Rightarrow 2^n - 2^{n-1}(1 - (\frac{1}{2})^n)2$

$\Rightarrow 2^n [1 - (1 - (\frac{1}{2})^n)]$

$\Rightarrow 2^n [1 - 1 + (\frac{1}{2})^n]$

$\Rightarrow 2^n (\frac{1}{2})^n = 1$

$\Rightarrow O(1)$

Ans

Ques5 -

$i = 1, 2, 3 \ldots$

$S = 1, 3, 6, 10 \ldots n$ —— ①

also $S = 1, 3, 6, 10 \ldots n$ —— ②

Subt ① - ②

$0 = 1 + 2 + 3 + \ldots n - T_n$

$T_n = 1 + 2 + 3 + \ldots k$

for k iteration

$1 + 2 + 3 \ldots + k <= n$

$$\frac{k(k+1)}{2} <= n$$

$$\frac{k^2 + k}{2} <= n$$

$$O(k^2) <= n$$

$$k = O(\sqrt{n})$$

Ques6 -

$$i^2 <= n$$

$$i <= \sqrt{n}$$

$$i = 1, 2, 3, \ldots \sqrt{n}$$

$$\sum_{i=1}^{n} 1 + 2 + 3 + \ldots \sqrt{n}$$

$$T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2}$$

$$T(n) = \frac{n + \sqrt{n}}{2}$$

$$= O(n)$$

Ans

Ques7 -

for $k = k * 2$

$k = 1, 2, 4, 8, \dots n$

GP, $a = 1$ $r = 2$

$$= \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(2^k - 1)}{1}$$

$n = 2^k$

$\Rightarrow \log n = k$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	⋮
3	⋮	⋮
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$O(n \log^2 n)$

Ques8 $T(n) = T(n/3) + n^2$

$a = 1,\ b = 3$ $f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow n^0 = 1 > [f(n) = n^2]$

$T(n) = \Theta(n^2)$

Ques9 -

for $i = 1 \Rightarrow j = 1, 2, 3, 4 \ldots\ldots n = n$

for $j = 2 \Rightarrow j = 1, 3, 5 \ldots\ldots n = n/2$

$\vdots$

$for(i = n) \Rightarrow j = 1 \ldots\ldots$

$$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} \ldots\ldots + 1$$

$$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \ldots\ldots + \frac{1}{n}\right]$$

$$\Rightarrow \sum_{j=n}^{1} n \log n$$

$$\Rightarrow O(n \log n)$$ Ans

Ques10 -

as given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n)$

as $n^k <= ac^n$

$\forall$ $n \geq n_0$ & some constant $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a2^1$

$n_0 = 1$ & $c = 2$ Ans