

# GraphFM: Graph Factorization Machines for Feature Interaction Modeling

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## Abstract

Factorization machine (FM) is a prevalent approach to modeling pairwise (second-order) feature interactions when dealing with high-dimensional sparse data. However, on the one hand, FM fails to capture higher-order feature interactions suffering from combinatorial expansion. On the other hand, taking into account interactions between every pair of features may introduce noise and degrade prediction accuracy. To solve the problems, we propose a novel approach, Graph Factorization Machine (GraphFM), by naturally representing features in the graph structure. In particular, we design a mechanism to select the beneficial feature interactions and formulate them as edges between features. Then the proposed model, which integrates the interaction function of FM into the feature aggregation strategy of Graph Neural Network (GNN), can model arbitrary-order feature interactions on the graph-structured features by stacking layers. Experimental results on several real-world datasets have demonstrated the rationality and effectiveness of our proposed approach. The code and data are available at <https://github.com/CRIPAC-DIG/GraphCTR>.

**Keywords:** Feature interaction, Factorization Machines, Graph Neural Network, Recommender System, Deep Learning.

## 1 Introduction

Predictive analytics is a fundamental task in machine learning (ML) and data mining (DM), which involves using input features to predict an output target, such as a real value for regression or categorical labels for classification. This is particularly important for web applications, such as online advertising and recommender systems [1, 2]. Distinct from continuous features which can be naturally found in images and audio, the features for web applications are mostly sparse and categorical. To accurately perform predictive analytics on these types of features, it is important to consider the interactions between them. As an example, consider a scenario in which we want to predict users' preferences for movies based on five categorical variables: (1) *Language* = {English, Chinese, Japanese, ...}, (2) *Genre* = {action, fiction, ...}, (3) *Director* = {Ang Lee, Christopher Nolan, ...}, (4) *Stars* = {Bruce Lee, Leonardo DiCaprio, ...} and (5) *Release Date* = {1991, 1992, ...}. To capture the impact of these feature interactions, a model might consider a 3-order cross feature such as (*Genre* = *fiction*, *Director* = *Christopher Nolan*, *Starring* = *Leonardo DiCaprio*) or (*Language* = *Chinese*, *Genre* = *action*, *Starring* = *Bruce Lee*) as potentially indicating higher user preferences.

Factorization machine (FM) [3, 4] are a popular and effective method for modeling feature interactions, which involve learning a latent vector for each one-hot encoded feature and modeling the pairwise (second-order) interactions between them through the inner product of their respective vectors. FM has been widely used in the field of recommender systems and click-through rate predictions due to its simplicity and effectiveness. However, because FM considers all feature interactions, it has two main drawbacks.

One of the main limitations of FM is that it is not able to capture higher-order feature interactions, which are interactions between three or more features. While higher-order FM (HOFM) has been proposed [3, 4] as a way to address this issue, it suffers from high complexity due to the combinatorial expansion of higher-order interactions. This makes HOFM difficult to use in practice. To address the limitations of FM in capturing higher-order feature interactions, several variants have been proposed that utilize deep neural networks (DNNs) [2, 5–7]. Factorisation-machine supported Neural Networks (FNNs) [5] apply DNN on top of pre-trained factorization machines to model high-order interactions. Neural Factorization Machines (NFM) [2] design a bi-interaction layer to learn the pairwise feature interaction and apply DNN to learn the higher-order ones. Wide&Deep [6] introduces a hybrid architecture containing both shallow and deep components to jointly learn low-order and high-order feature interactions. DeepFM [7] similarly combines a shallow component with a deep one to learn both types of interactions. While these

DNN-based models can effectively learn high-order feature interactions, they do so in an implicit, bit-wise manner. Consequently, they may lack the ability to provide persuasive rationales for their outputs.

In addition to not being able to effectively capture higher-order feature interactions, FM is also suboptimal because it considers the interactions between every pair of features, even if some of these interactions may not be beneficial for prediction [8, 9]. These unhelpful feature interactions can introduce noise and lead to overfitting, as they do not provide useful information but make it harder to train the model. For example, in the context of predicting movie preferences, the feature interactions between *Language* and *Release Date* might not be relevant and, therefore, not provide useful information for prediction. Ignoring these irrelevant feature interactions can improve model training. To solve this problem, the Attentional Decomposition Machine (AFM) [10] distinguishes the importance of the factorized interaction by reweighing each cross-feature using the attentional score [11], i.e. the influence of useless cross-features is reduced by assigning lower weights. However, it requires a predefined maximum order, which limits the potential of the model to find discriminative crossing features. Therefore, the Adaptive Factorization Network (AFN) [12] uses a logarithmic neural transformation layer composed of multiple vector-wise logarithmic neurons to automatically learn the powers (i.e. the order) of features in a potentially useful combination, thereby adaptively learning cross-features and their weights from the data.

Currently, Graph Neural Networks (GNN) [13–15] have recently emerged as an effective class of models for capturing high-order relationships between nodes in a graph and have achieved state-of-the-art results on a variety of tasks such as computer vision [16], neural language processing [17, 18], and recommender systems [19, 20]. At their core, GNNs learn node embeddings by iteratively aggregating features from the neighboring nodes, layer by layer. This allows them to explicitly encode high-order relationships between nodes in the embeddings. GNNs have shown great potential for modeling high-order feature interactions for click-through rate prediction. Fi-GNN [21] proposes to connect each pair of features and treat the multi-field features as a fully-connected graph, using a Gated Graph Neural Network (GGNN) [22] to model feature interactions on the graph. Graph Factorizer Machine (GFM) [23] utilizes FM to aggregate second-order neighbour messages, and utilizes the superposition of multiple GFM layers to aggregate higher-order neighbour messages to achieve multi-order interaction from neighborhood for recommendation. Graph-Convolved Factorization Machines (GCFM) [24] developed the Graph-Convolved Feature Crossing (GCFC) layer to traverse all features for each input example and leveraged the features of each sample to compute the corresponding multi-feature interaction graph and propagated its influence on other features. KD-DAGFM [25] proposes a directed acyclic graph based model, which can be aligned with the DP [26] algorithm to improve knowledge distillation (KD)[27] capability. However, not all feature interactions are

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beneficial, and GNNs rely on the assumption that neighboring nodes share similar features, which may not always hold in the context of feature interaction modeling.

In summary, when dealing with feature interactions, FM suffers intrinsic drawbacks. We thus propose a novel model Graph Factorization Machine (GraphFM), which takes advantage of GNN to overcome the problems of FM for feature interaction modeling. By treating features as nodes and feature interactions as the edges between them, the selected beneficial feature interactions can be viewed as a graph. We thus devise a novel technique to select the beneficial feature interactions, which is also to infer the graph structure. Then we adopt an attentional aggregation strategy to aggregate these selected beneficial interactions to update the feature representations. Specifically, to accommodate the polysemy of feature interactions in different semantic spaces, we utilize a multi-head attention mechanism [15, 28]. Each layer of our proposed model produces higher-order interactions based on the existing ones and thus the highest-order of interactions is determined by layer depth. Since our proposed approach selects the beneficial feature interactions and models them in an explicit manner, it has high efficiency in analyzing high-order feature interactions and thus provides rationales for the model outcome. Through extensive experiments conducted on CTR benchmark and recommender system datasets, we verify the rationality, effectiveness, and interpretability of our proposed approach.

Overall, the main contributions of this work are threefold: (1) We analyze the shortcomings and strengths of FM and GNN in modeling feature interactions. To solve their problems and leverage strengths, we propose a novel model GraphFM for feature interaction modeling. (2) By treating features as nodes and their pairwise feature interactions as edges, we bridge the gap between GNN and FM, and make it feasible to leverage the strength of GNN to solve the problem of FM. (3) Extensive experiments are conducted on CTR benchmark and recommender system datasets to evaluate the effectiveness and interpretability of our proposed method. We show that GraphFM can provide persuasive rationales for the feature interaction modeling and prediction-making process.

## 2 Related Work

In this work, we proposed a graph neural network-based approach to modeling feature interactions. We design a feature interaction selection mechanism, which can be seen as learning the graph structure by viewing the feature interactions as edges between features. In this section, we review three lines of research that are relevant to this work: 1) techniques for learning feature interactions, 2) Graph Neural Networks, and 3) graph structure learning methods.

## 2.1 Feature Interaction Modeling

Modeling feature interactions is a crucial aspect of predictive analytics and has been widely studied in the literature. FM [3] is a popular method that learns pairwise feature interactions through vector inner products. Since its introduction, several variants of FM have been proposed, including Field-aware factorization machine (FFM) [29] which takes into account field information and introduces field-aware embeddings, and AFM [10], which considers the weight of different second-order feature interactions. FmFM [30] model the interactions of field pairs as a matrix and utilizes kernel product to capture field interaction. However, these approaches are limited to modeling second-order interactions, which may not be sufficient in some cases.

As deep neural networks (DNNs) have proven successful in a variety of fields, researchers have begun using them to learn high-order feature interactions due to their deeper structures and nonlinear activation functions. The general approach is to concatenate the representations of different feature fields and feed them into a DNN to learn the high-order feature interactions. Factorization-machine supported Neural Networks (FNNs) [5] use pre-trained factorization machines to create field embeddings before applying a DNN, while Product-based Neural Networks (PNNs) [31] model both second-order and high-order interactions through the use of a product layer between the field embedding layer and the DNN layer. Like PNNs, Neural Factorization Machines (NFMs) [2] also use a separate layer to model second-order interactions, but they use a Bi-Interaction Pooling layer instead of a product layer and follow it with summation rather than concatenation. Other approaches to modeling second-order and high-order interactions jointly use hybrid architectures. The Wide&Deep [6] and DeepFM [7] contain a wide part modeling the low-order interaction and a deep part modeling the high-order interaction. However, like the other DNN-based approaches, these models learn high-order feature interactions in an implicit, bit-wise manner and may lack transparency in their feature interaction modeling process and model outputs. As a result, some studies have attempted to learn feature interactions in an explicit fashion through the use of specifically designed networks. Deep&Cross [32] introduces a CrossNet that takes the outer product of features at the bit level, while xDeepFM [33] uses a Compressed Interaction Network(CIN) to take the outer product at the vector level and then compresses the resulting feature maps to update the feature representations. However, xDeepFM has been found to have issues with generalization and scalability, and it has relatively high complexity due to its consideration of all pairwise bit-level interactions. DCNV2 [34] similarly uses CIN to learn efficient explicit and implicit feature intersections, but it additionally leverages low-rank techniques to approximate feature crosses in subspace for better performance and latency trade-offs.

More recently, some studies have attempted to use attention mechanisms to model feature interactions in a more interpretable way. HoAFM [35] updates feature representations by attentively aggregating the representations of co-occurring features, while AutoInt [36] uses a multi-head self-attention

mechanism to explicitly model feature interactions. InterHAt [37] is another model that uses an attentional aggregation strategy with residual connections to learn feature representations and model feature interactions. However, even with the use of attention mechanisms to account for the weight of each pair of feature interactions, aggregating all interactions together can still introduce noise and degrade prediction accuracy. To address these issues, some recent studies have attempted to identify beneficial feature interactions automatically. AutoFIS [38] is a two-stage algorithm that uses a gate operation to search and model beneficial feature interactions, but there is a loss of information between the stages, and the modeling process is not interpretable. AFN [12] uses a logarithmic neural network to adaptively learn high-order feature interactions, and SIGN [9] utilizes mutual information to detect beneficial feature interactions and a linear aggregation strategy to model them. However, these approaches may not be expressive or interpretable enough.

## 2.2 Graph Neural Networks

The graph is a kind of data structure that reflects a set of entities (nodes) and their relations (edges). Graph neural networks (GNNs), as deep learning architectures on graph-structured data, have attracted increasing attention. The concept of GNNs is first proposed by [39], and further elaborated in [40]. Nowadays, most of the prevailing GNN models follow the neighborhood aggregation strategy, that is, to learn the latent node representations via aggregating the features of neighborhoods layer by layer. The high-order relations between nodes can be modeled explicitly by stacking layers. Gated Graph Neural Networks (GGNN) [22] uses GRU [41] to update the node representations based on the aggregated neighborhood feature information. Though based on graph spectral theory [42], the learning process of graph convolutional networks (GCN) [13] also can be considered as a mean-pooling neighborhood aggregation. GraphSAGE [14] concatenates the node features and introduces three mean/max/LSTM aggregators to pool the neighborhood information. Graph attention network (GAT) [15] incorporates the attention mechanism to measure the weights of neighbors when aggregating neighborhood information of a node.

Due to the strength in modeling relations on graph-structured data, GNN has been widely applied to various applications like neural machine translation [43], semantic segmentation [44], image classification [45], situation recognition [16], recommendation [19, 20, 46], script event prediction [47], and fashion analysis [48]. Fi-GNN [21] is the first attempt to exploit GNN for feature interaction modeling. It first proposes to connect all the feature fields, and thus the multi-field features can be treated as a fully-connected graph. Then it utilizes GGNN [22] to model high-order feature interactions on the feature graph. KD-DAGFM [25] uses knowledge distillation and proposes a lightweight student model, directed acyclic graph FM, to learn arbitrary explicit high-order feature interactions from teacher networks. Other graph-based work, like GFM [23], utilizes the popular Factorization Machine to effectively aggregate multi-order

interactions in GNN. And GCFM [24] uses the multifilter graph-convolved feature crossing (GCFC) layer to learn the neighbor feature interactions.

Nevertheless, GNN was originally designed for graph classification tasks, which is based on the assumption that neighbors share similar features. As a result, GNN inherits unnecessary and unsuitable operations for feature interaction modeling. Our proposed model GraphFM introduces the interaction function of FM into the neighborhood aggregation strategy of GNN to effectively capture the beneficial factorized interaction.

### 3 Preliminaries

In this section, we first introduce the background of feature embeddings, which is fundamental for most feature interaction models based on deep learning. To help understand our proposed model GraphFM, which are based on FMs and GNNs, we then describe these two lines of work.

#### 3.1 Feature Embeddings

In many real-world predictive tasks, such as CTR prediction, input instances consist of both sparse categorical and numerical features. By tradition, we represent each input instance as a sparse vector:

$$\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n], \quad (1)$$

where  $n$  is the number of *feature fields* and  $\mathbf{x}_i$  is the representation of the  $i$ -th feature field (aka *feature*). Since the categorical features are very sparse and high-dimensional, a common way is to map them into a low-dimensional latent space. Specifically, a categorical feature  $\mathbf{x}_i$  is mapped to dense embedding  $\mathbf{e}_i \in \mathbb{R}^d$  as:

$$\mathbf{e}_i = \mathbf{V}_i \mathbf{x}_i, \quad (2)$$

where  $\mathbf{V}_i$  denotes the embedding matrix of field  $i$ .

For a numerical feature  $\mathbf{x}_j$  which is a scalar  $x_j$ , we also represent it in the  $d$ -dimensional embedding space:

$$\mathbf{e}_j = \mathbf{v}_j x_j \quad (3)$$

where  $\mathbf{v}_j$  is the embedding vector for the numerical field  $j$ . Therefore, we can obtain a feature embedding matrix consisting of these feature embeddings:

$$\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_n]^{\top}. \quad (4)$$

#### 3.2 Factorization Machines

Factorization machine (FM) is originally proposed for collaborative recommendation [3, 4]. It estimates the target by modeling all interactions between each

pair of features:

$$\hat{y}_{\text{FM}} = \langle \mathbf{w}, \mathbf{x} \rangle + \sum_{i_2 > i_1}^n \langle \mathbf{e}_{i_1}, \mathbf{e}_{i_2} \rangle, \quad (5)$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product operation. Intuitively, the first term  $\langle \mathbf{w}, \mathbf{x} \rangle$  is the linear regression of raw features, and the second term is the sum of all pairwise interactions, i.e., inner products of feature embeddings.

In principle, FMs can be extended to higher-order feature combinations [3, 4]. Let  $k \in \{2, \dots, K\}$  denote the order or degree of feature interactions considered and  $\mathbf{e}_i^{(k)}$  denotes the embedding of feature  $i$  for order  $k$ , the  $K$ -order higher-order FM (HOFM) can be defined as

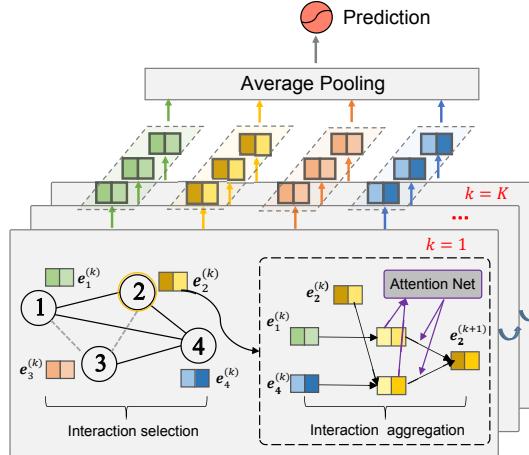
$$\begin{aligned} \hat{y}_{\text{HOFM}} = & \langle \mathbf{w}, \mathbf{x} \rangle + \sum_{i_2 > i_1}^n \langle \mathbf{e}_{i_1}^{(2)}, \mathbf{e}_{i_2}^{(2)} \rangle + \sum_{i_3 > i_2 > i_1}^n \langle \mathbf{e}_{i_1}^{(3)}, \mathbf{e}_{i_2}^{(3)}, \mathbf{e}_{i_3}^{(3)} \rangle \\ & + \dots + \sum_{i_K > \dots > i_1}^n \langle \mathbf{e}_{i_1}^{(K)}, \dots, \mathbf{e}_{i_K}^{(K)} \rangle, \end{aligned} \quad (6)$$

where  $\langle \mathbf{e}_{i_1}^{(K)}, \dots, \mathbf{e}_{i_K}^{(K)} \rangle = \text{sum}(\mathbf{e}_{i_1}^{(K)} \odot \dots \odot \mathbf{e}_{i_K}^{(K)})$  (sum of element-wise products). Since all the feature interactions of order up to  $K$  are included, its time complexity increases exponentially with, resulting in high computational complexity. Considering that not all feature interactions are beneficial, FMs have trouble modeling higher-order feature interactions in terms of both efficiency and effectiveness.

Although several recent researches have enhanced FMs with DNNs to model higher-order feature interactions, like NFM [2] and DeepFM [7], they model the higher-order feature interaction in an implicit manner, which lack persuasive rationales on the model outcome.

### 3.3 Graph Neural Networks

Given a graph  $G = \{V, E\}$  denote a graph, GNNs learn the representation vectors of nodes by exploring their correlations with neighboring nodes. Modern GNNs follow a neighborhood aggregation strategy, where we iteratively update the representation of a node by aggregating the feature of its neighbors [49]. After  $k$  iterations of aggregation, a node's representation encodes its interaction with neighbors within  $k$  hops. The choice of aggregation strategy in GNNs is crucial. A number of architectures have been proposed. In this work, we adopt the attentional aggregation strategy in [15], which will be further elaborated in Section 4.



**Fig. 1** The overview of GraphFM. The input features are modeled as a graph, where nodes are feature fields, and edges are interactions. At each layer of GraphFM, the edges (beneficial interactions) are first selected by the *interaction selection* component. Then these selected feature interactions are aggregated via the attention net to update feature embeddings in the *interaction aggregation* component. The learned feature embeddings at every layer are used for the final prediction jointly.

## 4 Graph Factorization Machine

### 4.1 Model Overview

The overview of GraphFM is shown in Fig. 1. Each input instance (multi-field feature) is represented as a graph where the nodes are feature fields, and the edges are interactions [9, 21]. Note that we will use nodes and features, edges, and interactions interchangeably in this paper. GraphFM updates feature representations layer by layer. The feature embeddings described in Section 3.1 are taken as the initial feature embeddings of GraphFM, i.e.,  $\mathbf{e}_i^{(1)} = \mathbf{e}_i$ , where  $\mathbf{e}_i^{(k)}$  stands for the updated feature embeddings at  $k$ -th layer. Since no edge information is given, we need to select the edges (beneficial interactions) by the interaction selection component first. Then we aggregate these selected feature interactions to update feature embeddings in the neighborhood aggregation component. Within each  $k$ -th layer, we are able to select and model only the beneficial  $k$ -th order feature interactions and encode these factorized interactions into feature representations. Finally, these learned feature embeddings encoded interactions of order up to  $K$  are concatenated to make the final prediction.

There are mainly two components in each layer of GraphFM. Next, we will introduce them in detail. As we focus on describing the detailed mechanism at every single layer, we omit the layer index  $k$  if not necessary.

## 4.2 Interaction Selection

To select the beneficial pairwise feature interactions, we devised the interaction selection mechanism. This can also be viewed as inferring the graph structure, which is to predict the links between nodes. However, the graph structure  $G = \{V, E\}$  is discrete, where an edge  $(v_i, v_j) \in E$  linking two nodes is either present or absent. This makes the process non-differentiable, therefore it cannot be directly optimized with gradient-descent-based optimization techniques.

To overcome this limitation, we replace the edge set  $E$  with weighted adjacency  $\mathbf{P}$ , where  $p_{ij}$  is interpreted as the probability of  $(v_i, v_j) \in E$ , which also reflects how beneficial their interaction is. It should be noted that we learn different graph structures  $\mathbf{P}^{(k)}$  at each  $k$ -th layer. Compared with using a fixed graph at each layer, we have more efficiency and flexibility in enumerating the beneficial higher-order feature interaction by this means. More specifically, by using a fixed graph structure at each layer, we can only obtain a fixed set of feature interactions. However, with the adaptive learned graph structure at each layer, the model is capable of modeling any potential feature interactions.

### 4.2.1 Metric function

We aim to design a metric function between each pair of feature interactions to measure whether they are beneficial. Formally, the weight  $p_{ij}$  of an edge  $(v_i, v_j)$  is computed via a metric function  $f_s(\mathbf{e}_i, \mathbf{e}_j)$ . We here adopt a Neural Matrix Factorization (NMF) [1] based function to estimate the edge weight. Formally, a Multi-Layer Perception (MLP) with one hidden layer is used to transform the element-wise product of these feature vectors to a scalar:

$$f_s(\mathbf{e}_i, \mathbf{e}_j) = \sigma(\mathbf{W}_2^s \delta(\mathbf{W}_1^s (\mathbf{e}_i \odot \mathbf{e}_j) + \mathbf{b}_1^s) + \mathbf{b}_2^s), \quad (7)$$

where  $\mathbf{W}_1^s$ ,  $\mathbf{W}_2^s$ ,  $\mathbf{b}_1^s$ ,  $\mathbf{b}_2^s$  are the parameters of MLP.  $\delta(\cdot)$  and  $\sigma(\cdot)$  are ReLU and sigmoid activation functions, respectively. It should be noted that  $f_s$  is invariant to the order of its input, i.e.,  $f_s(\mathbf{e}_i, \mathbf{e}_j) = f_s(\mathbf{e}_j, \mathbf{e}_i)$ . Therefore, the estimated edge weights are identical to the same pair of nodes. Such continuous modeling of graph structure enables backpropagation of the gradients. Since we do not have a ground-truth graph structure, the gradients come from the errors between the model output with the target.

Intuitively, we treat the element-wise product of each pair of feature embeddings as a term and estimate its weight using MLP. One can also choose the Euclidean distance [50] or other distance metrics.

### 4.2.2 Graph sampling

From the estimated edge weighted matrix  $\mathbf{P}^{(k)}$  at each layer, we then sample the beneficial feature interactions, which is also to sample the neighborhood for each feature field. In this work, we uniformly sample a fixed-size set of neighbors. For each feature node  $v_i$  at  $k$ -th layer, we select  $m_k$  edges according

to the first  $m_k$  elements of  $\mathbf{P}^{(k)}[i, :]$ , which can be illustrated as follows:

$$\begin{aligned} & \text{for } i = 1, 2, \dots, n \\ & \quad \text{idx}_i = \text{argtop}_{m_k}(\mathbf{P}^{(k)}[i, :]) \\ & \quad \mathbf{P}^{(k)}[i, -\text{idx}] = 0, \end{aligned} \tag{8}$$

where  $\mathbf{P}^{(k)}[i, :]$  denotes the  $i$ -th column of matrix  $\mathbf{P}^{(k)}$  at  $k$ -th layer,  $\mathbf{P}^{(k)}[i, -\text{idx}_i]$  contains a subset of columns of  $\mathbf{P}^{(k)}$  that are not indexed by  $\text{idx}_i$ .  $\text{argtop}_{m_k}$  is an operator that performs the  $m_k$ -most important nodes selection for the query node  $i$  to attend. We only keep these  $m_k$  feature nodes, and the others are masked. Thus the neighborhood set of node  $v_j$  is defined as:

$$\mathcal{N}_i^{(k)} = \left\{ v_j \mid p_{ij}^{(k)} > 0, j = 1, 2, \dots, n \right\}. \tag{9}$$

Practically speaking, we found that our approach could achieve high performance when  $k = 3$ ,  $m_1$  equals the number of feature fields, which means that in the first layer, we model all pairs of feature interactions.

It is worth mentioning that we have also tried to set a threshold to select the edges in the graph, i.e., setting a minimum value for the edge probability of cutting edges off. But the performance is not as good as using a fixed-degree graph. This is reasonable as the edge weights of different nodes' neighbors are at different scales. Setting a single threshold on all the nodes will lead to the situation that the numbers of nodes' neighbors vary a lot. Some nodes will have barely any adjacent nodes after cutting off, while some may still have many.

### 4.3 Interaction Aggregation

Since we have selected the beneficial feature interactions, or in other words, learned the graph structure, we perform the interaction (neighborhood) aggregation operation to update feature representations.

For a target feature node  $v_i$ , when aggregating its beneficial interactions with neighbors, we also measure the attention coefficients of each interaction. To measure the attention coefficients, we use a learnable projection vector  $\mathbf{a}$  and apply a LeakyReLU non-linear activation function. Formally, the coefficients are computed as:

$$c_{ij} = \text{LeakyReLU}(\mathbf{a}^\top (\mathbf{e}_i \odot \mathbf{e}_j)). \tag{10}$$

This indicates the importance of the interactions between feature  $v_i$  and features  $v_j$ .

Note that we only compute  $c_{ij}$  for nodes  $j \in \mathcal{N}_i$ , where  $\mathcal{N}_i$  denotes the neighborhood of node  $v_i$ , which is also the set of features whose interaction with  $v_i$  are beneficial. To make coefficients easily comparable across different feature nodes, we normalized them across all choices of  $j$  using a softmax

function:

$$\alpha_{ij} = \frac{\exp(c_{ij})}{\sum_{j' \in \mathcal{N}_i} \exp(c_{ij'})}. \quad (11)$$

Once obtained the normalized attention coefficients, we compute the linear combination of these feature interactions with a nonlinearity as the updated feature representations:

$$\mathbf{e}'_i = \sigma \left( \sum_{j \in \mathcal{N}_i} \alpha_{ij} p_{ij} \mathbf{W}_a (\mathbf{e}_i \odot \mathbf{e}_j) \right), \quad (12)$$

where  $\alpha_{ij}$  measures the attention coefficients of each feature interaction between feature  $i$  and  $j$ , while  $p_{ij}$  stands for the probability of this feature interaction being beneficial. The attention coefficient  $\alpha_{ij}$  is calculated by the soft attention mechanism, while the  $p_{ij}$  is calculated by the hard attention mechanism. By multiplying them together, we control the information of selected feature interactions and make the parameters in the interaction selection component trainable with gradient back-propagation.

To capture the diversified polysemy of feature interactions in different semantic subspaces [37] and also stabilize the learning process [15, 28], we extend our mechanism to employ multi-head attention. Specifically,  $H$  independent attention mechanisms execute the update of Equation 12, and then these features are concatenated, resulting in the following output feature representation:

$$\mathbf{e}'_i = \parallel_{h=1}^H \sigma \left( \sum_{j \in \mathcal{N}_i} \alpha_{ij}^h p_{ij} \mathbf{W}_a^h (\mathbf{e}_i \odot \mathbf{e}_j) \right), \quad (13)$$

where  $\parallel$  denotes concatenation,  $\alpha_{ij}^h$  is the normalized attention coefficient computed by the  $h$ -th attention mechanism, and  $\mathbf{W}_a^h$  is the corresponding linear transformation matrix. One can also choose to employ average pooling to update the feature representations:

$$\mathbf{e}'_i = \sigma \left( \frac{1}{H} \sum_{h=1}^H \sum_{j \in \mathcal{N}_i} \alpha_{ij}^h p_{ij} \mathbf{W}_a^h (\mathbf{e}_i \odot \mathbf{e}_j) \right). \quad (14)$$

#### 4.4 Prediction and Optimization

The output of each  $k$ -th layer, is a set of  $n$  feature representation vectors encoded feature interactions of order up to  $k$ , namely  $\{\mathbf{e}_1^{(k)}, \mathbf{e}_2^{(k)}, \dots, \mathbf{e}_n^{(k)}\}$ . Since the representations obtained in different layers encode the interactions of different orders, they have different contributions to the final prediction. As such, we concatenate them to constitute the final representation of each

**Table 1** Statistics of evaluation datasets.

Dataset	#Instances	#Fields	#Features (sparse)
Criteo	45,840,617	39	998,960
Avazu	40,428,967	23	1,544,488
MovieLens-1M	739,012	7	3,529

feature [19]:

$$\mathbf{e}_i^* = \mathbf{e}_i^{(1)} \parallel \cdots \parallel \mathbf{e}_i^{(K)}, \quad (15)$$

At last, We employ average pooling on the vectors of all features to obtain a graph-level output and use a projection vector  $\mathbf{p}$  to make the final prediction:

$$\mathbf{e}^* = \frac{1}{n} \sum_{i=1}^n \mathbf{e}_i^*, \quad (16)$$

$$\hat{y} = \mathbf{p}^\top \mathbf{e}^*. \quad (17)$$

GraphFM can be applied to various prediction tasks, including regression, classification, and ranking. In this work, we conduct experiments on CTR prediction, a binary classification task. We thus use log loss as the loss function:

$$\mathcal{L} = -\frac{1}{N} \sum_{i=1}^N y_i \log \sigma(\hat{y}_i) + (1 - y_i) \log(1 - \sigma(\hat{y}_i)), \quad (18)$$

where  $N$  is the total number of training instances, and  $\sigma$  denotes the sigmoid function.  $y_i$  and  $\hat{y}_i$  denote the label of instance  $i$  and the prediction of GraphFM for it, respectively. The model parameters are updated using Adam[51].

## 5 Experiments

This section presents an empirical investigation of the performance of GraphFM on two CTR benchmark datasets and a recommender system dataset. The experimental settings are described, followed by comparisons with other state-of-the-art methods. An ablation study is also conducted to verify the importance of each component of the model and evaluate its performance under different hyperparameter settings. Finally, the question of whether GraphFM can provide interpretable explanations for its predictions is examined.

### 5.1 Experimental Settings

Our experiments are conducted on three real-world datasets, two CTR benchmark datasets, and one recommender system dataset. Details of these datasets are illustrated in Table 1. The data preparation follows the strategy in [25]. We randomly split all the instances in 8:1:1 for training, validation, and testing. We adopt the two most popular metrics, **AUC** and **Logloss** to measure the probability that one prediction diverges from the ground truth.

### 5.1.1 Baselines

We compare GraphFM with four classes of state-of-the-art methods: (A) the linear approach that only uses individual features; (B) FM-based methods that consider second-order feature interactions; (C) DNN-based methods that model high-order feature interactions; (D) aggregation-based methods that update features' representation and model their high-order feature interactions via aggregation strategy.

The models associated with their respective classes are listed as follows:

- **LR** (A) refers to the linear/logistics regression, which can only model linear interactions.
- **FM** [4] (B) is the official implementation for FM, which can only model second-order interactions.
- **AFM** [10] (B) is an extension of FM, which considers the weights of different second-order feature interactions by using attention mechanisms.
- **AFN** [12] (B) learns arbitrary-order feature interactions adaptively from data, instead of explicitly modeling all the cross features within a fixed maximum order.
- **FmFM** [30] (B) uses a field matrix between two feature vectors to model their interactions and learns the matrix separately for each field pair.
- **NFM** [2] (C) devises a bi-Interaction layer to model second-order interactions and uses DNN to introduce non-linearity and model high-order interactions.
- **HOFM** (C) [52] is the implementation of the higher-order FM [52]. It is a linear model.
- **DeepCrosssing** [53] (C) utilizes DNN with residual connections to model non-linear feature interactions in an implicit manner.
- **CrossNet** [32] (C) is the core of Deep&Cross model, which models feature interactions explicitly by taking the outer product of concatenated feature vector at the bit-wise level.
- **xDeepFM** [33] (C) takes the outer product of the stacked feature matrix at a vector-wise level to model feature interactions explicitly. It can be also combined with DNN which models implicit and explicit interactions simultaneously.
- **DCNV2** [34] (C) utilizes cross network from DCN to learning explicit and bounded-degree cross features.
- **AutoInt** [36] (D) uses self-attentive network to learn high-order feature interactions explicitly. It can also be seen as performing the multi-head attention mechanism [28] on the fully-connected graph.
- **Fi-GNN** [21] (D) models the features as a fully-connected graph and utilizes a gated graph neural network to model feature interactions.
- **InterHAt** [37] (D) utilizes an attention mechanism to aggregate features, which will then be multiplied by the original features to produce the higher-order feature interactions.

**Table 2** Performance comparison of different methods on three datasets. The four model classes (A, B, C, D) are defined in Section 5.1.1. The last two columns are average improvements of our proposed model GraphFM compared with corresponding base models (“+”: increase, “-”: decrease). We highlight the best performances on each dataset. Further analysis is provided in Section 5.2.

Model	Criteo		Avazu		MovieLens-1M	
	AUC	LogLoss	AUC	LogLoss	AUC	LogLoss
LR	0.7820	0.4695	0.7560	0.3964	0.7716	0.4424
FM [3]	0.7836	0.4700	0.7706	0.3856	0.8252	0.3998
AFM[10]	0.7938	0.4584	0.7718	0.3854	0.8227	0.4048
AFN [12]	0.8079	0.4433	0.7786	0.3799	0.8771	0.4721
FmFM [30]	0.8083	0.4434	0.7746	0.3859	0.8821	0.3279
NFM [2]	0.7957	0.4562	0.7708	0.3864	0.8357	0.3883
HOFM [52]	0.8005	0.4508	0.7701	0.3854	0.8304	0.4013
DeepCrossing [53]	0.8009	0.4513	0.7643	0.3889	0.8448	0.3814
CrossNet [32]	0.7907	0.4591	0.7667	0.3868	0.7968	0.4266
xDeepFM [33]	0.8009	0.4517	0.7758	0.3829	0.8286	0.4108
DCNV2 [34]	0.8074	0.4436	0.7666	0.3865	0.8833	0.4885
AutoInt [36]	0.8084	0.4427	0.7781	0.3795	0.8823	0.3463
Fi-GNN [21]	0.8077	0.4413	0.7778	0.3811	0.8792	0.3537
InterHAt [37]	0.8076	0.4446	0.7758	0.3860	0.8769	0.3591
GraphFM (ours)	<b>0.8091</b>	<b>0.4399</b>	<b>0.7798</b>	<b>0.3781</b>	<b>0.8902</b>	<b>0.3259</b>

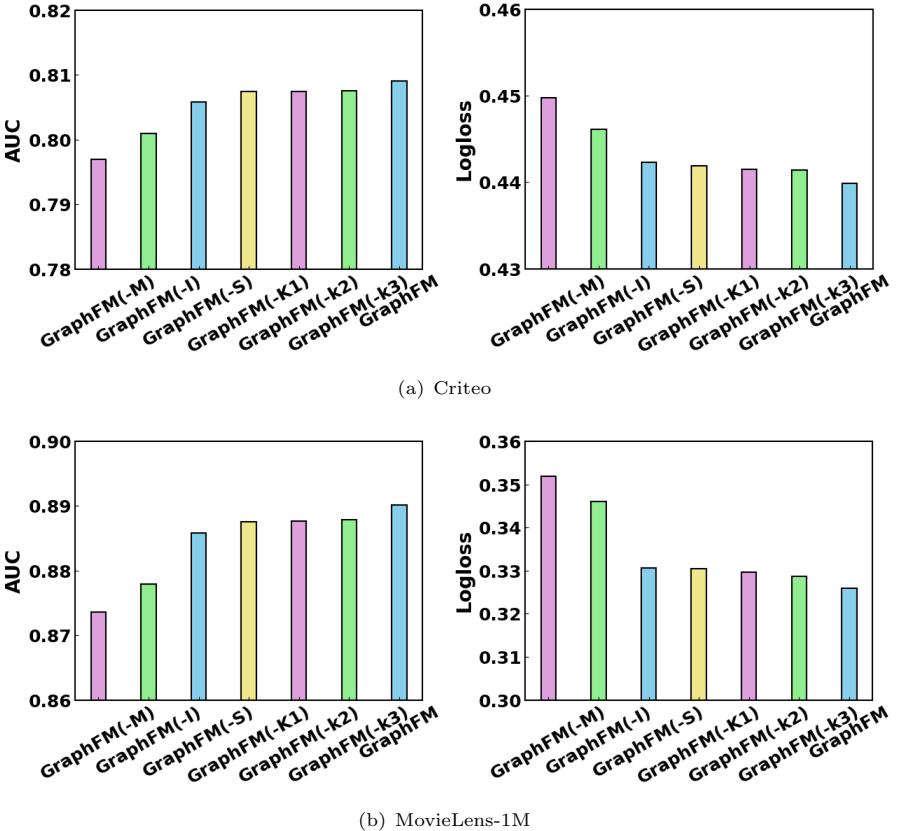
### 5.1.2 Implementation Details

We implement the method using Tensorflow [54] and Pytorch [55]. The feature embedding size is set as 16 for all methods. For a fair comparison, we set three layers for AutoInt, FiGNN, and also GraphFM. There are two attention heads in AutoInt and GraphFM. Implementation of the other compared baselines follows [36] and [25]. The optimal hyper-parameters are found by grid search.  $[m_1, m_2, m_3]$  are set as [39, 20, 5], [23, 10, 2], and [7, 4, 2] for Criteo, Avazu and MovieLens-1M dataset respectively. We use Adam [51] to optimize all these models. The experiments were conducted over a server equipped with 8 NVIDIA Titan X GPUs.

## 5.2 Model Comparison

### 5.2.1 Evaluation of Effectiveness

The performance comparison of these methods on three datasets is presented in Table 2, from which we have the following observations. Our proposed GraphFM achieves best performance among all these four classes of methods on three datasets. The performance improvement of GraphFM compared with the three classes of methods (A, B, C) is especially significant, above **0.01**-level.



**Fig. 2** The performance comparison of GraphFM with different components on Criteo and MovieLens-1M datasets. Further analysis is provided in Section 5.3.

The aggregation-based methods including InterHAt, AutoInt, Fi-GNN and our GraphFM consistently outperform the other three classes of models, which demonstrates the strength of the aggregation strategy in capturing high-order relations. Compared with the strong aggregation-based baselines AutoInt and Fi-GNN, GraphFM still advances the performance by a large margin, especially on MovieLens-1M dataset. The performance improvement on the other two datasets are also at **0.001**-level, which can be regarded as significant for CTR prediction task [6, 7, 36]. Such improvement can be attributed to its combination with FM, which introduces feature interactions operations, and also the interaction selection mechanism, which selects and models only the beneficial feature interactions. GraphFM outperforms the compared baselines by the largest margin on MovieLens-1M dataset, whose feature size is smallest among the three datasets. I suppose this is because the feature embedding size is not large enough for the other two datasets.

### 5.3 Ablation Studies

To validate the effectiveness of each component in GraphFM, we conduct ablation studies and compare several variants of it:

- **GraphFM(-S):** *interaction selection* is the first component in each layer of GraphFM, which selects only the beneficial feature interactions and treat them as edges. As a consequence, we can model only these beneficial interactions with the next *interaction aggregation* component. To check the necessity of this component, we remove this components, so that all pair of feature interactions are modeled as a fully-connected graph.
- **GraphFM(-I):** in the *interaction aggregation* component, we aggregate the feature interactions instead of the neighbors' features as in the standard GNNs. To check its rationality, we test the performance of directly aggregating neighborhood features instead of the feature interactions with them.
- **GraphFM(-M):** in the *interaction aggregation* component, we use a multi-head attention mechanism to learn the diversified polysemy of feature interactions in different semantic subspaces. To check its rationality, we use only one attention head when aggregating.
- **GraphFM(-K<sub>i</sub>):** before obtaining the final representation of the feature, we concatenate and average the feature representation  $e_k$  output from layer  $k$ . In order to study the degree of contribution of the features learned at different layers to the results, we use the feature representation of  $k$ -th layer for direct prediction, and there are  $K = 3$  layers.

The performances of GraphFM and these Four variants are shown in Fig. 2. We observe that GraphFM outperforms all the ablative methods, which proves the necessity of all these components in our model. The performance of GraphFM(-M) suffers from a sharp drop compared with GraphFM, proving that it is necessary to transform and aggregate the feature interactions in multiple semantic subspaces to accommodate the polysemy. Note that although we did not present the statistics here, we also tested the influence of number of attention heads  $H$ . The performance of using only one head, i.e., GraphFM(-M), is worse than that of using two, and more attention heads don not lead to improvement of performance but introduce much higher time and space complexity. GraphFM(-I) does not perform well either. This is reasonable, as without the interaction between features, the neighborhood aggregation operation will only make the neighboring features similar. As a consequence, no feature interactions are guaranteed to be captured. This interaction part is also the most significant difference between GraphFM and GNN, and the resulting gap in term of performance indicates that GraphFM is able to leverage the strength of FM to solve the drawbacks of GNN in modeling feature interactions. GraphFM(-S) achieves slightly worse performance than GraphFM, demonstrating that selecting and modeling only the beneficial interactions instead of all of them can avoid the noise and make it easier to train the model.

The GraphFM(- $K_i$ ) results show that each layer can learn features that benefit the results. Although different features can have a positive effect on the task prediction, the difference in effect is not large and the prediction results are worse than those obtained by combining all features, demonstrating the need to merge the features of the  $k$ -th layers.

## 5.4 Study of Neighborhood Sampled Size

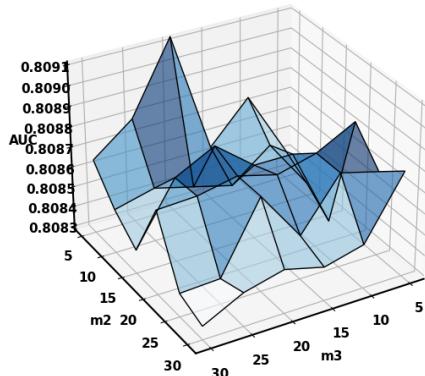
The number of selected neighbors for each feature node is an important hyper-parameter, which controls the number of features that each feature interacts with. We thus investigate how the neighborhood sampled size effects the model performance. As the total search space is too large, we here only show the performance of our model with  $K = 3, m_1 = n$  and varying value of  $m_2$  and  $m_3$ . The results on three datasets are summarized in Fig. 3.

On Criteo dataset, there are totally 39 feature fields. We found that our model achieves best performance with  $m_1 = 39, m_2 \times m_3 = 100$ . The performances vary in the range of [0.8084, 0.8091], which proves that our model is quite robust, and not very sensitive to the size of neighborhood sampled size. On Avazu dataset, the model performance peaks with  $m_1 = 23, m_2 = 10, m_3 = 2$  or  $m_2 = 15, m_3 = 4$ . On MovieLens-1M dataset, the model performance peaks when  $m_2 \times m_3$  at around 9. We also found a diminishing trend for sampling larger or smaller neighborhoods. In a word, the optimal neighborhood sampled sizes depend on the sizes of datasets.

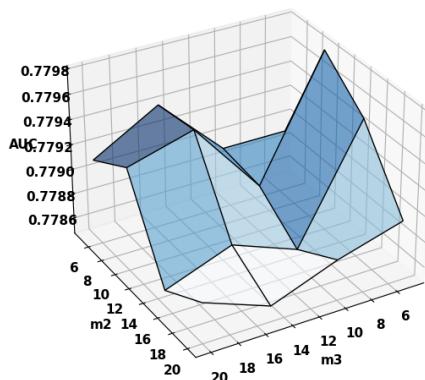
## 5.5 Visualization of Interaction Graph

Since the features along with selected beneficial feature interactions are treated as a graph, it can provide human readable interpretations on the prediction. Here we visualize heat maps of estimated edge weights of two cherry-pick instances on MovieLens-1M dataset in Fig. 4. We show the measured edge weights of each instance in the three layers, which selects the order-2, order-3, and order-4 feature interactions respectively. The positive edge weights indicate how beneficial feature interactions are. We set  $S_1 = 7, S_2 = 4, S_3 = 2$ , which means that we only keep 7, 4, and 2 pairs of beneficial order-2, order-3, and order-4 feature interactions respectively. Therefore, there are only 7, 4, and 2 interacted feature fields for each feature fields each row for heat maps of order-2, order-3, and order-4. The axes represent feature fields (*Gender*, *Age*, *Occupation*, *Zipcode*, *ReleaseTime*, *WatchTime*, *Genre*). *Gender*, *Age*, *Occupation* and *Zipcode* are users' demographic information. Note *Zipcode* indicates the users' place of residence. *textitReleaseTime* and *Gender* are the movie information. *WatchTime* (*Timestamp*) represents the time when users watched the movies.

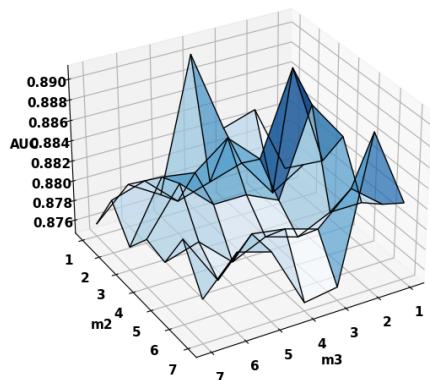
From the two instances in Fig. 4, we can obtain some following interesting observations. We find that in the first layer, which models the second order feature interactions, these feature fields are hard to distinguish when selecting



(a) Criteo

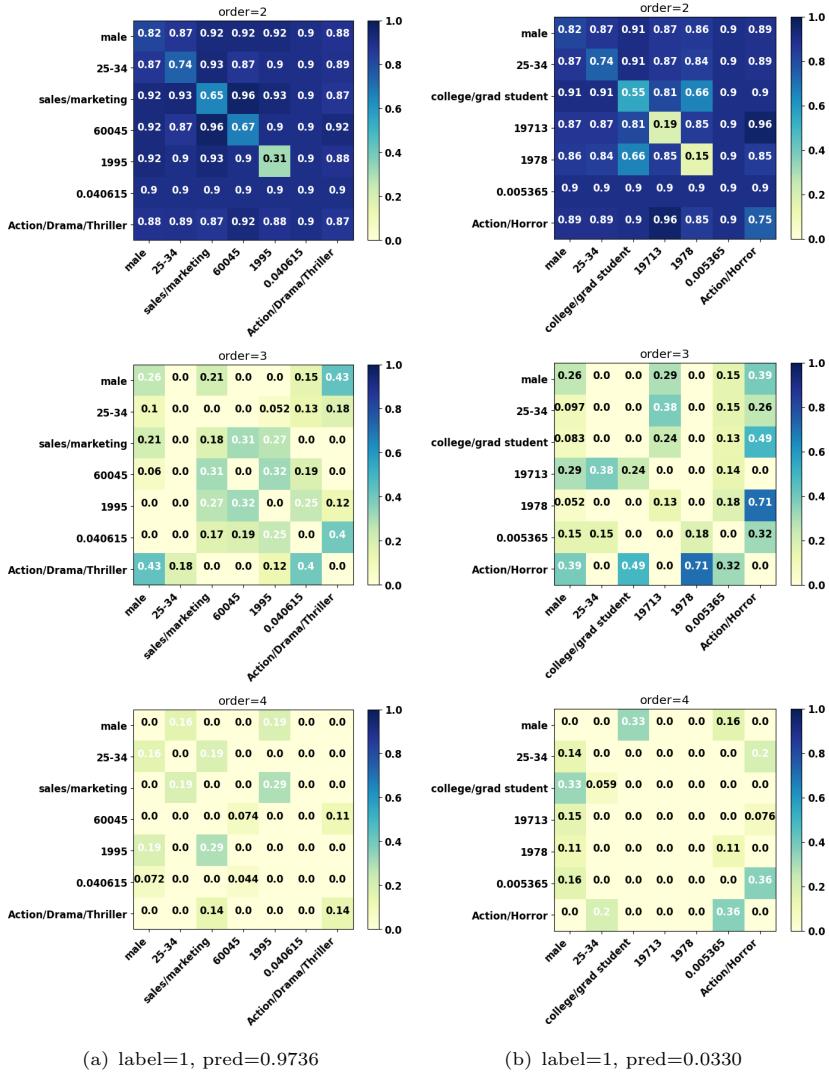


(b) Avazu



(c) MovieLens

**Fig. 3** Model performance with respect to the size of the sampled neighborhood, where the “neighborhood sample size” refers to the number of neighbors sampled at each depth for  $K = 3$  with  $m_1 = n$ , and  $m_2, m_3$  with varying values.



**Fig. 4** Heat maps of estimated edge weights of correctly predicted instance (a) and wrongly predicted instance (b) on MovieLens-1M dataset, where positive edge weights indicate beneficial feature interactions. The axes represent feature fields (*Gender*, *Age*, *Occupation*, *Zipcode*, *ReleaseTime*, *WatchTime*, *Genre*).

the beneficial interactions. This suggests that almost all the second-order feature interactions are useful, which also why we sample all of them in the first layer, i.e.,  $m_1 = n$ . Except that the diagonal elements are with the smallest values, which suggests that our designed interaction selection mechanism can classify the redundant self-interacted feature interactions, even though we keep and model all pairs of feature interactions. The selected feature interactions of order-3 and order-4 are mostly not overlapped in the correctly predicted

instance (a). In instance (a), our model selects relevant feature fields (*Gender*, *Age*, *ReleaseTime*, *WatchTime*) for *Genre* in order-3, while selects the other two feature fields (*Occupation*, *Gender*) in order-4. However, in the wrongly predicted instances (b), the feature interactions of order-3 and order-4 are mostly not overlapped.

This proves that our model can indeed select meaningful feature combination and model feature interactions of increasing orders with multiple layers in most cases, rather than select the redundant feature combinations of same feature fields. We can also find some meaningful feature combinations in common cases. For examples, *Gender* are usually relevant with feature fields *Age*, *occupation*, and *WatchTime*, while *Age* are usually relevant with feature fields *Gender*, *WatchTime*, and *Genre*. This provides some rationales for the model prediction.

## 6 Conclusion and Future Work

In this work, we disclose the relationship between FM and GNN, and seamlessly combine them to propose a novel model GraphFM for feature interaction learning. The proposed model leverages the strength of FM and GNN and also solve their respective drawbacks. At each layer of GraphFM, we select the beneficial feature interactions and treat them as edges in a graph. Then we utilize a neighborhood/interaction aggregation operation to encode the interactions into feature representations. By design, the highest order of feature interaction increases at each layer and is determined by layer depth, and thus the feature interactions of order up to the highest can be learned. GraphFM models high-order feature interactions in an explicit manner, and can generate human readable explanations on model outcome. Experimental results show that GraphFM outperforms the state-of-the-art baselines by a large margin. In addition, we conduct extensive experiments to analyze how the highest order of feature interactions and number of modeled feature interactions influence the model performance, which can help us gain deeper insight into feature interaction modeling. In the future, we aim to investigate whether the proposed method can also benefit graph representation learning, and graph/node classification tasks.

**Acknowledgements.** We would like to thank the anonymous reviewers for their valuable comments and suggestions allowing to improve the quality of this paper. This work is sponsored by National Science Foundation of China (62141608).

## References

- [1] He, X., Liao, L., Zhang, H., Nie, L., Hu, X., Chua, T.-S.: Neural collaborative filtering. In: WWW (2017). International World Wide Web Conferences Steering Committee

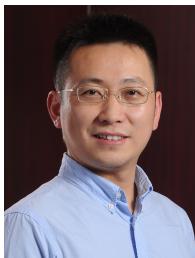
- [2] He, X., Chua, T.-S.: Neural factorization machines for sparse predictive analytics. In: SIGIR (2017)
- [3] Rendle, S.: Factorization machines. In: ICDM (2010)
- [4] Rendle, S.: Factorization machines with libfm. ACM TIST (2012)
- [5] Zhang, W., Du, T., Wang, J.: Deep learning over multi-field categorical data: A case study on user response prediction. arXiv preprint arXiv:1601.02376 (2016)
- [6] Cheng, H.-T., Koc, L., Harmsen, J., Shaked, T., Chandra, T., Aradhye, H., Anderson, G., Corrado, G., Chai, W., Ispir, M., *et al.*: Wide & deep learning for recommender systems. In: Proceedings of the 1st Workshop on Deep Learning for Recommender Systems (2016). ACM
- [7] Guo, H., Tang, R., Ye, Y., Li, Z., He, X.: Deepfm: a factorization-machine based neural network for ctr prediction. In: IJCAI (2017)
- [8] Zhang, C., Bengio, S., Hardt, M., Recht, B., Vinyals, O.: Understanding deep learning requires rethinking generalization. arXiv preprint arXiv:1611.03530 (2016)
- [9] Su, Y., Zhang, R., Erfani, S., Xu, Z.: Detecting beneficial feature interactions for recommender systems via graph neural networks. arXiv e-prints, 2008 (2020)
- [10] Xiao, J., Ye, H., He, X., Zhang, H., Wu, F., Chua, T.-S.: Attentional factorization machines: Learning the weight of feature interactions via attention networks. arXiv preprint arXiv:1708.04617 (2017)
- [11] Bahdanau, D., Cho, K., Bengio, Y.: Neural machine translation by jointly learning to align and translate. arXiv preprint arXiv:1409.0473 (2014)
- [12] Cheng, W., Shen, Y., Huang, L.: Adaptive factorization network: Learning adaptive-order feature interactions. In: Proceedings of the AAAI Conference on Artificial Intelligence, vol. 34, pp. 3609–3616 (2020)
- [13] Kipf, T.N., Welling, M.: Semi-supervised classification with graph convolutional networks. In: ICLR (2017)
- [14] Hamilton, W., Ying, Z., Leskovec, J.: Inductive representation learning on large graphs. In: NIPS (2017)
- [15] Veličković, P., Cucurull, G., Casanova, A., Romero, A., Lio, P., Bengio, Y.: Graph attention networks. In: ICLR (2018)
- [16] Li, R., Tapaswi, M., Liao, R., Jia, J., Urtasun, R., Fidler, S.: Situation

- recognition with graph neural networks. In: ICCV (2017)
- [17] Marcheggiani, D., Titov, I.: Encoding sentences with graph convolutional networks for semantic role labeling. arXiv preprint arXiv:1703.04826 (2017)
  - [18] Yao, L., Mao, C., Luo, Y.: Graph convolutional networks for text classification. In: AAAI (2019)
  - [19] Wang, X., He, X., Wang, M., Feng, F., Chua, T.-S.: Neural graph collaborative filtering. In: SIGIR (2019)
  - [20] Wu, S., Tang, Y., Zhu, Y., Wang, L., Xie, X., Tan, T.: Session-based recommendation with graph neural networks. In: AAAI (2019)
  - [21] Li, Z., Cui, Z., Wu, S., Zhang, X., Wang, L.: Fi-gnn: Modeling feature interactions via graph neural networks for ctr prediction. In: CIKM (2019). ACM
  - [22] Li, Y., Tarlow, D., Brockschmidt, M., Zemel, R.: Gated graph sequence neural networks. arXiv preprint arXiv:1511.05493 (2015)
  - [23] Xi, D., Zhuang, F., Zhu, Y., Zhao, P., Zhang, X., He, Q.: Graph factorization machines for cross-domain recommendation. arXiv preprint arXiv:2007.05911 (2020)
  - [24] Zheng, Y., Wei, P., Chen, Z., Cao, Y., Lin, L.: Graph-convolved factorization machines for personalized recommendation. IEEE Transactions on Knowledge and Data Engineering (2021)
  - [25] Tian, Z., Bai, T., Zhang, Z., Xu, Z., Lin, K., Wen, J.-R., Zhao, W.X.: Directed acyclic graph factorization machines for ctr prediction via knowledge distillation. In: Proceedings of the Sixteenth ACM International Conference on Web Search and Data Mining, pp. 715–723 (2023)
  - [26] Dudzik, A.J., Veličković, P.: Graph neural networks are dynamic programmers. Advances in Neural Information Processing Systems **35**, 20635–20647 (2022)
  - [27] Hinton, G., Vinyals, O., Dean, J.: Distilling the knowledge in a neural network. arXiv preprint arXiv:1503.02531 (2015)
  - [28] Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L., Polosukhin, I.: Attention is all you need. In: Advances in Neural Information Processing Systems, pp. 5998–6008 (2017)
  - [29] Juan, Y., Zhuang, Y., Chin, W.-S., Lin, C.-J.: Field-aware factorization machines for ctr prediction. In: RecSys (2016). ACM

- [30] Sun, Y., Pan, J., Zhang, A., Flores, A.: Fm2: Field-matrixed factorization machines for recommender systems. In: Proceedings of the Web Conference 2021, pp. 2828–2837 (2021)
- [31] Qu, Y., Cai, H., Ren, K., Zhang, W., Yu, Y., Wen, Y., Wang, J.: Product-based neural networks for user response prediction. In: ICDM (2016)
- [32] Wang, R., Fu, B., Fu, G., Wang, M.: Deep & cross network for ad click predictions. In: ADKDD (2017)
- [33] Lian, J., Zhou, X., Zhang, F., Chen, Z., Xie, X., Sun, G.: Xdeepfm: Combining explicit and implicit feature interactions for recommender systems. In: SIGKDD (2018)
- [34] Wang, R., Shivanna, R., Cheng, D., Jain, S., Lin, D., Hong, L., Chi, E.: Dcn v2: Improved deep & cross network and practical lessons for web-scale learning to rank systems. In: Proceedings of the Web Conference 2021, pp. 1785–1797 (2021)
- [35] Tao, Z., Wang, X., He, X., Huang, X., Chua, T.-S.: Hoafm: A high-order attentive factorization machine for ctr prediction. Information Processing & Management, 102076 (2019)
- [36] Song, W., Shi, C., Xiao, Z., Duan, Z., Xu, Y., Zhang, M., Tang, J.: Autoint: Automatic feature interaction learning via self-attentive neural networks. In: CIKM (2019)
- [37] Li, Z., Cheng, W., Chen, Y., Chen, H., Wang, W.: Interpretable click-through rate prediction through hierarchical attention. In: Proceedings of the 13th International Conference on Web Search and Data Mining, pp. 313–321 (2020)
- [38] Liu, B., Zhu, C., Li, G., Zhang, W., Lai, J., Tang, R., He, X., Li, Z., Yu, Y.: Autofis: Automatic feature interaction selection in factorization models for click-through rate prediction. In: Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pp. 2636–2645 (2020)
- [39] Gori, M., Monfardini, G., Scarselli, F.: A new model for learning in graph domains. In: IJCNN (2005)
- [40] Scarselli, F., Gori, M., Tsoi, A.C., Hagenbuchner, M., Monfardini, G.: The graph neural network model. IEEE Transactions on Neural Networks (2009)
- [41] Cho, K., Van Merriënboer, B., Gulcehre, C., Bahdanau, D., Bougares, F., Schwenk, H., Bengio, Y.: Learning phrase representations using

- rnn encoder-decoder for statistical machine translation. arXiv preprint arXiv:1406.1078 (2014)
- [42] Bruna, J., Zaremba, W., Szlam, A., LeCun, Y.: Spectral networks and locally connected networks on graphs. arXiv preprint arXiv:1312.6203 (2013)
- [43] Beck, D., Haffari, G., Cohn, T.: Graph-to-sequence learning using gated graph neural networks. arXiv preprint arXiv:1806.09835 (2018)
- [44] Qi, X., Liao, R., Jia, J., Fidler, S., Urtasun, R.: 3d graph neural networks for rgbd semantic segmentation. In: ICCV (2017)
- [45] Marino, K., Salakhutdinov, R., Gupta, A.: The more you know: Using knowledge graphs for image classification. In: CVPR (2017)
- [46] Chen, L., Wu, L., Hong, R., Zhang, K., Wang, M.: Revisiting graph based collaborative filtering: A linear residual graph convolutional network approach. arXiv preprint arXiv:2001.10167 (2020)
- [47] Li, Z., Ding, X., Liu, T.: Constructing narrative event evolutionary graph for script event prediction. arXiv preprint arXiv:1805.05081 (2018)
- [48] Cui, Z., Li, Z., Wu, S., Zhang, X.-Y., Wang, L.: Dressing as a whole: Outfit compatibility learning based on node-wise graph neural networks. In: WWW, pp. 307–317 (2019)
- [49] Xu, K., Hu, W., Leskovec, J., Jegelka, S.: How powerful are graph neural networks? arXiv preprint arXiv:1810.00826 (2018)
- [50] Kazi, A., Cosmo, L., Navab, N., Bronstein, M.: Differentiable graph module (dgm) graph convolutional networks. arXiv preprint arXiv:2002.04999 (2020)
- [51] Kingma, D., Ba, J.: Adam: A method for stochastic optimization. Computer Science (2014)
- [52] Blondel, M., Fujino, A., Ueda, N., Ishihata, M.: Higher-order factorization machines. In: NIPS (2016)
- [53] Shan, Y., Hoens, T.R., Jiao, J., Wang, H., Yu, D., Mao, J.: Deep crossing: Web-scale modeling without manually crafted combinatorial features. In: SIGKDD (2016)
- [54] Abadi, M., Barham, P., Chen, J., Chen, Z., Davis, A., Dean, J., Devin, M., Ghemawat, S., Irving, G., Isard, M., et al.: Tensorflow: A system for large-scale machine learning. In: OSDI (2016)

- [55] Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., et al.: Pytorch: An imperative style, high-performance deep learning library. *Advances in neural information processing systems* **32** (2019)



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