5. Logarithm

FORMULAS

1. $\log_a (m*n) = \log_a m + \log_a n$
2. $\log_a (m/n) = \log_a m + \log_a n$
3. $\log_a m^n = n\log_a m$
4. $\log_a a = 1$
5. $\log_a 1 = 0$; $\log_a 10 = 1$; $\log_a 100 = 2$
6. Change of base: $\log_b m * \log_a b = \log_a m$
7. $\sqrt{x} = x^{1/2}$; $\sqrt[3]{} = x^{1/3}$; $\sqrt[4]{x} = x^{1/4}$

1. Express the following relations in the logarithmic forms: (i) $3^4 = 81$ (ii) $5^0 = 1$ (iii) $\sqrt[5]{32} = 2$ (iv) $5^{-3} = \frac{1}{125}$ (v) $a^b = c$ [ans.:(i) $\log_3 81 = 4$ (ii) $\log_5 1 = 0$ (iii) $\log_{32} 2 = \frac{1}{5}$ (iv) $\log_5 \frac{1}{125} = -3$ (v) $\log_a c = b$]
2. Express the following logarithmic forms in the exponential forms: $(i) log_2 64 = 6$ $(ii) log_3 \frac{1}{81} = -4$ $(iii) log_a 1 = 0$ $(iv) log_{25}\left(\frac{1}{125}\right) = -\frac{3}{2}$ $(v) log_Q P = R$ [ans.:$(i) 2^6 = 64$ $(ii) 3^{-4} = \frac{1}{81}$ $(iii) a^0 = 1$ $(iv) (25)^{-\frac{3}{2}} = \frac{1}{125}$ $(v) Q^R = P$]
3. Find the value of x: $(i) x = log_2 128$ $(ii) log_x 81 = 4$ $(iii) log_5 x = -3$ $(iv) log_x 243 = 10$ $(v) log_{\sqrt{7}} 49 = x$ $(vi) log_{10}(7x - 5) = 2$ $(vii) x = log_{10}(0.0001)$ $(viii) log_{\sqrt{x}} 0.25 = 4$ [ans.:$(i) 7$ $(ii) 3$ $(iii) \frac{1}{125}$ $(iv) \sqrt{3}$ $(v) 4$ $(vi) 15$ $(vii) - 4$ $(viii) \frac{1}{2}$]
4. Find the logarithms of : (i)625 to the base 5 (ii)343 to the base $\sqrt{7}$ (iii) 0.1 is the base $9\sqrt{3}$ (iv) 1728 to the base $2\sqrt{3}$ (v) 2401 to the base $\sqrt[3]{7}$ (vi)2^{-8} to the base (vii)81 to the base $\sqrt[3]{9}$ (viii) $\sqrt{5}$ to the base 0.008 (ix) 1728 to the base $2\sqrt{3}$ (x) 0.000001 to the base 0.01. [**ans.: (i) 4 (ii) 6 (iii) $-4/5$ (vi) 6 (v)12 (vi) – 4 (vii) 6 (viii) $-1/6$ $ix)$6, (x)3]**
5. Find the base when: (i) 3 is the logarithm of 343 (ii)4 is the logarithm of 144 (iii) $-\frac{1}{3}$ is the logarithm of $\frac{1}{3}$ (vi) -1 is the logarithm of $\frac{1}{a}$ [ans.: (i) 7 (ii)$2\sqrt{3}$ (iii)27 (iv) a]
6. Find the simplest values of : (i) $log_3 5 \times log_{25} 27$ (ii) $log_8 27$ if $log_2 3 = a$ (iii) $log_{2\sqrt{2}} x$ if $log_{\sqrt{2}} x = a$ [ans.: (i) $\frac{3}{2}$ (ii) a (iii) $\frac{a}{3}$]
7. Prove that, log(1+2+3)= log1+log2+log3 [CU(H)'97]
8. Express M in terms of N: (i) $\frac{1}{2} log_3 M + 3 log_3 N = 1$ (ii) $log_{10} N = 3 - 2 log_{10} M$ [ans.: (i) $M = \frac{9}{N^6}$ (ii) $M = \sqrt{\frac{1000}{N}}$]

9. Prove that, (i)$a^{log_a x} = x$ (ii) $x^{2log_x a} = a^2$ (iii) $x^{log_a y} = y^{log_a x}$ (iv) $log_a m \times log_b n = log_a n \times log_b m$ (v) $log_2 3 \times log_3 2 = 1$ (vi) $log_a b \times log_b c \times log_c a = 1$ [CU(H)'93] (vii) $(log x)^2 - (log y)^2 = \log(xy) \log\left(\frac{x}{y}\right))$ (viii) $a^{\log_a 2^x} \times b^{\log_b 2^y} \times c^{\log_c 2^z} = \sqrt{xyz}$.

10. Find the values of : (i) $log_3 \sqrt[4]{729 \sqrt[3]{9^{-1}.(27)^{-4/3}}}$ (ii) $log_3 log_2 log_{\sqrt{3}} 81$

(iii) $log_{\sqrt{2}} 16 + log_{\sqrt{3}} 9$ (iv) $log_a b \times log_b c \times log_c d \times log_d a$ (v) $\frac{log\sqrt{27} + log 8 - log\sqrt{1000}}{log 1.2}$

(vi) $log_3 4 \times log_4 5 \times log_5 6 \times log_6 7 \times log_7 3$ (vii) $log_6 \sqrt{6\sqrt{6\sqrt{6 \ldots \infty}}}$ (viii) $\log_5 \sqrt{5\sqrt{5\sqrt{5 \ldots \infty}}}$

[ans.: (i)1 (ii)1 (iii)2 (iv)1 (v) 3/2 (vi)1 (vii)1 (viii) 1]

11. Prove that, (i)$log 2 + 16 \log\frac{16}{15} + 12 log\frac{25}{24} + 7\log\frac{81}{80} = 1$

(ii) $7\log\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80} = log 2$ (iii)$7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

(iv)$\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = log 2$ (v)$7\log\frac{10}{9} + 3\log\frac{81}{80} = 2\log\frac{25}{24} + \log 2$.

(vi) $7\log\frac{10}{9} + 3\log\frac{81}{80} = 2\log\frac{25}{24} + \log 2$. (vii)$x^{logy - logz} \times y^{logz - logx} \times z^{logx - logy} = 1$

(viii) $\frac{1}{log_{xy}(xyz)} + \frac{1}{log_{yz}(xyz)} + \frac{1}{log_{zx}(xyz)} = 2$ (ix) $log_{b^3} a \times log_{c^3} b \times log_{a^3} c = \frac{1}{27}$

(x) $\log a + \log a^2 + \log a^3 + \cdots \log a^n = \frac{n(n+1)}{2} \log a$

(xi) $log_a x \times log_b y \times log_c z = log_b x \times log_c y \times log_a z$

(xii) $log_{\frac{1}{y}} x \times log_{\frac{1}{z}} y \times log_{\frac{1}{x}} z = -1$ (xiii) $log_{x^2} x \times log_{y^2} y \times log_{z^2} z = \frac{1}{8}$

(xiv) $\log_b a \times \log_c b \times \log_d c = \log_d a$. (xv) $\log_2 \log_2 \log_2 16 = 1$. (xvi) $\log_2 10 - \log_8 125 = 1$.

12.(a) If $log_{30} 2 = .3010$,find the value of $log_8 25$. [ans.: 1.548]

(b) If $log_{30} 3 = a$ and $log_{30} 5 = b$, find the value of $log_{30} 8$. [ans.: 3(1- a-b)]

(c) If $\log_{10} 2 = 0.30103$ find the value of $\log_5 32$. [ans.:2.15(approx.)

13.(i) If $a^2 + b^2 = 7ab$ show that, $\log\left[\frac{1}{3}(a+b)\right] = \frac{1}{2}(log a + log b)$

(ii) If $\log\frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$ show that, $\frac{x}{y} + \frac{y}{x} = 23$.

(iii) If $a^{3-x}.b^{5x} = a^{5+x}.b^{3x}$ show that, $x log\left(\frac{a}{b}\right) = \log a$

(iv) If $a^4 + b^4 = 14a^2b^2$ show that, $log_e(a^2 + b^2) = log_e a + log_e b + 2log_e 2$

14.(a) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ prove that, $xyz = 1$

(b) If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ prove that, (i) $x^a y^b z^c = 1$ (ii) $x^{b+c}.\,y^{c+a}.\,z^{a+b} = 1$

(iii) If $\frac{\log x}{ry-qz} = \frac{\log y}{pz-rx} = \frac{\log z}{qx-py}$ prove that, $x^p y^q z^r = 1$

15.If log 2 =0.3010, log 3 = 0.4771, log 7=08451, find the value of (i) log 45 (ii)log 108 (iii) log 84 (iv) log 294 (v) log 21.6

16.If $\log_{10}2 = 0.30103$, $\log_{10}3 = 0.47712$ and $\log_{10}7 = 0.84510$, find the values of (i) $\log_{10}45$ and (ii) $\log_{10}105$. [ans.: (i) $\log_{10}45 = 1.65321$ and (ii) $\log_{10}105 = 2.02119$]

17.If $x = log_a(bc),\ y = log_b(ca)$ and $z = log_c(ab)$ show that,

(i) $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$ (ii) $x + y + z + 2 = xyz$

18.If $x = log_{2a}a, y = log_{3a}2a$ and $z = log_{4a}3a$, show that, $xyz + 1 = 2yz$.

19.If $log_p x = a$ and $log_q x = b$, prove that, $log_{\frac{p}{q}} x = \frac{ab}{(b-a)}$.

20.If $\log(x^2y^3) = a$ and $\log\left(\frac{x}{y}\right) = b$, find $\log x$ and $\log y$ in terms of a and b. [ans.: $\log_x = \frac{a+3b}{5}$ and $\log_y = \frac{a-2b}{5}$]

21.If $x = \frac{e^y - e^{-y}}{e^y + e^{-y}}$ show that, $y = \frac{1}{2} log_e \frac{1+x}{1-x}$.

22.If $log_a b = 10$ and $\log_{6a}(32b) = 5$, find the value of a. [ans.: 3]

23.Solve: (i)$\log_{10} x - \log_{10}\sqrt{x} = \frac{2}{\log_{10} x}$ (ii) $\log_2\log_2\log_2 x = 1$ (iii) $\log_8 x + \log_4 x + \log_2 x = 11$ (iv)$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$ [ans.: (i) 100 or, 1/100 (ii) 16 (iii) 64 (iv)8]

24. If a, b, c, are three consecutive positive integers, show that, log(1+ac)=2logb

C.U. QUESTIONS

1. Proved that the $\log_2\log_2\log_2 16 = 1$ **[2012,'14]**
2. if x = $\log_a bc$, y = $\log_b ca$, z = $\log_c ab$ than prove that $\frac{1}{x-1} + \frac{1}{y+1} + \frac{1}{z+} = 1$. **[1999,2013]**
3. Solve: $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$. [**ans.:x = 23 =8.]** **[2013,'14]**
4. Find the value of $\log_3\log_2\log_2 256$. **[2013]**
5. If $a^2+b^2=23ab$, then prove that $\log\left\{\frac{1}{5}(a+b)\right\} = \frac{1}{2}(\log a + \log b)$. **[2013]**
6. If $x=1+\log_a bc$, $y = 1+\log_b ca$, $z = 1+\log_c ab$ than prove that xyz =xy+yz+zx. **[2014]**
7. If If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ prove that, $x^2y^2z^2 = 1$. **[2014]**
8. Show that $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$. **[2015]**

9. Find the value of $\log_6\sqrt{6\sqrt{6\sqrt{6 \ldots \circ}}}$. **[2015]**

10. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ prove that, $x^{b+c}.y^{c+a}.z^{a+b} = 1$ **[2015]**

11. Prove that $\log_b a.\log_c b.\log_a c = 1$. **[2015]**

12. If $a^2+b^2=23ab$, then prove that $\log\frac{a+b}{2} = \frac{1}{2}(\log a+\log b)$. **[2016]**

13. Find the value of $\log_3\log_2\log_2 256$. **[ans.:1]** **[2016]**

14. If $\frac{\log x}{1} = \frac{\log y}{2} = \frac{\log z}{3}$, prove that x, y, z are in GP. **[2016]**

15. If $a^2+b^2=27ab$, then prove that $\log\frac{a-b}{5} = \frac{1}{2}(\log a+\log b)$. **[2016]**

16. Prove that $\log_3\left(\sqrt{\sqrt[3]{\sqrt[3]{3 \ldots \infty}}}\right) = 1$. **[2017]**

17. If $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a+\log b)$, show that $\frac{a}{b} + \frac{b}{a} = 7$. **[2017]**

18. If $x = log_a(bc),\ y = log_b(ca)$ and $z = log_c(ab)$ show that, $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$. **[2017]**

19. Solve for x if $\log_x 2+\log_x 4+\log_x 8 = 6$. **[2017]**