

### FORMULAS:

#### 1. Method of least squares:

(a) Linear trend: Equation of straight line  $y = a + bx$ ..... (1)

The normal equations:  $\sum y = na + b\sum x$ ..... (2)

$\sum yx = a\sum x + b\sum x^2$ ..... (3)

The value of  $a$  and  $b$  in (1) is obtained by solving (2) and (3).

### EXERCISES

#### 1. Show graphical representation of the following time series data and fit the trend line by free-hand method.

Year	1971	'72	'73	'74	'75	'76	'77	'78	'79	'80	'81
Production of tea ('000 kg.)	140	162	166	180	160	138	152	160	140	148	155

#### 2. Determine the trend by semi-average method.

(i)

Year	1967	'68	'69	'70	'71	'72	'73	'74	'75	'76
Annual value	40.6	42.5	43.8	39.6	41.0	43.2	40.5	39	44	45

(ii)

Year	1950	'51	'52	'53	'54	'55	'56	'57	'58
Production ('000 tons.)	79	90	96	87	94	102	98	104	100

#### 3. (i) Given the numbers 1, 0, -1, 0, 1, 0, -1, 0, 1; find the moving average of order four.

(ii) Given the numbers 2, 6, 1, 5, 3, 7, 2. Obtain a moving average of order 3. [CU(H)'84]

#### 4. Find the trend for the following series using a three-years moving average:

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

[CU(H)'83]

#### 5. Calculate 5-yearly moving average of the number of students studying in a college shown below:

[CU(H)'81]

Year	1951	'52	'53	'54	'55	'56	'57	'58	'59	60
No. of student	332	317	357	392	402	405	410	427	405	431

#### 6. Calculate 4-yearly moving average (centered) for the following time series data:

Year	1971	'72	'73	'74	'75	'76	'77	'78	'79	'80	'81	'82
Profit('000)	101	93	79	98	106	96	83	103	110	101	88	106

7. Find the trend of bank clearance by the method of moving average (assume a four year cycle).  
[CU(H)'78]

Year	Bank clearances (₹ crores)	Year	Bank clearances (₹ crores)
1966	53	1973	97
1967	79	1974	104
1968	76	1975	97
1969	69	1976	101
1970	94	1977	107
1971	105	1978	110
1972	108		

8. Compute 5-year moving average of the following time series data:

Year	1	2	3	4	5	6	7	8	9	10	11	10
Rainfall(inches)	96	101	103	91	89	103	121	123	118	117	137	151

Represent the data and the trend values on the same graph paper.

9. (i) Find the trend for the following series using a three-year weighted moving average with weight 1, 2, 1.

Year	1	2	3	4	5	6	7
Values	2	4	5	7	8	10	13

(ii) Given the numbers 2, 6, 1, 5, 3, 7, 2; write down the weighted moving average of period 3 the weight s being 1, 4, 1.

10. For the following series of observations, verify that the 6-year centered moving average is equivalent to a 7-year weighted moving average with weights 1, 2, 2, 2, 2, 2, 1 respectively.

Year	1970	'71	'72	'73	'74	'75	'76	'77	'78	'79	'80
Sales(in '00000)	2	4	3	6	7	9	4	6	7	8	10

11. Determine the period of the moving average for the following data and calculate moving average for that period.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Value	51	26	32	47	53	21	30	36	46	54	28	31	36	46	54

12. Find the quarterly trend values from the following data by the moving average method , using an appropriate period.

Quarterly output (million tons)

Year Quarterly	1964	1965	1966
I	52	59	57
II	54	63	61
III	67	75	72

IV	55	65	60
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13. Fit a straight line trend equation by the method of least squares: [CU(H)'86]

Year	1974	'75	'76	'77	'78	'79	'80	'81
Value	80	90	92	83	94	99	92	104

14. Fit a straight line trend to the following data:

Year	1965	'66	'67	'68	'69	'70	'71
Gross ex-factory value of output (₹ crores)	672	824	967	1204	1464	1758	2057

15. Fit a straight line trend to the following series of production data: [CU(H)'80]

Electricity generated (monthly average) in West Bengal

Year	1951	'52	'53	'54	'55	'56
Electricity generated (million. Kw)	101	107	113	121	136	148

16. The average yearly death in a certain city is given below. Fit a straight line trend by the method of least squares. Tabulate the trend value. [CU(H)'91]

Year	1954	'55	'56	'57	'58	'59	'60
No. of death	940	912	1055	1002	977	961	888

17. Fit a straight line trend equation by the method of least squares and estimate the value for 1969.

Year	1960	'61	'62	'63	'64	'65	'66	'67
Value	380	400	650	720	690	600	870	930

18. The production in million tons for various years are given below:

Year	1956	'57	'58	'59	'60	'61	'62	'63
Prod.(y)	50.3	52.7	49.3	57.3	56.8	60.7	62.1	58.6

Fit a straight line trend by the method of least squares and find by what year the production will reach 62.1 million tons.

[CU(H)'72]

19. Fit a suitable straight line trend to the following data by the method of least squares and estimate the percentage of insured people in 1997. [CU(H)'01]

Year	1989	1990	1991	1992	1993
Percentage of insured people	11.3	13.0	9.7	10.6	10.7

20. Using 1964 as the origin, obtain a straight line trend equation by the method of least squares. Find the trend value of the missing year 1961.

Year	1960	1962	1963	1964	1965	1966	1969
Value	140	144	160	152	168	176	180

21. The following table given the annual profit (in thousand rupees) in a certain factory.

Year	1971	1972	1973	1974	1975	1976	1977
Profit ('000₹)	60	72	75	65	80	85	90

(i) Fit a straight line trend by the method of least squares.

(ii) State the gradient of the trend line.

(iii) Calculate the projected profit for 1978.

[CU(H)'99]

22. Fit an appropriate trend line to the following data on production of pure sulphuric acid in India in tons and estimate trend values for the years 1961-63.

Year	1951	1952	1953	1954	1955	1956
Production	108.7	97.6	110.8	153.3	168.9	169.7

23. Below are given the figures of production (in thousand tons) of a sugar factory:

Year	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72
Production	77	88	94	85	91	98	90

Fit a straight line trend by the method of least squares and estimate the trend value for 1974-75.

24. Obtain the straight line trend equation and tabulate against each year after estimation the trend and short-term fluctuations.

Year	1960	'61	'62	'63	'64	'65	'66	'67	'68
Value	380	400	650	720	690	620	670	950	1040

25. Using 3-year moving average method determine the trend and short-term fluctuations for the following data: [CU(H)'99]

Year	1961	'62	'63	'64	'65	'66	'67
Values	21	34	45	28	40	57	73

26. Fit a parabolic curve of second degree to the data given below and estimate the value for 1979 and comment on it.

Year	1973	1974	1975	1976	1977
Sales (in '000 ₹)	10	12	13	10	8

27. The prices of a commodity during 1970-75 are given below. Fit a parabola  $Y = a + bX + cX^2$  to these data. Estimate the price of the commodity for the year 1976.

Year	1970	'71	'72	'73	'74	'75
Price	100	107	128	140	181	292

28. The sales of company in thousands of rupees for the year 1965 through 1971 are given below:

Year	1965	'66	'67	'68	'69	'70	'71
Sales	32	47	65	92	132	190	275

Estimate the figure for the year 1972 using an equation of the form  $Y = a.b^X$  where  $X$  = year and  $Y$  = sales.

29. Fit a trend equation  $\log y = A + Bx$  to the series of sales data given below: [CU(H)'71]

Year(x)	1943	'44	'45	'46	'47	'48	'49	'50	'51
Sales(y)	97	113	129	202	195	193	192	237	235

- 30.(i) Given the trend equation,  $Y = 35 + 5X + 3X^2$  [Origin: 1968, X unit = 1year]. Change the origin of the equation to 1974.
- (ii) Given the trend equation,  $Y = 10. (1.5)^X$  [Origin: 1968, X unit =1year]. Shift the origin forward by two years.
- (iii)the trend of the annual sales of Bharat Aluminium Company is described by the following equation:  
 $Y = 12 + 0.7X$  [Origin:1970, X unit = 1year and Y unit = annual production].  
 Step the equation down to a month to month basis and shift the origin to January, 1970.
- 31.The trend equation fitted to annual average sales is given by  $y=230+20x$ , unit of  $x$ -one year, origin- 30<sup>th</sup> June, 1960. Adjust the trend equation for finding the monthly trend values and find trend values for the months of January –March, 1972. [CU M.COM.'73]
- 32.A straight line trend equation was fitted to a series of 12 annual production figures from 1968 onwards. It is given as  $Y = 40.08+4.64t$  (origin at the middle of the series, unit of time = 6 months). Obtain annual trend values for the years 1967, 1976 and 1980. Get the trend equation adjusted so as to apply to quarterly production.
- 33.Given tha  $Y = 500 + 25t$  is the trend equation for production in some firm, with origin at January 1, 1975 and one year as the unit; determine percentage increase in production from 1968 to 1978.
- 34.The sales of a company rose from ₹60000 in the month of August to ₹69000 in the month of September. The seasonal indices for this two month are 105 and 140 respectively. The owner of the company was not at all satisfied with the rise of the sale in the month of September by ₹ 9000. He expected much more because of the seasonal index for the that month. What was his estimate of sales for the month of September?

## Answers

3.(i)0, 0, 0, 0, 0. (ii) 3, 4, 3, 5, 4 (4) 3.67, 5.33, 6.67, 8.33, 10.33 for the year 2-6  
 (5) 360, 374.6, 393.2, 407.2, 409.8, 415.6 for 1953-58 (6)93.38, 94.38, 95.25, 96.38, 97.50, 98.63, 99.88, 100.88 ('000 ₹) for 1973-80 (7)74.375, 82.75, 90, 97.5, 102.25, 102.5, 100.625, 101, 103 (₹ crores) for 1968-76 (8)96, 97.4, 101.4, 105.4, 110.8, 116.4, 123.2, 129.2 inches for the years 3-10 (9)(i)3.75, 5.25, 6.75, 8.25, 10.25 for the years 2-6 (ii) 4.5, 2.5, 4.0, 4.0, 5.5 (11) The given data exhibits a cycle of period 5. Hence, the period of moving average may be taken as 5; five-yearly moving average are: 41.8, 35.8, 36.6, 37.4, 37.2, 37.4, 38.8, 39, 39, 39, 39 for the years 3-13 (12) 4-quarter moving average for III 1964 to II 1966: 57.9, 59.9, 62, 64.2, 65.2, 64.8, 64.1, 63.1 (million tons) (13)  $y = 91.75 + 1.25x$  (origin: mid-point of 1977-1978,  $x$  unit =6 months and  $y$ =represents value) (14)  $y = 1278 + 236.43X$  (origin: 1968, X unit =1year and Y in ₹ crores (15) $Y = 121 + 4.714X$  [origin: mid-point of 1953-54, X unit=6month and Y= Electricity generated (in million kw)] (16)  $Y = 962.14 - 4.86X$



(origin: 1957, unit of  $X = 1$  year and  $Y =$  no. of death) (17)  $Y = 655 + 35.83X$  (origin: mid-point of 1963-64, unit of  $X = 6$  months), estimated values for 1969 is 1049. (18)  $Y = 55.975 + 0.826X$  (origin: mid-point of 1959-60,  $X$  units = 6 months and  $Y =$  Production in million tons), 1963-64. (19)  $Y = 21 + 0.1X$  (origin: 1979,  $X$  units = 1 year and  $Y$  is production in '000 tons) respectively (20)  $y = 11.06 - 0.36x$  (origin: 1991,  $X$  unit = 1 year); estimated percentage of insured people in 1997 is 8.9. (21) (i)  $Y = 75.29 + 4.32X$  (origin: 1974,  $X$  unit = 1 year and  $Y$  is production in '000 tons); (ii) 4.32 (iii) ₹92,570. (22) Straight line trend:  $Y = 134.83 + 8.02X$  (origin: mid-point of 1953-54,  $X$  unit = 6 months and  $Y$  is production of sulphuric acid in tons); 255.13, 271.17 and 287.21 (tons) for 1961-63. (23)  $y = 89 + 2X$  (origin: 1968-69,  $X$  unit = 1 year and  $Y$  is production of sugar in '000 tons; the trend value for 1974-75 is 101 ('000 tons) (24)  $Y = 680 + 70.5X$  (origin: 1964,  $X$  unit = 1 year); trend value, 398, 468.5, 539, 609.5, 680, 750.5, 821, 851.5, 962; short term fluctuations: -18, -68.5, 111, 110.5, 10, -130.5, -151, 58.5, 78. (25) 3-year moving average : 33.33, 35.67, 37.67, 41.67, 56.67 for 1962-66; short-term fluctuation: 0.67, 9.33, -9.67, -1.67, 0.33 for 1962-66. (26)  $Y = 6.47 - 0.6X - 0.86X^2$  (origin: 1975,  $X$  unit = 1 year and  $Y =$  sales in '000 ₹); estimated value for 1979 is (-9.69) ('000 ₹); the second degree parabola is not an appropriate trend measure for the given data. (27)  $Y = 126.55 + 18.04X + 1.786X^2$  (origin: mid-point of 1972-73,  $X$  unit = 6 months); the price of the commodity for the year 1976 is 340 units (approx.) (28) 387.3 ('000 ₹) (29)  $\log y = 2.229 + 0.0471x$  (origin: 1947,  $x$  unit = 1 year) (30) (i)  $Y = 173 + 41X + 3X^2$  (origin: 1974,  $X$  unit = 1 year) (ii)  $Y = 22.5(1.5)^X$  (origin: 1970,  $X$  unit = 1 year) (iii)  $Y = 0.9733 + 0.0049X$  (origin: January, 1970 and  $X$  unit = 1 month) (31)  $y = 230.833 + 1.667x$  (origin: July, 1960 and  $x$  unit = 1 month); 460.83, 462.50 and 464.17 for January-march, 1972 (32) Annual trend values for the year 1967, 1976 and 1980 are (-20.24), 63.28 and 100.4 respectively; Quarterly trend equation is:  $Y = 10.31 + 0.58t$  (origin: 3<sup>rd</sup> quarter, 1968 and  $t$  unit = 1 quarter) (33) 76.92% (34) ₹80000

## C.U. QUESTIONS

1. Define time series. Explain different component of time series with examples.

[2012, 2013(H), '14(G)(H), '15(H), '16(G), '17(G)]

2. Fit a straight line trend to the following data by the method of least square and estimate the percentage of insured people in 2012:

[2012]

Year	2007	2008	2009	2010	2011
Percentage of insured people	11.3	13.0	14.5	16.6	18.7

3. Find the trend for the following series using a 3 year weighted moving average with weight 1, 2, 1:

Year	1	2	3	4	5	6	7
Values	2	4	5	7	8	10	13

[ans.:3.75, 5.25, 6.75, 8.25, 10.25]

[2012,2016(G)]

4. Write down four point of advantages and disadvantages for the calculation of moving average in time series.

[2013,2016(H)]

5. You are given the annual profit figures for a certain firm for the years 2005 to 2011. Fit a straight line trend to the data and estimate the expected profit for the year 2012:

[2016]

Year	2005	2006	2007	2008	2009	2010	2011
Profit(lakhs)	30	36	38	32	40	42	48

[ans.:76.84]

6. Find the 3-year (certred) moving average by moving average method form the following table:

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Prod.	506	620	1036	673	588	696	1116	738	663	773	1189

[ans.:719, 738.75, 758.25, 776.375, 793.875, 812.875, 831.625]

[2013,2014(H)]

7. You are given annual profit figure for a certain firm for the year 2006 to2012. Fit a least straight line trend to the data and estimate the expected profit for the year 2015:

[2013]

Year	2006	2007	2008	2009	2010	2011	2012
Profit (₹lakhs)	60	72	75	78	80	85	95

[ans.: x=6, y=107,02]

8. for the following series of observation, verify that the 4-year centred moving average is equivalent to a 5-year moving average with weight 1,2, 2, 2, 1 respectively.

Year	2002	'03	'04	'05	'06	'07	'08	'09	'10	'11	'12
Sales(₹'000)	2	6	1	5	3	7	2	6	4	8	3

[ans.: 3.63, 3.9, 4.13, 4.38, 4.63, 4.88, 5.13]

[2013,2015(G)(H); 2017(G)]

9. Fit straight line trend equation by the method of Last Square. [ans.:y=95.4+4.5x]

[2014]

Year	2009	'10	'11	'12	'13
Values	83	94	99	97	104

10. Using moving average method, calculate 3-yearly moving average from the following data: **[2014]**

Year	2005	'06	'07	'08	'09	'10	'11	'12	'13
Percentage of insured people	11.3	13.0	14.5	16.6	18.7	19.3	21.3	24.3	2.2

**[ans.: 12.93, 14.70, 16.6, 18.2, 19.8, 21.6, 24.23]**

**[2014]**

11. From the following data of the production in India, using moving average method, calculate 4-yearly moving average:

**[2014]**

Year	2004	'05	'06	'07	'08	'09	'10	'11	'12	'13
Percentage (million pound)	464	515	518	467	502	540	557	571	586	612

**[ans.: 495.75, 503.625, 511.625, 529.50, 553, 572.5]**

**[2014]**

12. Fit a straight line trend by least squares method to the following data:

Year(x)	2008	'09	'10	'11	'12	'13
Sales (in ton)(y)	210	225	275	220	240	235

**[ans.:  $y = 234.17 + 1.643x$  [origin = middle of 2010-11 and unit of x, 6 months = 1]]**

**[2014, 1989]**

13. Fit a linear trend equation to the following data:

Year	2008	'09	'10	'11	'12	'13	'14
Production ('000 tons)	117	123	130	139	143	151	158

Hence estimate the production for the year 2015. **[ans.: 164.73(app.)]**

**[2015, 2017]**

14. Fit a linear trend equation to the following data:

**[2015]**

Year	2008	'09	'10	'11	'12	'13	'14
Production ('000 tons)	17	23	30	39	43	51	58

Hence estimate the production for the year 2015. **[ans.: 76.45]**

15. Fit a straight line trend equation by the method of least square from the following data and estimate the profit for the year 2015: **[ans.: 41.43]**

**[2016]**

Year	1975	'80	'85	'90	'95	2000	'05	'10
Profit (lakh)	10	13	15	20	22	28	33	40

16. Fit a square trend line to the following data and estimate the demand for 2016:

**[2016]**



Year	2010	2011	2012	2013	2014	2015
Demand ('000 units)	8	12	15	17	22	24

17. Construct seasonal indices from the following time series data:

[2017]

Quarter \ Year	I	II	III	IV
2014	90	75	87	70
2015	75	80	78	75
2016	80	75	75	72

[ans.: Grand total = 77.67; 105.14, 98.71, 102.99, 93.12]

18. Fit a least square trend line to the following data:

[2017]

Year	2008	'09	'10	'11	'12	'13	'14
Av. Prod. per month ('000 tonnes)	20	22	21	24	25	23	28

Hence find the Av. Production per month in the year 2017. [ans.: a=23.28, b=1.07, y=29.7]

19. Construct 5-year moving average of the number of students studying in a collage shown below: [2017]

Year	2004	'05	'06	'07	'08	'09	'10	'11	'12	'13	'14	'15
No. of stu.	332	317	357	392	402	405	410	427	405	431	467	483

[ans.: 360, 374.6, 393.2, 407.2, 409.8, 415.6, 428, 442.6]

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