**Assignment 2** - **Nitish Ramkumar, Stephan Du Toit, Yvonne Tong Yi, Baihan Chen**

1a 1

1b 2

1c 3

2a 3

2b 4

2c 5

3a 6

3b 7

3c 7

3d 8

4a 8

4b 9

4c 9

4d 10

4e 11

4f 12

5a 13

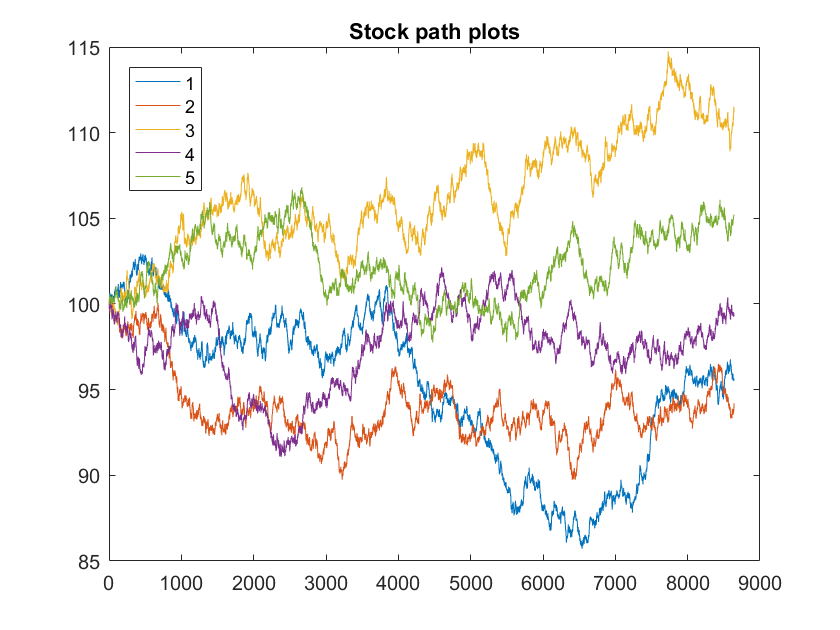
5b 14

5c 15

5d 16

## 1a

clear  
clc  
S0 = 100;%Initial stock price  
T = 0.25;%Maturity years  
K = 100;%Strike  
r = 0.05; %IR  
sigma=0.2; %SD  
delta = 0;  
h = 1/(360\*96); %every 5 minutes, which is 1/96 of a day  
  
randn('seed',0);  
  
NoOfSim = 5;  
stockPrices = zeros((T/h)+1,5);  
for counter = 1:5  
 stockPrices(:,counter) = GenerateStockPath(S0,r,T,h,sigma);  
end  
plot(stockPrices)  
title('Stock path plots')  
legend('1','2','3','4','5','Location','northwest')



## 1b

S0 = 100;%Initial stock price  
T = 0.25;%Maturity in days  
K = 100;%Strike  
r = 0.05; %IR  
sigma=0.2; %SD  
delta = 0;  
h = 1/(360\*96); %every 5 minutes, which is 1/96 of a day  
c1 = BlackScholes(S0,K,r,sigma,T,'Call')

c1 =  
  
 4.6150

## 1c

S0 = 100;%Initial stock price  
T = 0.25;%Maturity in days  
K = 100;%Strike  
r = 0.05; %IR  
sigma=0.2; %SD  
delta = 0;  
  
randn('seed',0);  
NoOfPaths = [100 1000 1000000 100000000];  
N = length(NoOfPaths);  
priceResult = zeros(N,3);  
for count = 1:size(priceResult,1)  
 priceResult(count,:) = MonteCarloSim(S0,r,T,K,sigma,NoOfPaths(count));  
end  
priceResult  
  
%The price approaches black scholes value  
%The spread reduces.

priceResult =  
  
 4.8522 3.6979 6.0065  
 4.7096 4.2788 5.1403  
 4.6196 4.6067 4.6326  
 4.6148 4.6135 4.6161

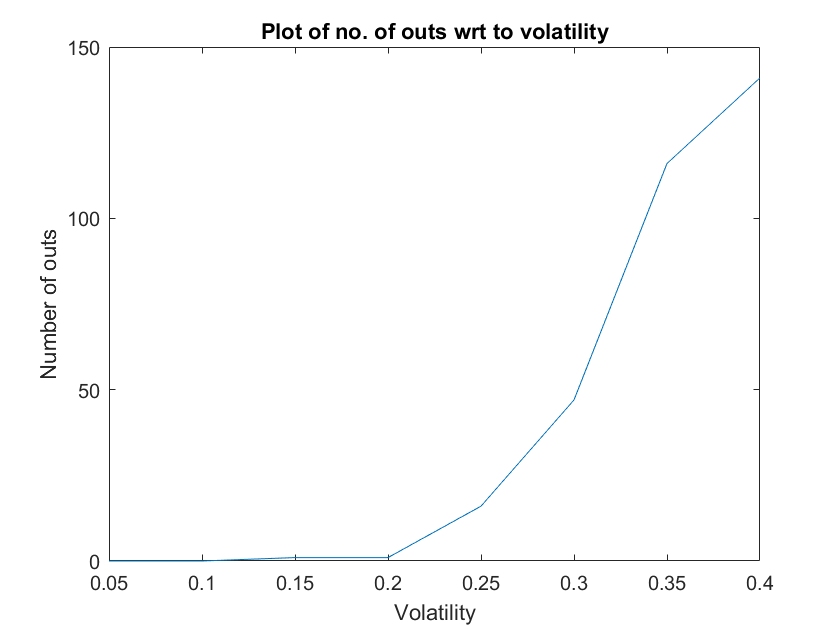
## 2a

S0 = 100;%Initial stock price  
T = 0.25;%Maturity in days  
K = 95;%Strike  
r = 0.05; %IR  
sigma=0.2; %SD  
delta = 0;  
Sb = 75;  
h = 1/(360\*96);  
NoOfSim = 1000;  
  
randn('seed',0);  
MCResult = MonteCarloDownOut(S0,r,T,K,sigma,NoOfSim,h,Sb)  
PutOption = BlackScholes(S0,K,r,sigma,T,'Put')

MCResult =  
  
 1.5858  
 1.3785  
 1.7930  
 5.0000  
  
  
PutOption =  
  
 1.5343

## 2b

S0 = 100;%Initial stock price  
T = 0.25;%Maturity in days  
K = 95;%Strike  
r = 0.05; %IR  
delta = 0;  
Sb = 75;  
h = 1/(360\*96);  
NoOfSim = 1000;  
sigma = [0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4];  
  
randn('seed',0);  
results = zeros(length(sigma),4);  
for count = 1:length(sigma)  
 results(count,:) = MonteCarloDownOut(S0,r,T,K,sigma(count),NoOfSim,h,Sb);  
end  
  
plot(sigma,results(:,4))  
title('Plot of no. of outs wrt to volatility')  
xlabel('Volatility')  
ylabel('Number of outs')  
  
%We can observe that the number of outs increases with volatility.  
%This makes sense because with an increase in volatility, we have a higher  
%change of going out of the cutoff.



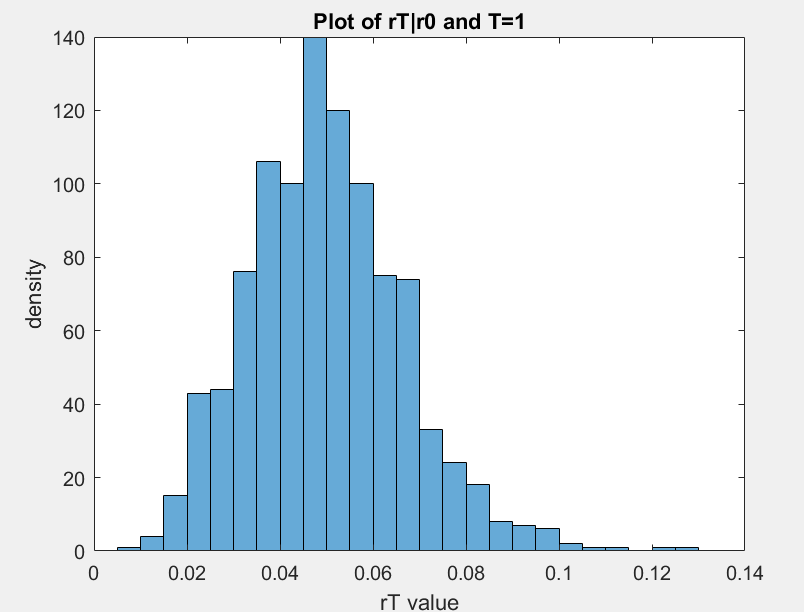
## 2c

S0 = 100;%Initial stock price  
T = 0.25;%Maturity in days  
K = 95;%Strike  
r = 0.05; %IR  
delta = 0;  
Sb = 75;  
h = 1/(360\*96);  
NoOfSim = 1000;  
sigma = [0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4];  
  
randn('seed',0);  
results = zeros(length(sigma),4);  
for count = 1:length(sigma)  
 results(count,:) = MonteCarloDownOut(S0,r,T,K,sigma(count),NoOfSim,h,Sb);  
end  
bsPrice = BlackScholes(S0,K,r,sigma,T,'Put');  
errorbar(sigma,results(:,1),results(:,3)-results(:,1))  
title('Plot of Monte Carlo put price with error bar and black scholes price')  
xlabel('Volatility')  
ylabel('price of Put')  
hold on  
plot(sigma,bsPrice)  
hold off

## 

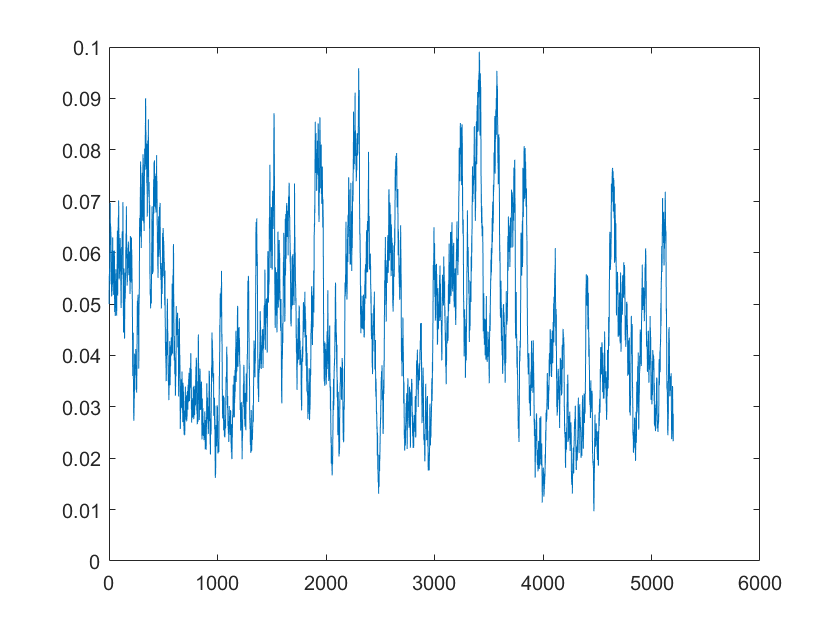
## 3a

r0 = 0.05;  
beta = 0.05;  
alpha = 0.6;  
delta = 0.1;  
T = 1;  
h = 1/250;  
  
randn('seed',0);  
NoOfSim = 1000;  
rTRes = zeros(NoOfSim,1);  
for i = 1:NoOfSim  
 db1 = randn(T/h,1).\*sqrt(h);  
 rt = IR(r0,alpha,beta,delta,db1,T,h);  
 rTRes(i) = rt(length(rt));  
end  
histogram(rTRes)



## 3b

r0 = 0.05;  
beta = 0.05;  
alpha = 0.6;  
delta = 0.1;  
T = 100;  
h = 1/52;  
randn('seed',0);  
db1 = randn(T/h,1).\*sqrt(h);  
rt = IR(r0,alpha,beta,delta,db1,T,h);  
plot(rt)



## 3c

r0 = 0.05;  
beta = 0.05;  
alpha = 0.6;  
sigma11 = 0.1;  
sigma12 = 0.2;  
S10 = 10;  
delta = 0.1;  
T = 0.5;  
h = 1/250;  
K = 10;  
NoofSim = 10000;  
  
randn('seed',0);  
results = MonteCarloExotic\_1(r0,S10,T,h,K,sigma11,sigma12,alpha,beta,delta,NoofSim);  
results(1)

ans =  
  
 0.6799

## 3d

r0 = 0.05;  
beta = 0.05;  
alpha = 0.6;  
sigma11 = 0.1;  
sigma12 = 0.2;  
sigma21 = 0.3;  
S10 = 10;  
S20 = 10;  
delta = 0.1;  
T = 0.5;  
h = 1/250;  
K = 10;  
NoofSim = 10000;  
  
randn('seed',0);  
results = MonteCarloMaxExotic(r0,S10,S20,T,h,K,sigma11,sigma12,sigma21,alpha,beta,delta,NoofSim);  
results(1)

ans =  
  
 1.2717

## 4a

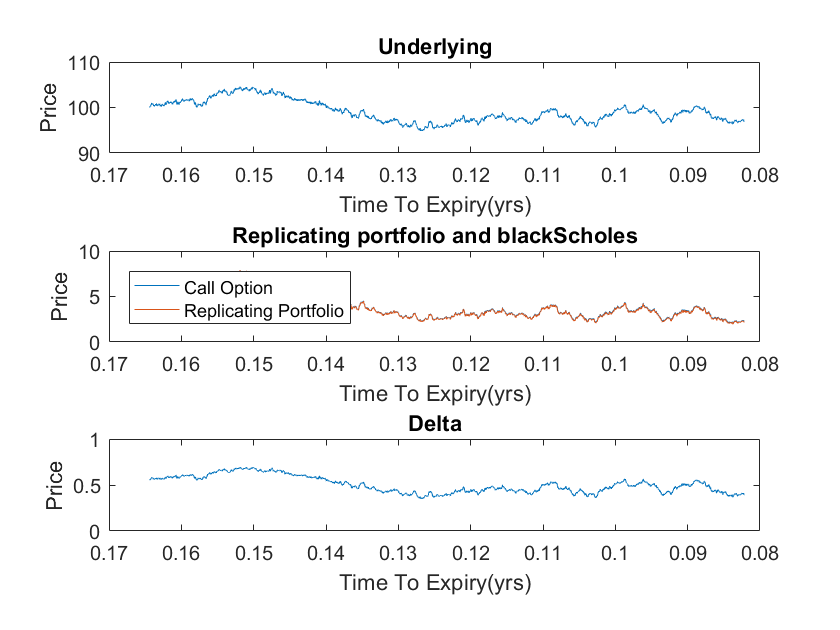
S0=100;  
K = 100;  
T=60/365;  
h = 1/(96\*365);  
r = 0.05;  
sigma = 0.3;  
mu = 0.2;  
delta = 0;  
randn('seed',0);  
stockPricesAll = GenerateStockPath(S0,mu,T,h,sigma);  
periodsReq = (30/365)/h;  
stockPrices = stockPricesAll(1:(periodsReq+1));

## 4b

blackScholesPrices = BlackScholes(stockPrices,K,r,sigma,(T:-h:30/365)','Call');

## 4c

deltas = BlackScholesDelta(stockPrices,K,r,sigma,(T:-h:30/365)','Call');  
[replicatingPort,riskFree] = DeltaHedging(stockPrices,blackScholesPrices,deltas,r,h,0);  
  
figure(1)  
subplot(3,1,1)  
plot(T:-h:30/365,stockPrices)  
title('Underlying')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )  
  
subplot(3,1,2)  
plot(T:-h:30/365,blackScholesPrices)  
hold on  
plot(T:-h:30/365,replicatingPort)  
legend('Call Option','Replicating Portfolio','location','southwest')  
title('Replicating portfolio and blackScholes')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )  
hold off  
  
subplot(3,1,3)  
plot(T:-h:30/365,deltas)  
title('Delta')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )

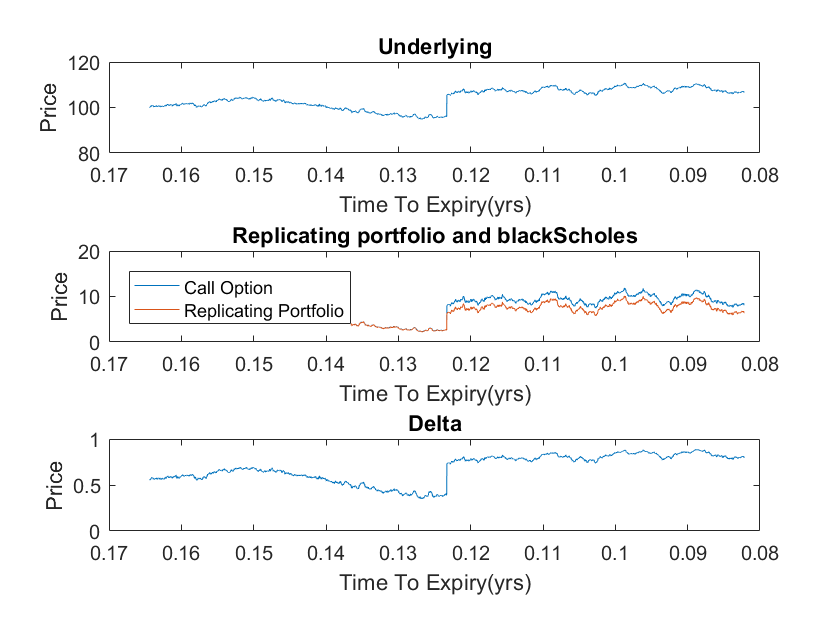


## 4d

randn('seed',0);  
stockJumpAll = GenerateStockPathWithJump(S0,mu,T,h,sigma,-0.1,0.25);  
stockJump = stockJumpAll(1:(periodsReq+1));  
BSJump= BlackScholes(stockJump,K,r,sigma,(T:-h:30/365)','Call');  
deltasJump = BlackScholesDelta(stockJump,K,r,sigma,(T:-h:30/365)','Call');  
[repPortJump,rfJump] = DeltaHedging(stockJump,BSJump,deltasJump,r,h,0);  
  
figure(1)  
subplot(3,1,1)  
plot(T:-h:30/365,stockJump)  
title('Underlying')  
xlabel('Time To Expiry(yrs)')  
set ( gca, 'xdir', 'reverse' )  
  
subplot(3,1,2)  
plot(T:-h:30/365,BSJump)  
xlabel('Time To Expiry(yrs)')  
hold on  
plot(T:-h:30/365,repPortJump)  
legend('Call Option','Replicating Portfolio','location','southwest')  
title('Replicating portfolio and blackScholes')  
set ( gca, 'xdir', 'reverse' )  
hold off  
  
subplot(3,1,3)  
plot(T:-h:30/365,deltasJump)  
title('Delta')  
xlabel('Time To Expiry(yrs)')  
set ( gca, 'xdir', 'reverse' )

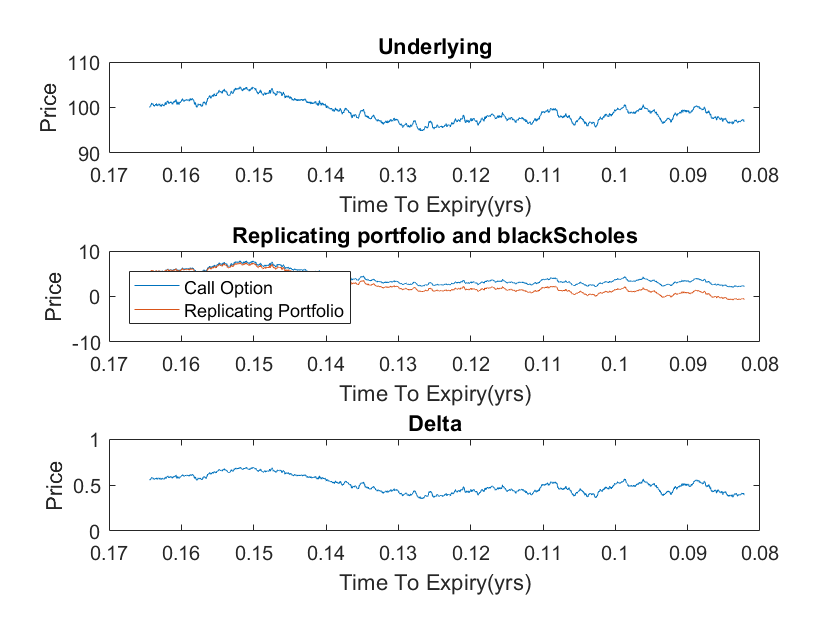
## 4e

randn('seed',0);  
stockJumpAll1 = GenerateStockPathWithJump(S0,mu,T,h,sigma,0.1,0.25);  
stockJump1 = stockJumpAll1(1:(periodsReq+1));  
BSJump1= BlackScholes(stockJump1,K,r,sigma,(T:-h:30/365)','Call');  
deltasJump1 = BlackScholesDelta(stockJump1,K,r,sigma,(T:-h:30/365)','Call');  
[repPortJump1,rfJump1] = DeltaHedging(stockJump1,BSJump1,deltasJump1,r,h,0);  
  
figure(1)  
subplot(3,1,1)  
plot(T:-h:30/365,stockJump1)  
title('Underlying')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )  
  
subplot(3,1,2)  
plot(T:-h:30/365,BSJump1)  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
hold on  
plot(T:-h:30/365,repPortJump1)  
legend('Call Option','Replicating Portfolio','location','southwest')  
title('Replicating portfolio and blackScholes')  
set ( gca, 'xdir', 'reverse' )  
hold off  
  
subplot(3,1,3)  
plot(T:-h:30/365,deltasJump1)  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
title('Delta')  
set ( gca, 'xdir', 'reverse' )



## 4f

[repPortTrans,rfTrans] = DeltaHedging(stockPrices,blackScholesPrices,deltas,r,h,0.002);  
figure(1)  
subplot(3,1,1)  
plot(T:-h:30/365,stockPrices)  
title('Underlying')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )  
  
subplot(3,1,2)  
plot(T:-h:30/365,blackScholesPrices)  
hold on  
plot(T:-h:30/365,repPortTrans)  
legend('Call Option','Replicating Portfolio','location','southwest')  
title('Replicating portfolio and blackScholes')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )  
hold off  
  
subplot(3,1,3)  
plot(T:-h:30/365,deltas)  
title('Delta')  
xlabel('Time To Expiry(yrs)')  
ylabel('Price')  
set ( gca, 'xdir', 'reverse' )



## 5a

When we run the Heston model with the given inputs, we get a price of **4.6833.**

The Black Scholes price (based on the assumed long run mean) is **5.017**

## 5b

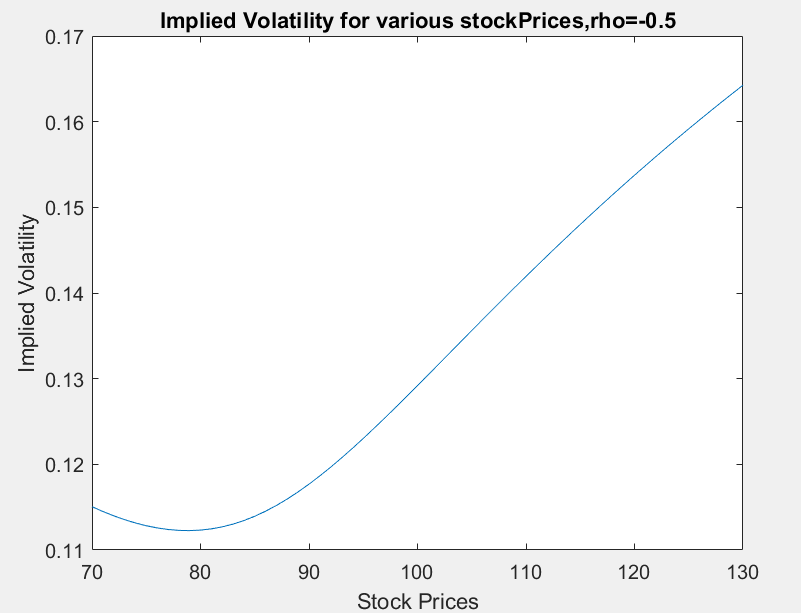
This is a graph between the difference in the Heston and Black Scholes price and the stock price.

As can be seen, the graph dips until just before the strike price, after that it increases until about 115 before starting to fall again.



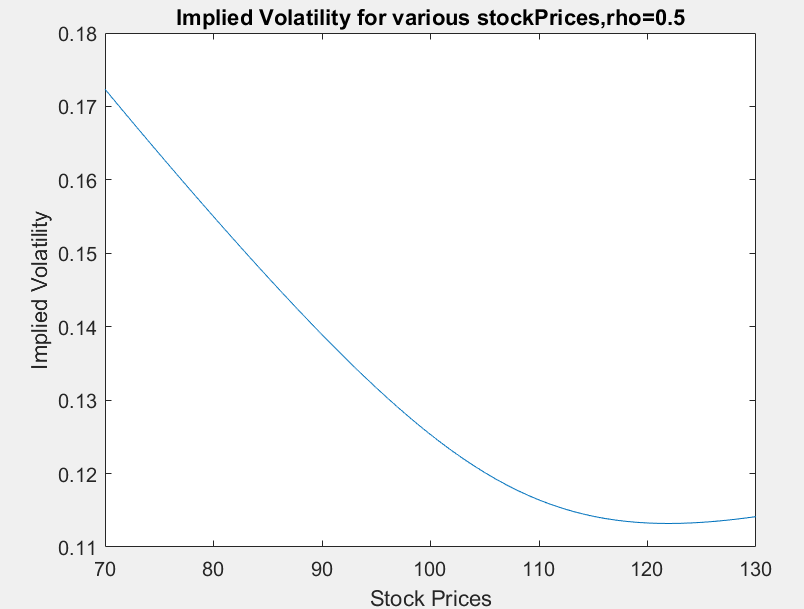
## 5c

The implied volatility for a correlation of -0.5 between Brownian motion 1 and 2 reduces until 80 and then increases with stock price.



## 5d

The implied volatility for a correlation of -0.5 between Brownian motion 1 and 2 reduces until 80 and then increases with stock price.



## R Code

5a

S0 = 100;%Initial stock price  
T = 0.5;%Maturity in days  
K = 100;%Strike  
r = 0.04; %IR  
sigma = 0.3;  
rho = -0.5;  
v0 = 0.01;  
k=6;  
theta = 0.02;  
lambda = 0;  
  
bsprice = BlackScholes(S0,K,r,sqrt(theta),T,'Call')  
price2 = HestonModel(S0,v0,r,T,0,K,rho,sigma,lambda,k,theta)  
%price = call\_heston\_cf(S0, v0, theta, k, sigma, r, rho, T, K)

5b

T = 0.5;%Maturity in days  
K = 100;%Strike  
r = 0.04; %IR  
sigma = 0.3;  
rho = -0.5;  
v0 = 0.01;  
k=6;  
theta = 0.02;  
lambda = 0;  
  
bsprices = BlackScholes(70:130,K,r,sqrt(theta),T,'Call');  
hestonprices = zeros(length(70:130),1);  
for St = 70:130  
 hestonprices(St-69) = HestonModel(St,v0,r,T,0,K,rho,sigma,lambda,k,theta);  
end  
  
diff = hestonprices - bsprices';  
plot(70:130,diff)  
title('Plot of Heston - BS prices based on underlying price')  
xlabel('Underlying price')  
ylabel('Heston - BS')

5c

S0 = 100;%Initial stock price  
T = 0.5;%Maturity in days  
K = 100;%Strike  
r = 0.04; %IR  
sigma = 0.3;  
rho = -0.5;  
v0 = 0.01;  
k=6;  
theta = 0.02;  
lambda = 0;  
St = 70:130;  
  
ImpliedVol = zeros(length(hestonprices),1);  
for i = 1:length(hestonprices)  
 ImpliedVol(i) = blsimpv(St(i),K,r,T,hestonprices(i));  
end  
  
plot(St,ImpliedVol)  
title('Implied Volatility for various stockPrices,rho=-0.5')  
xlabel('Stock Prices')  
ylabel('Implied Volatility')

5d

T = 0.5;%Maturity in days  
K = 100;%Strike  
r = 0.04; %IR  
sigma = 0.3;  
rho = 0.5;  
v0 = 0.01;  
k=6;  
theta = 0.02;  
lambda = 0;  
StockPrices = 70:130;  
  
hestonprices = zeros(length(St),1);  
for St = 70:130  
 hestonprices(St-69) = HestonModel(St,v0,r,T,0,K,rho,sigma,lambda,k,theta);  
end  
  
ImpliedVol = zeros(length(hestonprices),1);  
for i = 1:length(hestonprices)  
 ImpliedVol(i) = blsimpv(StockPrices(i),K,r,T,hestonprices(i));  
end  
  
plot(StockPrices,ImpliedVol)  
title('Implied Volatility for various stockPrices,rho=0.5')  
xlabel('Stock Prices')  
ylabel('Implied Volatility')