Assigment 1
Mgmt 237E: Empirical Methods
Ian Laker, PrasanthKumar, Nitish Ramkumar

Problem 1

1

Let us generate the moment generating function for r_t

Moment generating function Mg(s) = $\mathbb{E}(expr_t s)$

$= \mathbb{E}(e^{[\mu+\sigma\epsilon_t+\beta_t(\mu_j+\sigma_j\delta_t)]s})$

$= \mathbb{E}(e^{(\mu+\sigma\epsilon_t)s}e^{(\beta_t(\mu_j+\sigma_j\delta_t))s})$

$= \mathbb{E}(e^{(\mu+\sigma\epsilon_t)s})\mathbb{E}(e^{\beta_t(\mu_j+\sigma_j\delta_t)s})$

$= e^{s\mu+\frac{1}{2}\sigma^2 s^2}(p\mathbb{E}(e^{(\mu_j+\sigma_j\delta_t)s}) + (1-p)e^0)$

$= \mathbf{e^{s\mu+\frac{1}{2}\sigma^2 s^2}[pe^{s\mu_j+\frac{1}{2}\sigma_j{}^2 s^2} + (1-p)]}$

Mean

Lets keep A = $s\mu + \frac{1}{2}\sigma^2 s^2 + s\mu_j + \frac{1}{2}\sigma_j{}^2 s^2$, B = $s\mu + \frac{1}{2}\sigma^2 s^2$,

so Mg(s) = $pe^A + (1-p)e^B$

We know Mean = 1st Moment (M_1) = $\frac{\partial Mg}{\partial s}(s=0)$

$\frac{\partial Mg}{\partial s} = pe^A\frac{\partial A}{\partial s} + (1-p)e^B\frac{\partial B}{\partial s}$

We can calculate $\frac{\partial A}{\partial s}(s=0) = \mu + \mu_j$, $\frac{\partial B}{\partial s}(s=0) = \mu$

$Mean(\mu_{r_t}) = M_1 = \frac{\partial Mg}{\partial s}(s=0) = \mu + \mathbf{p\mu_j}$

Variance

We know Variance = M_2 - ${\mu_{r_t}}^2$

$M_2 = \frac{\partial^2 Mg}{\partial s^2}(s=0)$

$\frac{\partial^2 Mg}{\partial s^2} = pe^A\frac{\partial A}{\partial s}^2 + pe^A\frac{\partial^2 A}{\partial s^2} + e^B\frac{\partial B}{\partial s}^2 + e^B\frac{\partial^2 B}{\partial s^2} - pe^A\frac{\partial B}{\partial s}^2 - pe^B\frac{\partial^2 B}{\partial s^2}$

We can calculate $\frac{\partial^2 A}{\partial s^2}(s=0) = \sigma^2 + \sigma_j{}^2$, $\frac{\partial^2 B}{\partial s^2}(s=0) = \sigma^2$

So, $M_2 = p\mu_j{}^2 + 2p\mu\mu_j + p\sigma^2 + p\sigma_j{}^2 + \mu^2 + \sigma^2$

Variance $({\sigma_{r_t}}^2) = \sigma^\mathbf{2} + \mathbf{p\sigma_j{}^2} + \mathbf{p\mu_j{}^2} - \mathbf{p^2\mu_j{}^2}$

Skewness

We know Skewness = $\frac{M_3 - 3\mu_{r_t}{\sigma_{r_t}}^2 - {\mu_{r_t}}^3}{{\sigma_{r_t}}^{\frac{3}{2}}}$

$M_3 = \frac{\partial^3 Mg}{\partial s^3}$

$\frac{\partial^3 Mg}{\partial s^3} = pe^A\frac{\partial A}{\partial s}^3 + 3pe^A\frac{\partial A}{\partial s}\frac{\partial^2 A}{\partial s^2} + pe^A\frac{\partial^3 A}{\partial s^3} + e^B\frac{\partial B}{\partial s}^3 + 3e^B\frac{\partial B}{\partial s}\frac{\partial^2 B}{\partial s^2} + e^B\frac{\partial^3 B}{\partial s^3} - pe^B(\frac{\partial B}{\partial s})^3 - 3pe^B\frac{\partial B}{\partial s}\frac{\partial^2 B}{\partial s^2} - pe^B\frac{\partial^3 B}{\partial s^3}$

We can calculate $\frac{\partial^3 A}{\partial s^3}(s=0) = 0$, $\frac{\partial^3 B}{\partial s^3}(s=0) = 0$

$M_3 = p(\mu+\mu_j)^3 + 3p(\mu+\mu_j)(\sigma^2+\sigma_j{}^2) + (1-p)(\mu^3 + 3\mu\sigma^2)$

So Skewness = $\frac{p\mu_j{}^3 + 3p\mu_j\sigma_j{}^2 - 3p^2\mu_j\sigma_j{}^2 - 3p^2\mu_j{}^3 + 2p^3\mu_j{}^3}{(\sigma^2 + p\sigma_j{}^2 + p\mu_j{}^2 - p^2\mu_j{}^2)^{\frac{3}{2}}}$

$= \frac{\mu_j \mathbf{p}(\mu_j{}^2(\mathbf{2p^2-3p+1})-\mathbf{3}\sigma_j{}^2(\mathbf{p-1}))}{(\sigma^2+\mathbf{p}\sigma_j{}^2+\mathbf{p}\mu_j{}^2-\mathbf{p^2}\mu_j{}^2)^{\frac{3}{2}}}$

Kurtosis

We know Kurtosis $= \frac{M_4-4\mu_{r_t}M_3+6\mu_{r_t}{}^2M_2-3\mu_{r_t}{}^4}{\sigma_{r_t}{}^2}$

$M_4 = \frac{\partial^4 Mg}{\partial s^4}$

$\frac{\partial^4 Mg}{\partial s^4} = pe^A \frac{\partial^1 Mg}{\partial s^1}^4 + 6pe^A \frac{\partial^1 Mg}{\partial s^1}^2 \frac{\partial^2 Mg}{\partial s^2} + 4pe^A \frac{\partial^1 Mg}{\partial s^1} \frac{\partial^3 Mg}{\partial s^3} + 3pe^A \frac{\partial^2 Mg}{\partial s^2}^2 + pe^A \frac{\partial^4 Mg}{\partial s^4}$

$\frac{\partial^4 A}{\partial s^4} = 0, \frac{\partial^4 B}{\partial s^4} = 0$

$M_4 = p(\mu+\mu_j)^4 + 6p(\mu+\mu_j)^2(\sigma^2+\sigma_j{}^2) + 3p(\sigma^2+\sigma_j{}^2)^2 + (1-p)(\mu^4+6\mu^2\sigma^2+3\sigma^3)$

So Kurtosis $= \frac{\mathbf{3}\sigma_j{}^4\mathbf{p(1-p)}+\mu_j{}^4\mathbf{p(-6p^3+12p^2-7p+1)}+\mu_j{}^3\sigma_j{}^3\mathbf{p(12p^2-18p_6)}}{(\sigma^2+\mathbf{p}\sigma_j{}^2+\mathbf{p}\mu_j{}^2-\mathbf{p^2}\mu_j{}^2)^2}$

2

The general stock returns are distributed with skew and thick tails. A log normal distribution doesn't cover the skew and the thick tails which are necessary to represent general stock returns.

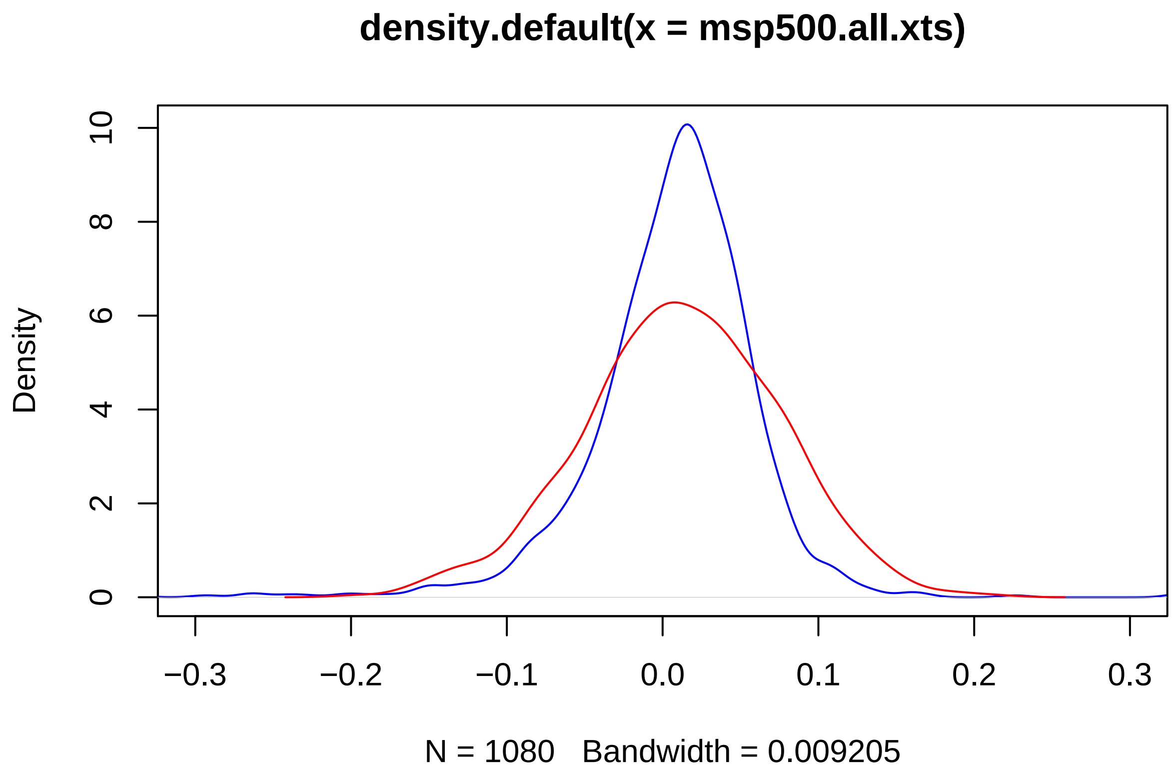
3

Get the SP500 monthly data to use as baseline

```r
# Function to get data #
getData <- function(sql, n = -1){
  #setup connection
  res <- dbSendQuery(wrds, sql)
  dbHasCompleted(res)

  #perform fetch
  returnData <- fetch(res, n)

  #clear memory
  dbClearResult(res)
  return(returnData)
}


sql3 <- "SELECT * FROM CRSP.MSP500"
msp500.all <- getData(sql3)
msp500.all$caldt <- as.Date(msp500.all$caldt)
msp500.all.xts <- xts::xts(log(1+msp500.all$vwretd[-1]),order.by = msp500.all$caldt[-1])
colnames(msp500.all.xts) <- "vwretd"
```

If the returns are normally distributed, the distribution will look like this compared to the monthly SP returns.

```r
plot(density(msp500.all.xts),col="blue", type="l",xlim=c(-0.3,0.3))

#only normal
lines(density(rnorm(n = 600,mean = 0.008,sd = 0.063)),col="red",type="l")
```



If we take the bernoulli-normal mix, the distribution will look like this compared to the monthly SP returns.

```
#normalBernoullimix
normalBernoulliMix <- function(normalMean,normalSD,bernProb,jumpMean,jumpSD,n)
{
  SecondTerm <- jumpMean + jumpSD*rnorm(n)
  jt <- rbinom(n,1,bernProb)*(SecondTerm)
  normalMean + normalSD * rnorm(n) + jt
}

normalBernoulliDist <- normalBernoulliMix(0.012,0.05,0.15,-0.03,0.1,600)

#mean, SD, skewness, Kurtosis
mean(normalBernoulliDist)
```

```
## [1] 0.006821567
```

```
sd(normalBernoulliDist)
```

```
## [1] 0.06773343
```

```
library(moments)
```

```
## Warning: package 'moments' was built under R version 3.3.2
```

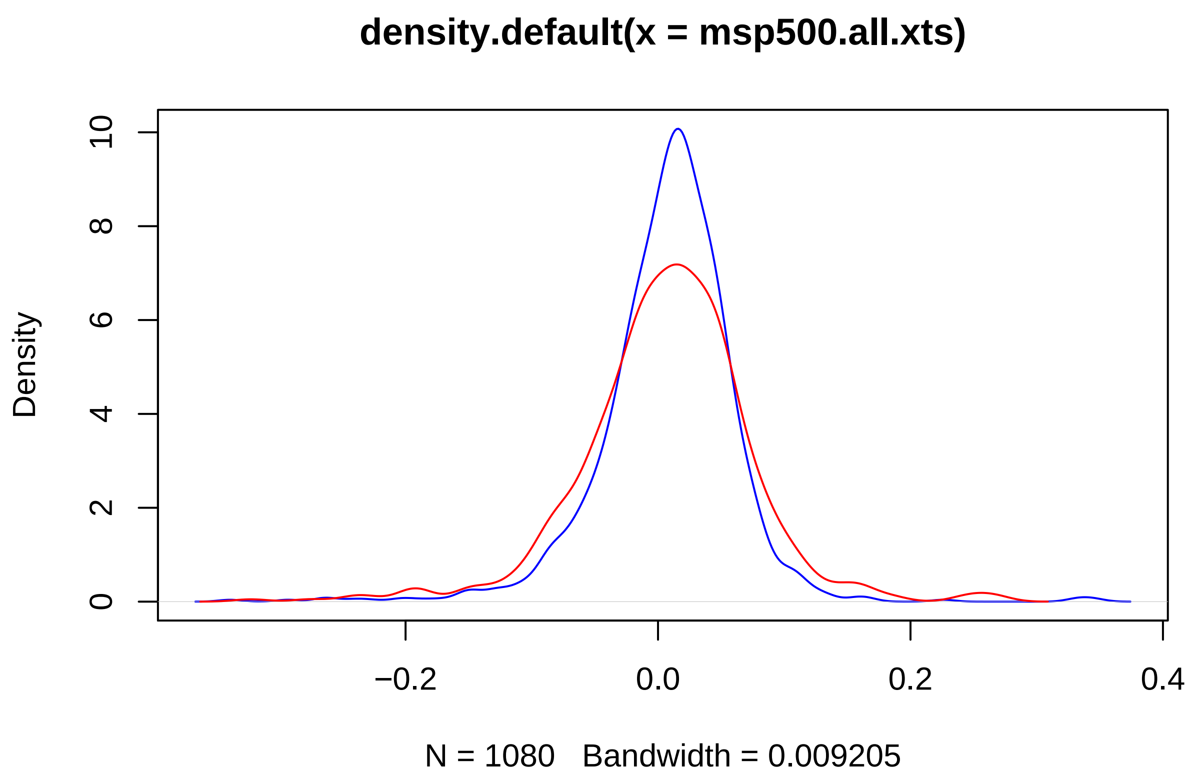
```r
skewness(normalBernoulliDist)
```

```
## [1] -0.3376926
```

```r
kurtosis(normalBernoulliDist)
```

```
## [1] 6.128758
```

```r
#plot sp500
plot(density(msp500.all.xts),col="blue", type="l")

#density
lines(density(normalBernoulliDist),type="l",col="red", ylim=c(0,9))

#actual values
lines(normalBernoulliDist,type="l",col="blue")
```



Problem 2

Get the necessary data

```r
#this.dir <- dirname(rstudioapi::getActiveDocumentContext()$path)
setwd("C:/_UCLA/237E_Empirical/Assigments/Assignment1")
library("readxl")
```

```
## Warning: package 'readxl' was built under R version 3.3.2
```

```r
library("xts")
```

```
## Warning: package 'xts' was built under R version 3.3.2
```

```
## Loading required package: zoo
```

```
## Warning: package 'zoo' was built under R version 3.3.2
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

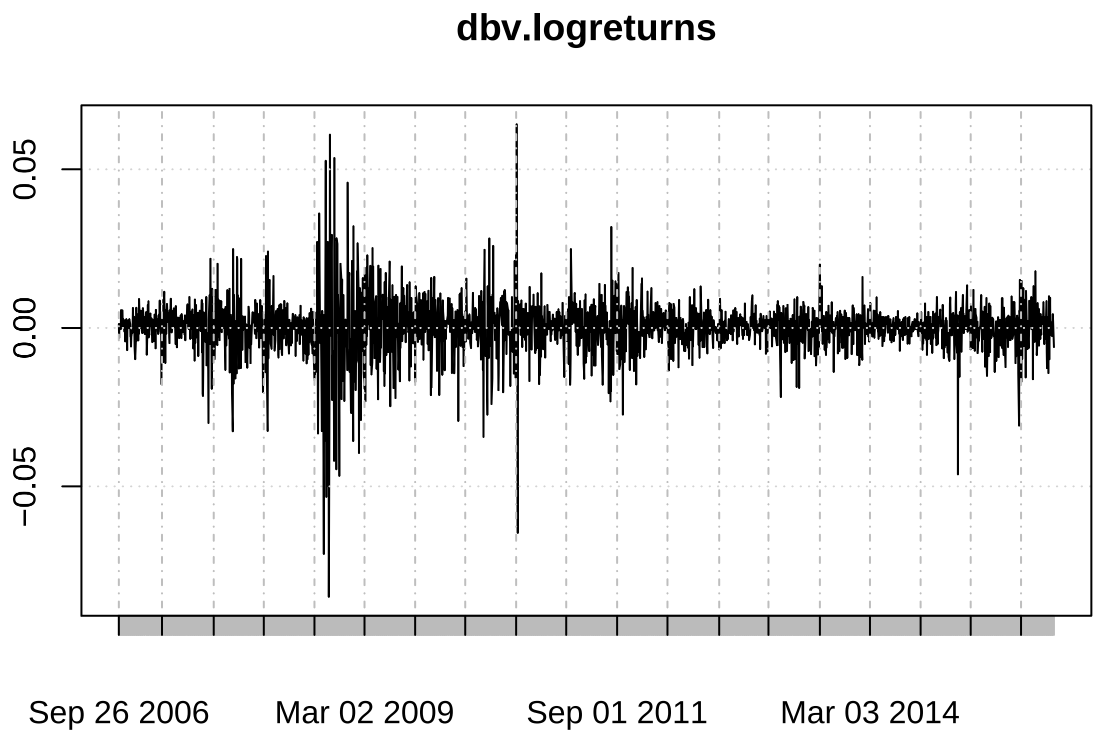
```r
dbv <- read_excel("DBV.xlsx")
gspc <- read_excel("GSPC.xlsx")

dbv$Date <- as.Date(dbv$Date)
gspc$Date <- as.Date(gspc$Date)
dbv.xts <- xts(dbv[,-1],order.by = dbv$Date)
gspc.xts <- xts(gspc[,-1],order.by = gspc$Date)
```

Calculate the log returns

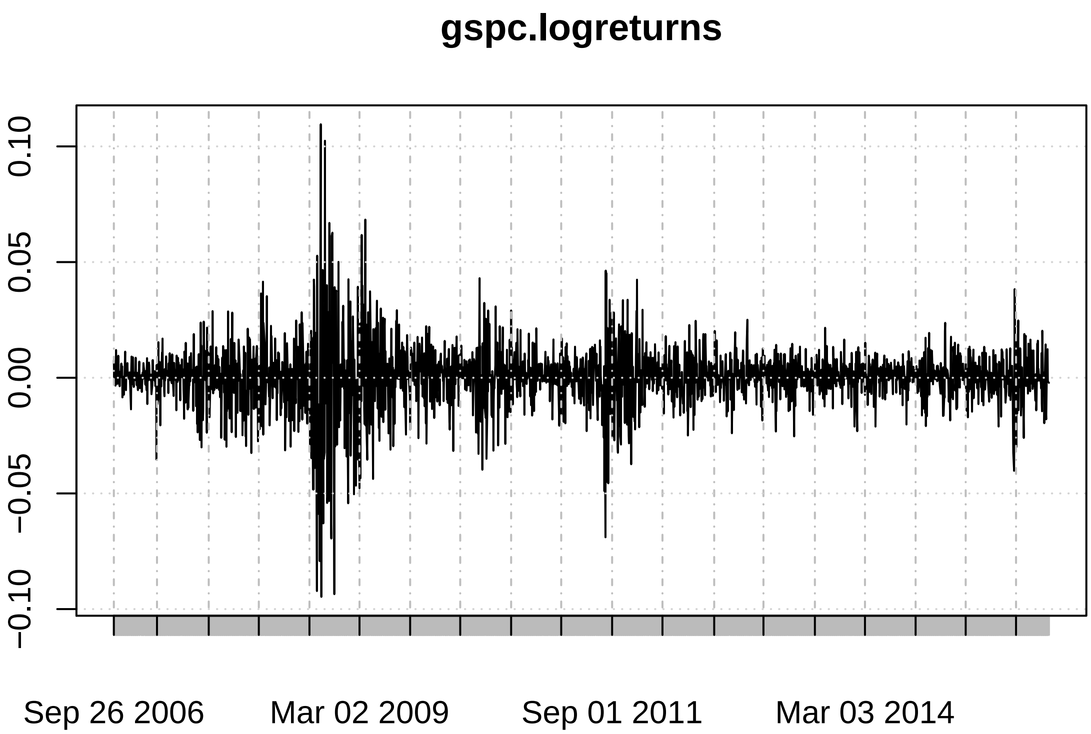
```r
dbv.logreturns <- log(dbv.xts$'Adj Close'[-1,]/lag(dbv.xts$'Adj Close')[-1,])
gspc.logreturns <- log(gspc.xts$'Adj Close'[-1,]/lag(gspc.xts$'Adj Close')[-1,])
```

1

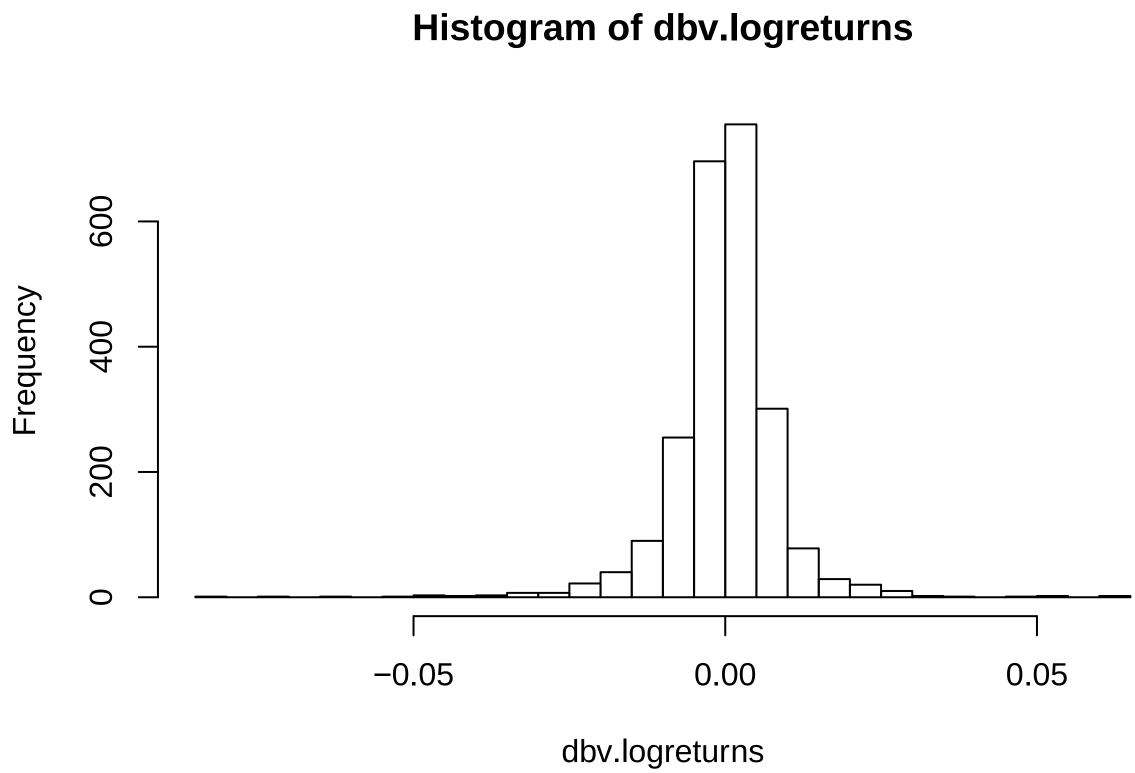
```r
#1
plot(dbv.logreturns)
```



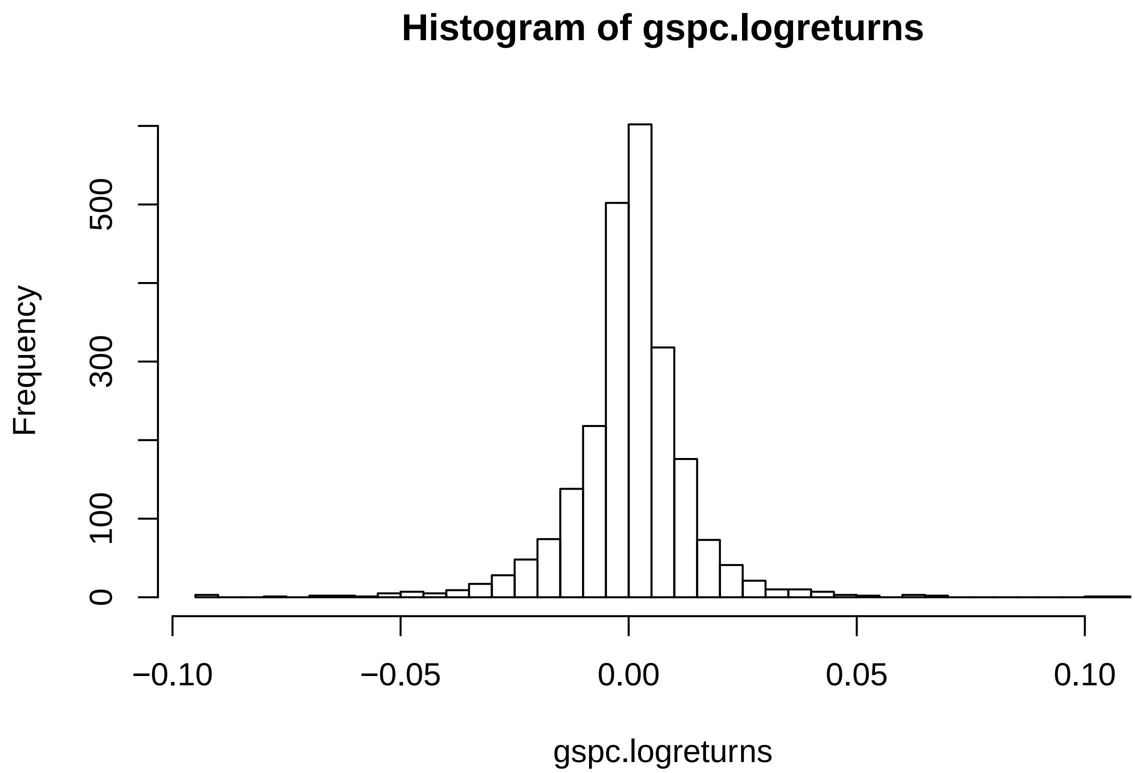

```r
plot(gspc.logreturns)
```




```
hist(dbv.logreturns,breaks = 50)
```




```
hist(gspc.logreturns, breaks = 50)
```



2

a

```
skewNullCheck <- function(returns,alpha=.05){
  criticalt <- qt(1-(alpha/2),df = length(returns))
  skewcap <- skewness(returns)
  skewt <- skewcap/(sqrt(6/length(returns)))
  returnVal <- c(skewcap-3,skewt,abs(skewt) > criticalt) #TRUE, so reject normal distribution, no skewn
  names(returnVal) <- c("Sample Skewness","Skewness t","Reject Null?")
  returnVal
}

skewNullCheck(dbv.logreturns)
```

```
## Sample Skewness      Skewness t    Reject Null?
##       -3.847038      -16.691872        1.000000
```

```
skewNullCheck(gspc.logreturns)
```

```
## Sample Skewness      Skewness t    Reject Null?
##       -3.324043       -6.385641        1.000000
```

```r
#b
kurtosisNullCheck <- function(returns,alpha=.05){
  criticalt <- qt(1-(alpha/2),df = length(returns))
  kurtosiscap <- kurtosis(returns)
  kurtosist <- (kurtosiscap-3)/(sqrt(24/length(returns)))
  returnVal <- c(kurtosiscap-3,kurtosist,abs(kurtosist) > criticalt) #TRUE, so reject normal distributi
  names(returnVal) <- c("Sample Kurtosis","Kurtosis t","Reject Null?")
  returnVal
}

kurtosisNullCheck(dbv.logreturns)
```

```
## Sample Kurtosis      Kurtosis t    Reject Null?
##        13.51674       133.18159         1.00000
```

```r
kurtosisNullCheck(gspc.logreturns)
```

```
## Sample Kurtosis      Kurtosis t    Reject Null?
##        9.797934       96.539900        1.000000
```

```r
#c
jbTest <- function(returns,alpha=.05){
  criticalchi <- qchisq(1-alpha,df = 2)
  skewcap <- skewness(returns)
  kurtosiscap <- kurtosis(returns)
  jbt <- skewcap^2/(6/length(returns)) + (kurtosiscap-3)^2/(24/length(returns))
  returnVal <- c(jbt,abs(jbt) > criticalchi)
  names(returnVal) <- c("JBt","Reject Null?")
  returnVal
}

jbTest(dbv.logreturns)
```

```
##          JBt Reject Null?
##     18015.95         1.00
```

```r
jbTest(gspc.logreturns)
```

```
##          JBt Reject Null?
##     9360.729        1.000
```

3

```r
sqlDaily <- "SELECT caldt, vwretd FROM CRSPQ.DSP500 WHERE year(caldt) >= 1973 AND year(caldt) < 2015"
dsp500 <- getData(sqlDaily)

skewKurtMat <- matrix(c(skewNullCheck(dbv.logreturns)[1],skewNullCheck(gspc.logreturns)[1],kurtosisNull(
colnames(skewKurtMat) <- c("Skewness","Kurtosis")
row.names(skewKurtMat) <- c("DBV","GSPC")
skewKurtMat
```

```
##       Skewness  Kurtosis
## DBV  -3.847038 13.516736
## GSPC -3.324043  9.797934
```

4

The expected return to standard deviation ratio covers only the first 2 moments of the return. It doesn't show the difference in skewness and kurtosis between the two investments.
As GSPC has lower negative skewness, there is lesser chance of getting a negative tail value.
It also has lower kurtosis, which means there is lesser chance of getting an extreme value

5

a

Both assumptions valid (i.e. homoskedastic and normal)

```r
lm <- lm(dbv.logreturns ~ gspc.logreturns)
reg1Summ <- summary(lm)
reg1Summ$coefficients[,2]
```

```
##     (Intercept) gspc.logreturns
##    0.0001361433    0.0101101940
```

```r
vcov(lm)
```

```
##                   (Intercept) gspc.logreturns
## (Intercept)      1.853500e-08   -1.923941e-08
## gspc.logreturns -1.923941e-08    1.022160e-04
```

```r
library(DataAnalytics)
reg2Summ <- lmSumm(lm(dbv.logreturns ~ gspc.logreturns),HAC = T)
```

```
## Multiple Regression Analysis:
##     2 regressors(including intercept) and 2330 observations
##     with heteroskedastic|autocorrelation consistent standard errors
##     Lag truncation = 0
##
## lm(formula = dbv.logreturns ~ gspc.logreturns)
##
## Coefficients:
##                    Estimate Std Error t value p value
## (Intercept)      -9.579e-05  0.000137   -0.70   0.484
## gspc.logreturns   4.309e-01  0.018280   23.57   0.000
## ---
## Standard Error of the Regression:  0.006571
## Multiple R-squared:  0.438  Adjusted R-squared:  0.438
```

```r
reg2Summ$coef.table[,2]
```

```
##     (Intercept) gspc.logreturns
##        0.000137        0.018280
```