Empirical_Assignment5

1

```r
#Sample Mean
Avg.Mkt <- 0.05

Avg.1 <- 0.01 + 0.9*Avg.Mkt
Avg.2 <- -0.015 + 1.2*Avg.Mkt
Avg.3 <- 0.005 + 1.0*Avg.Mkt
Avg <- c(Avg.1,Avg.2,Avg.3)
Avg
```

```
## [1] 0.055 0.045 0.055
```

```r
#Standard Deviation
Variance_matrix <- matrix(c(0.1^2,0,0,0,0.15^2,0,0,0,0.05^2),nrow=3)
betas <- c(0.9,1.2,1)

Variance.Mkt <- 0.15^2
Variance.1 <- betas[1]^2 * Variance.Mkt +  Variance_matrix[1,1]
Variance.2 <- betas[2]^2 * Variance.Mkt +  Variance_matrix[2,2]
Variance.3 <- betas[3]^2 * Variance.Mkt +  Variance_matrix[3,3]
Sd <- c(sqrt(Variance.1),sqrt(Variance.2),sqrt(Variance.3))
Sd
```

```
## [1] 0.1680030 0.2343075 0.1581139
```

```r
#SharpeRatio
Avg/Sd
```

```
## [1] 0.3273752 0.1920553 0.3478505
```

2

To hedge out the market risk, we go short the market based on the beta of the market provided in the regression equation.

```r
#Sample Mean
Avg.Hedged.1 <- 0.01
Avg.Hedged.2 <- -0.015
Avg.Hedged.3 <- 0.005
Avg.Hedged <- c(Avg.Hedged.1,Avg.Hedged.2,Avg.Hedged.3)
Avg.Hedged
```

```
## [1]  0.010 -0.015  0.005
```

```
#Standard Deviation
Variance.Hedged.1 <- Variance_matrix[1,1]
Variance.Hedged.2 <- Variance_matrix[2,2]
Variance.Hedged.3 <- Variance_matrix[3,3]
Sd.Hedged <- c(sqrt(Variance.Hedged.1),sqrt(Variance.Hedged.2),sqrt(Variance.Hedged.3))
Sd.Hedged
```

```
## [1] 0.10 0.15 0.05
```

```
#SharpeRatio
Avg.Hedged/Sd.Hedged
```

```
## [1]  0.1 -0.1  0.1
```

3

The maximum sharpe ratio squared based on the mean variance efficiency = $(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

Proof

The aim is to minimize portfolio variance ($w'\Omega w$, where w is the weights and Ω is variance-covariance matrix), such that the portfolio returns reach the necessary value of m

The objective function from the lagrangian form is

$min\ \frac{1}{2}w'\Omega w - k(w'\bar{R}^e - m)$

First order differential w.r.t w and set it to 0 to minimize

$\Omega w - k\bar{R}^e = 0$

so $w^{MVE} = k\Omega^{-1}\bar{R}^e$

so, $\bar{R}^e_{MVE} = (w^{MVE})'\bar{R}^e = k(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

$var({R^e}_{MVE}) = (w^{MVE})'\Omega w^{MVE} = k^2(\bar{R}^e)'\Omega^{-1}\Omega\Omega^{-1}\bar{R}^e$
$= k^2(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

So, the Sharpe Ratio squared for MVE is
${SR^2}_{MVE} = \frac{\bar{R}^e_{MVE}}{var({R^e}_{MVE})} = (\bar{R}^e)'\Omega^{-1}\bar{R}^e$

```
SharpeRatioSq.Max <- t(Avg.Hedged)%*%chol2inv(chol(Variance_matrix))%*%Avg.Hedged
sqrt(SharpeRatioSq.Max)
```

```
##           [,1]
## [1,] 0.1732051
```

4

```
SharpeRatio.Market <- 1/3
SharpeRatio.Combined <- sqrt(SharpeRatioSq.Max + SharpeRatio.Market^2)
SharpeRatio.Combined
```

```
##           [,1]
## [1,] 0.3756476
```

5a

```r
AllReturns <- c(Avg,Avg.Mkt)

#Calculate systematic variance and covariance (Betai * Betaj * market variance)
betas5 <- c(betas,1)
systematicVar.5 <- (betas5%*%t(betas5))*Variance.Mkt
fullCovarianceMatrix.5 <- rbind(cbind(Variance_matrix,0),0) + systematicVar.5

SharpeRatio.Max.Combined <- t(AllReturns)%*%chol2inv(chol(fullCovarianceMatrix.5))%*%AllReturns
Sd <- 0.15
k <- Sd/sqrt(SharpeRatio.Max.Combined)

weights_combined <- (chol2inv(chol(fullCovarianceMatrix.5))%*%AllReturns) * as.numeric(k)
weights_combined
```

```
##             [,1]
## [1,]  0.39931043
## [2,] -0.26620695
## [3,]  0.79862086
## [4,]  0.04880461
```

5b

```r
#Mean
Mean5 <- AllReturns%*%weights_combined

#SD
Sd5 <- sqrt(t(weights_combined)%*%fullCovarianceMatrix.5%*%weights_combined)

#Sharpe Ratio
SR5 <- Mean5/Sd5

output <- c(Mean5,Sd5,SR5)
names(output) <- c("Mean","SD","Sharpe Ratio")
output
```

```
##         Mean           SD Sharpe Ratio
##   0.05634714   0.15000000   0.37564759
```

6

a

```r
mimick.Weights <- ((betas - mean(betas))/(length(betas)*var(betas)))
mimick.Return <- mimick.Weights%*%Avg
```

```
systematicVar.stocks <- (betas%*%t(betas))*Variance.Mkt
fullCovarianceMatrix.stocks <- systematicVar.stocks +Variance_matrix

mimick.Sd <- sqrt(t(mimick.Weights)%*%fullCovarianceMatrix.stocks%*%mimick.Weights)
mimick.sharpe <- mimick.Return/mimick.Sd
c(mimick.Return,mimick.Sd,mimick.sharpe)
```

```
## [1] -0.02380952  0.41761117 -0.05701362
```

b

```
cor.mimick.market <- mimick.Weights%*%(betas*sqrt(Variance.Mkt)/sqrt(diag(fullCovarianceMatrix.stocks))
cor.mimick.market
```

```
##            [,1]
## [1,] -0.1532408
```

c

```
eigens <- eigen(fullCovarianceMatrix.stocks)
eigens$values/sum(eigens$values)
```

```
## [1] 0.80813177 0.13952986 0.05233837
```

d

```
eigens$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.4685584  0.7003778 -0.5384460
## [2,] -0.7451113 -0.6407531 -0.1850531
## [3,] -0.4746180  0.3144940  0.8220896
```

e

The first PCA is the market. As it can be noticed, though the first PCA explains 80.81%, the second and third PCA still explains a significant portion of the variance. Also the second PCA is close to a long short strategy, which means that without any exposure to market, it is explaining variance of the returns.