Empirical_Assignment5

1

```
#Sample Mean
Avg.Mkt <- 0.05

Avg.1 <- 0.01 + 0.9*Avg.Mkt
Avg.2 <- -0.015 + 1.2*Avg.Mkt
Avg.3 <- 0.005 + 1.0*Avg.Mkt
Avg <- c(Avg.1,Avg.2,Avg.3)
Avg
```

```
## [1] 0.055 0.045 0.055
```

```
#Standard Deviation
Variance_matrix <- matrix(c(0.1^2,0,0,0,0.15^2,0,0,0,0.05^2),nrow=3)
Variance.Mkt <- 0.15^2
Variance.1 <- Variance.Mkt +  Variance_matrix[1,1]
Variance.2 <- Variance.Mkt +  Variance_matrix[2,2]
Variance.3 <- Variance.Mkt +  Variance_matrix[3,3]
Sd <- c(sqrt(Variance.1),sqrt(Variance.2),sqrt(Variance.3))
Sd
```

```
## [1] 0.1802776 0.2121320 0.1581139
```

```
#SharpeRatio
Avg/Sd
```

```
## [1] 0.3050851 0.2121320 0.3478505
```

2

To hedge out the market risk, we go short the market based on the beta of the market provided in the regression equation.

```
#Sample Mean
Avg.Hedged.1 <- 0.01
Avg.Hedged.2 <- -0.015
Avg.Hedged.3 <- 0.005
Avg.Hedged <- c(Avg.Hedged.1,Avg.Hedged.2,Avg.Hedged.3)
Avg.Hedged
```

```
## [1]  0.010 -0.015  0.005
```

```r
#Standard Deviation
Variance.Hedged.1 <- Variance_matrix[1,1]
Variance.Hedged.2 <- Variance_matrix[2,2]
Variance.Hedged.3 <- Variance_matrix[3,3]
Sd.Hedged <- c(sqrt(Variance.Hedged.1),sqrt(Variance.Hedged.2),sqrt(Variance.Hedged.3))
Sd.Hedged
```

```
## [1] 0.10 0.15 0.05
```

```r
#SharpeRatio
Avg.Hedged/Sd.Hedged
```

```
## [1]  0.1 -0.1  0.1
```

3

The maximum sharpe ratio squared based on the mean variance efficiency = $(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

Proof

The aim is to minimize portfolio variance ($w'\Omega w$, where w is the weights and Ω is variance-covariance matrix), such that the portfolio returns reach the necessary value of m

The objective function from the lagrangian form is

$min\ \frac{1}{2}w'\Omega w - k(w'\bar{R}^e - m)$

First order differential w.r.t w and set it to 0 to minimize

$\Omega w - k\bar{R}^e = 0$

so $w^{MVE} = k\Omega^{-1}\bar{R}^e$

so, $\bar{R}^e_{MVE} = (w^{MVE})'\bar{R}^e = k(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

$var(R^e{}_{MVE}) = (w^{MVE})'\Omega w^{MVE} = k^2(\bar{R}^e)'\Omega^{-1}\Omega\Omega^{-1}\bar{R}^e$
$= k^2(\bar{R}^e)'\Omega^{-1}\bar{R}^e$

So, the Sharpe Ratio squared for MVE is
$SR^2{}_{MVE} = \frac{\bar{R}^e_{MVE}}{var(R^e{}_{MVE})} = (\bar{R}^e)'\Omega^{-1}\bar{R}^e$

```r
SharpeRatioSq.Max <- t(Avg.Hedged)%*%chol2inv(chol(Variance_matrix))%*%Avg.Hedged
sqrt(SharpeRatioSq.Max)
```

```
##           [,1]
## [1,] 0.1732051
```

4

```r
SharpeRatio.Market <- 1/3
SharpeRatio.Combined <- sqrt(SharpeRatioSq.Max + SharpeRatio.Market^2)
SharpeRatio.Combined
```

```
##           [,1]
## [1,] 0.3756476
```

5a

```
Avg.Hedged.Market <- c(Avg.Mkt,Avg.Hedged)
Variance_matrix_combined <- rbind(c(Variance.Mkt,0,0,0),cbind(rep(0,3),Variance_matrix))

SharpeRatio.Max.Combined <- t(Avg.Hedged.Market)%*%chol2inv(chol(Variance_matrix_combined))%*%Avg.Hedge
Sd <- 0.15
k <- Sd/sqrt(SharpeRatio.Max.Combined)

weights_combined <- (chol2inv(chol(Variance_matrix_combined))%*%Avg.Hedged.Market) * as.numeric(k)
weights_combined
```

```
##            [,1]
## [1,]  0.8873565
## [2,]  0.3993104
## [3,] -0.2662070
## [4,]  0.7986209
```

5b

```
#Mean
Mean5 <- Avg.Hedged.Market%*%weights_combined

#SD
Sd5 <- sqrt(t(weights_combined)%*%Variance_matrix_combined%*%weights_combined)

#Sharpe Ratio
Mean5/Sd5
```

```
##           [,1]
## [1,] 0.3756476
```

6

a

```
betas <- c(0.9,1.2,1)
mimick.Return <- ((betas - mean(betas))/(length(betas)*var(betas)))%*%Avg
mimick.Sd <- Variance.Mkt^0.5
mimick.sharpe <- mimick.Return/mimick.Sd
c(mimick.Return,mimick.Sd,mimick.sharpe)
```

```
## [1] -0.02380952  0.15000000 -0.15873016
```

b

```
corr.mimick.market <- mimick.Return * var(betas)
corr.mimick.market
```

```
##               [,1]
## [1,] -0.0005555556
```

c

```
var.matrix <- Variance_matrix
var.matrix[1,1] = var.matrix[1,1] + Variance.Mkt
var.matrix[2,2] = var.matrix[2,2] + Variance.Mkt
var.matrix[3,3] = var.matrix[3,3] + Variance.Mkt

eigens <- eigen(cov(var.matrix))
eigens$values/sum(eigens$values)
```

```
## [1] 6.675365e-01 3.324635e-01 6.143203e-16
```

d

```
eigens$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,]  0.4089859  0.7220524 -0.5580061
## [2,] -0.8941748  0.1950357 -0.4030044
## [3,]  0.1821592 -0.6637781 -0.7254079
```

e

The first PCA is the market. As it can be noticed, though the first PCA explains 66.7%, the second and third PCA still explains a significant portion of the variance. Also the second PCA is close to a long short strategy, which means that without any exposure to market, it is explaining variance of the returns.