CDS on CDO Portfolio A Fundamental Factors Model for Hedging, Collateralization and Capital Reserves

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Abstract
In this study, we have identified the cause of AIG's failure in 2008 to be large unhedged CDS exposures on CDO portfolios. We built a fundamental factors model to price CDS-CDOs and created hedging strategies. We specified subprime mortgage default and prepayment hazard rates to be dependent on common fundamental factors including interest rates and housing prices as well as deal specific idiosyncratic factors. We calibrated our base fundamental factor parameters with loan level ARM mortgages and made certain assumptions for the CDOs. We then studied the hedging performance with instruments like ABX.HE indices, Eurodollar futures and Home Price Index futures. We propose a new collateral scheme based on CVA values of both counterparties. We also demonstrate a framework to calculate the VaR on the CVA and the expected loss distributions of the CDS after collateralization. We then propose a method to determine the capital reserves needed for CDS contracts based on two VaRs.

Keywords
Fundamental Factors, Hedging, Collateralization, Capital Reserve, CDS-CDOs

Contents

1. Introduction

The near collapse of the American International Group (AIG), the largest insurance company at the time, was one of the most severe events during the subprime-mortgage crisis. The bailout of AIG, along with other concurrent collapses of some major financial institutions, led both practitioners and regulators to reflect upon their legacy risk management practices. In the context of emphasis on deregulation and financial innovations after Financial Services Modernization Act of 1999 (FSMA) and the Commodity Futures Modernization Act of 2000 (CFMA), disputes had arisen over how to strike a balance between the cost of stringent risk management, the regulatory burden and the benefits from rolling out structured financial instruments in a more inter-related financial system. Thereafter, we saw a market inclination towards vanilla instruments. The new regulatory frameworks, Dodd Frank Act and BASEL III accord (Basel Committee, 2010), both aim at ensuring the wellness of the financial system.

In this study, we will revisit the bailout of AIG, based on which we will propose a fundamental factor approach to mitigate the risks of AIG's most problematic position - the credit default swap portfolio on collateralized debt obligations. As we shall see later, our factor approach will consistently offer a three layers defense - hedging, collateral management and capital reserve - each of which will address a particular problem of AIG's collapse and is rooted deeply in economics.

The rest of the study is organized as follows. Section 2 reflects upon the AIG collapse and pinpoints the major causes for AIG's fall in 2008. In Section 3, the framework of the fundamental factor model is introduced. Section 4 uses the model to develop the possible cross-market hedging strategies for CDS-CDOs. Section 5 discusses the collateral arrangement based on CVA metrics. In Section 6, we show how capital reserve for CDS should be determined based on our fundamental factor model. Section 7 concludes the report.

2. A Reflection on How AIG Collapsed

2.1 AIG Bail-out Recap

AIG's fall epitomizes the aforementioned issue and the ongoing debate on managing complicated financial products. It is generally agreed that AIG's credit default swap portfolio on super-senior tranches of multi-sector collateralized debt obligations (CDS-CDOs) was the fatal cause of AIG's "death" (Sjostrom Jr, 2009; Boyd, 2011). The disruption in the housing market in 2007 seriously devalued the mortgages and mortgage-based derivatives (American International Group, 2007, 2008). Through both direct investments in RMBS and the CDS-CDOs position, AIG maintained a gigantic exposure to the mortgage market and hence had to record billions of realized and unrealized losses. Questioning AIG's ability to fulfill its obligations, counterparties to AIG initiated one collateral call after another, demanding liquid assets amounting to more than $50 billion - a disastrous number that pushed AIG towards the verge of bankruptcy which was avoided only after the Federal Reserve put together an $85 billion credit facility to bail out AIG (Sjostrom Jr, 2009)

However, it is inaccurate to solely attribute the consequence to CDS-CDOs or the abrupt decline of housing market. The design of the CDO aims to benefit from risk diversification and risk transfer through securitization (Lucas et al., 2007). CDS arose from the need for credit risk protection. Both of these instruments, in theory, achieve better risk allocation among players in the financial markets. As long as the risks are priced appropriately to compensate the firms who bear the risk, nothing is economically wrong with the CDS-CDOs themselves. Hence, AIG fell not because they maintained a CDS-CDOs portfolio on their book but rather because they took on risks which exceeded their capability to manage effectively.

2.2 Cause I: Gigantic Unhedged Exposure

Firstly, the most identifiable problem was AIG's gigantic exposure to the mortgage market through both direct investments and the unfunded CDS portfolios. In 1998 AIG started dabbling in the CDS business through its subsidiary, AIG Financial Products (AIGFP). AIG gradually became the major protection seller in the market, mainly due to AIGFP's AAA credit ratings inherited from AIG via the contractual guarantee agreement (TPM, 2009). Such a AAA-rating business model enabled AIGFP to price its CDS competitively and accumulate market share quickly.

Throughout the early 2000s, the market was deregulated and financial innovations were encouraged, AIGFP expanded its CDS coverage extensively from conventional corporate bonds to more exotic OTC contracts on CDOs. According to American International Group (2007), AIGFPs CDS position had grown quickly from $203 billion in 2003 to as large as $527 billion by the end of 2007. Of the $527 billion, $378 billion was sold to mainly European banks with the purpose of reducing the capital requirements ("Regulatory Capital Relief") and $149 billion, including the toxic $78 billion protection on CDOs, was contracted with clients, such as Goldman Sachs, who were interested in then-popular negative-basis arbitrage ("Arbitrage Portfolio") (American International Group, 2008).

Such a position, with hindsight, was too large given that AIG had only $2.3 billion in cash and $95 billion in equity to absorb losses. The question of their capability to handle such a large position was even more confounded as AIG directly invested in mortgage securities in its ordinary business and securities lending program (Sjostrom Jr, 2009). Yet, as written in AIG's 2007 Annual report, the management decided not to hedge their position in CDS with the preconception that the super senior tranches were safe given their simulation and modified BET models. When the mortgage market did go against their bet, the loss was significant in the sense that CDS-CDOs solely contributed a $11.2 and $25.7 billion unrealized loss in 2007 and 2008, in addition to other realized impairment costs ($20 - $30 billion) in 2008 (American International Group, 2008).

However, had AIG intended to hedge their position some time before the financial crisis, the situation might not have been more favorable to AIG. CDS are insurance-like contracts. As CDS is the most common and thus the cheapest source of credit protection, it would have been difficult to look for cheap alternatives to remove exposure from CDS. This is especially true when (1) the secondary market of both the reference asset and the CDS are not liquid enough, and (2) the reinsurance market is not large. To cut or hedge billions of exposure in any market during a distressed time is intrinsically hard, not to mention their contracts were mostly OTC. Hence, perhaps, AIG should have not put itself into a huge CDS-CDO pool in the first place.

2.3 Cause II: Problematic Credit Support Annex

The second problem, which is also the feature that differentiates the CDS-CDOs position from other direct mortgage investments, is the collateral posting obligation embedded in the CDS contracts. Such types of obligations impose contingent demand on the firms liquidity and, in contrast to asset devaluation, the inability to fulfill the cash obligation will lead to the immediate bankruptcy of the firm. After the Goldman Sachs collateral calls as the CDO market worsened, major counterparties to AIG started following the practices, putting AIG into cash woes. The subsequent downgrade of AIG automatically applied another multiplicative factor to the already-huge collateral base. By September 2008, AIG was required to post almost $31 billion collateral solely against CDS on CDOs positions (American International Group, 2009). Adding the collateral from their security lending business and their Guaranteed Investment Agrement (GIA) obligations, the number was as high as $54 billion - an amount that was filled only after the Fed stepped in (Sjostrom Jr, 2009; Boyd, 2011)

The collateral crisis on one hand was the result of management negligence but, fairly speaking, the terms of the Credit Support Annexes itself made collateral management difficult for AIG. According to AIG's FY2008 10-K, the calculation of collateral delivery amount against CDS-CDOs position is a two-step process:

1. Determine the exposure: computed as the difference between the net notional amount and the *market value* of the underlying CDOs;
2. Determine the delivery amount: the collateral to be delivered equals the net exposure which is the exposure less the collateral which has already been posted.

In additional to the above framework, AIG is allowed to

- post no collateral when the underlying CDO loss is below a certain threshold, typically 4% as in the contracts with Goldman Sachs.
- post less collateral with a multiplicative factor based on AIG's credit rating.

There are at least 4 potential problems associated with the aforementioned CSA. Firstly, instead of using the replacement value of the contract, the collateral is "market-price based" in which the "market value of the relevant CDO security is the price at which a marketplace participant would be willing to purchase such CDO security in a market transaction on such date". For an illiquid reference asset, such as a CDO security, whether the market quote is an accurate proxy of the true economic value of the asset is questionable. This is especially evident in a distressed financial market. Stanton and Wallace (2011) shows that the ABX.HE index during the crisis was traded at extremely low levels, implying an unrealistically high default rate and low recovery rate which have never been observed during the Great Depression. A collateral mechanism based on market information can be overly volatile due to market sentiment. Moreover, the purpose of collateral is to ensure the "payment of CDS when the reference actually defaults" instead of "hedging the value of instrument". Marked-to-market at an extremely distressed market is overly stringent for AIG.

Secondly, the mechanism of the collateral threshold and the dependency of the collateral amount on their credit rating directly resulted in the risk of a "jump in liquidity needs". In other words, when the CDO market loss hits a threshold or their credit rating is lowered, the collateral call can often be sudden and large. Moreover, the unexpected cash needs often coincide with financial distress which could lead to considerable difficulties in raising additional funds. Hence, such a jump can be detrimental as the cash-based obligation is probably one of the hardest types of obligation to fulfill and leads to immediate default no matter how many assets the firm has on its book.

Thirdly, the trigger rule, despite it appearing to be beneficial to AIG, tends to hide the liquidity risk and can lead managers to overlook the collateral management practices. The super-senior tranches of CDOs were considered to be of really low default risk before the crisis. Hence the super-senior tranches were trading near par and AIG was not required to post collateral as CSA was not triggered. This induced the management to accumulate a large position while being unaware of their liquidity exposure, in that their position was almost free of a cash burden. However, when the underlying asset went bad, as in the case of AIG, the liquidity risk was be too large to be managed.

Lastly, the CSA design is loosely related to the credit risk of both counterparties and the variability of the underlying. On one hand, the determination of the collateral trigger appears arbitrary and rigid over the life of the CDS contract. On the other hand, the infrequent adjustment of the credit rating means the counterparty risks are not captured timely. Both of the aforementioned disadvantages suggest the lack of an economic interpretation of these CSA settings, hide the true nature of counterparty risk and hence make the collateral hard to manage.

2.4 Moral Hazard of the Bailout

The FRBNY thought it would be unethical to leverage this threat of bankruptcy, since it knew it was not willing to let AIG default. By imposing onerous conditions to the $85 billion RCF, the Fed did a good job of minimizing the moral hazard in the short run. Unfortunately, this did signal that they were not willing to let AIG default, which ultimately led to the much larger moral hazard of bailing out the AIG counterparties.

As acknowledged by Fed Vice Chairman Donald Kohn at a Senate Banking Committee hearing, the aid to these counterparties contributed to moral hazard and "will reduce their incentive to be careful in the future". The fact that the counterparties did not take a haircut has been widely criticized, including by Neil Barofsky, the former inspector general of the Troubled Asset Relief Program (TARP). Barofsky told congress that "No lessons were learned from the counterparties, other than, if you do business with a giant, too-big-to-fail institution, you don't need to worry about it because Uncle Sam is going to sit there and backstop all of your bad bets."[1] This could led to a market distortion where investors, creditors and counterparties don't bother with due diligence of these large, interconnected financial institutions, which in turn could lead to lower borrowing costs for these financial institutions.

[1] Quote retrieved from http://hereandnow.wbur.org/2013/09/13/tarp-watchdog-banks

According to Senator Elizabeth Warren[2], the Too-Big-to-Fail problem has in fact gotten worse since the financial crisis. In a speech promoting the 21st Century Glass-Steagall Act, she said

> *Today, the four biggest banks are 30% larger than they were five years ago. And the five largest banks now hold more than half of the total banking assets in the country. One study earlier this year showed that the Too-Big-to-Fail status is giving the 10 biggest U.S. banks an annual taxpayer subsidy of $83 billion.*

3. Fundamental Factor Model

The aforementioned analysisl of AIG's fall suggests that at least three lines of defense can be establish to mitigate, if not avoid, the downside risk of a huge mortgage position. The firm should (1) reduce their exposure by either trimming their position or hedging their losses, which could avoid the abrupt loss of credibility and hence maintain their funding capability in a disrupted market; (2) establish a better collateral agreement that will facilitate a better understanding of credit exposure and collateral management, and (3) set aside an adequate amount of firm-wide capital to serve as the last defense should the above trade-specific risk management fail.

However, as the protection seller of bonds, looking for other alternatives to directly short the credit risk is difficult, as CDS is already the cheapest credit instrument. Hence, to avoid enormous hedging costs, especially when hedging a large position, cross-hedging tends to be a more feasible solution. Candidates for these instruments are the housing price index, ABX indices as well as put options on firms which have exposure to the mortgage market. These choices are based on the economic reasoning that the dynamics of these assets are more or less driven by the mortgage market.

3.1 Factors Specifications

This idea suggests the use of a fundamental factor model instead of the common statistical model, such as copula or BET. The fundamental models can identify inherent risks and establish sound hedges. Specifically, we assume that the economy of mortgages is primary driven by housing prices (housing index) and the interest rate. Mathematically, we adopted the log-normal model to capture the dynamics of housing prices

$$dH_t = (r_t - q_t)H_t + \sigma^H dW_1 \tag{1}$$

and the interest rate r_t is given using the one-factor Hull-White model (Hull and White, 1993).

$$dr_t = \kappa(\theta_t - r_t)r_t + \sigma^R dW_2 \tag{2}$$

[2]Quote Retrieved from https://www.commondreams.org/headline/2013/11/12-11

where q_t in equation (1) is the rental yield and assumed to be a constant $q_t = 0.25$ following Downing et al. (2005); W_1 and W_2 are Wiener processes under the risk-neutral measure and are assumed to be independent for simplicity. That is, we let

$$dW_1 dW_2 = 0, \quad q_t = 0.25 \quad \forall t > 0 \tag{3}$$

3.2 From Factors to Cash Flows of Mortgages

Proportional Hazard Rate Model

It is known that the cash flows of mortgages, and thereby mortgage-backed securities, depend heavily on prepayment risk and default probability. To model the influence of the housing index and the interest rate on mortgage payments, we adopted the proportional hazard rate model proposed by Schwartz and Torous (1989) in which (1) prepayment / default intensities are directly captured, and (2) the economic variables are explicitly incorporated. Mathematically, either the prepayment or the default hazard will follow

$$\lambda(t, X_i) = \lambda_0(t)\exp\{\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n\} \tag{4}$$

where $\lambda_0(t)$ is the baseline hazard controlling the shape of term structure, and the linear combination of covariates X_i's determines loan-specific shift of prepayment / default intensities. For the baseline hazard, we chose the log-logistic function

$$\lambda_0(t;\lambda,p,\gamma) = \frac{\lambda p(\lambda t)^{p-1}}{1+(\lambda t)^p} \tag{5}$$

The major benefits of such a functional form is its flexibility in fitting a variety of shapes of prepayment or default pattern, such as the well-known burn-out phenomenom of mortgages.

The selection of the covariates will determine how mortgage cash flows depend on the macro economy and the deal specifications. In this project, the following covariates are assumed in the proportional hazard rate model:

- **Coupon Differential**, which is defined as the difference between the coupon on the loan and the 3-month lagged 10-year interest rate. That is

  $$X_1(t) := \mathbf{CD}(t) := C - R^{(3)} \tag{6}$$

  A large coupon differential induces mortgage borrowers to prepay the loan and refinance the mortgage at a lower rate;

- **Loan-to-Value Ratio**, which is defined as

  $$X_2(t) := \mathbf{LTV}(t) = L_t/V_t \tag{7}$$

  where L_t is the remaining principal of the loan at time t and V_t is the value of the property at time t. To model the dynamics of the property value, we further assume that the return of V_t coincides with the return of housing index

  $$\frac{dV_t}{V_t} = \frac{dH_t}{H_t} \tag{8}$$

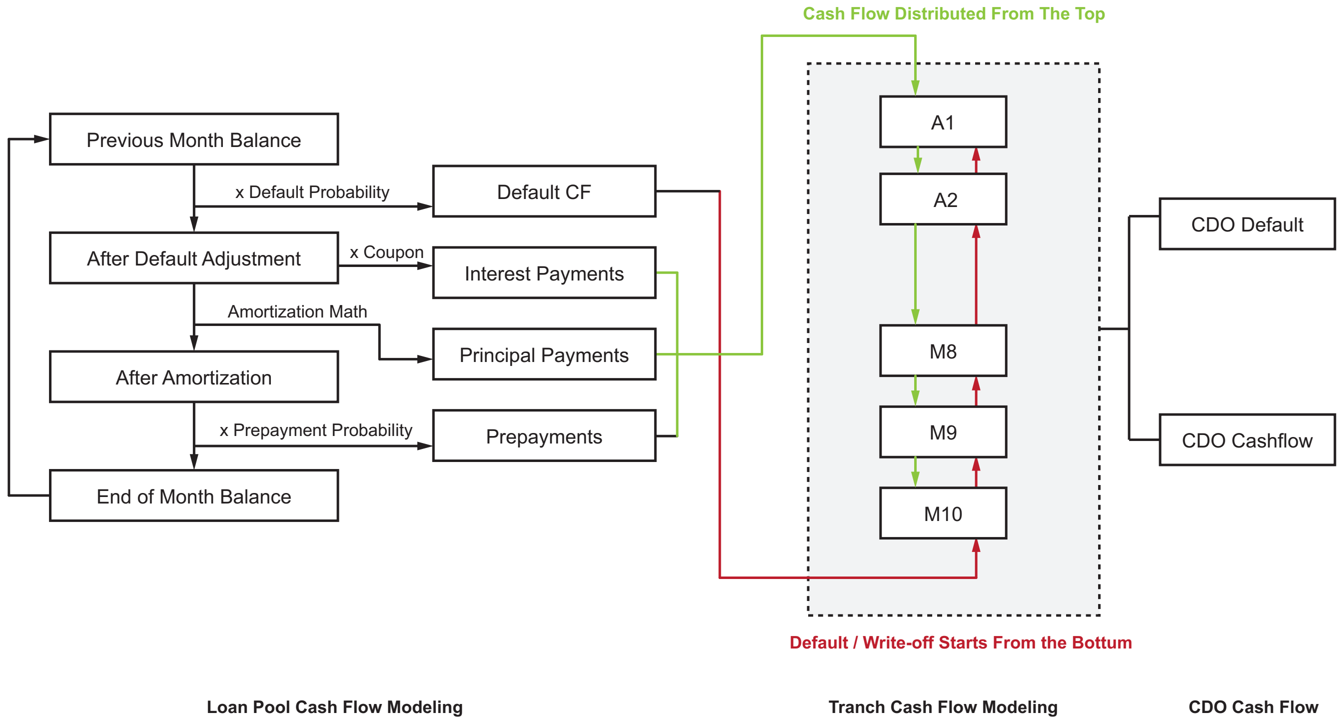

Figure 1. Schematic Design of MBS Cash Flow Modeling

The two covariates only explain the impact of the macroeconomic factors on the mortgage cash flows. However, each loan pool can also be influenced by deal-specific variables, such as credit scores, property location etc. To account for these idiosyncratic components, we add a time-varying "factor" to proxy the residual information. Mathematically, the "third factor" takes the form

$$X_3(t) := \beta_0 + \varepsilon(t) \quad \varepsilon(t) \sim \text{i.i.d}\, N(0, \sigma^2) \tag{9}$$

and we further assume that the idiosyncratic factors are independent among deals. Combining the equation (6), (7), (9) with (4). Our modified hazard rate is

$$\lambda(t) = \lambda_0(t)\exp\{\beta_0 + \beta_1 \mathbf{CD}(t) + \beta_2 \mathbf{LTV}(t) + \varepsilon(t)\} \tag{10}$$

which can be obtained through simulation when the parameter is calibrated.

Model Calibration

To calibrate the model, we used the mortgage loan-level data. With the hazard rate model above, we adopted the MLE to obtain the estimates.

Mortgage Cash Flows and Defaults

With the hazard rate term structure, $\lambda(t)$, the prepayment or the default probability in a given period is

$$PD(T_1, T_2) = \exp\left\{\int_{T_1}^{T_2} \lambda(t)dt\right\} \approx \exp\{\lambda(T_2)(T_2 - T_1)\} \tag{11}$$

where the last term approximates the probability with a piecewise constant hazard rate. Let $PD_P(t)$, $PD_D(t)$ denote the prepayment probability and the default probability respectively, then the mortgage payment can be computed through the following four steps (See Figure 1)

1. Given the last period ending balance $L(t-1)$, we can compute the average default amount of the mortgage pool with

$$\mathbf{Default}(t) = L(t-1) \times PD_D(t)$$

2. Then the amortization math can be applied to the remaining non-default balance of the bond to compute the principal and the interest portion of the payments. Denoted with $P(t)$ and $I(t)$ respectively.

3. After the amortization is determined, the prepayment is obtained by applying $PD_P(t)$ to the balance after the default and principal payment are deducted.

$$\mathbf{Prepayment}(t) = [L(t-1) - P(t) - \mathbf{Default}(t)] \times PD_P(t)$$

4. The ending balance is then

$$L(t) = L(t-1) - P(t) - \mathbf{Default}(t) - \mathbf{Prepayment}(t)$$

Iterating the above four steps until the mortgage pools terminate will give us the default amount and the cash flows available for distribution for each period t.

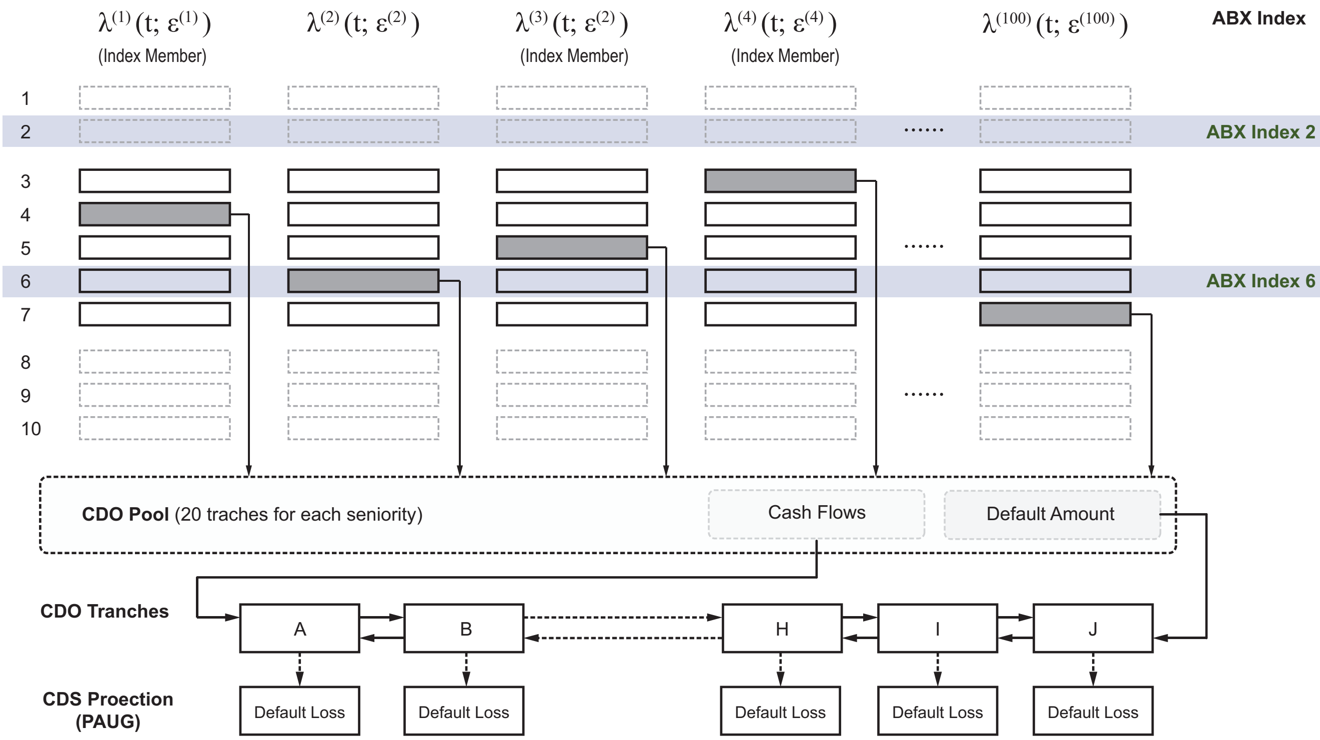

Figure 2. Schematic Design of CDOs, ABX Cash Flows Modeling

3.3 From Mortgage Pool to MBS Cash Flows

Given the cash flows and the amount defaulted of the mortgage pool, we can determine the cash flow that each tranche receives based on the structure of the MBS. Here, for simplicity of illustration, we assume that all MBSs have a simple waterfall structure *without* the accrued interest class (Z), overcollateralization (X) and the residual class (R) which are often observed in real deals.

With this simple deal structure, as shown in Figure 2, cash flows are distributed according to the seniority of the tranches, with the most senior bond paid off first. On the contrary, the default amount will cause the least senior tranche to be written off first. After obtaining the period-by-period cash flows and default amounts, we can model the cash flows of CDO and other mortgage related securities.

4. Cross Market Hedging of CDS-CDOs

4.1 Hedging with ABX.HE Indexes

In this study, we simulated 100 MBS deals by modeling at the pool level the cash flows of defaults and prepayments. The MBS subordination structures in those deals are all identical and given in Table 1. We randomly selected 100 tranches with 20 tranches from tranche 3 to 7 each and one tranche from each MBS deal to form a CDO. The CDO subordination structure is the same as the MBS deals. We performed Monte Carlo simulation with 2000 interest and HPI paths, and analyzed the joint distribution of the loss of the senior CDO tranche and the average loss of the MBS tranches. The average losses from the MBS tranches were calculated from 20 randomly selected deals. The CDO senior tranche incurred losses on 727 paths out of 2000.

Table 1. Subordination Level of the Representative Deal

Tranche	Subordination (in %)	Spread (in bps)
1	22.1	18
2	18.4	26
3	13.5	33
4	11.5	38
5	9.8	41
6	8.2	47
7	6.7	80
8	5.3	100
9	4.2	200

In our simulation, we observed that the senior tranche of the CDO, formed from equal numbers of tranche 3 to 7 of the MBS deals, behaves generally in par with MBS tranche 3 and 4. The CDO senior tranche loss cash flows are very similar to those of the MBS tranche 3 and 4. They start to incur losses similarly. This is consistent with the diversification effect achieved with the CDO structuring. However, contrary to the intention of most CDO deals in reality, we found the

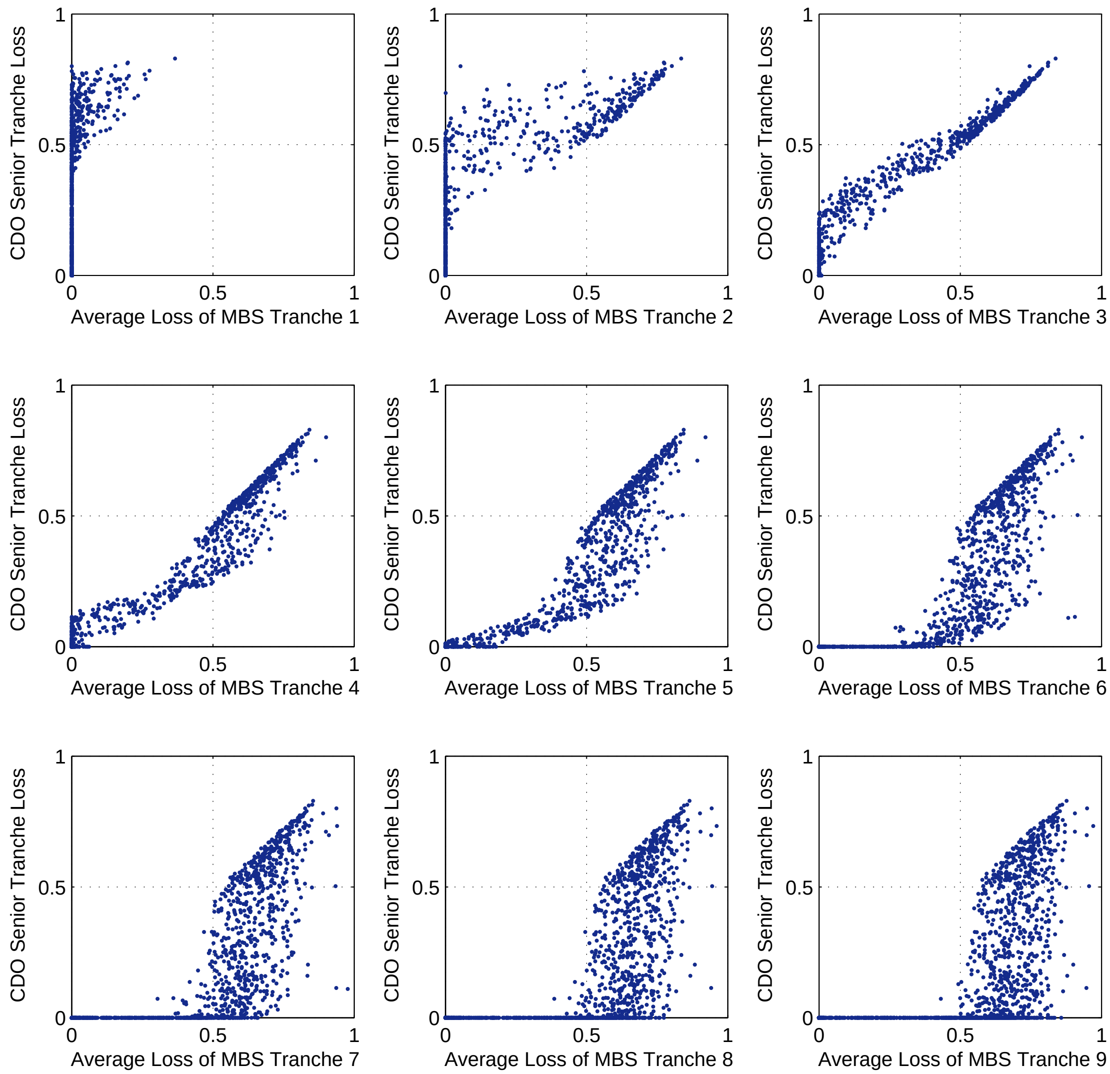

Figure 3. The joint distribution of the loss on the CDO senior tranche and average loss of different MBS tranches

senior CDO tranche is inferior to the most senior tranche of the MBS deals. Our simulation shows that the senior tranche of our CDO incurred about 50% loss before the most senior MBS tranche started to incur losses. This is likely due to the fact we calibrated parameters with subprime loans with high default rates, therefore the junior tranches had higher loss correlations, undermining the effect of diversification in the CDO deal.

In reality, a viable hedging strategy for the CDS on the senior tranche of CDO deal can be constructed as follows. First, the cash flows of the MBS deals are modeled. This should preferably be done at the loan level, utilizing as much as loan level information as possible. Then the cash flows to the CDO (and thus CDS) and the 20 ABX deals are simulated under the same sets of scenarios. Their joint loss distributions are analyzed, and the corresponding ABX class with the highest loss correlation should be chosen as the hedging instrument. In our simulation, the average of MBS tranche 4 and 5 are arguably the best hedging instruments, since they incur losses at the same points as the senior CDO tranche and the expected losses exhibit a linear relationship when they start to occur.

4.2 Hedging with Interest Rate Futures and HPI Futures

Any financial instruments used to hedge large short CDS positions should have the following desirable features:

1. the instrument should have minimal counterparty risk so that the hedge does not fail in times of stress when large default payments need to be made. Given this consideration, we prefer exchange-traded instruments for hedging over-the-counter instruments.

2. instruments used in dynamic trading strategies for hedging large positions should be highly liquid and have low bid-ask spreads so that transaction costs are not prohibitive, and the market does not move against the trade. The inherent standardization in exchange-traded contracts lends itself to this consideration.

We describe a dynamic hedging strategy in which the hedge needs to be rebalanced every period. Interest rates affect CDS prices in 2 different ways: 1. cash flows are discounted by interest rates, and 2. default rate changes with interest rate due to which cash flows from the CDS change. However, for CDS contracts, the default effect is expected to dominate the discounting effect. Since ARM default rates increase as interest rates increase, we expect the CDS value from the seller's point of view to decrease when the interest rate increases. Similarly, during a housing market boom, we expect the default rate to decrease which would increase the CDS value from the seller's point of view.

To obtain interest rate duration and housing price index (HPI) duration of the CDS contract, we run a regression of the form

$$CDS_i = \alpha + \beta \cdot r_i + \gamma \cdot HPI_i + \varepsilon_i$$

where i indexes the Monte Carlo paths, CDS_i represents CDS MTM along path i at time $t = 1$, r_i represents interest rate along path i at time $t = 1$, HPI_i represents housing price index value along path i at time $t = 1$ and ε_i is the error term. To avoid in-sample fitting bias, we only use half of our Monte Carlo paths to generate this regression fit. Given estimates for the parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$, our estimate for interest rate duration at time $t = 0$ is $D^{ir}_{cds} = -\frac{\partial CDS}{\partial r} = -\hat{\beta}$ and for HPI duration at time $t = 0$ is $D^{hpi}_{cds} = -\frac{\partial CDS}{\partial HPI} = -\hat{\gamma}$. Based on our simulation, we obtain $\hat{\alpha} = 4.67 * 10^6$, $\hat{\beta} = -4.29 * 10^8$, $\hat{\gamma} = 2.33 * 10^6$ and $R^2 = 58.69\%$. The high negative value of $\hat{\beta}$ and high positive value of $\hat{\gamma}$ both make intuitive sense as described previously. In Figure 4, we show a fit of actual CDS MTM at time $t = 1$ v/s estimated CDS MTM at time $t = 1$.

We now need to choose appropriate hedging instruments. Since a wide variety of exchange-traded interest rate contracts are available in the market, we run a regression between interest rates and CDS prices at time $t = 1$ and notice a clear linear relationship between the two. Hence, we choose CME Eurodollar futures as the interest rate hedging instrument. CME Eurodollar futures contracts are highly liquid standardized exchange traded contracts with almost no counterparty risk and well-suited to act as a hedging instrument. Since Eurodollar future prices are given by $100 - r$, they have an almost linear exposure to interest rates and hence, are well-suited for hedging interest rate exposure of CDS. The notional amount of Eurodollar futures to enter into is calculated as $N_{ir} = -\frac{D^{ir}_{cds}}{D^{ir}_{fut}} = \hat{\beta}$ so as to make the hedge portfolio duration-neutral.

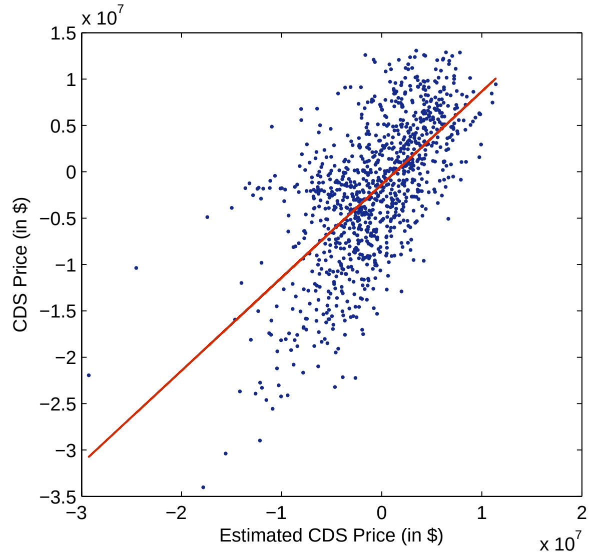

Figure 4. Regression of Actual CDS MTM vs Estimated CDS MTM

The regional diversification of mortgages underlying MBS and CDO instruments means that CDS instruments have exposure to US-wide housing market. As such, we choose CME Housing Composite Index futures to hedge house price exposure in CDS contracts. For clarity, we describe the contractual details of the instrument briefly below.

The S&P/Case-Shiller home price indices are designed to be a consistent benchmark of housing prices in the United States. They measure the average change in single-family home prices in a particular geographic market. The indices are calculated with the repeat sales method, which uses data on properties that have sold at least twice, in order to capture the true appreciated value of constant quality homes. The main variable used for index construction is the price change between two arms-length sales of the same single-family home. The S&P/Case-Shiller National Home Price index is a composite of single-family home price indices for the nine US Census divisions and is calculated quarterly (S&P Indices, 2011).

The S&P/Case-Shiller home price index is tradeable through futures on the Chicago Mercantile Exchange, known as Housing Composite Index (HCI) Futures. These futures are cash-settled and the underlying is 250 times the S&P/Case-Shiller National Home Price index. Typically, contracts expiring during the next 6 quarters are available to trade at any given point of time.

From empirical data, we observe that the bid-ask spread for front-end expiring HCI Futures is approximately 4% of notional. Using trading volume data for 2012 and 2013, we observe that the approximate annual trading volume is 300 contracts. The bid-ask spread and annual volume most likely make HCI Futures infeasible as a hedging instrument but we hope that greater awareness, more quotes from other traders, and a willingness by traders and hedgers to dabble in these products will improve the liquidity of futures on this critically important financial market. (Dolan, 2011)

Since HCI futures duration can be written as $D_{hpi}(t) = \exp((r_t - q)*(T-t))$, where q = dividend yield on HPI and T = settlement time of HCI future, we can calculate the notional amount of HCI futures to enter into as $N_{hpi} = -\frac{D^{hpi}_{cds}}{D_{hpi}} = \exp(-(r_0 - q)*T)*\hat{\gamma}$. We obtain $N_{ir} = -4.29*10^8$ and $N_{hpi} = -2.34*10^6$.

Finally, the hedge portfolio MTM can be calculated as $CDS^H_i = CDS_i + N_{ir} * IR^{MTM}_i + N_{hpi} * HPI^{MTM}_i$, where IR^{MTM}_i= MTM of Eurodollar futures contract along path i at time $t = 1$ and HPI^{MTM}_i= MTM of HCI futures contract along path i at time $t = 1$. We use the Monte Carlo paths not used to calculate durations to evaluate the performance of our hedge portfolio over the next period. We observe that 1. the standard deviation of the MTM portfolio value at time $t = 1$ decreases from $8.02*10^6$ to $4.69*10^6$, and 2. we eliminate fat tails to quite a large extent. The strategy described above can now be reused to update the hedge portfolio at time $t = 1$. Figure 5 shows the probability distribution of the hedged and unhedged portfolio.

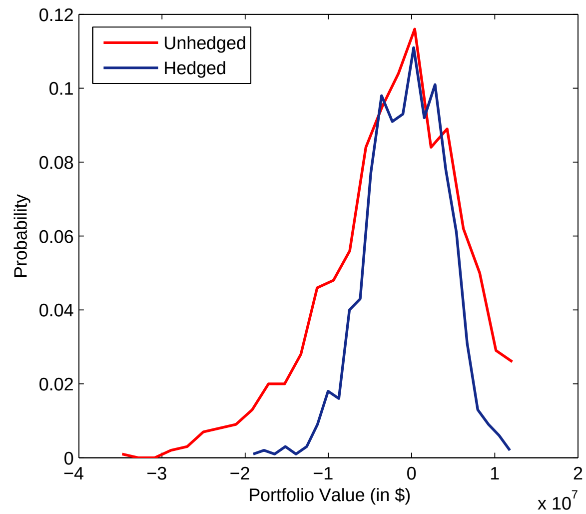

Figure 5. Probability Distribution of Hedged and Unhedged Portfolio Value

4.3 Hedging with Puts on Equity

AIG's leveraged positions accumulated a large exposure to the mortgage market. The total credit exposure of the CDS position represents a large portion of subprime mortgages trading in the market. On the other hand, the supply chain of the subprime mortgage market in the US can be split into just a few parts: origination, aggregation and securitization. The subprime residential mortgages are created through two major sources: wholesale and retail. The US subprime mortgage market is highly concentrated in the sense that more than 60% of the originations are dominated by a few large wholesale originators: banks, thrifts and unaffiliated mortgage originators. (Stanton et al., 2014)

According to a report of The Home Mortgage Disclosure Act (HMDA), the top forty lenders account for more than 90% of the residential mortgage origination in 2006. The top 10 lenders accounted for more than 60% origination in 2006 . These mortgage originators were highly leveraged on short term financing using Repo or Asset Backed Commercial Paper market. Because asset backed securities account for a large portion of their balance sheet before securitization. Therefore essentially they have similar credit exposures to the CDS AIG had underwritten. For some independent mortgage companies, such as Countrywide Financial Corp and New Century Financial Corp, the entire business was based on subprime mortgage origination. A natural hedging bet would be that when the subprime mortgage market suffers, the stock prices of these companies will also drop. We want to buy a basket of 10% out of the money put options to hedge AIG's large CDS exposure to subprime mortgage CDOs. Considering the size of the CDS exposure that AIG had, there were very few instruments that could hedge the credit risk of subprime mortgages.

We collected the following data for the study of this empirical relationship.We downloaded the ABX AAA 2005-2 weekly index from Bloomberg during period between 2007/8/31 to 2013/9/27. We obtained the publicly traded put option data for companies in HMDA's top 40 originators: Countrywide Financial Corp. , Wells Fargo Co., Washington Mutal Bank, Citigroup, JPMortgage Chase Corp., Bank of American Corporation, Wachovia Corp., GMAC Residential Capital Group, Indymac Bank, GMAC Residential Holding Corp, EMC, SunTrust Bank, PHH group, Capital One Group, BB&T, New Century Financial, National City Corp, US Bancorp and Mortgage IT. The option data were obtained from OptionMetrics through WRDS. For each company, we selected all the put options to be at least 10% out of the money and have open interest larger than 600.

The empirical study in Figure 6 shows a clear negative correlation between the ABX prices and put option prices for the full sample period from 2007. Since we do not have the real market data on the CDS position AIG held, this study will at least tell us how robust the hedge is. For the subprime mortgage market in general, we notice that our data may be biased after 2008 financial crisis as a lot of companies have shrunk their holdings of asset backed securities. For example, the private label MBS market is still very small. But in general, we see ABX as an important global economic indicator of the US housing market and it should be positively correlated with the financial sector. Based on the regression, put options can achieve hedging to cover 1.5 times the ABX index loss on average. The adjusted R-squareds are between 1% and 16%. If we choose a subset of the companies that had more than 10% R-squared and perform a multi variate regression of ABX returns on put options, we essentially create a minimized variance hedging portfolio. Our empirical study shows that the residual variance of the hedged portfolio is less than 40% of the unhedged portfolio. Although there are not enough DOOM put options on the market to cover the entire huge CDS exposure, we still believe this hedging method should at least cover some exposure at a reasonable prices.

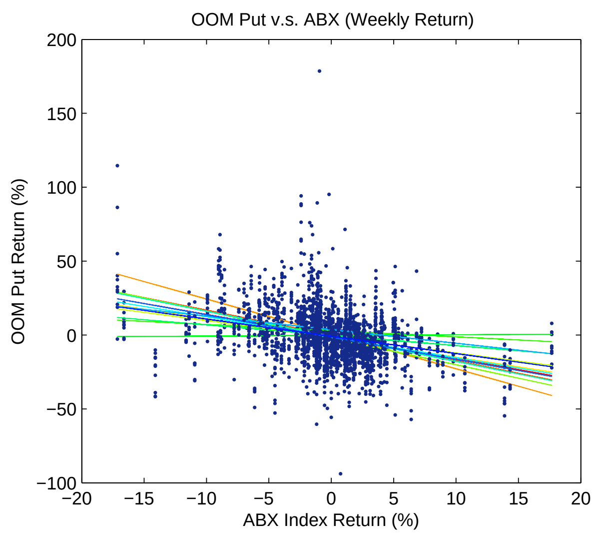

Figure 6. ABX Put Return

5. CVA-based Collateral Agreement

5.1 Collateral, The Economics Behind

Let's consider a simple contract with counterparty risk. We denote the two counterparties A and B. The contract pays a lump sum at time T. The lump sum payoff is stochastic, with a symmetric distribution with zero mean. For example, the payoff can be a driftless Wiener process at T. The expected payoffs for both parties are 0 at T without considering the counterparty risks.

Now let's consider the scenario that A is default-risk free, but B has a default risk. Now, A faces the counterparty risk from B, with a non-zero probability that B defaults when the contract matures in money for A at time T. It follows that the value of the contract to A is less than 0 at $t = 0$. The present value of A's expected loss for this contract, which is the definition of the credit value adjustment, is

$$\mathbf{CVA}_A = DF(0,T)\cdot\mathbf{LGD}\cdot PD_B^Q(0,T)\mathbb{E}^Q[\max(V_T,0)] \quad (12)$$

where $\mathbf{CVA}_A$ is the credit value adjustment to party A, $\mathbf{LGD}$ stands for the loss given default, $PD(0,t)$ is the probability of default and V_T is the value of the contact.

At time $t = 0$, the value of this contract from A's point of view is $-\mathbf{CVA}_A$, since the expected payoff is 0 without counterparty risk, and there is no initial cash exchange. However, from B's point of view the contract still has value 0. In other words, the contract is unfair for A.

There are several ways to remedy B's credit risk. The first way is that B pays half of $\mathbf{CVA}_A$ to A at $t = 0$, so that the values of the contract to both parties are the same.

The second way is that B can post full mark-to-market collateral to A. We assume that the collateral is posted continuously, default of B is recognized immediately, and there is no additional cost to the mark-to-market price for A to replace the contract. In case B defaults, A can always replace the original contract with another party using the collateral B posted. In this case, the expected loss for A is 0, and so is $\mathbf{CVA}_A$. The contract value is 0 for both parties again.

Now we consider the case of both A and B having default risk. B's expected loss is similar to A, with reversed sign of payoff.

$$\mathbf{CVA}_B = DF(0,T)\cdot\mathbf{LGD}\cdot PD_A^Q(0,T)\mathbb{E}^Q[-\min(-V_T,0)] \quad (13)$$

The contract value for A and B is

$$V_A = V_0 - \mathbf{CVA}_A \quad (14)$$

$$V_B = -V_0 - \mathbf{CVA}_B \quad (15)$$

For a fair trade, we have $V_A = V_B$, or

$$2V_0 - \mathbf{CVA}_A + \mathbf{CVA}_B = 0 \quad (16)$$

If A and B have different probability of default, $\mathbf{CVA}_A$ and $\mathbf{CVA}_B$ will not be equal. For equation (16) to hold, there are several ways. For example, A and B can arrange initial cashflows, so the V_0 is non-zero. Alternatively, they can adjust their collateral, so that $\mathbf{CVA}_A = \mathbf{CVA}_B$, and V_0 can be kept at 0. By adjusting their collateral, we mean adjusting the collateral scheme. A collateral scheme is a rule decided at inception of the contract which determines the amount of collateral that each party has to post under different market conditions.

There can be different collateral schemes, for example, a fixed percentage of the mark-to-market value of the contract. A more complex collateral scheme will consider not only the mark-to-market value but also the counterparty risk value adjustments. However, the value of the contract becomes hard to estimate under this scheme, because the counterparty risk value adjustment depends on the collateral, which in turn depends on the counterparty risk value adjustment. In this study, we only consider posting fixed percentages of mark-to-market value as collaterals, without considering the counterparty risk value adjustment.

Now we return to our simple contract, and determine how much collateral each party needs to post. Suppose A has decided to post a fraction of α_A of mark-to-market value as collateral, the question is, what fraction α_B of mark-to-market value B should post. We consider the simplest case, in which the default hazard rates of A and B are constant during the contract time.

From $\mathbf{CVA}_A = \mathbf{CVA}_B$, we can rearrange

$$(1-\alpha_A)\left(1-e^{\lambda_A T}\right) = (1-\alpha_B)\left(1-e^{\lambda_B T}\right) \tag{17}$$

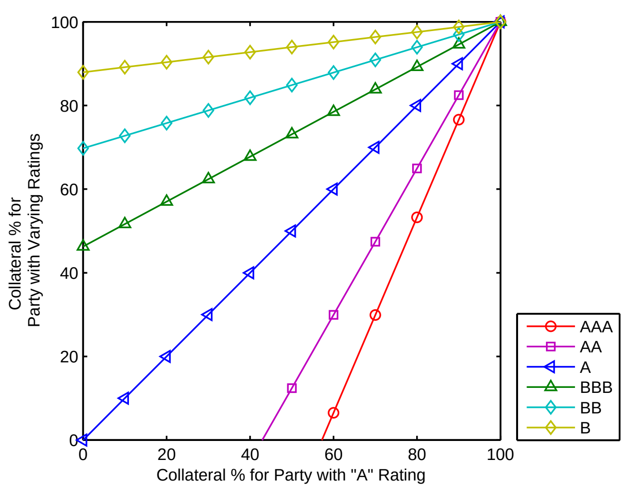

Figure 7. The Collateral Percentage Relationship of an A-rating Party Facing Counterparty With Different Ratings

We calculated the relationship of α_A and α_B, with different combinations of counterparty risks. Figure 7 shows the relationship of collateral percentages when an "A" rated party is facing a counterparty with a different rating.

5.2 Implementation of CVA

Discretizing the CVA expression in Equation 12, we get

$$\mathbf{CVA}(0) = (1-R)\sum_{i=0}^{T-1}\mathbb{E}^{Q}\left[\frac{1}{2}\left(E(t_i)DF(0,t_i)\right.\right. \tag{18}$$

$$\left.\left.+E(t_{i+1})DF(0,t_{i+1})\right)\left(S(t_i)-S(t_{i+1})\right)\right] \tag{19}$$

From the above relationships, it is evident that we need the value of the CDS contract at future times to calculate CVA. The most obvious but computationally expensive methodology (and hence not possible in practice) to calculate CVA is simulating multiple paths from each future node of the original simulated paths (originating at $t = 0$) and using equation 21 to get the CVA at each node. In order to avoid simulations inside simulations we propose an application of the Least Squares Monte Carlo (LSM) approach proposed by Longstaff and Schwartz (2001) to calculate CVA at future times.

The LSM approach proposes that the expected continuation value of an American contract at time t and node i of the simulated paths can be estimated by doing least squares regression on the pathwise discounted values of the realized cash flow. The fitted values from this regression are the expected continuation values. This approach is generic and can be used to calculate the expected future values of any contract. Therefore, in our study, we use the LSM approach to calculate the expected future value V_t of the CDS contract.

Choosing the right regressors is the key to improving the accuracy of the fitted values in the LSM approach. We note that the value of our CDS contract fundamentally depends on the House Price Index (HPI) and interest rate (r). We therefore perform the following regression at each time t, to get V_t at each node i:

$$\mathbf{DCF}(t) = \alpha + \beta_1 H_t + \beta_2 H_t^2 + \beta_3 H_t^3 + \tag{20}$$

$$\gamma_1 r_t + \gamma_2 r_t^2 + \gamma_3 r_t^3 + \tag{21}$$

$$\theta_1 \log H_t + \theta_2 (\log H_t)^2 + \theta_3 H_t r_t \tag{22}$$

where **DCF** is the discounted sum of the realized cash flows from time t to T (maturity of the contract) along the given path. It's worth mentioning that adding the log terms in the above regression significantly improves the regression quality in comparison to that of the regression done without log terms.

In addition to V_t at each node n of the simulated paths, we also need the risk neutral probabilities of the counterparty default. In practice, these can be calculated from the CDS on the counterparty. However, for simplicity, we have used constant hazard rates for default probabilities calculation. Denoting probability of default between t and Δt by $PD(t, t+\Delta t)$ and survival probability uptil time t by $SP(t)$, we note that,

$$PD(t, t+\Delta t) = SP(t) - SP(t+\Delta t) \tag{23}$$

where we assume a constant hazard rate over the life the firm, hence

$$SP(t) = \exp(-\lambda t) \tag{24}$$

We calculate PD between different intervals using the constant hazard rates obtained from the 4.5 years implied PD for different rating in the paper by Terry and Andrew.

Using the above methodology, the CVA for the CDS buyer is \$77485.61 and that for the CDS seller is \$19724.19 assuming that both have a AAA rating and neither party posts collateral.

As described in the previous section, we calculate the collateral for the CDS contract at time t as x percentage of the value at time t where x is calculated such that the CVA for the seller equals the CVA for the buyer. In our example, as expected, the CVA for the buyer is greater than the CVA for the seller. Therefore, the seller of the CDS contract will post collateral such that the CVA of the buyer reduces and becomes equal to the CVA of the seller.

The graph demonstrates that as the credit rating of the CDS seller deteriorates, it is necessary to post a greater percentage of the contract value as collateral. The buyer rating is assumed to be AAA for this example.

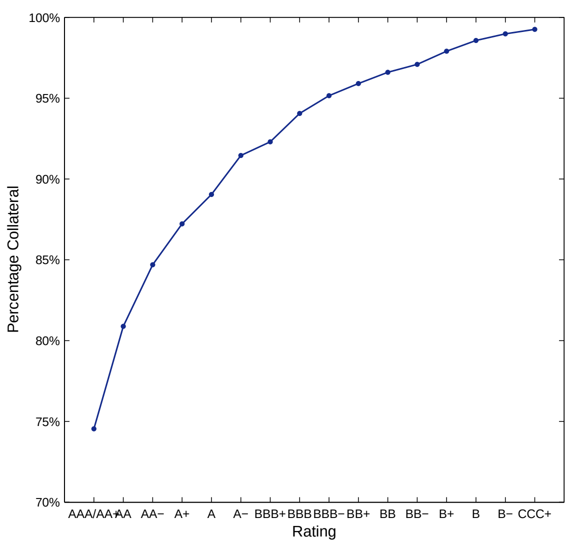

Figure 8. Collateral Percentage Variation with Seller Rating

6. Capital Reserve of CDS

A firm faces two kinds of losses on the CDS contract: i) the mark to market losses on the contract value and ii) mark to market of the CVA. Denoting time by t, value of the contract at t by V_t, collateral posted by counterparty at t by C_t, we can write the Profit and Loss equations:

$$PnL_1 = V_{t+1} - V_t - C_t$$

$$PnL_2 = CVA_t - CVA_{t+1}$$

Note that CVA is a loss by definition and hence $PnL_2$is defined by subtracting the latest CVA value from the previous value. For the purpose of illustration, we do the capital reserve calculation only for the seller of the CDS contract for a horizon of a quarter. Note that the two losses described above are correlated as both depend upon contract value. We already have 2000 samples of V_{t+1} (using the LSM approach described in the above section). For each V_{t+1}^i where i represents the *ith* path we calculate the total loss $PnL^i = PnL_1^i + PnL_2^i$ and get the total loss distribution at t+1. In order to calculate PnL_2^iwe need to get the CVA values at t+1. We once again use the LSM approach to calculate CVA at each node n at $t = 3$ months. CVA at $t = 3$ months is the predicted value of the regression below:

$$\begin{aligned} \mathbf{CVA}(t) = & \alpha + \beta_1 H_t + \beta_2 H_t^2 + \beta_3 H_t^3 + & (25)\\ & \gamma_1 r_t + \gamma_2 r_t^2 + \gamma_3 r_t^3 + & (26)\\ & \theta_1 \log H_t + \theta_2 (\log H_t)^2 + \theta_3 H_t r_t & (27) \end{aligned}$$

For estimation of the regression coefficients we use the pathwise CVA values based upon the realized cash flows along a particular path.

Capital Reserve is 1% percentile (i.e. 99% VaR) of the total PnL distribution.The graphs below illustrates that with our collateral scheme the variation of the CVA loss distribution and the hence the CVA capital reserve requirement are stable for various possible ratings of the CDS seller. This result is expected as our collateral scheme requires low rated CDS sellers to post more collateral than is posted by high rated CDS sellers. Eventually the CDS buyer faces the same CVA irrespective of the rating of the CDS seller. However if we consider a fixed collateral posting (Figure 9), the variation in the CVA losses increases as the rating of the CDS seller depreciates. Hence the capital reserve requirements for the CDS buyer will increase.

It is worth mentioning that the PnL distribution in Figure 12 looks the same. However, they are similar but not the same. The graph below shows the 1% VaR of the PnL distribution and clarifies the presence of some variation amongst the PnL distribution. In the Figure 9, The red vertical line represents 99% VaR.

We illustrated the concept of capital reserve only for one product. However, in practice, we need to take into account the net PnL of the firm taking into consideration all the positions the firm is holding and carefully netting while modeling correlation between various positions.

7. Conclusion

In this study, we revisited the series of events that led to the bailout of AIG. AIG's near-collapse is rooted in its huge

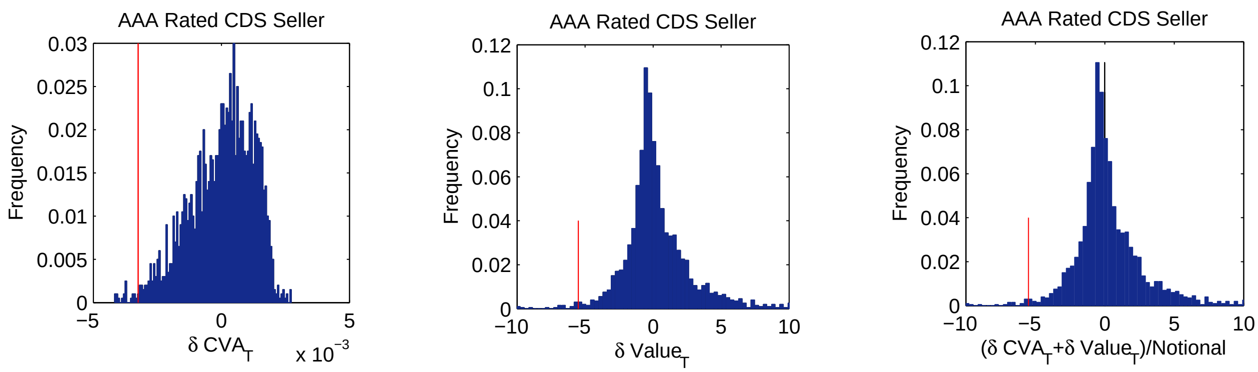

Figure 11. Decomposition of Net PnL into PnL_1 and PnL_2

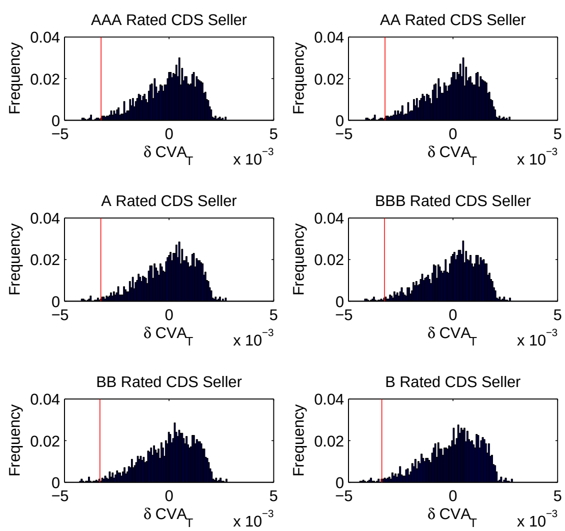

Figure 9. Distribution of CVA PnL for Different Seller Ratings with the Proposed Collateral Scheme

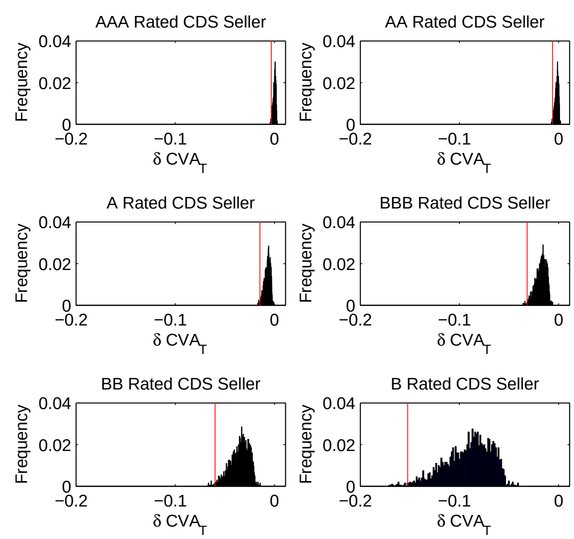

Figure 10. Distribution of CVA PnL for Different Seller Ratings with the 75% Collateral Scheme

accumulated position in CDS contracts on the super senior tranches of CDOs, which were primarily backed by subprime MBS. We implemented a macro-economic factor model to simulate the cash flows of such CDS contracts based on the HPI and the interest rate. We proposed schemes to hedge CDS contracts on MBS by ABX.HE indices, put options on mortgage market participants, and vanilla Eurodollar and HPI futures. Simulations and hedging results are presented. We modeled the counterparty risk of this type of CDS contract and the effects of collateral schemes on the CVA valuation. We demonstrated a framework to calculate the VaR on the CVA, and proposed a model using the CVA VaR as capital requirements for counterparty risk.

Acknowledgment

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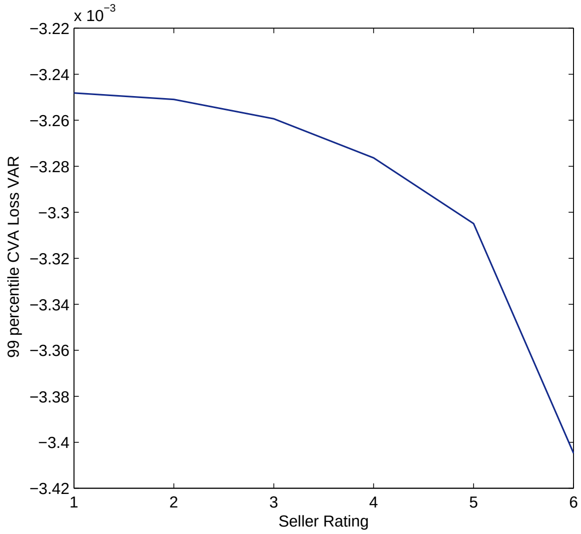

Figure 12. 99% VaR of CVA with Different Sellers Rating

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