Derivation of belief propagation equations for community / core-periphery structure

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$\psi_r^{u\to v}$ is the marginal probability that node u is type r in the absence of v, given the marginal probabilities for all other nodes besides v and knowledge about the adjacency matrix. Let I be the information: $(\forall w \neq u, v, \forall t : \psi_t^{w\to u}) \wedge A \wedge v$ is absent. Then

$$\psi_r^{u\to v} = P(t_u = r \mid I)$$

We sum over all possible type configurations ($\{w'\}$) for nodes which don't include u and v. This is OK because each $\{w'\}$ is mutually exclusive.

$$= \sum_{\{w'\}} P(t_u = r \mid \{w'\} \wedge I) P(\{w'\} \mid I)$$

We first expand the first probability as a product of independent probabilities (by our tree assumption)., where f is the function we described last Friday.

$$= \sum_{\{w'\}} \left[\prod_{w \in \{w'\}} f(t_u = r \mid t_w \wedge A_{uw}) \right] P(\{w'\} \mid I)$$

We next expand the right term as a product of $\psi' s$. Each $\psi_t^{w\to u}$ is unaffected by I: the only difference between I and the information $\psi_t^{w\to u}$ is conditional on, I', is that I includes the absence of v and I' includes the absence of u. However, $\psi_t^{w\to u}$ is independent of u by assumption and independent of v by our treelike factor graph assumption (note the community detection factor graph is definitely not treelike: the influence of non-edges causes the factor graph to be a clique. However, the BP result is only for treelike factor graphs. I believe this is where our derivation fell through. Since our factor graph isn't treelike, the derivation isn't exact (but is pretty close, because the influence of v should be minimal).

$$= \sum_{\{w'\}} \left[\prod_{w \in \{w'\}} f(t_u = r \mid t_w \wedge A_{uw}) \psi_{t_w}^{w\to u} \right]$$

The rest of the derivation should just be rearranging.