All questions carry equal marks.

Answer all the questions.

No part of a question may be answered separately.

1. If $P(A \cap B^c) = 0.2, P(A \cap B) = 0.1$ and $P(A^c \cap B) = 0.3$, find the value of $P(A \mid A \cup B^c)$.
2. A die is rolled till a 5 appears. Let X be the number rolls required in this process. Find the probability function of X and also find the probability of obtaining 5 in six or fewer rolls?
3. Suppose that in an automatic process of filling oil into cans, the content of a can (in liters) is $Y = 100 + X$, where X is a random variable with density $f(x) = 1 - |x|$ when $|x| \leq 1$ and $f(x) = 0$ otherwise. Find and sketch $F(x)$.
4. Find the skewness of the distribution with density $f(x) = 2e^{-2x}$ if $x \geq 0$ and $f(x) = 0$ otherwise.
5. If the distribution of X is binomial with parameters n and p then prove that for any given $\epsilon > 0$, $P\left\{\left|\frac{X}{n} - p\right| > \epsilon\right\} \to 0$ as $n \to \infty$.
6. Find the smallest positive root of the equation $x = \tan x$ by fixed point iteration method correct to four decimals.
7. When nodes are equally spaced, prove that $f[x_0, x_1, \cdots, x_n] = \frac{\Delta^n f(x_0)}{n! h^n}$.
8. Interpolate $f(x) = x^5$ on the interval $-1 \leq x \leq 1$ by the cubic spline $g(x)$ using the nodes $-1, 0, 1$ and satisfying the conditions $g'(-1) = f'(-1)$ and $g'(1) = f'(1)$.
9. Find a polynomial $f(x)$ satisfying $f(-1) = -3, f(0) = 1, f(1) = 5, f(3) = 115$ and $f(4) = 197$.
10. Prove that $\int_{x_0}^{x_2} f(x)dx$ can be approximated by $\frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$ where $x_i = x_0 + ih$, $i = 1,2$.

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