National Institute of Technology, Rourkela
Mid Semester Examination
Session: 2009–10 Spring Semester
Programme: B.Tech. IV Semester
Course Code: MA 202 Course Name: Complex Analysis and Partial Differential Equations
Full Marks: 30 Duration of Examination: 2 hours

Answer all questions
All questions are of equal value
All parts of a question should be answered at one place.

(1) Solve the wave equation $u_{tt} = c^2 u_{xx}$ with initial conditions satisfying $u(x,0) = \phi(x), u_t(x,0) = \psi(x)$ by using D'Alembert's method.

(2) Find $u(x,t)$ of the string of length $L = \pi$ when $c^2 = 1$ and the initial velocity is zero and initial deflection is $k(\sin x - 0.5 \sin 2x)$.

(3) Find solution of the equation $u_{xx} - u_{yy} = 0$ by separation of variables.

(4) Find the solution of $u_{xx} - 2u_{xy} + u_{yy} = 0$ via normal form.

(5) Find the temperature $u(x,t)$ in a bar of silver, whose length 10 cm, constant cross section of area 1 cm^2, density 10.6 gm/cm^3, thermal conductivity 1.04 cal/(cm sec °C), specific heat 0.056 cal /(gm °C), that is perfectly insulated laterally, whose ends are kept at 0°C and whose initial velocity is $\sin 0.1\pi x$°C.

(6) Find linear fractional transformation that maps i,0,1 onto 2+i, 2, 3. Find the fixed points of $f(z) = z^2$.

(7) Evaluate $\int_C z e^{z^2} dz$, where C from 1 along axes to i.

(8) If $f(z) = \frac{\bar{z}^2}{z}$, $z \neq 0$ and $f(0) = 0$, check whether Cauchy-Riemann equations are satisfied at $z = 0$ and what do you conclude about the analyticity of $f(z)$ at $z = 0$?

(9) If $f(z) = u + iv$ is differentiable in a domain D then the partial derivatives of u and v exists and satisfy the Cauchy-Riemann equations in a domain D.

(10) State and prove the conformality of mapping by analytic functions.