NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

B. Tech. End Semester Examination – April, 2011

(FOURTH SEMESTER)

Subject: CA & PDE **Subject Code – MA 202**

Full Marks: 50 **Duration of Examination : 3 hours**

This question paper consists of two pages.

Question number 6 and 12 are of 5 marks and all other questions are of 4 marks each.

Answer all the questions.

No part of a question may be answered separately.

Answer to any question must not be duplicated.

1. Find and sketch the images of the circles $|z| = 1$ and $|z| = 2$ under the transformation $w = z + \frac{1}{z}$.

2. Find $\oint_C z^2\, dz$ where C is the boundary of the square with vertices $1 + i$, $1 - i$, $-1 - i$, and $-1 + i$ (counterclockwise) using (a) line integral and (b) Cauchy's integral theorem.

3. Let $f(z)$ be an analytic function in a simply connected domain D. Prove that for any $a \in D$ and any simply closed path C in D that encloses a, $\oint_C \frac{f(z)}{z-a} dz = 2\pi i \cdot f(a)$.

4. Let $f(z) = \frac{2z-1}{z^2-z-6}$. Find a Taylor series for $f(z)$ valid in $|z| < 2$ and a Laurrent series valid in $|z| > 3$.

5. Find the value of $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ using residue method.

6. Using residue method evaluate $\int_0^{\infty} \frac{1}{x^6+1} dx$.

7. A thin vibrating bar of length π has insulated ends and a initial temperature is x^2. If the bar is perfectly insulated laterally, find the temperature distribution $u(x,t)$ in the bar when $c = 1$.

8. Find the vibration of a thin square membrane of sides $a = b = 1$ ft if the tension is 12.5 lb/ft, the density is 2.5slugs/ft^2, ends fixed, initial velocity is zero and initial displacement $f(x,y) = \sin 3\pi x \cdot \sin 4\pi y$.

9. Find the temperature $u(x,t)$ in a thin semi circular plate $r < 1, y > 0$ if the segment $-1 < x < 1$ is kept at 0^0C and the semicircular boundary is kept at $\theta(\pi - \theta)^0C$ assuming that $c = 1$.

10. Derive the steady-state equation for the two dimensional heat equation in polar form.

11. Using the Fourier transform method, find the temperature in an infinite homogeneous bar if the initial temperature is $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ if $|x| > 1$.

12. Using Laplace transform method, solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = \sin t$ if $0 \leq t \leq 2\pi$ and $u(0,t) = 0$ otherwise, $u(x,0) = 0$ and $\frac{\partial u}{\partial t} = 1$ at $t = 0$.