



NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA
B. Tech. Mid Semester Examination – February, 2011
(FOURTH SEMESTER)

Subject: CA & PDE
Full Marks: 30

Subject Code – MA 202
Duration of Examination : 2 hours

All questions carry equal marks.

Answer all the questions.

No part of a question may be answered separately.

Answer to any question must not be duplicated.

1. Is $u = \frac{x}{x^2+y^2}$ harmonic? If your answer is yes, find the conjugate harmonic function v such that $f(z) = u + iv$ is analytic. Also find $f(z)$ as a function of z .
2. Prove that an analytic function with constant absolute value is a constant.
3. Find the linear fractional transformation that maps $i, 0, 1$ onto $2+i, 2, 3$ respectively.
4. (a) For what value(s) of z the mapping $f(z) = e^z - iz$ is not conformal?
(b) Find the images of the lines $x = 1$ and $y = 2$ under the mapping $w = z^2$.
5. Evaluate $\int_C \operatorname{Re} z^2 dz$ where $C : |z| = 1$ counterclockwise.
6. Find the deflection $u(x, t)$ of a vibrating string of length $l = \pi$ whose ends are fixed, wave speed $c^2 = 1$, initial velocity 0 and initial deflection $x(\pi^2 - x^2)$.
7. Reduce $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ to normal form and solve it.
8. Find the temperature distribution $u(x, t)$ in a laterally insulated thin copper bar of length 100 cm with constant cross section whose end points $x = 0$ and $x = 100$ are kept at 0°C and initial temperature $u(x, 0) = x$ if $0 < x < 50$ and $u(x, 0) = 100 - x$ if $50 < x < 100$.
9. Find the temperature $u(x, y)$ in a thin metallic square plate of sides $a = 12$ with insulated faces if the left, lower and right sides are kept at 0°C and the upper side at $\sin \frac{\pi x}{4}$.
10. Find the solution $u(x, y)$ of $u_x + u_y = (x + y)u$ by separating the variables.

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