EC-203

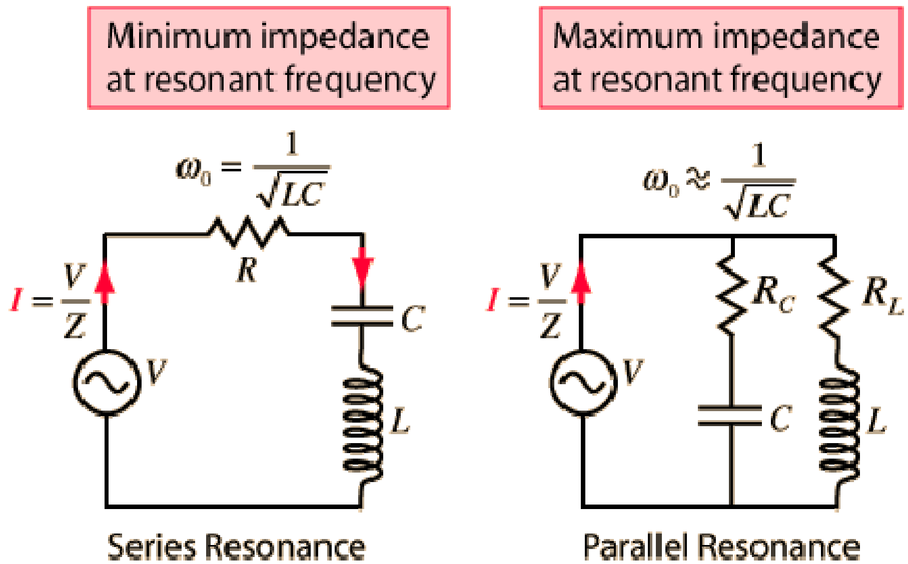
Networks

Chapter-1
Resonance in AC circuits

Resonance in AC circuits implies a special frequency determined by the values of the resistance , capacitance , and inductance .

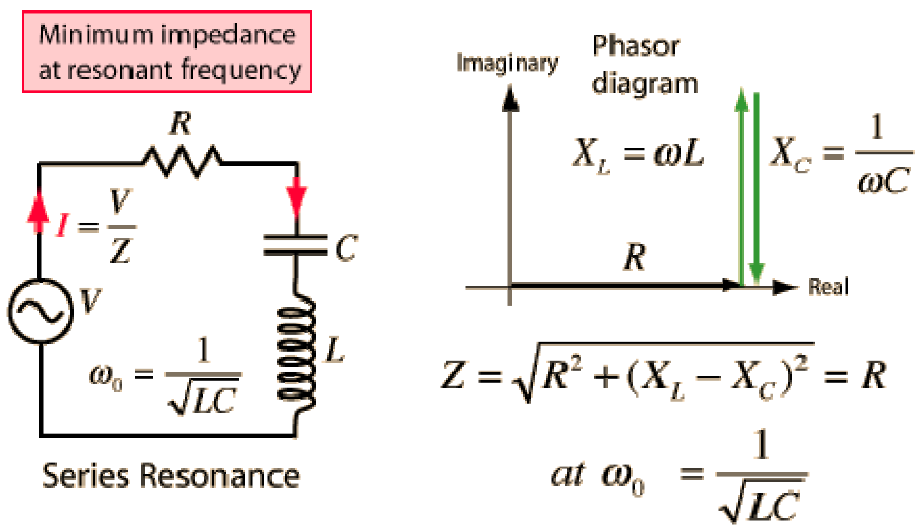
For series resonance the condition of resonance is straightforward and it is characterized by minimum impedance and zero phase.

Parallel resonance , which is more common in electronic practice, requires a more careful definition.



Series Resonance

The resonance of a series RLC circuit occurs when the inductive and capacitive reactances are equal in magnitude but cancel each other because they are 180 degrees apart in phase. The sharp minimum in impedance which occurs is useful in tuning applications. The sharpness of the minimum depends on the value of R and is characterized by the "Q" of the circuit.

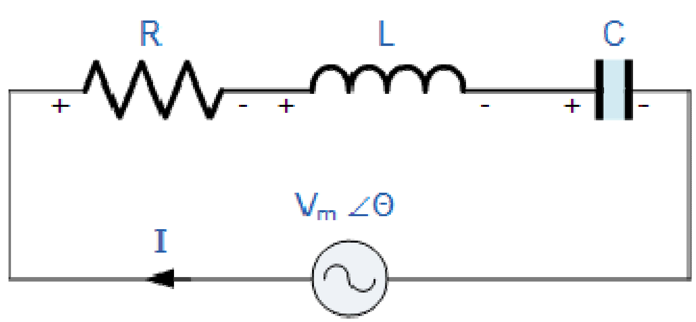


The Series Resonance Circuit

In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor.

The point at which this occurs is called the **Resonant Frequency**, (f_r) and the circuit is called a **Series Resonance** circuit.

Series resonance circuits are one of the most important circuits used in electronics. They can be found in various forms in mains AC filters, and also in radio and television sets producing a very selective tuning circuit for the receiving the different channels.



- Inductive reactance: $X_L = 2\pi f L = \omega L$

- Capacitive reactance: $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$

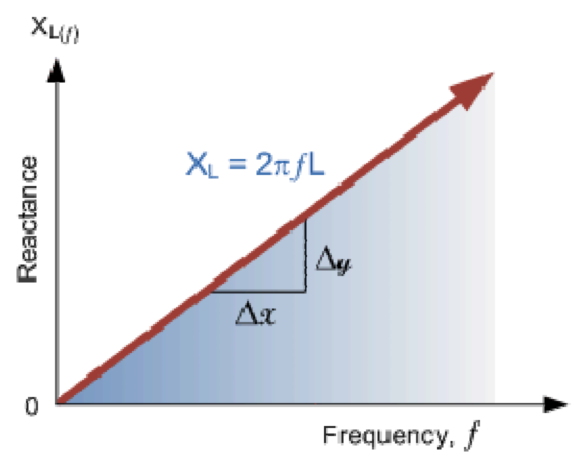
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive

- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

From the above equation for inductive reactance, if either the **Frequency** or the **Inductance** is increased the overall inductive reactance value of the inductor would also increase. As the frequency approaches infinity the inductors reactance would also increase towards infinity with the circuit element acting like an open circuit. However, as the frequency approaches zero or DC, the inductors reactance would decrease to zero, causing the opposite effect acting like a short circuit. This means then that inductive reactance is "**Proportional**" to frequency and is small at low frequencies and high at higher frequencies and this demonstrated in the following curve:

Inductive Reactance against Frequency

The graph of inductive reactance against frequency is a straight line linear curve. The inductive reactance value of an inductor increases linearly as the frequency across it increases. Therefore, inductive reactance is positive and is directly proportional to frequency ($X_L \propto f$)



Capacitive Reactance against Frequency

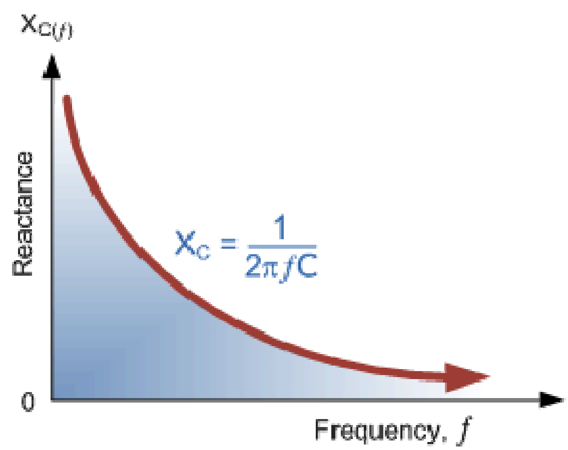
The same is also true for the capacitive reactance formula above but in reverse. If either the **Frequency** or the **Capacitance** is increased the overall capacitive reactance would decrease.

As the frequency approaches infinity the capacitors reactance would reduce to zero causing the circuit element to act like a perfect conductor of 0 Ω's.

However, as the frequency approaches zero or DC level, the capacitors reactance would rapidly increase up to infinity causing it to act like a very large resistance acting like an open circuit condition. This means then that capacitive reactance is "**Inversely proportional**" to frequency for any given value of capacitance and this shown below:

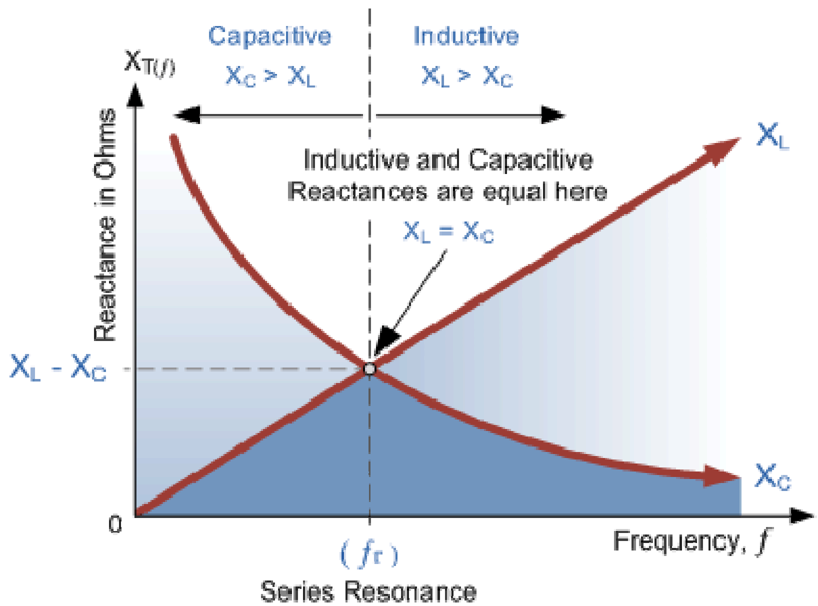
Capacitive Reactance against Frequency

The graph of capacitive reactance against frequency is a hyperbolic curve. The Reactance value of a capacitor has a very high value at low frequencies but quickly decreases as the frequency across it increases. Therefore, capacitive reactance is negative and is inversely proportional to frequency ($X_C \propto f^{-1}$)



Series Resonance Frequency

We can see that the values of these reactances depends upon the frequency of the supply. At a higher frequency X_L is high and at a low frequency X_C is high. Then there must be a frequency point were the value of X_L is the same as the value of X_C and there is. If we now place the curve for inductive reactance on top of the curve for capacitive reactance so that both curves are on the same axes, the point of intersection will give us the series resonance frequency point, (f_r or ω_r) as shown below.



where: f_r is in Hertz, L is in Henries and C is in Farads.

Series Resonance (Cont.)

Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is were the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

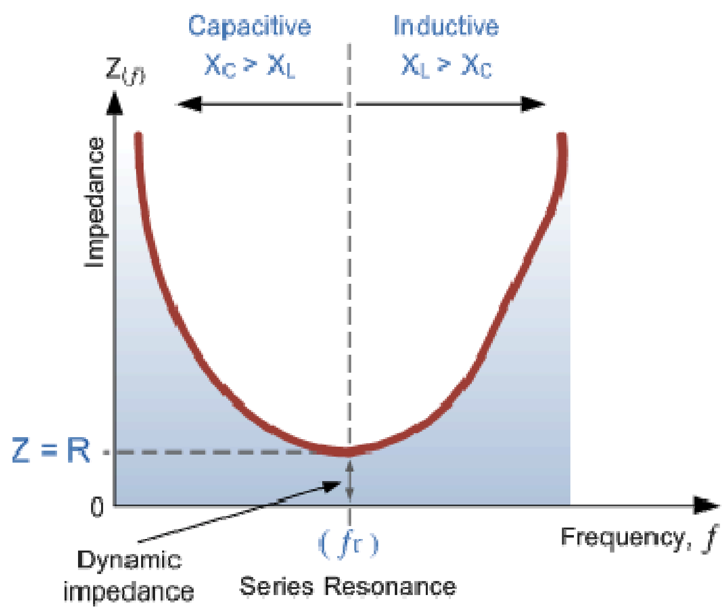
$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \,(\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \,(\text{rads})$$

Series Resonance (Cont.)

- We can see then that at resonance, the two reactances cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R.
- In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit is purely *"real"* as no imaginary impedances exist, they are cancelled out, so the total impedance of the series circuit becomes just the value of the resistance and: $Z = R$.
- Therefore, at resonance the impedance of the circuit is at its minimum value and equal to the resistance of the circuit. The impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, X_C (typically at high frequencies) or X_L (typically at low frequencies) will dominate either side of resonance as shown below.

Impedance in a Series Resonance Circuit



Impedance in a Series Resonance Circuit

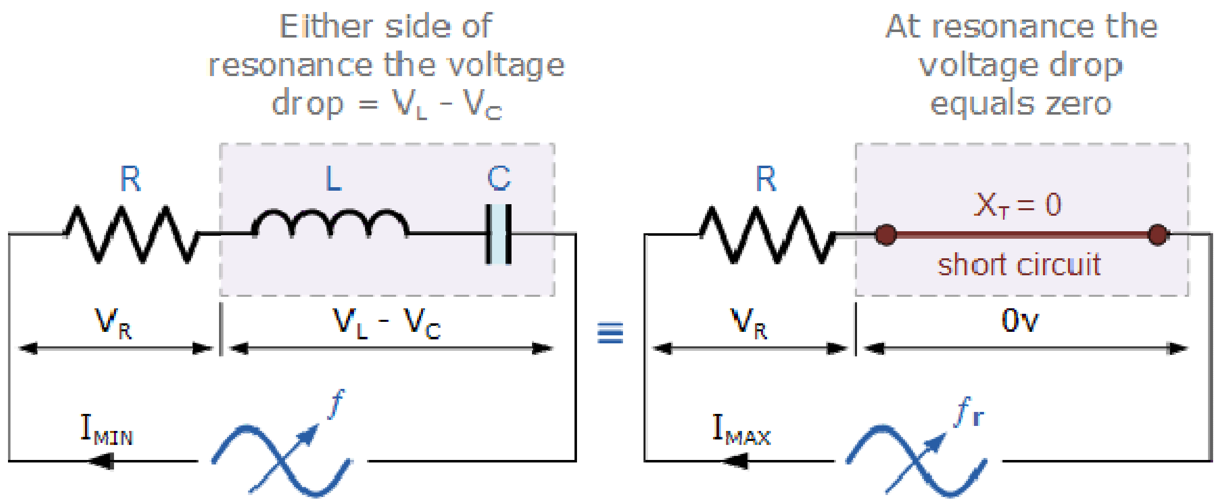
Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L.

You may also note that if the circuits impedance is at its minimum at resonance then consequently, the circuits **admittance** must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the circuits current may be dangerously high.

In series RLC circuits the voltage across a series combination is the phasor sum of V_R, V_L and V_C. Then if at resonance the two reactances are equal and cancelling, the two voltages representing V_L and V_C must also be opposite and equal in value thereby cancelling each other out because with pure components the phasor voltages are drawn at $+90^o$ and -90^o respectively.

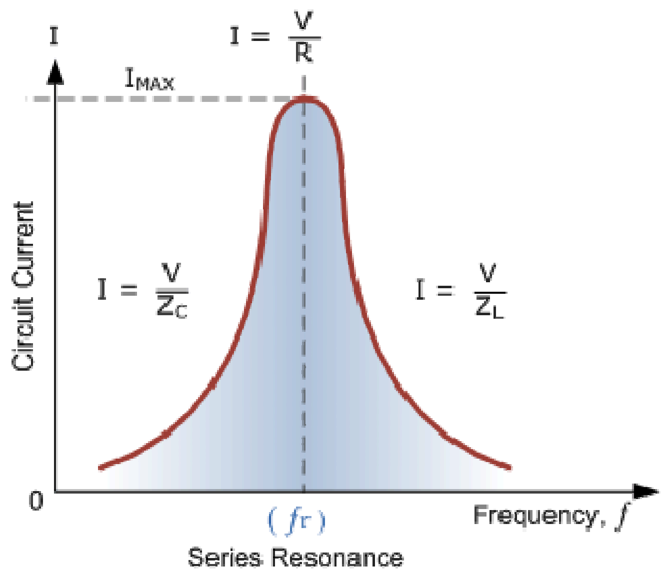
Then in a **series resonance** circuit $V_L = -V_C$ therefore, $V = V_R$.

Series RLC Circuit at Resonance



Since the current flowing through a series resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its minimum value, (= R). Therefore, the circuit current at this frequency will be at its maximum value of V/R as shown below.

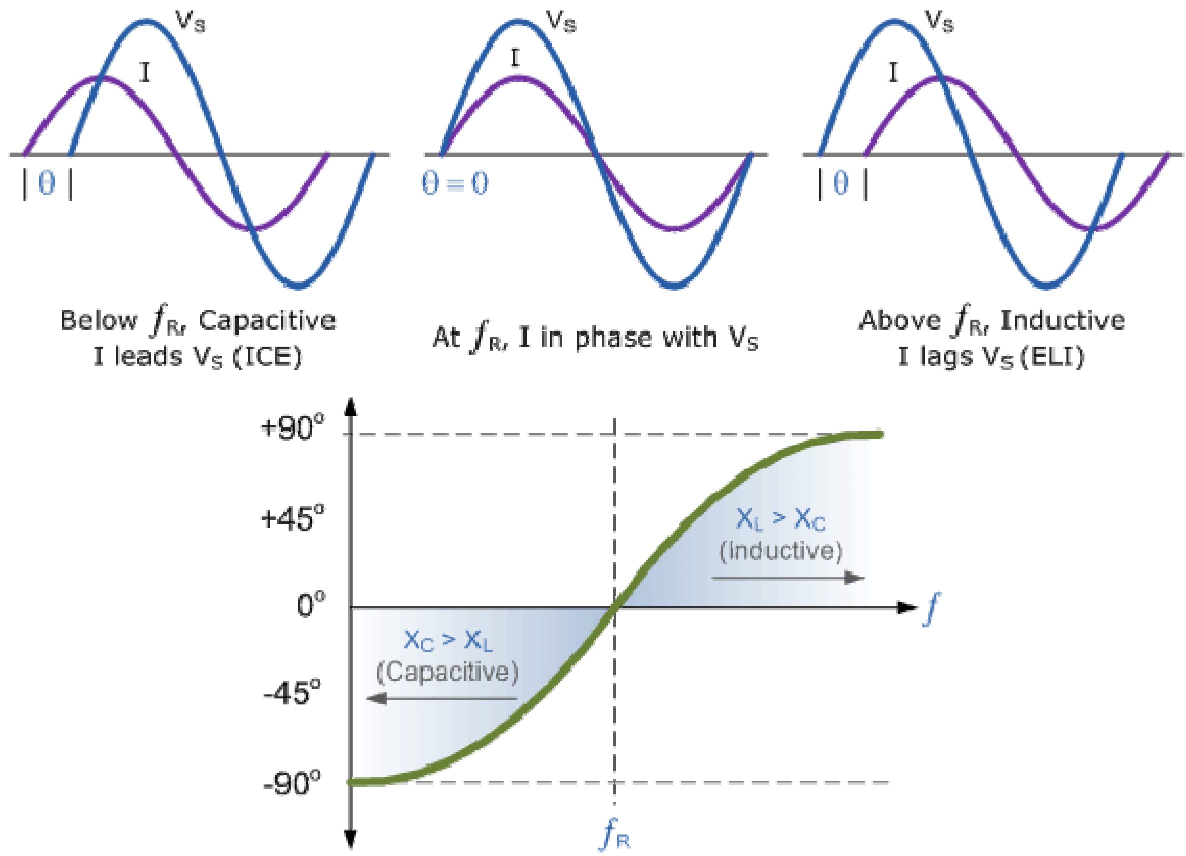
Series RLC Circuit at Resonance



Series RLC Circuit at Resonance

- The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite.
- The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition they cancel each other out.
- As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Acceptor Circuit** because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency. The effect of resonance in a series circuit is also called "voltage resonance“.
- You may also notice that as the maximum current through the circuit at resonance is limited only by the value of the resistance (a pure and real value), the source voltage and circuit current must therefore be in phase with each other at this frequency.
- Then the phase angle between the voltage and current of a series resonance circuit is also a function of frequency for a fixed supply voltage and which is zero at the resonant frequency point when: V, I and V_R are all in phase with each other as shown below. Consequently, if the phase angle is zero then the power factor must therefore be unity.

Phase Angle of a Series Resonance Circuit



Phase Angle of a Series Resonance Circuit

The phase angle is positive for frequencies above f_r and negative for frequencies below f_r

$$\tan\phi = \frac{X_L - X_C}{R} = 0^{\circ} \quad \text{(all real)}$$

Power in a Series Resonant Circuit

The average power dissipated in a series resonant circuit can be expressed in terms of the rms voltage and current as follows:

$$P_{avg} = I_{rms}^2 R = \frac{V_{rms}^2}{Z^2} R = \frac{V_{rms}^2 R}{R^2 + (X_L - X_C)^2}$$

Using the forms of the inductive reactance and capacitive reactance, the term involving them can be expressed in terms of the frequency.

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

where use has been made of the resonant frequency expression

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

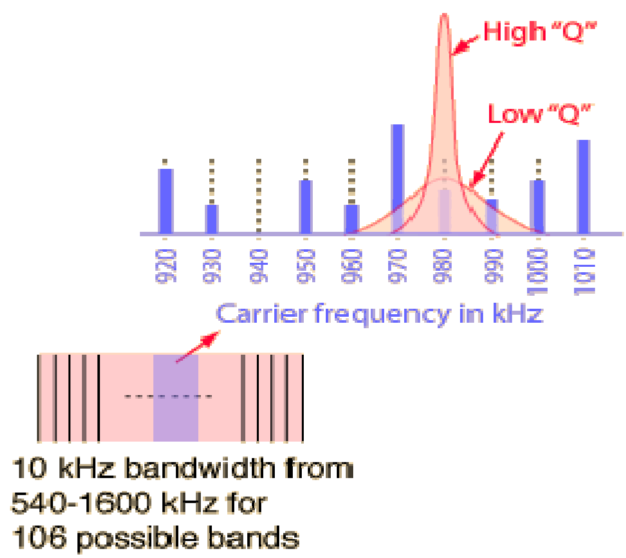
Substitution now gives the expression for average power as a function of frequency.

$$P_{avg} = \frac{V_{rms}^2 R \omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2}$$

Selectivity and Q factor of a Circuit

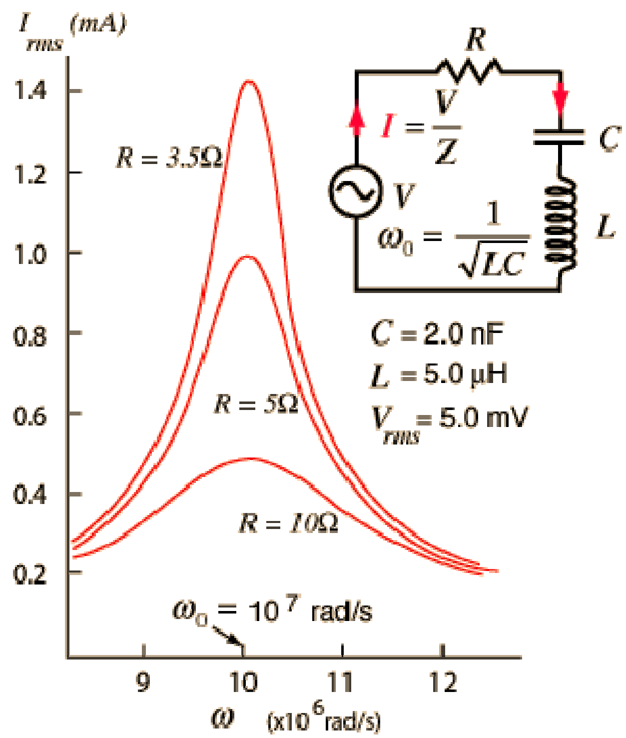
Resonant circuits are used to respond selectively to signals of a given frequency while discriminating against signals of different frequencies. If the response of the circuit is more narrowly peaked around the chosen frequency, we say that the circuit has higher "selectivity". A "quality factor" Q, as described below, is a measure of that selectivity, and we speak of a circuit having a "high Q" if it is more narrowly selective.

An example of the application of resonant circuits is the selection of AM radio stations by the radio receiver. The selectivity of the tuning must be high enough to discriminate strongly against stations above and below in carrier frequency, but not so high as to discriminate against the "sidebands" created by the imposition of the signal by amplitude modulation.



AM Radio

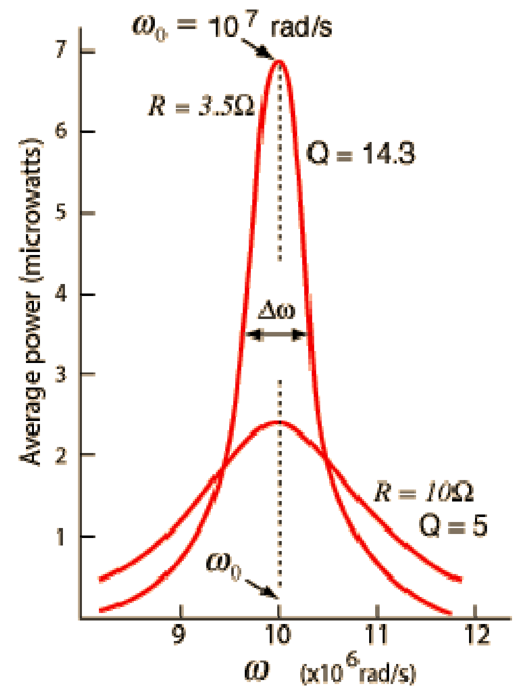
The selectivity of a circuit is dependent upon the amount of resistance in the circuit. The variations on a series resonant circuit below follow an example in Serway & Beichner. The smaller the resistance, the higher the "Q" for given values of L and C. The parallel resonant circuit is more commonly used in electronics, but the algebra necessary to characterize the resonance is much more involved.



Using the same circuit parameters, the illustration shows the power dissipated in the circuit as a function of frequency. Since this power depends upon the square of the current, these resonant curves appear steeper and narrower than the resonance peaks for current above. The quality factor Q is defined by

$$Q = \frac{\omega_0}{\Delta\omega}$$

where Δω is the width of the resonant power curve at half maximum.



Since that width turns out to be Δω =R/L, the value of Q can also be expressed as

$$Q = \frac{\omega_0 L}{R}$$

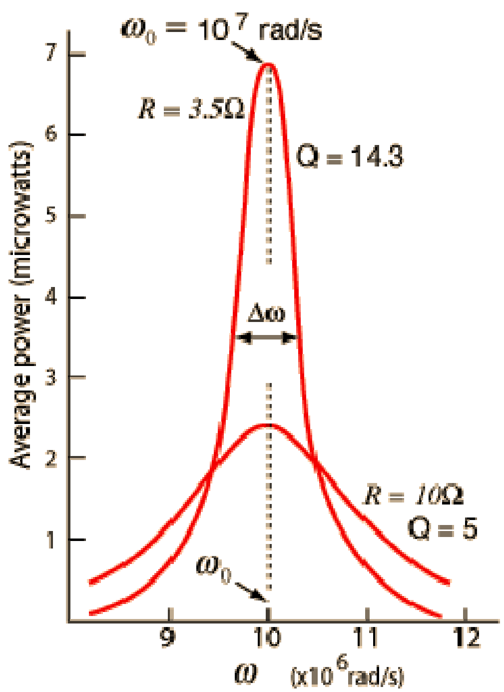
The Q is a commonly used parameter in electronics, with values usually in the range of Q=10 to 100 for circuit applications.

This power distribution is plotted using the same circuit parameters as were used in the example on the Q factor of the series resonant circuit

The average power at resonance is just

$$P_{avg} = \frac{V_{rms}^2}{R}$$

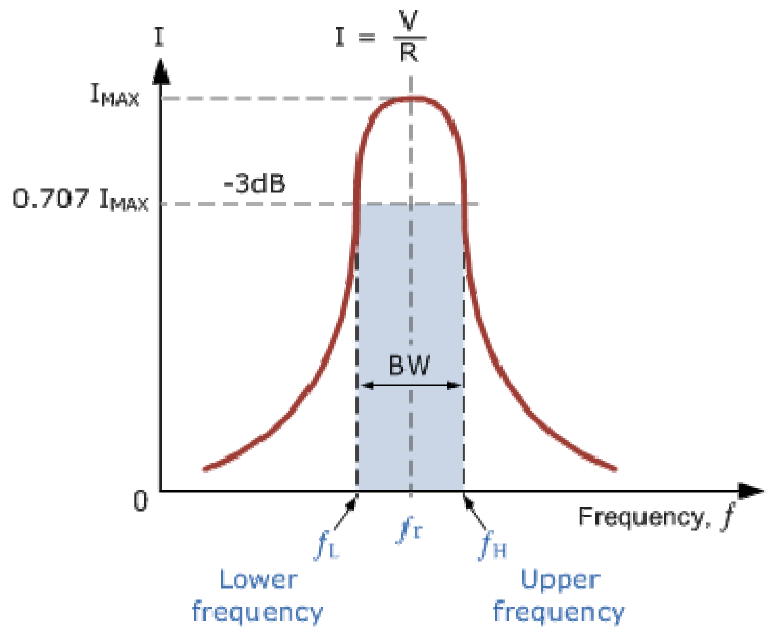
since at the resonant frequency ω_0 the reactive parts cancel so that the circuit appears as just the resistance R.



Bandwidth of a Series Resonance Circuit

- If the series RLC circuit is driven by a variable frequency at a constant voltage, then the magnitude of the current, I is proportional to the impedance, Z, therefore at resonance the power absorbed by the circuit must be at its maximum value as $P = I^2Z$.
- If we now reduce or increase the frequency until the average power absorbed by the resistor in the series resonance circuit is half that of its maximum value at resonance, we produce two frequency points called the **half-power points** which are -3dB down from maximum, taking 0dB as the maximum current reference.
- These -3dB points give us a current value that is 70.7% of its maximum resonant value as: $0.5(I^2 R) = (0.707 \times I)^2 R$. Then the point corresponding to the lower frequency at half the power is called the "lower cut-off frequency", labelled f_L with the point corresponding to the upper frequency at half power being called the "upper cut-off frequency", labelled f_H.
- The distance between these two points, i.e. ($f_H - f_L$) is called the **Bandwidth**, (BW) and is the range of frequencies over which at least half of the maximum power and current is provided as shown.

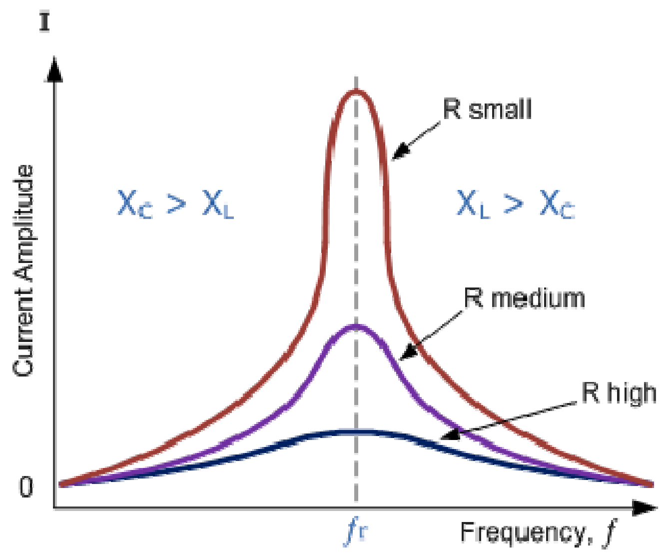
Bandwidth of a Series Resonance Circuit



The frequency response of the circuits current magnitude above, relates to the "sharpness" of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the **Quality factor, Q** of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, $Q = f_r /BW$.

As the bandwidth is taken between the two -3dB points, the **selectivity** of the circuit is a measure of its ability to reject any frequencies either side of these points. A more selective circuit will have a narrower bandwidth whereas a less selective circuit will have a wider bandwidth. The selectivity of a series resonance circuit can be controlled by adjusting the value of the resistance only, keeping all the other components the same, since $Q = (X_L \text{ or } X_C)/R$.

Bandwidth of a Series Resonance Circuit



Then the relationship between resonance, bandwidth, selectivity and quality factor for a series resonance circuit being defined as:

1) Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2) Current, (I)

$$\text{at } \omega_r \quad Z_T = \text{min}, \quad I_S = \text{max}$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). **Lower cut-off frequency, (f_L)**

At half power, $\frac{P_m}{2}$, $I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$Z = \sqrt{2}R,\ X = -R\ (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). **Upper cut-off frequency, (f_H)**

At half power, $\frac{P_m}{2}$, $I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$Z = \sqrt{2}R,\ X = +R\ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q}, \quad f_H - f_L, \quad \frac{R}{L}\,(\text{rads}) \quad \text{or} \quad \frac{R}{2\pi L}\,(\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{X_L}{R} = \frac{1}{R.X_C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Series Resonance Summary

- For resonance to occur in any circuit it must have at least one inductor and one capacitor.
- Resonance is the result of oscillations in a circuit as stored energy is passed from the inductor to the capacitor.
- Resonance occurs when $X_L = X_C$ and the imaginary part of the transfer function is zero.
- At resonance the impedance of the circuit is equal to the resistance value as Z = R.
- At low frequencies the circuit is capacitive as $X_C > X_L$ giving a leading power factor.
- At low frequencies the circuit is inductive as $X_L > X_C$ giving a lagging power factor.
- The high value of current at resonance produces very high values of voltage across the inductor and capacitor.
- Series resonance circuits are useful for constructing highly frequency selective filters. However, its high current and very high component voltage values can cause damage to the circuit.
- The most prominent feature of the frequency response of a resonant circuit is a sharp resonant peak in its amplitude characteristics.
- Because impedance is minimum and current is maximum, series resonance circuits are also called **Acceptor Circuits**.