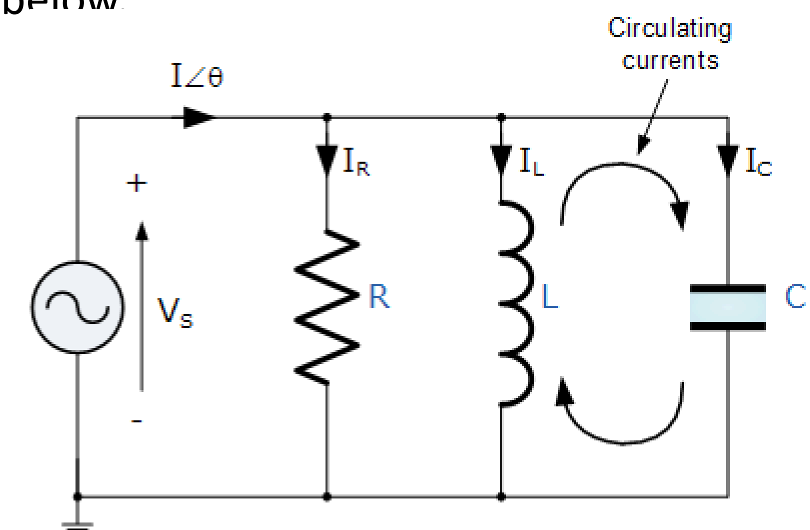
# The Parallel Resonance

# The Parallel Resonance Circuit

In many ways a **parallel resonance** circuit is exactly the same as the series resonance circuit. Both are 3-element networks that contain two reactive components making them a second-order circuit, both are influenced by variations in the supply frequency and both have a frequency point where their two reactive components cancel each other out influencing the characteristics of the circuit. Both circuits have a resonant frequency point.

The difference this time however, is that a parallel resonance circuit is influenced by the currents flowing through each parallel branch within the parallel LC tank circuit. A **tank circuit** is a parallel combination of L and C that is used in filter networks to either select or reject AC frequencies. Consider the parallel RLC circuit below.

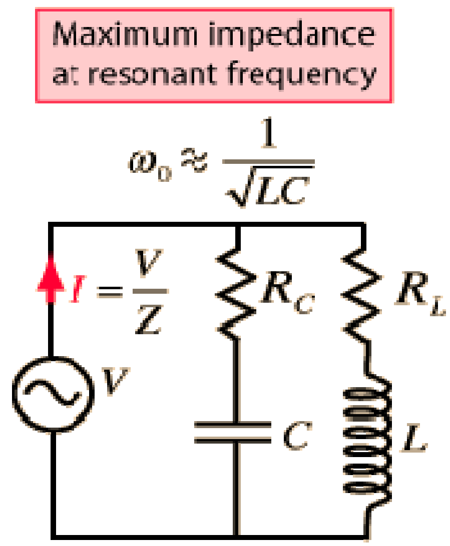


# The Parallel Resonance Circuit

- The resonance of a parallel RLC circuit is a bit more involved than the series resonance. The resonant frequency can be defined in three different ways, which converge on the same expression as the series resonant frequency if the resistance of the circuit is small.

Different possible definitions of the resonant frequency for a parallel resonant circuit:

1. The frequency at which $\omega L = 1/\omega C$, i.e., the resonant frequency of the equivalent series RLC circuit. This is satisfactory if the resistances are small.
2. The frequency at which the parallel impedance is a maximum.
3. The frequency at which the current is in phase with the voltage, unity power factor.




Parallel Resonance

## Resonance: Impedance Maximum

One of the ways to define resonance for a parallel RLC circuit is the frequency at which the impedance is maximum. The general case is rather complex, but the special case where the resistances of the inductor and capacitor are negligible can be handled readily by using the concept of admittance.

## Resonance: Phase Definition

Defining the parallel resonant frequency as the frequency at which the voltage and current are in phase, unity power factor, gives the following expression for the resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}}\left[\frac{R_L^2 C - L}{R_C^2 C - L}\right]^{\frac{1}{2}}$$

The above resonant frequency expression is obtained by taking the impedance expressions for the parallel RLC circuit and setting the expression for $X_{eq}$ equal to zero to force the phase to zero. After about a page of algebra, the above expression emerges. Note that for small values of the resistances, this approaches the series resonant frequency.

A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce a **parallel resonance** (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations.

A parallel resonant circuit stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply. This is because the corresponding instantaneous values of $I_L$ and $I_C$ will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in $I_R$.

$$\text{Admittance, } Y = \frac{1}{Z} = \sqrt{G^2 - B^2}$$

$$\text{Conductance, } G = \frac{1}{R}$$

$$\text{Inductive Susceptance, } B_L = \frac{1}{2\pi f L}$$

$$\text{Capacitive Susceptance, } B_C = 2\pi f C$$

- In the solution of AC parallel resonance circuits we know that the supply voltage is common for all branches, so this can be taken as our reference vector.
- Each parallel branch must be treated separately as with series circuits so that the total supply current taken by the parallel circuit is the vector addition of the individual branch currents.
- There are two methods available for the analysis of parallel resonance circuits. We can calculate the current in each branch and then add together or calculate the admittance of each branch to find the total current.
- We know from the previous series resonance tutorial that resonance takes place when $V_L = -V_C$ and this situation occurs when the two reactances are equal, $X_L = X_C$. The admittance of a parallel circuit is given as:

$$Y = G + B_L + B_C$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Resonance occurs when $X_L = X_C$ and the imaginary parts of Y become zero. Then:

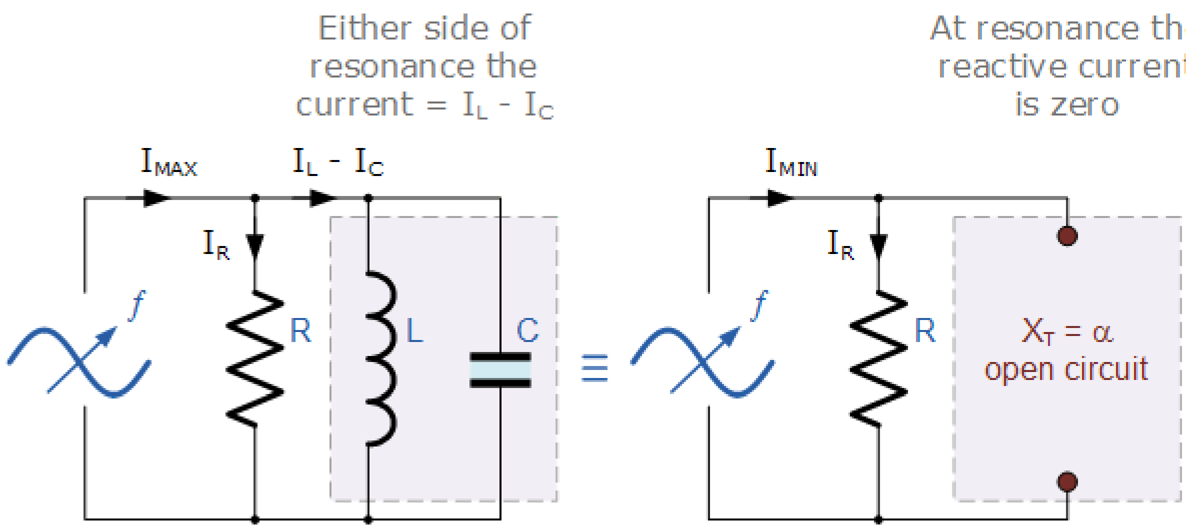
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} (\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} (\text{rads})$$

# Impedance in a Parallel Resonance Circuit

Notice that at resonance the parallel circuit produces the same equation as for the series resonance circuit. Therefore, it makes no difference if the inductor or capacitor are connected in parallel or series. Also at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being limited by the resistor, R only. So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and $Z = R$ as shown.



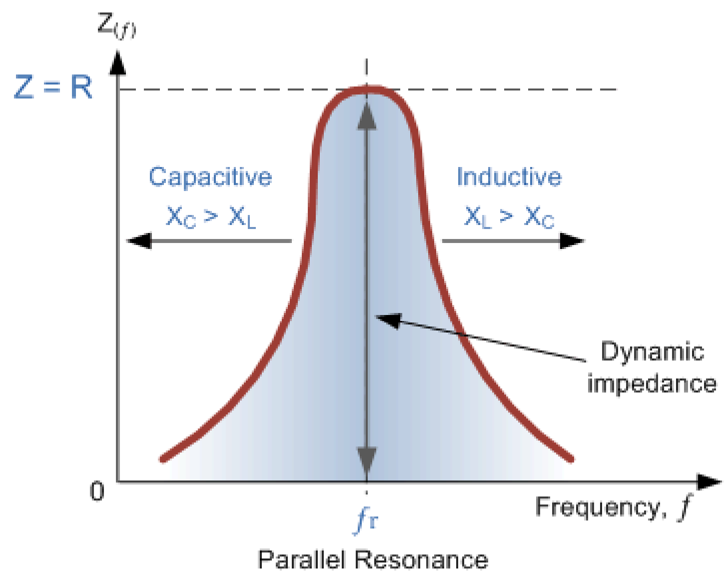
# Impedance in a Parallel Resonance Circuit

At resonance, the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit and we can change the circuit's frequency response by changing the value of this resistance.

Changing the value of R affects the amount of current that flows through the circuit at resonance, if both L and C remain constant.

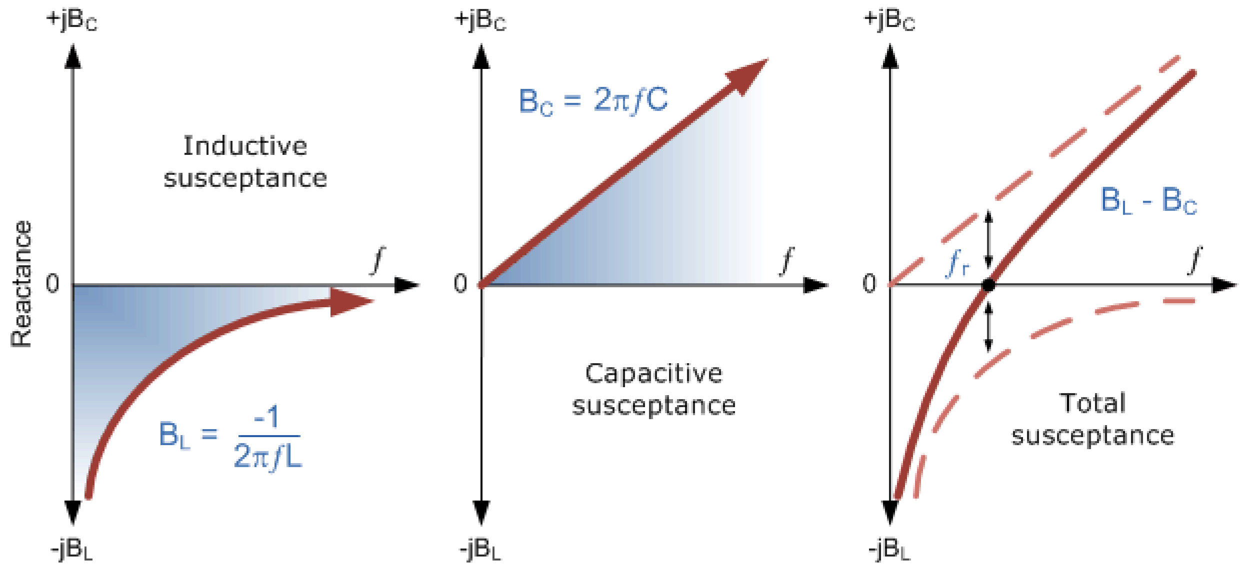
The impedance of the circuit at resonance $Z = R_{MAX}$ is called the "dynamic impedance" of the circuit.

# Impedance in a Parallel Resonance Circuit



- Note that if the parallel circuits impedance is at its maximum at resonance then consequently, the circuits **admittance** must be at its minimum and one of the characteristics of a parallel resonance circuit is that admittance is very low limiting the circuits current.
- Unlike the series resonance circuit, the resistor in a parallel resonance circuit has a damping effect on the circuits bandwidth making the circuit less selective. Also, since the circuit current is constant for any value of impedance, Z, the voltage across a parallel resonance circuit will have the same shape as the total impedance and for a parallel circuit the voltage waveform is generally taken from across the capacitor.
- We now know that at the resonant frequency, $f_r$ the admittance of the circuit is at its minimum and is equal to the conductance, G given by 1/R because in a parallel resonance circuit the imaginary part of admittance, i.e. the susceptance, B is zero because $B_L = B_C$ as shown.

# Susceptance at Resonance




From above, the *inductive susceptance*, $B_L$ is inversely proportional to the frequency as represented by the hyperbolic curve. The *capacitive susceptance*, $B_C$ is directly proportional to the frequency and is therefore represented by a straight line. The final curve shows the plot of total susceptance of the parallel resonance circuit versus the frequency and is the difference between the two susceptance's. We can see that the total susceptance is zero at the resonant frequency point were it crosses the horizontal axis. Below the resonant frequency point, the inductive susceptance dominates the circuit producing a "lagging" power factor, whereas above the resonant frequency point the capacitive susceptance dominates producing a "leading" power factor. Evidently, at the resonant frequency the circuits current is in phase with the applied voltage as there effectively there is only the resistance in the circuit so the power factor becomes one or unity, ( $\theta = 0^o$ ).

# Current in a Parallel Resonance Circuit

- As the total susceptance is zero at the resonant frequency, the admittance is at its minimum and is equal to the conductance, G. Therefore at resonance the current flowing through the circuit must also be at its minimum as the inductive and capacitive branch currents are equal ( $I_L = I_C$ ) and are 180° out of phase.
- We remember that the total current flowing in a parallel RLC circuit is equal to the vector sum of the individual branch currents and for a given frequency is calculated as:

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V.2\pi f C$$

Therefore, $I_T$ = vector sum of $(I_R + I_L + I_C)$

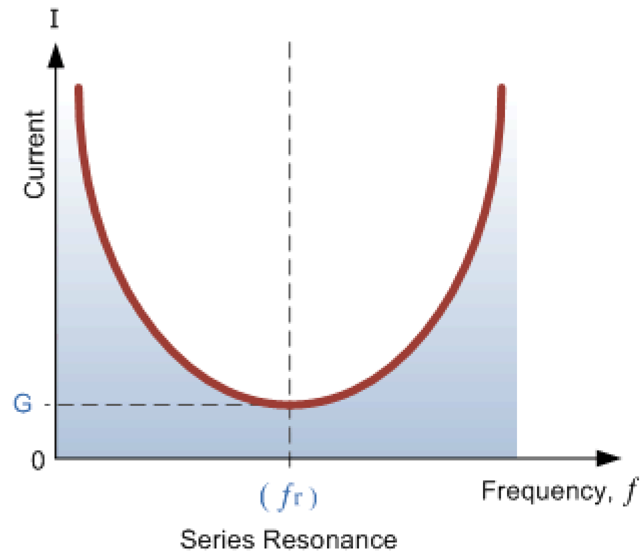
$$I_T = \sqrt{I_R^2 + (I_L + I_C)}$$

At resonance, currents $I_L$ and $I_L$ are equal and cancelling giving a net reactive current equal to zero. Then at resonance the above equation becomes

$$I_T = \sqrt{I_R^2 + 0^2} = I_R$$

Since the current flowing through a parallel resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its maximum value, ( =R ). Therefore, the circuit current at this frequency will be at its minimum value of V/R and the graph of current against frequency for a parallel resonance circuit is given as.

# Parallel Circuit Current at Resonance



# Parallel Circuit Current at Resonance

- The frequency response curve of a parallel resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at its maximum value, reaches its minimum value at the resonance frequency when $I_{MIN} = I_R$ and then increases again to maximum as $f$ becomes infinite.
- The result of this is that the magnitude of the current flowing through the inductor, L and the capacitor, C tank circuit can become many times larger than the supply current, even at resonance but as they are equal and at opposition ( $180^o$ out-of-phase ) they effectively cancel each other out.
- As a parallel resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Rejector Circuit** because at resonance, the impedance of the circuit is at its maximum thereby suppressing or rejecting the current whose frequency is equal to its resonant frequency. The effect of resonance in a parallel circuit is also called "current resonance".
- The characteristics and graphs drawn for a parallel circuit are exactly opposite to that of series circuits with the parallel circuits maximum and minimum impedance, current and magnification being reversed. Which is why a parallel resonance circuit is also called an **Anti-resonance** circuit.

# Bandwidth & Selectivity of a Parallel Resonance Circuit

- The selectivity or **Q-factor** for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as:

$$\text{Quality Factor, } Q = \frac{R}{2\pi f L} = 2\pi f CR = R\sqrt{\frac{C}{L}}$$

- Note that the Q-factor of a parallel resonance circuit is the inverse of the expression for the Q-factor of the series circuit. Also in series resonance circuits the Q-factor gives the voltage magnification of the circuit, whereas in a parallel circuit it gives the current magnification.

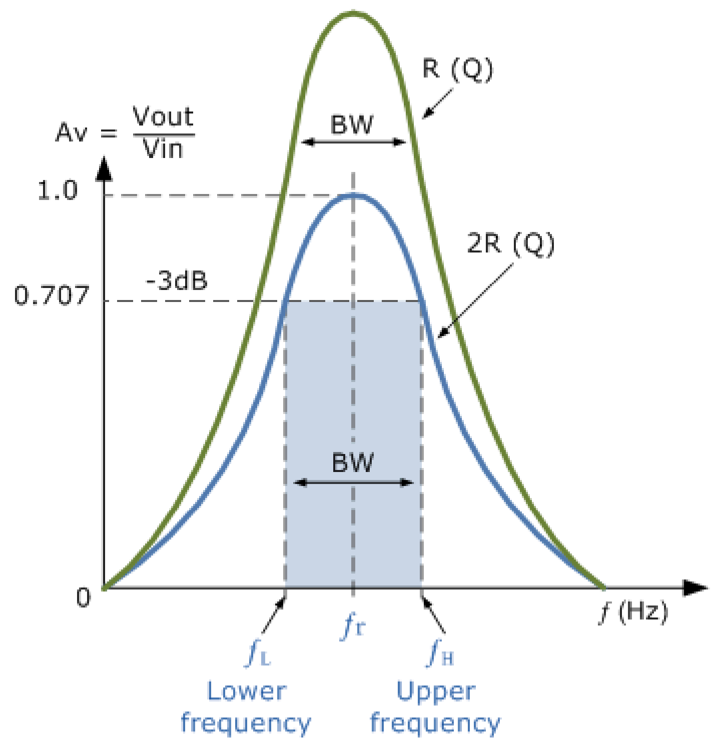
# Bandwidth & Selectivity of a Parallel Resonance Circuit

- The bandwidth of a parallel resonance circuit is defined in exactly the same way as for the series resonance circuit. The upper and lower cut-off frequencies given as: $f_{upper}$ and $f_{lower}$ respectively denote the half-power frequencies where the power dissipated in the circuit is half of the full power dissipated at the resonant frequency 0.5( $I^2$ R ) which gives us the same -3dB points at a current value that is equal to 70.7% of its maximum resonant value, $(0.707 \times I)^2$ R.

- As with the series circuit, if the resonant frequency remains constant, an increase in the quality factor, **Q** will cause a decrease in the bandwidth and likewise, a decrease in the quality factor will cause an increase in the bandwidth as defined by: BW = $f_r$ /Q or BW = $f_2$ - $f_2$.

- Also changing the ratio between the inductor, L and the capacitor, C, or the value of the resistance, R the bandwidth and therefore the frequency response of the circuit will be changed for a fixed resonant frequency. This technique is used extensively in tuning circuits for radio and television transmitters and receivers.

# Bandwidth of a Parallel Resonance Circuit



# Parallel Resonance Summary

- **Parallel Resonance** circuits are similar to series resonance circuits. Resonance occurs in a parallel RLC circuit when the total circuit current is in-phase with the supply voltage as the two reactive components cancel each other out.
- At resonance the admittance of the circuit is at its minimum and is equal to the conductance of the circuit. Also at resonance the current drawn from the supply is also at its minimum and is determined by the value of the parallel resistance.
- The equation used to calculate the resonant frequency point is the same for the previous series circuit.
- However, while the use of either pure or impure components in the series RLC circuit does not affect the calculation of the resonance frequency, but in a parallel RLC circuit it does.
- In our analysis above we have assumed pure inductive and capacitive components with negligible resistance, but in reality the coil will contain some resistance. Then the equation for calculating the resonant frequency of a parallel circuit is modified too.