ANALOG COMMUNICATION ASSIGNMENT-II

4.2-1 For each of the following baseband signals: (i) $m(t) = \cos 1000t$; (ii) $m(t) = 2\cos 1000t + \cos 2000t$; (iii) $m(t) = \cos 1000t \cos 3000t$:

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10{,}000t$.

(c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

(d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

4.2-2 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] if: **(i)** $m(t) = \text{sinc}\,(100t)$; **(ii)** $m(t) = e^{-|t|}$; **(iii)** $m(t) = e^{-|t-1|}$. Observe that $e^{-|t-1|}$ is $e^{-|t|}$ delayed by 1 second. For the last case you need to consider both the amplitude and the phase spectra.

4.2-3 Repeat Prob. 4.2-1 [parts **(a)**, **(b)**, and **(c)** only] for $m(t) = e^{-|t|}$ if the carrier is $\cos(10{,}000t - \pi/4)$. *Hint*: Use Eq. (3.36).

4.2-4 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$, where $m(t)$ is a signal band-limited to B Hz. Figure P4.2-4 shows a DSB-SC modulator available in the stock room. The carrier generator available generates not $\cos\omega_c t$, but $\cos^3\omega_c t$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

(a) What kind of filter is required in Fig. P4.2-4?

(b) Determine the signal spectra at points b and c, and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of ω_c?

(d) Would this scheme work if the carrier generator output were $\cos^2\omega_c t$? Explain.

(e) Would this scheme work if the carrier generator output were $\cos^n\omega_c t$ for any integer $n \geq 2$?

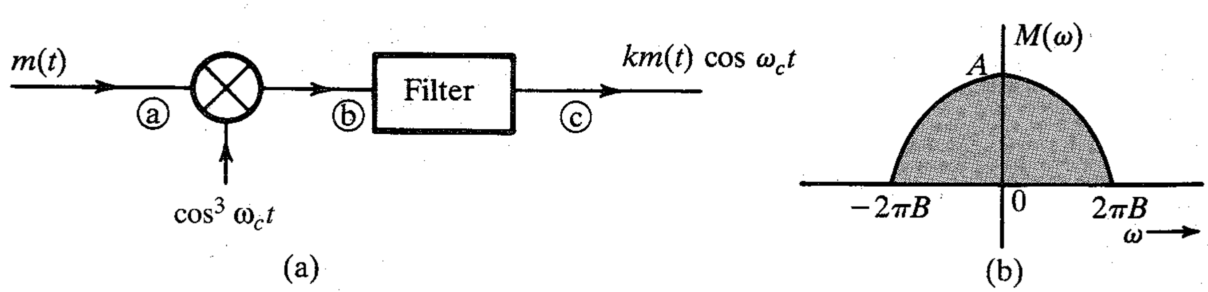


Figure P4.2-4

4.2-5 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$ with the carrier frequency $f_c = 300$ kHz ($\omega_c = 2\pi \times 300,000$). The following equipment is available in the stock room: **(i)** a signal generator of frequency 100 kHz; **(ii)** a ring modulator; **(iii)** a bandpass filter tuned to 300 kHz.

(a) Show how you can generate the desired signal.

(b) If the output of the modulator is $km(t)\cos\omega_c t$, find k.

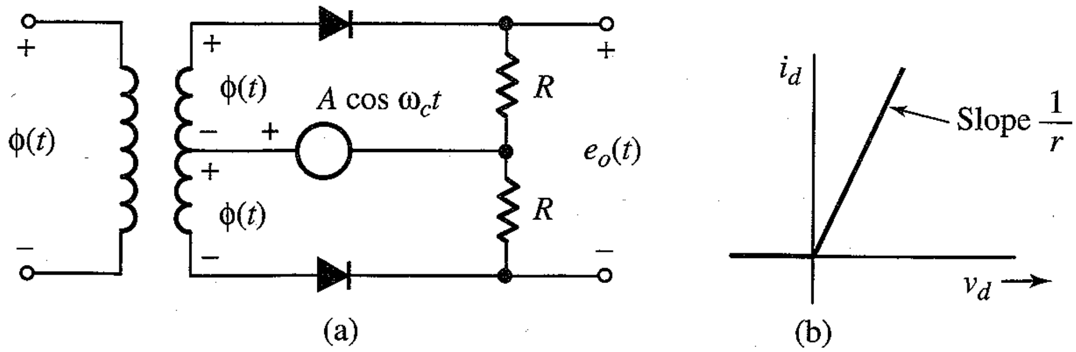


Figure P4.2-6

4.2-6 In Fig. P4.2-6, the input $\phi(t) = m(t)$, and the amplitude $A \gg |\phi(t)|$. The two diodes are identical with a resistance r ohms in the conducting mode and infinite resistance in the cutoff mode. Show that the output $e_o(t)$ is given by

$$e_o(t) = \frac{2R}{R+r}\, w(t)\, m(t)$$

where $w(t)$ is the switching periodic signal shown in Fig. 2.22a with period $2\pi / W_c$ seconds.

(a) Hence, show that this circuit can be used as a DSB-SC modulator.

(b) How would you use this circuit as a synchronous demodulator for DSB-SC signals.

4.2-7 In Fig. P4.2-6, if $\phi(t) = \sin(\omega_c t + \theta)$, and the output $e_0(t)$ is passed through a low-pass filter, then show that this circuit can be used as a phase detector, that is, a circuit that measures the phase difference between two sinusoids of the same frequency (ω_c). *Hint*: show that the filter output is a dc signal proportional to $\sin\theta$.

4.2-8 Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Fig. P4.2-8. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

(a) Sketch signal spectra at points a, b, and c.

(b) What must be the bandwidth of the channel?

(c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.

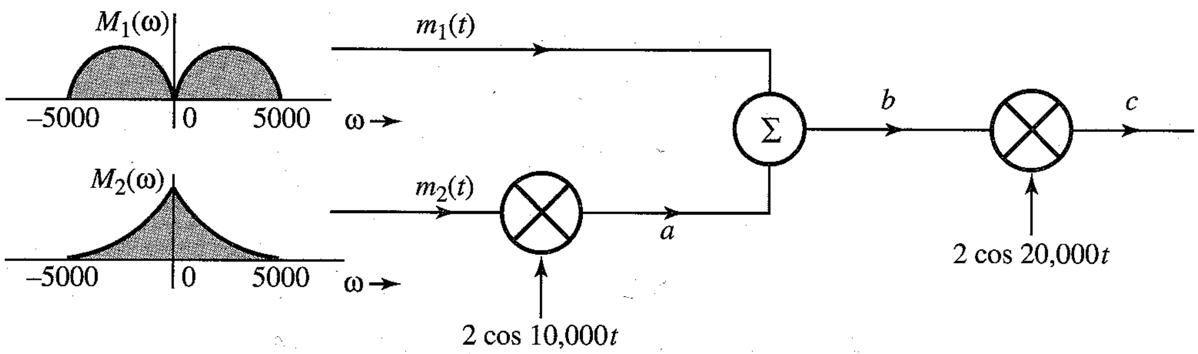


Figure P4.2-8

4.2-10 A DSB-SC signal is given by $m(t) \cos (2\pi)10^6 t$. The carrier frequency of this signal, 1 MHz, is to be changed to 400 kHz. The only equipment available is one ring modulator, a bandpass filter centered at the frequency of 400 kHz, and one sine wave generator whose frequency can be varied from 150 to 210 kHz. Show how you can obtain the desired signal $cm(t) \cos (2\pi \times 400 \times 10^6 t)$ from $m(t) \cos (2\pi)10^6 t$. Determine the value of c.

4.3-1 Figure P4.3-1 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal $[A + m(t)] \cos \omega_c t$ regardless of the value of A.

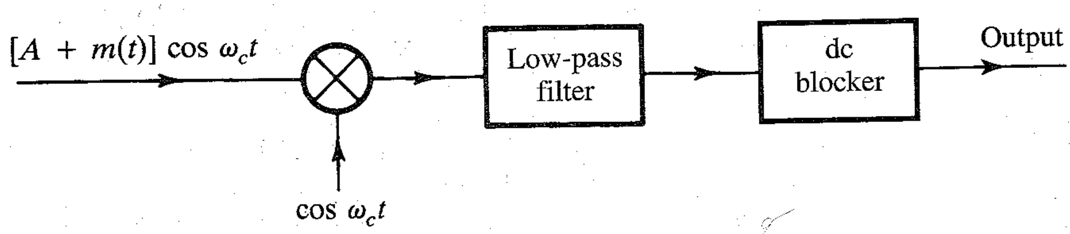


Figure P4.3-1

4.3-5 Show that any scheme that can be used to generate DSB-SC can also generate AM. Is the converse true? Explain.

4.3-6 Show that any scheme that can be used to demodulate DSB-SC can also demodulate AM. Is the converse true? Explain.

4.3-7 In the text, the power efficiency of AM for a sinusoidal $m(t)$ was found. Carry out a similar analysis when $m(t)$ is a random binary signal as shown in Fig. P4.3-7 and $\mu = 1$. Sketch the AM signal with $\mu = 1$. Find the sideband's power and the total power (power of the AM signal) as well as their ratio (the power efficiency η).

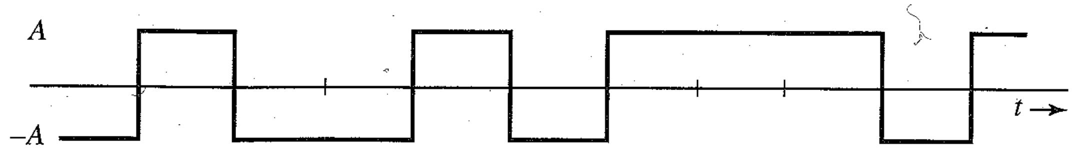


Figure P4.3-7

4.3-8 In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a dc blocker, as shown in Fig. P4.3-8. Assume a crystal detector to be basically a squaring device. Determine the signals at points *a*, *b*, *c*, and *d*. Point out the distortion term in the output $y(t)$. Show that if $A \gg |m(t)|$, the distortion is small.

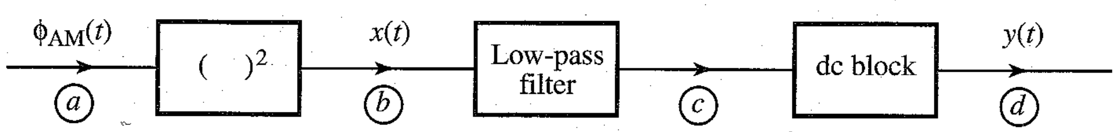


Figure P4.3-8

4.4-1 In a QAM system (Fig. 4.14), the locally generated carrier has a frequency error $\Delta\omega$ and a phase error δ; that is, the receiver carrier is $\cos[(\omega_c + \Delta\omega)t + \delta]$ or $\sin[(\omega_c + \Delta\omega)t + \delta]$. Show that the output of the upper receiver branch is

$$m_1(t)\cos[(\Delta\omega)t + \delta] - m_2(t)\sin[(\Delta\omega)t + \delta]$$

instead of $m_1(t)$, and the output of the lower receiver branch is

$$m_1(t)\sin[(\Delta\omega)t + \delta] + m_2(t)\cos[(\Delta\omega)t + \delta]$$

instead of $m_2(t)$.

4.5-1 A modulating signal $m(t)$ is given by:

(a) $m(t) = \cos 100t$

(b) $m(t) = \cos 100t + 2\cos 300t$

(c) $m(t) = \cos 100t \cos 500t$

In each case:

(i) Sketch the spectrum of $m(t)$.

(ii) Find and sketch the spectrum of the DSB-SC signal $2m(t)\cos 1000t$.

(iii) From the spectrum obtained in **(ii)**, suppress the LSB spectrum to obtain the USB spectrum.

(iv) Knowing the USB spectrum in **(ii)**, write the expression $\varphi_{\text{USB}}(t)$ for the USB signal.

(v) Repeat **(iii)** and **(iv)** to obtain the LSB signal $\varphi_{\text{LSB}}(t)$.

4.5-3 Find $\varphi_{\text{LSB}}(t)$ and $\varphi_{\text{USB}}(t)$ for the modulating signal $m(t) = B\,\text{sinc}\,(2\pi Bt)$ with $B = 1000$ and carrier frequency $\omega_c = 10,000\pi$. Follow these do-it-yourself steps:

(a) Sketch spectra of $m(t)$ and the corresponding DSB-SC signal $2m(t)\cos\omega_c t$.

(b) To find the LSB spectrum, suppress the USB in the DSB-SC spectrum found in **(a)**.

(c) Find the LSB signal $\varphi_{\text{LSB}}(t)$, which is the inverse Fourier transform of the LSB spectrum found in part **(b)**. Follow a similar procedure to find $\varphi_{\text{USB}}(t)$.

4.5-5 An LSB signal is demodulated synchronously, as shown in Fig. P4.5-5. Unfortunately, the local carrier is not $2\cos\omega_c t$ as required, but is $2\cos[(\omega_c + \Delta\omega)t + \delta]$. Show that:

(a) When $\delta = 0$, the output $y(t)$ is the signal $m(t)$ with all its spectral components shifted (offset) by $\Delta\omega$. *Hint*: Observe that the output $y(t)$ is identical to the right-hand side of Eq. (4.17a) with ω_c replaced with $\Delta\omega$.

(b) When $\Delta\omega = 0$, the output is the signal $m(t)$ with phases of all its spectral components shifted by δ. *Hint*: Show that the output spectrum $Y(\omega) = M(\omega)e^{j\delta}$ for $\omega \geq 0$, and equal to $M(\omega)e^{-j\delta}$ when $\omega < 0$.

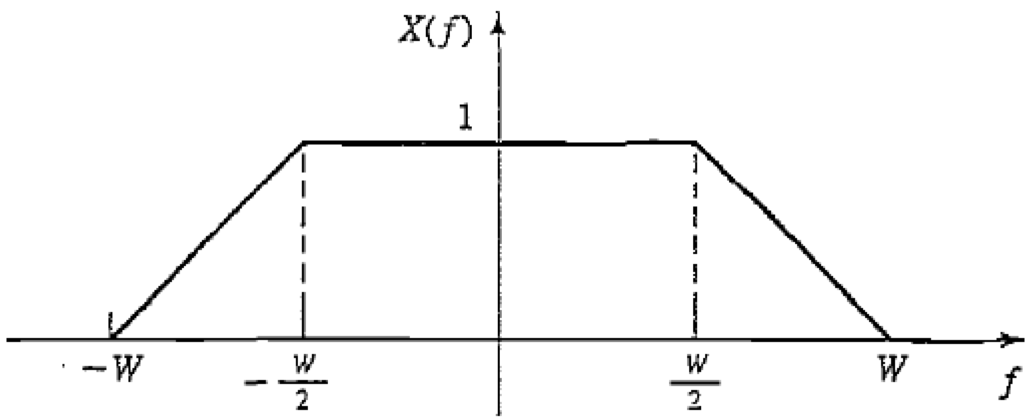
4.8-1 A transmitter transmits an AM signal with a carrier frequency of 1500 kHz. When an inexpensive radio receiver (which has a poor selectivity in its RF-stage bandpass filter) is tuned to 1500 kHz, the signal is heard loud and clear. This same signal is also heard (not as strong) at another dial setting. State, with reasons, at what frequency you will hear this station. The IF frequency is 455 kHz.

4.8-2 Consider a superheterodyne receiver designed to receive the frequency band of 1 to 30 MHz with IF frequency 8 MHz. What is the range of frequencies generated by the local oscillator for this receiver? An incoming signal with carrier frequency 10-MHz is received at the 10 MHz setting. At this setting of the receiver we also get interference from a signal with some other carrier frequency if the receiver RF stage bandpass filter has poor selectivity. What is the carrier frequency of the interfering signal?

2.43 Prove that

$$x_c(t) = x(t)\cos(2\pi f_0 t) + \hat{x}(t)\sin(2\pi f_0 t)$$
$$x_s(t) = \hat{x}(t)\cos(2\pi f_0 t) - x(t)\sin(2\pi f_0 t)$$

2.45 A lowpass signal $x(t)$ has a Fourier transform shown in Figure P-2.45(a). This signal is applied to the system shown in Figure P-2.45(b). The blocks marked by



(a)

2.48 Show that the signal $x(t)$ and its Hilbert transform are orthogonal; i.e.,

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\, dt = 0$$

2.49 Let $x(t)$ represent a bandpass signal and $m(t)$ denote a lowpass signal with non-overlapping spectra. Show that the Hilbert transform of $c(t) = m(t)x(t)$ is equal to $m(t)\hat{x}(t)$.

2.59 Let $m(t) = \text{sinc}^2(t)$ and let $x(t) = m(t)\cos 2\pi f_0 t - \hat{m}(t)\sin 2\pi f_o t$ represent a bandpass signal.

1. Find the pre-envelope, $z(t)$, and the lowpass equivalent signal to $x(t)$.
2. Determine and plot the Fourier transform of the signal $x(t)$. What is the bandwidth of $x(t)$?
3. Repeat for $x(t) = m(t)\cos 2\pi f_0 t + \hat{m}(t)\sin 2\pi f_o t$.

3.1 The message signal $m(t) = 2\cos 400t + 4\sin(500t + \frac{\pi}{3})$ modulates the carrier signal $c(t) = A\cos(8000\pi t)$, using DSB amplitude modulation. Find the time domain and frequency domain representation of the modulated signal and plot the spectrum (Fourier transform) of the modulated signal. What is the power content of the modulated signal?

3.2 In a DSB system the carrier is $c(t) = A\cos 2\pi f_c t$ and the message signal is given by $m(t) = \text{sinc}(t) + \text{sinc}^2(t)$. Find the frequency domain representation and the bandwidth of the modulated signal.

3.3 The two signals (a) and (b) shown in Figure P-3.3 DSB modulate a carrier signal $c(t) = A\cos 2\pi f_0 t$. Precisely plot the resulting modulated signals as a function of time and discuss their differences and similarities.

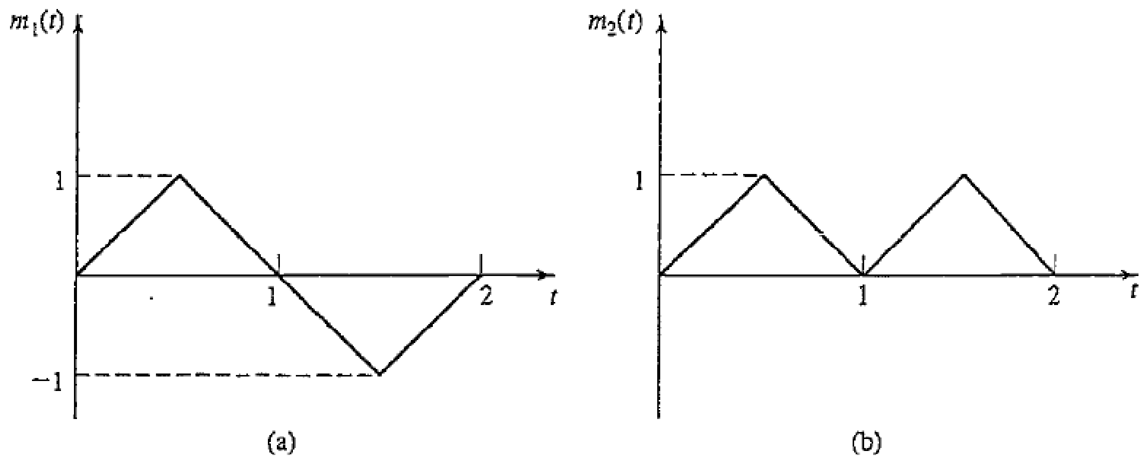


Figure P-3.3

3.4 Suppose the signal $x(t) = m(t) + \cos 2\pi f_c t$ is applied to a nonlinear system whose output is $y(t) = x(t) + \frac{1}{2}x^2(t)$. Determine and sketch the spectrum of $y(t)$ when $M(f)$ is as shown in Figure P-3.4 and $W \ll f_c$.

3.5 The modulating signal

$$m(t) = 2\cos 4000\pi t + 5\cos 6000\pi t$$

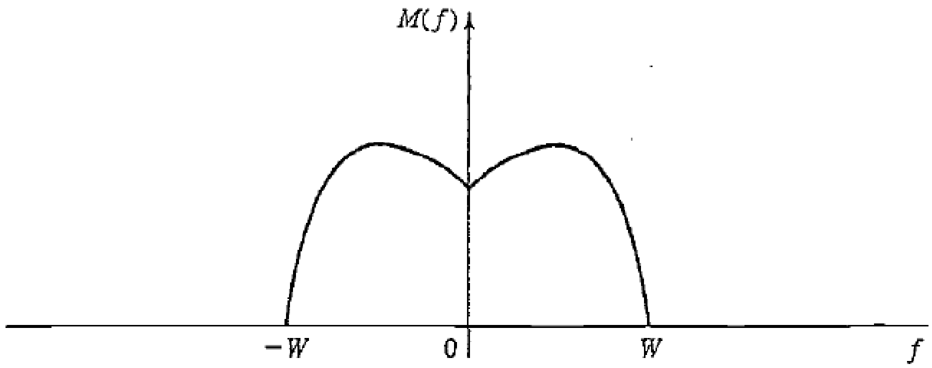


Figure P-3.4

is multiplied by the carrier

$$c(t) = 100 \cos 2\pi f_c t$$

where $f_c = 50$ kHz. Determine and sketch the power-spectral density of the DSB signal.

3.6 A DSB-modulated signal $u(t) = Am(t) \cos 2\pi f_c t$ is mixed (multiplied) with a local carrier $x_L(t) = \cos(2\pi f_c t + \theta)$ and the output is passed through a LPF with a bandwidth equal to the bandwidth of the message $m(t)$. Denoting the power of the signal at the output of the lowpass filter by P_{out} and the power of the modulated signal by P_U, plot $\frac{P_{out}}{P_U}$ as a function of θ for $0 \le \theta \le \pi$.

3.7 An AM signal has the form

$$u(t) = [20 + 2 \cos 3000\pi t + 10 \cos 6000\pi t] \cos 2\pi f_c t$$

where $f_c = 10^5$ Hz.

1. Sketch the (voltage) spectrum of $u(t)$.
2. Determine the power in each of the frequency components.
3. Determine the modulation index.
4. Determine the power in the sidebands, the total power, and the ratio of the sidebands power to the total power.

3.8 A message signal $m(t) = \cos 2000\pi t + 2 \cos 4000\pi t$ modulates the carrier $c(t) = 100 \cos 2\pi f_c t$ where $f_c = 1$ MHz to produce the DSB signal $m(t)c(t)$.

1. Determine the expression for the upper sideband (USB) signal.
2. Determine and sketch the spectrum of the USB signal.

3.9 A DSB-SC signal is generated by multiplying the message signal $m(t)$ with the periodic rectangular waveform shown in Figure P-3.9 and filtering the product with a bandpass filter tuned to the reciprocal of the period T_p, with bandwidth $2W$, where W is the bandwidth of the message signal. Demonstrate that the output

$u(t)$ of the BPF is the desired DSB-SC AM signal

$$u(t) = m(t) \sin 2\pi f_c t$$

where $f_c = 1/T_p$.

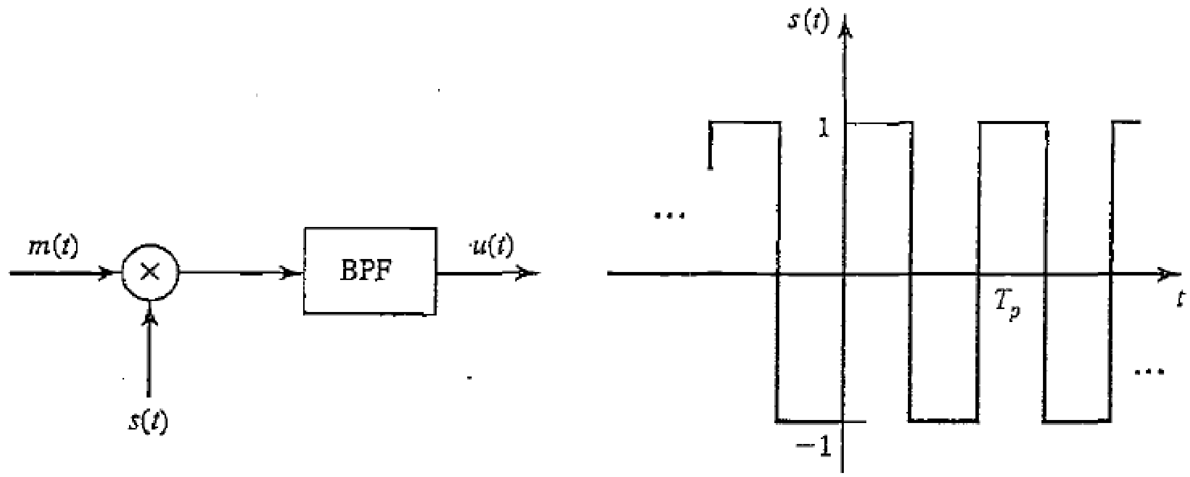


Figure P-3.9

3.10 Show that in generating a DSB-SC signal as in Problem P-3.9, it is not necessary that the periodic signal be rectangular. This means that any periodic signal with period T_p can substitute for the rectangular signal in Figure P-3.9.

3.13 An AM signal is generated by modulating the carrier $f_c = 800$ kHz by the signal

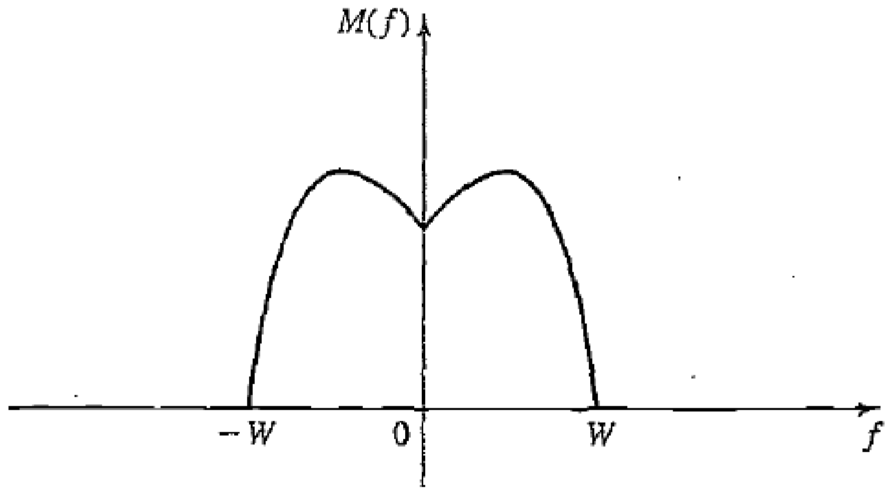
$$m(t) = \sin 2000\pi t + 5 \cos 4000\pi t t$$

The AM signal

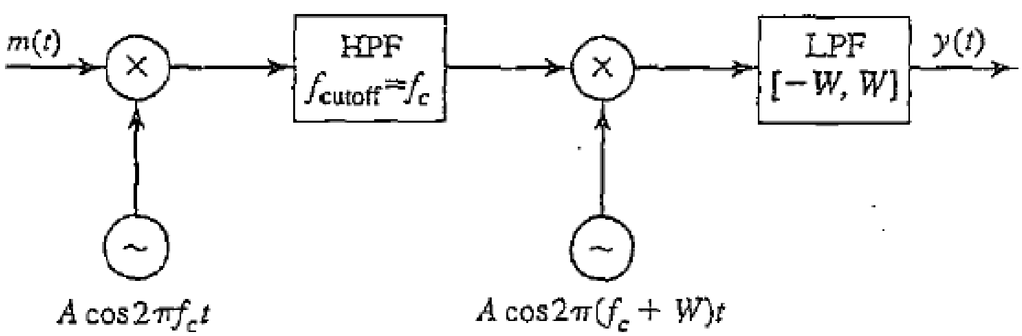
$$u(t) = 100[1 + m(t)] \cos 2\pi f_c t$$

is fed to a 50 Ω load.

1. Determine and sketch the spectrum of the AM signal.
2. Determine the average power in the carrier and in the sidebands.



(a)



(b)

Figure P-3.11

3. What is the modulation index?
4. What is the peak power delivered to the load?

3.14 The output signal from an AM modulator is

$$u(t) = 5\cos 1800\pi t + 20\cos 2000\pi t + 5\cos 2200\pi t$$

1. Determine the modulating signal $m(t)$ and the carrier $c(t)$.
2. Determine the modulation index.
3. Determine the ratio of the power in the sidebands to the power in the carrier.

3.15 A DSB-SC AM signal is modulated by the signal

$$m(t) = 2\cos 2000\pi t + \cos 6000\pi t$$

The modulated signal is

$$u(t) = 100m(t)\cos 2\pi f_c t$$

where $f_c = 1$ MHz.

1. Determine and sketch the spectrum of the AM signal.
2. Determine the average power in the frequency components.

3.16 A SSB AM signal is generated by modulating an 800-kHz carrier by the signal $m(t) = \cos 2000\pi t + 2\sin 2000\pi t$. The amplitude of the carrier is $A_c = 100$.

1. Determine the signal $\hat{m}(t)$.
2. Determine the (time domain) expression for the lower sideband of the SSB AM signal.
3. Determine the magnitude spectrum of the lower sideband SSB signal.

3.18 The message signal $m(t)$ whose spectrum is shown in Figure P-3.18 is passed through the system shown in the same figure.

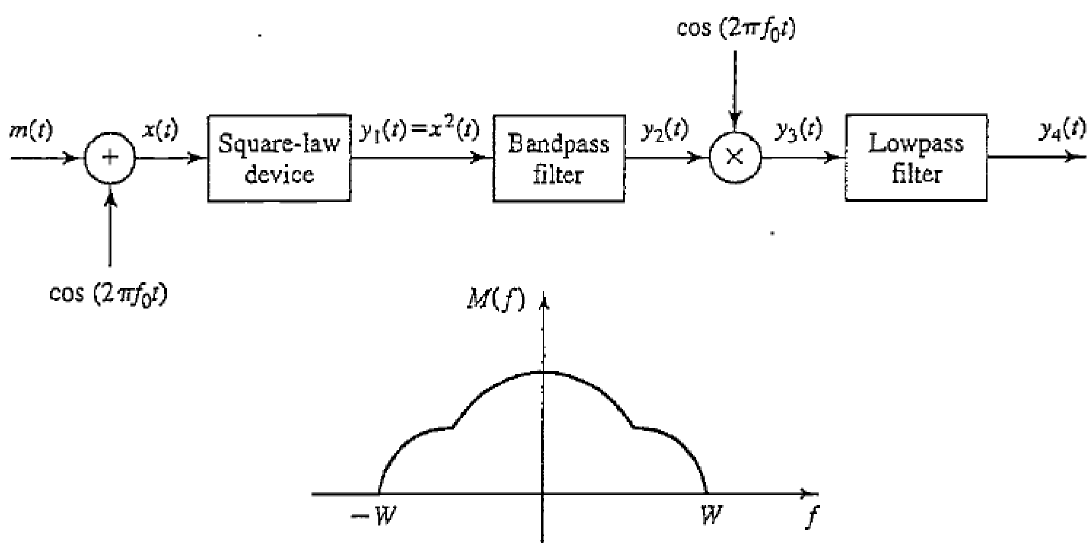


Figure P-3.18

The bandpass filter has a bandwidth of $2W$ centered at f_0 and the lowpass filter has a bandwidth of W. Plot the spectra of the signals $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$, and $y_4(t)$. What are the bandwidths of these signals?

3.19 The system shown in Figure P-3.19 is used to generate an AM signal. The modulating signal $m(t)$ has zero mean and its maximum (absolute) value is $A_m = \max |m(t)|$. The nonlinear device has a input–output characteristic

$$y(t) = ax(t) + bx^2(t)$$

1. Express $y(t)$ in terms of the modulating signal $m(t)$ and the carrier $c(t) = \cos 2\pi f_c t$.
2. What is the modulation index?
3. Specify the filter characteristics that yield an AM signal at its output.

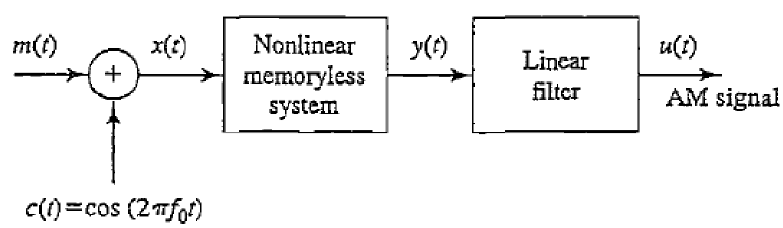
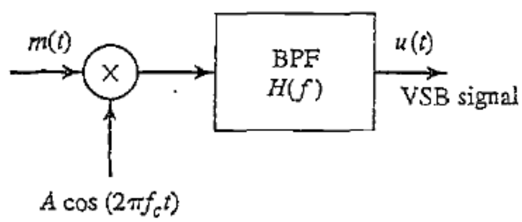
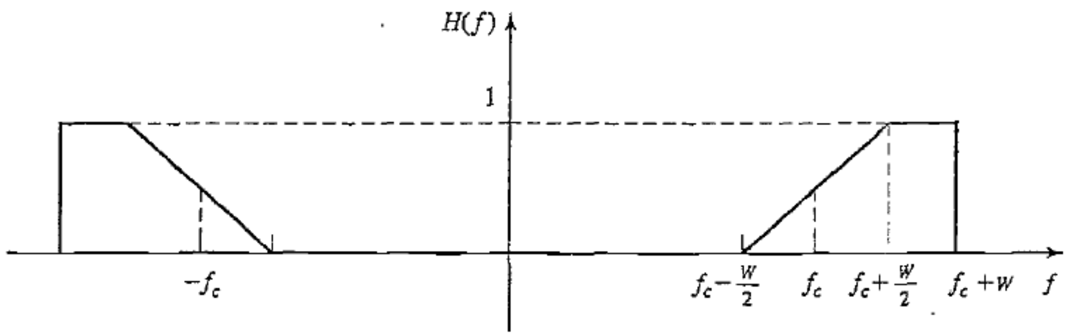


Figure P-3.19

3.21 A vestigial sideband modulation system is shown in Figure P-3.21. The bandwidth of the message signal $m(t)$ is W and the transfer function of the bandpass filter is shown in the figure.

1. Determine $h_l(t)$ the lowpass equivalent of $h(t)$, where $h(t)$ represents the impulse response of the bandpass filter.
2. Derive an expression for the modulated signal $u(t)$.





3.22 Find expressions for the in-phase and quadrature components, $x_c(t)$ and $x_s(t)$, and envelope and phase, $V(t)$ and $\Theta(t)$, for DSB, SSB, Conventional AM, USSB, LSSB, FM, and PM.

3.23 The normalized signal $m_n(t)$ has a bandwidth of 10,000 Hz and its power content is 0.5 W. The carrier $A \cos 2\pi f_0 t$ has a power content of 200 W.

1. If $m_n(t)$ modulates the carrier using SSB amplitude modulation, what will be the bandwidth and the power content of the modulated signal?
2. If the modulation scheme is DSB-SC, what will be the answer to part 1?
3. If the modulation scheme is AM with modulation index of 0.6, what will be the answer to part 1?

3.24 The message signal $m(t) = 10\,\text{sinc}(400t)$ frequency modulates the carrier $c(t) = 100 \cos 2\pi f_c t$. The modulation index is 6.

1. Write an expression for the modulated signal $u(t)$?
2. What is the maximum frequency deviation of the modulated signal?
3. What is the power content of the modulated signal?
4. Find the bandwidth of the modulated signal.