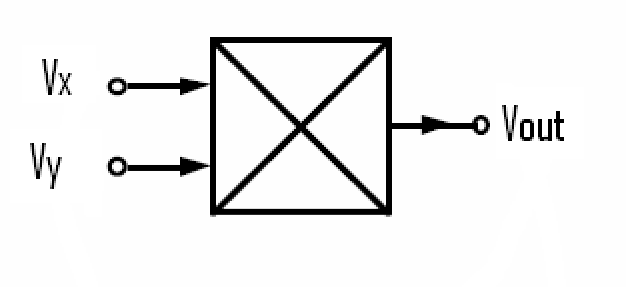
Microelectronics

Chapter 2:

(Lecture 2 and 3)

BJT and MOS Analog Multiplier



Analog Multiplier

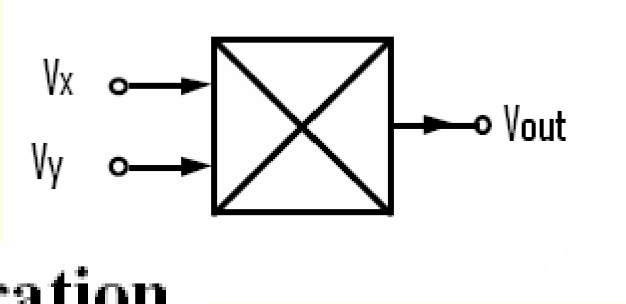
- Objectives:

1. Introduction
2. Revision of Simple Emitter Coupled Circuit .
3. Gilbert Cell As a 4-quadrant Multiplier
4. Complete Gilbert Multiplier without any restriction on the I/P Signals.
5. The application of the Gilbert Cell to realize a Phase Detector (PD)
6. Gilbert Cell as a DSB-SC Modulator
7. MOS Gilbert Cell

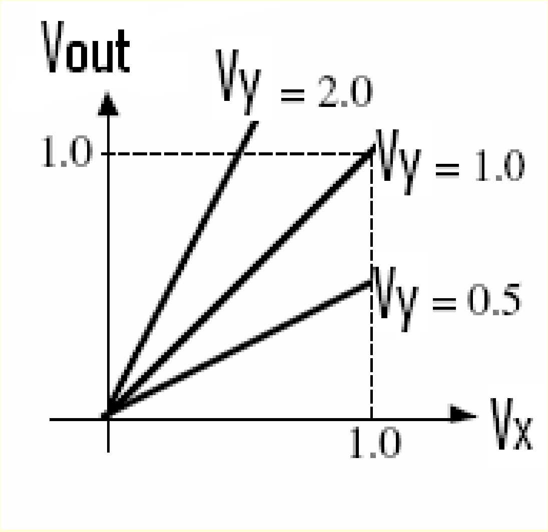
Introduction

- One of the fundamental building blocks in analog circuit design is the analog multiplier.
- Multipliers are particularly important in communication and signal processing circuits.
- **Applications of Multipliers**
  - Nonlinear analog signal processing
  - Mixing
  - Phase difference detection
  - Modulation and demodulation
  - Frequency translation

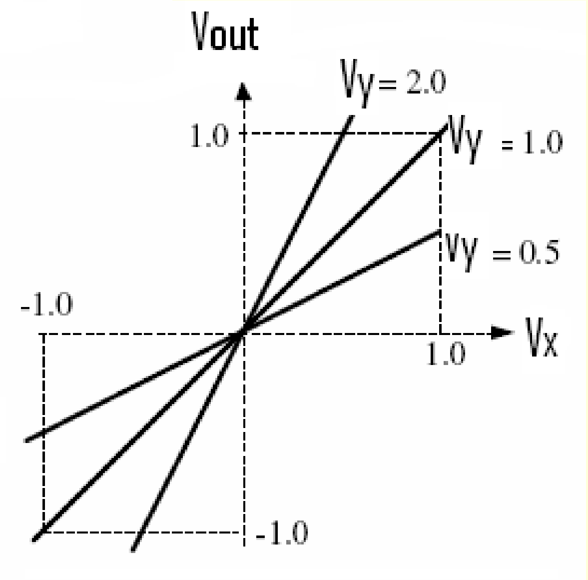
Introduction



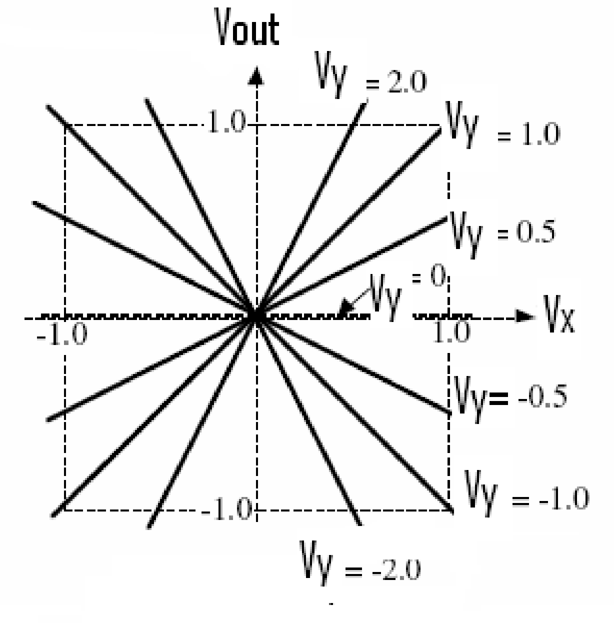
Types of Multiplication



One-Quadrant Multiplier



Two-Quadrant Multiplier



Four-Quadrant Multiplier

Simple Emitter Coupled Circuit (ECC)

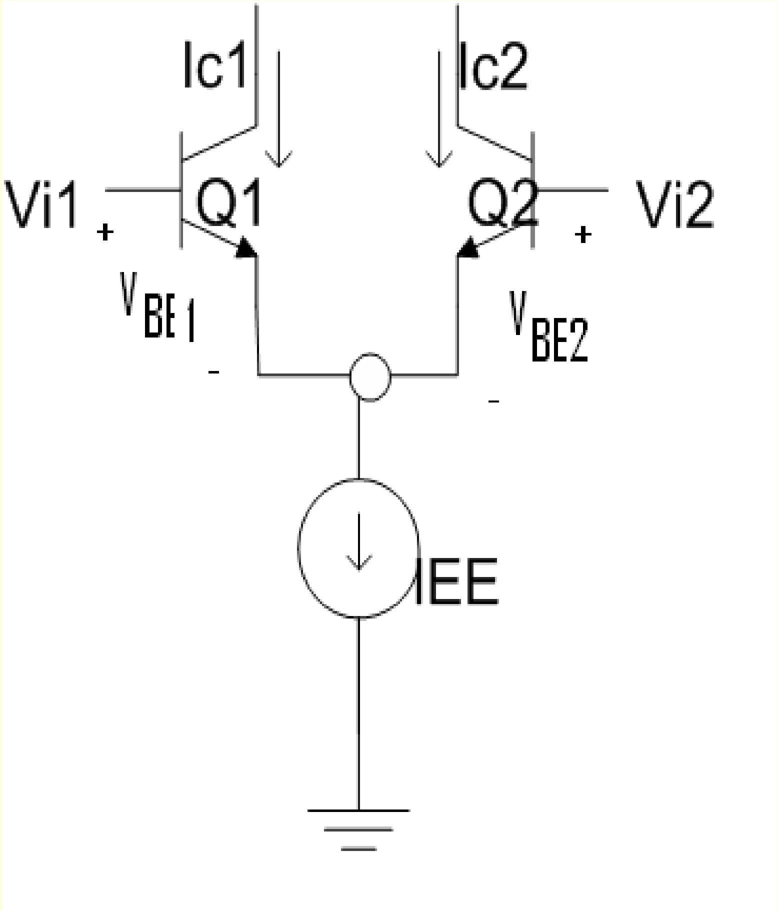
The Basic equation of the ECC is :

$$V_{i1} - V_{BE1} = V_{i2} - V_{BE2}$$

But in the active region, the collector current of the npn transistor is given by:

$$I_C = I_o e^{\frac{V_{BE}}{V_T}}$$

$$\Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_o}\right)$$



Simple Emitter Coupled Circuit (ECC)

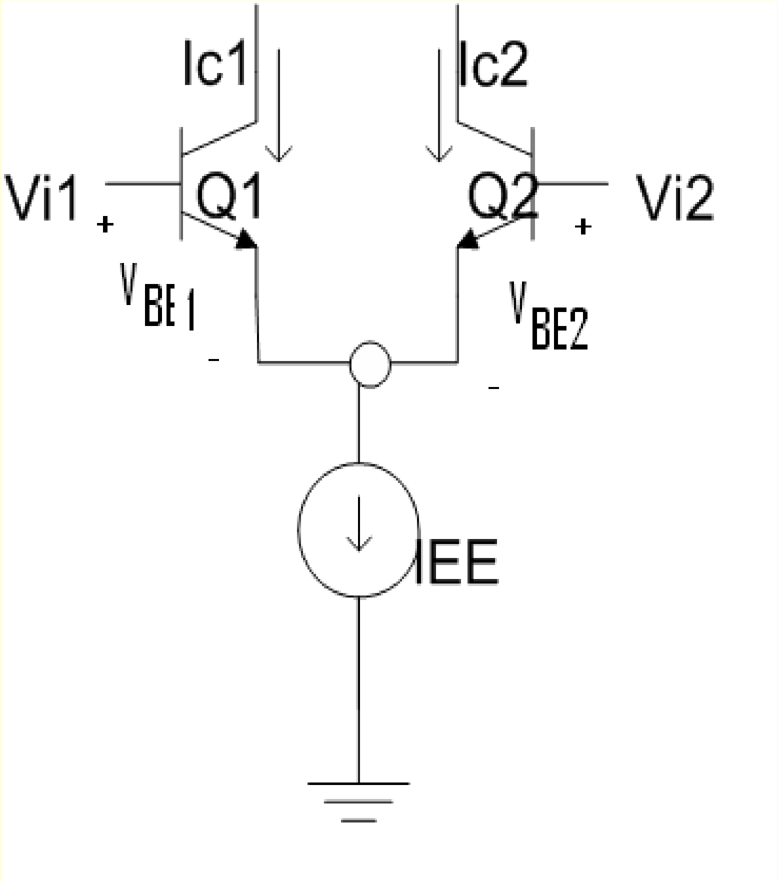
Therefore

$$V_{i1} - V_{i2} = V_{id} = V_T \ln(\frac{I_{C1}}{I_{o1}}) - V_T \ln(\frac{I_{C2}}{I_{o2}})$$

For a matched transistors

$$I_{o1} = I_{o2}$$

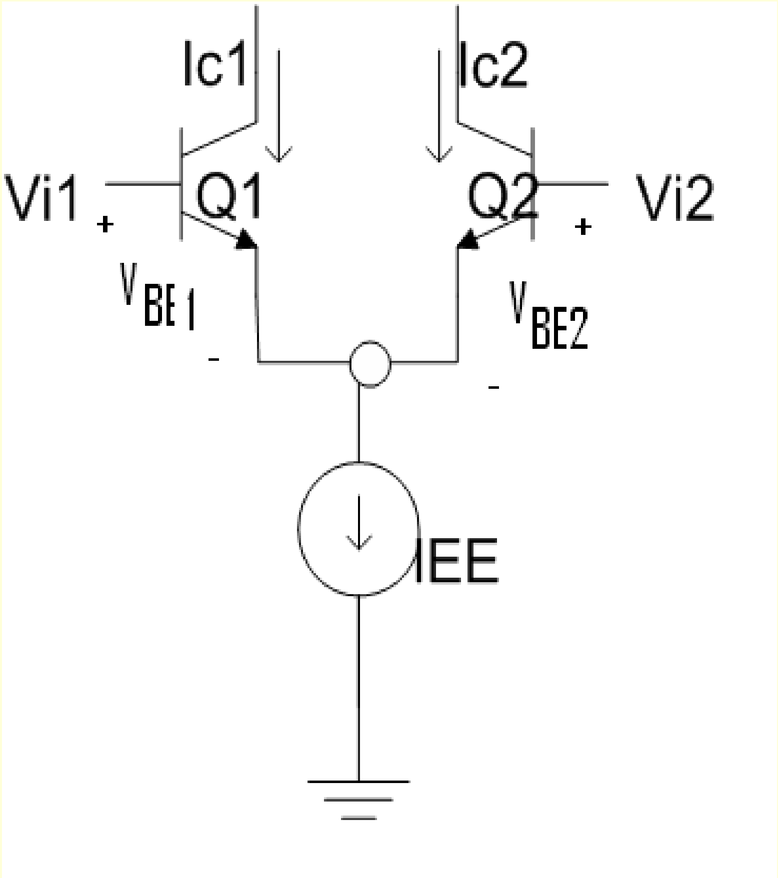
$$V_{id} = V_T \ln(\frac{I_{C1}}{I_{C2}})$$



Simple Emitter Coupled Circuit (ECC)

Therefore:

$$\frac{I_{C1}}{I_{C2}} = e^{\frac{V_{id}}{V_T}} \qquad (1)$$



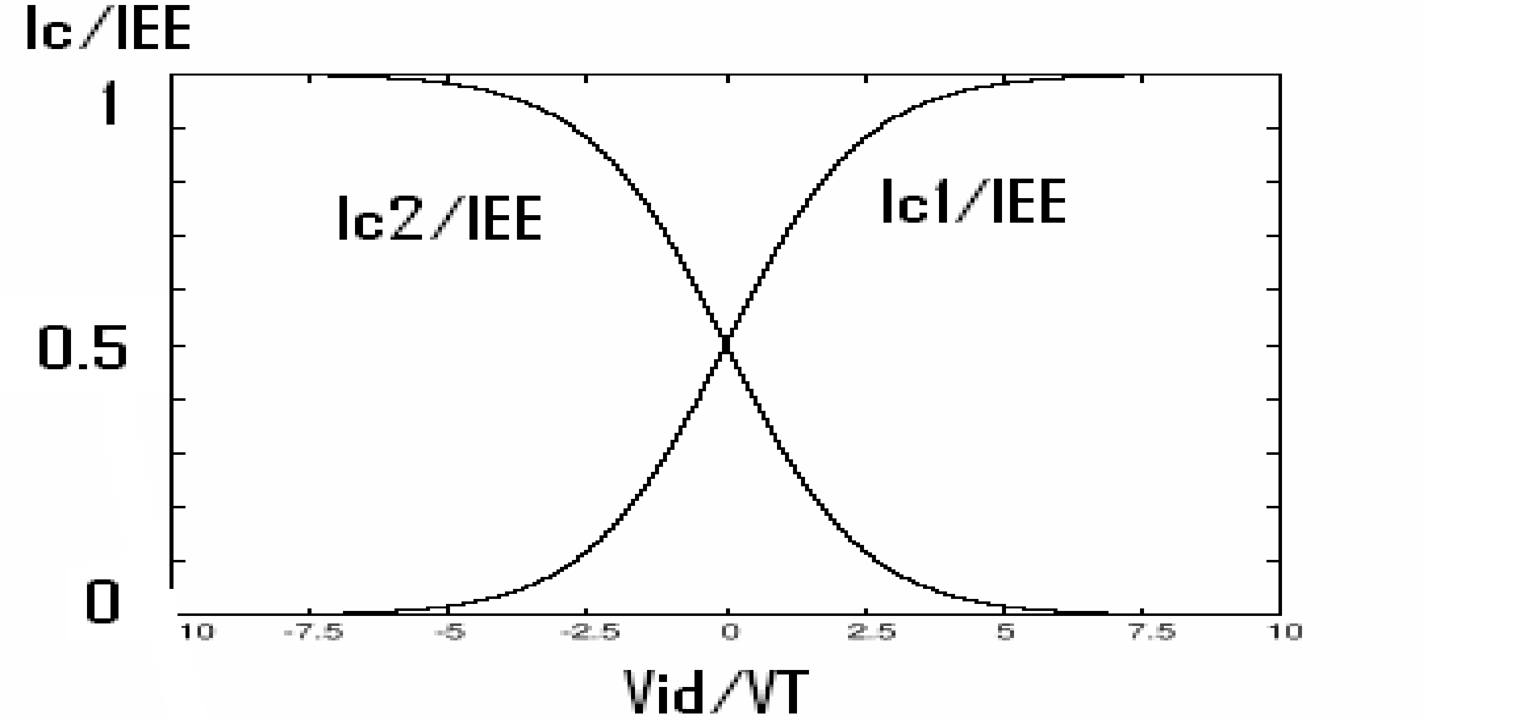
And assuming high β

$$I_{C1} + I_{C2} = IEE \qquad (2)$$

Simple Emitter Coupled Circuit (ECC)

From (1) and (2):

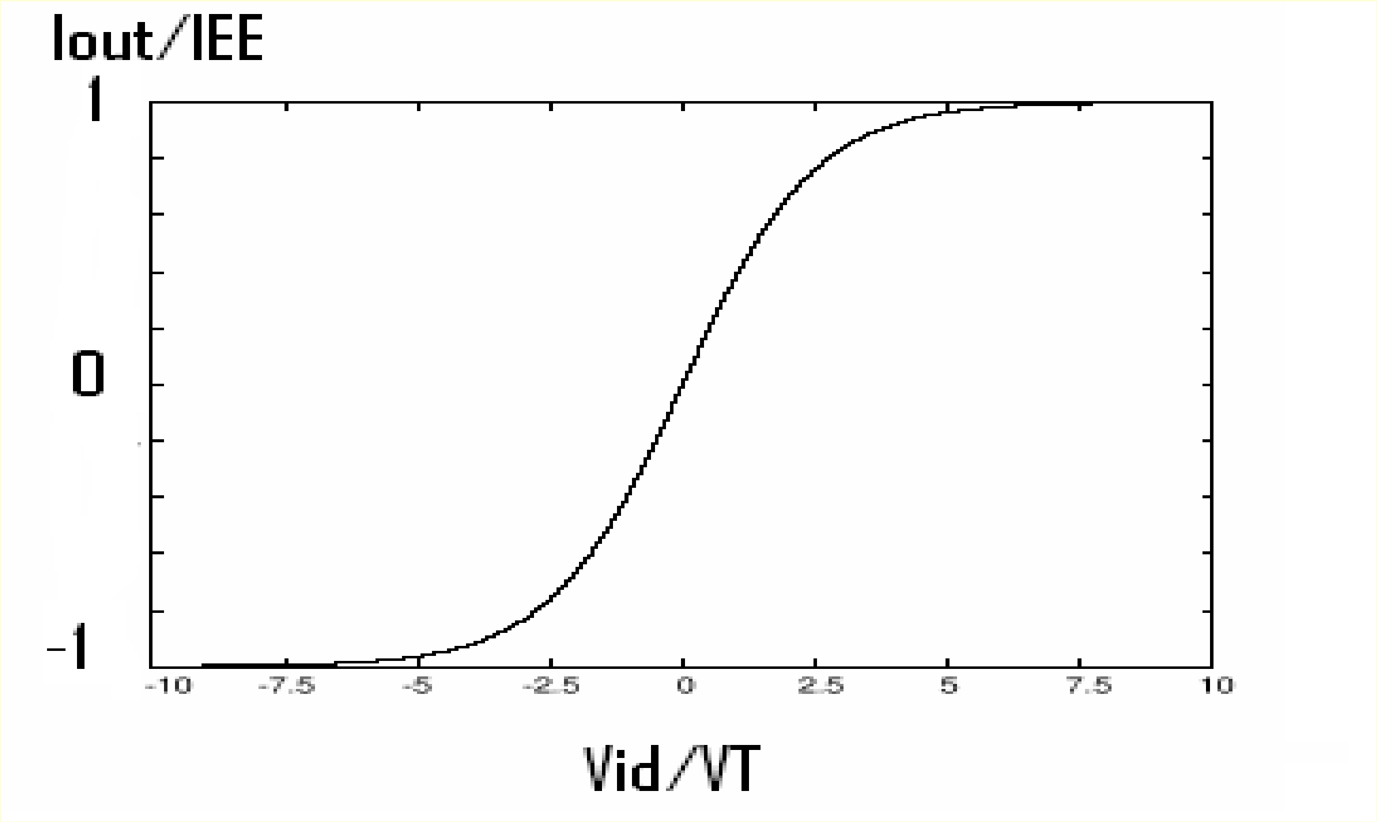
$$I_{C1} = \frac{IEE}{1+e^{-\frac{V_{id}}{V_T}}} \qquad \& \qquad I_{C2} = \frac{IEE}{1+e^{\frac{V_{id}}{V_T}}}$$



Simple Emitter Coupled Circuit (ECC)

And also:

$$I_{out} = IEE \tanh(\frac{V_{id}}{2V_T})$$



Simple Emitter Coupled Circuit (ECC)

Note That:

For $|V_{id}| << V_T$

The output current of the ECC is given by:

$$I_{out} \cong K(V_{id}) * (IEE)$$

Therefore, the circuit can operates as a multiplier with two inputs V_{id} **and** IEE

Where K is the constant of the proportionality and is given by:

$$K = \frac{1}{2V_T} \quad [V^{-1}]$$

Simple Emitter Coupled Circuit (ECC)

- **<u>The disadvantages of the ECC as an multiplier:</u>**

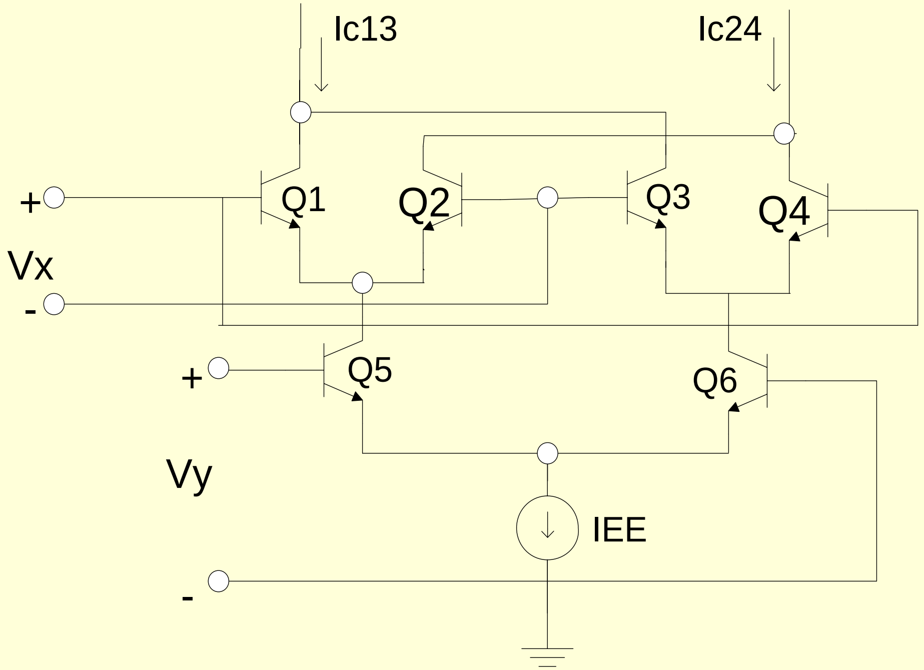
1. The circuits operates as 2- quadrant multiplier
   (V_{id} may be positive or negative but IEE must be positive)

2. The two inputs of the multiplier are not the same type
   (one of the inputs is voltage V_{id} and the another is current)

3. The circuits operates as 2- quadrant multiplier under the restriction: $|V_{id}| << V_T$

Gilbert Cell As a 4-quadrant Multiplier

The Gilbert Cell is a circuit used to overcome the disadvantages of the ECC.



The output differential current of the Gilbert Cell is defined as:

$$I_{out} = (I_{C13} - I_{C24}) = (I_{C1} + I_{C3}) - (I_{C2} + I_{C4})$$

Gilbert Cell As a 4-quadrant Multiplier

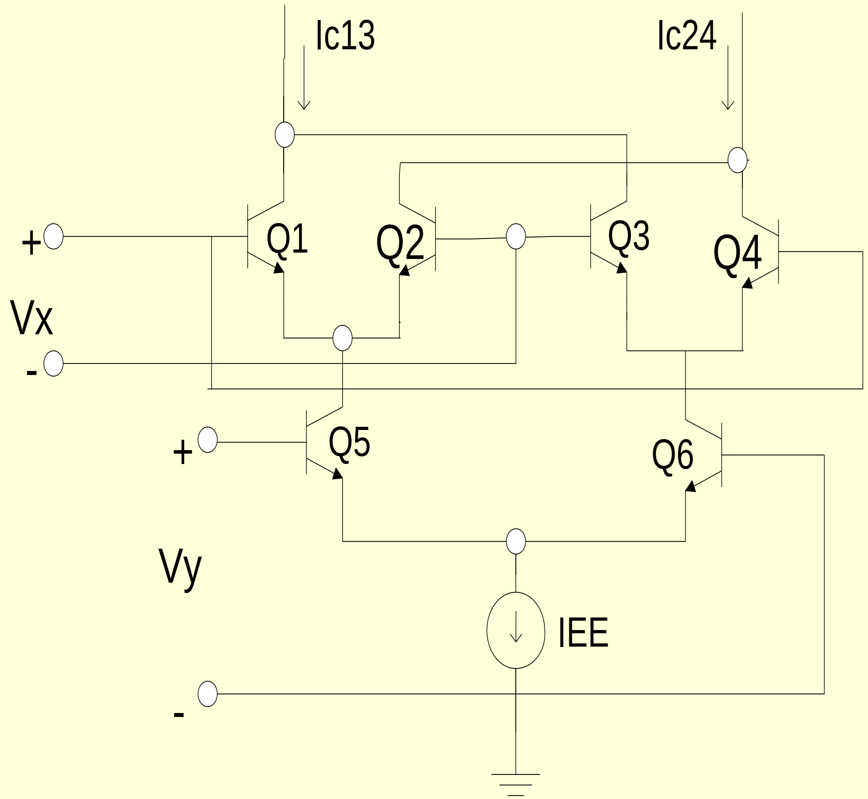
The output current can be written as:

$$I_{out} = (I_{C1} - I_{C2}) + (I_{C3} - I_{C4})$$

$$Iout = I_{C5} \tanh \frac{V_X}{2V_T} + I_{C6} \tanh(\frac{-V_X}{2V_T})$$

$$Iout = (I_{C5} - I_{C6}) \tanh(\frac{V_X}{2V_T})$$

$$Iout = IEE \tanh(\frac{V_Y}{2V_T}) \tanh(\frac{V_X}{2V_T})$$



Gilbert Cell As a 4-quadrant Multiplier

Note that:

From the expression of the output current of the Gilbert Cell:

$$Iout = IEE \tanh(\frac{V_Y}{2V_T}) \tanh(\frac{V_X}{2V_T})$$

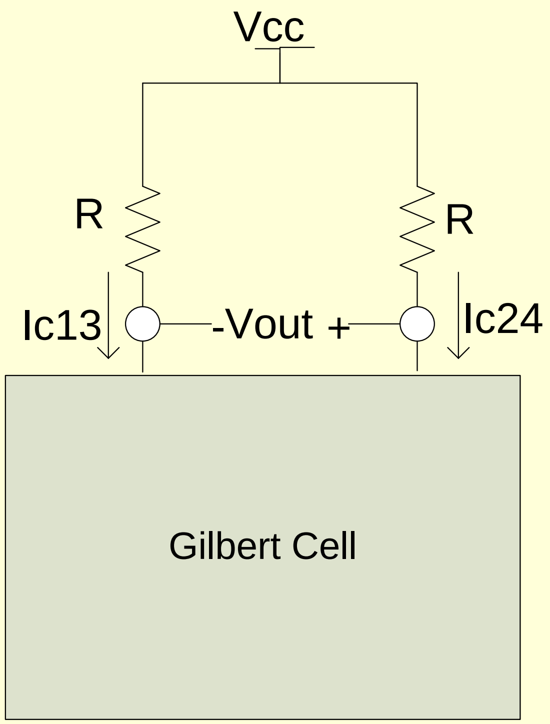
For

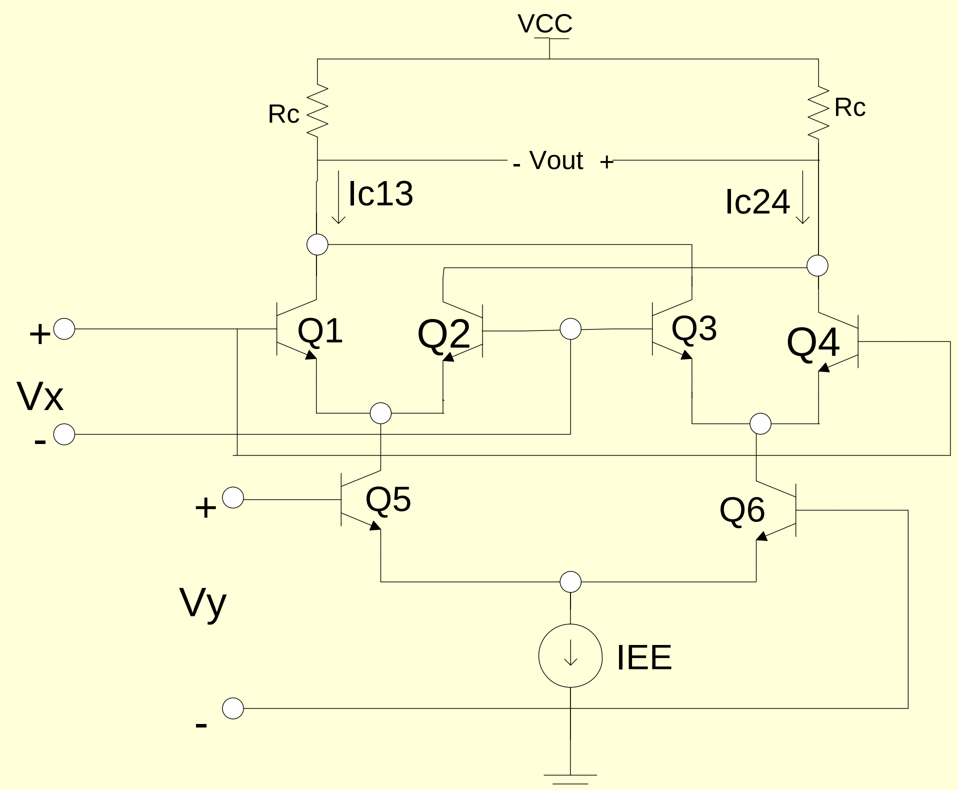
$$|V_X| \& |V_Y| << V_T \quad \Rightarrow \quad I_{out} \cong K(V_X * V_Y)$$

Therefore Gilbert Cell operates as 4-quadrant multiplier because $Vx \& Vy$ can take positive and negative values, Where $K = \frac{IEE}{(2V_T)^2}[mA/V^2]$

Gilbert Cell As a 4-quadrant Multiplier with Output Voltage

Note that : The output of the Gilbert Multiplier is a current, and if it is required to have an output voltage, a differential current to a voltage converter is needed. This can be realized by using the two resistors connected to Vcc as shown.





Gilbert Cell As a 4-quadrant Multiplier with Output Voltage

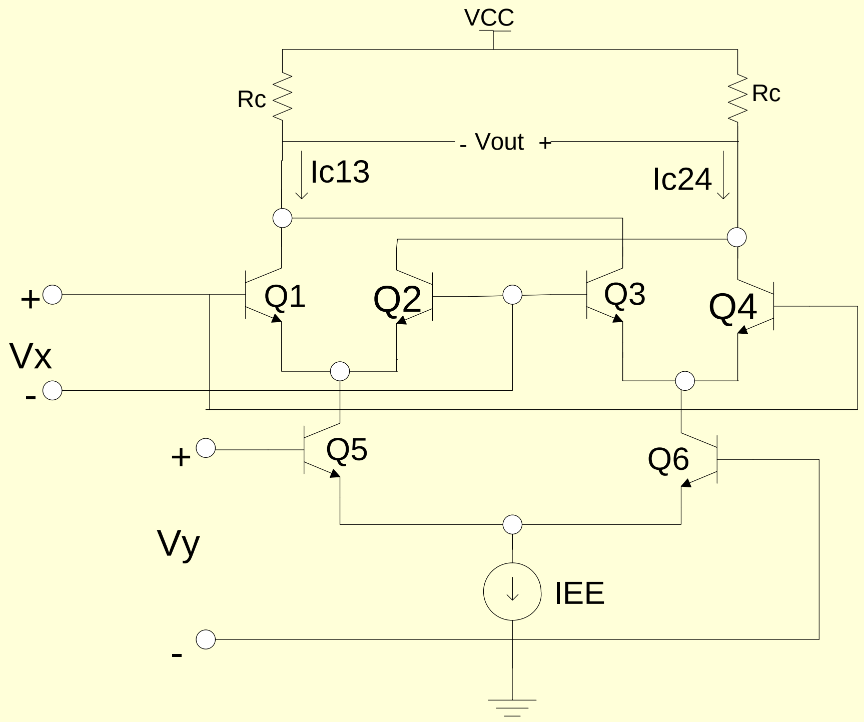
$$V_{out} = (V_{CC} - I_{C24}R_C) - (V_{CC} - I_{C13}R_C)$$

$$V_{out} = (I_{C13} - I_{C24})R_C$$

$$V_{out} = I_{EE}R_C \tanh(\frac{V_X}{2V_T})\tanh(\frac{V_Y}{2V_T})$$

$$V_{out} \cong K(V_X * V_Y) \quad \text{For } |V_X| \& |V_Y| << V_T$$

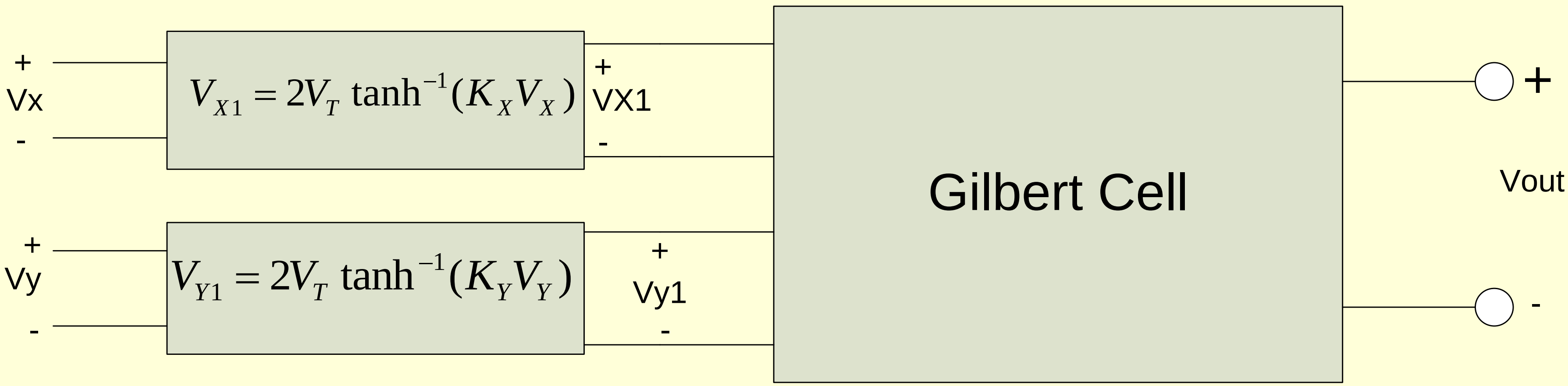
Where $$K = I_{EE}R_C / (2V_T)^2 [V^{-1}]$$



Note that: For large signals, the Gilbert cell is not operate correctly as a multiplier, therefore we try to make the I/P signals pass through certain circuits that eliminate the nonlinearity of the tanh function.

Complete Gilbert Multiplier without any restriction on the I/P Signals

- Basic Idea



$$V_{out} = I_{EE} R_C \tanh\left(\frac{V_{X1} = 2V_T \tanh^{-1}(K_X V_X)}{2V_T}\right) \tanh\left(\frac{V_{Y1} = 2V_T \tanh^{-1}(K_Y V_Y)}{2V_T}\right)$$

$$V_{out} = K(V_X * V_Y) \quad \text{where } K = I_{EE} R_C K_X K_Y [V^{-1}]$$

$\tanh^{-1}$ circuit

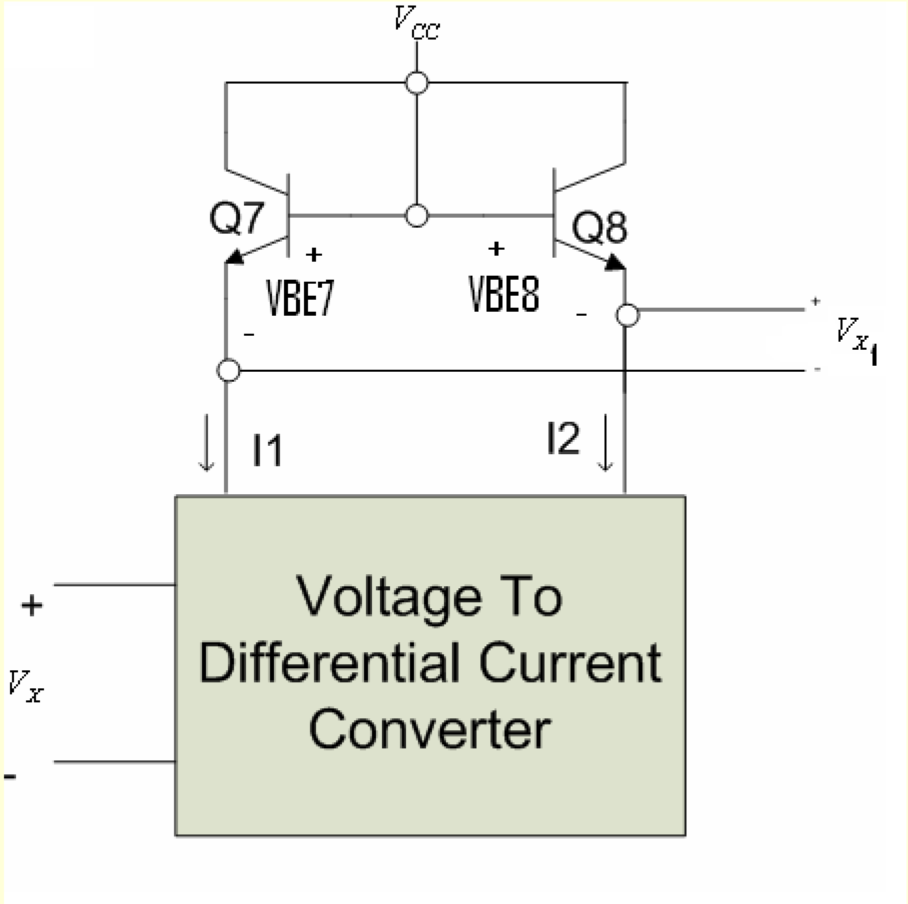
From the shown circuit:

$$V_{X1} = V_{BE7} - V_{BE8}$$

$$V_{X1} = V_T \ln(\frac{I_1}{I_{o7}}) - V_T \ln(\frac{I_2}{I_{o8}})$$

Assuming Q7 and Q8 are matched

$$V_{X1} = V_T \ln(\frac{I_1}{I_2})$$



$\tanh^{-1}$ circuit

Assuming the voltage to differential current converter produces two Output currents given by:

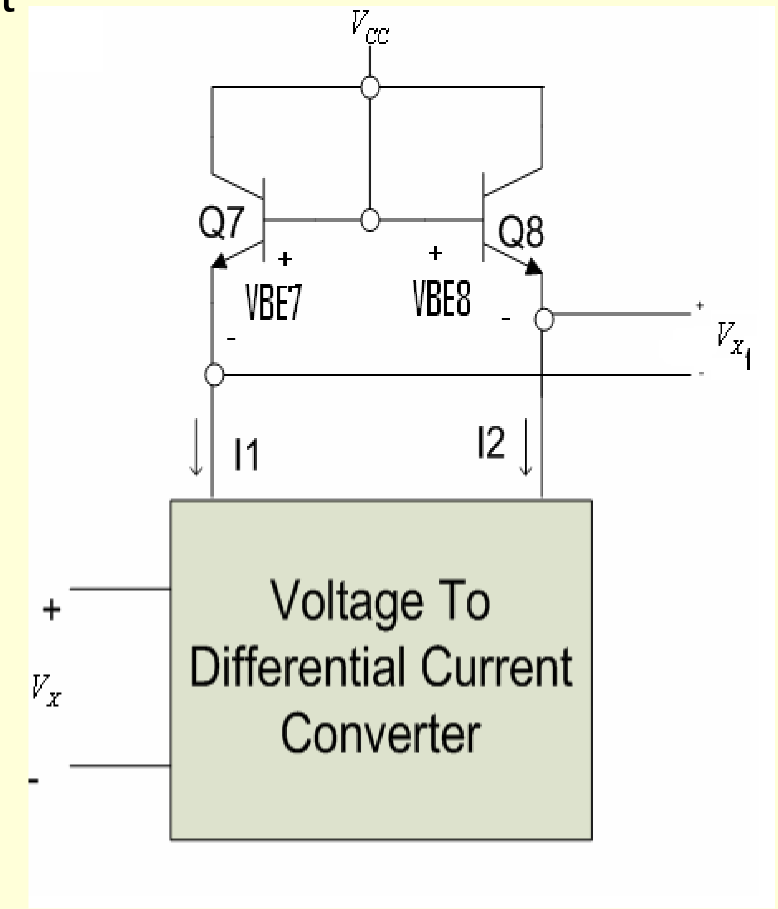
$$I_1 = I_{dc}(1 + K_X V_X) \qquad \&$$

$$I_2 = I_{dc}(1 - K_X V_X)$$

Therefore, the output voltage of the $\tanh^{-1}$ circuit is given by:

$$V_{X1} = V_T \ln\left(\frac{1 + K_X V_X}{1 - K_X V_X}\right)$$

$$\Rightarrow \quad V_{X1} = 2V_T \tanh^{-1}(K_X V_X)$$



Voltage to Differential Current Converter

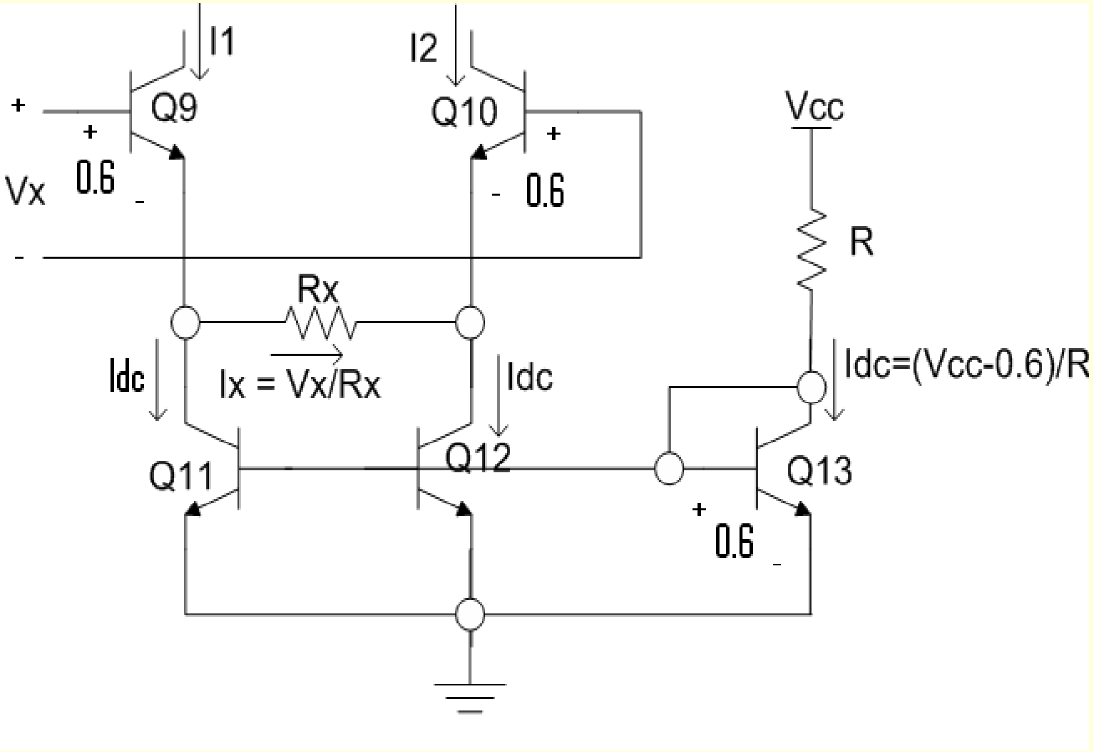
From the shown circuit, we have:

$$I_1 = I_{dc} + \frac{V_X}{R_X}$$

$$I_1 = I_{dc}\left(1 + \frac{V_X}{I_{dc}R_X}\right)$$

$$\Rightarrow I_1 = I_{dc}(1 + K_X V_X)$$

and $$K_X = \frac{1}{I_{dc}R_X}$$



Similarly $$I_2 = I_{dc}(1 - K_X V_X)$$

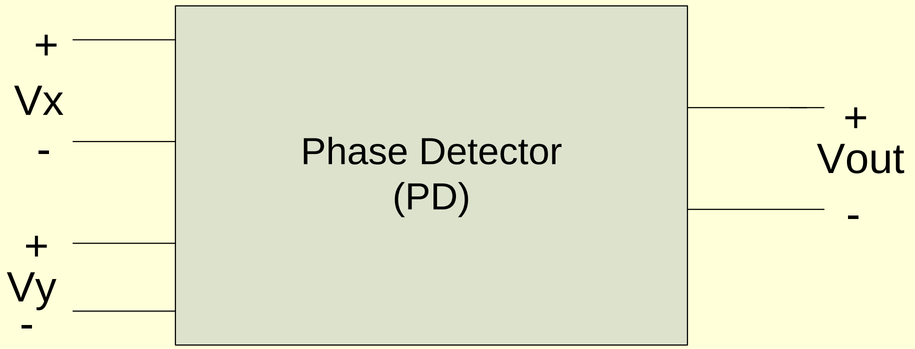
The application of the Gilbert Cell to realize a Phase Detector (PD)

Phase Detector is a circuit provide output voltage proportional to the phase difference between two input signals.

Types of PD:

Linearized phase detector: The output is linearly proportional to the phase difference between the two input signal.

Sinusoidal phase detector: The output is proportional to the Sin or Cosine the phase difference between the two input signal.



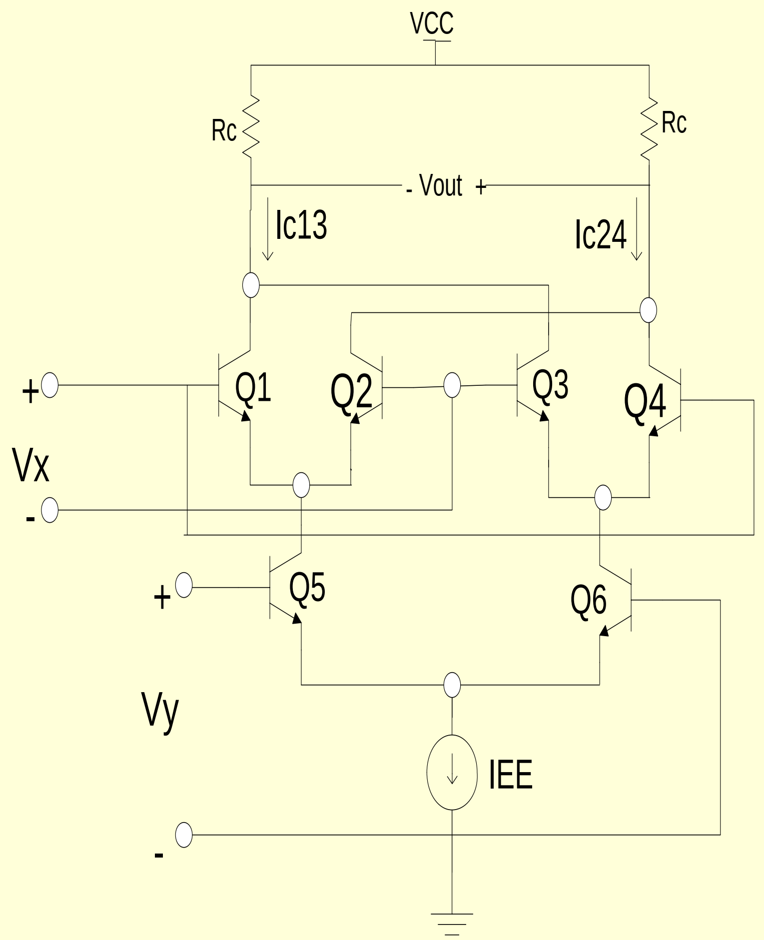
Linearized PD

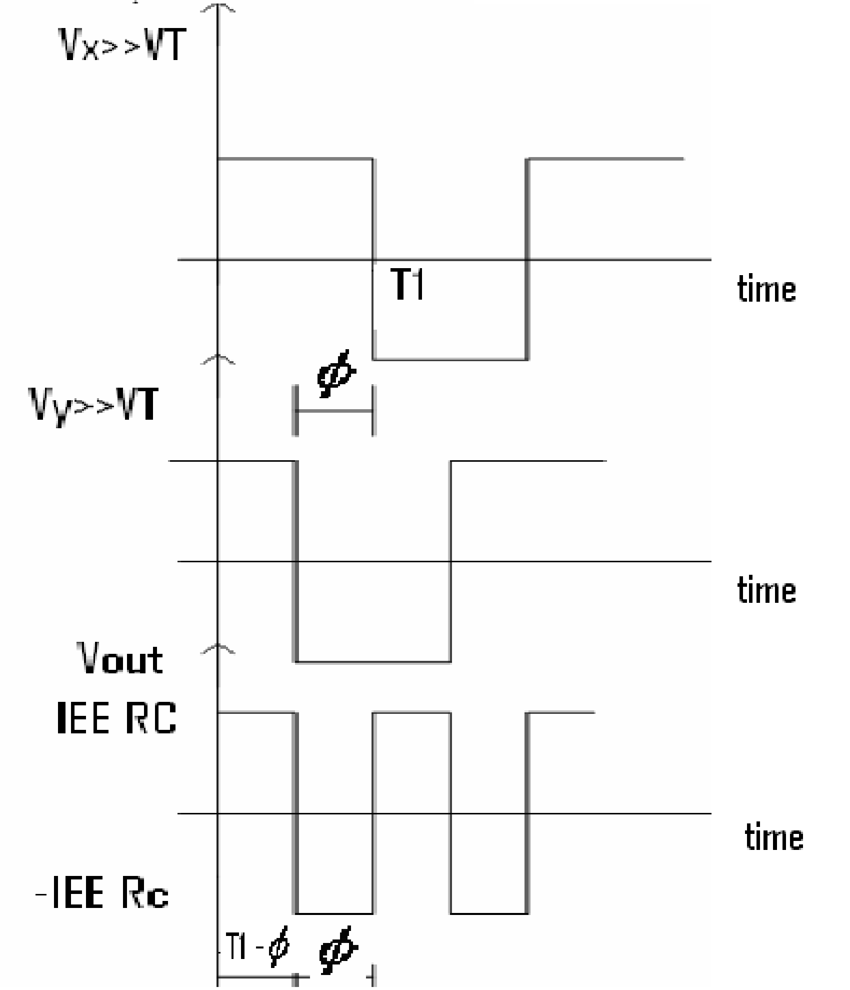
Condition on the inputs for proper operations:

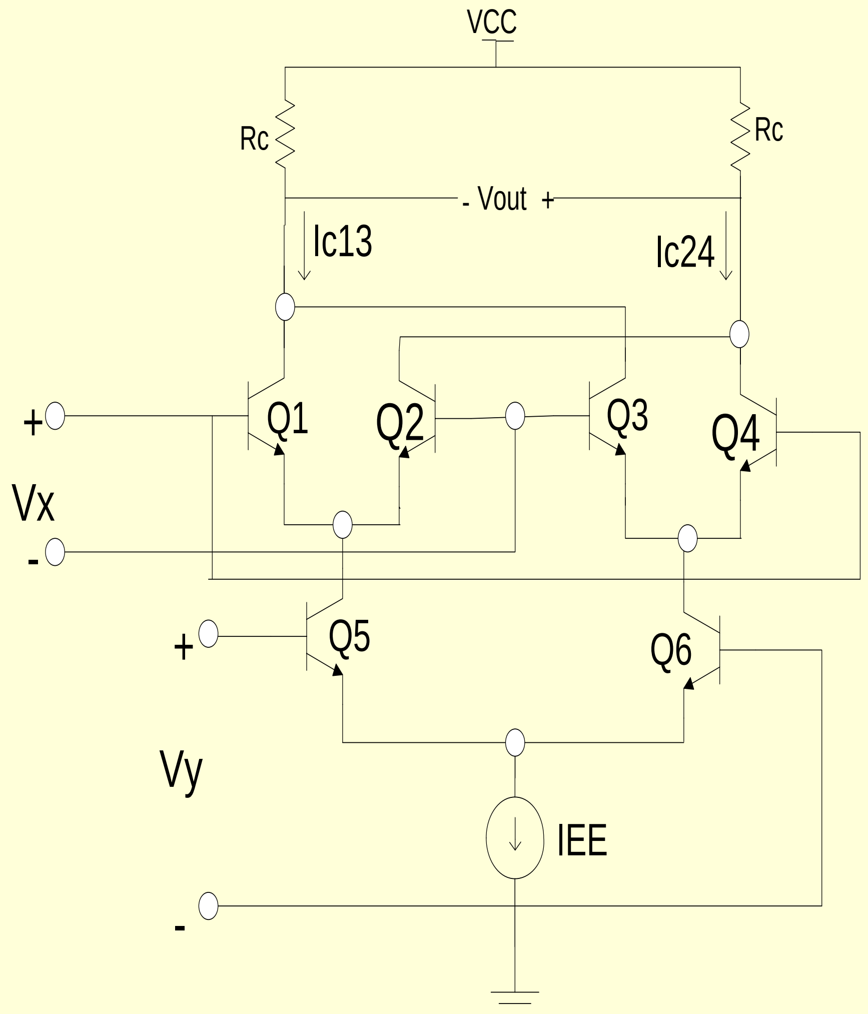
The amplitudes of $|V_X| \& |V_y| >> V_T$

Therefore, the transistors operate as a switch (ON/OFF)

Example: Draw the output waveform and calculate its average value for the input signal waveforms shown below:







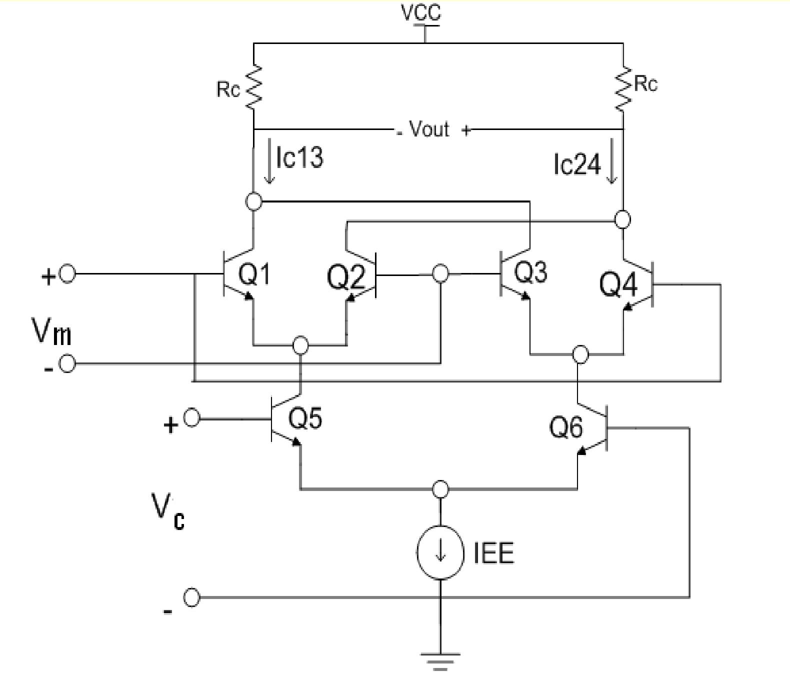
$$\bar{V}_{out} = \int_0^{T_1} v_{out}(t)dt = -\frac{1}{T_1}[I_{EE}R_C\phi - I_{EE}R_C(T_1-\phi)]$$

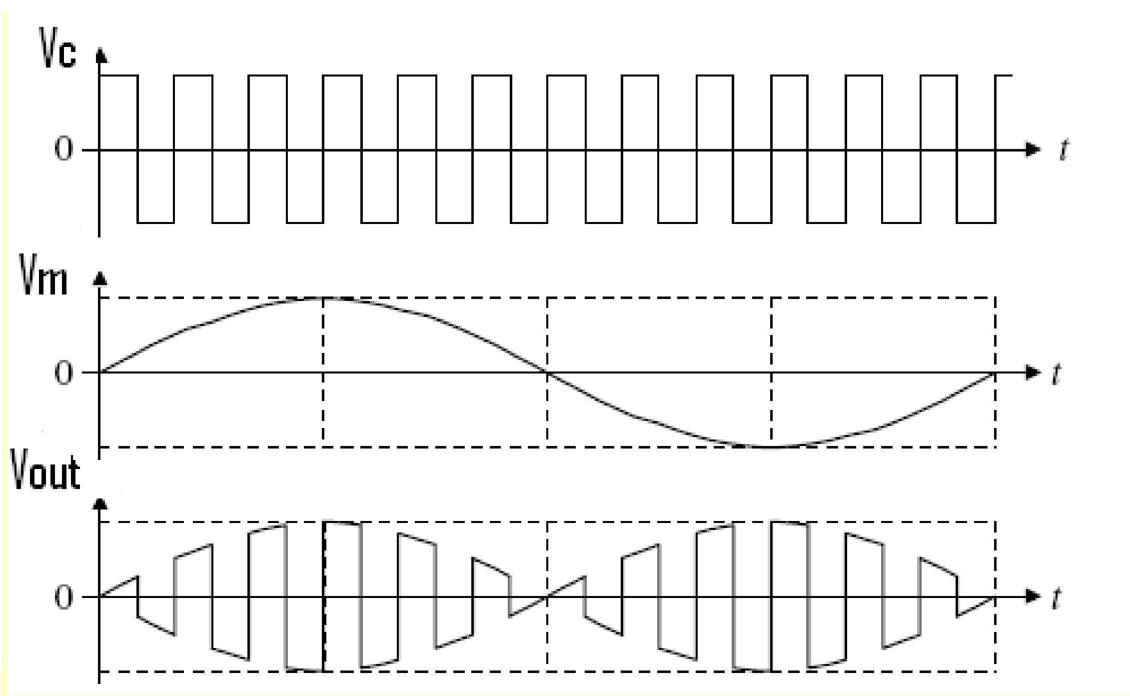
$$\bar{V}_{out} = -I_{EE}R_C[2\phi/T_1 - 1]$$

Gilbert Cell as a DSB-SC Modulator

Condition on the inputs for proper operations:

The Gilbert Cell can be used as a switching modulator under the condition, One of the input signal(Carrier) is >> VT and the another input(modulating Signal) is <<VT

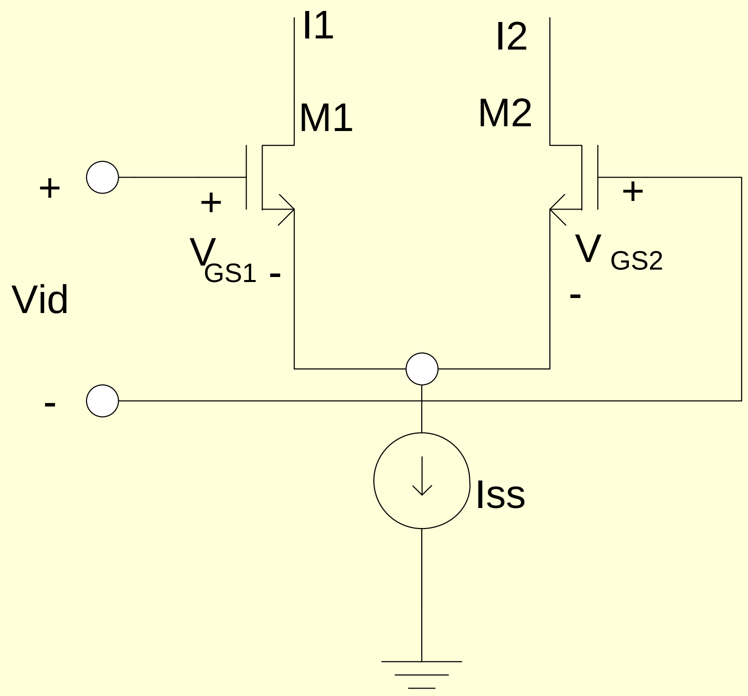




MOS Multiplier: NMOS Differential Pair

The output current of the NMOS Differential pair is given:

$$I_{out} = I_1 - I_2$$



Where the currents of M1 and M2 in the saturation region are given by:

$$I_1 = \frac{K_1}{2}(V_{GS1} - V_T)^2 = \frac{K_1}{2}A^2$$

$$I_2 = \frac{K_2}{2}(V_{GS2} - V_T)^2 = \frac{K_2}{2}B^2$$

And assuming M1 and M2 are matched (K_1=K_2= $\mu_n C_{ox}\dfrac{W}{L}$)

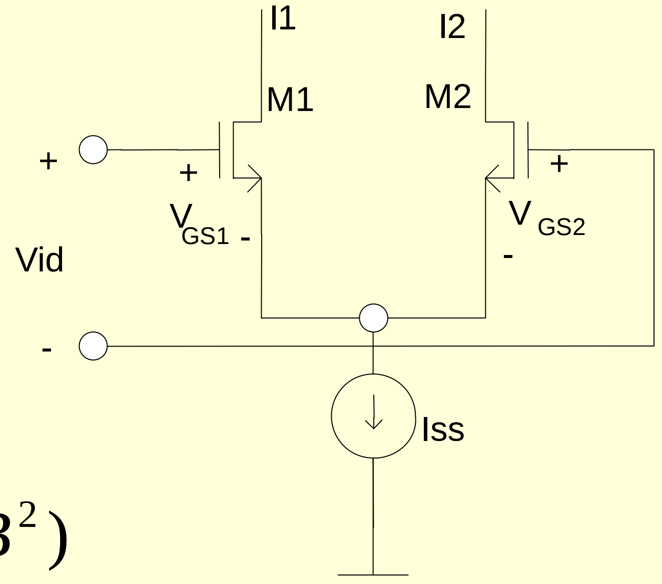
MOS Multiplier: NMOS Differential Pair

Therefore:

$$I_{out} = \frac{K}{2}(A^2 - B^2) = \frac{K}{2}(A-B)(A+B)$$

And $I_1 + I_2 = \frac{K}{2}(A^2 + B^2) = I_{SS}$

Where $A - B = V_{GS1} - V_{GS2} = V_{id}$



Note: $(A+B)^2 + (A-B)^2 = 2(A^2 + B^2)$

$$(A+B)^2 = 4\frac{I_{SS}}{K} - V^2{}_{id} \quad \Longrightarrow \quad (A+B) = \sqrt{4\frac{I_{SS}}{K}}\left(\sqrt{1 - \frac{V^2{}_{id}}{4I_{SS}/K}}\right)$$

$$I_{out} = \sqrt{I_{SS}K}V_{id}\left(\sqrt{1 - \frac{V^2{}_{id}}{4I_{SS}/K}}\right)$$

MOS Multiplier: NMOS Differential Pair

Note:

$$I_{out} = I_1 - I_2 = \sqrt{I_{SS}K}V_{id}\left(\sqrt{1 - \frac{V^2{}_{id}}{4I_{SS}/K}}\right)$$

and

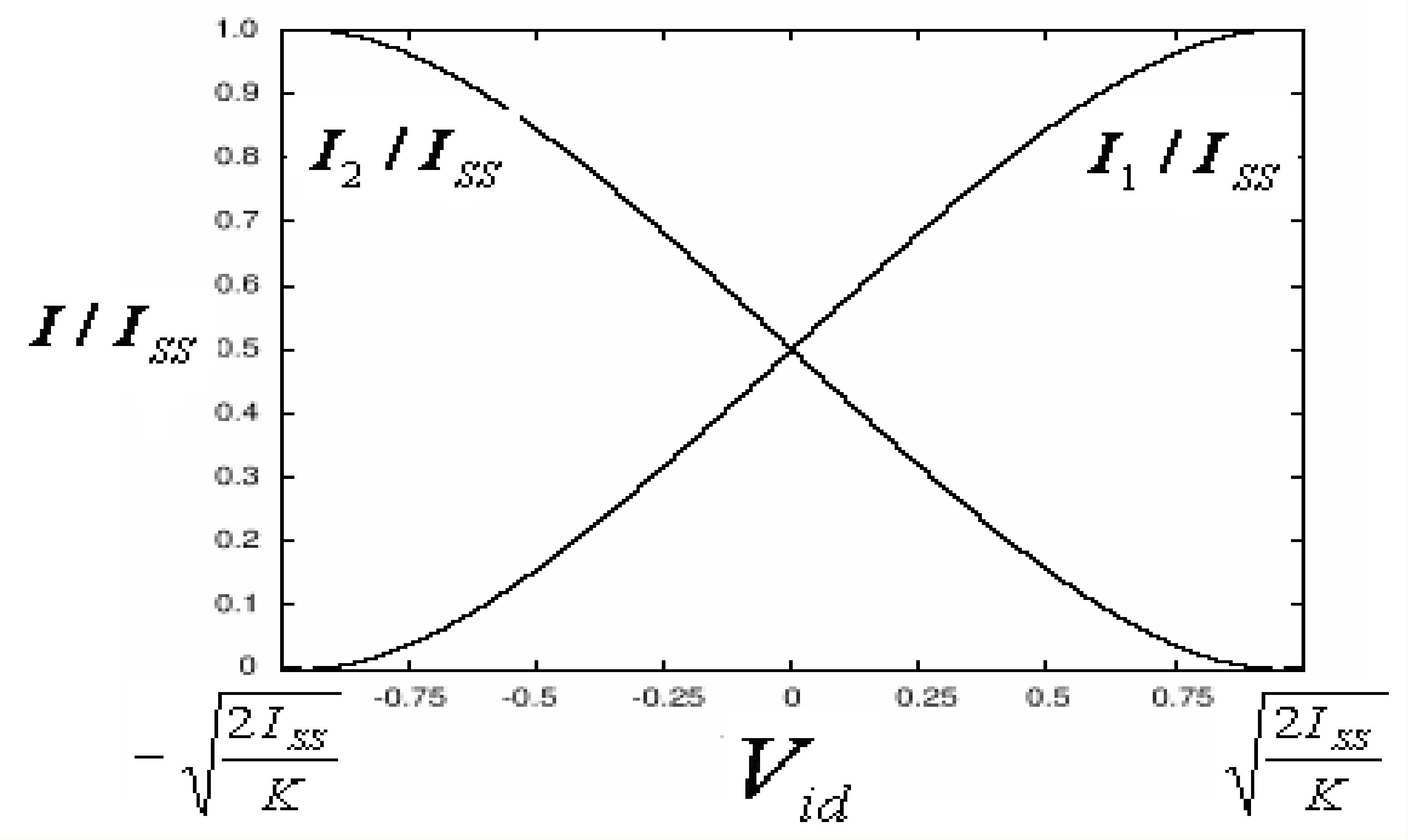
$$I_1 + I_2 = I_{SS}$$

Therefore:

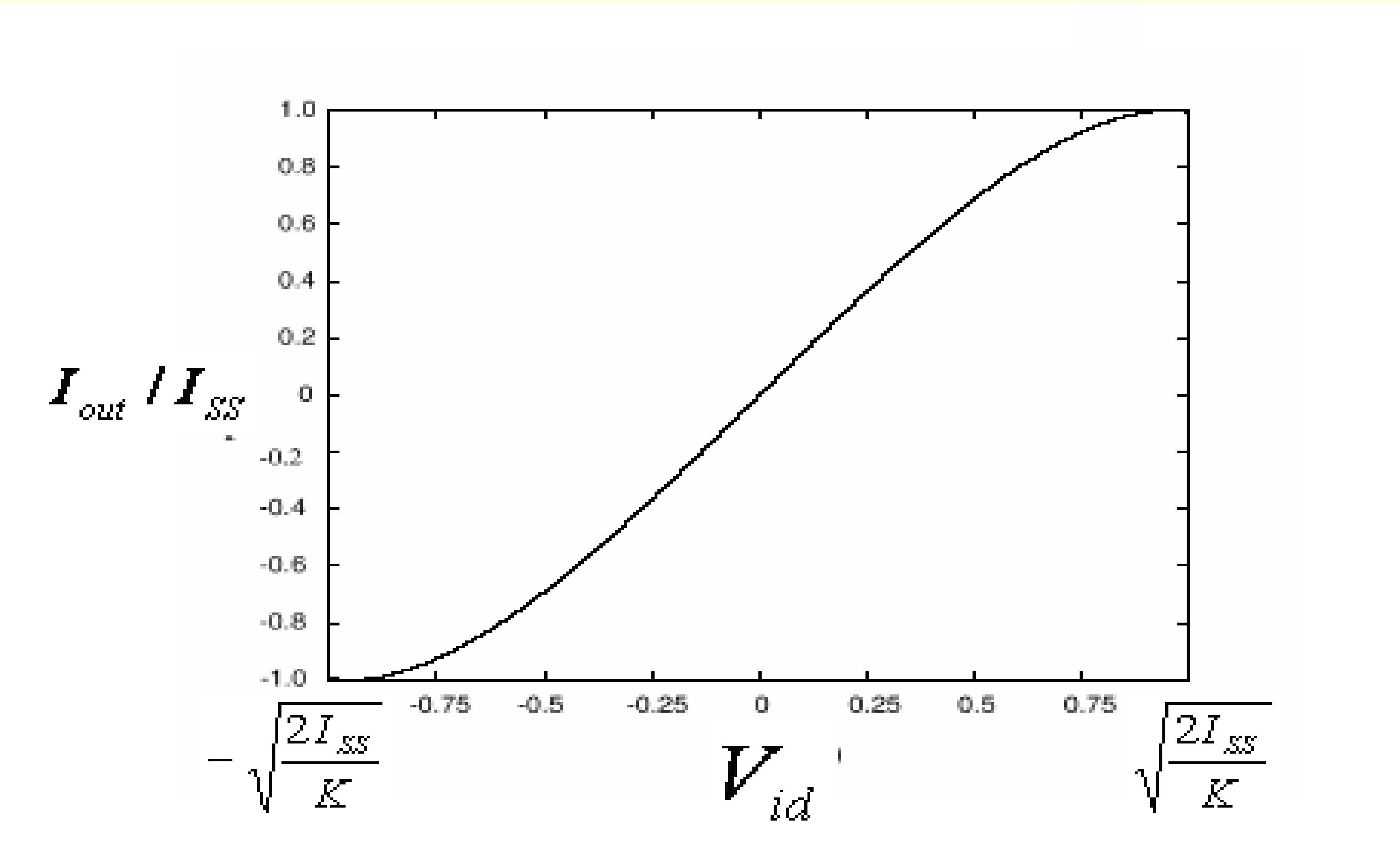
$$I_1 = \frac{I_{SS}}{2} + \frac{1}{2}\sqrt{I_{SS}K}V_{id}\left(\sqrt{1 - \frac{V^2{}_{id}}{4I_{SS}/K}}\right)$$

$$I_2 = \frac{I_{SS}}{2} - \frac{1}{2}\sqrt{I_{SS}K}V_{id}\left(\sqrt{1 - \frac{V^2{}_{id}}{4I_{SS}/K}}\right)$$

MOS Multiplier: NMOS Differential Pair



MOS Multiplier: NMOS Differential Pair



MOS Multiplier: NMOS Differential Pair

Note: if $V_{id} < \sqrt{\frac{2I_{SS}}{K}}$

The nonlinear term of output current can be neglected and the output current is given by:

$$I_{out} \cong \sqrt{I_{SS}K}V_{id}$$

and

$$I_1 \cong \frac{I_{SS}}{2} + \frac{1}{2}\sqrt{I_{SS}K}V_{id}$$

$$I_2 \cong \frac{I_{SS}}{2} - \frac{1}{2}\sqrt{I_{SS}K}V_{id}$$

MOS Gilbert Cell

The output current can be written as:

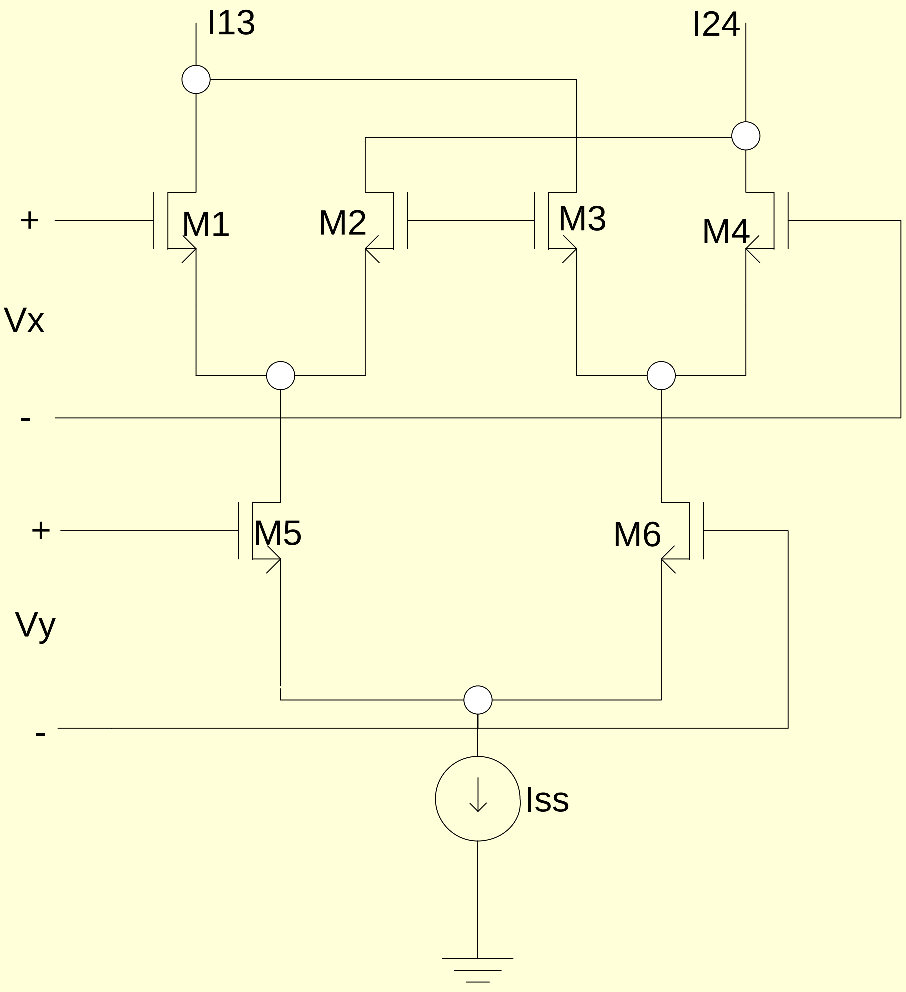
$$I_{out} = I_{13} - I_{24} = (I_1 - I_3) + (I_2 - I_4)$$

$$I_{out} \cong \sqrt{I_5 K} V_X - \sqrt{I_6 K} V_X$$

$$I_{out} \cong \sqrt{K}(\sqrt{I_5} - \sqrt{I_6}) V_X$$

and $(\sqrt{I_5} - \sqrt{I_6}) = \sqrt{\frac{K}{2}} V_y$

$$\Rightarrow \quad I_{out} \cong \frac{K}{\sqrt{2}}(V_X * V_y)$$



END of Chapter 2