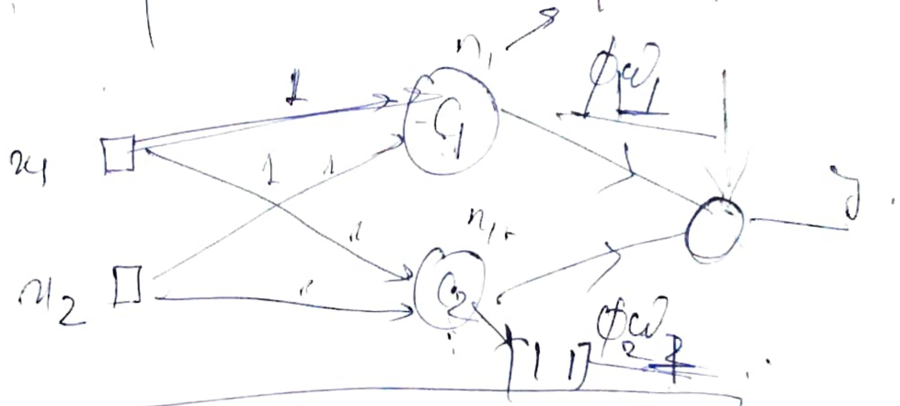
XNOR RBF

i/p 1	i/p 2	Y
0	0	1
0	1	0
1	0	0
1	1	1



$$\boxed{\phi = \exp(-\|x - c_i\|^2)}$$

1st case

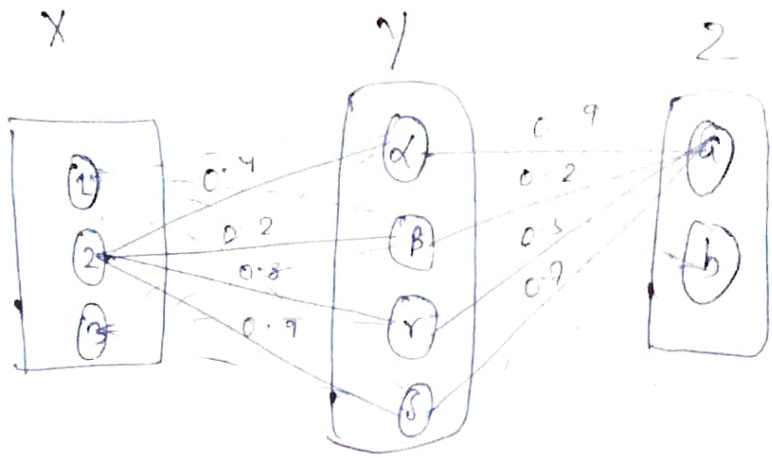
$$\phi_1 = \phi(\|0-0\|^2 + \|0-0\|^2) = e^0$$

$$= \phi(\|x - c_1\|^2)$$

$$\phi_2 = \phi(\|x - c_2\|^2)$$

$$= \phi(\|0-1\|^2 + \|0-1\|^2)$$

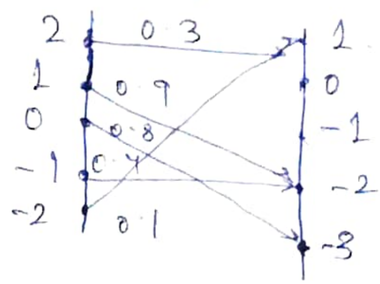
$$w_1 e^0 + w_2 e^{-2} + \theta = 1 \quad \text{———} \quad (i)$$



3Q. Apply extension principle to the fuzzy set with discrete universe

$$A = 0.1/-2 + 0.4/-1 + 0.8/0 + 0.9/1 + 0.3/2$$

$$f(x) = x^2 - 3$$



Ans:

By applying extension principle

$$B = 0.1/1 + 0.4/-2 + 0.8/-3 + 0.9/-2 + 0.3/1$$

$$= 0.8/-3 + (0.4 \vee 0.9)/-2 + (0.1 \vee 0.3)/1$$

$$= 0.8/-3 + 0.9/-2 + 0.3/1$$

4Q. Application of the extension principle to the fuzzy sets with continuous universes

$$\mu_A(x) = \text{bell}(x; 1.5, 2, 0.5)$$

$$f(x) = \begin{cases} (x-1)^2 - 1, & \text{if } x \geq 0 \\ x, & \text{if } x < 0 \end{cases}$$

Primary terms (young, middle, aged, old).
negation ("not").
hedges (very, more or less, quite, extremely, etc.).

Concentration operation

$CON(A) = A^2$

$$A^K = \int_X [\mu_A(x)]^K / x$$

Dilation

$DIL(A) = A^{0.5}$

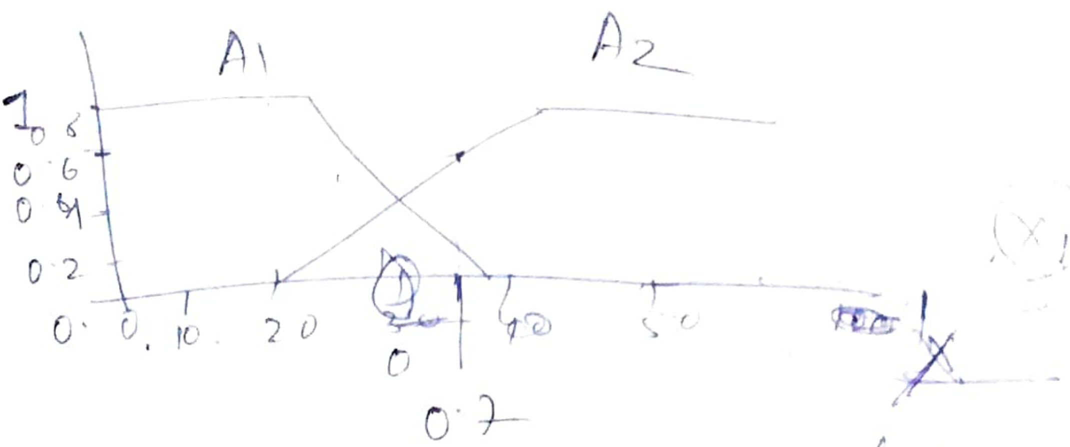
$NOT(A) = \neg A = \int_X [1 - \mu_A(x)] / x$

$A \text{ AND } B = A \cap B = \int_X [\mu_A(x) \wedge \mu_B(x)] / x$

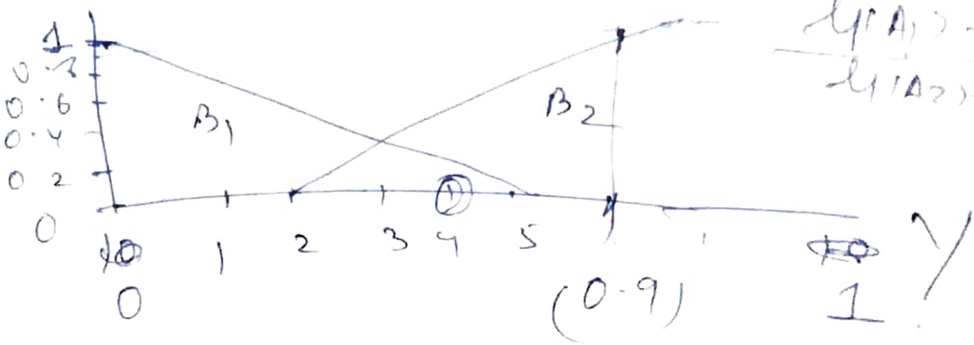
$A \text{ OR } B = A \cup B = \int_X [\mu_A(x) \vee \mu_B(x)] / x$

$$\mu_{young}(x) = bell(x, 20, 2, 0) = \frac{1}{1 + \left(\frac{x}{20}\right)^4}$$

$$\mu_{old}(x) = bell(x, 30, 3, 100) = \frac{1}{1 + \left(\frac{x - 100}{30}\right)^6}$$



$\mu(A_1) = 0.2$
$\mu(A_2) = 0.6$



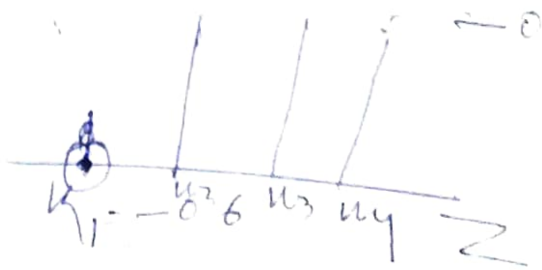
$\mu(B_1) = 0$
$\mu(B_2) = 0.8$

Rule-1

$\mu(z_1) = 0 = \text{min}\ \cancel{\text{max}}\ (0.2, 0)$

$K_1 = 0.7 - 2\times 0.9 + 0.5 = 1.2 - 1.8$
$= -0.6$

$\mu(z_2) =$

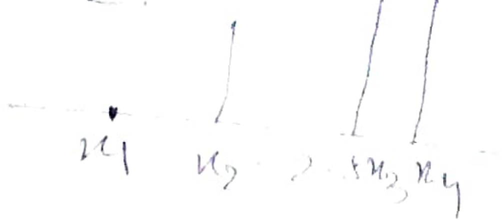


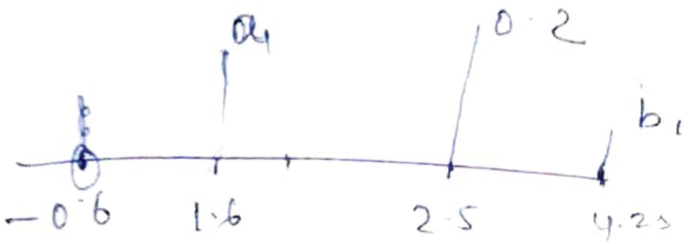
Rule 2

$\mu(z_2) = \min(0.2, 0.8) = 0.2$

$K_2 = 2\times 0.7 + 0.9 + 0$?

0.2   2.5



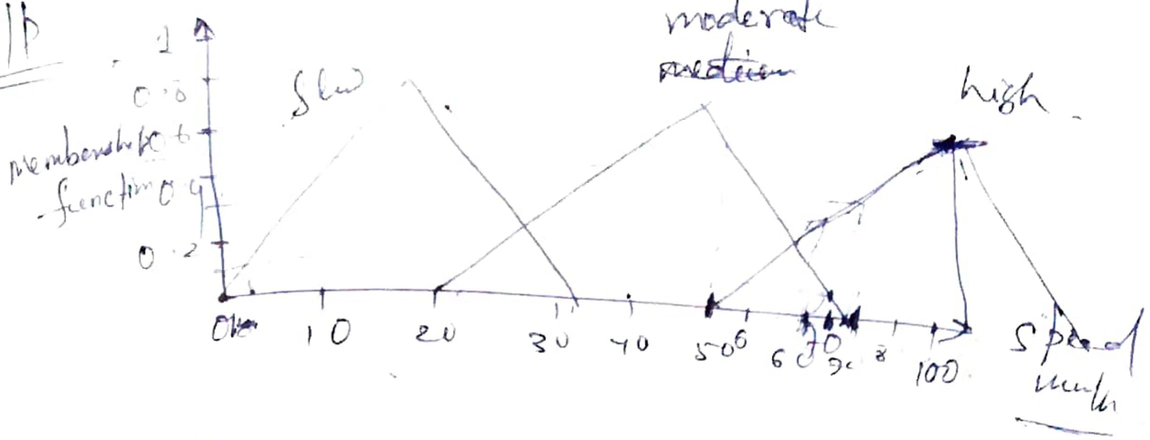


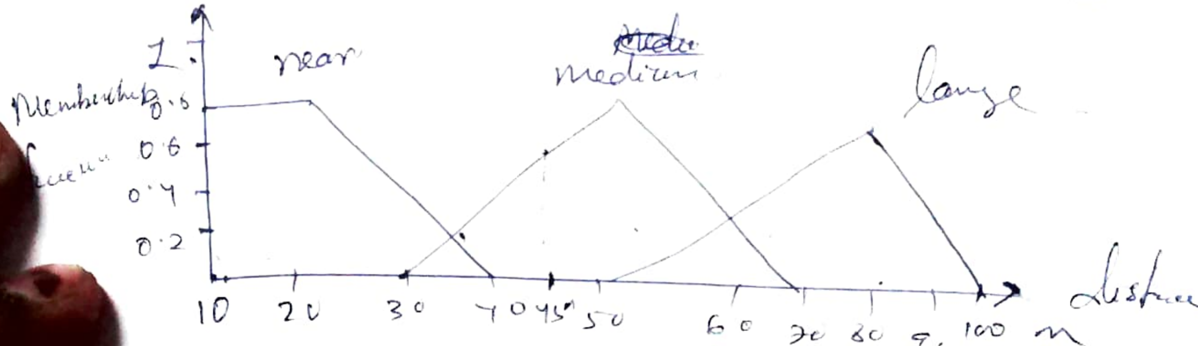
min

$$= \frac{(-0.6)\times 0 + (2.5)0.2 + (1.6)a_1 + (4.25)b_1}{(0 + a_1 + 0.2 + b_1)}$$

=

i/p





i/p-1 when speed is slow, $\mu = 0$

moderate, $\mu_{mod} = 0.1$

high, $\mu_h = 0.3$

i/p-2 when distance ~~speed~~ is near, $\mu_{near} = 0$

when distance medium, $\mu_{medium} = 0.6$

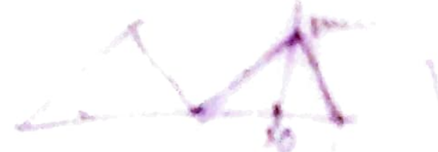
when " large, $\mu_{large} = 0$.

If speed is slow & distance near, br = low.

If speed is slow or distance low, br = low.

If speed is high distance near, br = high

If speed is high distance medium, br = high



Mamdani fuzzy inference

It performs four steps

→ fuzzification of the i/p variables

→ Rule evaluation

→ Aggregation of the rule outputs &

→ defuzzification

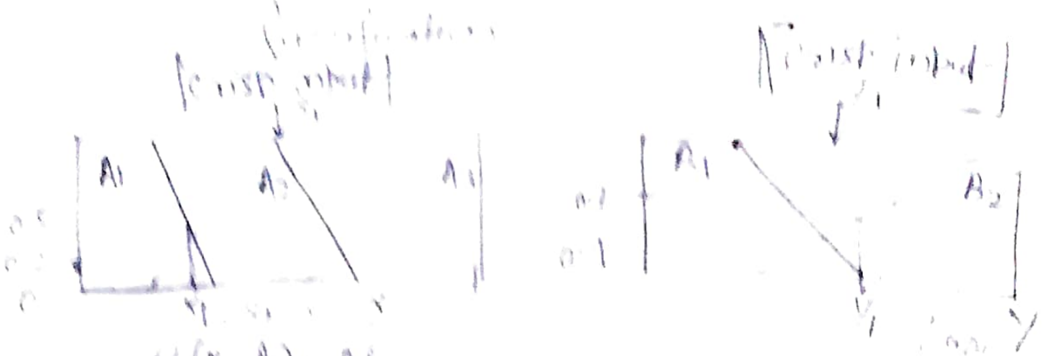
(80%)

(60%)

Two i/ps & one o/p problem

ad - A_3
ma - A_2
in - A_1

Rule 1	**Rule 1**
IF x is A_3	If Project funding is adequate
OR y is B_1	OR Project staffing is small
THEN z is C_1	THEN risk is low
Rule 2	**Rule 2**
If x is A_2	If project funding is marginal
AND y is B_2	AND project staffing is large
then z is C_2	THEN risk is normal
Rule 3	**Rule 3**
If x is A_1	If project funding is inadequate
then z is C_3	then risk is high



$\mu(x = A_1) = 0.5$
$\mu(x = A_2) = 0.2$
$\mu(x = A_3) = 0$

$\mu(y = B_1) = 0.1$
$\mu(y = B_2) = 0.7$

A_1 = inadequate B_1 = small
A_2 = marginal B_2 = medium
A_3 = adequate B_3 = large

Step 2

Rule Evaluation

with C_1 = low, C_2 = normal, C_3 = high

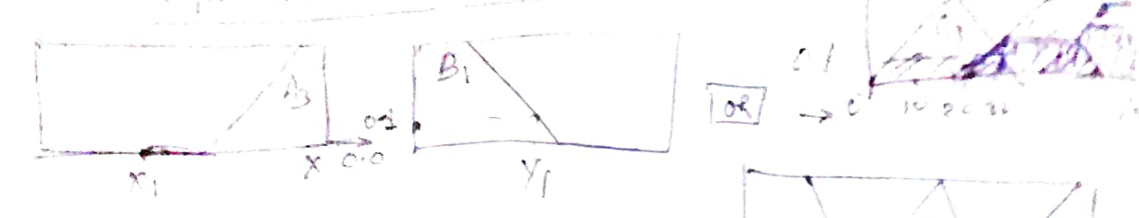
The 2nd step is to take the fuzzified inputs $\mu(x = A_1) = 0.5$, $\mu(x = A_2) = 0.2$, $\mu(x = A_3) = 0$, $\mu(y = B_1) = 0.1$ & $\mu(y = B_2) = 0.2$

Apply them to the antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents the fuzzy operator is used (OR or AND) to obtain a single number that represents the result of the antecedent evaluation. This number is then applied to the consequent membership function.

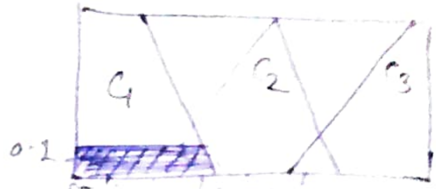
Union $\mu_A(x) \vee \mu_B(x) = \max[\mu_A(x), \mu_B(x)]$

AND $\mu_A(x) \wedge \mu_B(x) = \min[\mu_A(x), \mu_B(x)]$

Mamdani Rule Evaluation

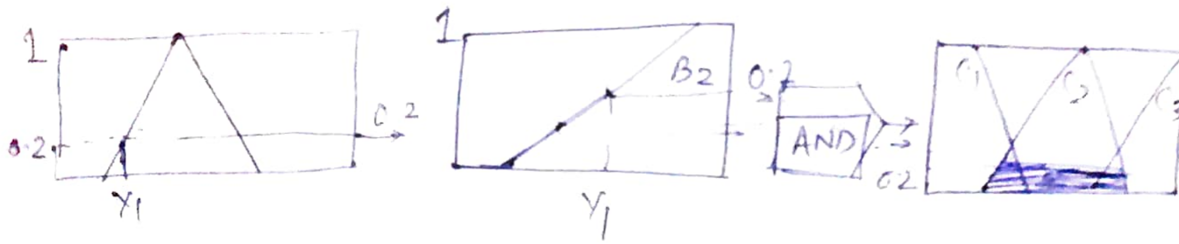


Rule-1



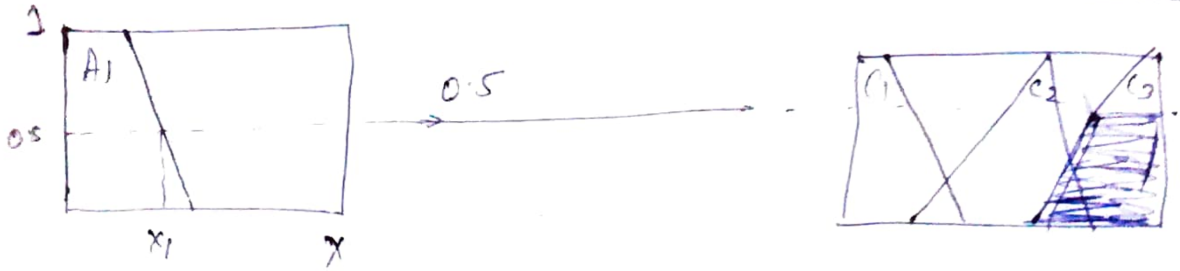
If x is $A_3(0.0)$ OR

y is $B_1(0.1)$ then z is $C_1(0.1)$



Rule-2

If x is $A_2(0.2)$ AND y is $B_2(0.7)$ THEN z is $C_2(0.2)$



Rule-3

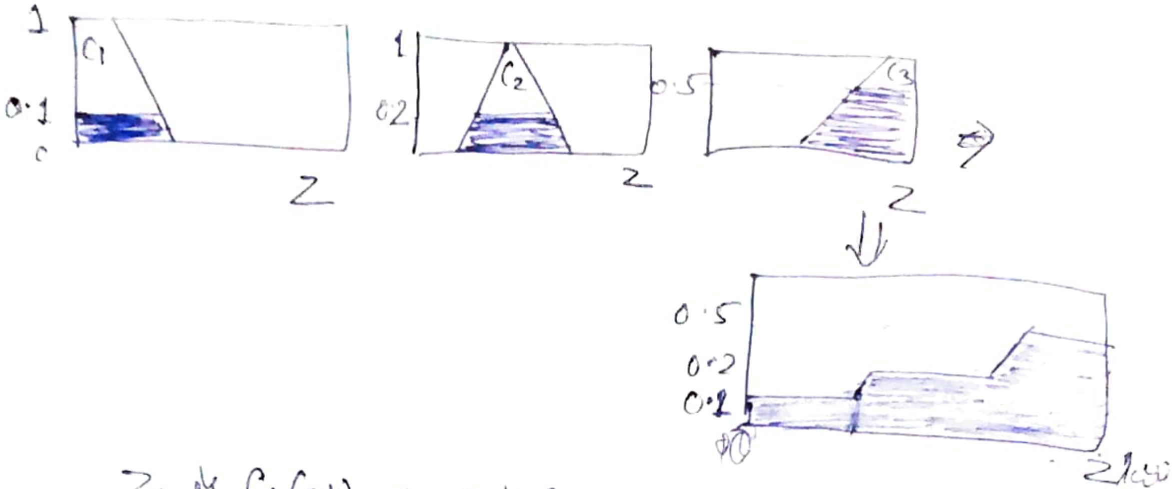
If x is $A_1(0.5)$ THEN z is $C_3(0.5)$.

Step-3

Aggregation of the rule outputs

Aggregation is the process of unification of the o/p of all rules.

Aggregation of the rule outputs



Z is $C_1(0.1) \to Z$ is $C_2(0.2) \to Z$ is $C_3(0.5) \Rightarrow \boxed{\Sigma}$.

II Defuzzification:

The last step of in the fuzzy inference process is defuzzification.

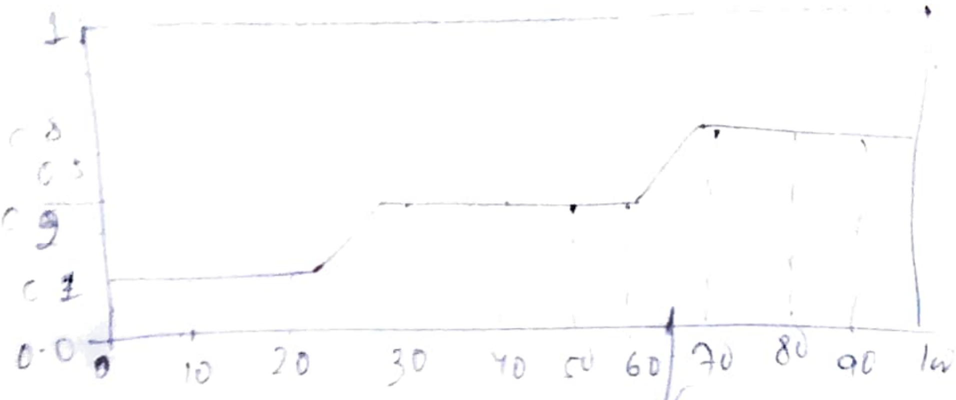
fuzziness helps to evaluate the rules but the final o/p of a fuzzy system has to be a crisp number.

The i/p for the defuzzification process is the aggregate o/p fuzzy set & the o/p is a single number.

Centroid technique

centre of gravity (COG).

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}.$$



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5}$$

$= 67.4$

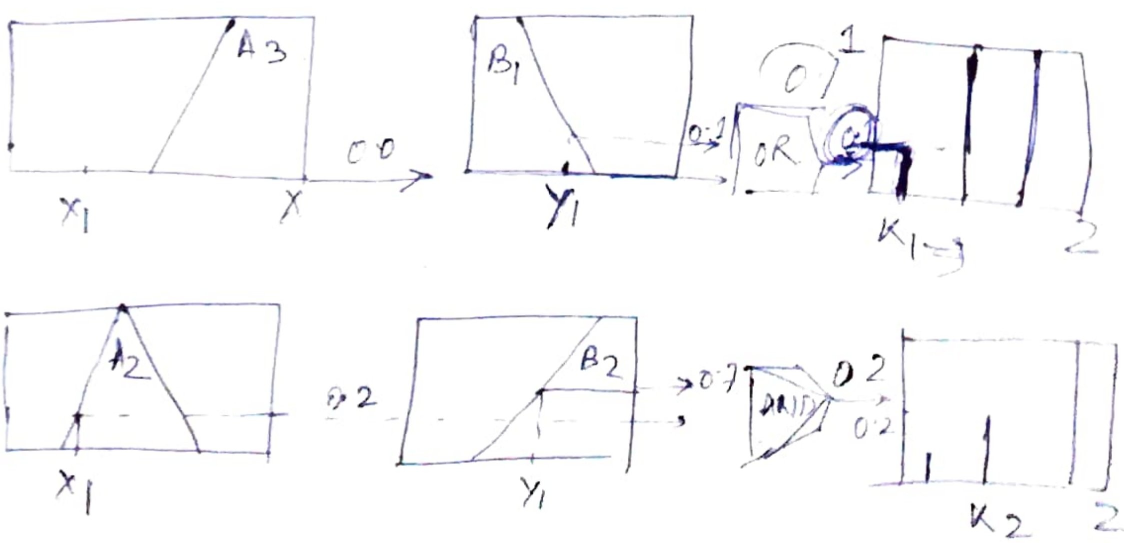
Case 2

Sugeno fuzzy inference

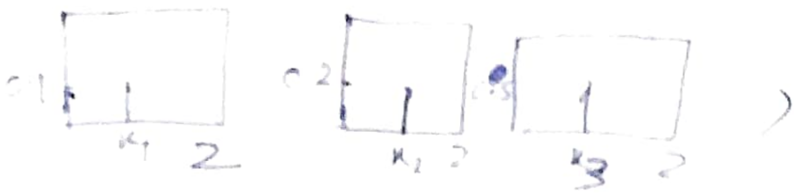
Mamdani's process is not computationally efficiency.

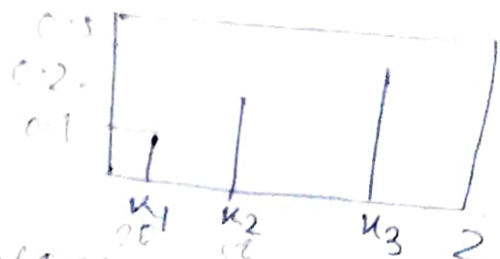
Sugeno model suggested to use a single spike, a singletone, as

Sugeno rule evaluation



Aggregation of the rule of





$2 \ldots x_1(0.1) \longrightarrow 2 \ldots x_2(0.2) \rightarrow 2 \ldots x_3(0.5)$

$\rightarrow \boxed{\Sigma}$

Weighted Aregate (WA)

$$WA = \frac{\mu(x_1) \times x_1 + \mu(x_2)\, x_2 + \mu(x_3) \times x_3}{\mu(x_1) + \mu(x_2) + \mu(x_3)}$$

$$= \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} \quad 65$$

Max-min Composition
Max-product "

Max - $\vee$
Min - $\wedge$

Let R_1 = "x is relevant to y"
R_2 = "y is relevant to z"

two fuzzy relations defined on $X \times Y$ & $Y \times Z$
$X = \{1, 2, 3\}$, $Y = \{\alpha, \beta, \gamma, \delta\}$, $Z = \{a, b\}$

$$R_1 = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 \\ 0.4 & 0.2 & 0.8 & 0.9 \\ 0.6 & 0.8 & 0.3 & 0.2 \end{bmatrix}$$

$$R_2 = \begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.3 \\ 0.5 & 0.6 \\ 0.7 & 0.2 \end{bmatrix}$$

Derive a fuzzy relation "x is relevant to z" based on R_1 & R_2

Degree of relevance between 2 ($\in X$) & a ($\in Z$)

using max-min composition & max product Composition

Ans

$\mu_{R_1 \circ R_2}(2, a) = \max(0.4 \wedge 0.9,\ 0.2 \wedge 0.2,\ 0.8 \wedge 0.5,$
$0.9 \wedge 0.7)$
$= \max(0.4, 0.2, 0.5, \ldots$
$= 0.7$ Max-min

$\mu_{R_1 \circ R_2}(2, a) = \max(0.4 \times 0.9 \ldots$
$= \max(0.36 \ldots$
$= 0.63$ (max product)