

energy function

$$E = -\frac{1}{2} \sum_j \sum_k \omega_{kj} \eta_k \eta_j$$

$$P(\eta_k \rightarrow -\eta_k) = \frac{1}{1 + \exp(-\Delta E_k / T)}$$

In clamped condition in which the visible neurons are clamped onto the specific state in which determine by environment.

In free-running condition in which all neurons are allowed to operate freely.

$$\Delta \omega_{kj} = \eta (s_{kj}^+ - s_{kj}^-) \quad , i \neq k.$$

$s_{kj}^+$  denotes correlation between  $k$  &  $j$  with the n/w ~~is~~ in its clamped condition.

$s_{kj}^-$  denotes correlation b/w neuron  $k$  &  $j$  with the n/w in its free running condition.

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## \* Credit-assignment Problem :-

1. Temporal credit assignment Prob.
2. Structural " " " "

## \* Memory :-

1. Short-term memory
2. Long-term memory

1. Short-term : a compilation knowledge representing the current state of environment is called short-term memory. Any discrepancies b/w knowledge stored in the short-term mem ~~is~~ and in world state is used to update the short-term memory.

Long time memory referred to knowledge stored

## \* Working memory :-

It has following ch -



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$$x = [x_1, x_2, \dots, x_n]^T$$

$$x = [2, 3, \dots]^T$$

$$\begin{bmatrix} x_1 & x_2 & \dots & x_n \\ x_1 & x_2 & \dots & x_n \\ \vdots & \vdots & \ddots & \vdots \\ x_1 & x_2 & \dots & x_n \end{bmatrix}$$

$$x = [x_1, x_2, \dots, x_n]^T$$

$$x = \sum_{k=1}^n x_k e_k$$

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Definition: Matrix Norm

$$\|A\| = \sum_{k=1}^n \|A_k\|$$

$$\vec{r} = \vec{r}_1 + \vec{r}_2$$

$$\vec{r}_1$$

$$\vec{r}_2 = \vec{r}_1 - \vec{r}_1$$

→ Result

$$\vec{r}_1 = \vec{r}_2$$

$$\vec{r}_1 = \vec{r}_2$$

$$\vec{r}_1 = \vec{r}_2$$

$$\vec{r}_1 = \vec{r}_2$$

$$\vec{r}_1 = \vec{r}_2$$

$$= \sum_{k=1}^m (\vec{x}_k^T \vec{x}_j) \vec{y}_k$$

$$= (\vec{x}_j^T \vec{x}_j) \vec{y}_j + \sum_{\substack{k=1 \\ k \neq j}}^m (\vec{x}_k^T \vec{x}_j) \vec{y}_k$$

$$= \vec{y}_j + \vec{v}_j$$

$v_j$  → Noise vector

$$v_j = \sum_{\substack{k=1 \\ k \neq j}}^m (\vec{x}_k^T \vec{x}_j) \vec{y}_k = \sum_{\substack{k=1 \\ k \neq j}}^m \cos(\vec{x}_k, \vec{x}_j) \vec{y}_k$$

$$\cos(\vec{x}_k, \vec{x}_j) = \frac{\vec{x}_k^T \vec{x}_j}{\|\vec{x}_k\| \|\vec{x}_j\|}$$

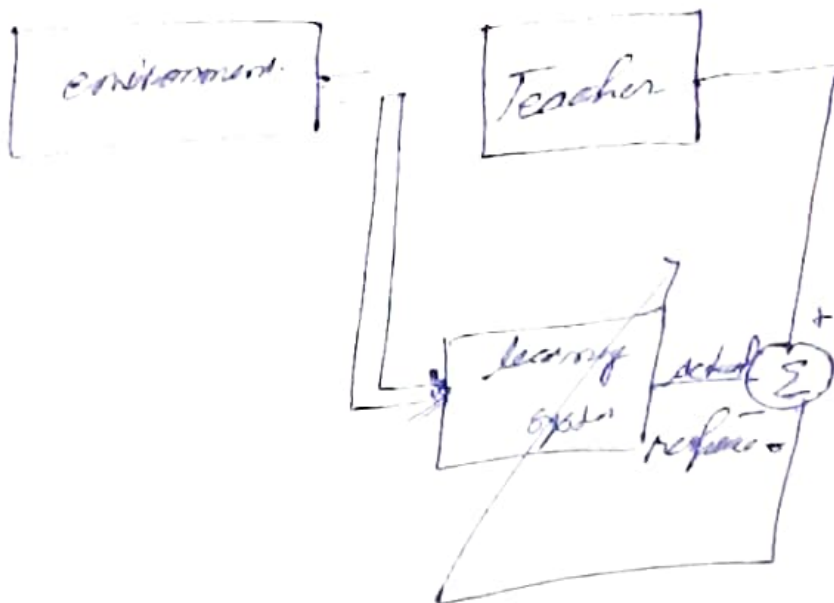
for perfect recall

$$\vec{x}_i \cdot \vec{x}_j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

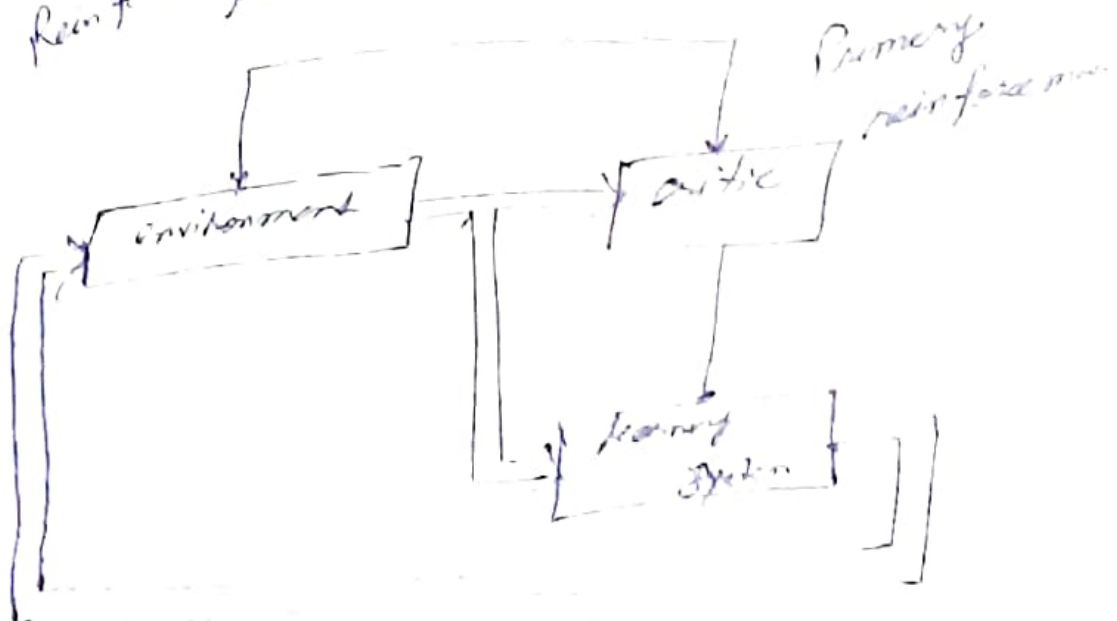
Limit of storage capacity of network

number of orthogonal memory vectors  $L \times m$   
 $m \rightarrow \infty$

# \* Learning with teacher (supervised learning)



Reinforcement learning



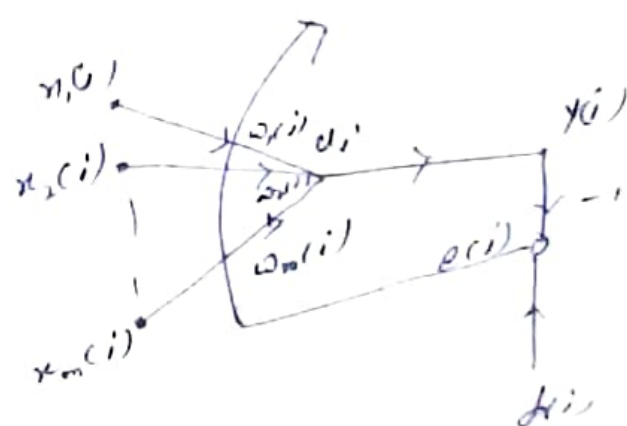
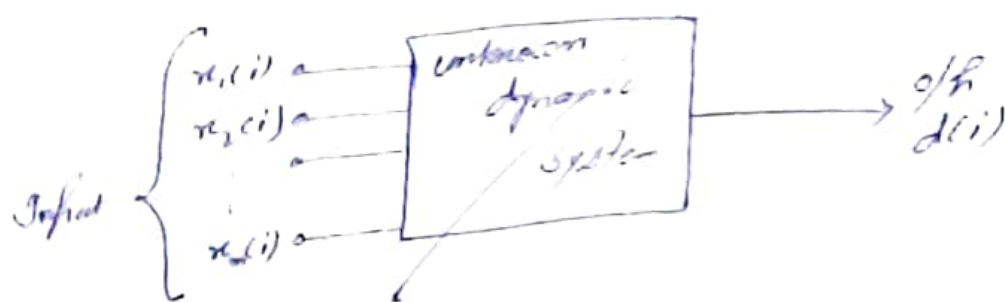
unsupervised



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# Single Layer Perceptron (SLP)

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Signal flow  
representation

Data:

$$S = \{ \vec{x}(i), d(i) : i = 1, 2, 3, \dots, m \}$$

$$\vec{x}(i) = [x_1(i) \quad x_2(i) \quad \dots \quad x_m(i)]^T$$

The neural model of water under the influence of an ~~environment~~ <sup>algorithm</sup> that controls the necessary adjustment to the complex system in response to the statistical variation in the outputs of the N/W. with the



Following Point.

1) The algo<sup>m</sup> start from arbitrary setting of neuron synaptic wt.

2) The adjustment to the synaptic wt. in response to statistical variation in system var are made on continuous basis

3) Learning - wt. are completed inside a window ... that is one sample period long. (mean mode)

mean mode described in fig is called adaptive filter. Its operation consists of two continuous process which involves computation of two signals and o/p denoted by  $y(n)$  that is produced in response to the  $n$  element to the stimulus vector  $x(n)$  or error signal denoted by  $e(n)$ . Adaptive process: which involve adjustment of adaptive wt. of neuron in accordance to reference error error signals.

$$y(i) = d(i) = \sum_{k=1}^m w_k(i) x_k(i)$$

$$y(i) = \vec{x}^T(i) \vec{w}$$

$$e(i) = d(i) - y(i)$$

★ unconstrained optimization technique  
to find  $\vec{w}^*$

Cost function:-

$$E(\vec{w})$$

$$E(\vec{w}^*) \leq E(\vec{w})$$

↓  
optimized

To minimize the Cost function of  $E(\vec{w})$  with respect to  $\vec{w}$ . The necessary ~~condition~~ ~~of~~ ~~minimization~~ condition is  $\nabla E(\vec{w}^*) = 0$

↓  
grad.

$$\nabla = \left\{ \frac{\partial}{\partial w_1}, \frac{\partial}{\partial w_2}, \dots, \frac{\partial}{\partial w_m} \right\}$$

$$\mathcal{L}(\vec{w}(n+1)) < \mathcal{L}(\vec{w}(n))$$

$\vec{w}(n) \rightarrow$  old value of wt. vector

$\vec{w}(n+1) \rightarrow$  updated val of wt. vector

### → Methods of Steepest Descent :-

$$\vec{J} = \nabla \mathcal{L}(\vec{w})$$

in this method the successive adjustments applied to the wt vector  $\vec{w}$  are in the dir<sup>n</sup> of steepest descent, that is in a direction opposite to the gradient vector  $\{\vec{g}\}$ .

$$\vec{w}(n+1) = \vec{w}(n) - \eta \vec{g}(n)$$

η is a constant value  
called step size or  
learning rate parameter.

$$\vec{w}(n) = \vec{w}(n) - \eta \vec{g}(n)$$

first order Taylor series approximation,

$$f(x_0 + h) \approx f(x_0) + f'(x_0)h$$

$$\xi(\vec{w}(n+1)) \approx \cancel{\xi(\vec{w}(n))} \xi(\vec{w}(n)) + g^T(h) \Delta w(n)$$

$$\begin{aligned}\xi(\vec{w}(n+1)) &= \xi(\vec{w}(n)) - \eta g^T(n) g(n) \\ &= \xi(\vec{w}(n)) - \eta \|\vec{g}(n)\|^2\end{aligned}$$

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## Newton's Method

- basic idea of this method is to minimize the quadratic ~~function~~ <sup>approx.</sup> of the Cost function ~~around~~ <sup>near</sup>  $x_0$ .

$$Q(x) = \frac{1}{2} (x - x_0)^T H(x_0) (x - x_0) + g(x_0)^T (x - x_0)$$

$$Q'(x) = (x - x_0)^T H(x_0) + g(x_0)$$

by setting  $Q'(x) = 0$

$$x = x_0 - H(x_0)^{-1} g(x_0)$$

$$f(x_0 + h) \approx f(x_0) + f'(x_0)h + \frac{1}{2}f''(x_0)h^2$$

$$\Delta \xi(\omega(n)) \approx g'(n) \Delta \omega(n) + \frac{1}{2} \Delta \omega^T(n) H(n) \Delta \omega(n)$$

$$H(n) = \begin{bmatrix} \frac{\partial^2 \xi(n)}{\partial \omega_1^2} & \frac{\partial^2 \xi(n)}{\partial \omega_1 \partial \omega_2} & \dots & \frac{\partial^2 \xi(n)}{\partial \omega_1 \partial \omega_m} \\ \frac{\partial^2 \xi(n)}{\partial \omega_2 \partial \omega_1} & \frac{\partial^2 \xi(n)}{\partial \omega_2^2} & \dots & \frac{\partial^2 \xi(n)}{\partial \omega_2 \partial \omega_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \xi(n)}{\partial \omega_m \partial \omega_1} & \frac{\partial^2 \xi(n)}{\partial \omega_m \partial \omega_2} & \dots & \frac{\partial^2 \xi(n)}{\partial \omega_m^2} \end{bmatrix}$$

$$\xi = g(n) + \vec{H}(n) \Delta \omega(n) = 0$$

$$\Delta \omega(n) = - \vec{H}^{-1}(n) \vec{g}(n)$$

$$\omega(n+1) = \omega(n) - \vec{H}^{-1} \vec{g}(n)$$

• Gauss Newton method

$$e(i) = \frac{1}{2} \sum_{j=1}^n e^2(i)$$

$$1 \leq i \leq n$$

$$e'(i, \omega) = e(i) + \left[ \frac{\partial e(i)}{\partial \omega} \right]^T [\omega(n+1) - \omega(n)]$$

$\omega \approx \omega(n)$

$$e'(n, \omega) = e(n) + J(n) [\omega(n+1) - \omega(n)]$$

$$J(n) = \begin{bmatrix} \frac{\partial e_1}{\partial \omega_1} & \frac{\partial e_1}{\partial \omega_2} & \dots & \frac{\partial e_1}{\partial \omega_m} \\ \frac{\partial e_2}{\partial \omega_1} & \frac{\partial e_2}{\partial \omega_2} & \dots & \frac{\partial e_2}{\partial \omega_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial e_n}{\partial \omega_1} & \frac{\partial e_n}{\partial \omega_2} & \dots & \frac{\partial e_n}{\partial \omega_m} \end{bmatrix}$$

Objective

$$\omega(n+1) = \arg \min_{\omega} \left\{ \frac{1}{2} \|e'(n, \omega)\|^2 \right\}$$

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$$\frac{1}{2} \| e'(n, \omega) \|^2 = \frac{1}{2} e'(n, \omega) e'^T(n, \omega)$$

$$= \frac{1}{2} [\bar{e}(n) + \bar{J}(n) (\omega(n+1) - \omega(n))]$$

$$[\bar{e}(n) + \bar{J}(n) (\omega(n+1) - \omega(n))]^T$$

$$= \frac{1}{2} \| e(n) \|^2 + e^T(n) \bar{J}(n) (\omega(n+1) - \omega(n))$$

$$+ \frac{1}{2} (\omega(n+1) - \omega(n))^T \bar{J}^T(n) \bar{J}(n) (\omega(n+1) - \omega(n))$$

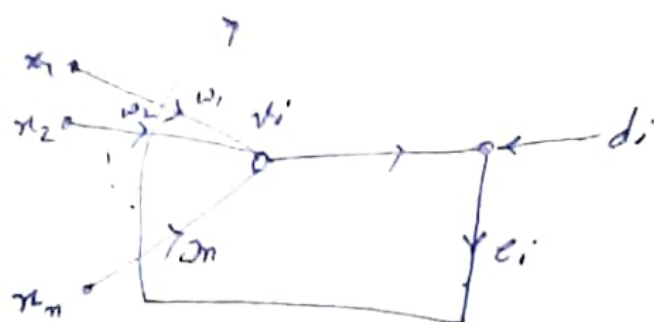
Derivative w.r.t.  $\omega(n)$  and setting it to zero.

$$= J^T e(n) + \bar{J}^T(n) \bar{J}(n) (\omega(n+1) - \omega(n))$$

$$\omega(n+1) = \omega(n) + (\bar{J}^T(n) \bar{J}(n))^{-1} \bar{J}^T(n) e(n)$$



# \* Linear least square filter :-



$$\hat{y}(\omega) = f(e\omega)$$

$$e(n) = d(n) - \hat{x}(n) \hat{z}(n)$$

diff. w.r.t  $\omega(n)$

$$\nabla e(n) = -x^T(n)$$

$$\hat{z}(n) = -x(n)$$

$$\omega(n+1) = \omega(n) + (x^T(n) x(n))^{-1} x^T(n) (d(n) - \hat{x}(n) \hat{z}(n))$$

$$= \omega(n) + (x^T(n) x(n))^{-1} x^T(n) (d(n) - \hat{x}(n) \hat{z}(n))$$

$$= (x^T(n) x(n))^{-1} (x^T(n) x(n)) \omega(n)$$

$$= \omega(n) + (x^T(n) x(n))^{-1} x^T(n) (d(n) - \hat{x}(n) \hat{z}(n))$$

$$\boxed{\omega(n+1) = (x^T(n) x(n))^{-1} x^T(n) d(n)}$$

$$P(n+1) = (X^T(n) X(n))^{-1} X^T(n) d(n)$$

$$X^+(n) = (X^T(n) X(n))^{-1} X^T(n)$$

$$W(n+1) = X^+(n) d(n) \quad \xrightarrow{\text{Pseudo inverse of data matrix } X(n)}$$

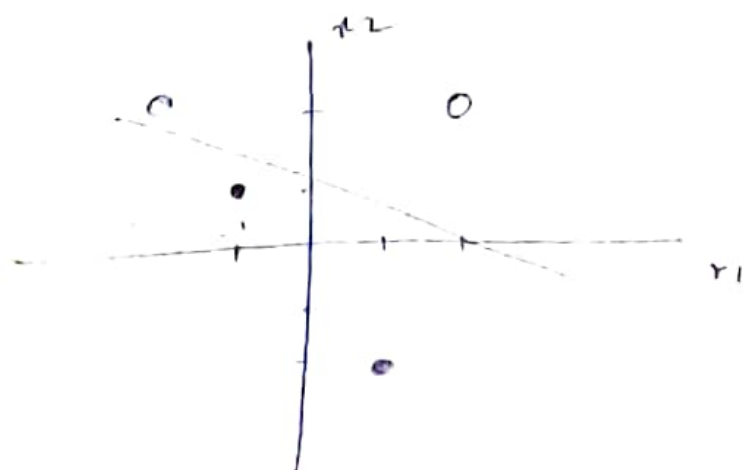
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$$p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$\left\{ p_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, t_1 = 0 \right\}, \left\{ p_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, t_2 = 0 \right\}$$

$$\left\{ p_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, t_3 = 0 \right\}, \left\{ p_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_4 = 1 \right\}$$



$$W(n+1) = 0$$

$$1$$

Derive

$$\begin{cases} \omega = [0 \ 0] \\ b = 0 \end{cases}$$

iteration - 1

$$\omega_0 = [0 \ 0], \quad b_0 = 0$$

$$\omega_{n+1} = \omega_n + \eta \epsilon \quad \text{assume } 1$$

$$\boxed{\begin{aligned} \omega_{n+1} &= \omega_n + \epsilon \rho \\ b_{n+1} &= b_n + \epsilon \end{aligned}}$$

$$\epsilon = \text{target} - a$$

↪ average value found  
by record  
weight.

$$\begin{aligned} a &= \text{function}(\omega_0, \tau, b_0) \\ &= \text{function}([0 \ 0] \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0) \end{aligned}$$

$$\text{error}(\omega) = 1$$

$$\epsilon = 1 - 0 = 1$$

$$b_1 = b_0 + \epsilon = 1$$

$$\omega_1 = \omega_0 + \eta \epsilon \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0 + 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$