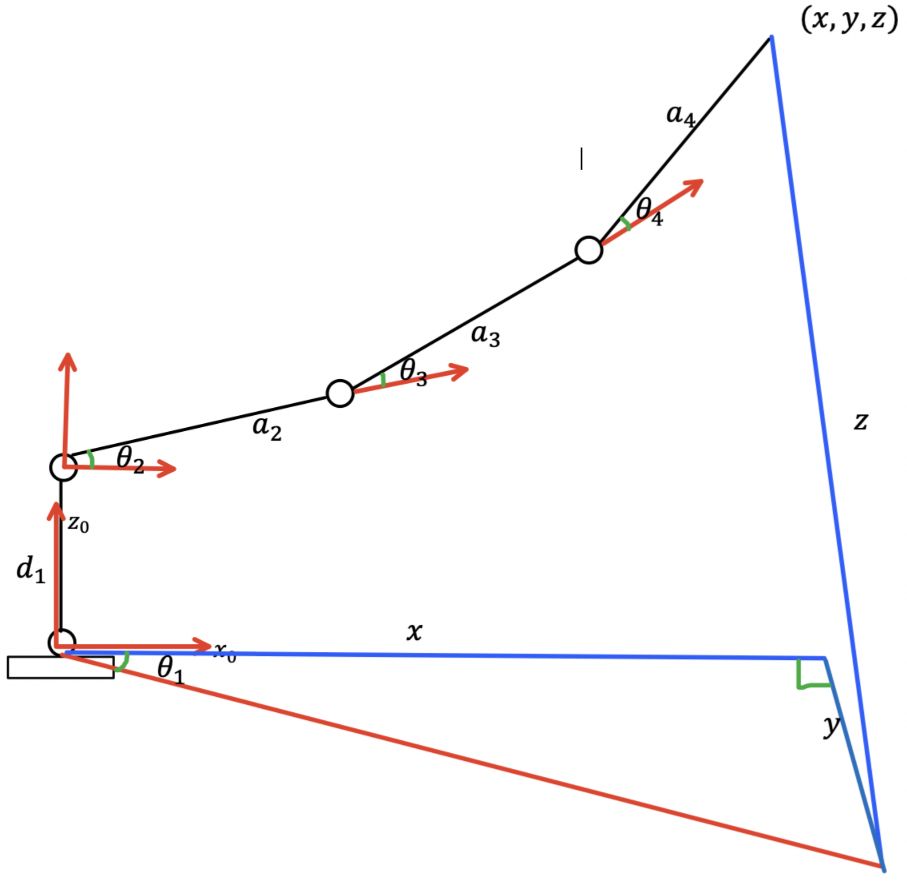
Robotics Lab 05

Syeda Manahil Wasti, Syed Mujtaba Hassan, Aatiqa Khalid

March 17, 2023

1. Given a desired position, (x, y, z), of the end-effector and orientation, ϕ, find mathematical expressions for all solutions to this inverse kinematics problem. Show all steps and specifically state how many solutions exist? Assuming that direction of $\hat{x}$ of end-effector is along the length of the last link, ϕ is the angle it makes with the x-axis of frame 1, i.e. $\phi = \theta_2 + \theta_3 + \theta_4$. When the gripper is parallel to the base board, then $\phi = 0°$.

Solution:



$$\alpha = \cos^{-1}\left(\frac{a_2^2 - a_3^2 + (\sqrt{y^2+x^2} - a_4\sin\phi)^2 + (z - d_1 - a_4\sin\phi)^2}{2a_2\sqrt{(\sqrt{y^2+x^2} - a_4\sin\phi)^2 + (z - d_1 - a_4\sin\phi)^2}}\right)$$

$$\beta = \cos^{-1}\left(\frac{a_2^2 + a_3^2 - (\sqrt{x^2+y^2} - a_4\cos\phi)^2 - (z - d_1 - a_4\sin\phi)^2}{2a_2a_3}\right)$$

$$\gamma = \tan^{-1}\left(\frac{z - d_1 - a_4 \sin\phi}{\sqrt{x^2+y^2} - a_4 \sin\phi}\right)$$

Where

$$a_2 = a_3 = a_4 = 10 \text{ cm}$$

$$d_1 = 13 \text{ cm}$$

$$\phi = \theta_2 + \theta_3 + \theta_4$$

First set of solutions

$$\theta_{11} = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_{21} = \pi - \beta$$

$$\theta_{31} = \gamma - \alpha$$

$$\theta_{41} = \phi - \theta_{21} - \theta_{31}$$

Second set of solutions

$$\theta_{12} = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_{22} = \beta - \pi$$

$$\theta_{32} = \gamma + \alpha$$

$$\theta_{42} = \phi - \theta_{22} - \theta_{32}$$

Third set of solutions

$$\theta_{13} = \pi + \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_{23} = \beta - \pi$$

$$\theta_{33} = \pi - \gamma + \alpha$$

$$\theta_{43} = \phi - \theta_{23} - \theta_{33}$$

Fourth set of solutions

$$\theta_{14} = \pi + \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_{24} = \pi - \beta$$

$$\theta_{34} = \pi - \gamma - \alpha$$

$$\theta_{44} = \phi - \theta_{24} - \theta_{34}$$

2. Say there are N possible solutions to the IK problem of our manipulator, in general. Write a MATLAB function findJointAngles(x,y,z,phi), which accepts the position and orientation of end-effector as arguments and returns an N × 4 matrix containing all the IK solutions. Row i of this matrix corresponds to solution i, and column j of the matrix contains the values for θ_j.

Solution:

```
function s = findJointAngles(x,y,z,phi)

a2 = 10;
a3 = 10;
a4 = 10;
d1 = 13;

x1 = sqrt(x^2 + y^2) - a4*sin(phi);
```

```
y1 = sqrt(x^2 + y^2) - a4*cos(phi);
z1 = z-d1-a4*sin(phi);

alpha = acos((a2^2 - a3^2 + x1^2 + z1^2)/(2*a2*(sqrt(x1^2 + z1^2))));
beta = acos((a2^2 + a3^2 - y1^2 - z1^2)/(2*a2*a3));
gamma = atan(z1/x1);

% first set:
theta_11 = atan2(y,x);
theta_31 = gamma - alpha;
theta_21 = pi - beta;
theta_41 = phi - theta_21 - theta_31;

% second set:
theta_12 = atan2(y,x);
theta_32 = gamma + alpha;
theta_22 = beta - pi;
theta_42 = phi - theta_22 - theta_32;

% third set:
theta_13 = pi + atan2(y,x);
theta_33 = pi - gamma + alpha;
theta_23 = beta - pi;
theta_43 = phi - theta_23 - theta_33;

% fourth set:
theta_14 = pi + atan2(y,x);
theta_34 = pi - gamma - alpha;
theta_24 = pi - beta;
theta_44 = phi - theta_24 - theta_34;

s = [theta_11, theta_21, theta_31, theta_41;
     theta_12, theta_22, theta_32, theta_42;
     theta_13, theta_23, theta_33, theta_43;
     theta_14, theta_24, theta_34, theta_44];
end
```

3. Write a MATLAB function findOptimalSolution(x,y,z,phi), which accepts the desired position and orientation as arguments and returns a vector [theta1, theta2, theta3, theta4] corresponding to the optimal and realizable inverse kinematics solution. Optimal solution is the IK solution closest to the current configuration of the robot, i.e. minimize $b_1|\Delta\theta_1| + b_2|\Delta\theta_2| + b_3|\Delta\theta_3| + b_4|\Delta\theta_4|$. You can choose $b_i = 1$.

Solution:

```
function optimal_solution = findOptimalSolution(x, y, z, phi)

arb = Arbotix('port', 'COM8', 'nservos', 5);
position = arb.getpos(); % get the current joint angles
position = position(1,1:4); % removing the gripper setting
position(1) = position(1) + pi/2;
position(2) = position(2) + pi/2;
position = position(:,1:4);

s1 = findJointAngles(x, y, z, phi); % get all possible solutions
```

```
s = real(s1); % take only real part

for i=1:4 % for setting angles in range -pi to pi
    position(i) = mod(position(i) + pi, 2*pi) - pi;
    s(1,i) = mod(s(1,i) + pi, 2*pi) - pi;
    s(2,i) = mod(s(2,i) + pi, 2*pi) - pi;
    s(3,i) = mod(s(3,i) + pi, 2*pi) - pi;
    s(4,i) = mod(s(4,i) + pi, 2*pi) - pi;
end

t1 = sum(abs((s(1, :) - position) + pi));
t2 = sum(abs(s(2, :) - position) + pi);
t3 = sum(abs(s(3, :) - position) + pi);
t4 = sum(abs(s(4, :) - position) + pi);

t = min([t1, t2, t3, t4]);

  if t == t1
        optimal_solution = s(1, :);
    elseif t == t2
        optimal_solution = s(2, :);
    elseif t == t3
        optimal_solution = s(3, :);
    elseif t == t4
        optimal_solution = s(4, :);
  end
end
```

4. (a) Select five points (x, y, z, ϕ) in the workspace of the robot and execute the optimal solution for each point, as determined by your findOptimalSolution function. Measure and note down the achieved point in each case.

Solution:

First configuration (100, 200, 200, $\frac{\pi}{2}$):

measuring physically we get:

x = 104 mm

y = 202 mm

z = 196 mm

$\text{error}_1 = \sqrt{(100-104)^2 + (200-202)^2 + (200-196)^2} = 6$

Second configuration (0, 0, 370, $\frac{3\pi}{4}$):

measuring physically we get:

x = 0 mm

y = 0 mm

z = 372 mm

$\text{error}_2 = \sqrt{(0-0)^2 + (0-0)^2 + (370-372)^2} = 2$

Third configuration (80, -100, 230, $\frac{3\pi}{4}$:)
measuring physically we get:
x = 78 mm
y = -98 mm
z = 232 mm
$\text{error}_3 = \sqrt{(80-78)^2 + (-100-(-98))^2 + (230-232)^2} = 2.83$
Fourth configuration (140, -50, 250, $\frac{\pi}{2}$):
measuring physically we get:
x = 136 mm
y = -52 mm
z = 248 mm
$\text{error}_4 = \sqrt{(140-136)^2 + (-50-(-52))^2 + (250-248)^2} = 4.9$
Fifth configuration (120, 120, 200, $\frac{\pi}{4}$):
using setPosition and measuring physically we get:
x = 124 mm
y = 118 mm
z = 194 mm
$\text{error}_5 = \sqrt{(120-124)^2 + (120-118)^2 + (200-194)^2} = 7.48$

mean error (e_m) = $\frac{error_1 + error_2 + error_3 + error_4 + error_5}{5}$

$e_m = \frac{6 + 2 + 2.83 + 4.9 + 7.48}{5}$

$e_m = 4.642$

(b) Determine the accuracy of your system and compare it to the value obtained in the previous lab. Comment on any possible statistically significant differences.

Solution:
The mean error is found to be 4.642 which is less than what was found in lab 4.

(c) Is there any (x, y, z, ϕ) in the workspace for which all possible solutions are realizable? Justify.

Solution:
There are points in the workspace for which all possible solutions are realizable meaning no singularity occurs i.e., there are no infinite solutions possible.
We can use the findJointAngles(x, y, z, phi) function to find solutions for different points and then use findPincher(theta_1, theta_2, theta_3, theta_4) function to check if all 4 of the solutions give back the same point or not. If yes, then the point is one for which all possible solutions are realizable.