CHAPTER 1

UNIT I

Introduction to Signals and Systems

Syllabus :

Definition of signals and systems, Communication and control systems as examples, Classification of signals : Continuous time and discrete time, Even, Odd, Periodic and non periodic, Deterministic and non deterministic, Energy and power, Operations on signals : Amplitude scaling, Addition, Multiplication, Differentiation, Integration (Accumulator for DT), Time scaling, Time shifting and folding, Precedence rule. Elementary signals : Exponential, Sine, Step, Impulse and its properties, Ramp, Rectangular, Triangular, Signum, Sinc.

1.1 Definition of Signal :

PU : May 07

University Questions

Q. 1 Define : Signal (May 07, 2 Marks)

- In a communication system, the word 'signal' is very commonly used. Therefore we must know its exact meaning.
- Mathematically, signal is described as a function of one or more independent variables.
- Basically it is a physical quantity. It varies with some dependent or independent variables.
- So the term signal is defined as "A physical quantity which contains some information and which is function of one or more independent variables."
- The signals can be one-dimensional or multidimensional.

One dimensional signals :

- When the function depends on a single variable, the signal is said to be one dimensional.
- Example of one dimensional signal is speech signal whose amplitude varies with time.

Multidimensional signals :

- When the function depends on two or more variables, the signal is said to be multidimensional.
- The example of a multidimensional signal is an image because it is a two dimensional signal with horizontal and vertical co-ordinates.

1.2 System :

PU : May 07

University Questions

Q. 1 Define : System (May 07, 4 Marks)

Definition :

- A system is defined as the entity that operates on one or more signals to accomplish a function, to produce new signals.

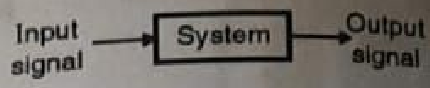


Fig. 1.2.1 : A system

  Fig. 1.2.1 demonstrates the interaction between signals and system.
- The types of input and output signals depends on the type of system being used.

1.2.1 Types (Examples) of Systems :

- Signals and systems have several applications. Some of the important types of systems are as follows :
  1. Communication system.
  2. Control system.
  3. Remote sensing system.
  4. Biomedical signal processing.
  5. Auditory system.

1.2.2 Communication System :

- There are three basic elements of any communication system :
  1. Transmitter 2. Channel 3. Receiver.
- Fig. 1.2.2 shows the basic communication system.

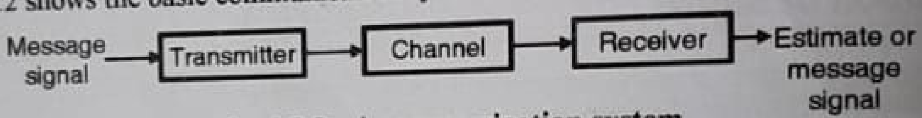


Fig. 1.2.2 : A communication system

- The transmitter and receiver are located far away from each other. The channel is a physical medium that connects them.
- Each of the three elements can be viewed as a separate system, with their own signals associated with them.
- The transmitter converts the message signal into a signal which is suitable for transmission. The message signal can be a speech signal, video signal or computer data.
- The channel can be a coaxial cable, optical fiber, satellite channel, or a mobile radio channel, depending on the area of application.
- When the transmitted signal travels over the channel transmitter to receiver, it gets distorted due to the physical characteristics of the channel. The noise and other interferences get added to the transmitted signal.
- The receiver operates on the received signal which is a corrupted version of the transmitted signal, to reconstruct an estimate of the original message signal.
- The role of the receiver is thus exactly opposite to that of the transmitter.
- The functions performed by the transmitter and receiver are dependant on the type of communication systems being considered.
- A communication system can be one of the following two types :
  1. Analog system. 2. Digital system.
- The **analog communication systems** are relatively simple. The transmitter in such systems contains a modulator whereas the receiver contains a demodulator.
- **Modulation** is a process which converts the message signal into a form that is compatible with the transmission characteristics of the channel.

- The **analog modulation** can be either AM (amplitude modulation), FM (frequency modulation), or PM (phase modulation).
- An appropriate demodulator needs to be used in order to recover the original signal back.
- The **digital communication systems** are more complex than the analog systems. The message signal at the transmitter input can be either an analog signal such as speech/video or a digital signal such as computer data.
- If the message is an analog signal then the transmitter performs the following operations to convert it to digital form :

  1. Sampling 2. Quantization 3. Encoding.
- Note that the sampling and encoding operations are reversible but the quantization is not reversible. Quantization is a process of approximation and introduces errors.
- For the digital message signal the sampling, quantization and encoding are not required.
- The transmitter may have to perform some additional operations such as data compression and channel encoding.
- The coded signal is used for modulating a high frequency carrier wave and the modulated signal is transmitted over the channel.
- The receiver performs all these operations in the reverse manner.

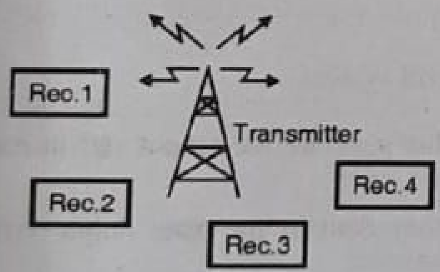
Modes of communication :

- There are two basic modes of communication :

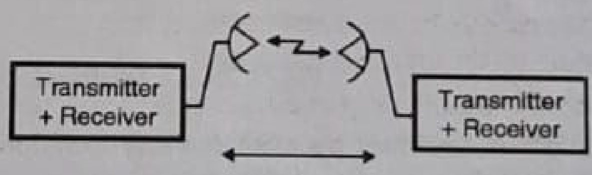
  1. Broadcasting 2. Point to point communication.

1. Broadcasting :

Here a transmitter transmits the information in many directions and the receivers receive it. As shown in Fig. 1.2.3(a), in broadcasting the flow of information is **unidirectional** i.e. from transmitter to receivers.



(a) Broadcasting



(b) Point to point communication

Fig. 1.2.3 : Modes of communication

2. Point to point communication :

- In this type of communication, the communication takes place between a single transmitter and a single receiver as shown in Fig. 1.2.3(b).
- Both the sides of the link have a transmitter and a receiver and the flow of information is **bi-directional**.
- The examples of broadcast communication are radio transmission or TV transmission. The examples of point to point communication system are the telephone system, and the deep space communication links between an earth station and space craft.

Problems :

- Every communication system suffers from the presence of **channel noise**, which puts a severe limitation on the quality of the received signal.
- It is possible to overcome this problem by using large antennas and error controlling techniques.

1.2.3 Control System :

- Control of a physical system is an important application of signals and systems. Some of the examples in which control is applied are as follows :
  1. Mass-transit vehicles
  2. Machine tools
  3. Paper mills
  4. Auto pilots
  5. Automobile engines
  6. Nuclear reactors
  7. Power plants
  8. Robots.
- The object that is to be controlled is called as the **plant**. The reasons for using control systems are :
  1. To attain a satisfactory response. 2. To get a robust performance.
- A plant is said to produce a satisfactory response if its output follows or tracks a specified reference value. The plant output should be held close to this reference value. This process is called as regulation.
- A control system is called as a **robust control system** if it has a good regulation.
- These desirable properties can be attained by using the feedback control system as shown in Fig. 1.2.4. This system is also called as closed-loop control system.

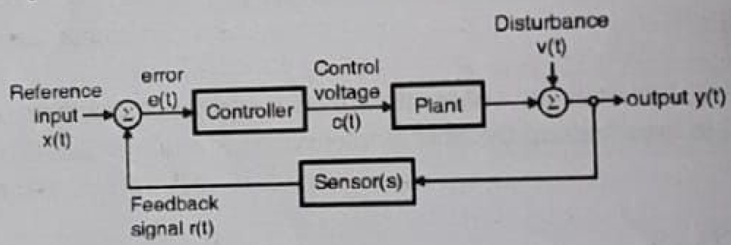


Fig. 1.2.4 : A feedback control system

- The plant is described by mathematical operations that generate the output y(t) in response to plant input c(t) and external disturbance v(t).
- The sensors are included in the feedback path and they convert the plant output y(t) into the feedback signal r(t).
- The feedback signal is compared with the reference signal x(t) and an error signal e(t) which is equal to the difference between x(t) and r(t).

$$e(t) = x(t) - r(t)$$

- The error signal is then applied to the controller which generates a control voltage c(t) which controls the plant.
- The control system of Fig. 1.2.4 has a single input and single output and it is known as a single input/single output (SISO) system.
- A control system with more inputs and outputs is known as a multiple input/multiple output (MIMO) system.

- The controller is in the form of a digital computer or microprocessor. Then the system is known as a digital control system.
- Due to the advantages such as flexibility, high accuracy etc., the digital control system is becoming more and more popular.
- The digital control system uses the operations such as sampling, quantization and coding, stated earlier.

1.3 Classification of Signals :

Fig. 1.3.1 shows the classification of signals.

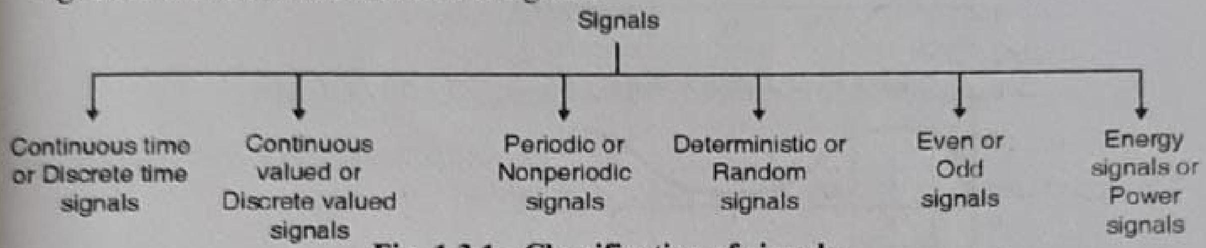


Fig. 1.3.1 : Classification of signals

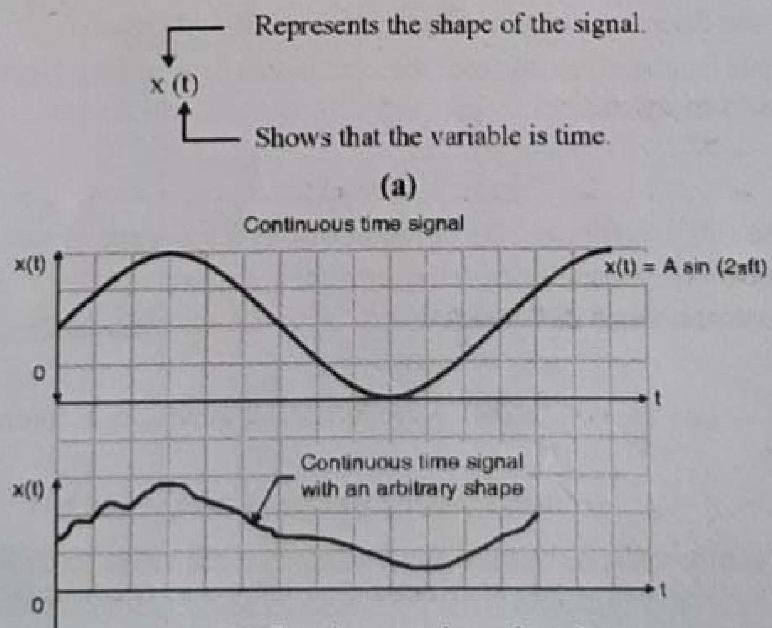
1.3.1 Continuous and Discrete Time Signals : **PU : May 07**

University Questions

Q. 1 With the help of neat sketches explain the difference between Analog continuous time signal and analog discrete time signal. **(May 07, 6 Marks)**

Continuous time (CT) signal :

- A signal of continuous amplitude and time is known as a continuous time signal or an analog signal. This signal will have some "value" at every instant of time.
- The electrical signals derived in proportion with the physical quantities such as temperature, pressure, sound etc. are generally continuous signals.
- The other examples of continuous signals are sine wave, cosine wave, triangular wave etc. Some of the continuous signals are as shown in Fig. 1.3.2(b).



(b) Continuous time signals

Fig. 1.3.2

- The continuous time signals are represented at x(t) where x represents the shape of the signal and t shows that the variable is time.

Discrete time signals :

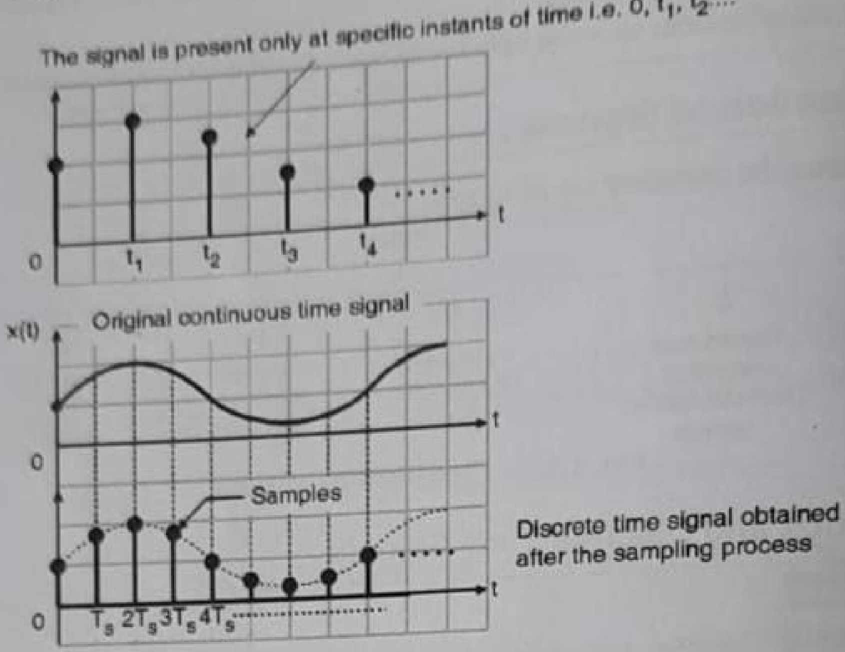


Fig. 1.3.2(c) : Discrete time signals

- If the signal is represented only at "discrete instants of time," then it is known as a discrete time signal. The discrete time signals have "values" only at certain instants of time.
- If we take the blood pressure readings of a patient after every one hour and plot the graph then the resultant signal will be a discrete signal.
- One more way of obtaining the discrete signals is by "sampling" the analog signals as shown in Fig. 1.3.2(c). The dotted line joining the "sample values" is an imaginary line.
- The sampled version of the continuous time signal is represented by $x_s(t)$. The signals which are discrete in time and discrete in amplitude are called as "digital signals".
- The digital signals can be obtained from the continuous time analog signal by a process called "analog to digital conversion".

Sequence of samples :

- The discrete time (DT) signal can also be visualised as a sequence of samples taken at uniform intervals and denoted as x(n).
- Where x represents the shape of the signal and "n" is the variable. Such a sequence is shown in Fig. 1.3.2(d).

x (n)
Represents the shape of the signal.
Shows that the variable is time.

- For example x(1) is the value of x(n) at n = 1, or x(−2) is the value of x(n) at x = − 2 as shown in Fig. 1.3.2(d).

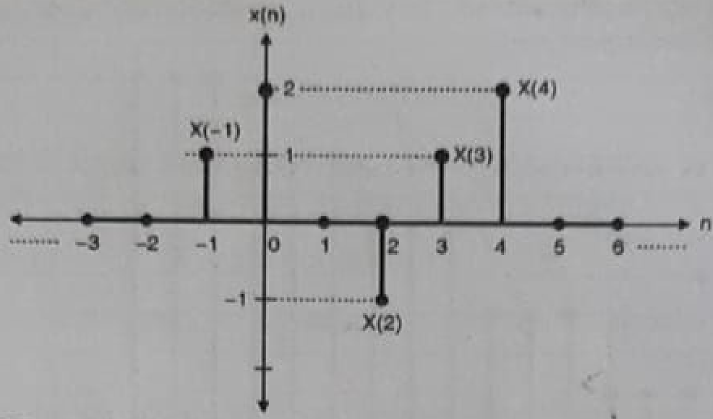


Fig. 1.3.2(d) : DT signal viewed as sequence of samples

- Mathematically it is denoted as follows :

$$x(n) = \{...... 0, 0, 1, \underset{\uparrow}{2}, 0, -1, 1, 2, 0, 0 ....\}$$

Where the arrow indicates the value of x(n) at n = 0.

1.3.2 Continuous Valued or Discrete Valued Signals :

Continuous valued signal :

If the variation in the amplitude of signal is continuous then, it is called as continuous valued signal. Such signal may be continuous or discrete in nature. Such signals are as shown in Fig. 1.3.3(a).

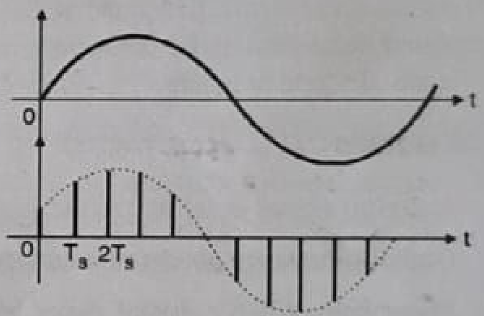


Fig. 1.3.3(a) : Continuous valued signals

Discrete valued signal :

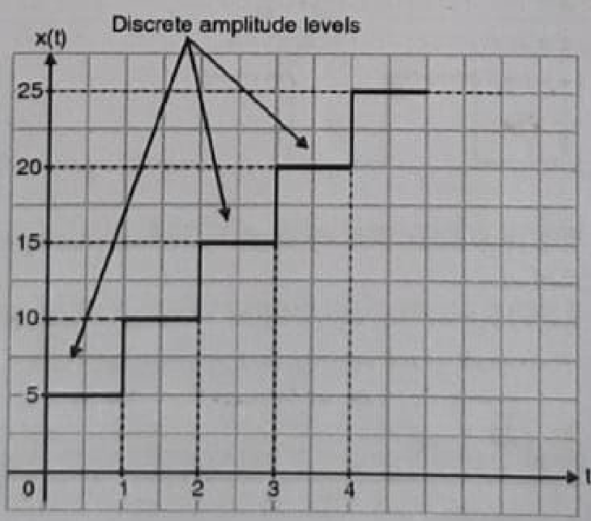


Fig. 1.3.3(b) : Discrete amplitude signal continuous in nature

- If the variation in the amplitude of signal is not continuous; but the signal has certain discrete amplitude levels then such signal is called as discrete valued signal. Such signal may be again continuous or discrete in nature as shown in Figs. 1.3.3(b) and 1.3.3(c).

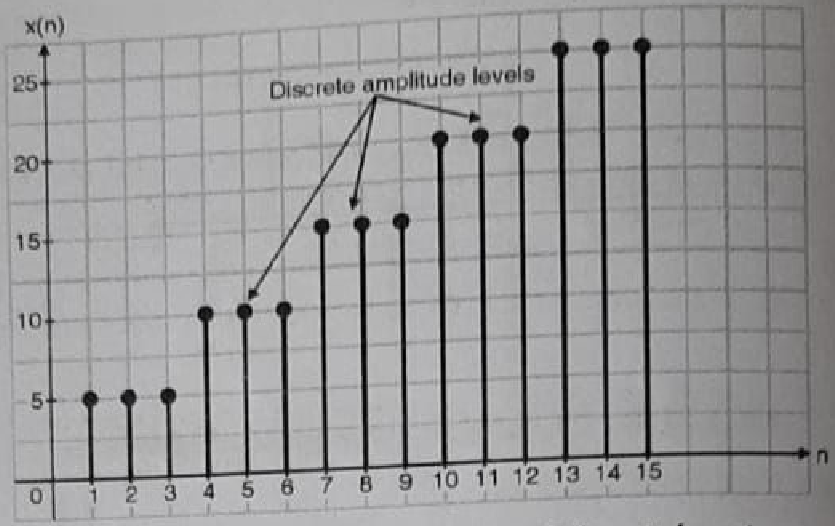


Fig. 1.3.3(c) : Discrete amplitude signal discrete in nature

- As shown in Fig. 1.3.3(b), the signal is defined at all instants of time. So it is continuous signal. But it takes only certain discrete amplitude levels. The amplitude is not continuously changing with time. So it is discrete amplitude signal continuous in nature.
- As shown in Fig. 1.3.3(c), the signal is defined only at discrete intervals of time. So it is discrete signal. And this signal takes only certain discrete amplitude levels. So it is discrete amplitude signal discrete in nature.

Digital signals :

- A **digital signal** is defined as the signal which has only a finite number of distinct values.
- Digital signals are not continuous signal. They are discrete signals as shown in Fig. 1.3.3(d).
- **Binary signal :** If a digital signal has only two distinct values, i.e. 0 and 1 then it is called as a binary signal.

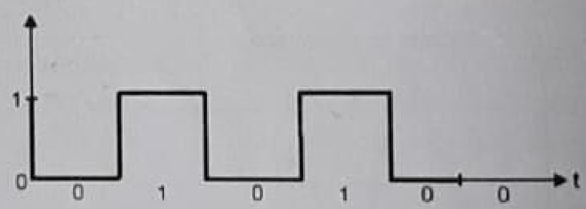


Fig. 1.3.3(d) : Binary signa: (Digital signal)

- **Octal signal :** A digital signal having eight distinct values is called as an octal signal.
- **Hexadecimal signal :** A digital signal having sixteen distinct values is called as the hexadecimal number.

Type of digital signal	Number of distinct values
Binary	2
Octal	8
Hexadecimal	16

1.3.3 Periodic and Non-periodic Signals :

Periodic CT signal :

- A CT signal which repeats itself after a fixed time period is called as a periodic signal. The periodicity of a CT signal can be defined mathematically as follows :

$$x(t) = x(t + T_o) \quad : \text{Condition of periodicity} \qquad ...(1.3.1)$$

- where T_o is called as the period of signal x(t), in other words, signal x(t) repeats itself after a period of T_o sec.
- Examples of periodic signals are sine wave, cosine wave, square wave etc. Fig. 1.3.4 shows a sine wave which is periodic because it repeats itself after a period T_0.

Non-periodic CT signal :

- A CT signal which does not repeat itself after a fixed time period or does not repeat at all is called as a non-periodic or aperiodic signal.
- The non-periodic signals do not satisfy the condition of periodicity stated in Equation (1.3.1).

$$\therefore \quad \text{For a non-periodic signal } x(t) \neq x(t + T_o) \qquad ...(1.3.2)$$

- Sometimes it is said that an aperiodic signal has a period $T_o = \infty$. Fig. 1.3.4 shows a decaying exponential signal.
- This exponential signal is non-periodic but it is deterministic because we can mathematically express it as $x(t) = e^{-\alpha t}$.

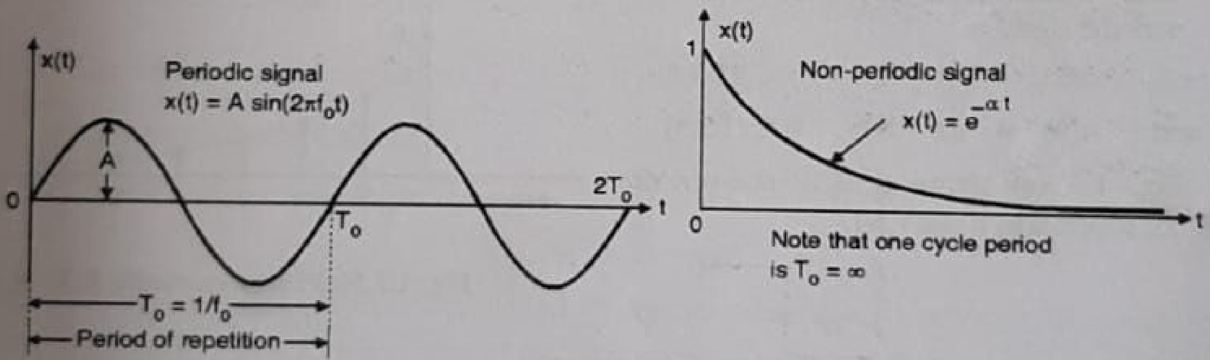


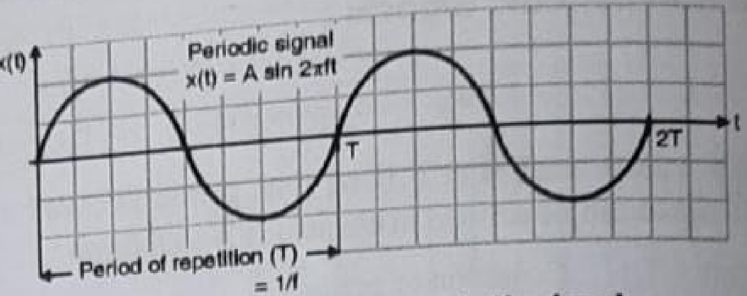
Fig. 1.3.4 : Periodic and non-periodic signals

Periodic discrete time signal :

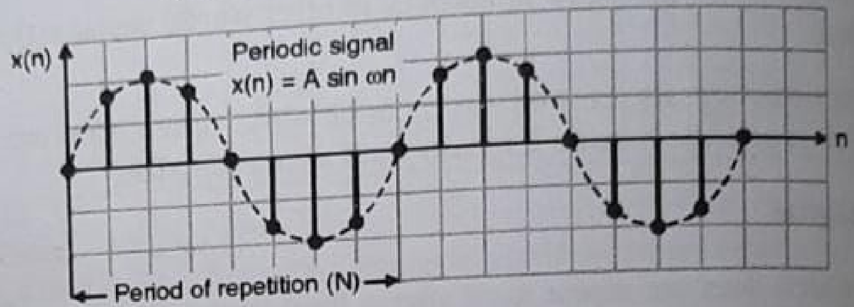
- For the discrete time signal, the condition of periodicity is,

$$x(n) = x(n + N) \qquad ...(1.3.3)$$

- Here number 'N' is the period of signal. The smallest value of N for which the condition of periodicity exists is called as fundamental period.
- Periodic signals are shown in Figs. 1.3.5(a) and (b).



(a) Continuous time periodic signal



(b) Discrete time periodic signal

Fig. 1.3.5

Non-periodic DT signal :

- A signal which does not repeat itself after a fixed time period or does not repeat at all is called as non-periodic or aperiodic signal. Thus mathematical expression for non-periodic signal is,

$$x(t) \neq x(t+T_0) \quad \text{...(1.3.4)}$$

and $$x(n) \neq x(n+N) \quad \text{...(1.3.5)}$$

Fig. 1.3.5(c) shows a D.T. non-periodic signal, for which $x(n) \neq x(n+N)$.

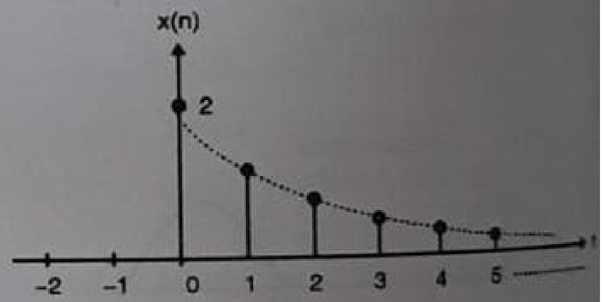


Fig. 1.3.5(c) : Non-periodic D.T. signal

Condition for periodicity of a discrete time signal :

A discrete time sinusoidal signal is periodic only if its frequency(f_0) is rational. That means frequency f_0 should be in the form of ratio of two integers.

Proof :

- For the discrete signal, the condition of periodicity is,

$$x(n+N) = x(n) \quad \text{...(1.3.6)}$$

- Let x(n) be the cosine wave. So it can be expressed as,

$$x(n) = A\cos(2\pi f_0 n + \theta) \quad \text{...(1.3.7)}$$

Here A = Amplitude

and θ = Phase shift

- Now the equation of $x(n+N)$ can be obtained by replacing 'n' by 'n + N' in Equation (1.3.7).

$$\therefore \quad x(n+N) = A\cos[2\pi f_0 (n+N)+\theta] \quad ...(1.3.8)$$

- According to condition of periodicity Equation (1.3.6); we can equate Equations (1.3.7) and (1.3.8).

$$\therefore \quad A\cos[2\pi f_0 (n+N)+\theta] = A\cos(2\pi f_0 n+\theta)$$

$$\therefore \quad A\cos(2\pi f_0 n + 2\pi f_0 N+\theta) = A\cos(2\pi f_0 n+\theta) \quad ...(1.3.9)$$

- To satisfy this equation,

$$2\pi f_0 N = 2\pi k \quad ...(1.3.10)$$

Where k is an integer

$$\therefore \quad f_0 = \frac{k}{N} \quad \textbf{....Proved} \quad ...(1.3.11)$$

- Here k and N both are integers. Thus discrete time (DT) signal is periodic if its frequency f_0 is rational.

Periodicity condition for $x(n) = x_1(n) + x_2(n)$:

- Here input sequence x(n) is expressed as summation of two discrete time sequences. We can calculate the values of f_1 and f_2 corresponding to $x_1(n)$ and $x_2(n)$.
- Let $x_1(n)$ and $x_2(n)$ both be periodic discrete time signals (sequences).
- So according to condition of periodicity,

$$f_1 = \frac{k_1}{N_1} \quad \text{and} \quad f_2 = \frac{k_2}{N_2}$$

- The resultant signal x(n) is periodic if $\frac{N_1}{N_2}$ is ratio of two integers. The period of x(n) will be least common multiple of N_1 and N_2.
- Similarly if continuous time signals is,

$$x(t) = x_1(t) + x_2(t)$$

- We can calculate the values of T_1 and T_2 corresponding to $x_1(t)$ and $x_2(t)$. Then the resultant x(t) is periodic if $\frac{T_1}{T_2}$ is ratio of two integers. The fundamental period of x(t) will be least common multiple of T_1 and T_2.

Solved examples :

Ex. 1.3.1 : Prove that the sinewave shown in Fig. 1.3.4 is a periodic signal.

Soln. : The sinewave shown in the Fig. 1.3.4 can be mathematically represented as,

$$x(t) = A\sin\omega_o t \quad ...(1)$$

Now, let us test if it satisfies the condition for periodicity i.e. if,

$$x(t) = x(t+T_o) \quad ...(2)$$

So, let us find the expression for $x(t+T_o)$

$$x(t+T_o) = A \sin \omega_o (t+T_o)$$
$$= A \sin [\omega_o t + \omega_o T_o] \quad ...(3)$$

But $\omega_o = 2\pi f_o$ and $T_o = \frac{1}{f_o}$. Therefore $\omega_o T_o = 2\pi f_o \times \frac{1}{f_o} = 2\pi$. Substitute this in Equation (3), to get,

$$x(t+T_o) = A \sin [\omega_o t + 2\pi]$$
$$= A [\sin(\omega_o t) \cos 2\pi + \cos(\omega_o t) \sin 2\pi]$$
$$\therefore x(t+T_o) = A \sin \omega_o t = x(t) \quad ...(4)$$

Therefore the sinewave shown in Fig. 1.3.4 is a periodic signal.

Ex. 1.3.2 : Prove that the exponential signal shown in Fig. 1.3.4 is non-periodic.

Soln. : The exponential signal shown in Fig. 1.3.4 is expressed mathematically as,

$$x(t) = e^{-\alpha t}$$
$$\text{Substitute } t = (t+T_o) \text{ to get,}$$
$$x(t+T_o) = e^{-\alpha(t+T_o)} = e^{-\alpha t} e^{-\alpha T_o}$$

But $T_o = \infty$

$$\therefore e^{-\alpha T_o} = e^{-\infty} = 0$$
$$\therefore x(t+T_o) = e^{-\alpha t} \cdot 0 = 0$$
$$\therefore x(t) \neq x(t+T_o)$$

Hence the exponential signal shown in Fig. 1.3.4 is a non-periodic signal.

Ex. 1.3.3 : What is the fundamental frequency of the waveform shown in Fig. P. 1.3.3, in Hz and rad/sec ?

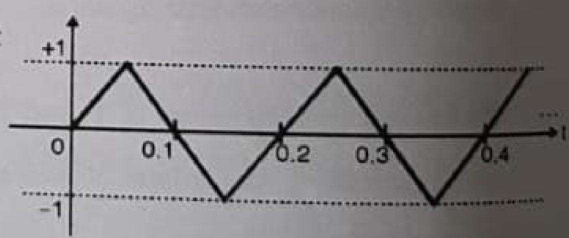


Fig. P. 1.3.3

Soln. :

- One cycle corresponds to 0.2 sec. Hence $T_o = 0.2$ sec.

$$\therefore \text{Frequency } f_o = \frac{1}{T_o} = \frac{1}{0.2} = 5 \text{ Hz} \quad ...\textbf{Ans.}$$

- Frequency in rad/sec. $= \omega_o = 2\pi f_o = 2 \times 3.14 \times 5 = 31.4$ rad/s ...Ans.

Ex. 1.3.4 : What is the fundamental frequency of the D.T. square wave shown in Fig. P. 1.3.4.

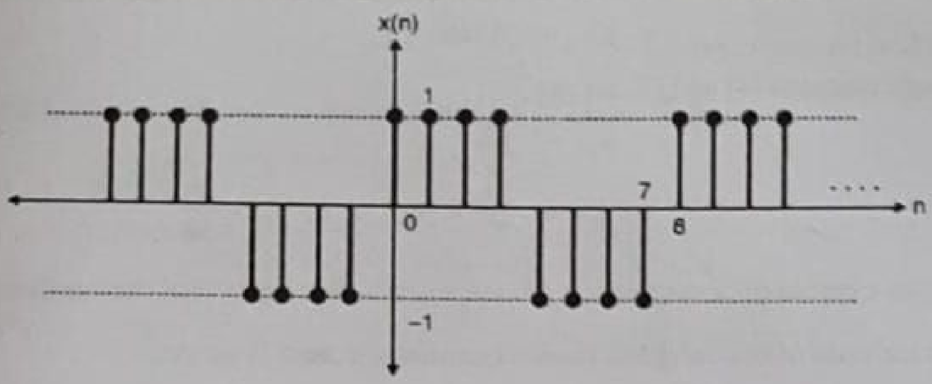


Fig. P. 1.3.4

Soln. :

- The fundamental angular frequency or simply fundamental frequency of x (n) is given by,

$$\Omega = \frac{2\pi}{N}$$

When N = a positive integer indicating number of samples in one cycle.

- For the given signal N = 8.

$$\therefore \; \Omega = \frac{2\pi}{8} = \frac{\pi}{4} \text{ radians} \qquad \text{...Ans.}$$

Ex. 1.3.5 : State whether the following signals x(t) is periodic or not, giving reasons. If it is periodic, find the corresponding period.

$x(t) = 2 \cos 100\pi t + 5 \sin 50 t$

Soln. :

1. The given signal is,

$$x(t) = 2 \cos 100\pi t + 5 \sin 50 t \qquad \text{...(1)}$$

$$\text{Let } x(t) = x_1(t) + x_2(t) \qquad \text{...(2)}$$

$$\text{Here } x_1(t) = 2 \cos 100\pi t \qquad \text{...(3)}$$

$$\text{and } x_2(t) = 5 \sin 50 t \qquad \text{...(4)}$$

The standard equation can be expressed as, $x_1(t) = A \cos \omega_1 t$ ...(5)

Comparing Equations (3) and (5) we get, $\omega_1 = 100\pi$

Now we have,

$$\omega_1 = 2\pi f_1 = \frac{2\pi}{T_1}$$

$$\therefore \; \frac{2\pi}{T_1} = 100\pi$$

$$\therefore \; T_1 = \frac{2\pi}{100\pi} = \frac{1}{50} \qquad \text{...(6)}$$

Similarly standard equation for $x_2(t)$ can be written as,

$$x_2(t) = A \sin \omega_2 t \quad ...(7)$$

Comparing Equations (4) and (7) we get,

$$\omega_2 = 50$$

$$\therefore 2\pi f_2 = \frac{2\pi}{T_2} = 50, \quad \therefore T_2 = \frac{2\pi}{50} \quad ...(8)$$

Here x (t) is expressed as summation of two signals. We know that the resultant signal x (t) is periodic if $\frac{T_1}{T_2}$ is the ratio of two integers. From Equations (6) and (7) we get,

$$\frac{T_1}{T_2} = \frac{1}{50} \cdot \frac{50}{2\pi} = \frac{1}{2\pi}$$

It is not the ratio of two integers. Thus x (t) is **non-periodic.**

Ex. 1.3.6 : Determine which of the following signals are periodic. If periodic determine the fundamental period.

1. $10 \sin (12 \pi t) + 4 \sin (18 \pi t)$ 2. $3 \sin 4t$ 3. $3 + t^2$

Ans. : 1. Periodic with F.P. = 1/3 sec. 2. Non periodic 3. Non periodic

Ex. 1.3.7 : Determine whether or not each of the following signal is periodic. If periodic, determine its fundamental period. **May 07, 4 Marks**

1. $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$ 2. $x(n) = \sin\left(\frac{6\pi}{7} n + 1\right)$

Soln. :

1. $$x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2 = \cos^2\left(2t - \frac{\pi}{3}\right)$$

$$x(t) = \cos\left(2t - \frac{\pi}{3}\right) \cos\left(2t - \frac{\pi}{3}\right) \quad ...(1)$$

The standard equation can be expressed as,

$$x(n) = \cos \omega_1 n \cos \omega_2 n \quad ...(2)$$

Comparing Equations (1) and Equation (2),

$$\omega_1 = \left(2t - \frac{\pi}{3}\right), \omega_2 = \left(2t - \frac{\pi}{3}\right)$$

$$\text{But } \omega = 2\pi f$$

$$\therefore 2\pi f_1 = \left(2t - \frac{\pi}{3}\right)$$

$$6\pi f_1 = (6t - \pi)$$

$$\therefore f_1 = \left(\frac{6t - \pi}{6\pi}\right)$$

$$\therefore \quad f_1 = \left(\frac{6t}{6\pi} - \frac{\pi}{6\pi}\right) = \left(\frac{1}{\pi} - \frac{1}{6}\right) = f_2$$

Hence f_1 and f_2 are ratio of non- integer values, so it is **nonperiodic.**

2. $$x(n) = \sin\left(\frac{6\pi}{7} n + 1\right)$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin\left(\frac{6\pi}{7} n + 1\right) = \sin \frac{6\pi}{7} n \cos 1 + \sin 1 \cos \frac{6\pi}{7} n$$

$$\sin\left(\frac{6\pi}{7} n + 1\right) = \sin \frac{6\pi}{7} n \cdot 1 + 0 \cdot \cos \frac{6\pi}{7} n$$

$$\therefore \quad \left(\frac{6\pi}{7} n + 1\right) = \sin \frac{6\pi}{7} n \qquad \ldots(3)$$

$$\text{Comparing with, } x(n) = \sin \omega_1 n \qquad \ldots(4)$$

Comparing Equations (3) and (4),

$$\therefore \quad \omega_1 = \frac{6\pi}{7}$$

$$\text{But} \quad \omega = 2\pi f$$

$$2\pi f_1 = \frac{6\pi}{7}$$

$$f_1 = \frac{6\pi}{7 \times 2\pi}$$

$$\therefore \quad f_1 = \frac{3}{7} \text{ cycles per samples} \qquad \ldots(5)$$

Since frequency 'f_1' is expressed as the ratio of two integers, this sequence is **periodic.**

To determine fundamental period :

$$f_1 = \frac{k}{N} \qquad \ldots(6)$$

Here 'N' indicates, the fundamental period.

Comparing Equations (5) and Equation (6),

Fundamental period = N = 7 samples

Ex. 1.3.8 : Determine which of the following signals are periodic.

1. $x_1(t) = \sin 15\pi t$ 2. $x_2(t) = \sin 20\pi t$ 3. $x_3(t) = x_1(t) + x_2(t)$ **Dec. 07, 4 Marks**

Soln. :

1. $x_1(t) = \sin 15\pi t$

For satisfying condition of periodicity,

$$x_1(t) = x_1(t + T_0)$$

$$\therefore \quad x_1(t + T_0) = \sin 15\pi(t + T_0)$$

$$\therefore \quad x_1(t + T_0) = \sin 15\pi t + \sin 15\pi T_0$$

$$\therefore \quad x_1(t) \neq x_1(t+T_0)$$

Therefore x_2 (t) signal is **nonperiodic.**

2. $\mathbf{x_2(t) = \sin 20 \pi t}$

For satisfying condition of periodicity,

$$\begin{aligned}
x_2(t) &= x_2(t+T_0) \\
T_0) &= \sin 20\pi (t+T_0) \\
x_2(t+T_0) &= \sin 20\pi t + \sin 20\pi T_0 \\
\therefore x_2(t) &\neq x_2(t+T_0)
\end{aligned}$$

Therefore x_2 (t) signal is **nonperiodic.**

3. $\mathbf{x_3(t) = x_1(t) + x_2(t)}$

$$\therefore x_3(t) = \sin 15\pi t + \sin 20\pi t$$

The standard equation can be expressed as,

$$\begin{aligned}
x_1(t) &= \sin \omega_1 t = \sin 15\pi t \\
\therefore \quad \omega_1 &= 15\pi \\
\therefore \quad 2\pi f_1 &= \frac{2\pi}{T_1} = 15\pi \\
\frac{2}{T_1} &= 15 \\
\therefore \quad T_1 &= \frac{2}{15} \quad \text{...(1}
\end{aligned}$$

Similarly,

$$\begin{aligned}
x_2(t) &= \sin \omega_2 t = \sin 20\pi t \\
\therefore \omega_2 &= 20\pi \\
\therefore \quad 2\pi f_2 &= \frac{2\pi}{T_2} = 20\pi \\
\frac{1}{T_2} &= 10 \\
T_2 &= \frac{1}{10} \quad \text{...(2}
\end{aligned}$$

Here x_3 (t) is expressed as summation of two signals. Therefore res[illegible]nt signal x_3 (t) is periodic if $\frac{T_1}{T_2}$ is the ratio cf two integer.

From Equations (1) and Equation (2),

$$\frac{T_1}{T_2} = \frac{2}{15} \times \frac{10}{1} = \frac{20}{15} = \frac{4}{3}$$

Therefore it is the ratio of two integers.

Thus x_3 (t) signal is **periodic.**

Ex. 1.3.9 : Find whether the following are periodic. If yes, find period also : **Dec. 06, 6 Marks**

1. $\sum_{k=-\infty}^{+\infty} (-1)^k \delta(t-2k)$ 2. $\cos\left(\frac{\pi n}{5}\right)\sin\left(\frac{\pi n}{5}\right)$

Soln. :

1. $$x(t) = \sum_{k=-\infty}^{+\infty} (-1)^k \delta(t-2k)$$

First we will draw the nature of signal by considering some range of k. Let k = – 2 to + 2

$$\therefore \quad x(t) = \sum_{k=-2}^{2} (-1)^k \delta(t-2k)$$

$$\therefore \quad x(t) = (-1)^{-2}\delta(t+4)+(-1)^{-1}\delta(t+2)+(-1)^0\delta(t)+(-1)^1\delta(t-2) +(-1)^2\delta(t-4)$$

$$\therefore \quad x(t) = \delta(t+4)-\delta(t+2)+\delta(t)-\delta(t-2)+\delta(t-4)$$

The given signal is existing from $-\infty$ to $+\infty$,

$$\therefore \quad x(t) = \ldots\delta(t+4)-\delta(t+2)+\delta(t)-\delta(t-2)+\delta(t-4)+\ldots$$

This signal is shown in Fig. P. 1.3.9.

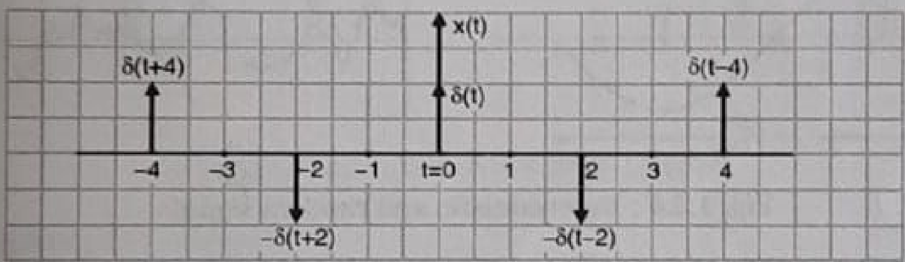


Fig. P. 1.3.9

Thus the signal is periodic and period = T = 2.

2. $$x(n) = \cos\left(\frac{\pi n}{5}\right)\sin\left(\frac{\pi n}{5}\right) \quad \ldots(1)$$

The standard equation can be written as,

$$x(n) = \cos 2\pi f_1 n \sin 2\pi f_2 n \quad \ldots(2)$$

Comparing Equations (1) and (2) we get,

$$2\pi f_1 = \frac{\pi}{5} \quad \text{and} \quad 2\pi f_2 = \frac{\pi}{5}$$

$$\therefore \quad f_1 = \frac{1}{10} \quad \text{and} \quad f_2 = \frac{1}{10}$$

Since f_1 and f_2 are ratio of two integers, the signal is periodic and period = N = 10

1.3.4 Deterministic and Random Signals :

Deterministic signal :

- A signal which can be described by a mathematical expression, look up table or some well defined rule is known as the deterministic signal.
- The examples of deterministic signals are sine wave, cosine wave, square wave etc. Fig. 1.3.6 shows a sine wave which is a deterministic signal because it can be represented mathematically as,

$$x(t) = A \sin(2\pi f t)$$

- Where A is the peak amplitude and f is the frequency of the signal.
- The deterministic signals such as sine, cosine etc. are periodic but some deterministic signals may not be periodic. Example of such signal is an exponential signal.

Random signals :

- A signal which cannot be described by any mathematical expression is called as a random signal. Due to this it is not possible to predict about the amplitude of such signals at a given instant of time.
- The example of random signal is "Noise" in the communication systems. The deterministic and random signals are as shown in Fig. 1.3.6.

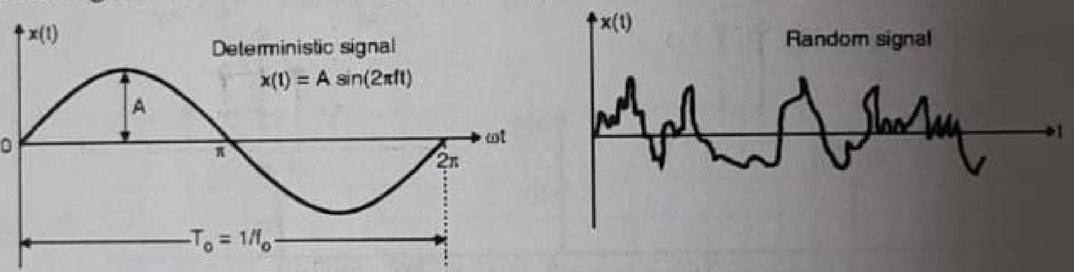


Fig. 1.3.6 : Deterministic and random signals

1.3.5 Symmetrical (Even) or Antisymmetrical (Odd) Signals :

Symmetrical signal (Continuous time) :

- A signal x (t) is said to be symmetrical or even if it satisfies the following condition,

Condition for symmetry : $x(t) = x(-t)$ ...(1.3.1

Where, $x(t)$ = Value of the signal for positive "t".

and $x(-t)$ = Value of the signal for negative "t".

An example of symmetrical signal is a cosine wave shown in Fig. 1.3.7.

Antisymmetrical signal (Continuous time) :

A signal x (t) is said to be antisymmetric or odd if it satisfies the following condition,

Condition for antisymmetry : $x(t) = -x(-t)$ ...(1.3.1

An example of odd signal is a sine wave shown in Fig. 1.3.7.

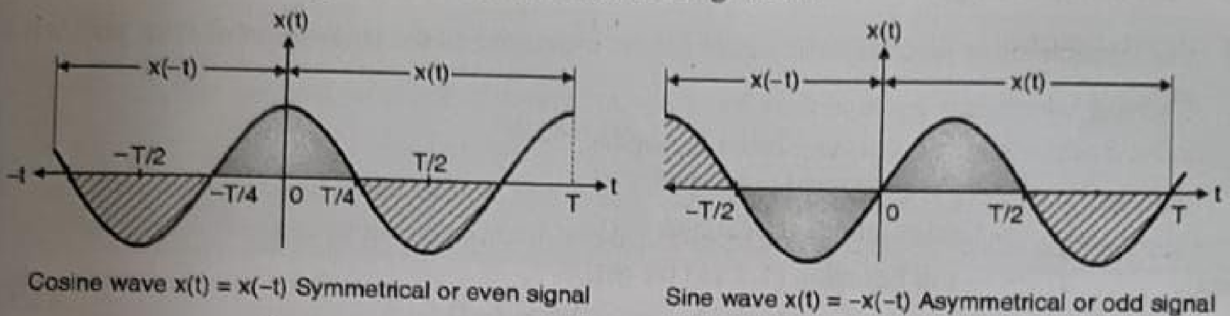


Fig. 1.3.7 : Symmetrical and antisymmetrical signals

Even and odd discrete time signals :

D.T. symmetric signals :

A discrete time real valued signal is said to be symmetric(even) if they satisfy the following condition.

Condition for symmetry : $x(n) = x(-n)$ ...For D.T. signal

Examples : Fig. 1.3.8(a) and (b) are examples of D.T. even signals.

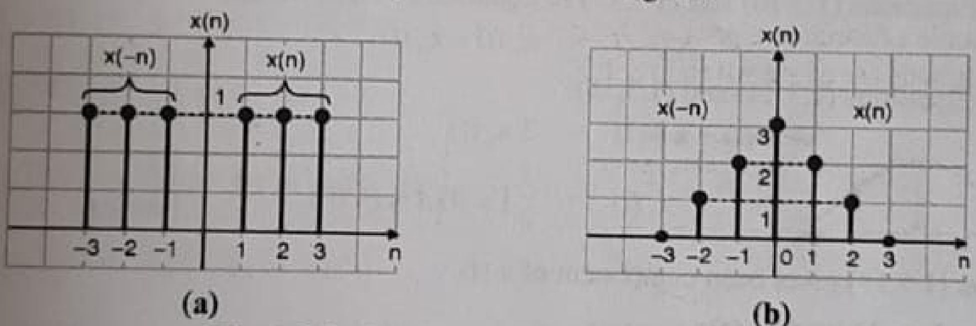


Fig. 1.3.8 : D.T. symmetric (even) signals

Odd (Antisymmetric) D.T. signal :

- Similarly discrete time (D.T.) signal x (n) is said to be antisymmetric or odd if it satisfies the following condition.

  Condition of antisymmetric : $x(-n) = -x(n)$ ... For D.T. signal

  Fig. 1.3.9 shows the antisymmetric discrete time signals.

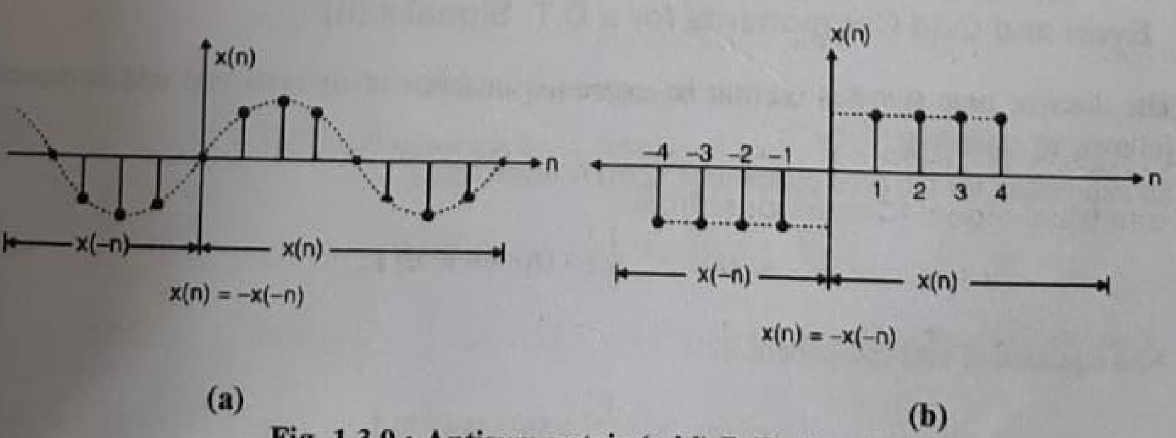


Fig. 1.3.9 : Antisymmetric (odd) D.T. signals

1.3.6 Decomposing a CT Signal into Even and Odd Parts :

- Any continuous or discrete time signal can be expressed as the summation of even part and odd part.

$$\therefore \; x(t) = x_e(t) + x_o(t) \qquad ...(1.3.14)$$

Here $x_e(t)$ = Even component of signal $x(t)$

and $x_o(t)$ = Odd component of signal $x(t)$

Putting 't' = $-t$ in Equation (1.3.14) we get,

$$x(-t) = x_e(-t) + x_o(-t) \qquad ...(1.3.15)$$

- Let us now obtain the expressions for the even and odd part $x_e(t)$ and $x_o(t)$.

Expression for the even part $x_e(t)$:

- For the even signal we have,

$$x_e(t) = x_e(-t) \qquad ...(1.3.16)$$

- And for odd signal we have,

$$x_o(-t) = -x_o(t) \qquad ...(1.3.17)$$

- Putting Equations (1.3.16) and (1.3.17) in Equation (1.3.15) we get,

$$x(-t) = x_e(t) - x_o(t) \qquad ...(1.3.18)$$

- Adding Equations (1.3.14) and (1.3.18),

$$x(t) + x(-t) = 2\,x_e(t)$$

$$\therefore \; x_e(t) = \frac{1}{2}[\,x(t) + x(-t)\,] \qquad ...(1.3.19)$$

Equation (1.3.19) gives even component of $x(t)$.

Expression for the odd part $x_o(t)$:

Now subtracting Equation (1.3.18) from Equation (1.3.14) we get,

$$x(t) - x(-t) = 2\,x_o(t)$$

$$\therefore \; x_o(t) = \frac{1}{2}[\,x(t) - x(-t)\,] \qquad ...(1.3.20)$$

Equation (1.3.20) gives odd component of $x(t)$.

1.3.7 Even and Odd Components for a D.T. Signal x (n) :

- The discrete time signal $x(n)$ can be expressed in terms of its even and odd components as follows, by replacing "t" by "n".
- So expression for the even component $x_e(n)$ is given by,

$$x_e(n) = \frac{1}{2}[\,x(n) + x(-n)\,] \qquad ...(1.3.21)$$

- And equation of odd component is,

$$x_o(n) = \frac{1}{2}[\,x(n) - x(-n)\,] \qquad ...(1.3.22)$$

Solved examples :

Ex. 1.3.10 : Find even and odd components of each of the following signals :

1. $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$
2. $x(t) = (1 + t^3)\cos^3(10t)$
3. $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$
4. $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$ **Dec. 05, 8 Marks**

Soln. :

Even part of a signal is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

And odd part of a signal is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

1. Given $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

$$\therefore x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$$

Now $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$$\therefore \quad x_e(t) = \frac{1}{2}[1 + t + 3t^2 + 5t^3 + 9t^4 + 1 - t + 3t^2 - 5t^3 + 9t^4]$$

$$\therefore \quad x_e(t) = \frac{1}{2}[2 + 6t^2 + 18t^4]$$

$$\therefore \quad x_e(t) = 1 + 3t^2 + 9t^4$$ ...Ans.

And $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$\therefore \quad x_o(t) = \frac{1}{2}[1 + t + 3t^2 + 5t^3 + 9t^4 - 1 + t - 3t^2 + 5t^3 - 9t^4]$$

$$\therefore \quad x_o(t) = \frac{1}{2}[2t + 10t^3]$$

$$\therefore \quad x_o(t) = t + 5t^3$$ ...Ans.

2.. Given $x(t) = (1 + t^3)\cos^3(10t)$

We have, $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos(3\theta)$

$$\therefore \quad x(t) = (1 + t^3)\left[\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t)\right]$$

$$\therefore \quad x(t) = \frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) + \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)$$

Thus $x(-t) = \frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) - \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)$

hus $x(-t) = \frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) - \frac{3}{4}t^3\cos(10t) - \frac{1}{4}t^3\cos(30t)$

Now $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$\therefore \quad x_e(t) = \frac{1}{2}\Big[\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) + \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)$
$+ \frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) - \frac{3}{4}t^3\cos(10t) - \frac{1}{4}t^3\cos(30t)\Big]$

$\therefore \quad x_e(t) = \frac{1}{2}\left[\frac{3}{2}\cos(10t) + \frac{1}{2}\cos(30t)\right]$

$\therefore \quad x_e(t) = \frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t)$ ...A

And $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$\therefore \quad x_o(t) = \frac{1}{2}\Big[\frac{3}{4}\cos(10t) + \frac{1}{4}\cos(30t) + \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)$
$- \frac{3}{4}\cos(10t) - \frac{1}{4}\cos(30t) + \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)\Big]$

$\therefore \quad x_o(t) = \frac{1}{2}\left[\frac{3}{2}t^3\cos(10t) + \frac{1}{2}t^3\cos(30t)\right]$

$\therefore \quad x_o(t) = \frac{3}{4}t^3\cos(10t) + \frac{1}{4}t^3\cos(30t)$ ...A

3. Given $x(t) = \cos t + \sin t + \sin t \cos t$

$\therefore \quad x(-t) = \cos t - \sin t - \sin t \cos t$

Now $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$\therefore \quad x_e(t) = \frac{1}{2}[\cos t + \sin t + \sin t\cos t + \cos t - \sin t - \sin t \cos t]$

$\therefore \quad x_e(t) = \frac{1}{2}[2\cos t]$

$\therefore \quad x_e(t) = \cos t$ ...A

And $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$\therefore \quad x_o(t) = \frac{1}{2}[\cos t + \sin t + \sin t \cos t - \cos t + \sin t + \sin t \cos t]$

$\therefore \quad x_o(t) = \frac{1}{2}[2\sin t + 2\sin t\cos t]$

$\therefore \quad x_o(t) = \sin t + \sin t \cos t$ ...A

4. Given $x(t) = 1 + t\cos t + t^2\sin t + t^3 \sin t \cos t$

$\therefore \quad x(-t) = 1 - t\cos t - t^2 \sin t + t^3 \sin t \cos t$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$\therefore \quad x_e(t) = \frac{1}{2}[1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t + 1 - t\cos t - t^2 \sin t + t^3 \sin t \cos t]$$

$$\therefore \quad x_e(t) = \frac{1}{2}[2 + 2t^3 \sin t \cos t]$$

$$\therefore \quad x_e(t) = 1 + t^3 \sin t \cos t$$ ...Ans.

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t - 1 + t\cos t + t^2 \sin t - t^3 \sin t \cos t]$$

$$= \frac{1}{2}[2t \cos t + 2t^2 \sin t]$$

$$\therefore \quad x_o(t) = t\cos t + t^2 \sin t$$ ...Ans.

Ex. 1.3.11 : Function x (t) is as shown in Fig. P. 1.3.11(a).

Draw even and odd parts of x (t).

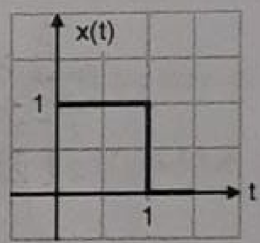


Fig. P. 1.3.11(a) : Given function x(t)

Soln. :

- The even part is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

- And the odd part is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

where x (– t) represents the folded version of x (t). The folding operation is discussed later on.

Steps to be followed :

Step 1 : Draw the signal x (t).
Step 2 : Draw its folded version x (– t).
Step 3 : Add x (t) and x (– t) or subtract x (– t) from x (t).
Step 4 : Divide the addition or subtraction by 2 to get $x_e(t)$ and $x_o(t)$.

- These steps are followed in Fig. P. 1.3.11(b) to (i) to obtain $x_e(t)$ and $x_o(t)$.

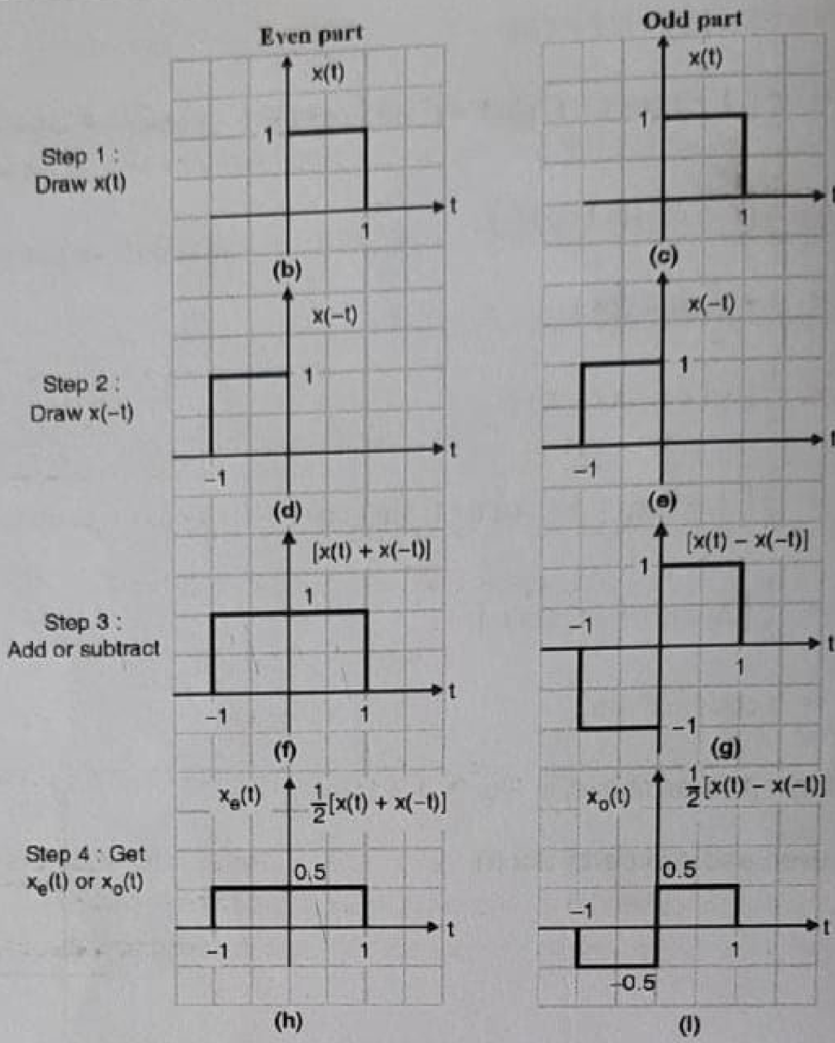


Fig. P. 1.3.11

- The above definitions of even and odd signals assume that the signals are real valued.
- If the signals are complex valued, then we have to talk in terms of conjugate symmetry.
- A complex valued signal is said to be having a conjugate symmetry if it satisfies the following condition,

$$x(-t) = x^*(t) \quad ...(1.3.23)$$

- Remember that if $x(t) = a + jb$ then its complex conjugate is given by,

$$x^*(t) = (a - jb)$$

- So the complex valued signal $x(t)$ is conjugate symmetric if its real part is even and its imaginary part is odd.

1.3.8 Energy and Power Signals :

PU : May 05

University Questions

Q. 1 Compare the following pairs : Energy signal and power signal (May 05, 8 Marks)

Power signal :

- A signal is called as a power signal if its "average normalized power" is non-zero and finite.
- It has been observed that almost all the periodic signals are power signals.

Energy signals :

- A signal having a finite non-zero total normalized energy is called as an energy signal.
- It is observed that almost all the non-periodic signals defined over a finite period, are energy signals.
- As these signals are defined over a finite period, they are called as time limited signals.

1.4 Energy and Power of the Signals :

PU : May 12

University Questions

Q. 1 Define energy signal and power signal. **(May 12, 2 Marks)**

1.4.1 Time Average (dc Value) of a Signal :

- The time average or dc value of a signal x (t) over all time is defined as,

$$\text{Time average or dc value : } \langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt \quad ...(1.4.1)$$

- The integration of x (t) from $-T/2$ to $T/2$ gives us the area under x (t) between the time limits $-T/2$ to $T/2$.
- This area is then divided by T to obtain the average value of x (t). And as the limit of $T \to \infty$ is taken, the average is calculated over the entire time range from $-\infty$ to $+\infty$.
- If the signal x (t) is periodic signal with period "T_o" then its average value is given by,

$$\text{Time average of periodic signal : } \langle x(t) \rangle = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x(t)\, dt \quad ...(1.4.2)$$

1.4.2 Power :

Instantaneous power :

- If x (t) is a voltage across a resistor R, the instantaneous power is defined as,

$$\text{Instantaneous power} = \frac{x^2(t)}{R} \quad ...(1.4.3)$$

- However if x (t) is a current signal then the expression for the instantaneous power is given by,

$$\text{Instantaneous power} = x^2(t) \times R \quad ...(1.4.4)$$

Normalized power :

- Everytime we may not know whether x (t) is a voltage signal or a current signal. Hence in order to make the expression for power independent of the nature of x (t), we normalize it by substituting R = 1 in Equations (1.4.3) and (1.4.4).
- Therefore the normalized instantaneous power is given by,

$$\text{Normalized instantaneous power} = x^2(t) \quad ...(1.4.5)$$

Average normalized power :

- Next step is to obtain the average value of this normalized power. From the basic expression for time average defined by Equation (1.4.1) we can write that,

Energy signals :

- A signal having a finite non-zero total normalized energy is called as an energy signal.
- It is observed that almost all the non-periodic signals defined over a finite period, are energy signals.
- As these signals are defined over a finite period, they are called as time limited signals.

1.4 Energy and Power of the Signals : PU : May 12

University Questions

Q. 1	Define energy signal and power signal.	(May 12, 2 Marks)

1.4.1 Time Average (dc Value) of a Signal :

- The time average or dc value of a signal x (t) over all time is defined as,

$$\text{Time average or dc value} : \langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt \quad ...(1.4.1)$$

- The integration of x (t) from $-T/2$ to $T/2$ gives us the area under x (t) between the time limits $-T/2$ to $T/2$.
- This area is then divided by T to obtain the average value of x (t). And as the limit of $T \to \infty$ is taken, the average is calculated over the entire time range from $-\infty$ to $+\infty$.
- If the signal x (t) is periodic signal with period "T_o" then its average value is given by,

$$\text{Time average of periodic signal} : \langle x(t) \rangle = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x(t)\, dt \quad ...(1.4.2)$$

1.4.2 Power :

Instantaneous power :

- If x (t) is a voltage across a resistor R, the instantaneous power is defined as,

$$\text{Instantaneous power} = \frac{x^2(t)}{R} \quad ...(1.4.3)$$

- However if x (t) is a current signal then the expression for the instantaneous power is given by,

$$\text{Instantaneous power} = x^2(t) \times R \quad ...(1.4.4)$$

Normalized power :

- Everytime we may not know whether x (t) is a voltage signal or a current signal. Hence in order to make the expression for power independent of the nature of x (t), we normalize it by substituting R = 1 in Equations (1.4.3) and (1.4.4).
- Therefore the normalized instantaneous power is given by,

$$\text{Normalized instantaneous power} = x^2(t) \quad ...(1.4.5)$$

Average normalized power :

- Next step is to obtain the average value of this normalized power. From the basic expression for time average defined by Equation (1.4.1) we can write that,

$$\text{Average normalized power} = P = \langle x^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \quad \text{...(1.4.6)}$$

- The above definition can be generalized for a complex signal x (t) as,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt \quad \text{...(1.4.7)}$$

- From Equations (1.4.6) and (1.4.7) it is observed that the signal power P is the time average (mean) of the signal amplitude squared that is the "mean squared" value of x (t).
- Therefore the square root of P is the root mean square (rms) value of x (t).

$$\therefore\ P = \text{Mean square value of } x(t) \qquad \therefore\ \sqrt{P} = \text{rms value of } x(t) \quad \text{...(1.4.8)}$$

- For a periodic signal with a period T_o, the Equations (1.4.6) and (1.4.7) get modified to,

$$P = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt \quad \text{...(1.4.9)}$$

- For a complex periodic signal x (t) the average normalized power is given by,

$$P = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} |x^2(t)|\, dt \quad \text{...(1.4.10)}$$

Based on these definitions of average normalized power, we have defined the power signal in the preceding section.

1.4.3 Energy :

- The total normalized energy for a "real" signal x (t) is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt \quad \text{...(1.4.11)}$$

- However if the signal is complex then the expression for total normalized energy is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt \quad \text{...(1.4.12)}$$

- These equations indicate that the energy is the area under $x^2(t)$ the curve over $(-\infty \le t \le \infty)$ hence it is always positive.

1.4.4 Comparison of Energy and Power Signals :

University Questions **PU : May 05**

Q. 1 Compare the following pairs : Energy signal and power signal **(May 05, 8 Marks)**

Table 1.4.1

Sr. No.	Power signals	Energy signals
1	The signal having finite non-zero power are called as power signals.	The signals having a finite non-zero energy are called as energy signals.

Sr. No.	Power signals	Energy signals
2	Almost all the periodic signals in practice are power signals.	Almost all the non-periodic signals are energy signals.
3	Power signals can exist over an infinite time. They are not time limited.	Energy signals exist over a short period of time. They are time limited.
4	Energy of a power signal is infinite.	Power of an energy signal is zero.

1.4.5 Average Power of a DT Signal :

For a discrete time signals, average power P is given by,

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n) \qquad \text{...(1.4.13)}$$

1.4.6 Energy of a DT Signal :

The energy of a D.T. signal is denoted by E and it is given by,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad \text{...(1.4.14)}$$

Note : The nonperiodic signals are generally energy signals whereas the periodic signals are generally the power signals.

Signal	Condition
Energy signal	$0 < E < \infty$
Power signal	$0 < P < \infty$

1.4.7 Power of the Energy Signals :

Let x (t) be an energy signal i.e. x (t) has a finite non-zero energy. Let us calculate the power of x (t). By definition, stated in Equation (1.4.10) the power of x (t) is given by,

$$P = \lim_{T_o \to \infty} \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} \frac{1}{T_o} \cdot \lim_{T_o \to \infty} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt$$

$$= \lim_{T_o \to \infty} \frac{1}{T_o} \cdot \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \lim_{T_o \to \infty} \frac{1}{T_o} \cdot E \qquad \text{Using Equation (1.4.12)}$$

$$\therefore \quad P = 0 \times E = 0, \qquad \text{As, } \lim_{T_o \to \infty} \frac{1}{T_o} = 0$$

Thus the power of an energy signal is zero over a finite time.

1.4.8 Energy of a Power Signal :

Let x (t) be a power signal. The normalized energy of this signal is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

This expression can be written in a different way as follows :

$$E = \lim_{T_o \to \infty} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} \left[T_o \cdot \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt \right]$$

$$= \lim_{T_o \to \infty} T_o \cdot \lim_{T_o \to \infty} \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} T_o \cdot [P]$$

$$\therefore \; E = \infty \qquad \text{As } \lim_{T_o \to \infty} T_o = \infty$$

Thus the energy of a power signal is infinite over a finite time.

Ex. 1.4.1 : What is the average power of the square wave shown in Fig. P. 1.4.1 ?

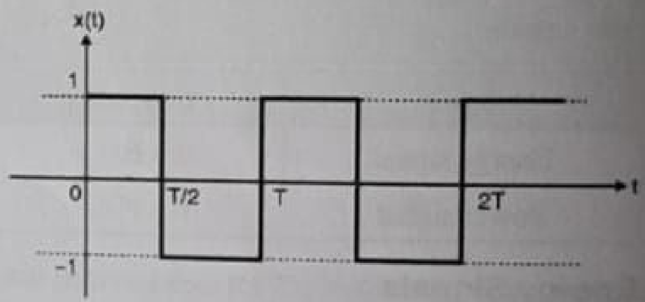


Fig. P. 1.4.1

Soln. : Given signal is periodic. So consider one cycle from 0 to T.

$$P = \frac{1}{T} \int_{0}^{T} x^2(t)\, dt$$

$$\text{But, } x(t) = 1 \qquad \text{For } 0 \le x(t) \le T/2$$

$$T = -1 \qquad \text{For } T/2 \le x(t) \le T$$

$$\therefore \; P = \frac{1}{T} \left[\int_{0}^{T/2} (1)^2\, dt + \int_{T/2}^{T} (-1)^2\, dt \right] = \frac{1}{T} \left[1\,(t)_0^{T/2} + 1\,(t)_{T/2}^{T} \right]$$

$$= \frac{1}{T} \left[\frac{T}{2} + \frac{T}{2} \right] = 1$$

Ex. 1.4.2 : Calculate the average power of the triangular wave shown in Fig. P. 1.4.2.

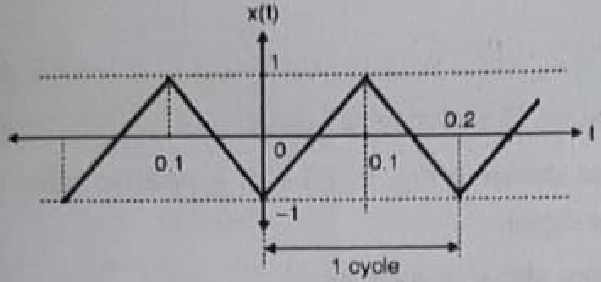


Fig. P. 1.4.2

Ans. : $P = 1/3$ W

Ex. 1.4.3 : Find the energy and power of the following signals :

$x(t) = A\cos(2\pi f_o t + \theta)$ **May 10, 2 Marks**

Soln. :

$$x(t) = A\cos(2\pi f_o t + \theta)$$

This is a periodic signal with period $T_o = 2\pi/\omega_o$ or $1/f_o$. Therefore the power is given by,

$$P = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} A^2\cos^2(2\pi f_o t + \theta)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\frac{A^2}{2}[1+\cos 2(2\pi f_o t+\theta)]\,dt$$

$$= \lim_{T\to\infty}\frac{A^2}{2T}\int_{-T/2}^{T/2}dt + \lim_{T\to\infty}\frac{A^2}{2T}\int_{-T/2}^{T/2}\cos(4\pi f_o t + 2\theta)\,dt$$

$$= \frac{A^2}{2T}\times(T/2+T/2)+0$$

$$\therefore\ P = \frac{A^2}{2}$$ **...Ans.**

The second term reduces to zero because the integral appearing in this second term represents the area under a sinusoid over a very large time period i.e. $T\to\infty$.

Conclusion : Thus the power of a sinusoid of peak amplitude A is $A^2/2$ irrespective of the value of its frequency f_o or phase angle θ.

Energy of this signal is given by,

$$E = \int_{-\infty}^{\infty} A^2\cos^2(2\pi f_o t+\theta)\,dt = \frac{A^2}{2}\int_{-\infty}^{\infty}[1+\cos 2(2\pi f_o t+\theta)]\,dt$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty}dt + \frac{A^2}{2}\int_{-\infty}^{\infty}\cos 2(2\pi f_o t+\theta)\,dt$$

$$\therefore\ E = \infty + 0 = \infty$$ **...Ans.**

Thus the energy of the signal is ∞. Hence the given signal $x(t) = A\cos(2\pi f_o t+\theta)$ is a power signal.

Ex. 1.4.6 : Find whether the signal sin ωt is energy or power signal.

Ans. : Power signal

Ex. 1.4.7 : Determine whether the signal shown in Fig. P. 1.4.7 is an energy signal or a power signal.

Dec. 10, 6 Marks

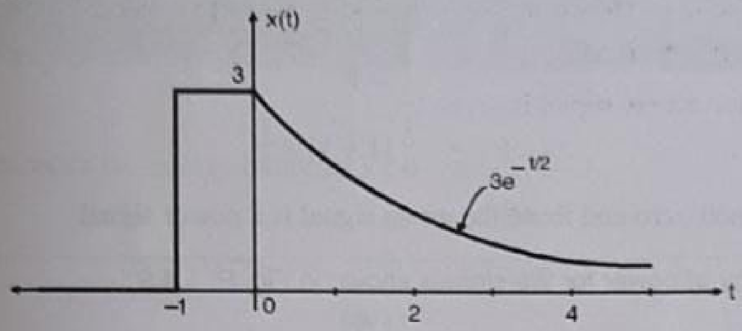


Fig. P. 1.4.7

Soln. : The signal shown in Fig. P. 1.4.7 tends to zero as $|t| \to \infty$. Therefore the average power content of this signal will be zero. Let us calculate the total normalized energy of this signal.

$$x(t) = 3 \qquad -1 \le t \le 0$$

$$= 3e^{-t/2} \qquad t > 0$$

$$\therefore \quad E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-1}^{0} x^2(t)\,dt + \int_{0}^{\infty} x^2(t)\,dt = \int_{-1}^{0} (3)^2\,dt + \int_{0}^{\infty} (3e^{-t/2})^2\,dt$$

$$= 9[t]_{-1}^{0} + 9\left[\frac{e^{-t}}{-1}\right]_0^{\infty} = 9[0+1] - 9[e^{-\infty} - e^0]$$

$$\therefore \quad E = 9 + 9 = 18 \qquad \text{...Ans.}$$

Thus the total normalized energy of the signal is non-zero and finite. Hence this signal is an energy signal.

Ex. 1.4.8 : Determine whether the signal shown in Fig. P. 1.4.8 is an energy signal or a power signal.

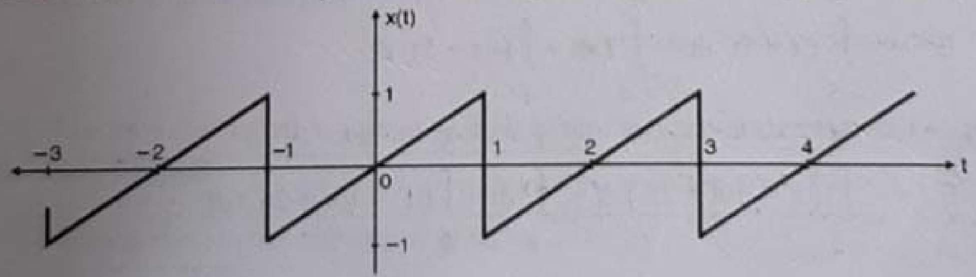


Fig. P. 1.4.8

Soln. : The signal shown in Fig. P. 1.4.8 does not tend to zero as $|t| \to \infty$. Therefore the energy of this signal will be ∞. Let us calculate the average normalized power of this signal. Referring to Equation (1.4.10) we can write that,

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt$$

From Fig. P. 1.4.8, it is clear that $T_0 = 2$ sec. and $x(t) = t$

$$\therefore \quad P = \frac{1}{2} \int_{-1}^{1} t^2\, dt = \frac{1}{2 \times 3} [t^3]_{-1}^{1}$$

$$\therefore \quad P = \frac{1}{6} [1 + 1] = \frac{1}{3}$$ ...Ans.

As the power is non-zero and finite the given signal is a power signal.

Ex. 1.4.9 : Find energy of power for the signals shown in Fig. P. 1.4.9 :

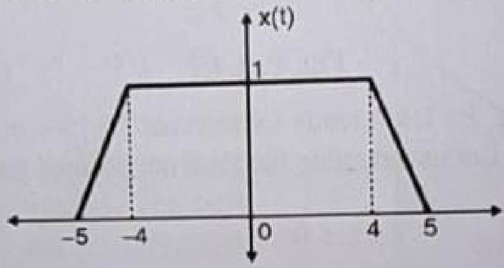


Fig. P. 1.4.9

Soln. :

1. The given signal can be mathematically written as,

$$x(t) = \begin{cases} t+5 & \text{for } -5 \le t \le -4 \\ 1 & \text{for } -4 \le t \le 4 \\ -t+5 & \text{for } 4 \le t \le 5 \end{cases}$$

Now energy of a signal is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$\therefore \quad E = \int_{-5}^{-4} (t+5)^2\, dt + \int_{-4}^{4} 1\, dt + \int_{4}^{5} (-t+5)^2\, dt$$

$$\therefore \quad E = \int_{-5}^{-4} (t^2 + 10t + 25)\, dt + \int_{-4}^{4} dt + \int_{4}^{5} (t^2 - 10t + 25)\, dt$$

$$\therefore \quad E = \left[\frac{t^3}{3} + \frac{10t^2}{2} + 25t\right]_{-5}^{-4} + [t]_{-4}^{4} + \left[\frac{t^3}{3} - \frac{10t^2}{2} + 25t\right]_{4}^{5}$$

$$\therefore \quad E = [20.33 - 45 + 25] + [4 + 4] + [20.33 - 45 + 25] = 8.66 \text{ J}$$

Thus given signal is an energy signal.

Ex. 1.4.10 : Obtain energy for the signal $x(n) = a^n u(n)$ where $|a| < 1$.

Ans. : The energy is given by $E = 1/(1 - a^2)$ if $|a^2| < 1$.

Ex. 1.4.11 : $x(n) = (0.5)^n u(n)$. State whether it is an energy or power signal. Justify.

May 10, Dec. 11, 2 Marks

Soln. :

First we will calculate the energy of signal $x(n)$. It is given by,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{... (1)}$$

The given signal is,

$$x(n) = (0.5)^n u(n) \quad \text{... (2)}$$

Since it is multiplied by unit step; this signal is present from $n = 0$ to $n = \infty$. Thus Equation (1) becomes,

$$E = \sum_{n=0}^{\infty} [(0.5)^n]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$E = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \quad \text{...(3)}$$

We have the standard geometric series formula,

$$\sum_{n=0}^{\infty} A^n = 1 + A + A^2 + \ldots = \frac{1}{1-A}$$

$$\therefore E = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

That means this is finite energy.

Justification :

If $0 < E < \infty$ then the signal is energy signal. Since the calculated value of energy is finite; the given signal is **energy signal**.

Ex. 1.4.12 : Determine the energy and power of signal given by :

$$x(n) = \left(\frac{1}{2}\right)^n \qquad n \geq 0$$
$$= 3^n \qquad n < 0$$

Soln. : The energy of signal is given by,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \dots(1)$$

The given signal is, $x(n) = \left(\frac{1}{2}\right)^n$ for the range $n \geq 0$. That means for the range $n = 0$ to $n = \infty$

And $x(n) = 3^n$ for $n < 0$. That means for the range $n = -1$ to $n = -\infty$. Thus Equation (1) becomes,

$$E = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n\right]^2 + \sum_{n=-1}^{-\infty} [3^n]^2$$

$$\therefore E = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^2\right]^n + \sum_{n=-1}^{-\infty} [3^2]^n = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n + \sum_{n=-1}^{-\infty} (9)^n \quad \dots(2)$$

Consider the first summation term. Using geometric series formula we can express it as follows :

$$\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} \quad \dots(3)$$

Now consider the second summation term. To make the limits of summation positive; put $n = -m$. Then the limits will change as follows.

When $n = -1 \Rightarrow -m = -1 \therefore m = 1$

When $n = -\infty \Rightarrow -m = -\infty \therefore m = \infty$

$$\therefore \sum_{n=-1}^{-\infty} (9)^n = \sum_{m=+1}^{\infty} (9)^{-m} = \sum_{m=1}^{\infty} \left(\frac{1}{9}\right)^m$$

Now use the standard summation formula,

$$\sum_{n=1}^{\infty} A^n = \frac{A}{1-A}$$

$$\therefore \sum_{m=1}^{\infty} \left(\frac{1}{9}\right)^m = \frac{1/9}{1-\frac{1}{9}} = \frac{\frac{1}{9}}{8/9} = \frac{1}{8} \quad \dots(4)$$

Putting Equations (3) and (4) in Equation (2) we get,

$$E = \frac{4}{3} + \frac{1}{8} \qquad \therefore E = \frac{35}{24}$$

This is the finite energy. That means the given signal is energy signal. We know that **if the energy of signal is finite then its power is zero**. Thus the power of given signal is zero.

$$\therefore P = 0$$

Ex. 1.4.13 : Find signal power of $x(n) = u(n)$

Ans. : **The unit step sequence is power signal P = 1/2 Watt**

Ex. 1.4.14 : Find whether the following signal is an energy signal or power signal.

$$x(n) = n \qquad n > 0$$
$$= 0 \qquad n < 0$$

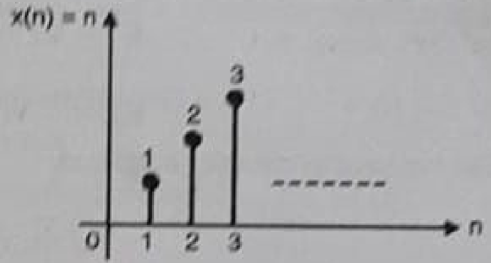


Fig. P. 1.4.14

Ans. : **E = ∞ and P = ∞ Hence unit ramp is neither an energy signal nor a power signal.**

Ex. 1.4.15 : Find whether the following signals are energy or power signals and find appropriate value.

1. $x[n] = \left(\frac{1}{2}\right)^n u[n]$ 2. $x[n] = u[n] - u[n-4]$ **May 05, 6 Marks**

Ans. : **1. Energy signal E = 4/3 J. 2. Energy signal E = 5 J**

Ex. 1.4.16 : Find whether the following signals are energy or power. Find appropriate value :

1. $x(n) = \cos(\pi n) \qquad -4 \le n \le 4$
   $= 0 \qquad$ Otherwise
2. $x(t) = (2e^{-t} - 6e^{-2t})\, u(t)$. **Dec. 07, 6 Marks**

Soln. :

1. $x(n) = \cos(\pi n) \qquad -4 \le n \le 4$

First we will check periodicity. The standard equation is, $x(n) = \cos(2\pi f n)$

$$\therefore \quad 2\pi f = \pi \qquad \therefore \quad f = \frac{1}{2}$$

$$\text{But,} \quad f = \frac{K}{N}$$

$\therefore$ It is periodic with period N = 2.

Thus we will calculate the power of signal. It is,

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{2} \sum_{n=0}^{1} |x(n)|^2 = \frac{1}{2}(1+1)$$

$$\therefore \quad P = 1 \text{ W} \qquad \text{...Ans.}$$

2. $x(t) = (2e^{-t} - 6e^{-2t})\, u(t) \qquad \therefore \; x(t) = 2e^{-t} u(t) - 6e^{-2t} u(t)$.

It is exponential signal which is non-periodic, so we will calculate energy of signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\therefore \quad E = \int_0^{\infty} |2e^{-t} s|^2 \, dt - \int_0^{\infty} |6e^{-2t}|^2 \, dt = \int_0^{\infty} 4e^{-2t} \, dt - \int_0^{\infty} 36e^{-4t} \, dt$$

$$\therefore \quad E = 4\left[\frac{e^{-2t}}{-2}\right]_0^{\infty} - 36\left[\frac{e^{-4t}}{-4}\right]_0^{\infty} = \frac{4}{-2}[e^{-\infty} - e^0] + \frac{36}{4}[e^{-\infty} - e^0]$$

$$\therefore \quad E = -2(-1) + 9(-1) = -7$$

But we have to consider magnitude.

$$\therefore E = 7 \text{ Joules.}$$ ...Ans.

1.4.9 Multichannel and Multidimensional Signals :

Multichannel signals :

- As the name indicates, multichannel signals are generated by multiple sources or multiple sensors.
- The resultant signal is the vector sum of signals from all channels.

Example :

- A common example of multichannel signal is ECG waveform. To generate ECG waveform different leads are connected to the body of a patient.
- Each lead is acting as individual channel. Since there are 'n' number of leads; the final ECG waveform is a result of multichannel signal. Mathematically final wave is expressed as,

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \text{; If three leads are used.}$$

Multidimensional signals :

- If a signal is a function of single independent variable, the signal is called as one-dimensional signal. On the other hand, if the signal is a function of multi (many) independent variables then it is called as multidimensional signal.
- A good example of multidimensional signal is the picture displayed on the TV screen. To localise a pixel (a point) on the TV screen two co-ordinates namely X and Y are required.
- Similarly this point is a function of time also. So to display a pixel, minimum three dimensions are required; namely x, y and t.
- Thus this is multidimensional signal. Mathematically it can be written as, P (x, y, t).

Comparison of Multichannel and Multidimensional signal :

Sr. No.	Multichannel signal	Multidimensional signal
1.	Such signals are generated by multiple sources or multiple sensors.	Such signals are function of many independent variables.
2.	Example : ECG signal. Mathematically it is represented by, $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$	Example : Picture displayed on TV screen. Mathematically it is represented by, P (x, y, t).

1.5 Elementary Signals :

- In signals and systems we need to use some standard or elementary signals.
- In this section we will show some important standard signal graphically and express them mathematically.
- Some of the standard continuous time and discrete time signals are :
  1. A dc signal
  2. Unit step signal
  3. Sinusoidal signal
  4. Signum function
  5. A rectangular pulse
  6. Delta or unit impulse function
  7. Exponential signals
  8. Sinc function.

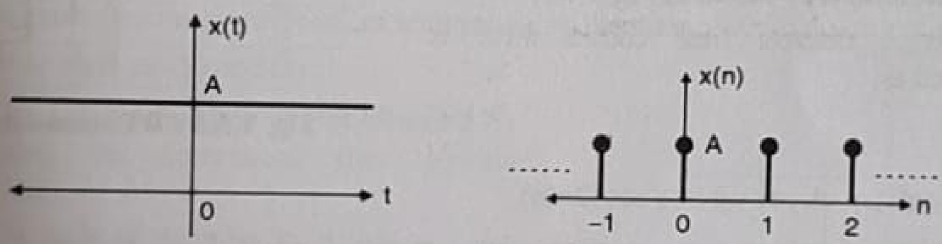
  Let us understand them one-by-one.

1.5.1 DC Signal :

Continuous time DC signal :

- A dc signal is as shown in Fig. 1.5.1(a). As seen from the waveform the amplitude "A" of a direct current (dc) signal remains constant independent of time.
- The dc signal can be represented mathematically as follows :

$$\text{A dc signal} : x(t) = A \qquad -\infty < t < \infty \qquad ...(1.5.1)$$



(a) A CT dc signal **(b) A discrete time dc signal**

Fig. 1.5.1

Discrete time DC signal :

- Fig. 1.5.1(b) shows the discrete time dc signal. It is a sequence of samples each of amplitude A and extending from $-\infty < n < \infty$.
- This signal can be mathematically represented as follows :

$$x(n) = A \qquad ... -\infty < n < \infty$$

- Or it can be represented in the infinite sequence form as follows :

$$x(n) = \{...... A, A, \underset{\uparrow}{A}, A, A, ...\}$$

1.5.2 Sinusoidal Signals :

C.T. sinusoidal signals :

- The sinusoidal signals include sine and cosine signals. They are as shown in the Fig. 1.5.2.

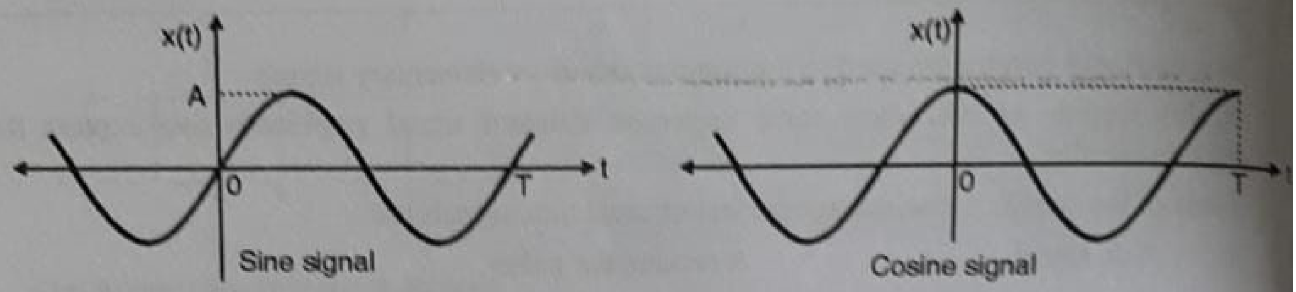


Fig. 1.5.2 : Sinusoidal signals

- Mathematically they can be represented as follows :

A sine signal : $x(t) = A \sin \omega t = A \sin (2 \pi f t)$ ...(1.

A cosine signal : $x(t) = A \cos \omega t = A \cos (2 \pi f t)$ ...(1.

Discrete time sinusoidal wave :

- A discrete time sinusoidal waveform is denoted by,

$$x(n) = A \sin \omega n = A \sin (2\pi f n)$$

Here A = Amplitude

ω = Angular Frequency = $2 \pi f$

- This waveform is as shown in Fig. 1.5.3.
- Similarly a discrete time cosine wave is expressed as,

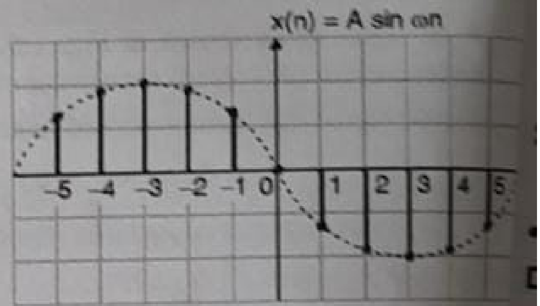


Fig. 1.5.3 : DT sinusoidal wavefor

$$x(n) = A \cos \omega n = A \cos (2\pi f n)$$

1.5.3 Unit Step Signal :

C.T. unit step signal :

- The unit step signal is as shown in Fig. 1.5.4. It has a constant amplitude of unity(1) for the zero or positive values of time "t". Whereas it has a zero value for negative values of t.
- The unit step signal is mathematically represented as,

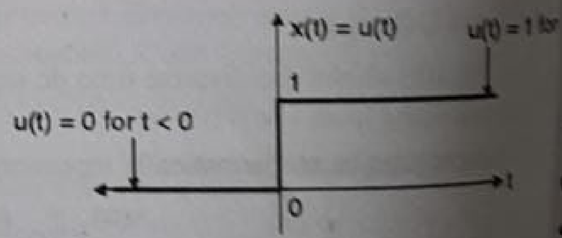


Fig. 1.5.4 : Unit step signal

Unit step signal : $u(t) = 1$ For $t \geq 0$

$= 0$ For $t < 0$ ...(1

A D.T. unit step signal :

- A discrete time unit step signal is denoted by u(n). Its value is unity(1) for all positive valu n. That means its value is one for $n \geq 0$. While for other values of n, its value is zero.

$$\therefore \quad u(n) = \begin{cases} 1 & \text{For } n \geq 0 \\ 0 & \text{For } n < 0 \end{cases}$$

- In the form of sequence it can be written as,

$$u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, ...\}$$

- This signal is graphically represented in Fig. 1.5.5.

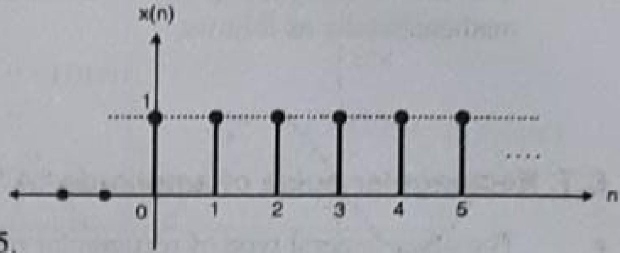


Fig. 1.5.5 : A D.T. unit step signal

1.5.4 Signum Function :

PU : Dec. 06

University Questions

Q. 1 Express signum function in terms of unit step function, and find the even part and odd part of unit step function. **(Dec. 06, 4 Marks)**

- The signum function is as shown in the Fig. 1.5.6.
- It is represented mathematically as follows :

$$\text{Signum function : sgn}(t) = 1 \qquad \text{For } t > 0$$
$$= -1 \qquad \text{For } t < 0 \quad ...(1.5.5)$$

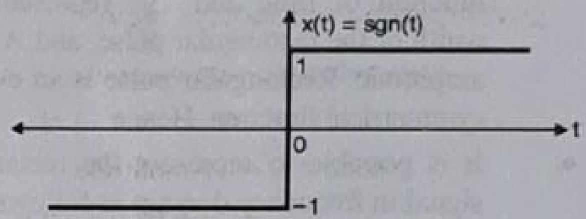


Fig. 1.5.6 : Signum function

- The signum function is an "odd" or antisymmetric function.

Discrete time signum function :

- A D.T. signum function can be obtained by sampling the continuous time signum function.
- It is a train of samples of values + 1 for positive "n" and – 1 for negative "n" as shown in Fig. 1.5.7.

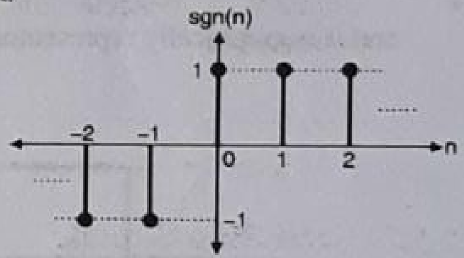


Fig. 1.5.7 : D.T. signum function

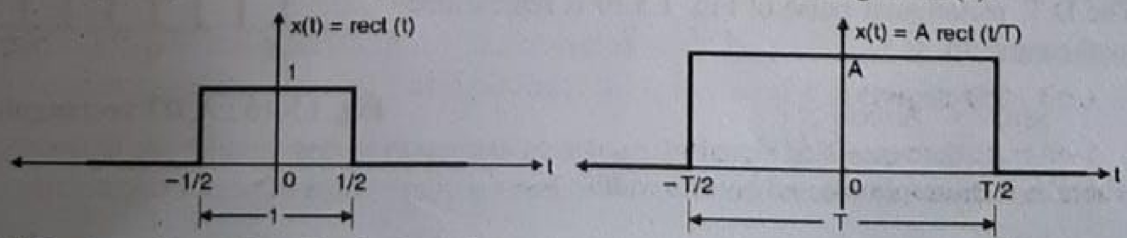
- The DT signum function is defined mathematically as follows :

$$x(n) = \text{sgn}(n) = 1 \qquad n \geq 0$$
$$= -1 \qquad n < 0 \quad ...(1.5.5(a))$$

1.5.5 Rectangular Signals :

C.T. Rectangular pulse of unit Amplitude and Unit duration :

- A rectangular pulse of unit amplitude and duration is as shown in Fig. 1.5.8(a).



(a) Rectangular pulse of unit duration and unit amplitude

(b) Rectangular pulse of duration T and amplitude A

Fig. 1.5.8

- The unit rectangular pulse is centered about the y-axis i.e. about t = 0. It is represente mathematically as follows,

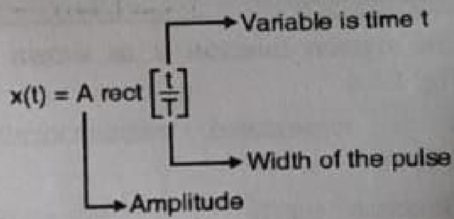
$$\text{rect}(t) = 1 \qquad -1/2 \le t \le 1/2$$
$$= 0 \qquad \text{Elsewhere} \qquad ...(1.5.6$$

C.T. Rectangular pulse of amplitude "A" and duration "T" :

- The other general type of rectangular pulse having an amplitude of "A" over a duration of "T" as shown in Fig. 1.5.8(b).
- This pulse also is centered about t = 0. It is mathematically represented as,

$$A\,\text{rect}\left[\frac{t}{T}\right] = A \qquad -T/2 \le t \le T/2$$
$$= 0 \qquad \text{Elsewhere} \qquad ...(1.5.$$

- In this expression, "t" shows that it is a function of time and "T" represents the width of the rectangular pulse, and A is the amplitude. Rectangular pulse is an even or symmetrical function. Hence



- It is possible to represent the rectangular signal in frequency domain as follows :

$$x(f) = A\,\text{rect}\left[\frac{f}{2W}\right]$$

and it is graphically represented as shown in Fig. 1.5.9.

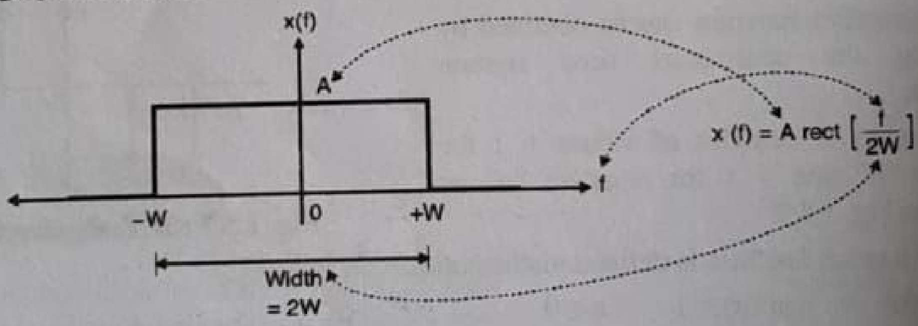


Fig. 1.5.9

D.T. rectangular pulse :

- Fig. 1.5.10 shows the discrete time rectangular pulse.
- The D.T. rectangular pulse of Fig. 1.5.10 is represented mathematically as :

$$x(n) = A\,\text{rect}\left[\frac{n}{N}\right]$$

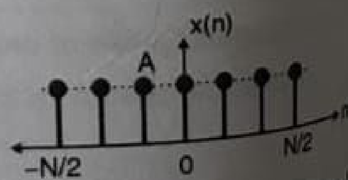


Fig. 1.5.10 : A DT rectangular pulse

where A is its amplitude and N is its width.

$$A\,\text{rect}\left[\frac{n}{N}\right] = A \qquad ..... -N/2 \le n \le N/2$$
$$= 0 \qquad ..... \text{elsewhere}$$

1.5.6 Delta or Unit Impulse Function [δ (t)] :

- The delta function δ (t) is an extremely important function used for the analysis of communication systems. The impulse response of a system is its response to a delta function applied at the input.

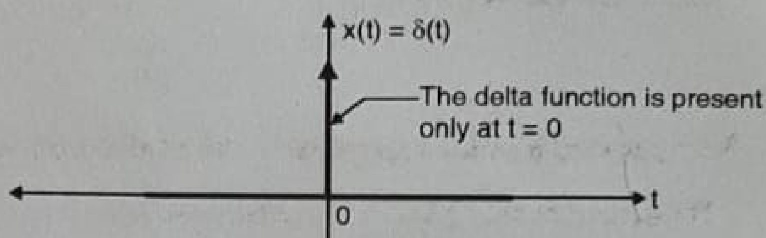


Fig. 1.5.11 : Delta function

- The delta function is as shown in Fig. 1.5.11. It is present only at t = 0, its width tends to 0 and its amplitude at t = 0 is infinitely large so that the area under the pulse is unity (i.e. 1). Due to unity area, it is called as a unit impulse function.

$$\text{Delta function : } \delta(t) = 0 \quad \text{For } t \neq 0 \qquad \text{OR} \quad \delta(t) \neq 0 \quad \text{For } t = 0 \qquad ...(1.5.8)$$

The area under the unit impulse is given as,

$$\text{Area under unit impulse : } \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad ...(1.5.9)$$

1.5.6.1 Important Properties of a Delta Function :

Here let us understand two important properties of the delta function. They are :

1. Sifting property 2. Replication property.

- The sifting property of delta function states that,

$$\textbf{Sifting property : } \int_{-\infty}^{\infty} x(t)\, \delta(t - t_m)\, dt = x(t_m) \qquad ...(1.5.10)$$

- where $\delta(t - t_m)$ represents the time shifted delta function. This delta function is present only at $t = t_m$. The RHS of Equation (1.5.10) represents the value of x (t) at $t = t_m$.
- This result indicates that the area under the product of a function with an impulse δ (t) is equal to the value of that function at the instant where the unit impulse is located.
- This property is also known as the **sampling property**.
- The other property of delta function is the replication property. It states that,

$$\textbf{Replication property : } x(t) * \delta(t) = x(t) \qquad ...(1.5.11)$$

- This property can be stated in words as : The convolution of any function x (t) with delta function yields the same function. The sign * in Equation (1.5.11) represents "convolution".
- The process of convolution and its properties have been discussed in section 1.7.

Ex. 1.5.1 : Write the properties of impulse signal $\delta(t)$ and evaluate the following using $\delta(t)$ properties for given x(t) shown in Fig. P. 1.5.1(a) ahead :

1. $\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt$
2. $\int_{-\infty}^{+\infty} x(t-1)\,\delta(t-1)\,dt$
3. $\int_{-\infty}^{+\infty} x(t)\,\delta(4t)\,dt$
4. $x(t)\cdot \sin t\cdot \delta(t)$
5. $\sin[\,x(t)\,\delta(t)\,]$.

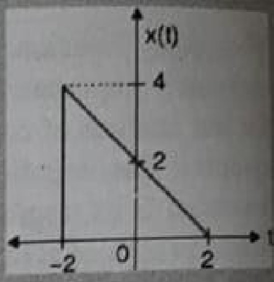


Fig. P. 1.5.1(a)

Dec. 09, 10 M

Soln. :

1. According to sifting property,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_o)\,dt = x(t_o)$$

Let $\quad t_o = 0$

$$\therefore \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0) = 2.$$

2. Let $x'(t) = x(t-1)$. It is shown in Fig. P. 1.5.1(b).

Now $\quad \int_{-\infty}^{\infty} x'(t)\,\delta(t-1)\,dt = x'(1) = 2$

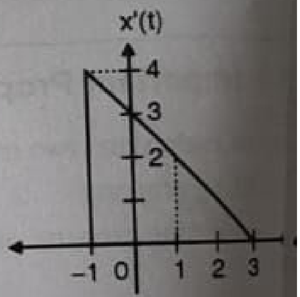


Fig. P. 1.5.1(b)

3. According to scaling property,

$$\delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t)$$

$$\therefore \delta(4t) = \frac{1}{4}\,\delta(t)$$

$$\therefore \int_{-\infty}^{\infty} x(t)\,\delta(4t)\,dt = \int_{-\infty}^{\infty} x(t)\cdot\left[\frac{1}{4}\,\delta(t)\right] dt$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} x(t)\,\delta(t)\cdot dt \qquad \text{Let } t_o = 0$$

Using shifting property,

$$\int_{-\infty}^{\infty} x(t)\,\delta(4t)\,dt = \frac{1}{4}\,x(t_o) = \frac{1}{4}\times 2 = \frac{1}{2}$$

4. Consider the term $\sin t\cdot\delta(t)$

According to product property,

$$y(t)\cdot\delta(t) = y(0)\,\delta(t)$$

$$\therefore \sin t\cdot\delta(t) = \sin(0)\cdot\delta(t) = 0\times\delta(t) = 0$$

$$\therefore \quad x(t) \cdot \sin t \cdot \delta(t) = 0$$

5. According to product property,

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t) = 2$$

$$\therefore \quad \sin[x(t) \cdot \delta(t)] = \sin(2) = 0.0348°$$

1.5.7 Unit Sample Signal δ (n) :

PU : May 12

University Questions

Q. 1. Define unit impulse function and write its relation with unit step in CT and DT. **(May 12, 5 Marks)**

- The D.T. version of unit impulse signal is the unit sample signal.
- A discrete time unit impulse function is denoted by δ (n). Its amplitude is 1 at n = 0 and for all other values of n; its amplitude is zero.

$$\therefore \quad \delta(n) = \begin{cases} 1 & \text{For } n = 0 \\ 0 & \text{For } n \neq 0 \end{cases}$$

- In the sequence form it can be represented as,

$$\delta(n) = \{\ldots, 0, 0, 0, \underset{\uparrow}{1}, 0, 0, 0\}$$

$$\text{or} \quad \delta(n) = \{ 1 \}$$

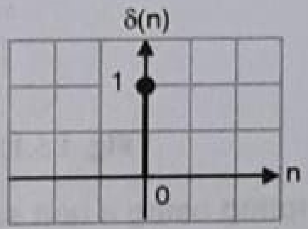


Fig. 1.5.12 : Unit sample signal δ(n)

- The graphical representation of unit sample signal is as shown in Fig. 1.5.12.

Relation between D.T. unit impulse and D.T. unit step signals :

- Fig. 1.5.12(a) shows how to obtain a D.T. unit impulse signal δ [n] from a D.T. unit step signal u [n].
- Mathematically this relation can be expressed as,

$$\delta[n] = u[n] - u[n-1] \qquad \ldots(1.5.11(a))$$

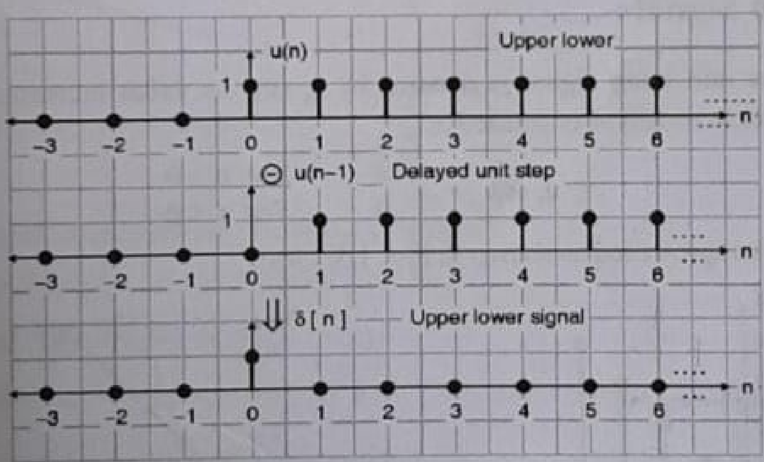


Fig. 1.5.12(a) : How to obtain a unit sample signal from a unit step signal

- We can obtain a D.T. unit step signal u[n] by taking the sum of unit samples. This is mathematically expressed as,

$$u[n] = \ldots + \delta[n] + \delta[n-1] + \delta[n-2] + \ldots$$

$$= \sum_{k=0}^{\infty} \delta[n-k] \qquad \ldots(1.5.11(b))$$

- The summation of unit impulses to obtain a unit step function is shown in Fig. 1.5.12(b).

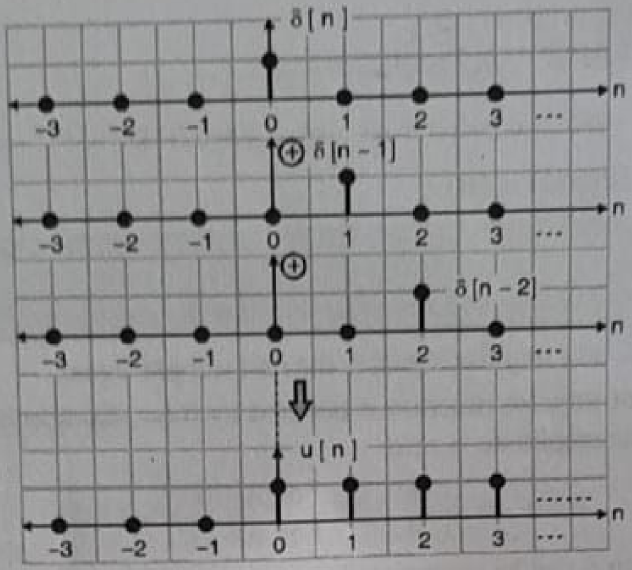


Fig. 1.5.12(b) : How to obtain a D.T. unit step from unit impulses

Sampling using a unit sample signal :

- The unit sample sequence can be used to sample the value of a signal.
- If signal x (n) is multiplied by a unit sample δ (n), then we get the value of x (n) at n = 0 as the product. That means,

$$x(n)\,\delta(n) = x(0) \qquad \text{...(1.5.11(a))}$$

- This happens because δ (n) = 1 only at n = 0. Similarly we can write that,

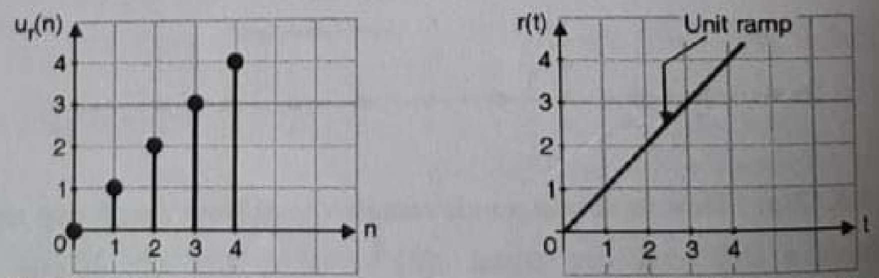
$$x(n)\,\delta(n-n_0) = x(n_0) \qquad \text{...(1.5.11(b))}$$

1.5.8 Unit Ramp Signal :

- A discrete time unit ramp signal is denoted by $u_r(n)$. Its value increases linearly with sample number n. Mathematically it is defined as,

$$u_r(n) = \begin{cases} n & \text{For } n \geq 0 \\ 0 & \text{For } n < 0 \end{cases}$$

- Graphically it is represented as shown in Fig. 1.5.13(a).



(a) Unit ramp signal $u_r(n)$ **(b) A CT ramp signal r(t)**

Fig. 1.5.13

- A continuous time ramp signal is denoted by r(t). Mathematically it is expressed as,

$$r(t) = \begin{cases} t & \text{For } t \geq 0 \\ 0 & \text{For } t < 0 \end{cases}$$

It is as shown in Fig. 1.5.13(b).

1.5.9 C.T. Complex Exponential Signals :

- The continuous time (C.T.) complex exponential signal is of the following form,

$$x(t) = C e^{\alpha t} \quad ...(1.5.12)$$

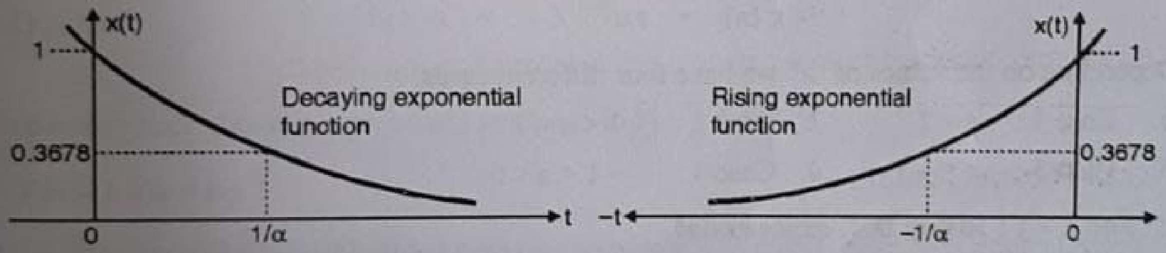
- Here C and α are in general complex numbers. Depending on the values of these parameters, the complex exponent can have several different characteristics.

Types of complex exponential signal :

- Depending on the type of C and α, the complex exponential signal can be classified as follows :
  1. Real exponential signals
  2. Periodic complex exponential signals

1. CT real exponential signals :

- If C and α both are real, then the corresponding exponential signal is called as the real exponential signal.
- The exponential functions also are used extensively in the signal analysis. There are two types of exponential functions viz., rising and decaying exponential functions as shown in the Fig. 1.5.14.



(a) Decaying exponential function **(b) Rising exponential function**

Fig. 1.5.14

- They are mathematically represented as follows :
  1. Decaying exponential function : $x(t) = e^{-\alpha t}$ ...(1.5.13)
  2. Rising exponential function : $x(t) = e^{\alpha t}$ ...(1.5.14)
- Note that we have assumed C = 1 for both the equations stated above.

2. CT periodic complex exponential signals :

- This is the second important type of complex exponential signals. For this type, α is assumed to be purely imaginary.
- Such an exponential is mathematically expressed as,

$$x(t) = e^{j\omega_0 t} \quad ...(1.5.15)$$

- The most important property of this signal is that it is a periodic signal. Applying the condition of periodicity we can write that,

- The summation of unit impulses to obtain a unit step function is shown in Fig. 1.5.12(b).

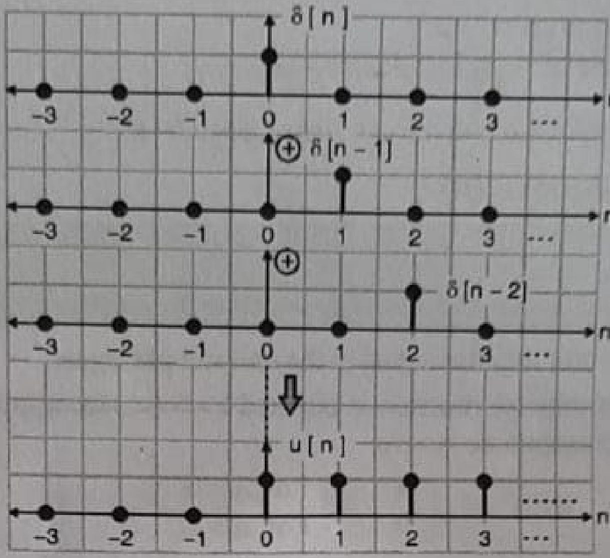


Fig. 1.5.12(b) : How to obtain a D.T. unit step from unit impulses

Sampling using a unit sample signal :

- The unit sample sequence can be used to sample the value of a signal.
- If signal x (n) is multiplied by a unit sample δ (n), then we get the value of x (n) at n = product. That means,

$$x(n)\,\delta(n) = x(0) \qquad ...(1.$$

- This happens because δ (n) = 1 only at n = 0. Similarly we can write that,

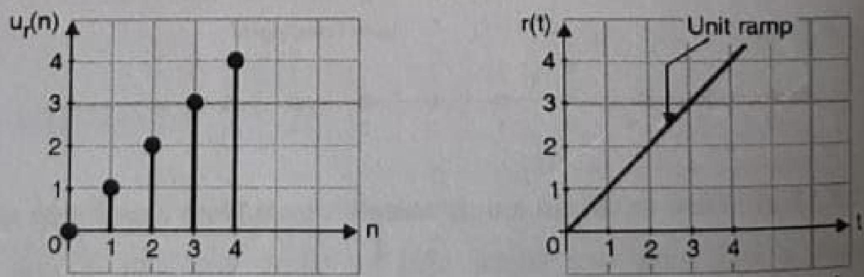
$$x(n)\,\delta(n-n_0) = x(n_0) \qquad ...(1.$$

1.5.8 Unit Ramp Signal :

- A discrete time unit ramp signal is denoted by u_r (n). Its value increases linearly with number n. Mathematically it is defined as,

$$u_r(n) = \begin{cases} n & \text{For } n \geq 0 \\ 0 & \text{For } n < 0 \end{cases}$$

- Graphically it is represented as shown in Fig. 1.5.13(a).



(a) Unit ramp signal u_r (n) **(b) A CT ramp signal r(t)**

Fig. 1.5.13

- A continuous time ramp signal is denoted by r(t). Mathematically it is expressed as,

$$r(t) = \begin{cases} t & \text{For } t \geq 0 \\ 0 & \text{For } t < 0 \end{cases}$$

It is as shown in Fig. 1.5.13(b).

1.5.9 C.T. Complex Exponential Signals :

- The continuous time (C.T.) complex exponential signal is of the following form,

$$x(t) = C e^{\alpha t} \quad ...(1.5.12)$$

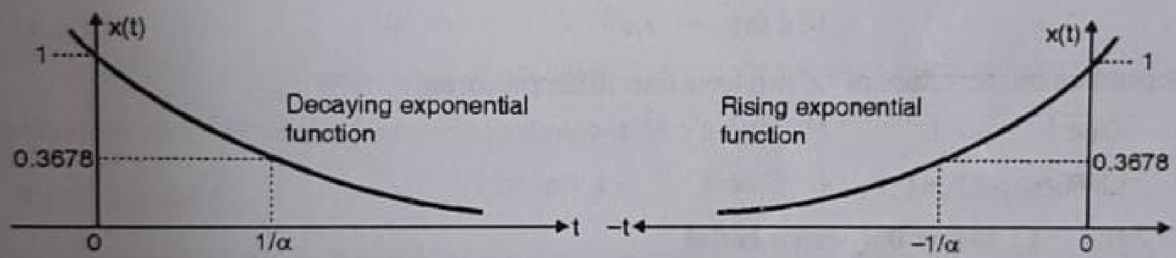
- Here C and α are in general complex numbers. Depending on the values of these parameters, the complex exponent can have several different characteristics.

Types of complex exponential signal :

- Depending on the type of C and α, the complex exponential signal can be classified as follows :
  1. Real exponential signals
  2. Periodic complex exponential signals

1. CT real exponential signals :

- If C and α both are real, then the corresponding exponential signal is called as the real exponential signal.
- The exponential functions also are used extensively in the signal analysis. There are two types of exponential functions viz., rising and decaying exponential functions as shown in the Fig. 1.5.14.



(a) Decaying exponential function (b) Rising exponential function

Fig. 1.5.14

- They are mathematically represented as follows :
  1. Decaying exponential function : $x(t) = e^{-\alpha t}$ ...(1.5.13)
  2. Rising exponential function : $x(t) = e^{\alpha t}$ ...(1.5.14)
- Note that we have assumed C = 1 for both the equations stated above.

2. CT periodic complex exponential signals :

- This is the second important type of complex exponential signals. For this type, α is assumed to be purely imaginary.
- Such an exponential is mathematically expressed as,

$$x(t) = e^{j\omega_0 t} \quad ...(1.5.15)$$

- The most important property of this signal is that it is a periodic signal. Applying the condition of periodicity we can write that,

$$x(t) = e^{j\omega_o t} = e^{j\omega_o(t+T)}$$

$$e^{j\omega_o t} = e^{j\omega_o t} \cdot e^{j\omega_o T} \quad ...(1.5.1$$

- The above expression will be true if and only if,

$$e^{j\omega_o T} = 1$$

- So the conclusion is that for $e^{j\omega_o t}$ to be periodic, $e^{j\omega_o T}$ has to be equal to 1.
- If $\omega_o = 0$, then $x(t) = 1$ for any value of T. If $\omega_o \neq 0$, then the fundamental period T_o o (t) that is the smallest positive value of T for which $e^{j\omega_o T} = 1$ is

$$T_o = \frac{2\pi}{|\omega_o|} \quad ...(1.5.$$

- Thus the signals $e^{j\omega_o t}$ and $e^{-j\omega_o t}$ will have the same fundamental period of T_o.

1.5.10 Discrete Time exponential signals :

- A discrete time exponential signal is expressed as,

$$x(n) = a^n \quad ...(1.5.$$

Here 'a' is some real constant.

- If 'a' is the complex number then x (n) is written as,

$$x(n) = r e^{j\theta} \quad ...(1.5.$$

- Depending on the values of "a" we have four different cases,

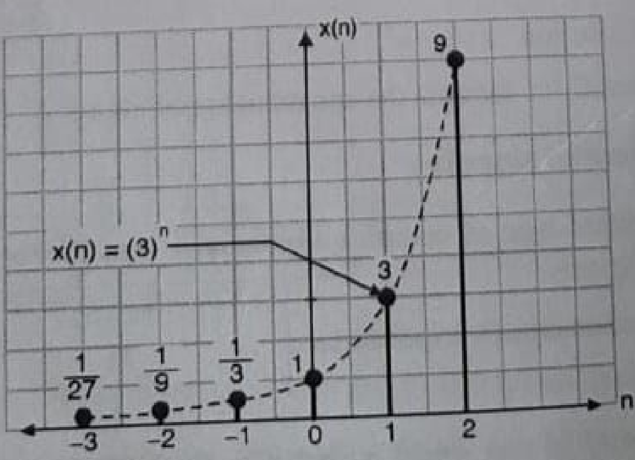
1. Case 1 : $a > 1$ 2. Case 2 : $0 < a < 1$
3. Case 3 : $a < -1$ 4. Case 4 : $-1 < a < 0$

Case 1 : For a > 1 : Rising D.T. exponential

- Let a = 3. Thus we have,

$$x(n) = a^n = 3^n$$

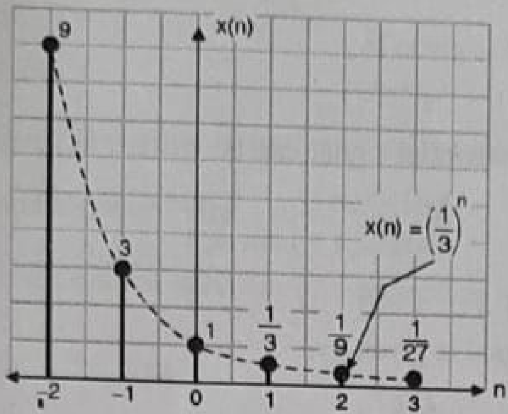
- Graphically such signal is represented as shown in Fig. 1.5.15(a).
- Since the signal is exponentially growing; it is called as rising exponential signal.



n	3^n
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9

Fig. 1.5.15(a) : Rising exponential signal (a > 1)

Case 2 : For $0 < a < 1$: Decaying exponential signal



n	$(1/3)^n$
-3	27
-2	9
-1	3
0	1
1	1/3
2	1/9
3	1/27

Fig. 1.5.15(b) : Decaying exponential signal

- In this case we will get decaying exponential sequence. Let $a = \frac{1}{3}$

$$\therefore \quad x(n) = a^n = \left(\frac{1}{3}\right)^n .$$

Graphically such signal is represented as shown in Fig. 1.5.15(b).

Case 3 : For $a < -1$:

- In this case we will get double sided rising exponential signal.

$$\text{Let } a = -3$$

$$\therefore \quad x(n) = (-3)^n$$

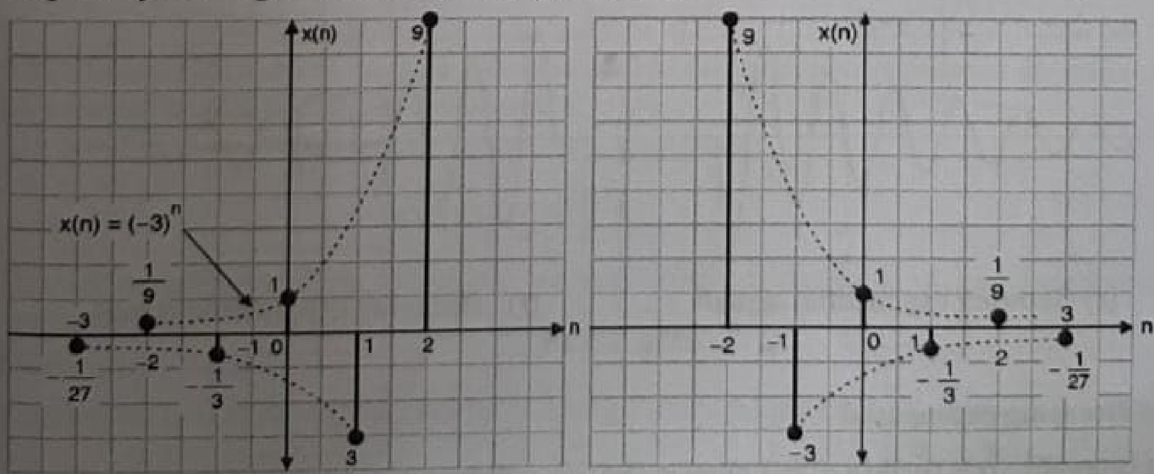
Graphically such signal is represented as shown in Fig. 1.5.15(c).

Case 4 : For $-1 < a < 0$:

In this case we will get double sided decaying exponential signal.

$$\text{Let } a = -\frac{1}{3} \qquad \therefore \; x(n) = a^n = \left(-\frac{1}{3}\right)^n$$

Graphically such signal is as shown in Fig. 1.5.15(d).



(c) Double sided growing exponential signal **(d) Double sided decaying exponential signal**

Fig. 1.5.15

Relation between the complex exponential and sinusoidal signals :

- The C.T. complex exponential signal is given by,

$$x(t) = e^{j\omega_0 t} \quad ...(1.5.$$

- By Euler's relation, the complex exponential signal can be written in terms of sinusoidal signals as,

$$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t \quad ...(1.5$$

General complex exponential signal :

- The general complex exponential signal is given by,

$$x(t) = C e^{\alpha t}$$

- If we express C in the polar form and α in the rectangular form, then

$$C = |C| e^{j\theta}$$

$$\text{and} \quad \alpha = r + j\omega_0$$

- Substituting we get

$$C e^{\alpha t} = |C| e^{j\theta} e^{(r+j\omega_0)t}$$

$$= |C| e^{rt} e^{j(\omega_0 t + \theta)} \quad ...(1.5$$

- Using the Euler's expression, we can expand this equation as follows :

$$C e^{\alpha t} = |C| e^{rt} \cos(\omega_0 t + \theta) + j |C| e^{rt} \sin(\omega_0 t + \theta) \quad ...(1.5$$

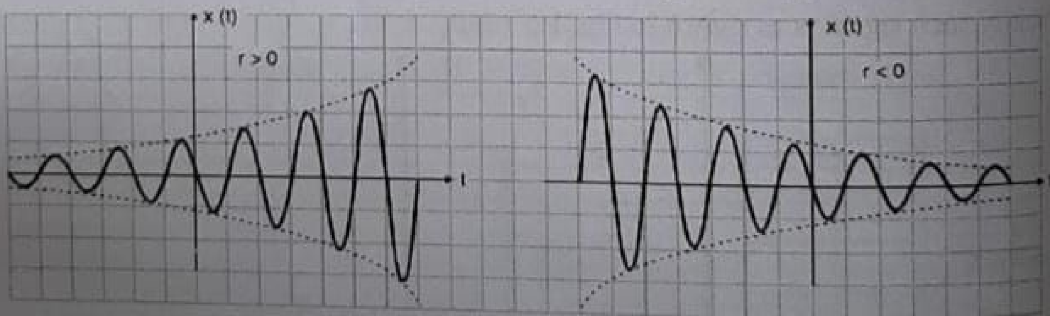
- If $r = 0$ then $e^{rt} = 1$

$$\therefore \; C e^{\alpha t} = |C| \cos(\omega_0 t + \theta) + j |C| \sin(\omega_0 t + \theta) \quad ...(1.5$$

- Thus for $r = 0$, the real and imaginary parts of the complex exponential are sinusoidal.

For $r > 0$: Growing exponential

- If $r > 0$, then e^{rt} in Equation (1.5.23) will be a growing exponential signal.
- So in Equation (1.5.24), the cosine and sine terms are multiplied by a growing exponential signal.
- Hence we get a growing sinusoidal signal as shown in Fig. 1.5.15(e).



(e) Growing exponential signal (f) Decaying exponential signal

Fig. 1.5.15

For $r < 0$: Decaying exponential

- If r is negative ($r < 0$), then e^{rt} in Equation (1.5.23) will be a decaying exponential.
- Hence each sinusoidal signal in Equation (1.5.24) is multiplied by a decaying exponential. So we get a decaying sinusoidal signal as shown in Fig. 1.5.15(f).

Damped sinusoids : Sinusoidal signals, multiplied by decaying exponentials are called as damped sinusoids.

1.5.11 Sinc Function :

- The sinc function or sinc pulse is mathematically expressed as,

$$\text{Sinc function : } \operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{... For } x \neq 0 \quad \text{...(1.5.25)}$$

where x is the independent variable. The procedure of plotting the sinc function is explained in Ex. 1.5.1.

- It is proved in Ex. 1.5.1, that

$$\operatorname{sinc}(x) = 1 \quad \text{... At } x = 0$$

$$\text{and} \quad \operatorname{sinc}(x) = 0 \quad \text{... At } x = \pm 1, \pm 2, \pm 3 \text{ ....}$$

- Hence the graphical representation of a sinc function is as shown in Fig. 1.5.16.

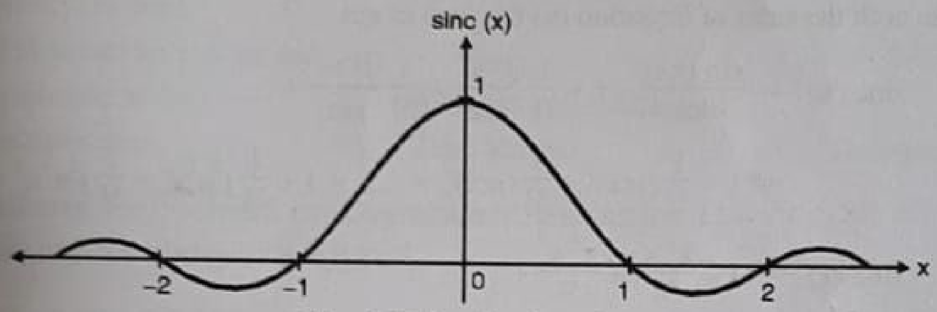


Fig. 1.5.16 : sinc function

- Fig. 1.5.16 shows that sinc function has the shape of a sinewave.
- Its magnitude goes on decreasing as the value of |x| increases.
- sinc $x \rightarrow 0$ when $|x| \rightarrow \infty$.

Ex. 1.5.2 : Plot graphically the sinc function.

Soln. :

- The sinc function is mathematically represented as,

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{... For } x \neq 0 \quad \text{...(1)}$$

- Here x is an independent variable. We have to find the value of sinc function for different values of x including zero and negative values of x.

$$\text{Substitute } x = 0 \text{ in Equation (1)}$$

$$\operatorname{sinc}(x) = \frac{\sin 0}{0}.$$

- This is an indefinite form. Therefore we must find out the value of sinc function at x = 0 separately.

To find sinc(x) at x = 0 :

- Express sin (πx) in the exponential form. Using Euler's theorem we can express sin (πx) as,

$$\sin(\pi x) = \frac{e^{j\pi x} - e^{-j\pi x}}{2j} \quad \text{...(2)}$$

- To solve Equation (2), let us take help of the standard exponential series. It is given as,

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \ldots$$

Substitute $t = j\pi x$ to get,

$$\therefore \; e^{(j\pi x)} = 1 + j\pi x + \frac{(j\pi x)^2}{2!} + \frac{(j\pi x)^3}{3!} + \frac{(j\pi x)^4}{4!} + \ldots$$

$$\text{Similarly, } e^{(-j\pi x)} = 1 - j\pi x + \frac{(-j\pi x)^2}{2!} + \frac{(-j\pi x)^3}{3!} + \frac{(-j\pi x)^4}{4!} + \ldots$$

- Subtracting Equation (4) from Equation (3), we get,

$$e^{(j\pi x)} - e^{(-j\pi x)} = 2\,j\pi x + \frac{2}{3\,!}(j\pi x)^3 + \frac{2}{5\,!}(j\pi x)^5 + \ldots$$

- Divide both the sides of Equation (5) by "2j" to get,

$$\sin(\pi x) = \frac{e^{(j\pi x)} - e^{(-j\pi x)}}{2j} = \pi xt + \frac{(j\pi x)^3}{j3!} + \frac{(j\pi x)^5}{j5!} + \ldots$$

- Now divide both the sides of Equation (6) by (πx) to get,

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = 1 + \frac{1}{3!}\frac{(j\pi x)^3}{j\pi x} + \frac{1}{5!}\frac{(j\pi x)^5}{j\pi x} + \ldots$$

$$= 1 + \frac{1}{3!}(j\pi x)^2 + \frac{1}{5!}(j\pi x)^4 + \ldots = 1 + \frac{1}{3!}j^2\pi^2x^2 + \frac{1}{5!}j^4\pi^4x^4 + \ldots$$

$$\therefore \; \text{sinc}(x) = 1 - \frac{\pi^2 x^2}{3!} + \frac{\pi^4 x^4}{5!} + \ldots$$

- Now substitute $x = 0$ in the Equation (7), to get,

$$\text{sinc}(0) = 1$$

Values of sinc(x) at other values of x :

At other values of x, find out sinc(x) using the Equation (1).

Table P. 1.5.2

x	sinc (x)	x	sinc (x)
0.25	0.9	– 0.25	0.9
0.50	0.6366	– 0.5	0.6366
0.75	0.3	– 0.75	0.3
1.5	– 0.2122	– 1.5	– 0.2122
2.5	0.1273	– 2.5	– 0.1273

By plotting these values, we get the waveform for sinc(x) as shown in Fig. P. 1.5.2.

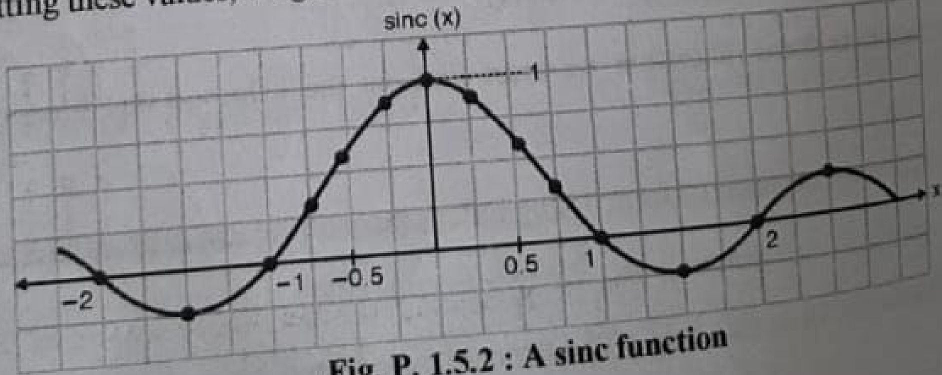


Fig. P. 1.5.2 : A sinc function

Conclusion from the Fig. P. 1.5.2 :

The sinc function passes through zero at the multiple values of x i.e. at $x = \pm 1, \pm 2, \pm 3$..... This is because $\sin(\pi x) = 0$ for multiple values of x and the amplitude of the "sinc" function decreases with increase in the magnitude of x.

1.6 Operations on a Signal :

- The independent variable for a C.T. signal is time "t" and for a D.T. signal is "n".
- The transformation of the signal is the central (important) concept in signal and system analysis.
- Transformation of independent variable involves simple modifications of "t" and "n".
- Some of the important transformations of the independent variable are :
  1. Time shifting 2. Time scaling 3. Folding or time reversal.
- In this section, we are going to discuss these three useful signal operations :
- However the discussion is valid for functions which have independent variables other than time (e.g. frequency) as well.
- Some useful signal operations are :
  1. Amplitude scaling 2. Addition and subtraction
  3. Multiplication 4. Time scaling 5. Time shifting.

1.6.1 Operations Performed on Dependent Variables :

1.6.1.1 Amplitude Scaling of a Signal :

Amplitude scaling of a CT signal :

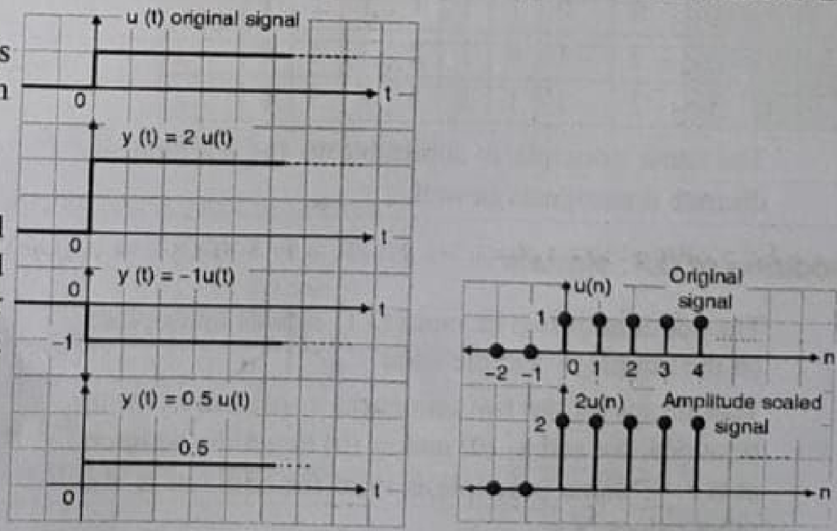
- Amplitude scaling means changing an amplitude of given continuous time signal.
- We will denote continuous time signal by x (t). If it is multiplied by some constant 'A' then the resulting signal is,

$$y(t) = A\,x(t)$$

- y (t) is the amplitude scaled version of x (t) and A is a constant.
- Depending on the value of A we can get different versions of x (t). This is illustrated in Fig. 1.6.1(a).
- Note that the scaling takes place on the y-axis and not on the x-axis (i.e. time axis).

Example :

Let x (t) = u (t) be the original signal. Then the time scaled signal y (t) = A u (t) for different values of A is shown in Fig. 1.6.1(a).



(a) Amplitude scaling (b) Amplitude scaling of DT signal

Fig. 1.6.1

- The same concept is applicable to the discrete time signals as well.

Amplitude scaling of a DT signal :

- Fig. 1.6.1(b) illustrates the amplitude scaling of a DT signal.
- The original signal is given by,

$$y(n) = u(n) = \{..., 0, 0, ..., 1, \underset{\uparrow}{1}, 1, 1, ...\}$$

- The amplitude scaled signal is given by,

$$y(n) = 2u(n) = \{..., 0, 0, ..., \underset{\uparrow}{2}, 2, 2, 2, ...\}$$

1.6.1.2 Addition of Signals :

Addition of CT signals :

- Let the two signals to be added by $x_1(t)$ and $x_2(t)$. Then their addition is given by,

$$y(t) = x_1(t) + x_2(t)$$

- Note that the addition is to be made on instant to instant basis.

Example :

Let $x_1(t) = u(t)$ and $x_2(t) = r(t)$

- Suppose we want to obtain $y(t) = x_1(t) + x_2(t)$ that means, $y(t) = u(t) + r(t)$. This operation is performed as follows :

Here $u(t)$ = Unit step = 1 for $t = 0$ to ∞.

and $r(t)$ = Unit ramp = t for $t = 0$ to ∞.

- The addition is shown in the tabulated form in Table 1.6.1 and graphically in Fig. 1.6.2(a).

Table 1.6.1

t	−2	−1	0	1	2	3	4	5	.....
$x_1(t) = u(t)$	0	0	1	1	1	1	1	1	.....
$x_2(t) = r(t)$	−2	−1	0	1	2	3	4	5	.....
y (t)	−2	−1	1	2	3	4	5	6	.....

- The same principle is applicable to the addition of discrete time signals as well.

Addition of D.T. signals :

- The addition of two or more D.T. signals takes place on the sample by sample basis.
- That means for the two sequences $x_1(n)$ and $x_2(n)$ to be added, we add $x_1(0)$ and $x_2(0)$ to get the addition at $n = 0$. Same principle is used for addition at the other values of n.

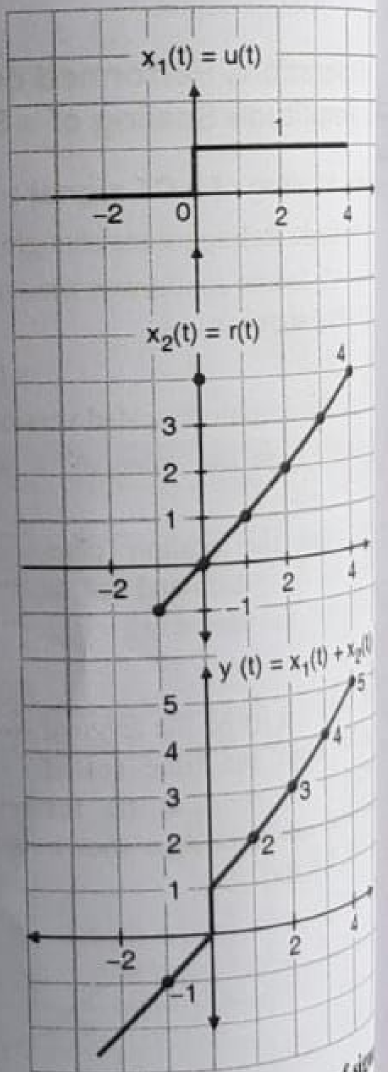


Fig. 1.6.2(a) : Addition of sig...

- Let $x_1(n) = u(n)$ and $x_2(n) = u(n)$. Table 1.6.2 shows their addition in the tabular form while Fig. 1.6.2(b) shows the addition graphically.

Table 1.6.2

n	− 2	− 1	0	1	2	3	4	5	.....
$x_1(n)$	0	0	1	1	1	1	1	1	.....
$x_2(n)$	− 2	− 1	0	1	2	3	4	5	.....
$y(n) = x_1(n) + x_2(n)$	− 2	− 1	1	2	3	4	5	6	.....

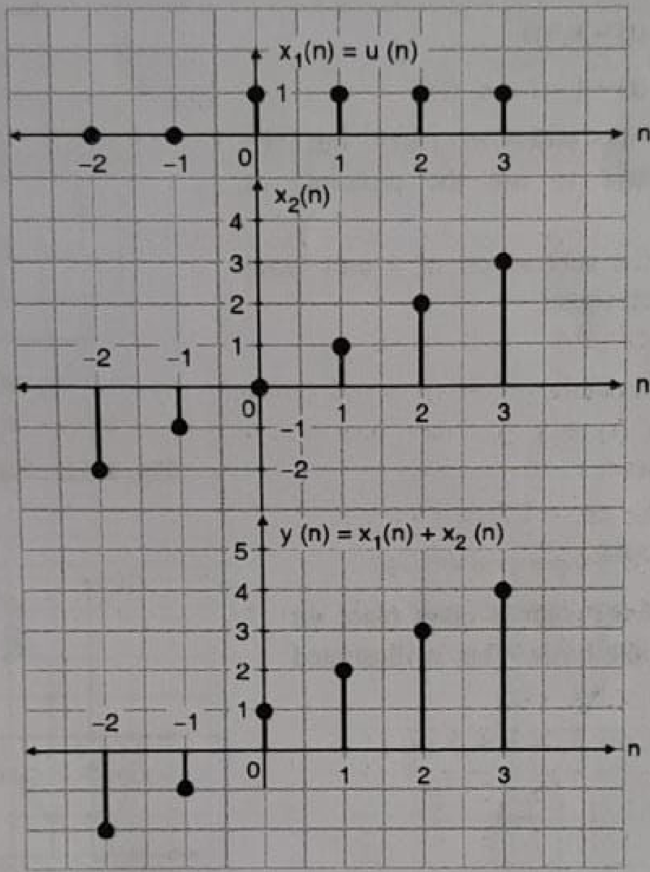


Fig. 1.6.2(b) : Addition of D.T. signals

- This addition can be mathematically represented as follows :

$$x_1(n) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1, \ldots\}$$

$$x_2(n) = \{\ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, 3, 4, \ldots\}$$

- And their addition is

$$y(n) = x_1(n) + x_2(n) = \{\ldots, -2, -1, \underset{\uparrow}{1}, 2, 3, 4, \ldots\}$$

Example :

A physical example of a device that adds two signals is an audio mixer which adds ... sound signal.

1.6.1.3 Subtraction of Signals :

Subtraction of C.T. signals :

- Similarly we can obtain the subtraction of two signals on the instantaneous basis.

$$y(t) = x_1(t) - x_2(t)$$
$$= x_1(t) + [-1 \times x_2(t)]$$

- This shows that in order to carry out the subtraction, we have to use the principle of addition only.
- Fig. 1.6.3 shows the subtraction of a unit ramp signal and a unit step signal.

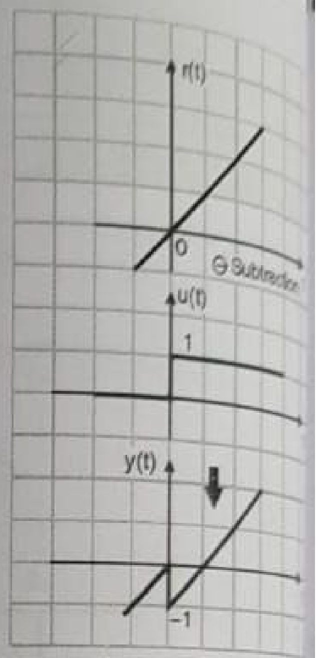


Fig. 1.6.3 : Subtraction of CT s

Subtraction of DT signals :

- The subtraction of DT signals takes place on the sample by sample basis. This is illustrated in Fig. 1.6.4.

$$x_1(n) = \{0, \underset{\uparrow}{0}0, 1, 2, 3, 4, 5 .....\}$$

$$\text{and } x_2(n) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1 ....\}$$

$$\therefore\ x_1(n) - x_2(n) = \{0, \underset{\uparrow}{-1}, 0, 1, 2, 3, 4, ...\}$$

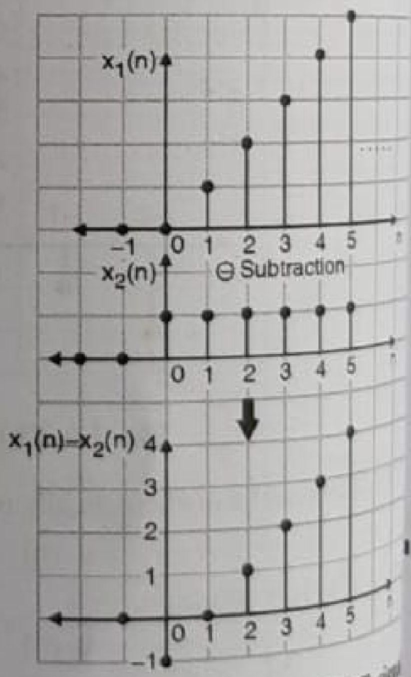


Fig. 1.6.4 : Subtraction of D.T. sig

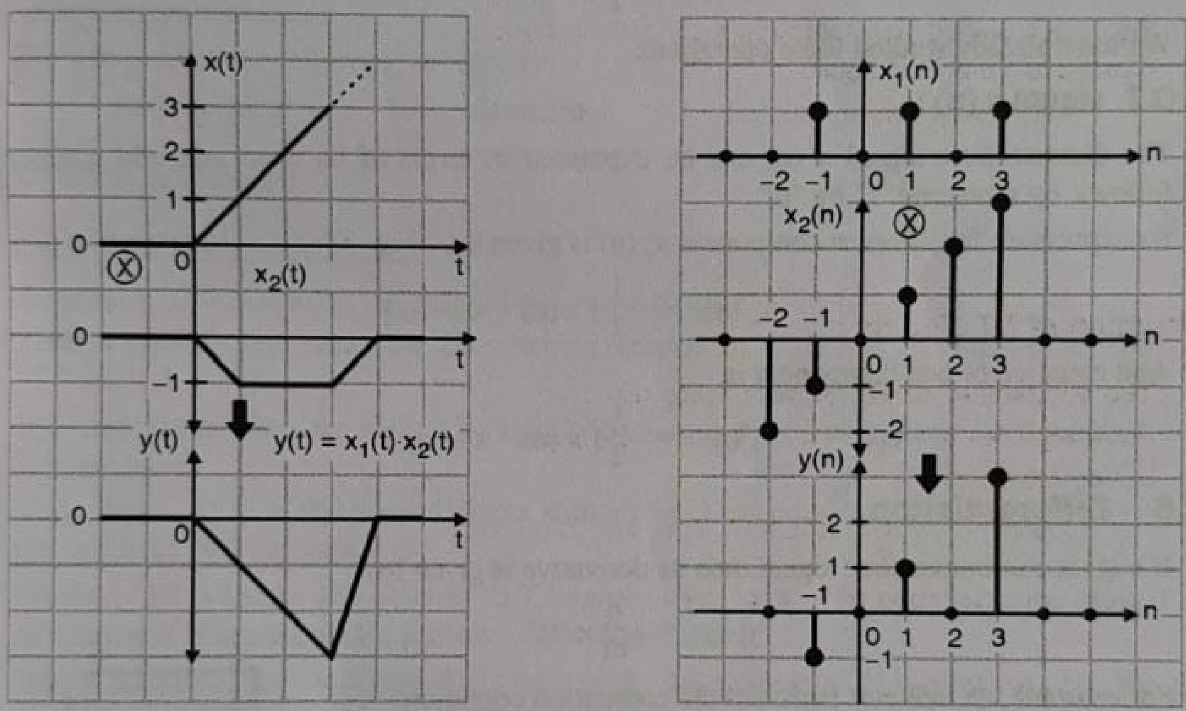
1.6.1.4 Multiplication of Two Signals :

Multiplication of CT Signals :

- If $x_1(t)$ and $x_2(t)$ are two continuous signals then the product of $x_1(t)$ and $x_2(t)$ is,

$$y(t) = x_1(t) \cdot x_2(t)$$

- In this case, the multiplication of amplitudes of two signals take place. For example $y(t) = u(t) \cdot r(t)$.
- Since the amplitude of $u(t)$ is 1; this multiplication will not change the ramp signal $r(t)$. But if we perform, $y(t) = 2u(t) \cdot r(t)$ then it will make the slope of $r(t)$ equal to 2.
- For each prescribed instant of time (t) the value of $y(t)$ is given by the product of corresponding values of $x_1(t)$ and $x_2(t)$ at the same instant.
- Fig. 1.6.5(a) shows the example of multiplication of two CT signals.



(a) Multiplication of CT signals **(b) Multiplication of DT signals**

Fig. 1.6.5 : Multiplication of signals

Multiplication of D.T. Signal :

- Similarly for D.T. signals

$$y(n) = x_1(n) \cdot x_2(n)$$

- Fig. 1.6.5(b) shows the multiplication of two sequences. Note that the multiplication of DT signals is performed on the sample by sample basis.
- In Fig. 1.6.5(b) $x_1(n) = \{0, 1, \underset{\uparrow}{0}, 1, 0, 1\}$ and $x_2(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2, 3\}$

Then their product sequence is given by,

$$y(n) = \{0, -1, \underset{\uparrow}{0}, 1, 0, 3\}$$

Physical example :

A physical example is AM radio signal, in which $x_1(t)$ consists of an audio signal component and $x_2(t)$ is a carrier wave (sine wave).

1.6.1.5 Even and Odd Parts :

For CT signal :

- Even part of signal x (t) is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

- and odd part of x (t) is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- We have already studied these operations.

For a D.T. signal x (n) :

- The discrete time signal x (n) can be expressed in terms of its even and odd components as follows, by replacing "t" by "n".
- So expression for the even component $x_e(n)$ is given by,

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

- And equation of odd component is,

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

1.6.1.6 Differentiation :

- If x (t) is continuous time signal then its derivative is given by,

$$y(t) = \frac{d}{dt}x(t)$$

- For example, an inductor performs differentiation operation.
- Let the current passing through inductor be denoted by i (t) as shown in Fig. 1.6.6.
- Then the voltage across an inductor is,

$$v(t) = L\frac{d}{dt}i(t)$$

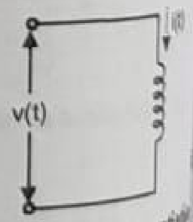


Fig. 1.6.6 : Differentiation

1.6.1.7 Integration :

- Similarly integration of x (t) with respect to time is given by,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

- Here τ is an integration variable.
- For example, a capacitor performs integration operation. Let i (t) be the current passing through capacitor as shown in Fig. 1.6.7.
- Then the voltage developed across the capacitor is,

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau$$

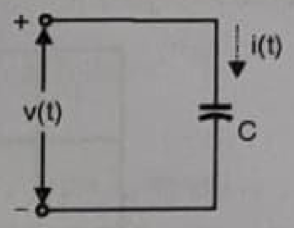


Fig. 1.6.7 : Integration

1.6.1.8 Accumulation for D.T. Signal :

- The integration is basically a process of summation. This process is equivalent to accumulation or summation of sample values over the specified interval of time.

1.6.2 Operations Performed on Independent Variable :

- The independent variable in our discussion is time "t" or "n". So the operations discussed in this section are performed on "t", or "n".
- We will discuss the following operations :

  1. Time shifting 2. Time scaling 3. Folding or time inversion.

1.6.2.1 Time Shifting :

- Let x (t) be the original signal.
- A signal x (t) is said to be "shifted in time" if we replace "t" by (t – T).
- Thus x (t – T) represents the time shifted version of x (t) and the amount of time shift is "T" second.
- If T is positive, then the shift is to right (delay) and if "T" is negative then the shift is to the left (advance).
- Thus x (t – 3) is x (t) delayed (right shifted) by 3 seconds and x (t + 2) is x (t) advanced (left shifted) by 2 seconds.
- Similarly let x (n) be the original D.T. signal. Then x (n – N) represents the shifted version of x (n), and N represents the amount of positional shift.
- It N is positive, then the signal shifts right or get delayed and if N is negative, then the signal is shifted left or it gets advanced.
- Thus x (n – 3) is x (n) delayed (right shifted) by 3 positions whereas x (n + 2) is x (n) advanced (left shifted) by 2 positions.

Time advance of CT Signals :

- Let x (t) = r (t) is the original signal which is present only for $t \geq 0$.

$$\therefore \quad x(t) = r(t) = t \quad \ldots.. \; t \geq 0$$
$$= 0 \quad \ldots. \; t < 0$$

- Then x (t + 2) represents the time advanced (left shifted) version of x (t).
- Table 1.6.3 shows the values of x (t) and x (t + 2) for different values of t and Fig. 1.6.8(a) shows the time advanced version.

Table 1.6.3 : Time advance

	t	-3	-2	-1	0	1	2	3	....
Original	x(t)	-3	-2	-1	0	1	2	3	....
Time advanced	x(t+2)	x(-3+2) = x(-1) = -1	x(-2+2) = x(0) = 0	1	2	3	4	5	....

Time advance by 2

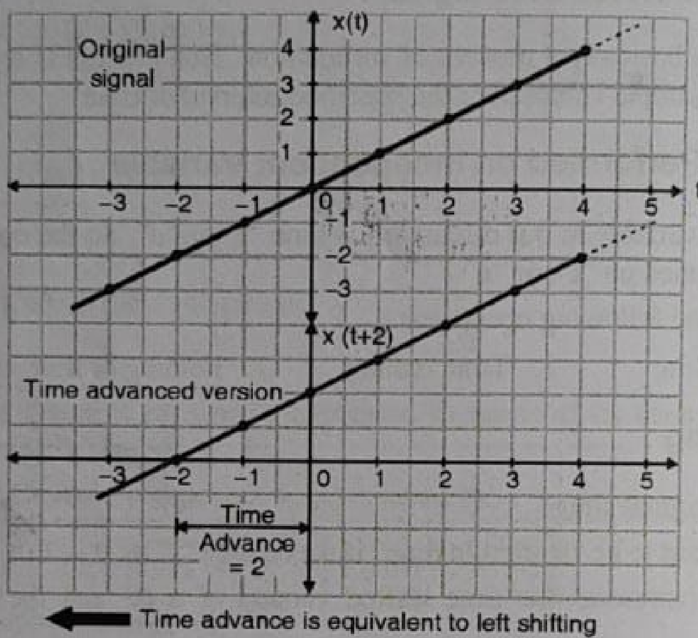


Fig. 1.6.8(a) : Time advancing

Note :	When time advanced the signal is shifted left on the time axis(x-axis). So left shifting and time advance are one and the same.

Time advance for a D.T. signal :

Let x (n) = u (n) be the original signal. Then x (n + 3) = u (n + 3) is the advanced version of u and the left shift is of three positions as shown in Fig. 1.6.8(b).

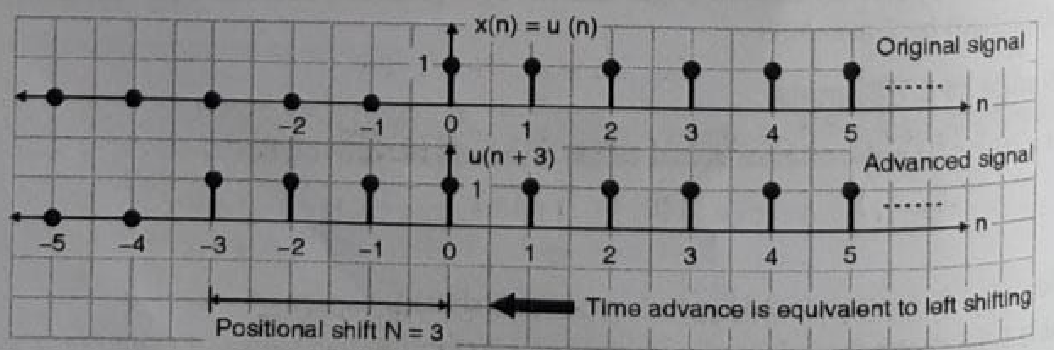


Fig. 1.6.8(b) : Positional advance of D.T. signal

Mathematical representation of shifting operation :

From Fig. 1.6.8(b) we can write the two sequences mathematically as follows :

Original signal : $x(n) = u(n) = \{ \dots 0, 0, 0, 0, \underset{\uparrow}{1}, 1, 1, 1, 1 \dots\dots \}$

Shifted signal : $u(n+3) = \{ \dots 0, \underbrace{1, 1, 1}_{N=3}, \underset{\uparrow}{1}, 1, 1, 1 \dots\dots \}$

Note that the arrow is shifted right by N = 3 positions

Note : The arrow indicating n = 0, has shifted three positions (N = 3) to the right as compared to the original sequence.

Time delay of CT signals :

- Let x (t) = u (t) be the original signal. Then x (t – 1) i.e. u (t – 1) represents the delayed version of x (t) with a delay equal to 1.
- Table 1.6.4 explains the concept of time delaying and Fig. 1.6.9(a) shows the waveforms.

Table 1.6.4 : Time delay

t	–1	0	1	2	3	4	.....
x(t) = u(t)	0	1	1	1	1	1	
x(t–1) = u(t–1)	u (–1–1) = u(–2) = 0	u (0–1) = u(–1) = 0	u (1–1) = u(0) = 1	u (2–1) = u(1) = 1	1	1	

➡ Time delay by 1

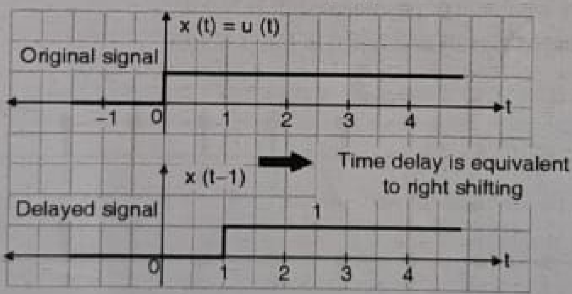


Fig. 1.6.9(a) : Time delay

Note : When time delayed, the signal is shifted right on the x-axis.

Ex. 1.6.1 : If the given signal $x(t) = e^{-at} u(t)$, draw the signals x (t + 2) and x (t – 3).

Soln. :

- x (t + 2) is a time advanced signal with a left shift of 2 places.
- x (t – 3) is a time delayed signal with a right shift of 3 places.
- These waveforms are shown in Fig. P. 1.6.1.

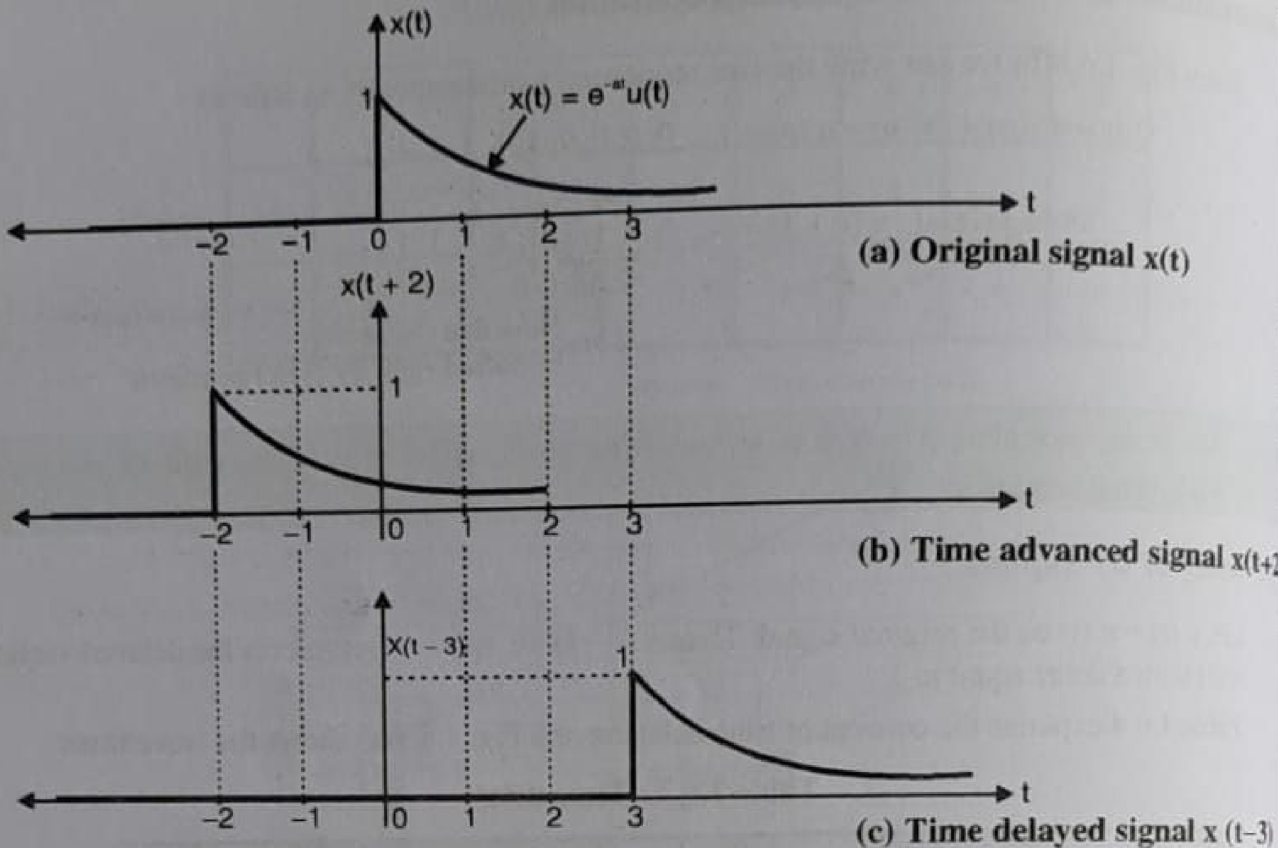


Fig. P. 1.6.1 : Time shifting

Time delay for a D.T. signal :

- For the original D.T. signal x (n), the delayed signal is denoted by x (n – N). The delayed si... is always the right shifted.
- Consider x (n) = u (n) as the original signal. Then x (n – 4) represents its delayed version or ... shifted version and the amount of positional shift is N = 4.
- The original and delayed signals are shown in Fig. 1.6.9(b).

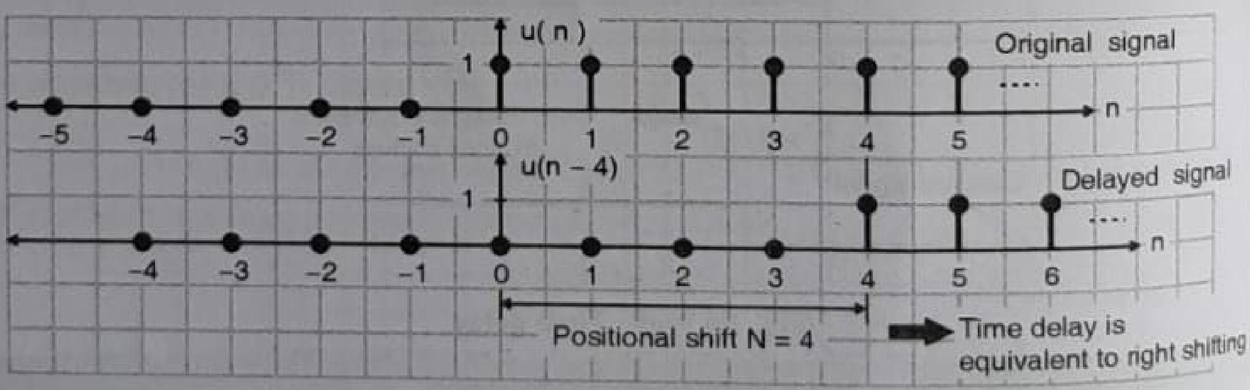


Fig. 1.6.9(b) : Positional delay of a D.T. signal

Mathematical representation :

From Fig. 1.6.9(b) we can express the two sequences mathematically as follows :

Original signal $x(n) = u(n) = \{ \ldots 0, 0, 0, 0, \underset{\uparrow}{1}, 1, 1, 1, 1, \ldots \}$

Delayed signal $u(n-4) = \{ \ldots \underset{\uparrow}{0}, 0, 0, 0, 1, 1, 1, 1, 1 \ldots \}$

Note that the arrow
is shifted left by N = 4 positions

Note : The arrow indicating n = 0 has shifted N = 4 positions to the left as compared to the original signal.

1.6.2.2 Time Scaling :

Time scaling of CT signals :

- It x (t) is the original signal, then x (at) is its time scaled version, where a is a constant.
- Depending on the value of "a" we can either have signal compression or signal expansion.

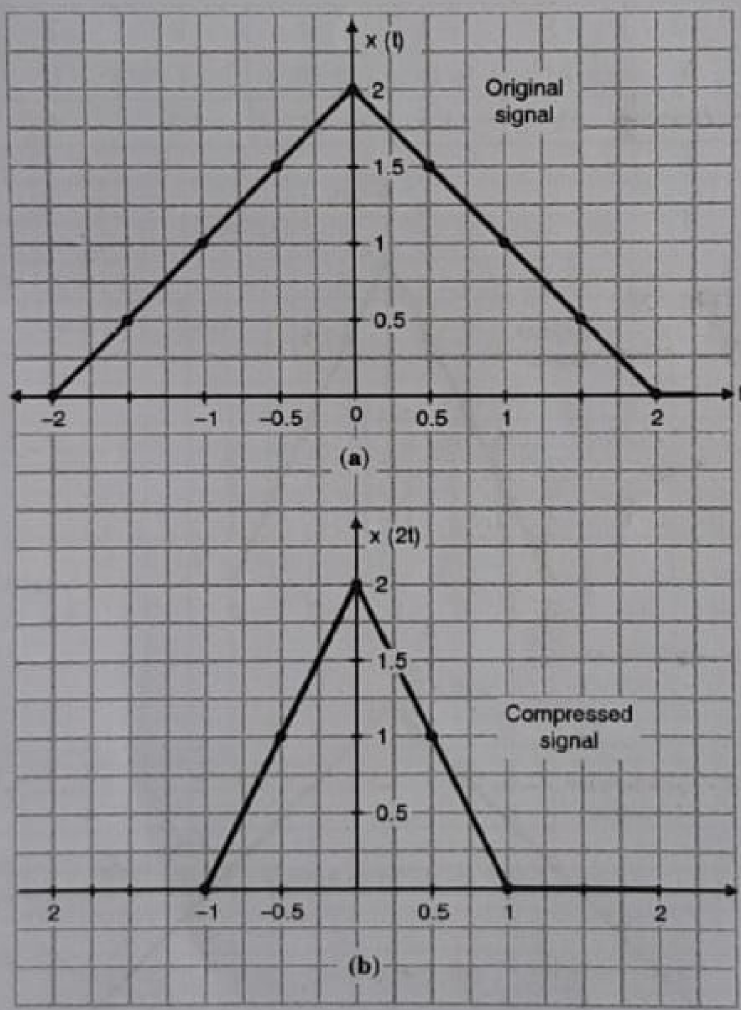


Fig. 1.6.10 : Signal compression

Signal compression a > 1 :

- For the values of "a" greater than 1, signal compression will take place.

- Let the signal x (t) be a triangular wave as shown in Fig. 1.6.10(a). Then for a = 2 the comp... version of x (t) i.e. x (2t) is shown in Fig. 1.6.10(b).
- The time scaling process is also illustrated in Table 1.6.5.

Table 1.6.5 : Signal compression

t	– 2	– 1.5	– 1	0.5	0	0.5	1	[illegible]
x (t)	0	0.5	1	1.5	2	1.5	1	[illegible]
x(2t)	x (2x – 2) = x (– 4) = 0	x(– 3) = 0	x(– 2) = 0	x(– 1) = 1	x(0) = 2	x(1) = 1	x(2) = 0	x(3)...

Signal expansion :

- In x (at) if a is positive and less than 1 (0 < a < 1) then the time scaled signal is the exp... version of x (t).
- This process is illustrated in Table 1.6.6 and in Fig. 1.6.11.
- Note that the value of a is assumed to be equal to 0.5.

Table 1.6.6 : Signal expansion

t	– 4	– 3	– 2	– 1	0	1	2	3	4
x (t)	0	0	0	1	2	1	0	0	[illegible]
x(0.5 t)	0	0.5	1	1.5	2	1.5	1	0.5	[illegible]

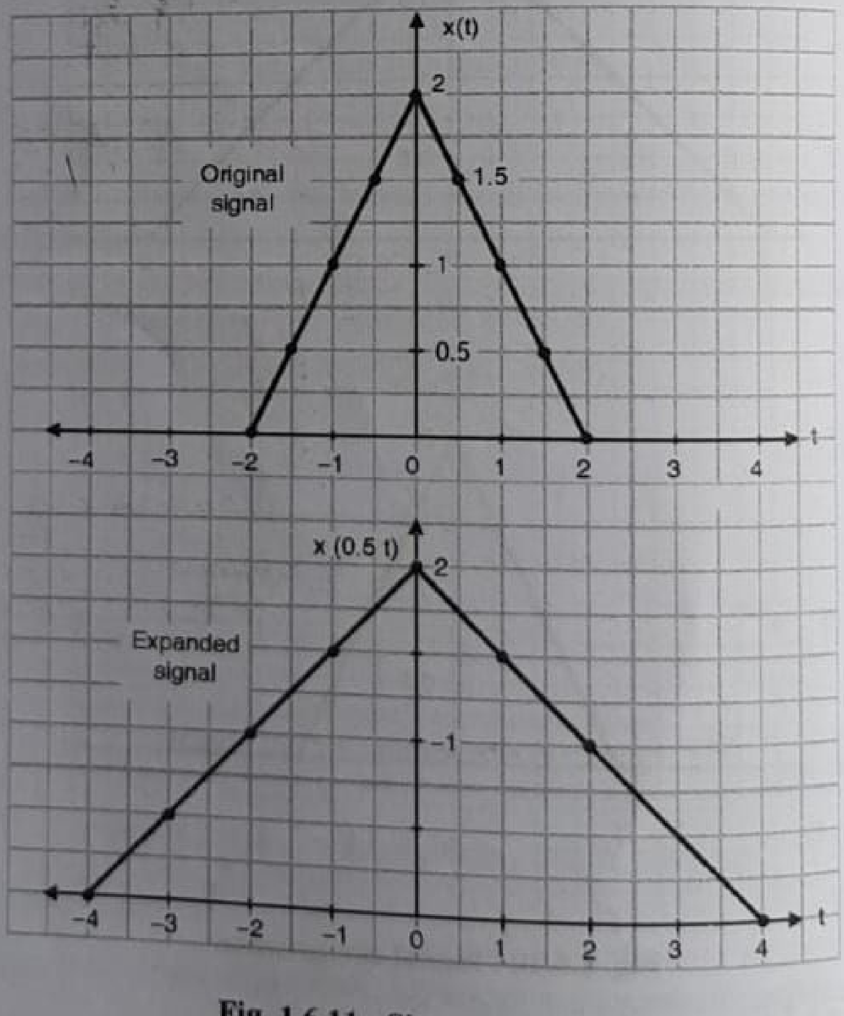


Fig. 1.6.11 : Signal expansion

Time scaling of DT signals :

- If x(n) is the original DT signal then x (an) is called as the time scaled version of x(n), where a is a constant.
- Depending on the value of "a", we can either have signal compression or signal expansion.

Signal compression (a > 1) :

- For a > 1, the signal compression will take place. Let the original signal be represented by,

$$x(n) = \{0, 0, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1, 0, 0, \ldots\}$$

This signal is as shown in Fig. 1.6.12(a).

- For a = 2 the compressed version of x(n) i.e. x (2n) is as shown in Fig. 1.6.12(b). The time scaling process is illustrated in Table 1.6.7. The time scaled signal is given by,

$$x(2n) = \{0, 0, 0, 0, 1, \underset{\uparrow}{1}, 1, 0, 0, 0, 0\}$$

Table 1.6.7 : Compression of a DT signal

n	– 4	– 3	– 2	– 1	0	1	2	3	4
x (n)	0	1	1	1	1	1	1	1	0
x (2n)	x (– 8) = 0	x (– 6) = 0	x (– 4) = 0	x (– 2) = 1	x (0) = 1	x (2) = 1	x (4) = 0	x (6) = 0	x (8) = 0

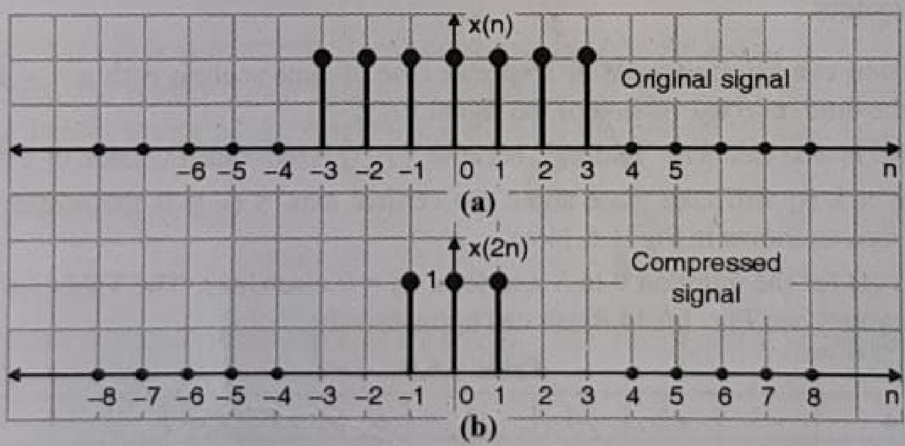


Fig. 1.6.12 : Compression of a DT signal

Signal expansion (a < 1) :

In x (an) if a is positive and less than 1 (0 < a < 1) then the time scaled signal x (an) is the expanded version of the original signal x(n).

This process is illustrated in Fig. 1.6.13 and Table 1.6.8 with a = 0.5.

The original signal $x(n) = \{0, 0, 0, 1, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\}$

Table 1.6.8 : Expansion of a DT signal

n	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
x (n)	0	0	0	1	1	1	1	1	1	1	0	0
x (0.5 n)	x (– 3) = 1	x (– 2.5) = 0	x (– 2) = 1	x (– 1.5) = 0	x (– 1) = 1	x (0.5) = 0	x (0) = 1	x (0.5) = 0	x(1) = 1	0	1	0

and the expanded signal is

$$x(0.5(n)) = \{0, 1, 0, 1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1, 0, 1, 0 \ldots\}$$

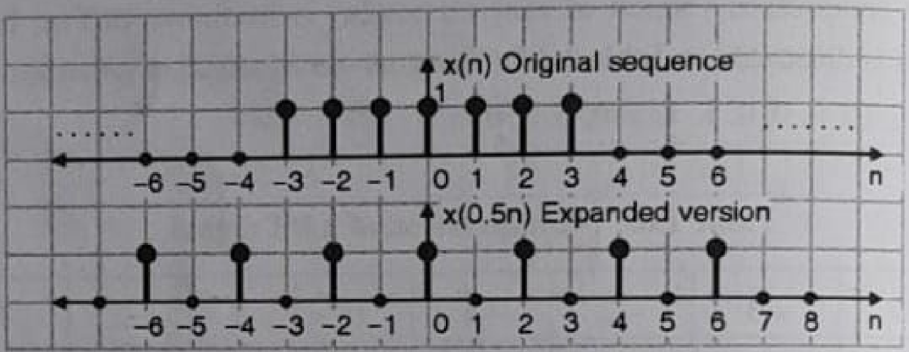


Fig. 1.6.13 : Expansion of a DT signal

1.6.2.3 Time Reversal (Time Inversion) or Folding :

Folding of CT signals :

- Time inversion can be considered as a special case of time scaling with a = – 1. Thus x represents the time reversed version of the signal x (t).
- Time reversal is also called as "folding", because x (– t) is the folded version of x (t).
- The folding of x (t) will take place about the vertical axis. x (– t) is the mirror image of about the y axis as shown in Fig. 1.6.14(a).
- Let x (t) = r (t) for the duration 0 to 3 and let x (t) = 0 elsewhere. The Table 1.6.9 explain folding operations and Fig. 1.6.14 illustrates it graphically.

Table 1.6.9

t	– 4	– 3	– 2	– 1	0	1	2	3	4
x (t)	0	0	0	0	0	1	2	3	0
x(– t)	x[– (– 4)] = x(4) = 0	x(3) = 3	x(2) = 2	x(1) = 1	x(0) = 0	x(– 1) = 0	x(– 2) = 0	x(– 3) = 0	x(– 4) = 0

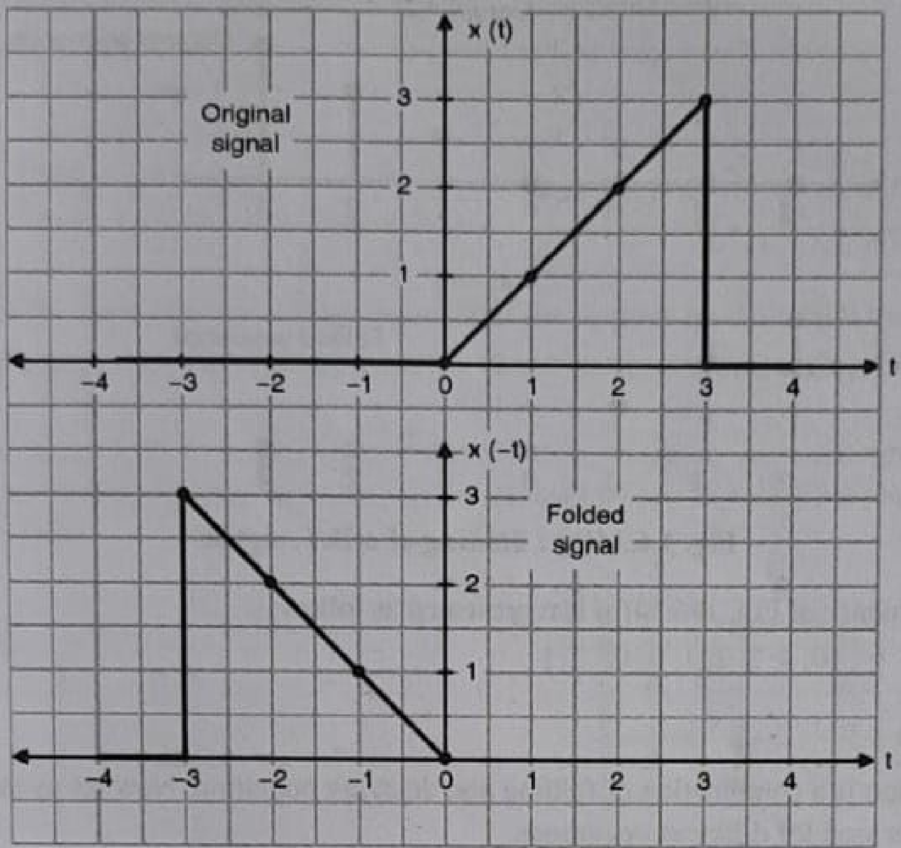


Fig. 1.6.14(a) : Process of folding

lding of D.T. signal :

If x (n) is the original sequence then x (– n) represents the folded version.

We can obtain the folded signal simply by replacing "n" by "– n". The folded sequence is actually the mirror image of the original signal, with the mirror assumed to be placed at the y-axis.

Let the original sequence be,

$$x(n) = \{..., 0, \underset{\uparrow}{0}, 1, 2, 3, 4, 0, 0, ...\}$$

Then the folded sequence is obtained as shown in Table 1.6.10 and it is graphically plotted as shown in Fig. 1.6.14(b).

Table 1.6.10 : Folding of a D.T. signal

	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6	...
)	0	0	0	0	0	0	1	2	3	4	0	0	...
n)	x(5) = 0	x(4) = 4	x(3) = 3	x(2) = 2	x(1) = 1	x(0) = 0	x(– 1) = 0	x(– 2) = 0	x(– 3) = 0	x(– 4) = 0	x(– 5) = 0	x(– 6) = 0	...

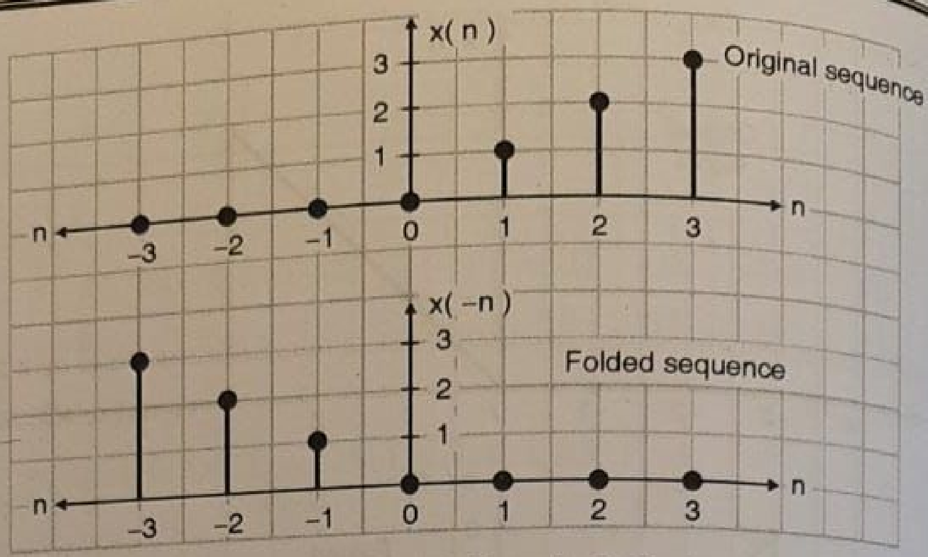


Fig. 1.6.14(b) : Folding of a D.T. signal

- The folded sequence of Fig. 1. 6.14(b) is represented as follows :

$$x(-n) \;=\; \{0, 4, 3, 2, 1, \underset{\uparrow}{0}, 0, \ldots.\}$$

Folding and Delay :

This operation is a combination of folding and delaying operation. Now let us take a qu at the modifications used for different operations.

1. **For drawing diagrams :**
   - To obtain delayed sequence, shift the original diagram towards right by 'k' sample
   - To obtain advanced sequence, shift the original diagram towards left by 'k' sample
   - To obtain the folded version, take the mirror image of the diagram at n = 0.
2. **For writing the sequence :**
   - The delayed version of x (n) is denoted by x (n – k)

     Negative sign always indicates delay operation of x (n).
   - The advanced version of x (n) is denoted by x (n + k)

     Positive sign always indicates advanced operation of x (n).
   - To obtain the folded version of x (n); replace n by – n

   Now folding and delay operation means :
   - First fold the sequence x (n); that means obtain x (– n)
   - Then delay the folded sequence by k samples.

One major difference between x (n) and x (– n) x (– n) denotes the mirror image of x know that in case of mirror image; left and right sides are reversed. Now if x (n) is original s then its delayed version is denoted by x (n – k). The folded sequence is denoted by x (– n). Si mirror image the delayed version of folded signal is denoted by x (– n + k).

$$\therefore \quad x(n) \xrightarrow{\text{Delay}} x(n-k)$$

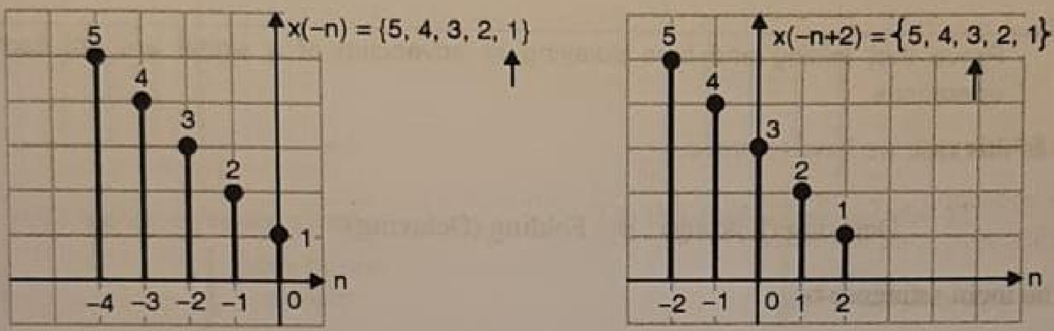
Folding ↓

$$x(-n) \xrightarrow{\text{Delay}} x[-(n-k)] = x(-n+k)$$

But stick to the basic concepts. Delay means shift the diagram towards right by k samples.

Consider the same original sequence $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ as shown in Fig. 1.6.15(a). The

Folded sequence $x(-n)$ is shown in Fig. 1.6.15(a). Suppose we want to delay this folded sequence $x(-n)$ by '2' samples then it will be denoted by $x(-n+2)$. This sequence is as shown in Fig. 1.6.15(b).



(a) Folded version x (– n) **(b) Delay of folded sequence x (– n + 2)**

Fig. 1.6.15

From Fig. 1.6.15(b), the sequence $x(-n+2)$ can be written as,

$$x(-n+2) = \{5, 4, \underset{\uparrow}{3}, 2, 1\}$$

olding and Advance :

The advanced version of original sequence $x(n)$ is denoted by $x(n+k)$. The folded version of : (n) is denoted by $x(-n)$. Since folding means mirror image of sequence; the advanced version of olded sequence is denoted by $x(-n-k)$.

$$\therefore \quad x(n) \xrightarrow{\text{Advance}} x(n+k)$$

Folding ↓

$$x(-n) \xrightarrow{\text{Advance}} x[-(n+k)] = x(-n-k)$$

Consider the same folded sequence $x(-n)$ shown in Fig. 1.6.15(a). Suppose we want to advance iis sequence by '2' samples, then the advanced version is denoted by $x(-n-2)$. Such a sequence is as hown in Fig. 1.6.16.

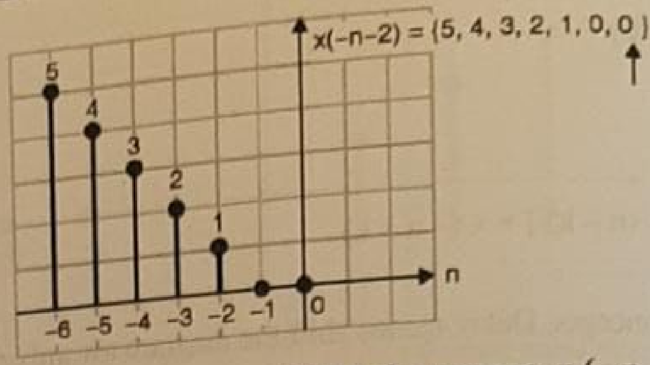


Fig. 1.6.16 : Advance of folded sequence, x (– n – 2)

Remember the basic rule. Advancing the sequence means shifting the diagram toward samples. From Fig. 1.6.16, we can write the sequence x (– n – 2) as,

$$x(-n-2) = \{5, 4, 3, 2, 1, 0, \underset{\uparrow}{0}\}$$

Ex. 1.6.1 : Prove that folding and time delaying or advancing of a signal are not c operations.

Soln. : In this case we have to prove,

$$\text{Delaying (Folding)} \neq \text{Folding (Delaying)}$$

Let the input sequence be,

$$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$$

This sequence is represented in Fig. P. 1.6.1(a).

Now consider L.H.S.

Step 1 : First fold the sequence x (n). The folded sequence x (– n) is shown in Fig. P. 1.6.1

Step 2 : Delay this sequence by 1 sample. This gives delaying of folding operation. This s is shown in Fig. P. 1.6.1(c).

Step 3 : Now consider the R.H.S.

First delay the sequence x (n) by one sample. This sequence is as shown in Fig. P

Step 4 : Now take the folded version of signal x (n – 1). This gives folding of delaying Such sequence is as shown in Fig. P. 1.6.1(e).

Compare Fig. P. 1.6.1(c) and Fig. P. 1.6.1(e). Since these two diagrams are not same s

Delaying (Folding) ≠ Folding (Delaying)

Hence proved.

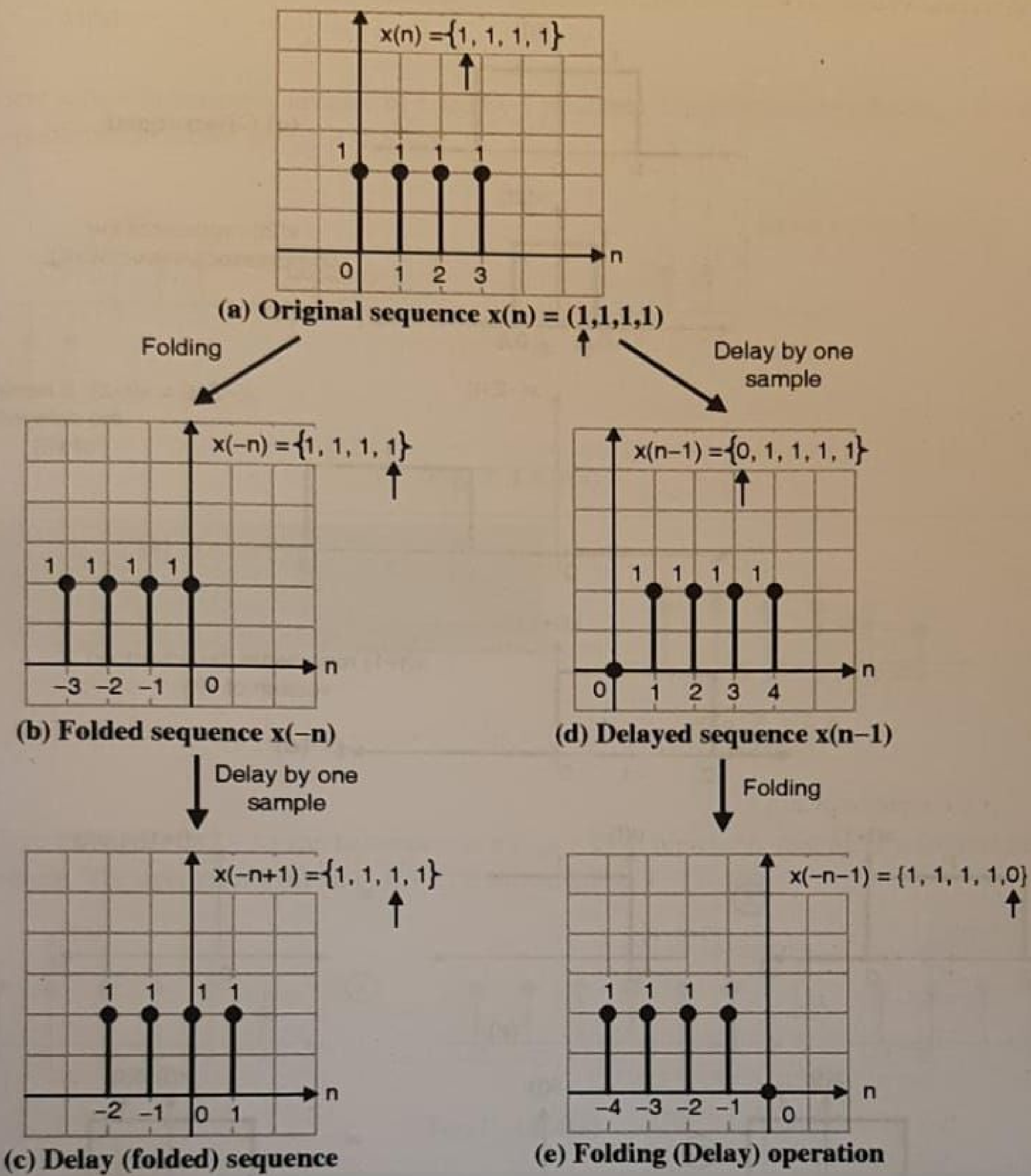


Fig. P. 1.6.1

Ex. 1.6.2 : For the signal defined as :

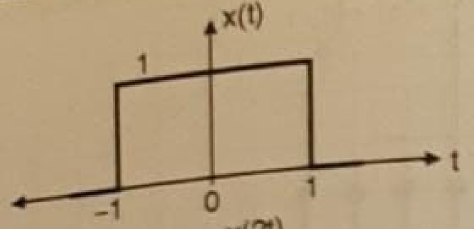
$$x(t) = 1 \quad \text{for } -1 \leq t \leq 1$$
$$= 0 \quad \text{otherwise}$$

Sketch the following.

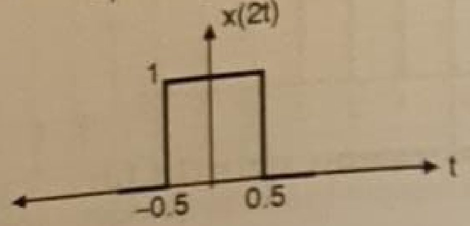
(a) $x(2t)$ (b) $x(-2+t)$ (c) $x(t+1)$

(d) $x(t+1)\,u(t)$ (e) $x(t)\cdot\delta(t)$.

Soln. : The required signals are as shown in Fig. P. 1.6.2.

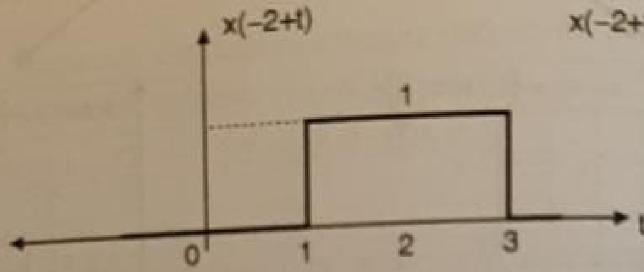


(a) Given signal



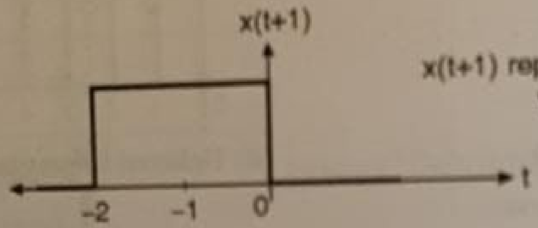
x(2t) represents the compressed version of x(t)

(b)



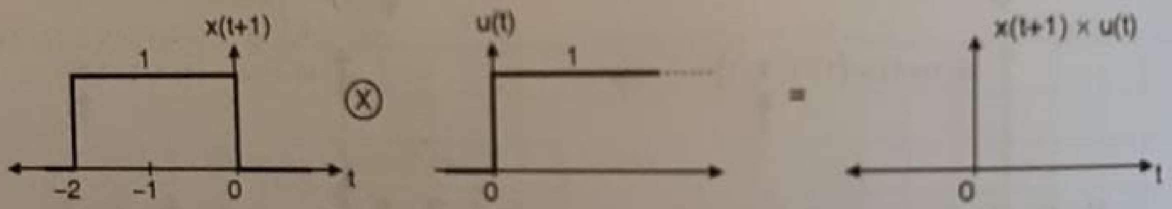
x(−2+t) = x(t−2). It represe the delayed v of x(t)

(c)



x(t+1) represents the advanced version of x(t)

(d)



(e)

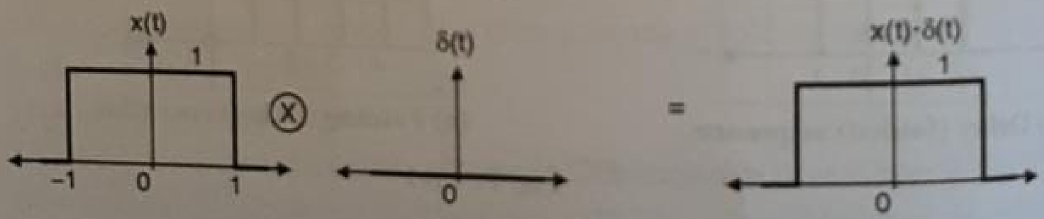


Fig. (f) 1.6.2

Ex. 1.6.3 : The discrete time signal :

$$x(n) = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 0. \end{cases}$$

Find and sketch the signal $y(n) = x(n+3)$

Dec. 12,

Soln. : The given sequence can be written as,

$$x(n) = \{-1, -1, \underset{\uparrow}{0}, 1, 1\}$$

Here x (n + 3) indicates advance of x (n) by 3 positions. It is obtained by shifting x (n) towards left by 3 positions, as shown in Fig. P. 1.6.3(a).

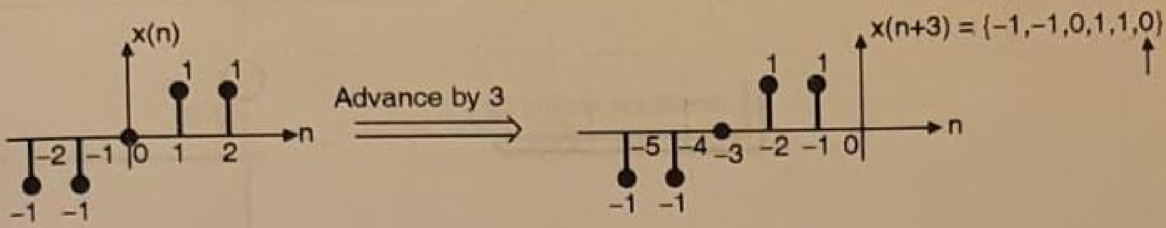


Fig. P. 1.6.3(a)

Ex. 1.6.4 : Sketch and label the following signal :

y (n) = x (n) · u (2 − n)

Dec. 12, 5 Marks

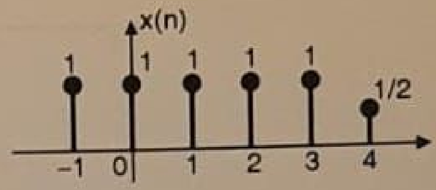


Fig. P. 1.6.4

Soln. :

Here the signal u (2 − n) can be written as u (− n + 2). It represents folding of u (n) and delaying by 2 positions. The operation x (n) · u (− n + 2) is shown in Fig. P. 1.6.4(a).

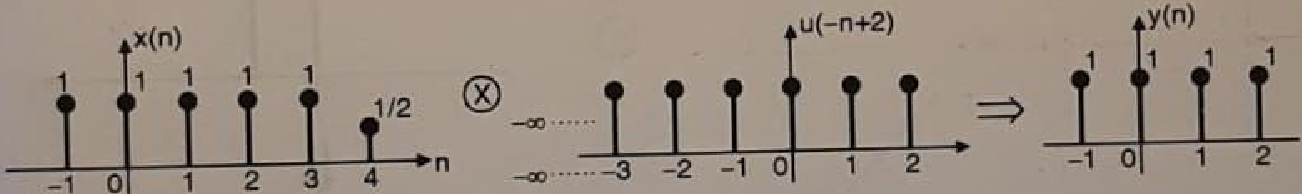


Fig. P. 1.6.4(a)

Ex. 1.6.5 : Sketch the following waveforms :

1. $x_1(t) = u(t+1) - 2u(t) - 2u(t-1)$
2. $x_2(t) = r(t+1) - r(t) + r(t-2)$

Dec. 10, 5 Marks

Soln. :

1. **Given :** $x_1(t) = u(t+1) - 2u(t) - 2u(t-1)$

The sketch of $x_1(t)$ is shown in Fig. P. 1.6.5(a).

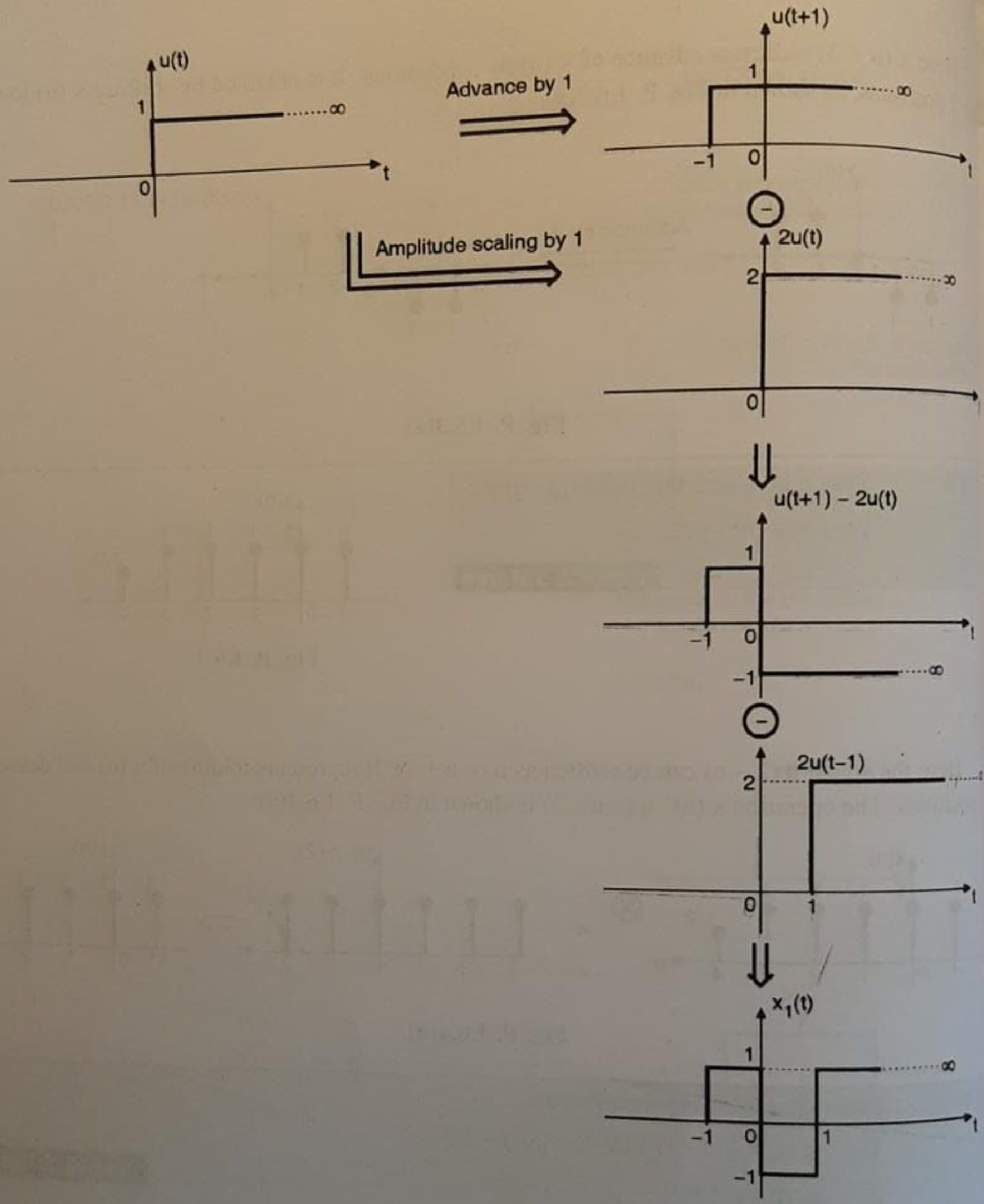


Fig. P. 1.6.5(a)

2. **Given :** $x_2(t) = r(t+1) - r(t) + r(t-2)$

The sketch of $x_2(t)$ is shown in Fig. P. 1.6.5(b).

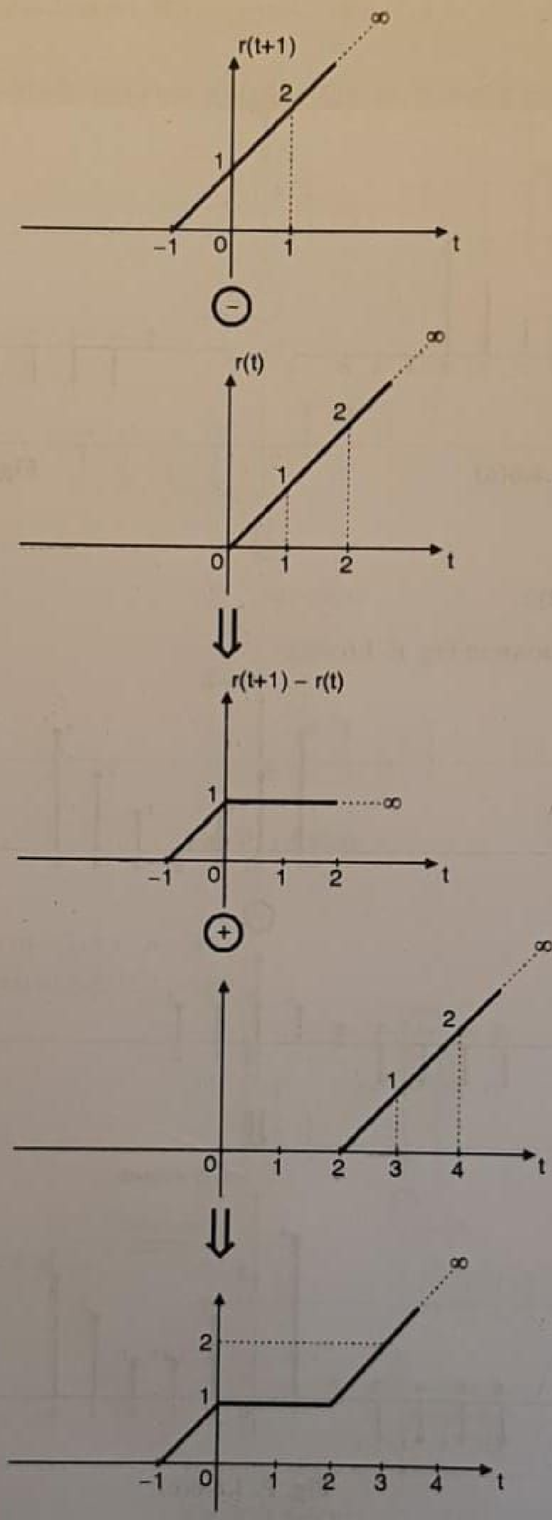


Fig. P. 1.6.5(b)

Ex. 1.6.6 : Let x [n] and y [n] be in Fig. P. 1.6.6(a) and P. 1.6.6(b) respectively. Sketch the fo... signals :

1. $x[n-2] + y[n+2]$ 2. $x[3-n]\,y[n]$ 3. $x[n+2]\,y[6-n]$

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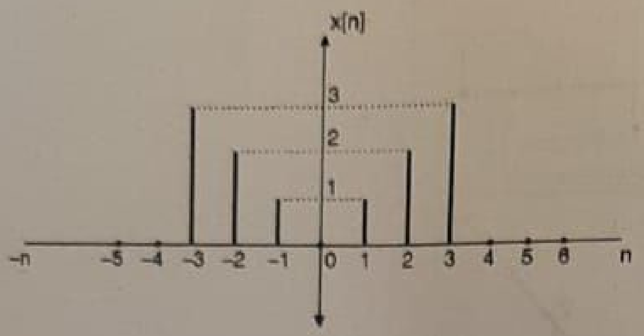

Fig. P. 1.6.6(a)

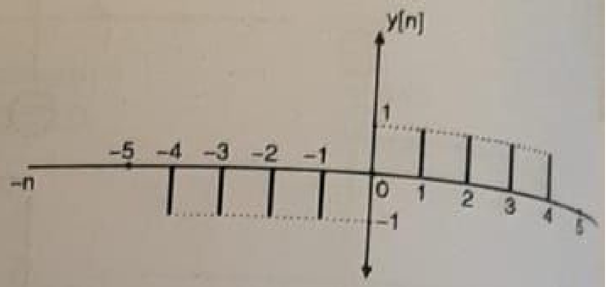

Fig. P. 1.6.6(b)

Soln. :

1. **x (n – 2) + y (n + 2) :**

This operation is shown in Fig. P. 1.6.6(c).

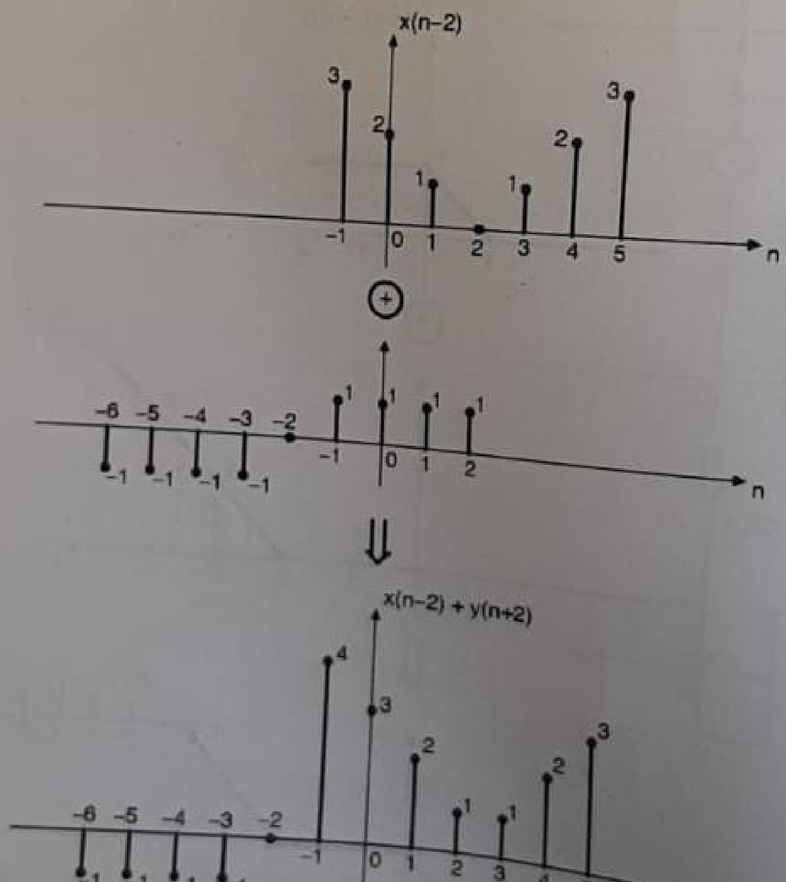

Fig. P. 1.6.6(c)

2. **x (3 – n) y (n) :**

It can be written as, x (– n + 3) y (n).

It is shown in Fig. P. 1.6.6(d).

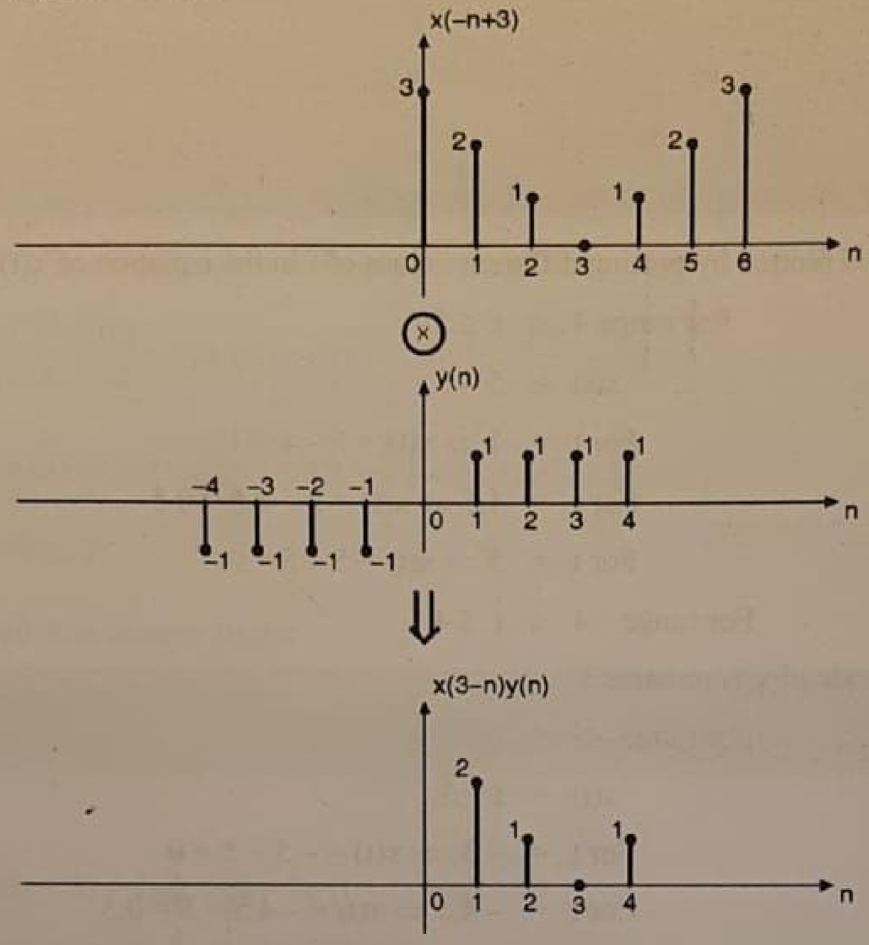

Fig. P. 1.6.6(d)

x (n + 2) y (6 – n) :

It can be written as, x (n + 2) y (– n + 6).

This operation is shown in Fig. P. 1.6.6(e).

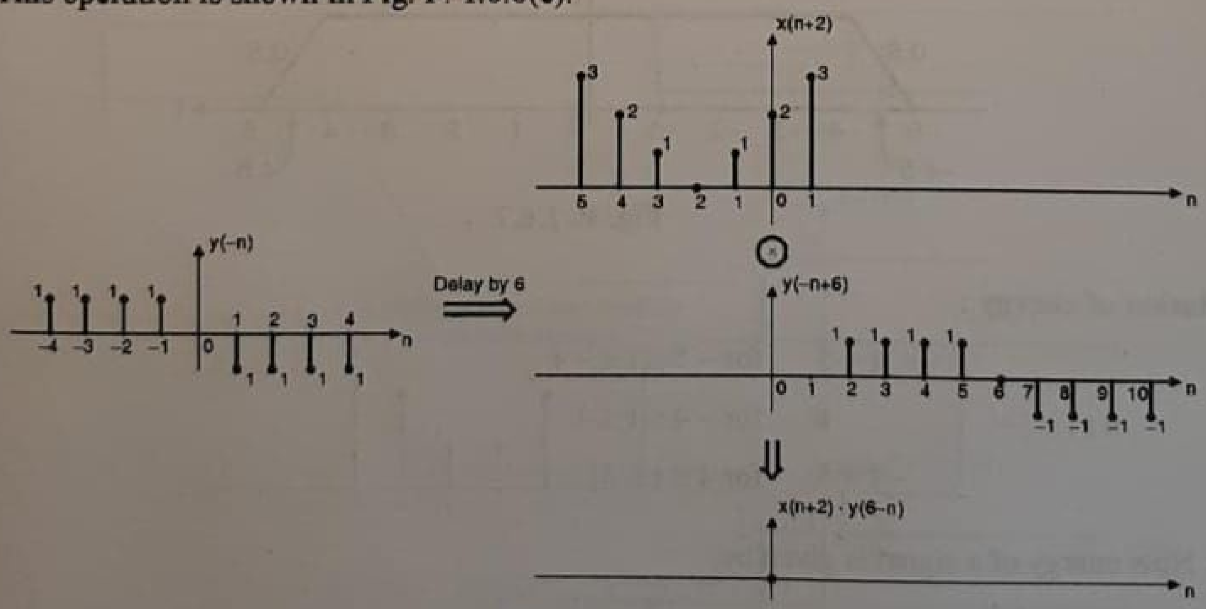

Fig. P. 1.6.6(e)

Ex. 1.6.7 : Sketch the following signal :

$$x(t) = \begin{cases} 5-t & 4 \le t \le 5 \\ 1 & -4 \le t \le 4 \\ t+5 & -5 \le t \le -4 \\ 0 & \text{otherwise} \end{cases}$$

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Also determine total energy of signal x(t).

Soln. : The signal is plotted by putting different values of t in the equation of x(t) as follows :

For range $4 \le t \le 5$.

$$x(t) = 5-t$$

For $t = 4 \Rightarrow x(t) = 5-4 = 1$

For $t = 4.5 \Rightarrow x(t) = 5-4.5 = 0.5$

For $t = 5 \Rightarrow x(t) = 5-5 = 0$

For range $-4 \le t \le 4$

Here amplitude of x(t) remains 1.

For range $-5 \le t \le -4$

$$x(t) = t+5$$

For $t = -5 \Rightarrow x(t) = -5+5 = 0$

For $t = -4.5 \Rightarrow x(t) = -4.5+5 = 0.5$

For $t = -4 \Rightarrow x(t) = -4+5 = 1$

The sketch of signal x(t) is shown in Fig. P. 1.6.7.

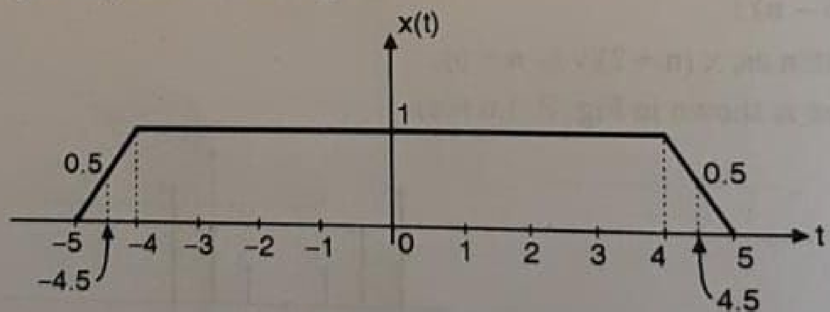


Fig. P. 1.6.7

Calculation of energy :

$$x(t) = \begin{cases} t+5 & \text{for } -5 \le t \le -4 \\ 1 & \text{for } -4 \le t \le 4 \\ -t+5 & \text{for } 4 \le t \le 5 \end{cases}$$

Now energy of a signal is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$\therefore \quad E = \int_{-5}^{-4} (t+5)^2\, dt + \int_{-4}^{4} 1\, dt + \int_{4}^{5} (-t+5)^2\, dt$$

$$\therefore \quad E = \int_{-5}^{-4} (t^2+10t+25)\, dt + \int_{-4}^{4} dt + \int_{4}^{5} (t^2-10t+25)\, dt$$

$$\therefore \quad E = \left[\frac{t^3}{3}+\frac{10t^2}{2}+25\,t\right]_{-5}^{-4} + [\,t\,]_{-4}^{4} + \left[\frac{t^3}{3}-\frac{10t^2}{2}+25\,t\right]_{4}^{5}$$

$$\therefore \quad E = [\,20.33-45+25\,]+[\,4+4\,]+[\,20.33-45+25\,]$$

$$\therefore \quad E = 8.66\ J$$

Thus given signal is an energy signal.

Ex. 1.6.8 : Sketch the following signal :

$x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$ **Dec. 11, 8 Marks**

Soln. :

1. Given $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$.

The sketch of x(t) is as shown in Fig. P. 1.6.8.

Step 1 :

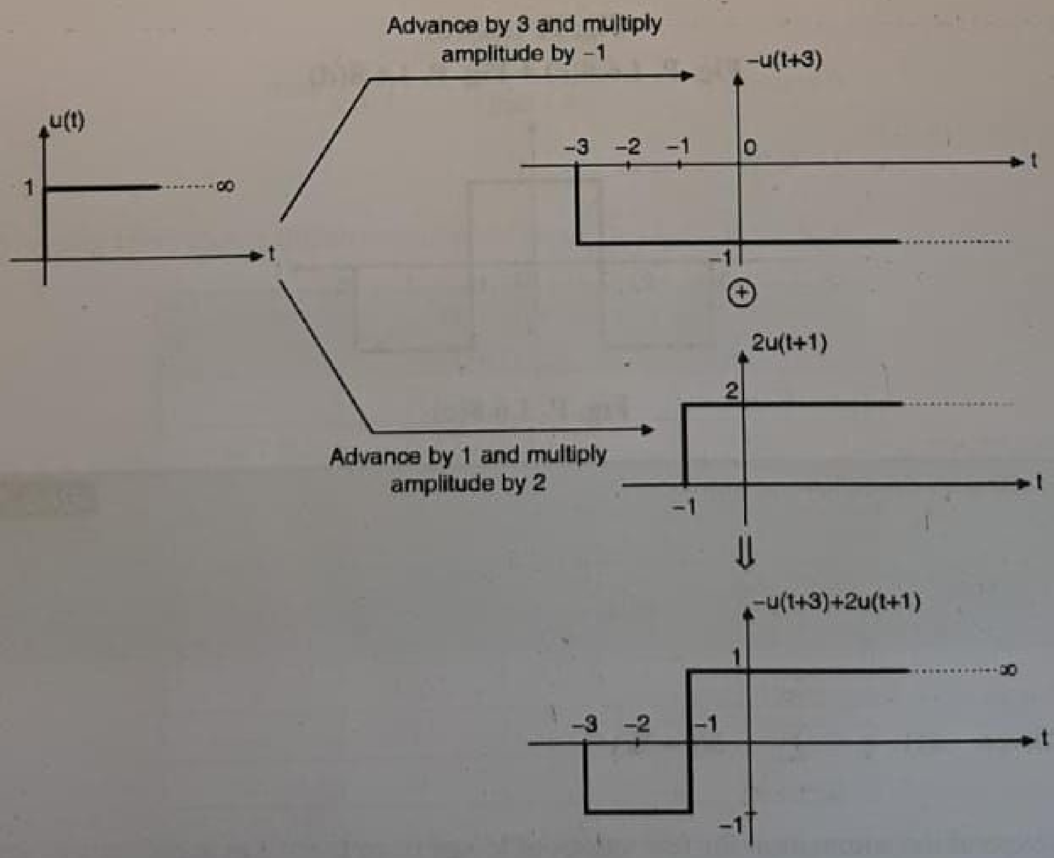


Fig. P. 1.6.8(a)

Step 2 :

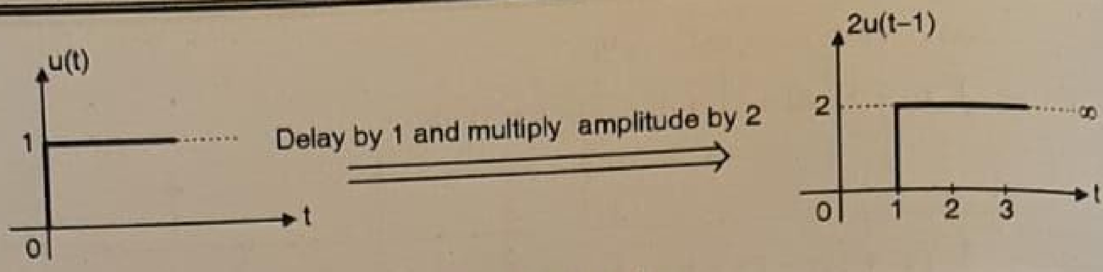


Fig. P. 1.6.8(b)

Step 3 :

Fig. P. 1.6.8(a) – Fig. P. 1.6.8(b)

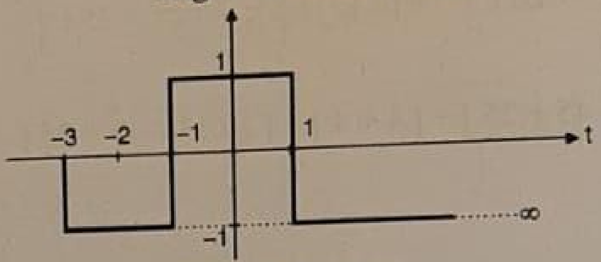


Fig. P. 1.6.8(c)

Step 4 :

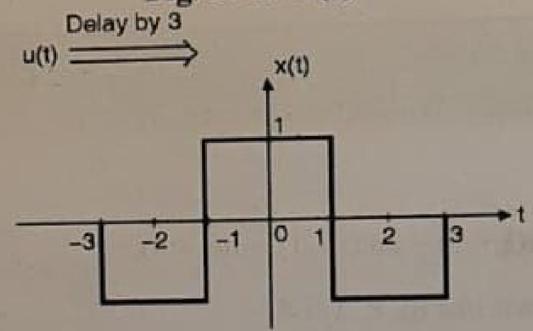


Fig. P. 1.6.8(d)

Fig. P. 1.6.8(c) + Fig. P. 1.6.8(d)

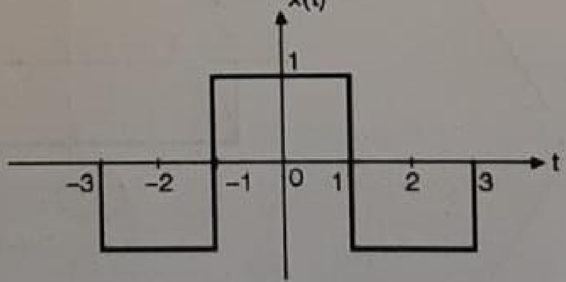


Fig. P. 1.6.8(e)

Ex. 1.6.9 : Sketch the following signal : **Dec. 11, 8 Mar**

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-3k)$$

Soln. : Given $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-3k)$

Let us expand the summation for few values of k, say from $k = -2$ to $+2$.

$$x(t) = \ldots\delta(t+6) + \delta(t+3) + \delta(t) + \delta(t-3) + \delta(t-6) + \ldots\ldots$$

The sketch of x(t) is shown in Fig. P, 1.6.9.

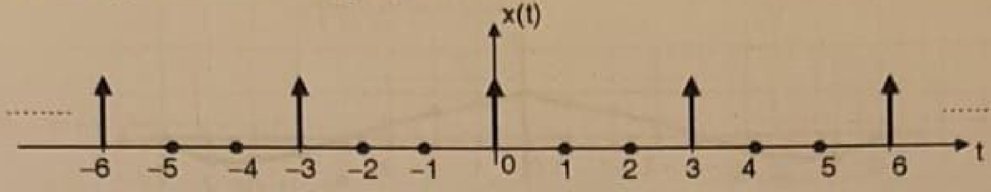


Fig. P. 1.6.9

Ex. 1.6.10 : Sketch the following signals to the scale : **May 06, 6 Marks**

1. $x(n) = \text{sinc}\left(\frac{n}{4}\right)$ for $-8 \le n \le 8$

2. $x(n) = \sum_{k=-4}^{+4} \delta(n-k)$ where $\delta(n)$ is an impulse signal.

Soln. :

1.
$$x(n) = \text{sinc}\left(\frac{n}{4}\right) \text{ for } -8 \le n \le 8$$

sinc function is defined as,

$$\begin{aligned} \text{sinc}(x) &= \frac{\sin(\pi x)}{\pi x} && x \ne 0 \\ &= 1 && \text{for } x = 0 \end{aligned}$$

$$\begin{aligned} \therefore \quad \text{sinc}\left(\frac{n}{4}\right) &= \frac{\sin\left(\frac{\pi n}{4}\right)}{(\pi n/4)} && \text{for } n \ne 0 \\ &= 1 && \text{for } n = 0 \end{aligned}$$

The following table shows different values of $\text{sinc}\left(\frac{n}{4}\right)$ for $-8 \le n \le \infty$

Value of n	$\text{sinc}\left(\frac{n}{4}\right)$	Value of n	sinc (n / 4)
– 8	0	1	0.9
– 7	– 0.12	–	0.63
– 6	– 0.21	3	0.3
– 5	– 0.18	4	0
– 4	0	5	– 0.18
– 3	0.3	6	– 0.21
– 2	0.63	7	– 0.12
– 1	0.9	8	0
0	1		

The plot of sinc function is shown in Fig. P. 1.6.10(a).

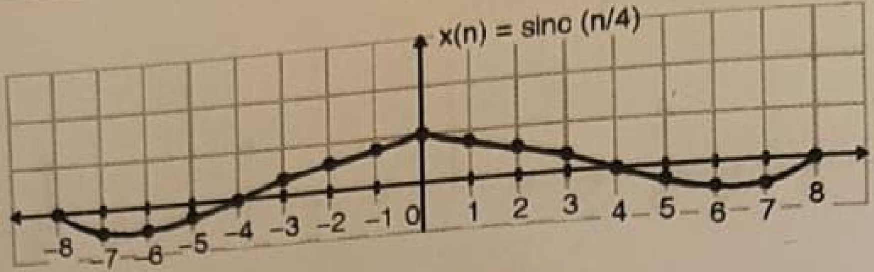


Fig. P. 1.6.10(a)

2.
$$x(n) = \sum_{k=-4}^{+4} \delta(n-k)$$

Expanding the summation we get,

$$x(n) = \delta(n+4)+\delta(n+3)+\delta(n+2)+\delta(n+1)+\delta(n)$$
$$+\delta(n-1)+\delta(n-2)+\delta(n-3)+\delta(n-4)$$

The plot of x (n) is shown in Fig. P. 1.6.10(b).

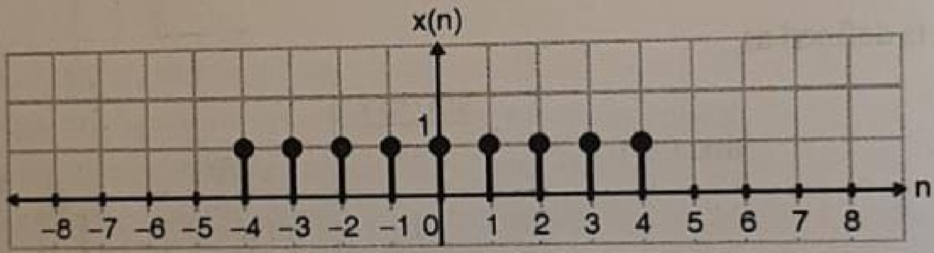


Fig. P. 1.6.10(b)

Ex. 1.6.11 : A discrete signal $x(n) = \{3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3\}$ then determine :

1. $y(n) = \sum_{k=-\infty}^{n+3} x(k)$ 2. $y(n) = x(n-1) + x(n) - x(n+1)$

May 06, 10 Mar

Soln. :

1. Given $x(n) = \{3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3\}$

$$\therefore x(-3) = 3, x(-2) = 2, x(-1) = 1, x(0) = 0, x(1) = 1, x(2) = 2, x(3) =$$

$$y(n) = \sum_{k=-\infty}^{n+3} x(k)$$

$$y(n) = \ldots x(-3) + x(-2) + x(-1) + x(0) + x(1) + x(2) + x(3) + \ldots + x(n+3)$$

$$\therefore \quad y(n) = 3+2+1+0+1+2+3 = 12$$

2. $y(n) = x(n-1) + x(n) - x(n+1)$

$= x(n) + x(n-1) - x(n+1)$

The plot of signal y (n) is shown in Fig. P. 1.6.11.

$$\therefore \quad y(n) = \{3, 1, 5, \underset{\uparrow}{0}, -1, 0, 5, 3\}$$

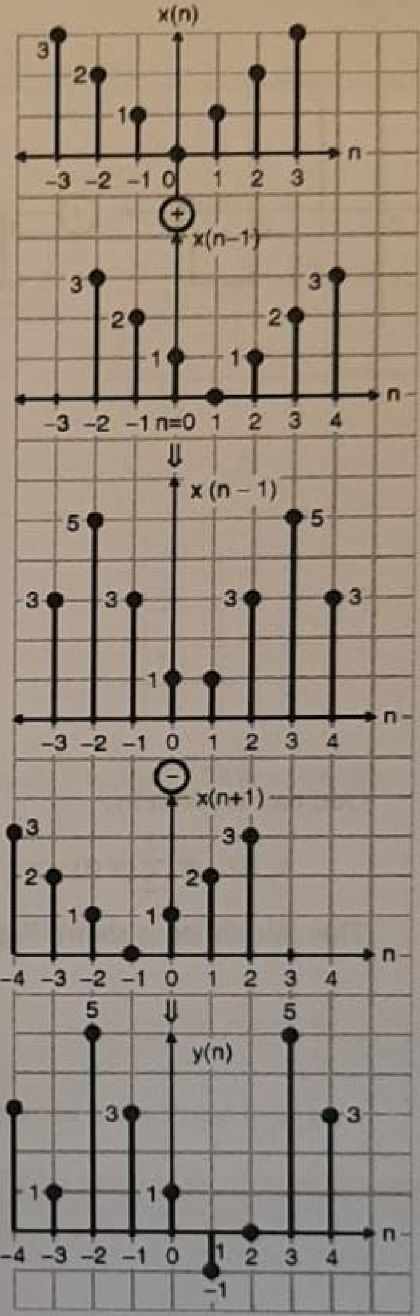


Fig. P. 1.6.11

Ex. 1.6.12 : Find the odd and even parts of the signal (Refer Fig. P. 1.6.12) :

Dec. 10, 6 Marks

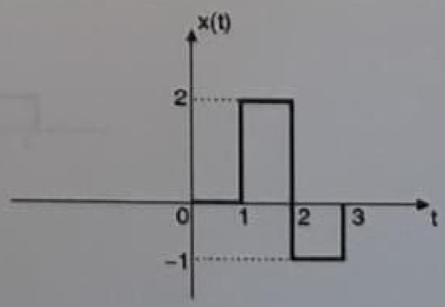


Fig. P. 1.6.12

Soln. :

Even part is given by $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

This calculation is shown in Fig. P. 1.6.12(a).

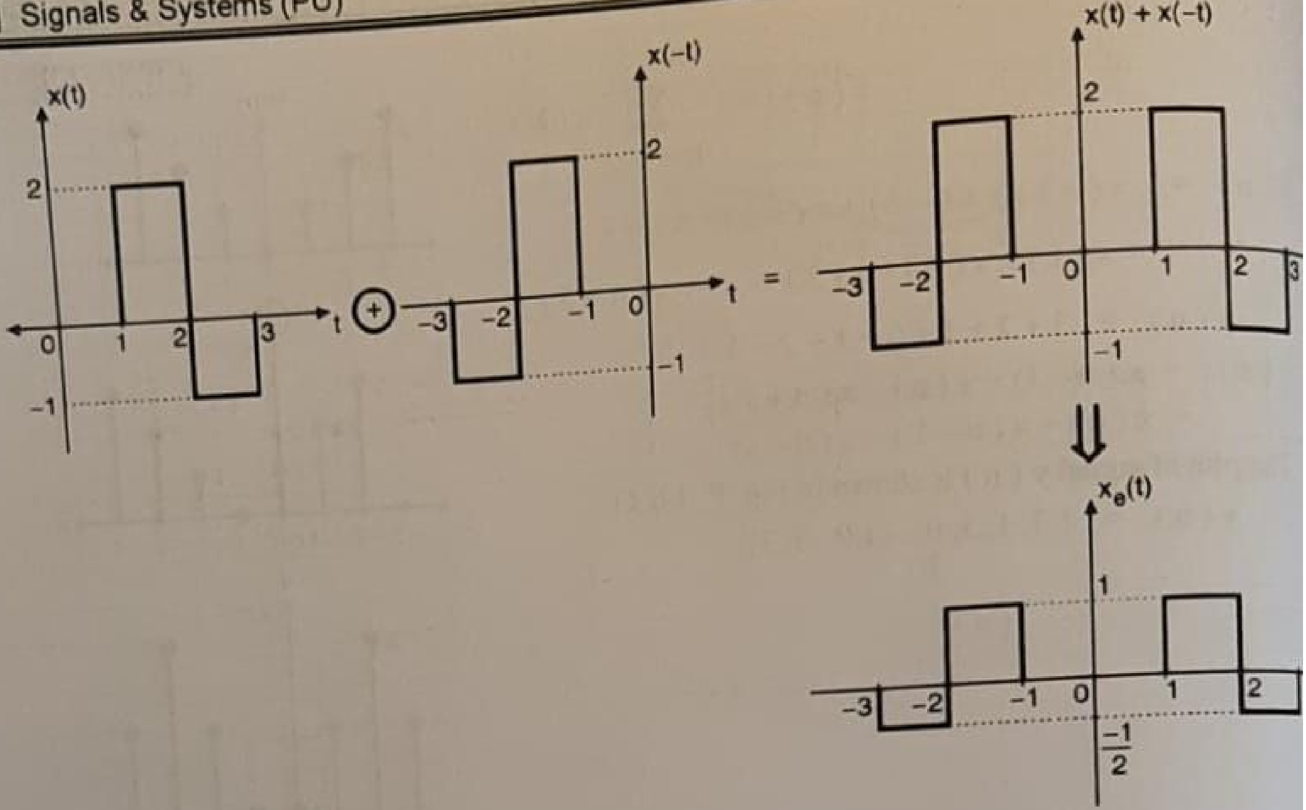


Fig. P. 1.6.12(a)

Odd part is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

This calculation is shown Fig. P. 1.6.12(b).

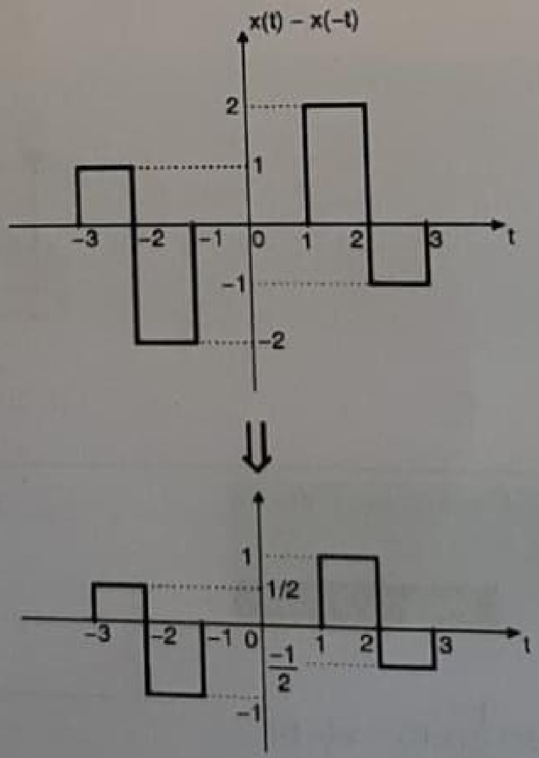


Fig. P. 1.6.12(b)

Ex. 1.6.13 : Sketch the following signal if : **Dec. 07, 4 Marks**

$x(n) = \{ \underset{\uparrow}{6}, 4, 2, 2 \}$, $y(n) = x(-n + 2)$.

Soln. :

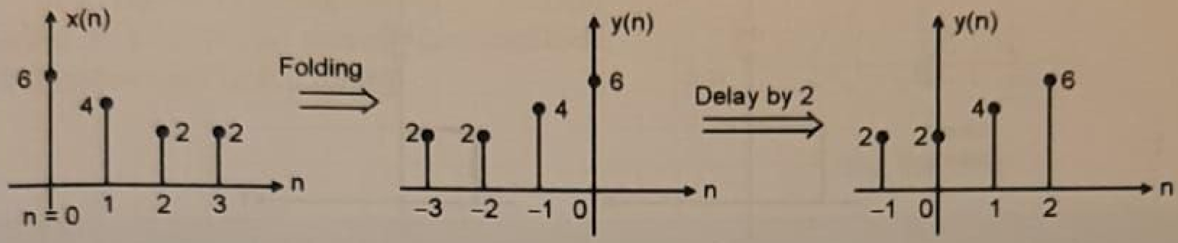


Fig. P. 1.6.13

Ex. 1.6.14 : Function x (t) is as shown in Fig. P. 1.6.14(a). Draw even and odd parts of x (t).

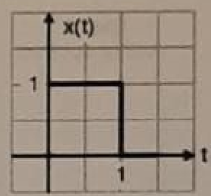


Fig. P. 1.6.14(a) : Given function x(t)

Soln. :

- The even part is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

- And the odd part is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

where x (– t) represents the folded version of x (t). The folding operation is discussed later on.

Steps to be followed :

Step 1 : Draw the signal x (t).
Step 2 : Draw its folded version x (– t).
Step 3 : Add x (t) and x (– t) or subtract x (– t) from x (t).
Step 4 : Divide the addition or subtraction by 2 to get $x_e(t)$ and $x_o(t)$.

- These steps are followed in Fig. P. 1.6.14(b) to (i) to obtain $x_e(t)$ and $x_o(t)$.

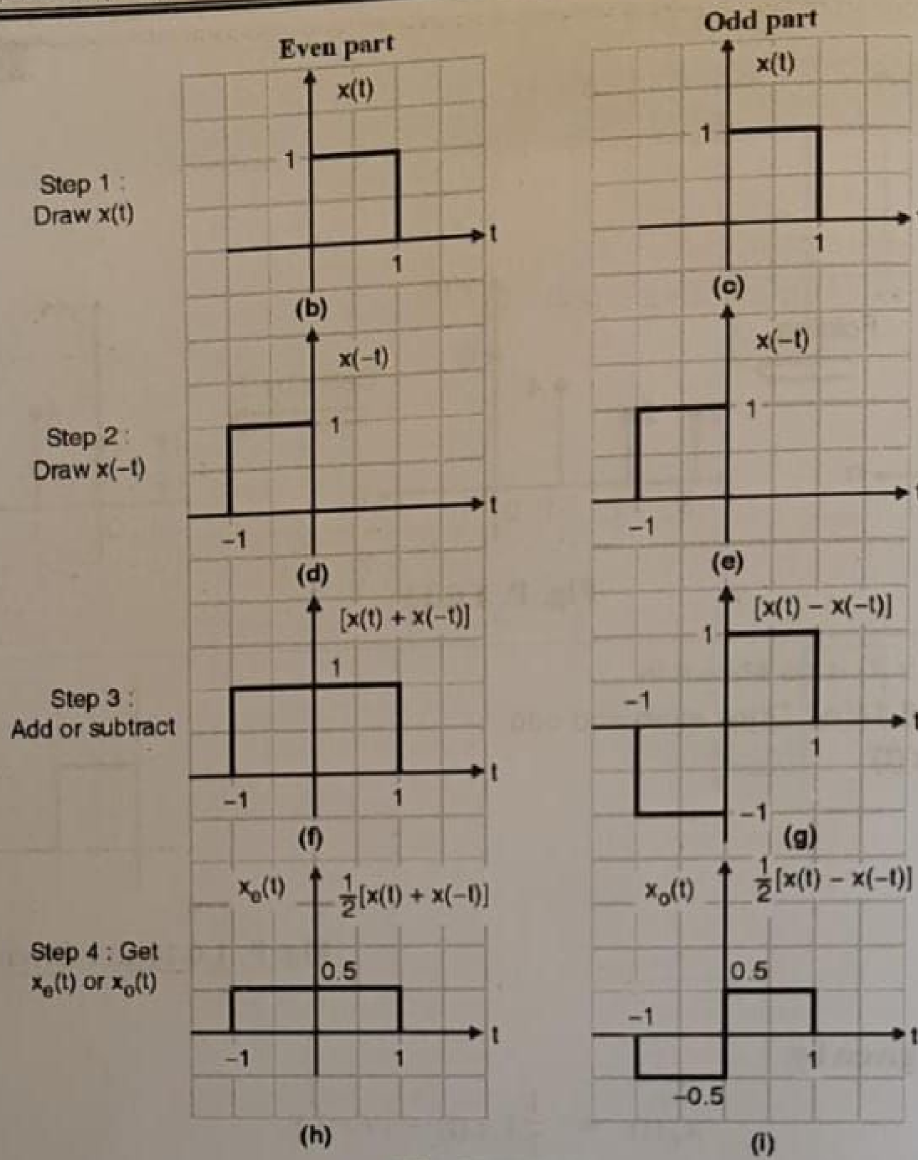


Fig. P. 1.6.14

Ex. 1.6.15 : Refer Fig. P. 1.6.15. Find even and odd part of C.T. signal.

Dec. 12, 4 Marks

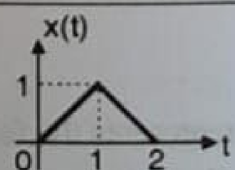


Fig. P. 1.6.15

Soln. : Even part is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

It is shown in Fig. P. 1.6.15(b)

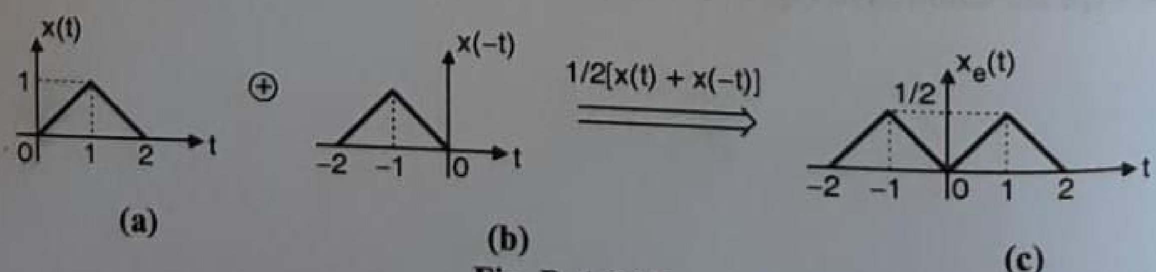


Fig. P. 1.6.15

Odd part is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

It is obtained by subtracting Fig. P. 1.6.15(b) from Fig. P. 1.6.15(a) and then dividing amplitude by 2. It is shown in Fig. P. 1.6.15(d)

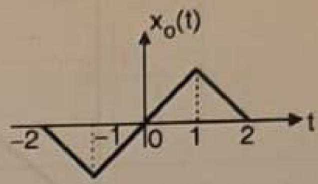


Fig. 1.6.15(d)

Ex. 1.6.16 : Sketch and label the even and odd components of the signals shown in Fig. P. 1.6.16.

May 10, 8 Marks

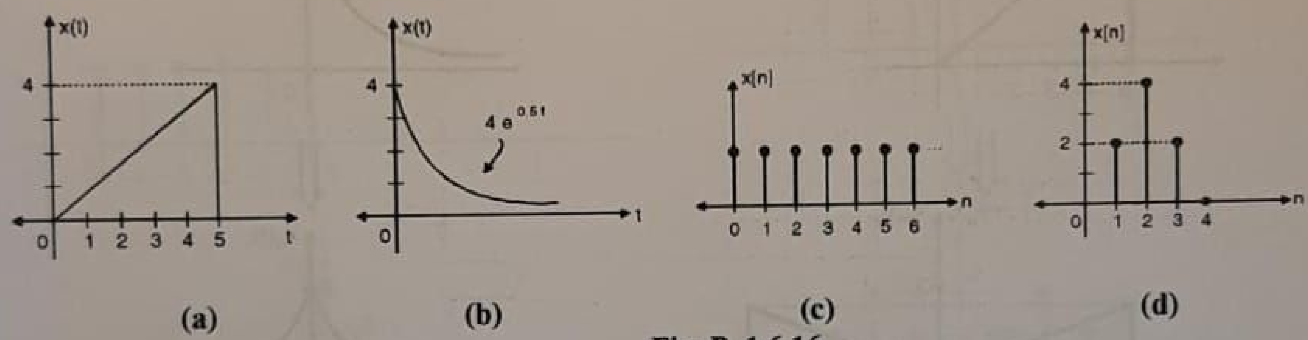


Fig. P. 1.6.16

Soln. :

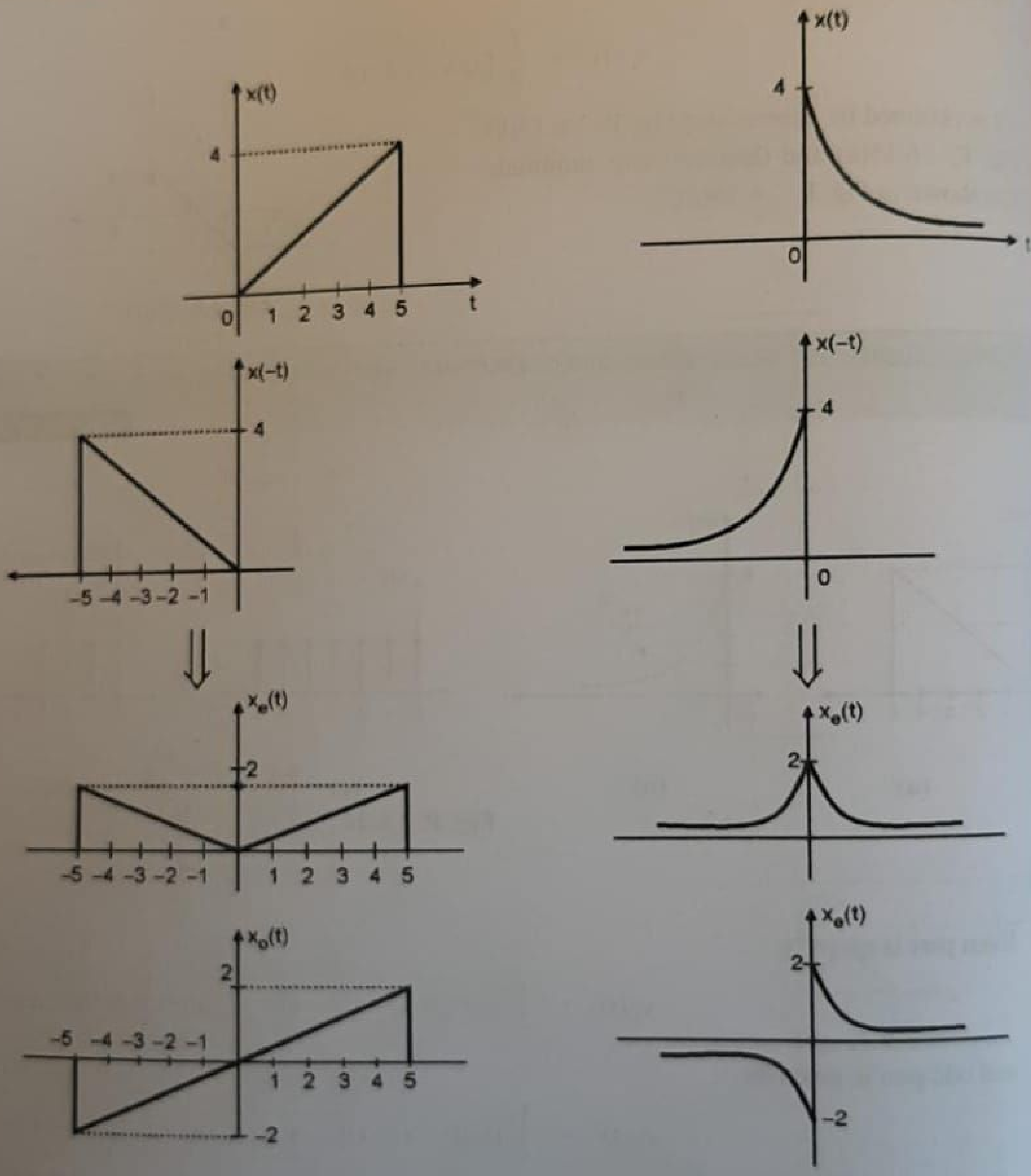
Even part is given by

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \quad \text{or} \quad x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

and odd part is given by

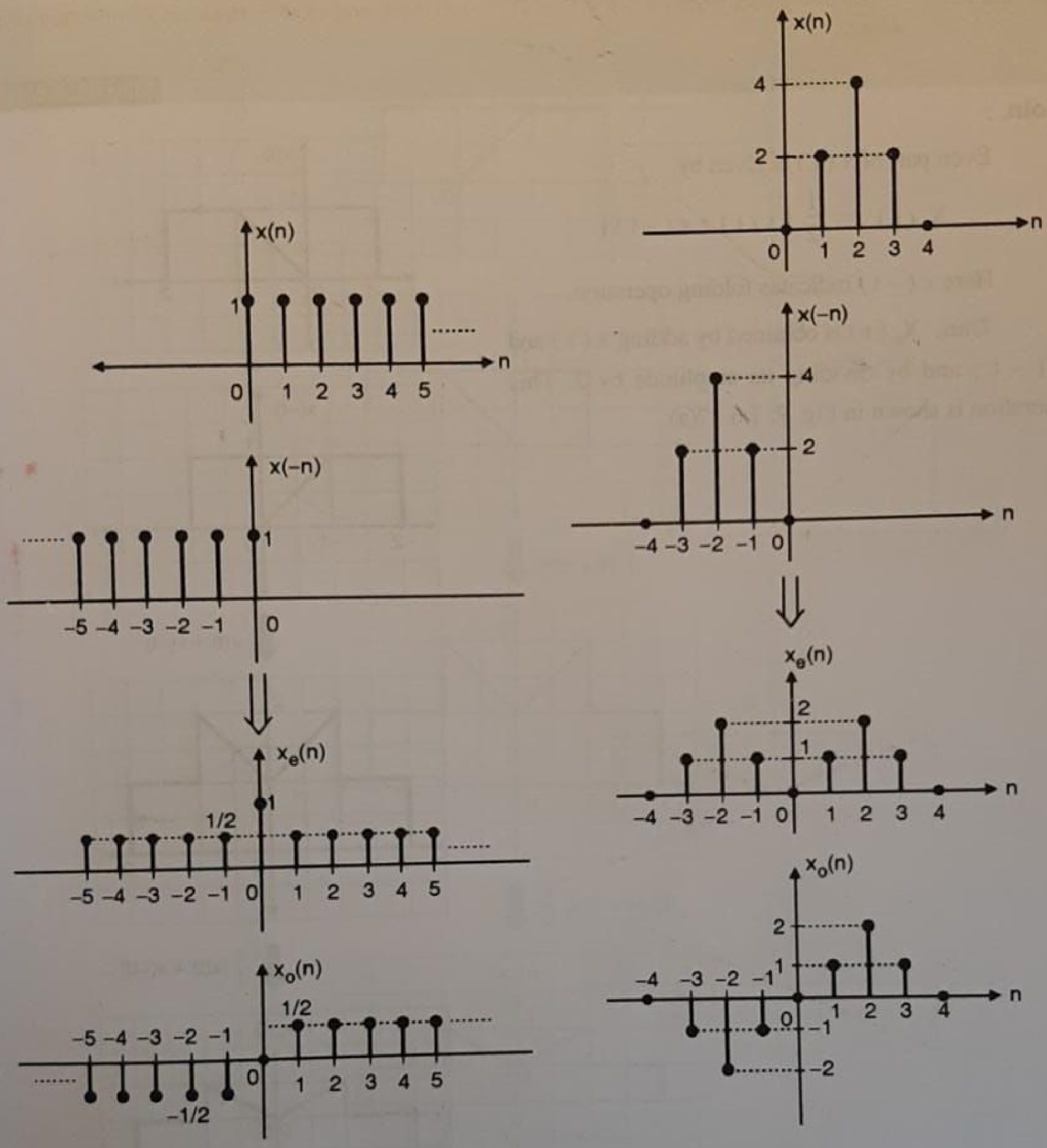
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] \quad \text{or} \quad x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

Even and odd parts are shown in following Figs. P. 1.6.16(e), P. 1.6.16(f), P. 1.6.16(g) and P. 1.6.16(h).


x(t)
4
0
1 2 3 4 5
t
x(−t)
4
−5 −4 −3 −2 −1
$x_e(t)$
2
−5 −4 −3 −2 −1 1 2 3 4 5
$x_o(t)$
2
−5 −4 −3 −2 −1
1 2 3 4 5
−2
x(t)
4
0
t
x(−t)
4
0
$x_e(t)$
2
$x_e(t)$
2
−2

(e) (f)

Fig. P. 1.6.16



(g) (h)

Fig. P. 1.6.16

Ex. 1.6.17 : From the given signal x (t), Fig. P. 1.6.17 find and sketch even and odd pairs of signal. **May 05, 4 M**

Soln. :

Even part of x (t) is given by,

$$X_e(t) = \frac{1}{2}[x(t)+x(-t)]$$

Here x (– t) indicates folding operation.

Thus, $X_e(t)$ is obtained by adding x (t) and x (– t) and by dividing its amplitude by 2. This operation is shown in Fig. P. 1.6.17(a).

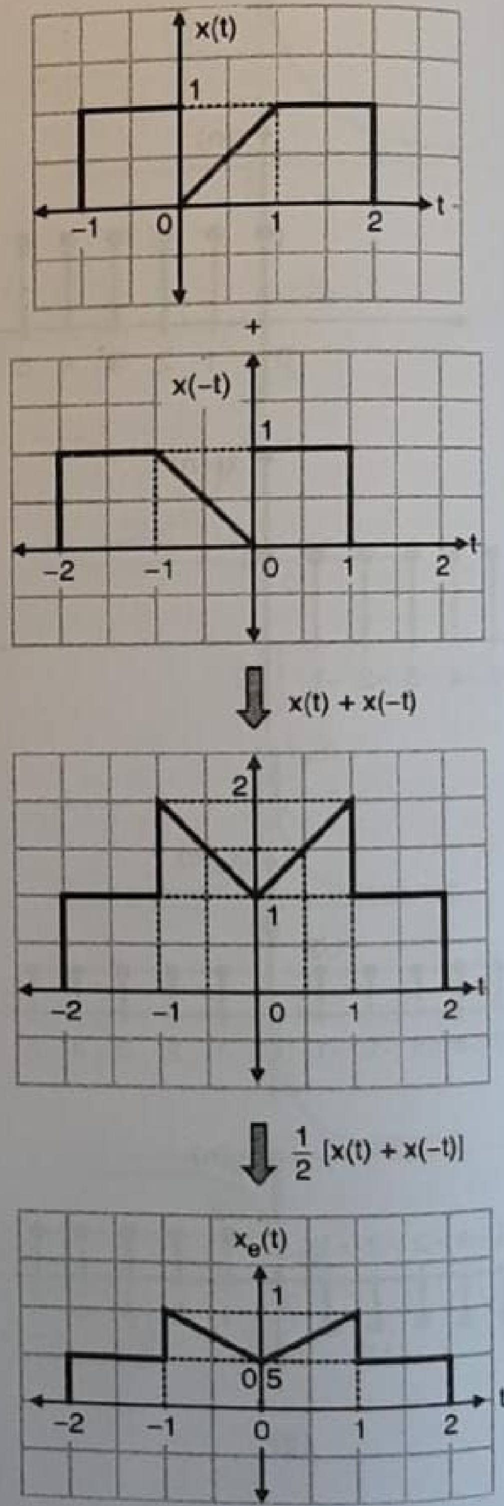


Fig. P. 1.6.17(a)

Odd part of x (t) is given as,

$$X_o(t) = \frac{1}{2}[x(t)+x(-t)]$$

Thus odd part of x (t) is obtained by adding x (t) and x (– t) and then by dividi amplitude by 2. This operation is

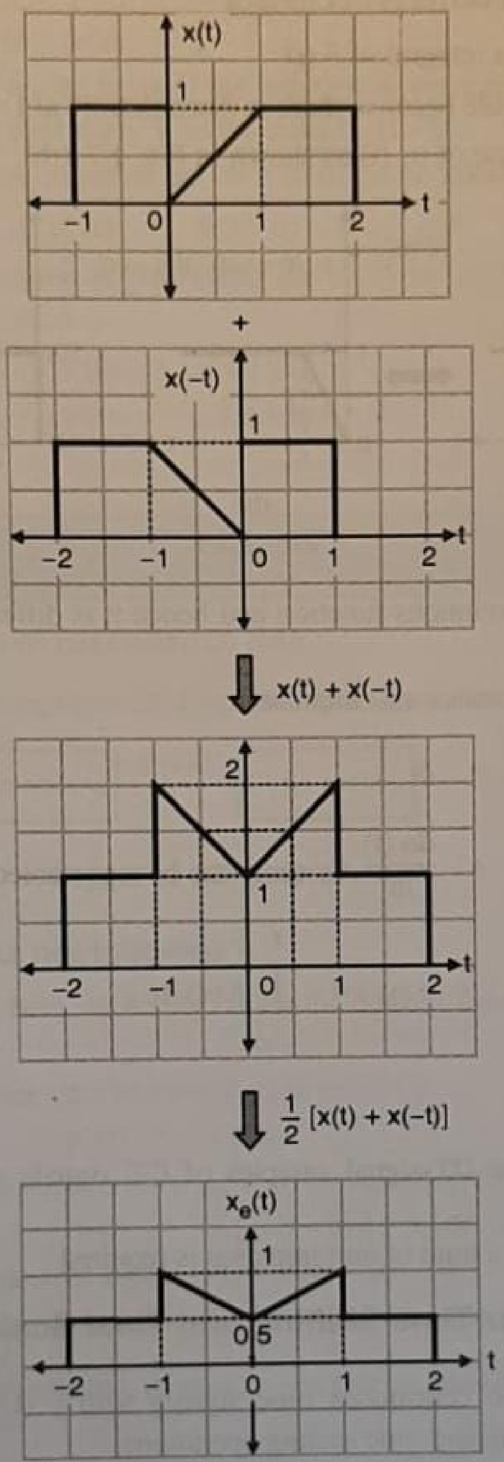


Fig. P. 1.6.17(b)

1.7 Relationship between Unit Step and Unit Impulse : **PU : May 12**

University Questions

Q. 1 Define unit impulse function and write its relation with unit step in CT and DT. **(May 12, 5 Marks)**

Unit step u (t) and unit impulse δ (t) of CT can be related as follows :

Mathematically $\delta(t)$ is derivative of $u(t)$ or

$u(t)$ is integral of $\delta(t)$

Unit step is not differentiable because there is discontinuity at $t = 0$ as shown in Fig. 1.7.1(a). Hence $u(t)$ has limiting case of $u_\Delta(t)$ as shown in Fig. 1.7.1(b).

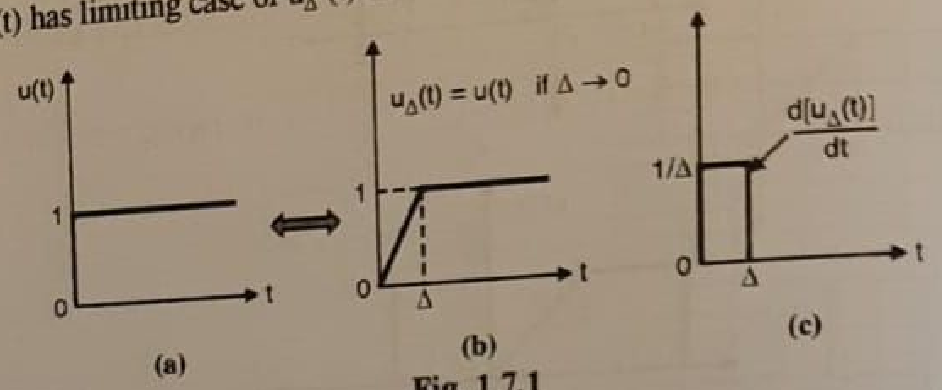


Fig. 1.7.1

- We know that $u_\Delta(t)$ is continuous function and hence it is differentiable. Derivative of u_Δ (t shown in Fig. 1.7.1(c).
- As $\Delta \to 0$ Fig. 1.7.1(c) becomes unit impulse.

$$\therefore \lim_{\Delta \to 0} \frac{d[u_\Delta(t)]}{dt} = \delta t$$

$$\therefore \delta(t) = \frac{du(t)}{dt}$$ Or same can be represented as

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

Importance of impulse :

- To convert CT-signal into DT-signal samples of CT signals are required to take at re interval of time.
- For sampling of CT signal a train of unit impulses is required.

1.7.1 Precedence Rule for Time Shifting and Time Scaling :

- Let $x(t)$ and $y(t)$ be two continuous time signals with $y(t)$ derived from $x(t)$ thro combination of time shifting and time scaling operations.
- Let the relation between $x(t)$ and $y(t)$ be given mathematically as follows :

$$y(t) = x(at - b) \quad ...(1$$

where a and b are constants.

- Let the relation between $x(t)$ and $y(t)$ satisfy the following two conditions :

Condition - 1 $\quad y(0) = x(-b)$

and

Condition - 2 $\quad y(b/a) = x(0)$

- In order to obtain y (t) correctly from x (t), it is necessary to perform the time shifting and ti scaling operations in the correct order.
- The proper order is decided on the basis of the fact that in the scaling operation "t" is replaced "at" and in the time shifting operation "t" is replaced by (t – b).
- So the time shifting operation is carried out on x (t) to get an intermediate signal y′ (t) as,

$$y'(t) = x(t-b) \quad \text{...(1.7}$$

- Then the time scaling operation is performed on y′ (t) by replacing t by "at". The result desired signal y (t).

$$\therefore \quad y(t) = y'(t)\big|_{t=at}$$

$$\therefore \quad y(t) = x(at-b) \quad \text{...(1.7}$$

Conclusion :

The precedence rule states that for obtaining y (t) from x (t) the proper order needs to followed. First time shifting and then the time scaling needs to be performed.

Diagrammatic representation of precedence rule :

The precedence rule can be represented diagrammatically as shown in Fig. 1.7.2.

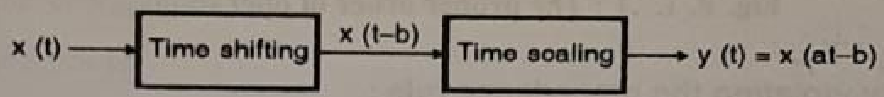


Fig. 1.7.2 : Precedence rule

What if the precedence rule is not followed ?

If the rule is not followed, then we do not obtain the desired signal y (t). This is demonstrated follows :

- First perform the time scaling operation on x (t) to get y′ (t)

$$y'(t) = x(at)$$

- Then perform the time shifting operation on y′ (t) by replacing t by (t – b).

$$\therefore \quad y(t) = x[a(t-b)] = x[at-ab]$$

- We have not obtained the desired signal y (t) = x (at – ab).
- To understand the precedence rule, solve the following example :

Ex. 1.7.1 : If x (t) = rect(t/3) then obtain y (t) = x (2t – 3) first by following the precedence rule and the by violating the rule.

Soln. :

Step 1 : Draw x (t) :

x (t) is given by,

$$x(t) = \text{rect}\left[\frac{t}{3}\right]$$

So it is a rectangular signal of width 3 time units and an amplitude equal to 1. It is plotted a shown in Fig. P. 1.7.1(a).

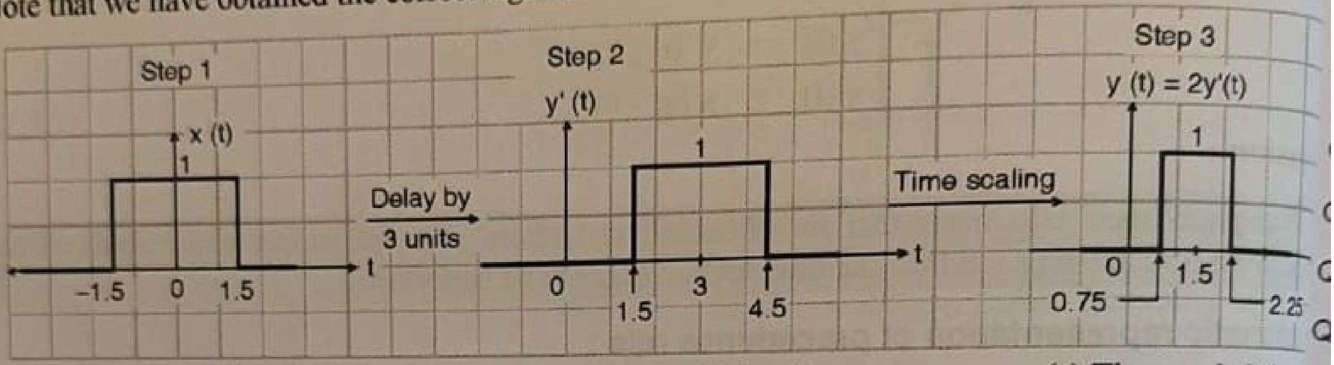
Part I : Solution by following the precedence rule :

Step 2 : Delay x (t) by 3 units :

As shown in Fig. P. 1.7.1(b), x (t) is delayed by 3 time units to obtain the interme[diate] signal $y'(t) = x(t-3)$.

Step 3 : Apply time scaling to y′ (t) :

Now apply time scaling to y′ (t) with a = 2 to get $y(t) = x(2t-3)$ as shown in Fig. P. 1.7.1 Note that we have obtained the correct signal.



(a) Original signal (b) Delayed signal (c) Time scaled sign[al]

Fig. P. 1.7.1 : The proper order of operations

Part II : Solution by violating the precedence rule :

Step 4 : Draw the original signal :

Refer Fig. P. 1.7.1(d).

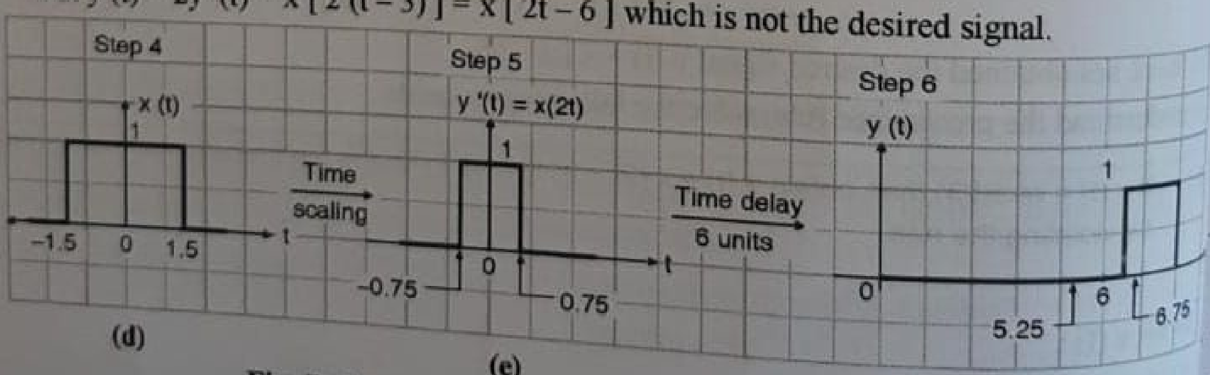
Step 5 : Time-scaling :

Refer Fig. P. 1.7.1(e). The time scaled signal is $y'(t) = x(2t)$.

Step 6 : Time-shifting :

Refer Fig. P. 1.7.1(f). The intermediate signal y′(t) has been delayed by 3 time un[its to] obtain y (t).

But $y(t) = 2y'(t) = x[2(t-3)] = x[2t-6]$ which is not the desired signal.



(d) (e) (f)

Fig. P. 1.7.1 : Solution by violation of precedence rule

Review Questions

Q. 1 Define signal.

Q. 2 Give the classification of signals.

Q. 3 Define continuous time (CT) and discrete time (DT) signals.

Q. 4 Differentiate between CT and DT signals.

Q. 5 How is a CT signal represented mathematically ?

Q. 6 How do you obtain a DT signal from a CT signal ?

Q. 7 Define a digital signal.

Q. 8 Define periodic and nonperiodic signals.

Q. 9 State the condition of periodicity.

Q. 10 Compare periodic and nonperiodic signals.

Q. 11 Prove that $x(t) = A \cos \omega_o t$ is a periodic signal.

Q. 12 Define deterministic and random signals.

Q. 13 Compare deterministic and random signals.

Q. 14 Define even and odd signals.

Q. 15 Compare energy and power signals.

Q. 16 Define signal energy.

Q. 17 Define signal power.

Q. 18 Define multichannel and multidimensional signals.

Q. 19 Define the following signals and write their mathematical expressions :

1. DC signal
2. Exponential signal
3. Rectangular pulse
4. Signum function.

Q. 20 State mathematical expression for unit step and rectangular signals.

Q. 21 State the important properties of delta function.

Q. 22 State the relation between DT unit impulse and DT unit step signals.

Q. 23 Define a unit ramp signal and draw it graphically.

Q. 24 State different types of complex exponential signal.

Q. 25 What is the relation between the complex exponential and sinusoidal signals.

Q. 26 Draw the $x(t) = \text{sinc}(t)$ signal graphically.

Q. 27 Define the following :

1. Time shifting.
2. Time scaling

Q. 28 Define folding or time reversal

Q. 29 What is the difference between amplitude scaling and time scaling ?

Q. 30 State the relationship between unit step and unit impulse signals.

Q. 31 State the precedence rule for time shifting and time scaling.

Q. 32 Define convolution and state its significance.

Q. 33 What is the practical application of convolution?

Q. 34 State convolution theorems.

Q. 35 Define a sinc signal and mathematically obtain its values at $t = 0, \pm 1, \pm 2$ .... Plot the function.

Q. 36 Explain the transformation of independent variables.