Discovering signals in fMRI data; a Bayesian nonparametric approach

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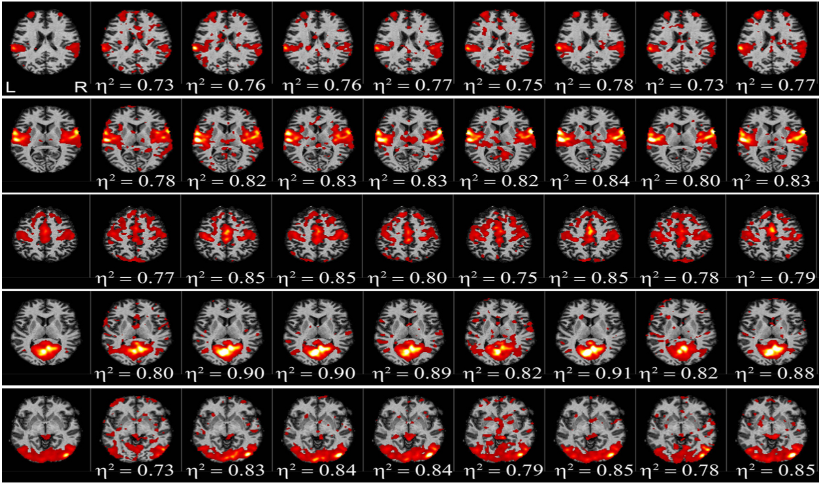
Outline

What is fMRI?

- fMRI: functional magnetic resonance imaging
- measures the change in brain blood flow while subject is stimulated
- data tends to look like (voxel, time, intensity of reading)
- e.g., to identify regions of the brain associated with hunger, fMRI readings can be taken while hungry subjects are shown pictures of food.

Example



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η² = 0.85

Problems

- multiple comparison problem - thousands of readings over time and space
- want to incorporate location and time information - want continuous significant regions, not voxels
- want to determine these significant regions adaptively

Outline

The idea

- focus on data with p-values rather than intensities
- assume that data is generated according to a prior which induces clustering of significant p-values
- get the posterior distribution (somehow)
- p-values which are in significant clusters are discoveries

The prior we assumed

- number of signal clusters: $k \sim$ Truncated Poisson$(\lambda, 1, k_{max})$
- signal centers: $c_j \sim$ Uniform over location-time space for $j = 1, ..., k$
- signal radius:
  $r_j \sim$ Truncated Normal$(\mu, \sigma, r_{min}, r_{max})$ for j = 1, . . . k.
- signal strength: $\beta_j \sim$ Uniform$(\beta_{min}, \beta_{max})$ for j = 1, . . . , k.
- p-values in signal clusters: $p_i \sim$ Beta$(\frac{1}{\beta_j}, \beta_j)$, when x_i is in cluster j.
- p-values not in signal clusters: $p_i \sim$ Uniform$(0, 1)$.

How do we write down the posterior??

- we don't - we sample from it
- we invented a Markov chain, inspired by Stephens (2000), with posterior distribution given by the likelihood of the data
- the type of chain is a birth-death chain (over an infinite dimensional state space!)

The rough algorithm

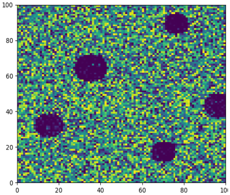
- initialize clusters and labels randomly by sampling from the prior
- repeat the following 10,000 times
  - flip a weighted coin to determine if a birth or death occurs
  - if birth, add a new cluster randomly
  - if death, delete a cluster which doesn't explain the data well
- take the average of the last 5,000 labels to determine if a p-value is signal or null.

Outline

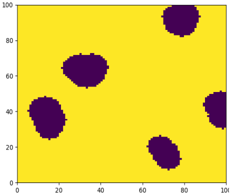
Toy Data: Preliminaries

- 100-by-100 grid with k=5 clusters of signals and the rest is null.
- the centers $C_k \sim$ uniform from the grid while being distinct for $k = 1, 2, ..., 5$
- the radius $R_k \sim TN(7, 2, 5, 10)$ for $k = 1, 2, ..., 5$
- signals in clusters $p_{ki} \sim Beta(1, \beta_k)$ where $\beta_k \sim TN(8, 5, 2, 200)$ for $k = 1, 2, ..., 5$

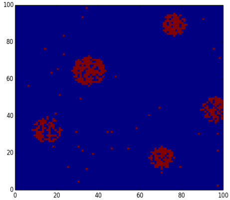
Toy Data (continued): Performance



(a) Actual grid



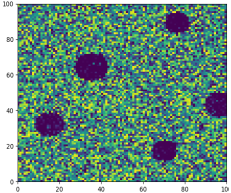
(b) Birth death chain



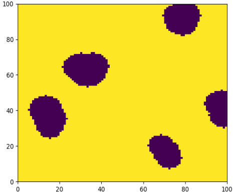
(c) BH

Figure: Simulated data

Toy Data (continued): More on Priors



(a) Actual grid

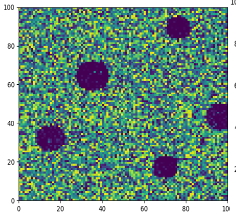


(b) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(8, 5, 2, 200)$

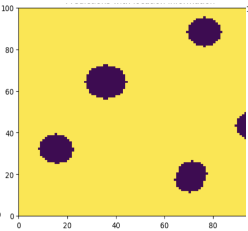
Figure: Simulated data

Toy Data (continued): More on Priors

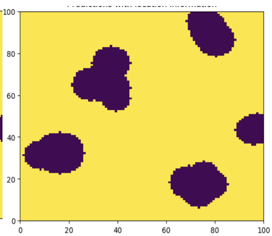
First, let's compare different priors on beta



(a) Actual grid



(b) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(30, 3, 2, 200)$

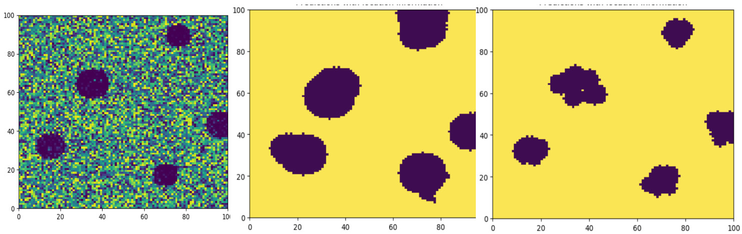


(c) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(2, 1, 2, 200)$

Figure: Simulated data

Toy Data (continued): More on Priors

Now, let's compare different priors on radius.



(a) Actual grid

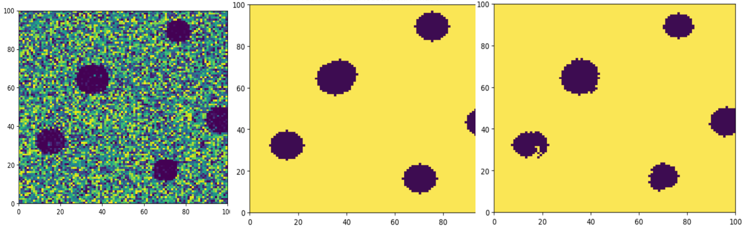
(b) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(10, 2, 7, 13)$; $\beta \sim TN(8, 5, 2, 200)$

(c) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(2, 2, 1, 10)$; $\beta \sim TN(8, 5, 2, 200)$

Figure: Simulated data

Toy Data (continued): More on Priors

What if we make signal stronger?



(a) Actual grid

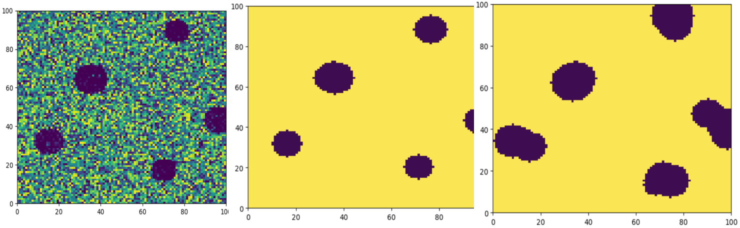
(b) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(10, 2, 7, 13)$; $\beta \sim TN(30, 3, 2, 200)$

(c) $k \sim Trun(Poi(50), 1, 100)$; $r \sim TN(2, 2, 1, 10)$; $\beta \sim TN(30, 3, 2, 200)$

Figure: Simulated data

Toy Data (continued): More on Priors

Next, we change the priors on number of clusters.



(a) Actual grid

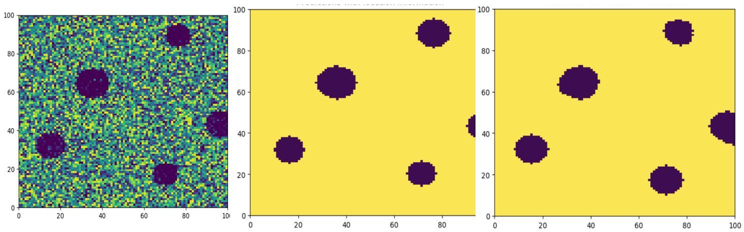
(b) $k \sim Trun(Poi(3), 1, 10)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(8, 5, 2, 200)$

(c) $k \sim Trun(Poi(300), 1, 1000)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(8, 5, 2, 200)$

Figure: Simulated data

Toy Data (continued): More on Priors

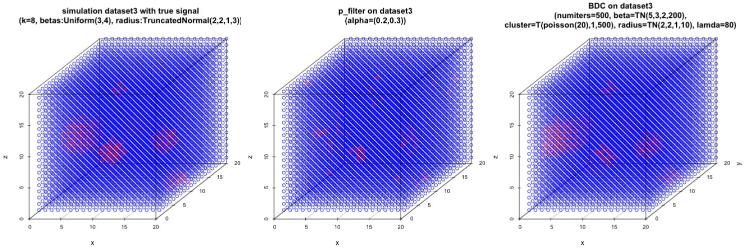
Again, let's make signal stronger



(a) Actual grid (b) $k \sim Trun(Poi(3), 1, 10)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(30, 3, 2, 200)$ (c) $k \sim Trun(Poi(300), 1, 1000)$; $r \sim TN(7, 2, 5, 10)$; $\beta \sim TN(30, 3, 2, 200)$

Figure: Simulated data

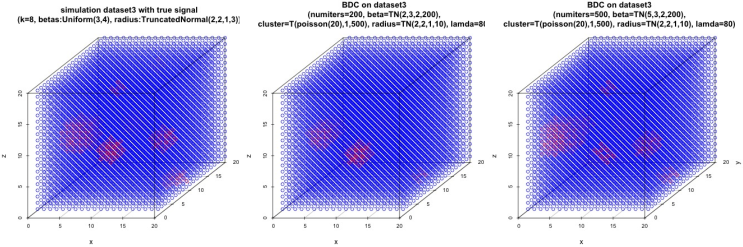
Toy Data 2 (3-D)



(a) (b) (c)

Figure: Simulated data

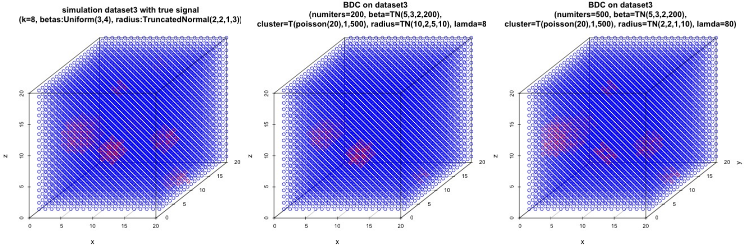
Toy Data 2 (3-D)



(a) (b) (c)

Figure: Simulated data

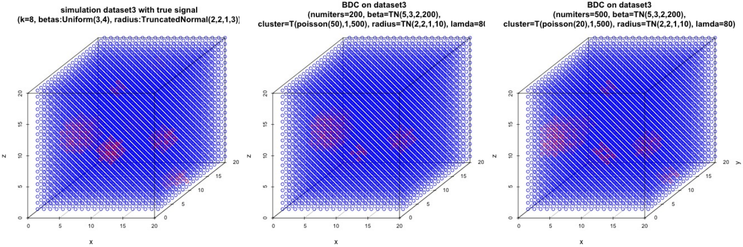
Toy Data 2 (3-D)



(a) (b) (c)

Figure: Simulated data

Toy Data 2 (3-D)



(a) (b) (c)

Figure: Simulated data

Conclusion and Future Work

- presented a nonparametric-bayesian method to adaptively identify clusters of signals.
- it showed promising results on both simulation and real fMRI data.
- possible extensions
  - work with intensities directly by specifying appropriate priors for null distributions and for signal distributions.
  - put priors on the hyper-parameters and maximize these priors using EM

Thanks!