Discovering signals in fMRI data; a Bayesian nonparametric approach

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Project Goal

- Formulate a method which can adaptively identify clusters of signals in functional magnetic resonance imaging (fMRI) data.
- Evaluate the proposed method by drawing comparison between it and the existing p-filter algorithm.

What is fMRI data?

- fMRI data measures the change in brain blood flow associated with mental activity [HSM04].
- fMRI data is in the form (voxel, time, intensity of reading).
- Example: To identify regions of the brain associated with hunger, fMRI readings can be taken while hungry subjects are shown pictures of food.
- Multiple comparison problem due to hundreds of thousands of voxels
- Identify significant clusters (not just individual voxels)

What's our method

- Inspired by Stephens (2000), we describe a bayesian nonparametric method by creating a Markov birth-death process with stationary distribution to detect clusters of signals.
- View each cluster as a point in parameter space.
- Posterior distribution of the parameters being stationary distribution.
- Theoretically, this method works for multiple-dimensional data which incorporates spatial and temporal information.

Details of the Method: Priors

- number of signal clusters: $k \sim$ Truncated Poisson$(\lambda, 1, k_{max})$.
- signal centers: $c_j \sim U(\mathcal{D})$ for $j = 1, ..., k$.
- signal radius:
  $r_j \sim$ Truncated Normal$(\mu, \sigma, r_{min}, r_{max})$ for j = 1, . . . k.
- signal strength: $\beta_j \sim U(\beta_{min}, \beta_{max})$ for j = 1, . . . , k.
- p-values in signal clusters: $p_i \sim Beta(\frac{1}{\beta_j}, \beta_j)$, when x_i is in cluster j.
- p-values not in signal clusters: $p_i \sim U(0, 1)$.

Details of the Method (continued): inventing the chain

- Birth: generating a new cluster.
- Death: "killing" an existing cluster.
- Birth rate: constant λ is pre-defined and independent of clusters.
- Death rate: μ_i depends on "current" clusters and is updated each step.
- Flip a weighted coin to decide birth (w/ prob $\frac{\lambda}{\lambda+\mu_i}$) or death (w/ prob $\frac{\mu_i}{\lambda+\mu_i}$).

Details of the Method (continued): death rate calculation using likelihoods

- K clusters with prior $Beta(\frac{1}{\beta_j}, \beta_j)$ for $j = 1, 2, ..., K$. K itself is random with prior F_K.
- Label specify which cluster each data point belongs.
- Current cluster likelihood: $l = logL(data|Beta(\frac{1}{\beta_j}, \beta_i)'s, labels)$; $c = logL(K|F_K)$
- Cluster likelihood after "killing" cluster j: $l_{-j} = logL(data|Beta(\frac{1}{\beta_j}, \beta_i)'s, labels_{-j})$; $c_{-j} = logL(K-1|F_K)$
- $u_j = log(\lambda) + (l_{-j} - l) + (c_{-j} - log(K) - c)$ for $j = 1, 2, ..., K$.
- $u = \sum_{j=1}^{K} e^{u_j}$.

Details of the Method (continued)

- At the end of each step, run metropolis-hasting algorithm to sample from the posterior of the beta distribution
- Purpose: TODO

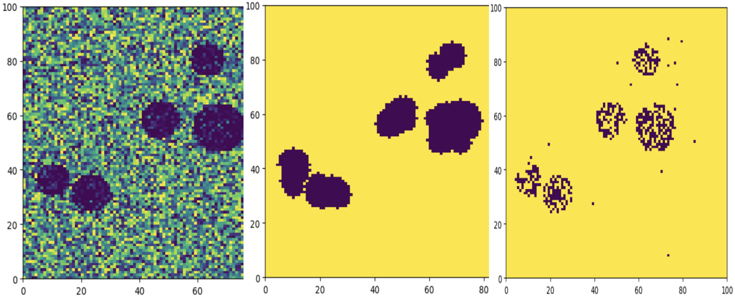
Details of the Method (continued)

- Run the chain long enough before starting collect sample labels.
- Sample labels from evenly space grid along the chain to avoid autocorrelation.
- Average over sample labels to determine if it is signal or null.

Toy Data: Preliminaries

- 100-by-100 grid with k=5 clusters of signals and the rest is null.
- The centers $C_k \sim$ uniform from the grid while being distinct for $k = 1, 2, ..., 5$
- The radius $R_k \sim TN(7, 2, 5, 10)$ for $k = 1, 2, ..., 5$
- Signals in clusters $p_{ki} \sim Beta(1, \beta_k)$ where $\beta_k \sim U(2, 5)$ for $k = 1, 2, ..., 5$

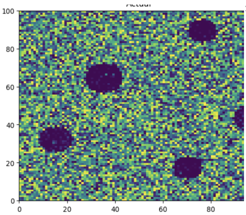
Toy Data (continued): Performance



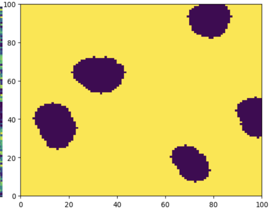
(a) Actual grid (b) Birth death chain (c) p-filter

Figure: Simulated data

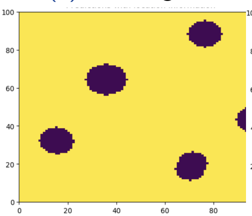
Toy Data (continued): More on Priors



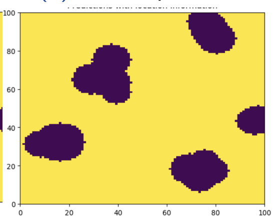
(a) Actual grid



(b) default priors



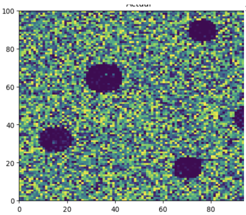
(c) $\beta \sim TN(30, 3, 2, 200)$



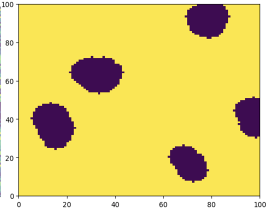
(d) $\beta \sim TN(2, 1, 2, 200)$

Figure: Simulated data

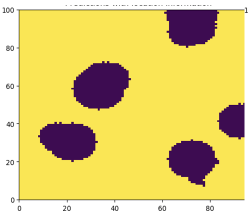
Toy Data (continued): More on Priors



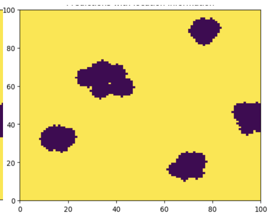
(a) Actual grid



(b) default priors



(c) $r \sim TN(10, 2, 7, 13)$

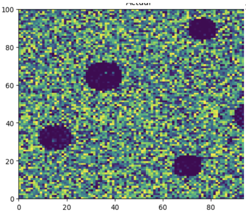


(d) $r \sim TN(2, 2, 1, 10)$

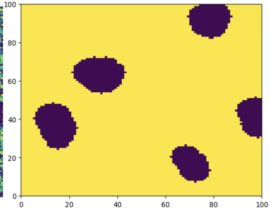
Figure: Simulated data

Toy Data (continued): More on Priors

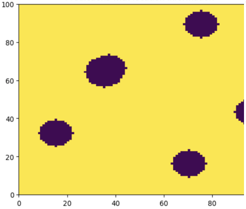
What if we make signal stronger?



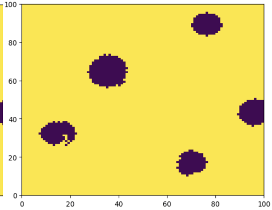
(a) Actual grid



(b) default priors

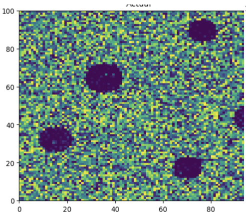


(c)
$\beta \sim TN(30, 3, 2, 200)$;
$r \sim TN(10, 2, 7, 13)$

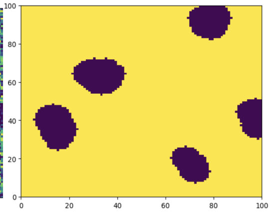


(d)
$\beta \sim TN(30, 3, 2, 200)$;
$r \sim TN(2, 2, 1, 10)$

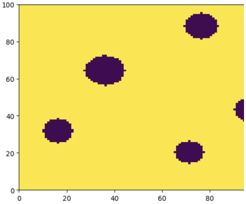
Toy Data (continued): More on Priors



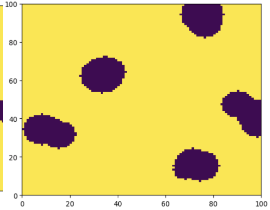
(a) Actual grid



(b) default priors



(c) $k \sim Trun(Poi(3), 1, 10)$

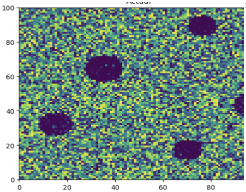


(d) $k \sim Trun(Poi(300), 1, 1000)$

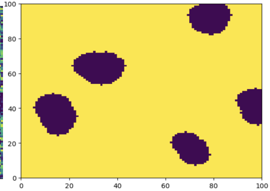
Figure: Simulated data

Toy Data (continued): More on Priors

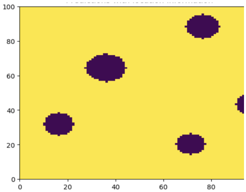
Again, let's make signal stronger



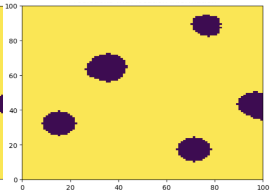
(a) Actual grid



(b) default priors

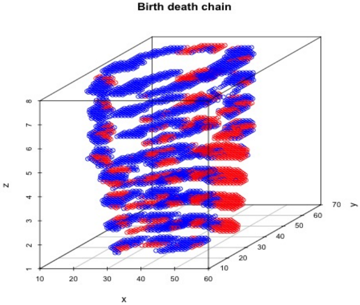


(c) $\beta \sim TN(30, 3, 2, 200)$; $k \sim Trun(Poi(3), 1, 10)$

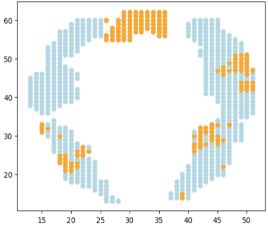


(d) $\beta \sim TN(30, 3, 2, 200)$; $k \sim Trun(Poi(300), 1, 1000)$

Performance on real fMRI data



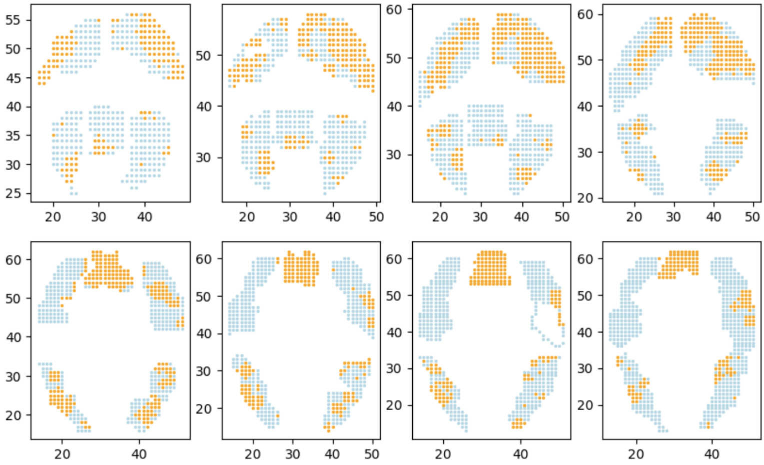
(a) 3-D view



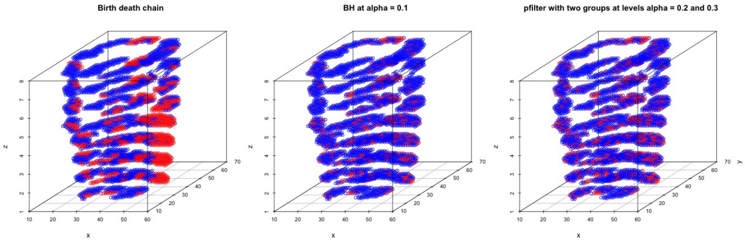
(b) 2-D view: one slice

Figure: fMRI data

Performance on real fMRI data (continued)



Performance on real fMRI data (continued): Comparison to p-filter



(a) Birth death chain (b) BH (c) p-filter with $\alpha =$ 0.2,0.3

Figure: fMRI data

Conclusion and Future Work

- Formulated and tested a nonparametric bayesian method to adaptively identify clusters of signals.
- Showed promising results on both simulation and real fMRI data.
- Extend from p-values to intensities directly by specifying appropriate priors for null distributions and for signal distributions.
- Put priors on the hyper-parameters and maximize this priors using EM. That is uniform prior over hyper-parameters.

Thanks!