Calculus on Surfaces

Ingo Nitschke

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Contents

1 Tensor Calculus

1.1 n-Tensors

	Full	Components	Mathemathica
n-Tensor[1]	$t = t^{(s_1,\ldots,s_n)}$	$t\,\overset{\uparrow}{\underset{\downarrow}{i_1}}\,\overset{\uparrow}{\underset{\downarrow}{i_2}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_n}}$ [2]	$\left\{\{t\,\overset{\uparrow}{\underset{\downarrow}{i_1}}\,\overset{\uparrow}{\underset{\downarrow}{i_2}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_n}}\}_{i_1,\ldots,i_n=1,2}, \{s_1,\ldots,s_n\}\right\}$
e. g.	$t = t^{(1,0,1,1,0)}$	$t^{i}{}_{j}{}^{kl}{}_{m}$	$\left\{\{t^{i}{}_{j}{}^{kl}{}_{m}\}_{i,j,k,l,m=1,2}, \{1,0,1,1,0\}\right\}$
Swap-Transpose $T_{k,l}$	$t^{T_{k,l}} = T_{k,l}(t)$	$t^{\cdots}{}_{i_k}{}^{\cdots}{}_{i_l}{}^{\cdots} \mapsto t^{\cdots}{}_{i_l}{}^{\cdots}{}_{i_k}{}^{\cdots}$ $t_{\cdots}{}^{i_k}{}_{\cdots}{}^{i_l}{}_{\cdots} \mapsto t_{\cdots}{}^{i_l}{}_{\cdots}{}^{i_k}{}_{\cdots}$	
Push-Transpose $T_{l\to k}$	$t^{T_{l\to k}} = T_{l\to k}(t)$	$t\cdots\overset{\uparrow}{\underset{\downarrow}{i_k}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_l}}\cdots \mapsto t\cdots\overset{\uparrow}{\underset{\downarrow}{i_l}}\,\overset{\uparrow}{\underset{\downarrow}{i_k}}\cdots$	`TransFromToTM[TensorMatrix,l,k]`
e. g.	$t^{T_{4\to 2}} = T_{4\to 2}(t)$	$t^{i}{}_{j}{}^{kl}{}_{m} \mapsto t^{il}{}_{j}{}^{k}{}_{m}$	
Contraction $C_{k,l}$	$C_{k,l}(t)$	$t\cdots\overset{\uparrow}{\underset{\downarrow}{i_k}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_l}}\cdots \mapsto t\cdots{}_{j}\cdots{}^{j}\cdots = t\cdots{}^{j}\cdots{}_{j}\cdots$	`ContractT[Tensor,k,l]`
Outer Product $\otimes$	$t\otimes s$	$t\,\overset{\uparrow}{\underset{\downarrow}{i_1}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_n}}\,,\, s\,\overset{\uparrow}{\underset{\downarrow}{j_1}}\cdots\overset{\uparrow}{\underset{\downarrow}{j_m}} \mapsto t\,\overset{\uparrow}{\underset{\downarrow}{i_1}}\cdots\overset{\uparrow}{\underset{\downarrow}{i_n}}\, s\,\overset{\uparrow}{\underset{\downarrow}{j_1}}\cdots\overset{\uparrow}{\underset{\downarrow}{j_m}}$	`OuterT[Tensor,Tensor]`

	Full	Components	Mathemathica
Partial Derivative ∂	$(\partial t)^{T_{1\to n}}$	$t\cdots{}_{,i} = \partial_i t\cdots$	`D[TensorMatrix,var[[i]]]`
Covariant Derivative ∇	$\nabla t^{(s_1,\ldots,s_n)} = (\partial t)^{T_{1\to n}} - \sum_{i=1}^{n}(-1)^{s_i}\left[C_{i,n-s_i+3}(t\otimes\Gamma)\right]^{T_{n+s_i\to i}}$	$t\cdots{}_{\vert i} = \nabla_i t\cdots$	`CoDT[Tensor]`

1.2 1-Tensors (vectors / 1-forms)

	Full	Components
∇	$\nabla t^{(0)}$	$t_{i\vert k} = t_{i,k} - \Gamma_{ik}{}^{l}t_l$
	$\nabla t^{(1)}$	$t^{i}{}_{\vert k} = t^{i}{}_{,k} + \Gamma_{lk}{}^{i}t^l$

[1] of type $(s_1,\ldots,s_n)$, with $s_i = 0, 1$. Hence, if $m = \sum_{i=1}^{n} s_i$, then $t \in \mathcal{T}^{m}_{n-m}(\mathcal{S})$

[2] $\uparrow$ if $s_k = 1$ (i_k is contravariant index); $\downarrow$ if $s_k = 0$ (i_k is covariant index)