1 Frank Oseen Energy

In $\mathbb{R}^3$:

$$E = \frac{1}{2} \int_\Omega K_1 \left(\nabla \cdot \mathbf{p} \right)^2 + K_2 \left(\mathbf{p} \cdot \left[\nabla \times \mathbf{p} \right] \right)^2 + K_3 \left\| \mathbf{p} \times \left[\nabla \times \mathbf{p} \right] \right\|^2 dV \tag{1}$$

With the Langrange identity for the K_3-term, we cann rewrite (1) to

$$E = \frac{1}{2} \int_\Omega K_1 \left(\nabla \cdot \mathbf{p} \right)^2 + \left(K_2 - K_3 \right) \left(\mathbf{p} \cdot \left[\nabla \times \mathbf{p} \right] \right)^2 + K_3 \left\| \mathbf{p} \right\|^2 \left\| \nabla \times \mathbf{p} \right\|^2 dV \tag{2}$$

If we restrict (2) to a 2-dimensional Manifold $M \subset \Omega$ and postulate that $\mathbf{p} \in T_X M$ is a normalized tangential vector in $X \in M$, we get

$$E = \frac{1}{2} \int_M K_1 \left(\mathrm{Div} \mathbf{p} \right)^2 + K_3 \left(\mathrm{Rot} \mathbf{p} \right)^2 dA \tag{3}$$

In terms of exterior calculus with the corresponding 1-form $\mathbf{p}^\flat \in \Lambda^1(M)$, we obtain

$$E = \frac{1}{2} \int_M K_1 \left(\mathbf{d}^* \mathbf{p} \right)^2 + K_3 \left(\mathbf{d} \mathbf{p} \right)^2 dA \tag{4}$$

where $\mathbf{d}^* := - * \mathbf{d} *$ is the L^2-orthogonal operator of the exterior derivative $\mathbf{d}$.