Notes On q-Projection and Helmholtz-deRham-Equation on ellipsoid

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1 q-Projection

We call $\mathcal{Q}(\mathcal{S})$ the space of trace-free symmetric surface 2-tensors, shortly surface q-tensors, i. e.

$$\mathcal{Q}(\mathcal{S}) = \left\{ \mathbf{q} \in \mathcal{T}^{(2)}(\mathcal{S}) \subset \mathcal{T}^{(2)}(\mathbb{R}^3) \cong \mathbb{R}^{3\times 3} \middle| \operatorname{Tr}(\mathbf{q}) = 0, \mathbf{q}^T = \mathbf{q} \right\} \tag{1}$$

$$= \left\{ \mathbf{q} \in \mathcal{T}^{(2)}(\mathbb{R}^3) \middle| \operatorname{Tr}(\mathbf{q}) = 0, \mathbf{q}^T = \mathbf{q}, \mathbf{q} \cdot \boldsymbol{\nu} = 0 (, \boldsymbol{\nu} \cdot \mathbf{q} = 0) \right\} . \tag{2}$$

With the surface projection

$$\pi_{\mathcal{S}} = \mathbf{I} - \boldsymbol{\nu} \otimes \boldsymbol{\nu} : \mathsf{T}\mathbb{R}^3 \cong \mathbb{R}^3 \to \mathsf{T}\mathcal{S} \tag{3}$$

we introduce the q-projection $\pi_{\mathcal{Q}(\mathcal{S})} : \mathcal{T}^{(2)}(\mathbb{R}^3) \cong \mathbb{R}^{3\times 3} \to \mathcal{Q}(\mathcal{S})$ with

$$\pi_{\mathcal{Q}(\mathcal{S})}(\mathbf{M}) := \pi_{\mathcal{S}} \cdot \left(\mathbf{M} + \mathbf{M}^T \right) \cdot \pi_{\mathcal{S}} - (\pi_{\mathcal{S}} : \mathbf{M}) \, \pi_{\mathcal{S}} \, . \tag{4}$$

It is clear, that $\pi_{\mathcal{Q}(\mathcal{S})}(\mathbf{M})$ is a surface tensor and symmetric, because $\pi_{\mathcal{S}}$ is a symmetric surface tensor. With $\operatorname{Tr}(\pi_{\mathcal{S}} \cdot \mathbf{M} \cdot \pi_{\mathcal{S}}) = \pi_{\mathcal{S}} : \mathbf{M} = \operatorname{Tr}(\pi_{\mathcal{S}} \cdot \mathbf{M}^T \cdot \pi_{\mathcal{S}})$ and $\operatorname{Tr}(\pi_{\mathcal{S}}) = 2$ we obtain the trace-freeness of $\pi_{\mathcal{Q}(\mathcal{S})}(\mathbf{M})$. Es waere zu ueberlegen ob man beim q-tensor Modell fr $\mathbb{R}^3$-Tensoren nur die schwaechere(?) Bedingung fordert, dass der surface-trace $\pi_{\mathcal{S}} : \mathbf{q}$ null sei statt der volle $\mathbb{R}^3$-Trace $\operatorname{Tr}(\mathbf{q})$ (vgl. Analogie zu Richtungsfelder: $\|\pi_{\mathcal{S}}\mathbf{p}\|_{\mathbb{R}^3} = \|\mathbf{p}\|_{\mathsf{T}\mathcal{S}} = 1$ vs. $\|\mathbf{p}\|_{\mathbb{R}^3} = 1$).

2 Notes on Laplace-deRham and other derivative stuff

3 Ellipsoid

Let be $\mathcal{S}$ a ellipsoid with semi-principal axes $(1, 0.5, 1.5)$.