

# Orientation Fields on Closed Surfaces

## A Discrete Exterior Calculus Primal Dual (DEC-PD) Approach

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## 1 Surface Discretization (Simplicial Complex)

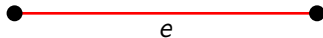
The surface mesh is made of simplices  $\sigma = v, e, f$ :

- **vertices**, edges, (triangle) faces



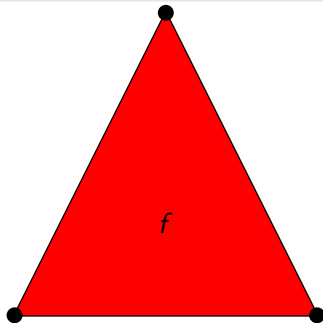
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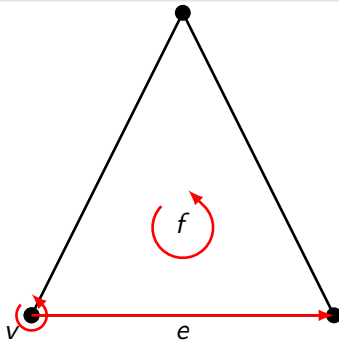
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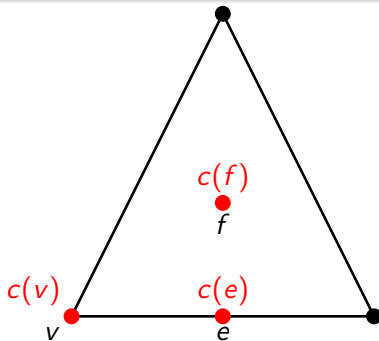
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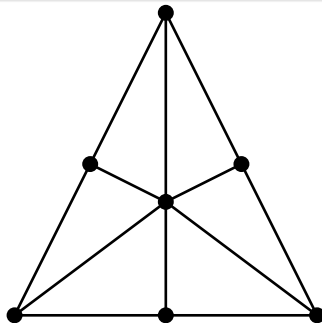
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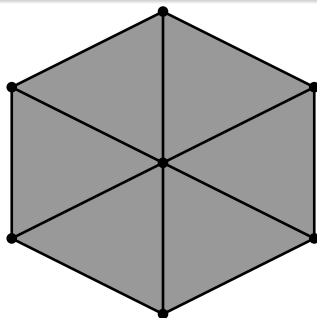
- vertices, edges, (triangle) faces
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- are refinable (circumcentric subdivision)





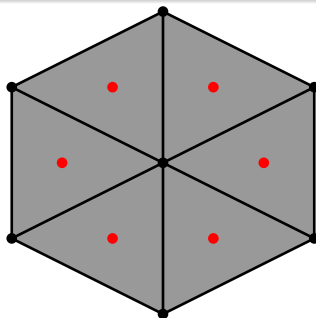
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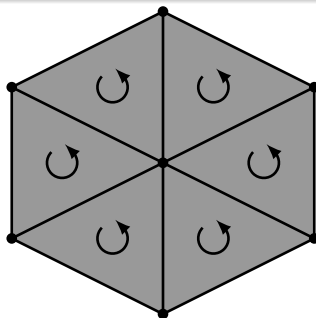
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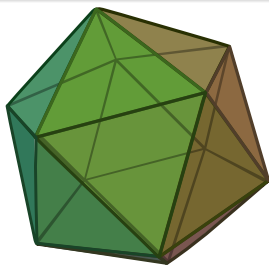
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- **oriented**: neighboured faces have the same orientation
- **manifold-like**: polyhedron  $\bigcup_{f \in \mathcal{F}} f$  is a  $C^0$ -manifold



<https://commons.wikimedia.org/wiki/File:Icosahedron.svg>