Curvature approximation on surfaces – A Discrete Exterior Calculus Approach

I. Nitschke[1], A. Voigt[1,2]

Abstract

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Keywords: Surfaces, Curvature, DEC

1. Introduction

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2. Discrete Exterior Calculus (DEC)

The Discrete Exterior Calculus [3, 2] defines discrete differential p-forms on a triangulated mesh (simplicial complex). For surface meshes, i.e. triangulated orientable 2-manifolds, the degree of the discrete p-forms is 0, 1 or 2 and their are represented by scalars on vertices, edges, triangles or chains of them. Operators for the differential forms, like the exterior derivation $\mathbf{d}$ or the hodge star $\star$, can be approximated by expressions on the discrete geometrical structure. E.g. the integral over a triangle of the exterior derivation $\mathbf{d}$ for a 1-form can be expressed as the integral of the 1-form over the boundaries edges of the triangle. This follows directly from the Stokes Theorem [1, Ch. 7].

2.1. Discrete manifolds

In our setting, the surface meshes are linear triangulations of orientable closed 2-manifolds. Such a Triangulation are sets of p-simplices $\{\sigma^p\}$ of the same degree p, e.g. sets of vertices, edges and triangles, and form a simplicial complex of dimension $n = 2$. A simplicial complex K comply two essential rules:

1. Every face of a simplex $\sigma^p \in K$ is in K.
2. The intersection of two simplices in K is also in K or empty.

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Email addresses: ingo.nitschke@tu-dresden.de (I. Nitschke), axel.voigt@tu-dresden.de (A. Voigt)

[1]Department of Mathematics, TU Dresden, 01062 Dresden, Germany

[2]Center of Advanced Modeling and Simulation, TU Dresden, 01062 Dresden, Germany