CHAPTER-4

RISK –RETURN ANALYSIS & MANAGEMENT

Lecture Flow:

- Concept of Uncertainty and Risk
- **Types of Risk**
- Measurement of Risk (Standard Deviation and Variance)
- Systematic and Unsystematic Risk
- Diversification and Portfolio Risk
- The Security Market Line(SML)
- **Markowitz's portfolio theory**
- The Capital Asset Pricing Model(CAPM) and Arbitrage Pricing Theory(APT)
- Fama-French 3-Factor Model

Learning Objectives

- ***After you read this chapter, you should be able to answer the following questions:***

✓What do we mean by ***risk, and what are some of the alternative measures*** *of risk used in* investments?

✓What do we mean by risk and *risk aversion*?

✓How do you compute the expected rate of return for an **individual risky asset** or **a portfolio of assets**?

Con't…

- How do you compute the standard deviation of rates of return for an individual risky asset?

- What do we mean by the ***covariance between rates of return, and how do you compute*** co-variance?

- What is the relationship between **covariance** **and** **correlation?**

- What is the formula for the **standard deviation for a portfolio** of risky assets, and how does it differ from the ***standard deviation of an individual risky asset?***

Con't…

- What are the basic assumptions behind the **Markowitz portfolio theory?**
- Given the formula for the standard deviation of a portfolio, ***why and how do you diversify a portfolio?***
- What happens to the **standard deviation of a portfolio** when you **change the correlation between the assets in the portfolio?**
- What happens to the **standard deviation of a portfolio with *Constant Correlation* between the assets in the portfolio and *with Changing Weights* assets in the portfolio?**

Con't…

- **Risk analysis can be confusing, but it will help if you remember the following:**
- **1.** All financial assets are expected to produce *cash flows, and the risk of an asset is* judged in terms of the risk of its cash flows.
- **2.** The risk of an asset can be considered in two ways: (1) on a *stand-alone basis,* or (2) in a *portfolio context,*
- **3.** In a portfolio context, an asset's risk can be divided into two components: (a) *diversifiable Risk,* and (b) *market risk,*

Con't…

- **4.** An asset with a **high degree of relevant (market) risk** must provide a relatively **high expected rate of return** to attract investors.
  - Investors in general are ***averse to risk,*** *so* they will not buy risky assets unless those assets have **high expected returns**.

- **5.** In this chapter, we focus on ***financial assets*** *such as* ***stocks and bonds,*** *but the concepts* discussed here also apply to *physical assets such as* ***computers, trucks, or even*** **whole plants.**

What is Uncertainty ?

- Whenever you make a **financing or investment** decision, there is some **uncertainty** about the outcome.
- ***Uncertainty means not knowing exactly*** what will happen in the future.
- There is uncertainty in most everything we do as financial managers, because no one knows precisely what changes will occur in such things as
  - tax laws,
  - consumer demand,
  - the economy, or
  - interest rates etc.

Con't….

- Though the terms "**risk**" and "**uncertainty**" are often used to mean the same thing, there is a distinction between them.
- **Uncertainty** is not knowing what's going to happen. ***Risk is how we characterize how*** *much uncertainty exists: The greater the uncertainty, the greater the* risk.
  - Risk is the degree of uncertainty.

Risk Analysis

Risk --- What is this?

- **Consider the two cases.**
- 1)Mr Ramesh has put his money in National Bank of Ethiopia (NBE) bond where **he is going to get 12%** p.a.. He is really happy with the rate of return. Will he have sleepless nights, if the economy goes into **recession**?.
- **Answer: Of course no.**
- 2) Mr. Ramesh is very bullish/Optimistic with the stock market **and invests money into equity diversified fund** with the expectation **that he will get 15% return.** Will he have sleepless nights if economy goes into deep recession, and now he feels that he may get **negative returns of say 5-7%?**
- **Answer: Of course yes.**
- **In the second situation, he has a fear, which is the result of huge difference in his expected return and the actual return, which he may get. This difference itself is the risk that he bears. Does he face this kind of difference in the first situation? No. So there is no risk.**

What is Risk ?

- Literally **risk** is defined as "exposing to **danger** or **hazard**".
  - ✓ Which is perceived as **negative terms.**

- Webster's dictionary, for instance, defines risk as "exposing to danger or hazard"

- **Risk:** refers to a set of circumstances regarding a given decision which can be assigned probabilities**"**

Con't…

✓In finance,

- **Risk** refers to the likelihood that we will receive a return on an investment that is **different** from the return we ***expected to make.***

- **Risk** is the probability or likelihood that **actual results (rates of return)** deviates from **expected returns.**

Con’t…

- Thus, **risk** includes not only the bad outcomes (returns that are lower than expected), but also **good outcomes (returns that are higher than expected**).

- In fact, we can refer to the former as **downside risk** and the latter as **upside risk**.

What is Risk ?

- ***Chinese Symbol for Risk:*** The first symbol is the symbol for "**danger**" while the second is the symbol for "**opportunity**", making risk a mix of danger and opportunity.

危機

- Hence, **risk** is both ***bad outcomes*** and ***good*** outcomes.

Sources of Risk

Sources of Risk Con't…

✓**Business Risk:**

- Uncertainty of income flows caused by the nature of a firm's business

- Sales volatility and operating leverage determine the level of business risk.

Source of risk Con't…

✓Financial Risk

- Uncertainty caused by the use of debt financing. (Level of Financial Leverage)

- Borrowing requires fixed payments which must be paid ahead of payments to stockholders.

- The use of debt increases uncertainty of **stockholder income** and causes an increase in the stock's risk premium.

Source of Risk Con't…

✓**Liquidity Risk**

- Uncertainty is introduced by the **secondary market** for an investment.
  – How long will it take to convert an investment into cash?
  – How certain is the price that will be received?

Source of Risk Con't…

- **Exchange Rate Risk:**
- Uncertainty of return is introduced by acquiring securities denominated in a currency different from that of the investor.

- Changes in exchange rates affect the investors return when converting an investment back into the "home" currency.

Source of Risk Con't…

- **Country Risk:**
- Political risk is the uncertainty of returns caused by the possibility of a major **change in the political or economic environment** in a country.

- Individuals who invest in countries that have unstable political-economic systems must include a country risk-premium when determining their required rate of return

Source of Risk Con't…

- ✓ ***Interest rate risk*** is the chance that changes in interest rates will adversely affect a security's value.

- ✓ ***Purchasing Power Risk*** refers to the chance that changing price levels (inflation or deflation) will adversely affect investment returns.

Classification of Risk:
Diversifiable and Non-diversifiable Risk

- Although there are many reasons why **actual returns** may differ from **expected returns**, we can group the reasons into two categories:

  A. **Market wide/Systematic Risk and**

  B. **Firm-specific/ Unsystematic Risk.**

- The risks that arise from **firm-specific actions** affect one or a few investments, while the **risks arising from market wide** reasons affect many or all investments.
- This distinction is critical to the way we assess risk in finance.

Con’t…

- **1) Systematic risk -** The risk inherent to the entire market or entire market segments is known as **systematic risk**. This is also known as:
  - **Un-diversifiable risk"** or **"market risk" or "uncontrollable risk. "**

- **Interest rates, inflation, economic policies, recession, wars** etc all represent sources of systematic risk because they affect the entire market and cannot be avoided through diversification.
- This risk can’t be **mitigated** through diversification, only **through hedging or** by using the correct asset allocation strategy..

Con’t…

- **What is 'Asset Allocation:**
- Asset allocation is an investment strategy that aims to balance **risk and reward** by apportioning a portfolio's assets according to an individual's goals, risk tolerance and investment horizon.

- The three main asset classes - equities, fixed-income, and cash and equivalents - have different levels of risk and return, so each will behave differently over time.

Examples of Systematic Risk

- The government changes the *interest rate policy*. The *corporate tax* rate is increased.
- The government resorts to massive *deficit financing*.
- The *inflation* rate increases.
- The RBI promulgates a restrictive *credit policy*.

- The government relaxes the *foreign exchange* controls and announces full *convertibility* of the Indian rupee.
- The government withdraws tax on dividend payments by companies.
- The government eliminates or reduces the capital gain tax rate.

Con't…

- **2) Unsystematic Risk** – The risk which is specific to a company or industry is known as **unsystematic risk**.

- This risk can be reduced through appropriate diversification. This is also known as "**specific risk", "diversifiable risk" or "residual risk" or " controllable risk.**

- **Examples of unsystematic risk:**
  - Employees strike
  - Key person leaving

Examples of Unsystematic Risk

- The company workers declare strike.
- The R&D expert leaves the company.
- A formidable competitor enters the market.
- The company loses a big contract in a bid.
- The company makes a breakthrough in process innovation.
- The government increases custom duty on the material used by the company.
- The company is unable to obtain adequate quantity of raw material

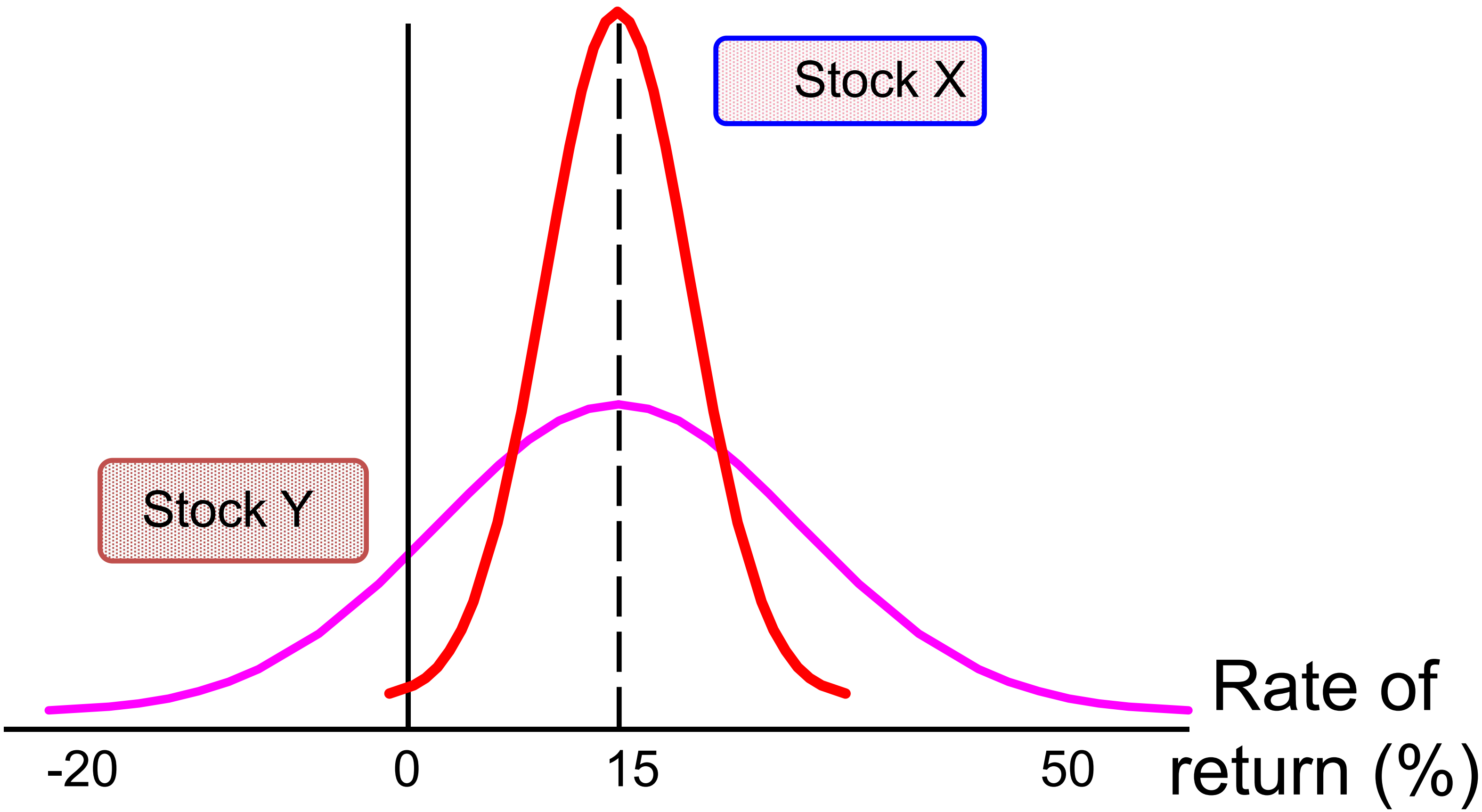
Con’t…

- Activity-1
- **Why Diversification Reduces or Eliminates Firm-Specific Risk: Give an Intuitive Explanation**

What is **investment** risk?

- Typically, investment returns are not known with certainty.
- **Investment risk** pertains to the probability of earning a return less than **that expected.**
- The greater the chance of a return far below the expected return, the **greater the risk**.

Probability Distribution



- Which stock is riskier? **Why**?

Answer: Con't…

- The tighter (or more peaked) the probability distribution, the more likely it is that the **actual outcome** will be close to the **expected value,** and hence the less likely it is that the actual return will end up far below the expected return.
  - Thus, the tighter the probability distribution, the **lower the risk assigned to a stock.**
- *Since **Stock X** has a relatively tight probability distribution, its actual return is likely to be closer to its 15% expected return than that of **Stock Y**.*
- *According to this definition, Stock **X** is less risky than Stock **Y** because there is a smaller chance that its actual return will end up far below its expected return.*

Measuring of Risk and Return

Two Sides of the Investment Coin.

Measuring the Expected Return and Risk of a Single Asset

1) Calculation of Expected Return (EX ANTE)

✓When we talk about **expectations,** we talk about **probability.**

✓The future or expected return of a security is uncertain; however it is possible to describe the future returns statistically as a probability distribution.

- **The mean of this distribution is the expected return.**

✓The **expected return** of the investment is the **probability weighted average of all the possible returns.**

Con't….

- ✓ If the **possible returns** are denoted by X_i and the related **probabilities** are $P(X_i)$, expected return may be represented as and can be calculated as:

$$E(Ri) = \Sigma X_i P(X_i).$$

- It is the sum of the products of possible returns with their respective probabilities. Consider the example below.

Con't…

- **Expected value of return** for asset, **E(Ri)** : Expected rate of return is the return expected to be realized from an investment.

$$E(Ri) = p_1r_1 + p_2r_2 \ldots\ldots + p_nr_n$$

where,

$p_1, p_2 \ldots p_n$ = is the probability of the i^{th} out come.

$r_1, r_2 \ldots\ldots r_n$ = is the i^{th} possible out come (return).

n = number of outcomes considered

or

$$E(R_i) = \Sigma\ (R_j \times Pr_j)$$

Where Rj = return for the jth outcome

Prj = probability of occurrence of the jth outcome

Con't…

- **Example:** Mr. X is considering the possible rates of return (dividend yield ~~plus cap~~ital gain or loss) that he might earn next year on a $10,000 investment in the stock of either **Alpha Company** or **Beta Company**. The rates of return probability distributions for the two companies are shown here under:

State of the economy	Probability of the state economy	Rate of return if the state economy occurs	
		Alpha Co	Beta Co.
Boom	0.35	20%	24%
Normal	0.40	15%	12%
Recession	0.25	5%	8%

<u>Required</u>: compute the expected rate of return on each company's stock and recommend where Mr "X" has to invest the $10,000 investable fund.

Solution

- $E(R_i) = \Sigma (R_j x\ Pr_j)$
- $E(R_{alpha}) = (0.35 * 20) + (0.4 * 15) + (0.25 * 5)$

  $E(R_{alpha}) = 7 + 6 + 1.25 = \underline{14.25\%}$

- $E(R_{Beta}) = (0.35 * 24) + (0.4 * 12) + (0.25 * 8)$

  $E(R_{Beta}) = 8.4 + 4.8 + 2 = \underline{15.2\%}$

2. *Measuring the Expected Risk of a Single Asset*

- Standard deviation is the most common statistical indicator of an asset's risk (stand alone risk).
- **S.D** measures the variability of a set of observations.
- The larger the standard deviation, the higher the probability that actual returns will be far below the expected return.
- Coefficient of variation is an alternative measure of stand-alone risk.

Con't…

- **Standard deviation** is indicator of risk asset (an absolute measure of risk) of that asset's expected return, **σ (Ri),** which measures the dispersion around its expected value.

▪The **standard deviation** considers the distance (deviation) of each possible outcome from the expected value and the probability associated with that distance.

▪This can be calculated using equation below:

$$\sigma\ (Ri) = \sqrt{\Sigma\,[R_j - E(R_i)\,]^2 \times Pr_j}$$

Con't…

- **<u>Steps to calculate the σ or sigma:</u>**

1. Calculate the expected rate of return:

   Expected rate of return, $E(R_i) = \Sigma\ (R_j \times Pr_j)$

2. Subtract the expected rate of return ($E(R_i)$) from each possible outcome (ri) to obtain a set of deviations about $E(R_i)$, Deviation$_i = r_i - E(R_i)$

Con't…

3. Square each deviation:

$$\text{Deviation}\boldsymbol{i} = (\mathrm{ri} - \boldsymbol{E}(\boldsymbol{R}_{\mathrm{i}}))\mathbf{2}$$

4. Multiply the squared deviations by the **probability of occurrence** for its related outcome.

$$\mathrm{Pi}(\mathrm{ri} - \boldsymbol{E}(\boldsymbol{R}_{\mathrm{i}})\ )\mathbf{2}$$

5. Sum these products to obtain the ***variance*** of the probability distribution:

6. $\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance } (\sigma^2)}$

Scenario-based Estimate of Risk

Example Using the Ex ante Standard Deviation – Raw Data

GIVEN INFORMATION INCLUDES:

- **Possible returns on the investment for different discrete states**
- **Associated probabilities for those possible returns**

State of the Economy	Probability	Possible Returns on Security A
Recession	25.0%	-22.0%
Normal	50.0%	14.0%
Economic Boom	25.0%	35.0%

Scenario-based Estimate of Risk

First Step – Calculate the Expected Return

Determined by multiplying the probability times the possible return.

State of the Economy	Probability	Possible Returns on Security A	Weighted Possible Returns
Recession	25.0%	-22.0%	-5.5%
Normal	50.0%	14.0%	7.0%
Economic Boom	25.0%	35.0%	8.8%
		Expected Return =	**10.3%**

Expected return equals the sum of the weighted possible returns.

Scenario-based Estimate of Risk

Second Step – Measure the Weighted and Squared Deviations

First calculate the dev... possible returns from the expected.

Now multiply the square deviations by their probability of occurrence.

State of the Economy	Probability	Possible Returns on Security A	Weighted Possible Returns	Deviation of Possible Return from Expected	Squared Deviations	Weighted and Squared Deviations
Recession	25.0%	-22.0%	-5.5%	-32.3%	0.10401	0.02600
Normal	50.0%	14.0%	7.0%	3.8%	0.00141	0.00070
Economic Boom	25.0%	35.0%	8.8%	24.8%	0.06126	0.01531
		Expected Return =	10.3%		Variance =	0.0420
					Standard Deviation =	20.50%

Second, square those deviations

The ... dev... squared terms.

The standard deviation is the square root of the variance (in percent terms).

Determining Standard Deviation (Risk Measure)

$$\sigma = \sqrt{\sum_{i=1}^{n} (R_i - \bar{R})^2 (P_i)}$$

Standard Deviation, s, is a statistical measure of the variability of a distribution around its mean.

It is the square root of variance.

Note, this is for a discrete distribution.

How to Determine the Expected Return and Standard Deviation

Stock BW			
R_i	P_i	$(R_i)(P_i)$	The expected return, $\overline{R}$, for Stock BW is .09 or 9%
-.15	.10	-.015	
-.03	.20	-.006	
.09	.40	.036	
.21	.20	.042	
.33	.10	.033	
Sum	*1.00*	*.090*	

How to Determine the Expected Return and Standard Deviation

Stock BW			
R_i	P_i	$(R_i)(P_i)$	$(R_i - \bar{R})^2(P_i)$
-.15	.10	-.015	.00576
-.03	.20	-.006	.00288
.09	.40	.036	.00000
.21	.20	.042	.00288
.33	.10	.033	.00576
Sum	*1.00*	*.090*	*.01728*

MEASURING EXPECTED (EX ANTE) RETURN AND RISK

EXPECTED RATE OF RETURN

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

STANDARD DEVIATION OF RETURN

$$\sigma = [\Sigma\, p_i\, (R_i - E(R))^2]$$

Bharat Foods Stock						
i. State of the Economy	p_i	R_i	$p_i R_i$	$R_i - E(R)$	$(R_i - E(R))^2$	$p_i(R_i - E(R))^2$
1. Boom	0.30	16	4.8	4.5	20.25	6.075
2. Normal	0.50	11	5.5	-0.5	0.25	0.125
3. Recession	0.20	6	1.2	-5.5	30.25	6.050

$E(R) = \Sigma p_i R_i = 11.5$ $\qquad$ $\Sigma p_i(R_i - E(R))^2 = 12.25$

$\sigma = [\Sigma p_i(R_i - E(R))^2]^{1/2} = (12.25)^{1/2} = 3.5\%$

Comments on Standard Deviation as a Measure of Risk

- Standard deviation (σ_i) measures total, or stand-alone, risk.

- The larger $\boldsymbol{\sigma_i}$ is, the lower the probability that actual returns will be **closer to expected returns.**

- Larger $\boldsymbol{\sigma_i}$ is associated with a wider **probability distribution of returns.**
- The larger standard deviation **($\boldsymbol{\sigma_i}$)** indicates a **greater variation of returns** and thus a ***greater chance that the expected return will not be realized.***

- The larger the Standard deviation **($\boldsymbol{\sigma_i}$),** the higher the risk, because Standard deviation (σ_i), is a ***measure of total risk.***

Coefficient of Variation: *A Relative Measure of Risk*

- If conditions for **two or more** investment alternatives are **not similar—that** is, if there are major differences in the ***expected rates of return or standard deviation***—it is necessary to use a measure of ***relative variability to indicate risk per unit of expected return***.
- *A* ***widely used relative measure*** *of* risk is the **coefficient of variation (CV), calculated as follows:**

- Formula for CV is:

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Average or Expected Return}}$$

$$\mathbf{CV} = \frac{\sigma\ (Ri)}{E(R_i)}$$

Coefficient of Variation *con't…*

- The **coefficient of variation** is a useful measure of risk when we are comparing the investment alternatives which have
  - (*i*) same standard deviations but different expected values, or
  - (*ii*) different standard deviations but same expected values, or
  - (*iii*) different standard deviations and different expected values.

Example-2

- consider the following two investments:

	Investment A	Investment B
Expected Return	0.07	0.12
Standard Deviation	0.05	0.07

- Comparing **absolute measures of risk**, investment **B appears to be riskier** because it has a standard deviation of 7 percent versus 5 percent for investment A.
- In contrast, the *CV figures show* that investment B has less relative variability or lower risk **per unit of expected return** because it has a substantially higher expected rate of return

$$\mathbf{CV_A = 0.05/0.07 = 0.714}$$

$$\mathbf{CV_B = 0.07/0.12 = 0.583}$$

PORTFOLIO THEORY AND ASSET PRICING MODELS

Introduction

- **Risk–return** trade-offs have an important role to play in corporate **finance theory** – from both a company and an investor perspective.
- Companies face variability in their **projected cash flows** whereas investors face variability in their **capital gains and dividends**.
- Assuming that companies and shareholders are rational, their aim will be to minimise the risk they face for a given return they expect to receive.
- So in this chapter we will examine how investors, by 'not putting all their eggs in one basket', are able to reduce the risk they face given the level of their expected return.

Con’t…

- **<u>PORTFOLIO</u>:** is a collection or a group of investment assets. If you hold only one asset, you suffer a loss if the return turns out to be very low.

- If you hold two assets, the chance of suffering a loss is reduced and returns on both assets must be low for you to suffer a loss.

- By diversifying, or investing in multiple assets that do not move proportionately in the same direction at the same time, you **reduce your risk.**

Con't…

- It is the **total portfolio risk** and **return** that is important.
- The risk and return of individual assets should not be analyzed in isolation; rather they should be analyzed in terms of how they affect the **risk and return of the portfolio** in which they are included.

- The goal of the financial manager should be to create an efficient portfolio, one that maximizes return for a given level of risk or minimizes risk for a given level of return.

Portfolio-Expected Return

The **expected return** on a portfolio, ˆrp, is simply the **weighted average of the expected returns on the individual assets in the portfolio,** with the weights being the fraction of the total portfolio invested in each asset:

$$\hat{r}_p = w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_n\hat{r}_n$$

Formula : E(Rp)= $\sum_{i=1}^{n} WiRi$

Where:

- ✓ *Rp = Expected Return of the Portfolio.*
- ✓ *Wi = The weights attached for each security.*
- ✓ ˆr1, ˆr2, ˆr3…… ˆrn = *Expected return of a security.*
- ✓ *n = No of stocks/ securities in a portfolio.*

Con't…

- *Note; "Wi" is the fraction of the portfolio's dollar value invested in stock/security i, that is, the value of investment in stock i divided by the total value of the portfolio. Thus, the sum of Wi's must be 1.00.*

- **Example:** Consider a portfolio of three stocks A, B, and C, with expected returns of 16%, 12%, and 20% respectively. The portfolio consists of 50% stock A, 25% stock B, and 25% stock C.
- **Required: what is the expected return on this portfolio?**
- *E(Rp)= .16*50+.12*25+.20*25=* ***16%*** *which is the expected return of a portfolio*

Con't…

Suppose we have the following projections on three stocks:

		Returns if state occurs		
State of Economy	*Probabilities*	*Stock A*	*Stock B*	*Stock C*
Boom	*0.40*	*10%*	*15%*	*20%*
Bust	*0.6*	*8%*	*4%*	*0%*

We want to calculate portfolio expected returns in two cases. First, what would be the expected return on a portfolio with equal amounts invested in each of the three stocks? Second, what would be the expected return if half of the portfolio were in A, with the remainder equally divided between B and C? (8pt)

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Another Example

- Consider the following information

State	Probability	X	Z
Boom	.25	15%	10%
Normal	.60	10%	9%
Recession	.15	5%	10%

- What is the expected return and standard deviation for a portfolio with an investment of \$6000 in asset X and \$4000 in asset Y?

PORTFOLIO EXPECTED RETURN Con't…

- **Example-2:** A portfolio consists of four securities with expected returns of 12%, 15%, 18%, and 20% respectively. The proportions of portfolio value invested in these securities are 0.2, 0.3, 0.3, and 0.20 respectively.

$$E(R_P) = \sum_{i=1}^{n} w_i \, E(R_i)$$

where $E(R_P)$ = expected portfolio return

w_i = weight assigned to security i

$E(R_i)$ = expected return on security i

n = number of securities in the portfolio

Then expected return on the portfolio of the above Example is:

$$E(R_P) = 0.2(12\%) + 0.3(15\%) + 0.3(18\%) + 0.2(20\%)$$

$$= \mathbf{16.3\%}$$

Ex Ante Portfolio Returns

Simply the Weighted Average of Expected Returns

Example-3:

	Relative Weight	Expected Return	Weighted Return
Stock X	0.400	8.0%	0.03
Stock Y	0.350	15.0%	0.05
Stock Z	0.250	25.0%	0.06
	Expected Portfolio Return =		**14.70%**

Risk in a Portfolio Context

- As we just saw, the expected return on a portfolio is simply the weighted average of the expected returns on the individual assets in the portfolio.

- However, unlike returns, the risk of a portfolio, **standard deviation of a portfolio**, is generally *not* the weighted average of the standard deviations of the individual assets in the portfolio; the portfolio's risk will almost always be *smaller* than the weighted average of the assets' S.D's.

Con't…

- **Portfolio risk** can be reduced by the simple kind of diversification, that is; when different assets are added to the portfolio, the **total risk tends to decrease.**

- **Total risk of portfolio** consists of **systematic risk & unsystematic risk** and this total risk is measured by the **variance** and **standard deviation** of the rate of return overtime.

- As the **portfolio size increases**, the **total risk decreases up to a certain** level. Beyond that limit, risk cannot be reduced.
  - This indicates that spreading out the assets beyond certain level can't be expected to reduce the portfolio's total risk below the level of **un-diversifiable risk.**

Modern Portfolio Theory (Markowitz's):

Harry Markowitz

- In the early 1960s, the investment community talked about **risk,** but there was no specific measure for the term.

- To build a portfolio model, however, investors had to quantify their **risk variable**.

- The basic portfolio model was developed by **Harry Markowitz**, who **derived the expected rate of return for a portfolio of assets** and an expected risk measure.

Con't…

- Modern portfolio theory was initiated by University of Chicago graduate student, **Harry Markowitz in 1952.**

- Markowitz showed how the risk of a portfolio is NOT just the weighted average sum of the risks of the individual securities…but rather, also a function of the degree of co movement of the returns of those individual assets.

Risk & Return – Modern Portfolio Theory (MPT)

- Prior to the establishment of Modern Portfolio Theory, most people only focused upon investment returns…they ignored **risk**.
- With MPT, investors had a tool that they could use to dramatically reduce the risk of the portfolio without a significant reduction in the expected return of the portfolio.

- **Markowitz** showed that the **variance of the rate of return** was a meaningful measure of portfolio risk under a reasonable set of **assumptions**, and he derived the formula for computing the **variance of a portfolio**.

- **The Markowitz model is based on several assumptions regarding investor behaviour:**

1. Investors consider each investment alternative as being represented by a **probability distribution of expected returns** over some holding period.
2. Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
3. Investors estimate the **risk of the portfolio on the basis of the variability of expected returns.**

Con't…

4. Investors base decisions solely on **expected return and risk**, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.

5. For a given **risk level**, investors prefer **higher returns to lower returns**. Similarly, for a given level of expected return, investors prefer less risk to more risk.

Con't…

- **Therefore:**

✓ Under these assumptions, a single asset or portfolio of assets is considered to be efficient if **no** other asset or portfolio of assets offers **higher expected return** with the same (or lower) risk, or lower risk with the **same (or higher) expected return.**

- The simplest portfolio to consider is that **containing two shares**.
- ***Diversification*** is the combination of assets whose returns do not vary with one another in the same direction at the same time.

- The extent to which a two-share portfolio will reduce unsystematic risk depends on the **correlation** between the two shares' returns.

- This correlation is quantified by the correlation coefficient (ρ) of the returns of the two shares, which can take any value in the range **-1 to +1.**

Correlation

- **Correlation (co-movement)**: refers to the association of movement between **two numbers**.

- It measures the **degree** of linear relationship to which two variables, such as **returns on two assets, move together.**

- Correlation takes on numerical values that range from **+1 to -1.**

Con't…

- While the **positive or negative sign** indicates the **direction of the co-movement**.
- The closer the correlation coefficient is to +1 or -1, the stronger the association.
  - **+1 is perfect positive correlation.**
  - **-1 is perfect negative correlation.**
  - 0.0 **no relationship**
- If the variables move together, they are positively correlated (a positive correlation coefficient).
- If the variables move in opposite direction, they are negatively correlated (a negative correlation).

Con't…

- **A correlation of +1.0** indicates that the variables **move up** and **down together**, the relative magnitude of the movements is exactly the same (perfect positive correlation).

- **A correlation of -1.0** implies that they move exactly opposite to each other. **(Perfect negative correlation).**

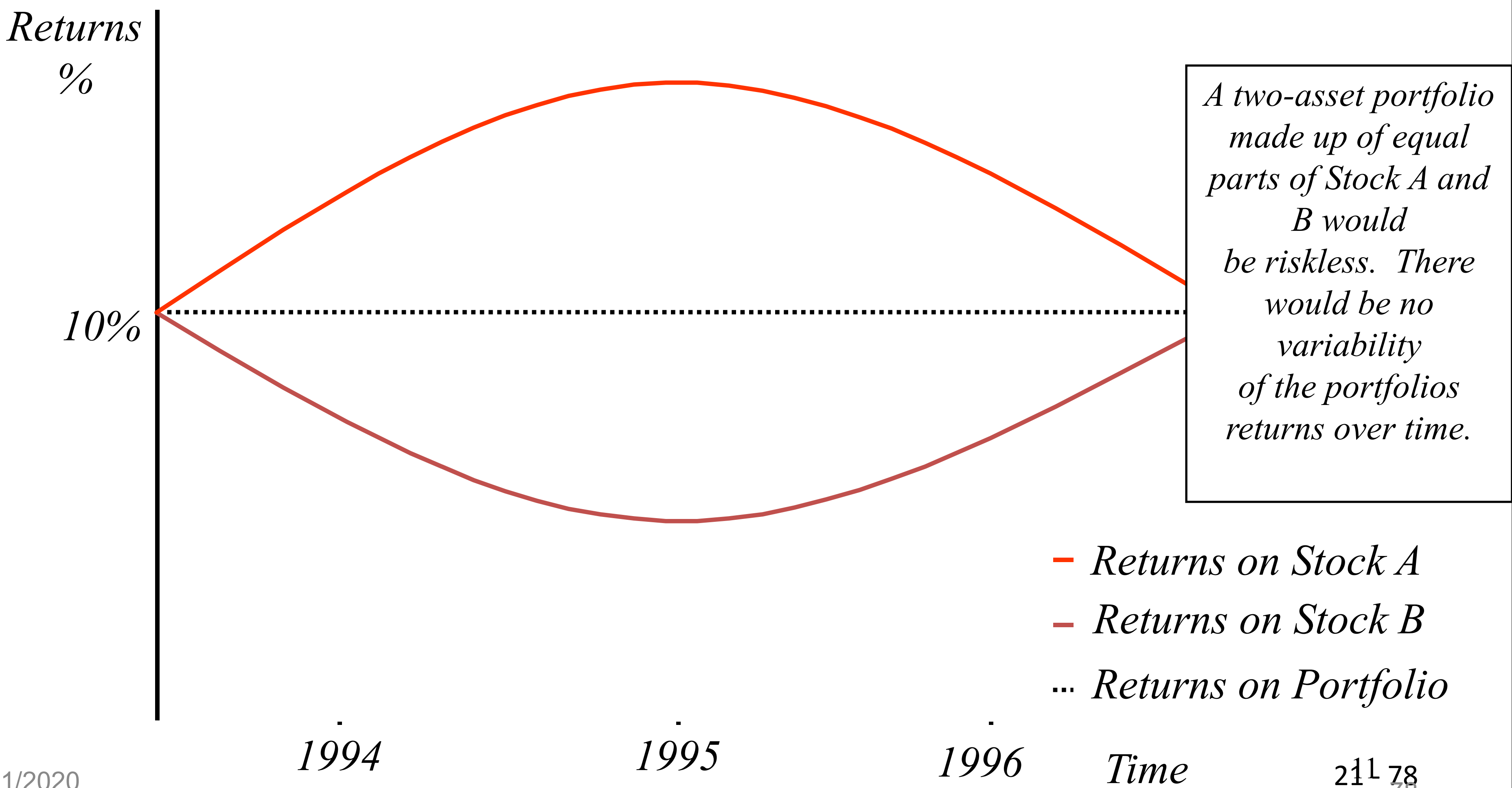
- **A correlation of 0.0** indicates that **no relationship** between the variables, that is they are unrelated.

Con't…

- If correlation is between 0.0 and +1.0, the returns usually **move up and down together,** but not all the time. The closer the correlation is to 0.0, the lesser the two sets of returns **move together.**

- A **correlation coefficient falling between –1 and 0** indicates negative, but not perfect negative correlation between the two assets' returns.

Perfect Negatively Correlated Returns over Time



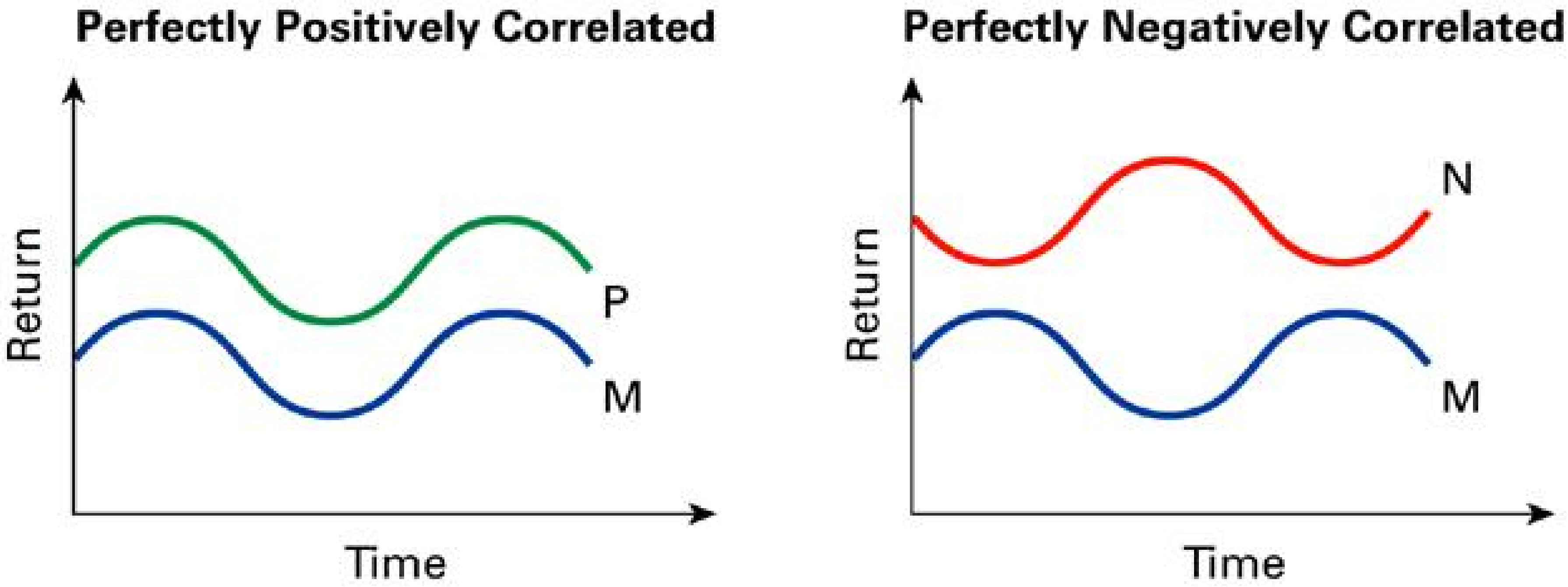
Con’t…

- ✓ If ρ *(A,B) =+1 no unsystematic risk can be diversified away*
- ✓ If ρ *(A,B =-1 all unsystematic risk will be diversified away*
- ✓ If ρ *(A,B)= 0 no correlation between the two securities’ returns*

- Therefore, when picking a two-share portfolio it is most beneficial to choose two shares whose **correlation coefficient** is as close to **-1** as possible.

- However, as long as the correlation coefficient is less than **+1**, some unsystematic risk will be **diversified away.**

The Correlation Between Series M, N, and P



Correlation:

Why Diversification Works!

- To reduce overall risk in a portfolio, it is best to combine assets that have a ***negative (or low-positive) correlation.***

- ***Uncorrelated*** assets reduce risk somewhat, but not as effectively as ***combining negatively correlated assets.***

- Investing in different investments with ***high positive correlation*** will not provide sufficient diversification.

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Con't…

- In practice it is difficult to find two securities whose correlation coefficient is exactly **-1,** but the most commonly quoted example is that of an **umbrella manufacturer** and **an ice cream company**.

- The classical **example** of the benefit of diversification is to consider the effect of combining investment in **an ice-cream producer** with the investment in a **manufacturer of umbrella.**

Con't…

- For simplicity assume that the return to the ice-cream producer is **+15% if the weather is sunny** and **-10% if it is rain.** Similarly the manufacturer of umbrella benefits when it **rains (+15)** and **looses when the sun shines (-10%).** Further assume that each of the two weather states occur with probability of 50%.

- *Thus,* ***diversification*** *does nothing to reduce risk if the portfolio consists of perfectly positively correlated stocks.*

Co-effient of Correlation

$$\text{Cor}(R_i, R_j) \text{ or } \rho_{ij} = \frac{\text{Cov}(R_i, R_j)}{\sigma_i \, \sigma_j}$$

$$= \frac{\sigma_{ij}}{\sigma_i \, \sigma_j}$$

$$\sigma_{ij} = \rho_{ij} \, . \, \sigma_i \, . \, \sigma_j$$

where ρ_{ij} = correlation coefficient between the returns on securities i and j

σ_{ij} = covariance between the returns on securities i and j

σ_i , σ_j = standard deviation of the returns on securities i and j

CO-VARIANCE OF RETURNS

- **Covariance** measures how closely security returns move together. The covariance between possible returns for securities J and K,
- When we consider two assets in the portfolio, we are concerned with the **co-movement** of security movement.
- **Co-movements between the returns of securities are measured by covariance (an absolute measure) and coefficient of correlation (a relative measure).**
- It can be:
  - **Positive Covariance,**
  - **Negative Covariance, and**
  - **Zero Covariance,**

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Con't…

- A **positive covariance** between the returns of two securities indicates that the returns of the two securities tend to move in the **same direction.**

- A **negative covariance** between the returns of two securities indicates that the returns of the two securities tend to move in **opposite directions**.

- A relatively **small or zero covariance** between the returns of two securities indicates that there is little or no relationship between the returns of the two securities;
- We denote the **covariance** between the return of security **i** and the return of security **j** by (the Greek letter **sigma,**σ_{ij} **or covariance ij**).

- **Covariance will flow with the following steps:**
- **Step 1:** Calculate the expected return for each assets
- **Step 2: For each scenario and investment, subtract the investment's** expected return from its possible outcome.
- **Step 3: For each scenario, multiply the deviations for the two investments.**
- **Step 4: Weight this product by the scenario's probability.**
- **Step 5: Sum these weighted products to arrive at** the covariance

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Con't….

$$\text{Cov}\,(R_i\,,\,R_j) = p_1\,[R_{i1} - E(R_i)]\,[\,R_{j1} - E(R_j)]$$

$$\vdots \qquad + p_2\,[R_{i2} - E(R_j)]\,[R_{j2} - E(R_j)]$$

$$+$$

$$+ p_n\,[R_{in} - E(R_i)]\,[R_{jn} - E(R_j)]$$

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The returns on assets 1 and 2 under five possible states of nature are given below

State of nature	Probability	Return on asset 1	Return on asset 2
1	0.10	-10%	5%
2	0.30	15	12
3	0.30	18	19
4	0.20	22	15
5	0.10	27	12

The expected return on asset 1 is :

$$E(R_1) = 0.10\,(-10\%) + 0.30\,(15\%) + 0.30\,(18\%) + 0.20\,(22\%) + 0.10\,(27\%) = 16\%$$

The expected return on asset 2 is :

$$E(R_2) = 0.10\,(5\%) + 0.30\,(12\%) + 0.30\,(19\%) + 0.20\,(15\%) + 0.10\,(12\%) = 14\%$$

The covariance between the returns on assets 1 and 2 is calculated below :

State of nature	*Probability*	*Return on asset 1*	*Deviation of the return on asset 1 from its mean*	*Return on asset 2*	*Deviation of the return on asset 2 from its mean*	*Product of the deviations times probability*
(1)	(2)	(3)	(4)	(5)	(6)	(2) x (4) x (6)
1	0.10	-10%	-26%	5%	-9%	23.4
2	0.30	15%	-1%	12%	-2%	0.6
3	0.30	18%	2%	19%	5%	3.0
4	0.20	22%	6%	15%	1%	1.2
5	0.10	27%	11%	12%	-2%	-2.2
						Sum = 26.0

Thus the covariance between the returns on the two assets is 26.0.

Grouping Individual Assets into Portfolios

- The riskiness of a portfolio that is made of different risky assets is a function of three different factors:
  - the riskiness of the individual assets that make up the portfolio
  - the relative weights of the assets in the portfolio
  - the degree of **co-movement** of returns of the assets making up the portfolio.
- The **standard deviation** of a **two-asset portfolio** may be measured using the Markowitz model:

$$\sigma_p = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2}$$

Expected Return and Risk For Portfolios

Standard Deviation of a Two-Asset Portfolio using Covariance

$$\sigma_p = \sqrt{\underbrace{(w_A)^2(\sigma_A)^2}_{\text{Risk of Asset A adjusted for weight in the portfolio}} + \underbrace{(w_B)^2(\sigma_B)^2}_{\text{Risk of Asset B adjusted for weight in the portfolio}} + \underbrace{2(w_A)(w_B)(COV_{A,B})}_{\text{Factor to take into account comovement of returns. This factor can be negative.}}}$$

Example

- Determine the expected return and standard deviation of the following portfolio consisting of two stocks that have a correlation coefficient of .75.

Portfolio	Weight	Expected Return	Standard Deviation
Apple	.50	.14	.20
Coca-Cola	.50	.14	.20

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Answer

- Expected Return of the Portfolio = .5 (.14) + .5 (.14)= **.14 or 14%**
- Standard deviation of the portfolio

  $= \sqrt{ (.5^2 x .2^2)+(.5^2 x .2^2)+(2 x .5 x .5 x .75 x .2 x .2)}$

  $= \sqrt{.035} =$ **.187 or 18.7%**

  - Lower than the weighted average of 20%.

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$$\sigma_p = [w_1^2\ \sigma_1^2 + w_2^2\ \sigma_2^2 + 2w_1w_2\ \rho_{12}\ \sigma_1\ \sigma_2]^{½}$$

Example : $w_1 = 0.6$, $w_2 = 0.4$,

$\sigma_1 = 10\%$, $\sigma_2 = 16\%$

$\rho_{12} = 0.5$

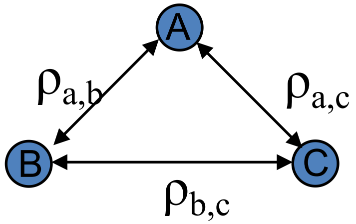
$$\sigma_p = [0.6^2 \text{ x } 10^2 + 0.4^2 \text{ x } 16^2 + 2 \text{ x } 0.6 \text{ x } 0.4 \text{ x } 0.5 \text{ x } 10 \text{ x } 16]^{½}$$

$$= 10.7\%$$

- The average standard deviation of two securities is 13, which is less than standard deviation of the portfolio, which is 10.

The data requirements for a three-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.

We need 3 (three) correlation coefficients between A and B; A and C; and B and C.



$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + \sigma_C^2 w_C^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B + 2 w_B w_C \rho_{B,C} \sigma_B \sigma_C + 2 w_A w_C \rho_{A,C} \sigma_A \sigma_C}$$

PORTFOLIO RISK : 3 – SECURITY CASE

$$\sigma_p = [\sum \sum w_i w_j \rho_{ij} \sigma_i \sigma_j]^{1/2}$$

Example : $w_1 = 0.5$, $w_2 = 0.3$, and $w_3 = 0.2$

$\sigma_1 = 10\%$, $\sigma_2 = 15\%$, $\sigma_3 = 20\%$

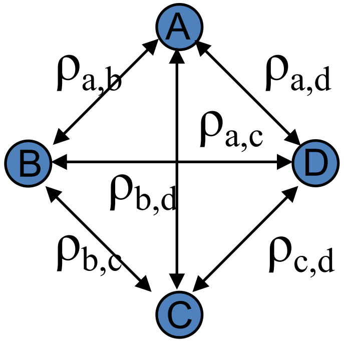
$\rho_{12} = 0.3$, $\rho_{13} = 0.5$, $\rho_{23} = 0.6$

$$\sigma_p = [w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$
$$+ 2w_2 w_3 \rho_{13} \sigma_1 \sigma_3 + 2w_2 w_3 \rho_{23} \sigma_2 \sigma_3]^{1/2}$$
$$= [0.5^2 \text{ x } 10^2 + 0.3^2 \text{ x } 15^2 + 0.2^2 \text{ x } 20^2$$
$$+ 2 \text{ x } 0.5 \text{ x } 0.3 \text{ x } 0.3 \text{ x } 10 \text{ x } 15$$
$$+ 2 \text{ x } 0.5 \text{ x } 0.2 \text{ x } 05 \text{ x } 10 \text{ x } 20$$
$$+ 2 \text{ x } 0.3 \text{ x } 0.2 \text{ x } 0.6 \text{ x } 15 \text{ x } 20]^{1/2}$$
$$= 10.79\%$$

Risk of a Four-asset Portfolio

✓The data requirements for a four-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.

✓We need 6 correlation coefficients between A and B; A and **C; A and D; B and C; C and D; and B and D.**



- Suppose Asset A has an expected return of 10 percent and a standard deviation of 20 percent. Asset B has an expected return of 16 percent and a standard deviation of 40 percent. If the correlation between A and B is 0.6, what are the expected return and standard deviation for a portfolio comprised of 30 percent Asset A and 70 percent Asset B?

$$\hat{r}_P = w_A \hat{r}_A + (1 - w_A)\hat{r}_B$$
$$= 0.3(0.1) + 0.7(0.16)$$
$$= 0.142 = 14.2\%.$$

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$$
\begin{aligned}
\sigma_p &= \sqrt{W_A^2 \sigma_A^2 + (1 - W_A)^2 \sigma_B^2 + 2W_A (1 - W_A) \rho_{AB} \sigma_A \sigma_B} \\
&= \sqrt{0.3^2 (0.2^2) + 0.7^2 (0.4^2) + 2(0.3)(0.7)(0.4)(0.2)(0.4)} \\
&= 0.309
\end{aligned}
$$

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RISK OF AN N - ASSET PORTFOLIO

$$\sigma^2_p = \Sigma\,\Sigma\, w_i w_j\, \rho_{ij}\, \sigma_i \sigma_j$$

n x n MATRIX

	1	2	3	…	n
1	$w_1^2\sigma_1^2$	$w_1w_2\rho_{12}\sigma_1\sigma_2$	$w_1w_3\rho_{13}\sigma_1\sigma_3$	…	$w_1w_n\rho_{1n}\sigma_1\sigma_n$
2	$w_2w_1\rho_{21}\sigma_2\sigma_1$	$w_2^2\sigma_2^2$	$w_2w_3\rho_{23}\sigma_2\sigma_3$	…	$w_2w_n\rho_{2n}\sigma_2\sigma_n$
3	$w_3w_1\rho_{31}\sigma_3\sigma_1$	$w_3w_2\rho_{32}\sigma_3\sigma_2$	$w_3^2\sigma_3^2$	…	
:	:				:
n	$w_nw_1\rho_{n1}\sigma_n\sigma_1$				$w_n^2\sigma_n^2$

Equal Risk and Return—Changing Correlation

- ***Consider first the case in which both*** assets have the same expected return and expected standard deviation of return. As an example, let us assume

$$E(R1) = 0.20$$

$$\sigma 1 = 0.10$$

$$E(R2) = 0.20$$

$$\sigma 2 = 0.10$$

- To show the effect of different covariance's, assume different levels of correlation between the two assets. Consider the following examples where the two assets have equal weights in the portfolio (*W1 = 0.50; W2 = 0.50*). *Therefore, the only value that changes in each example is the correlation* between the returns for the two assets. Recall that, $Cov_{ij} = \rho_{ij}\sigma_i\sigma_j$

Con't…

- Consider the following alternative correlation coefficients and the covariance's they yield. The covariance term in the equation will be equal to $r_{1,2}$ *(0.10)(0.10) because both standard deviations* are 0.10.

a. *r1,2 = 1.00; Cov1,2 = (1.00)(0.10)(0.10) = 0.010*

b. *r1,2 = 0.50; Cov1,2 = (0.50)(0.10)(0.10) = 0.005*

c. *r1,2 = 0.00; Cov1,2 = 0.000(0.10)(0.10) = 0.000*

d. *r1,2 = –0.50; Cov1,2 = (–0.50)(0.10)(0.10) = –0.005*

e. *r1,2 = –1.00; Cov1,2 = (–1.00)(0.10)(0.10) =–0.01*

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Con't…

- Now let us see what happens to the standard deviation of the portfolio under these five conditions.
- **Recall from previous Equation that:**

$$\sigma_p = \sqrt{\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2 w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2}$$

Con’t…

or

$$\sigma_{\text{port}} = \sqrt{x_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}_{1,2}}$$

Thus, in Case a,

$$\begin{aligned}\sigma_{\text{port(a)}} &= \sqrt{(0.5)^2(0.10)^2 + (0.5)^2(0.10)^2 + 2(0.5)(0.5)(0.01)} \\ &= \sqrt{(0.25)(0.01) + (0.25)(0.01) + 2(0.25)(0.01)} \\ &= \sqrt{0.01} \\ &= 0.10\end{aligned}$$

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Con't…

- In this case, where the returns for the two assets are perfectly positively correlated ($r_{1,2} = +1.0$), the standard deviation for the portfolio is, in fact, the weighted average of the individual standard deviations.
- The important point is that we **get no real benefit** from combining two assets that are perfectly correlated; they are like one asset already because their returns move together. Now consider Case b, where $r_{1,2}$ equals 0.50:

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Now consider Case b, where $r_{1,2}$ equals 0.50:

$$\sigma_{\text{port(b)}} = \sqrt{(0.5)^2(0.10)^2 + (0.5)^2(0.10)^2 + 2(0.5)(0.5)(0.005)}$$
$$= \sqrt{(0.0025)(0.0025) + 2(0.25)(0.005)}$$
$$= \sqrt{0.0075}$$
$$= 0.0868$$

The only term that changed from Case a is the last term, Cov1,2, which changed from 0.01 to 0.005. As a result, the standard deviation of the portfolio declined by about 13 percent, from 0.10 to 0.0868. Note that *the* ***expected return did not change*** because it is simply the weighted average of the individual expected returns; it is equal to 0.20 in both cases.

- You should be able to confirm through your own calculations that the standard deviations for Portfolios c and d are as follows:

c. 0.0707

d. 0.05

The final case where the correlation between the two assets is –1.00 indicates the ultimate benefits of diversification:

$$\sigma_{port(e)} = \sqrt{(0.5)^2(0.10)^2 + (0.5)^2(0.10)^2 + 2(0.5)(0.5)(-0.01)}$$

$$= \sqrt{(0.0050) + (-0.0050)}$$

$$= \sqrt{0}$$

$$= 0$$

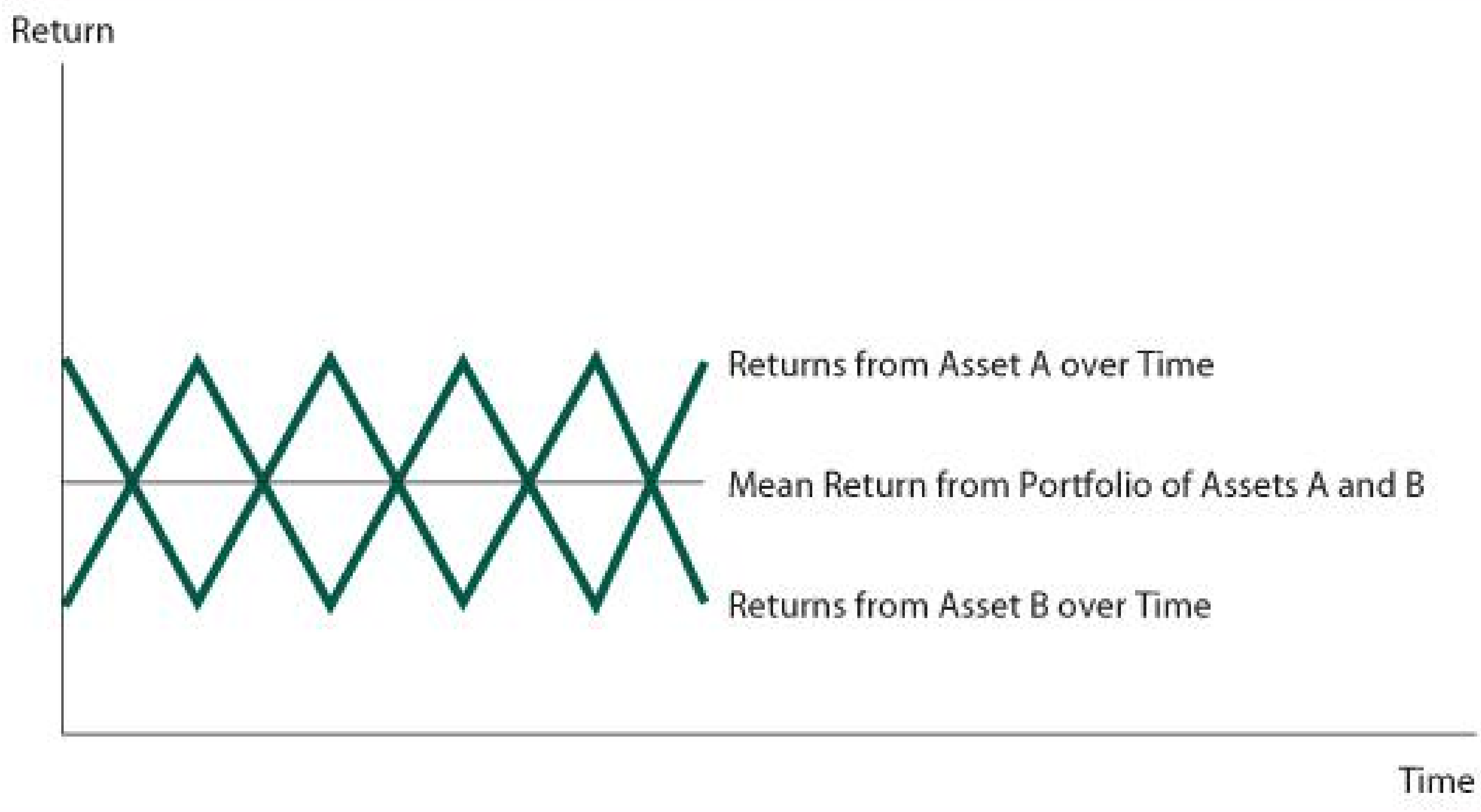
Here, the negative covariance term exactly offsets the individual variance terms, leaving an overall standard deviation of the portfolio of zero. *This would be a risk-free portfolio.*

Standard Deviation of a Portfolio

- Assets may differ in expected rates of return and individual standard deviations
- Negative correlation reduces portfolio risk
- Combining two assets with +1.0 correlation will not reduces the portfolio standard deviation
- Combining two assets with -1.0 correlation may reduces the portfolio standard deviation to zero
- See next Exhibits:

Exhibit

Exhibit 7.10 Time Patterns of Returns for Two Assets with Perfect Negative Correlation



Standard Deviation of a Portfolio

With *Different Returns and Risk*

- ***Combining Stocks with Different Returns and Risk :***

- The previous discussion indicated what happens when only the correlation coefficient (covariance) differs between the assets.

- We now consider two assets (or portfolios) with different expected rates of return and individual standard deviations.

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Standard Deviation of a Portfolio: *Different Returns and Risk*

- Two Stocks with Different Returns and Risk

Asset	$E(R_i)$	W_i	σ^2_i	σ_i
1	.10	.50	.0049	.07
2	.20	.50	.0100	.10

Case	Correlation Coefficient	Covariance
a	+1.00	.0070
b	+0.50	.0035
c	0.00	.0000
d	-0.50	-.0035
e	-1.00	-.0070

Because we are assuming the same weights in all cases (0.50 – 0.50), the expected return in every instance will be

$$E(R_{\text{port}}) = 0.50(0.10) + 0.50(0.20)$$
$$= 0.15$$

The standard deviation for Case a will be

$$\sigma_{\text{port(a)}} = \sqrt{(0.5)^2(0.07)^2 + (0.5)^2(0.10)^2 + 2(0.5)(0.5)(0.0070)}$$
$$= \sqrt{(0.001225) + (0.0025) + (0.5)(0.0070)}$$
$$= \sqrt{0.007225}$$
$$= 0.085$$

For Cases b, c, d, and e, the standard deviation for the portfolio would be as follows:

$$\sigma_{\text{port(b)}} = \sqrt{(0.001225) + (0.0025) + (0.5)(0.0035)}$$

$$= \sqrt{(0.005475)}$$

$$= 0.07399$$

$$\sigma_{\text{port(c)}} = \sqrt{(0.001225) + (0.0025) + (0.5)(0.00)}$$

$$= 0.0610$$

$$\sigma_{\text{port(d)}} = \sqrt{(0.001225) + (0.0025) + (0.5)(-0.0035)}$$

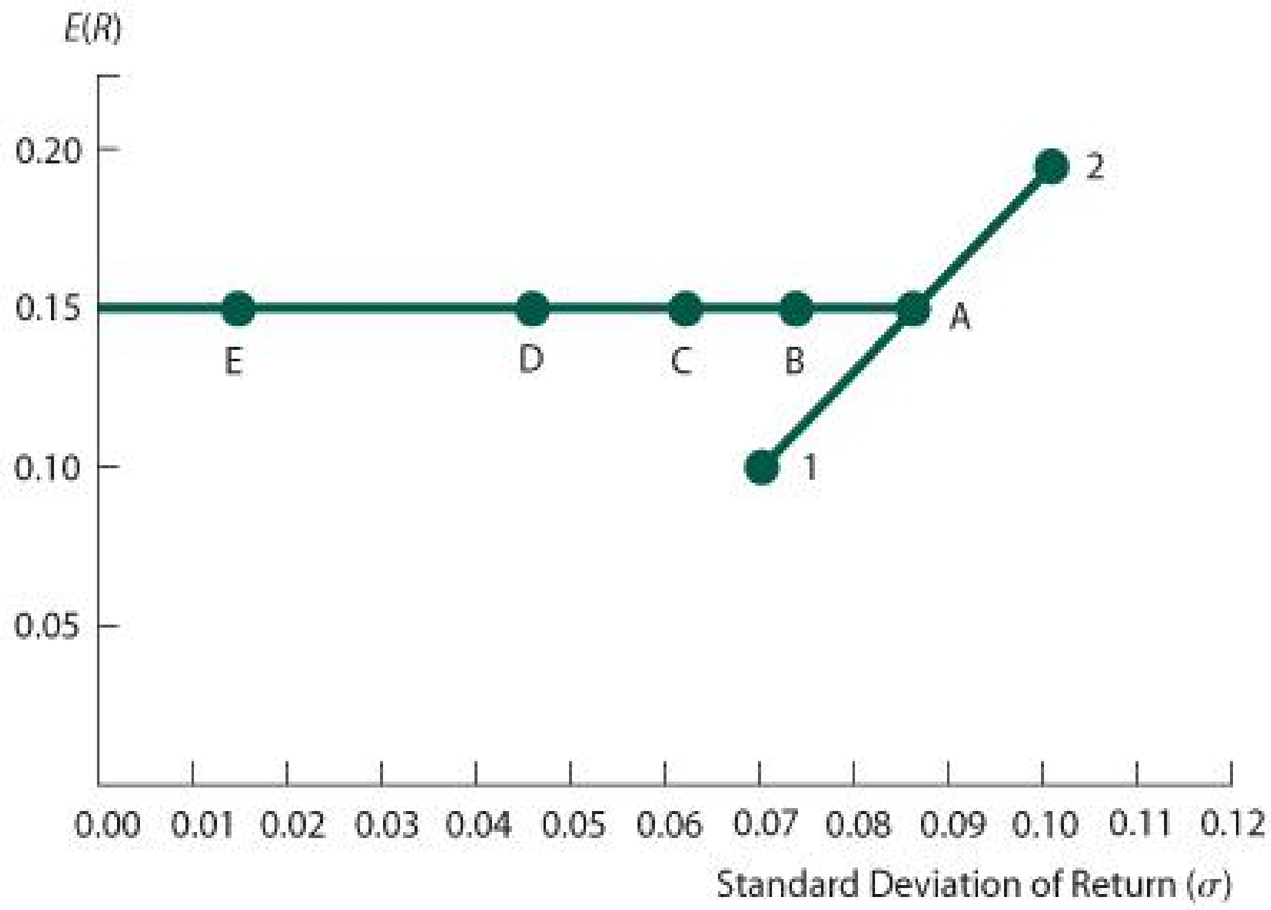
$$= 0.0444$$

$$\sigma_{\text{port(e)}} = \sqrt{(0.003725) + (0.5)(-0.0070)}$$

$$= 0.015$$

Exhibit

Exhibit 7.12 Risk–Return Plot for Portfolios with Different Returns, Standard Deviations, and Correlations



Standard Deviation of a Portfolio:
With Constant Correlation but Changing Weights of Assets

- Constant Correlation with *Changing Weights of Assets*
  - Two Stocks with Different Returns and Risk

Asset	$E(R_i)$	σ^2_i	σ_i
1	.10	.0049	.07
2	.20	.0100	.10

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Constant Correlation with Changing Weights of Assets

- Constant Correlation with Changing Weights
  - Assume the correlation is 0 in the earlier example and let the weight vary as shown below.
  - Portfolio return and risk are (See Exhibit 7.13):

Case	W_1	W_2	$E(R_i)$	$E(\sigma_{port})$
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700

We already know the standard deviation (σ) for Portfolio i. In Cases f, g, h, j, k, and m, the standard deviations would be[10]

$$\sigma_{port(g)} = \sqrt{(0.20)^2(0.07)^2 + (0.80)^2(0.10)^2 + 2(0.20)(0.80)(0.00)}$$
$$= \sqrt{(0.04)(0.0049) + (0.64)(0.01) + (0)}$$
$$= \sqrt{(0.006596)}$$
$$= 0.0812$$

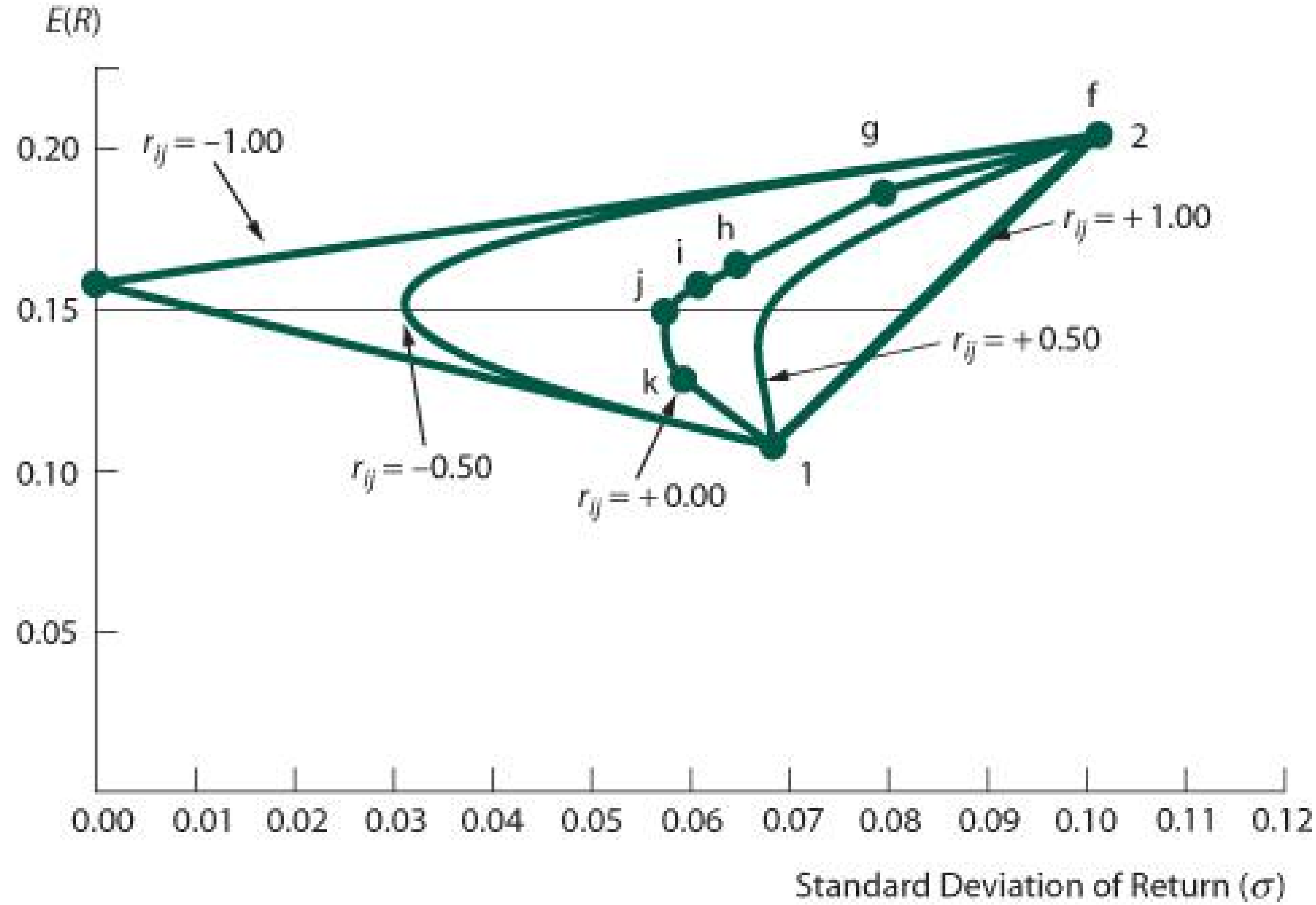
$$\sigma_{port(h)} = \sqrt{(0.40)^2(0.07)^2 + (0.60)^2(0.10)^2 + 2(0.40)(0.60)(0.00)}$$
$$= \sqrt{(0.004384)}$$
$$= 0.0662$$

$$\sigma_{port(j)} = \sqrt{(0.60)^2(0.07)^2 + (0.40)^2(0.10)^2 + 2(0.60)(0.40)(0.00)}$$
$$= \sqrt{(0.003364)}$$
$$= 0.0580$$

$$\sigma_{port(k)} = \sqrt{(0.80)^2(0.07)^2 + (0.20)^2(0.10)^2 + 2(0.80)(0.20)(0.00)}$$
$$= \sqrt{(0.003536)}$$
$$= 0.0595$$

Exhibit

Exhibit 7.13 Portfolio Risk–Return Plots for Different Weights When $r_{i,j}$ = +1.00; +0.50; 0.00; −0.50; −1.00



Diversification Potential

- The potential of an asset to diversify a portfolio is dependent upon the degree of co-movement of returns of the asset with those other assets that make up the portfolio.
- In a simple, two-asset case, if the returns of the two assets are perfectly negatively correlated it is possible (depending on the relative weighting) to eliminate all portfolio risk.
- This is demonstrated through the following chart.

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Constant Correlation with Changing Weights of Assets

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	1
B	14.0%	40.0%	

Perfect Positive Correlation – no diversification

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	17.5%
80.00%	20.00%	6.80%	20.0%
70.00%	30.00%	7.70%	22.5%
60.00%	40.00%	8.60%	25.0%
50.00%	50.00%	9.50%	27.5%
40.00%	60.00%	10.40%	30.0%
30.00%	70.00%	11.30%	32.5%
20.00%	80.00%	12.20%	35.0%
10.00%	90.00%	13.10%	37.5%
0.00%	100.00%	14.00%	40.0%

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0.5
B	14.0%	40.0%	

Positive Correlation – weak diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	15.9%
80.00%	20.00%	6.80%	17.4%
70.00%	30.00%	7.70%	19.5%
60.00%	40.00%	8.60%	21.9%
50.00%	50.00%	9.50%	24.6%
40.00%	60.00%	10.40%	27.5%
30.00%	70.00%	11.30%	30.5%
20.00%	80.00%	12.20%	33.6%
10.00%	90.00%	13.10%	36.8%
0.00%	100.00%	14.00%	40.0%

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0
B	14.0%	40.0%	

No Correlation – some diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	14.1%
80.00%	20.00%	6.80%	14.4%
70.00%	30.00%	7.70%	15.9%
60.00%	40.00%	8.60%	18.4%
50.00%	50.00%	9.50%	21.4%
40.00%	60.00%	10.40%	24.7%
30.00%	70.00%	11.30%	28.4%
20.00%	80.00%	12.20%	32.1%
10.00%	90.00%	13.10%	36.0%
0.00%	100.00%	14.00%	40.0%

Lower risk than asset A

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-0.5
B	14.0%	40.0%	

Negative Correlation – greater diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	12.0%
80.00%	20.00%	6.80%	10.6%
70.00%	30.00%	7.70%	11.3%
60.00%	40.00%	8.60%	13.9%
50.00%	50.00%	9.50%	17.5%
40.00%	60.00%	10.40%	21.6%
30.00%	70.00%	11.30%	26.0%
20.00%	80.00%	12.20%	30.6%
10.00%	90.00%	13.10%	35.3%
0.00%	100.00%	14.00%	40.0%

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-1
B	14.0%	40.0%	

Perfect Negative Correlation – greatest diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	9.5%
80.00%	20.00%	6.80%	4.0%
70.00%	30.00%	7.70%	1.5%
60.00%	40.00%	8.60%	7.0%
50.00%	50.00%	9.50%	12.5%
40.00%	60.00%	10.40%	18.0%
30.00%	70.00%	11.30%	23.5%
20.00%	80.00%	12.20%	29.0%
10.00%	90.00%	13.10%	34.5%
0.00%	100.00%	14.00%	40.0%

Risk of the portfolio is almost eliminated at 70% asset A

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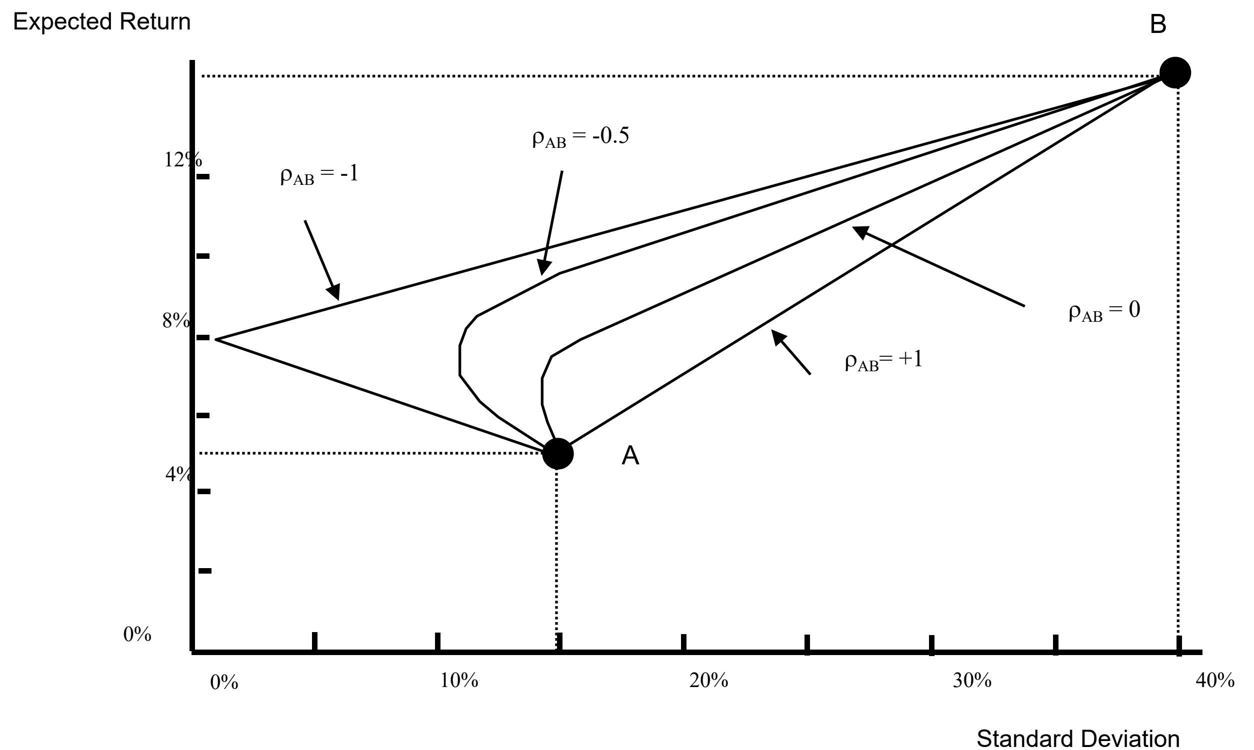
Diversification of a Two Asset Portfolio Demonstrated Graphically

The Effect of Correlation on Portfolio Risk: The Two-Asset Case



- **ASSIGNMENT With Presentation:**
  - ✓ Capital Asset Pricing Model (CAPM)
  - ✓ Arbitrage pricing theory
  - ✓ Fama-French 3-factor model

THANK U
AND
HAVE A NICE DAY!