UNIT FOUR

The Assignment Model

The Assignment Model

Characteristics

• Special form of linear programming model similar to the transportation model.

• Supply at each source and demand at each destination limited to one unit.

• In a balanced model supply equals demand.

• In an unbalanced model supply does not equal demand.

The Assignment Model
Example Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

The Assignment Model
Example Problem Model Formulation

Minimize $Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to

$$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$$

$$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$$

$$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$$

$$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$$

$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$$

$$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$$

$$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$$

$$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$$

$$x_{ij} \geq 0$$

Solution of the Assignment Model
(1 of 7)

- An *assignment problem* is a special form of the transportation problem where all supply and demand values equal one.

- Example: assigning four teams of officials to four games in a way that will minimize distance traveled by the officials.

Officials	Game Sites			
	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

The Travel Distances to Each Game for Each Team of Officials

- An *opportunity cost table* is developed by first subtracting the minimum value in each row from all other row values *(row reductions)* and then repeating this process for each column.

Officials	Game Sites			
	Raleigh	Atlanta	Durham	Clemson
A	120	0	90	70
B	30	0	60	130
C	70	0	35	65
D	15	0	40	55

The Assignment Tableau with Row Reductions

 - The minimum value in each column is subtracted from all column values (*column reductions*).

 - Assignments can be made in the table wherever a zero is present.

 - An *optimal solution* results when each of the four teams can be assigned to a different game.

 - Table 36 does not contain an optimal solution

	Game Sites			
Officials	Raleigh	Atlanta	Durham	Clemson
A	105	0	55	15
B	15	0	25	75
C	55	0	0	10
D	0	0	5	0

The Tableau with Column Reductions

- An optimal solution occurs when the number of independent unique assignments equals the number of rows and columns.

- If the number of unique assignments is less than the number of rows (or columns) a line test must be used.

Officials	Game Sites			
	Raleigh	Atlanta	Durham	Clemson
A	105	0	55	15
B	15	0	25	75
C	35	0	0	10
D	0	0	5	0

The Opportunity Cost Table with the Line Test

- In a line test all zeros are crossed out by horizontal and vertical lines; the minimum uncrossed value is subtracted from all other uncrossed values and added to values where two lines cross.

Officials	Game Sites			
	Raleigh	Atlanta	Durham	Clemson
A	90	0	40	0
B	0	0	10	60
C	55	15	0	10
D	0	15	5	0

The Second Iteration

- At least four lines are required to cross out all zeros in table 38.

- This indicates an optimal solution has been reached.

- Assignments and distances:

<u>Assignment Distance</u>	<u>Distance</u>	<u>Assignment</u>	
Team A → Atlanta	90	Team A → Clemson	160
Team B → Raleigh	100	Team B → Atlanta	70
Team C → Durham	140	Team C → Durham	140
Team D → Clemson	120	Team D → Raleigh	80
Total	450 miles	Total	450 miles

- If in initial assignment team A went to Clemson, result is the same; resulting assignments represent multiple optimal solutions.

- When supply exceeds demand, a dummy column is added to the tableau.

- When demand exceeds supply, a dummy row is added to the tableau.

- The addition of a dummy row or column does not affect the solution method.

- A prohibited assignment is given a large relative cost of M so that it will never be selected.

Officials	Raleigh	Atlanta	Durham	Clemson	Dummy
A	210	90	180	160	0
B	100	70	130	200	0
C	175	105	140	170	0
D	80	65	105	120	0
E	95	115	120	100	0

Game Sites

An Unbalanced Assignment Tableau with a Dummy Column

Solution of the Assignment Model
Summary of Solution Steps

1. Perform row reductions.

2. Perform column reductions.

3 In the completed opportunity cost table, cross out all zeros using the minimum number of horizontal and/or vertical lines.

4. If fewer than m lines are required, subtract the minimum uncrossed value from all other uncrossed values, and add the same value to all cells where two lines intersect.

5. Leave all other values unchanged and repeat step 3.

6. If m lines are required, the tableau contains the optimal solution. If fewer than m lines are required, repeat step 4.

Exercise: Make the optimal assignment of the following typists to the appropriate jobs.

Typist	Job				
	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	45	66	57	114	69
D	80	32	60	120	72
E	76	64	56	112	68