CHAPTER FIVE

Decision Theory Model

Introduction

- ☞ Brainstorming question
- Why do some organizations succeed and some others fail?

Decision theory (cont'd)

- Making of a decision requires enumeration of feasible and viable alternatives, the consequences associated with different alternatives, and measure of effectiveness by which the most preferred alternative is identified.
- Decision theory provides an analytical and systematic approach to the study of decision making.
- It provides a method of natural decision making wherein data concerning the occurrence of different outcomes may be evaluated to enable the decision maker to identify suitable alternative.

Decision theory (cont'd)

- Decision models useful in helping decision makers make the best possible decisions are classified according to the degree of certainty. The scale of certainty can range from complete certainty to complete uncertainty. The region which falls between this two is decision making under risk or probabilistic problems.

Some important terminologies

- Decision alternatives- courses of action, acts, or strategies and are under the control of the decision maker and known to the decision maker
- State of nature- A possible future condition resulting from the choice of the decision. States of nature are mutually exclusive and collectively exhaustive.
- Payoff- A numerical value resulting from each possible combination of alternatives and state of nature is called the payoffs

Steps in decision making

- Decision making process involves the following steps
  - Identify and define the problem
  - Listing of possible future events which can occur in the context of the decision problems
  - Identification of all the courses of action which are available to the decision maker
  - Expressing the payoffs resulting from the pair of state of nature and alternative courses of action
  - Apply an appropriate mathematical decision theory model to select the best course of action from the given list on the basis of some criteria that results in the optimal payoff.

Decision making environment

- Decision making under certainty
- Decision making under risk
- Decision making under uncertainty
  - What do they mean?

Decision making under uncertainty

- In the absence of knowledge about the probability of any state of nature occurring, the decision maker must arrive at decision only on the actual conditional payoff values, together with the policy.
- There are several different criteria of decision making uner such situation:
  - Maximax
  - Maximin
  - Equally likely/laplace
  - Criterion of realism
  - Criterion of regret

Criterion of optimism

- The decision maker ensures that he should not miss the opportunity to achieve the largest possible profit (maximax) or lowest possible cost (minimin). Thus he selects the alternative that represents the maximum of the maximums or minimum of minimums.
- Since this criterion selects the alternative with the largest payoff value, it is called an optimism decision criterion. The method is summarized as follows:
  - Select the maximum payoff value corresponding to each alternative
  - Select the alternative with largest anticipated pay of value

Criterion of pessimism

- The decision maker ensures that he would earn no less than or pay no more than some specified amount. He selects the alternative that represents the maximum of the minima (minimum of the maxima in case of lose) pay off in the case of profits. The method can be summarized as;
- Locate the minimum pay off value in the case of loss and maximum in the case of profit corresponding to each alternative course of action
- Select an alternative with the best anticipated payoff value
- This criterion is conservative about the future and always anticipates the worst possible outcome. It is called pessimistic decision criterion or weld's criterion.

Equally likely decision (laplace) criterion

- Some probabilities of state of nature are not known and therefore it is assumed that all states of nature is assigned an equal probability. As states of nature are mutually exclusive and collectively exhaustive, the probability of each of these must be one divided by number of states of nature. The working method is summarized as follows.
  - Assign equal probability value to each state of nature
  - Compute the expected payoff for each alternative as $\Sigma PjPij$
  - Select the best expected payoff
  - This criterion is also called the criterion of insufficient reason

Criterion of realism (Hurwicz criterion)

- This suggests that rational decision maker should not be neither completely optimistic or pessimistic; therefore, must display a mixture of both. Hurwicz introduced the concept of a coefficient of Optimism to measure the decision maker's degree of optimism. This coefficient lies between 0 and 1 where 0 represents complete pessimistic and 1 represents complete optimistic attitude about the future. Thus, α shows degree of optimism and $1-\alpha$ shows the coefficient of pessimism.
- Hurwicz approach suggests that the decision maker must select the alternative that maximizes:
  - H (α) (maximum in the column) + (1 minus α) (minimum in the column)
- The method can summarized as:
  - Decide the coefficient of optimism
  - For each alternative select the largest and the lowest payoff value and multiply it with the coefficient of optimism and the coefficient of pessimism respectively and calculate the weighted average.
  - Select the alternative with the largest anticipated weighted average payoff.

Criterion of regret

- It is also called opportunity loss decision criterion or minimax regret decision criterion because decision maker feels regret after adopting a wrong course of action resulting from resulting In an opportunity loss of payoffs. Thus he always intends to minimize this regret. The method can summarized as:
- From the given payoff matrix develop an opportunity loss matrix as follows
  - Find the best payoff corresponding to each sate of nature
  - Subtract all other entries in that row from this value
  - For each course of action, identify the worst or maximum regret value
  - Select the course of action with the smallest anticipated opportunity loss value

Illustration

Payoff table

Decision alternatives	State of demand		
	High	Medium	Low
Apartment bldg.	100,000	70,000	25,000
Office bldg.	50,000	40,000	30,000
Warehouse bldg.	150,000	25,000	(30,000)

Decision making under risk

- The essential difference between decision making under complete uncertainty and decision making under partial uncertainty is the presence of probabilities for the occurrence of the various states of nature under partial uncertainty.
- The term risk is often used in conjunction with partial uncertainty.
- The probabilities can be subjective estimates form managers or experts or it can be the reflection of historical frequencies. The sum of the probabilities of all states of nature must be 1.

Decision making criteria

- Expected monetary value
- Expected opportunity loss
- Expected value of perfect information

	State of nature	
Decision alternative	Strong demand P= 0.8	Weak demand P= 0.2
Small complex	8	7
Medium complex	14	5
Large complex	20	-9

Illustration

Payoff table

Decision alternatives	State of demand		
	High (0.5)	Medium(0.3)	Low(0.2)
Apartment bldg.	100,000	70,000	25,000
Office bldg.	50,000	40,000	30,000
Warehouse bldg.	150,000	25,000	(30,000)

Expected monetary value

- provides the decision maker with a value which represents an average payoff for each alternative.
- The best alternative is the one that has highest expected monetary value.
- The average or expected payoff of each alternative is a weighted average: the state of nature probabilities are used to weight the respective payoffs (long run average amount one could reasonably anticipate

Expected opportunity loss

- An alternative method to incorporate probabilities in to decision is to use expected opportunity loss.
- The approach is nearly identical to the EMV approach provided that the table of opportunity loss is used rather than the table of payoffs.
- Hence opportunity losses are weighted by the probabilities of their respective states of nature to compute a long run average opportunity loss, and the alternative with the smallest expected loss is selected (Opportunity loss results in the alternative as EMV approach).

Expected value of perfect information

- It can sometimes be useful for decision maker to determine the potential benefit of knowing for certain which state of nature is going to prevail.
- It is the measure of the difference between certain payoff that could be realized under a condition of certainty and expected payoff under condition involving risk.
- The expected value of perfect information presents an upper bound on the amount of money that the decision maker would be justified to spend to obtain perfect information

Expected value of perfect information

- It is, however, not always possible to remove uncertainties utterly.
- In such cases the decision maker must weigh the cost to reduce uncertainty (obtain better estimates of probabilities) against the expected benefits that would result from the improved estimation.
- EVPI is exactly equal to EOL. The EOL indicates the expected opportunity loss due to imperfect information which is equal to the expected payoff of the perfect information.

	State of nature	
Decision alternative	Strong demand P= 0.8	Weak demand P= 0.2
Small complex	8	7
Medium complex	14	5
Large complex	20	-9

P (strong d= 0.8)

- EMV 1= 7.8
- EMV 2= 12.2
- EMV 3= 14.2

EOL1= 12(0.8)= 9.6
EOL2= 6(0.8) + 2(0.2) =5.2
EOL3= 16(0.2) =3.2

EVWPI= 20*0.8+7*0.2= 17.4

EVPI=EVWPI-EMV=17.4-14.2=3.2

Example Question:

- Roba wants to establish the factory in Ethiopia. He has two outcomes or states of nature which are favorable market and unfavorable market. He can either establish last factory, small factory or he won't go for the factory that means do nothing. If he establishes a large factory, he could earn profit of BIRR 200,000 or loose BIRR 180,000 in unfavorable market. If he establishes the small factory, he could earn BIRR 100,000 in favorable market. He could lose BIRR 20,000 in unfavorable market. Now the purpose of establishing the factory is to earn maximum profit.
- Now if we shape the above example in a tabular form, we will get this kind of table as given below.

	STATES OF NATURE	
ALTERNATIVES	**FAVORABLE MARKET (BIRR)**	**UNFAVORABLE MARKET (BIRR)**
ESTABLISH LARGE FACTORY	**200,000**	**-180,000**
ESTABLISH SMALL FACTORY	**100,000**	**-20,000**
DO NOTHING	**0**	**0**

- **Decision Making Under Certainty:**
- In this environment, we already know what is best for us. We know the alternatives and results and we choose the best naturally. Like if we have 100,000 Birr and we want to invest, then we would like to open a bank's saving account which will give us more interest. Like we have the options of 5% annually, 10% annually and 12% annually. So we would automatically and naturally choose 12%.

- **Decision Making Under Uncertainty:**

In this environment, probability is not given. So we have the five situations under which we can make decisions.

- Maximax
- Maximin
- Criterion of Realism
- Equally Likely
- Minimax Regret

Maximax:

We usually use the optimistic or positive thinking approach here. We will choose which will give us maximum payoff

	States Of Nature		
Alternatives	**Favorable Market**	**Unfavorable Market**	**Maximum in a Row**
Establish Large Factory	**200,000**	**-180,000**	**200,000**
Establish Small Factory	**100,000**	**-20,000**	**100,000**
Do Nothing	**0**	**0**	**0**

Maximin:

Maximin is the totally opposite to the Maximax. In this environment, we take the pessimistic approach. In this scenario, we will take the lowest payoff which will be like decreasing the risk factor.

	States Of Nature		
Alternatives	**Favorable Market**	**Unfavorable Market**	**Minimum in a Row**
Establish Large Factory	**200,000**	**-180,000**	**-180,000**
Establish Small Factory	**100,000**	**-20,000**	**-20,000**
Do Nothing	**0**	**0**	**0**

Criterion of Realism

- In this situation, we are given the percentage of either pessimistic or optimistic. Like a question may contain that the person is 80% optimistic. So the probability will be 0.80 and 0.20. 0.80 of optimistic and 0.20 of pessimistic. We have to complete the 1 or 100% like 0.80+0.20=1

	States Of Nature		
Alternatives	**Favorable Market**	**Unfavorable Market**	**Criterion Of Realism**
Establish Large Factory	**200,000**	**-180,000**	**124,000**
Establish Small Factory	**100,000**	**-20,000**	**76,000**
Do Nothing	**0**	**0**	**0**

Equally Likely (Laplace):

- Equally likely criterion uses the average outcome. One criterion that uses all the payoffs for each alternative is equally likely.
-

	States Of Nature		
Alternatives	Favorable Market	Unfavorable Market	Laplace Criterion
Establish Large Factory	200,000	-180,000	10,000
Establish Small Factory	100,000	-20,000	40,000
Do Nothing	0	0	0

Minimax Regret:

- This criterion is based on opportunity loss. Opportunity loss refers to the difference between the optimal profit or payoff for a given state of nature and the actual payoff received for a particular decision. In other words, it is the amount lost by not picking the best alternative in a given outcome.

	Opportunity Loss Table		
	States Of Nature		
	Favorable Market	Unfavorable Market	
	200,000 - 200,000	0 - (-180,000)	
	200,000 - 100,000	0 - (-20,000)	
	200,000 - 0	0 - 0	
	States Of Nature		
Alternatives	Favorable Market	Unfavorable Market	Maximum in a Row
Establish Large Factory	0	180,000	180,000
Establish Small Factory	100,000	20,000	100,000
Do Nothing	200,000	0	200,000

Decision Making Under Risk:

Expected Monetary Value:

- The EMV means the long run average value of that decision. The EMV for an alternative is just the sum of possible payoffs of the alternatives, each weighted by the probability of that payoff occurring the expected value material.

- We can calculate EMV with the following formula.

- EMV (alternative i) = (Payoff of first state of nature) X (probability of first state of nature) + (Payoff of second state of nature) X (probability of second state of nature) + …..(Payoff of i state of nature) X (probability of i state of nature)

- Now let's calculate the EMV of every alternative. The probability is given that is 0.5
- EMV (Large Factory) = (0.5) (200,000) + (0.5) (-180,000)
  = **10,000**

- EMV (Small Factory) = (0.5) (100,000) + (0.5) (-20,000)
  =**40,000**

- EMV (Do Nothing) = (0.5) (0) + (0.5) (0)
  = **0**

	States Of Nature		
Alternatives	Favorable Market	Unfavorable Market	EMV
Establish Large Factory	200,000	-180,000	10,000
Establish Small Factory	100,000	-20,000	40,000
Do Nothing	0	0	0
Probabilities	0.5	0.5	

Expected Value Of Perfect Information (EVPI):

- Roba wants to produce another product and he is not aware of real market situation. Roba was suggested by the scientific marketing company to avail their services which will tell the whole scenario after conducting the survey that whether is it good for Roba to produce another product or not according to market situation? The firm is charging 65,000 birr. The term EVPI is used to check the worth of the information that the firm is giving in a particular amount.
- The formula for EVPI:
- EVPI = expected value *with* perfect information – maximum EMV

- ***Now here comes another question that what the hell Expected Value With Perfect Information is?***

- So here is the formula for EVwPI.

- EVwPI = (best payoff for first state of nature) X (probability of first state of nature) + (best payoff for second state of nature) X (probability of second state of nature) ……. (best payoff for i state of nature) X (probability of i state of nature)

- Look at the table given above. We have two states of nature. One is favorable market and the other one is unfavorable market. The best alternative for favorable market is Establishing a large factory with the payoff of 200,000. The best alternative for unfavorable market is Do Nothing with the payoff of 0 only. By using this information, we can put the values in our EVwPI formula.

- EVwPI = (200000)(0.5) + (0)(0.5) = 100,000

We have the EVwPI 100,000 birr. Now we have also the formula of EVPI where the value of EVwPI is used.

- EVPI = EVwPI – Maximum EMV

  = 100000 – 40000

  = 60,000

- Thus, Roba would be willing to pay for perfect information is 60,000 birr and the firm is charging 65,000 birr. So Roba won't go for the scientific marketing firm because the value of information is not 65,000.

Expected Opportunity Loss:

- The alternative approach to maximizing the EMV is to minimize expected opportunity loss. We will construct opportunity loss table first.
- Then the EOL is computed for each alternative by multiplying the opportunity loss by the probability and adding these together.

	States Of Nature	
Alternatives	**Favorable Market**	**Unfavorable Market**
Establish Large Factory	**0**	**180,000**
Establish Small Factory	**100,000**	**20,000**
Do Nothing	**200,000**	**0**
Probabilities	**0.5**	**0.5**

- The above table is describing the Opportunity Loss Table. Now we will calculate the Expected Opportunity Loss for every alternative.
- EOL For Large Factory = (0.5) (0) + (0.5) (180000) = **90000**
- EOL For Small Factory = (0.5) (100,000) + (0.5) (20,000)= **60000**
- EOL For Do Nothing = (0.5) (200,000) + (0.5) (0) = **100000**

- Now we will again construct the table of opportunity loss and will put the values of EOL in front of each alternative.

	States Of Nature		
Alternatives	Favorable Market	Unfavorable Market	EOL
Establish Large Factory	0	180,000	90,000
Establish Small Factory	100,000	20,000	60,000
Do Nothing	200,000	0	100,000

- Here we have the results in a tabular form. We have already told you that our decision will be based on the minimum EOL. So the choice that Roba will select is to establish a small factory as the Expected Opportunity Loss is less than the EOL of Large factory and if he don't construct a factory.

- One thing to remember is that minimum EOL will always result in the same decision as maximum EMV and that the EVPI will always equal to minim EOL. You can compare the EOL of small factory with the EVPI that we have calculated. The values of both the things are 60,000 birr.