UNIT FOUR

# The Assignment Model

# The Assignment Model

## Characteristics

- Special form of linear programming model similar to the transportation model.
- Supply at each source and demand at each destination limited to one unit.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

# The Assignment Model
## Example Problem Definition and Data

**Problem:** Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | RALEIGH | ATLANTA | DURHAM | CLEMSON |
| A | 210 | 90 | 180 | 160 |
| B | 100 | 70 | 130 | 200 |
| C | 175 | 105 | 140 | 170 |
| D | 80 | 65 | 105 | 120 |

# The Assignment Model
# Example Problem Model Formulation

Minimize $Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to

$$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$$

$$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$$

$$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$$

$$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$$

$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$$

$$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$$

$$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$$

$$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$$

$$x_{ij} \geq 0$$

# Solution of the Assignment Model (1 of 7)

- An *assignment problem* is a special form of the transportation problem where all supply and demand values equal one.

- Example: assigning four teams of officials to four games in a way that will minimize distance traveled by the officials.

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson |
| A | 210 | 90 | 180 | 160 |
| B | 100 | 70 | 130 | 200 |
| C | 175 | 105 | 140 | 170 |
| D | 80 | 65 | 105 | 120 |



The Travel Distances to Each Game for Each Team of Officials

# Solution of the Assignment Model (2 of 7)

- An *opportunity cost table* is developed by first subtracting the minimum value in each row from all other row values *(row reductions*) and then repeating this process for each column.

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson |
| A | 120 | 0 | 90 | 70 |
| B | 30 | 0 | 60 | 130 |
| C | 70 | 0 | 35 | 65 |
| D | 15 | 0 | 40 | 55 |

The Assignment Tableau with Row Reductions

# Solution of the Assignment Model (3 of 7)

- The minimum value in each column is subtracted from all column values (*column reductions*).

- Assignments can be made in the table wherever a zero is present.

- An *optimal solution* results when each of the four teams can be assigned to a different game.

- Table 36 does not contain an optimal solution

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson |
| A | 105 | 0 | 55 | 15 |
| B | 15 | 0 | 25 | 75 |
| C | 55 | 0 | 0 | 10 |
| D | 0 | 0 | 5 | 0 |

The Tableau with Column Reductions

# Solution of the Assignment Model (4 of 7)

- An optimal solution occurs when the number of independent unique assignments equals the number of rows and columns.

- If the number of unique assignments is less than the number of rows (or columns) a line test must be used.

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson |
| A | 105 | 0 | 55 | 15 |
| B | 15 | 0 | 25 | 75 |
| C | 35 | 0 | 0 | 10 |
| D | 0 | 0 | 5 | 0 |

The Opportunity Cost Table with the Line Test

# Solution of the Assignment Model (5 of 7)

- In a line test all zeros are crossed out by horizontal and vertical lines; the minimum uncrossed value is subtracted from all other uncrossed values and added to values where two lines cross.

| | Game Sites | | | |
|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson |
| A | 90 | 0 | 40 | 0 |
| B | 0 | 0 | 10 | 60 |
| C | 55 | 15 | 0 | 10 |
| D | 0 | 15 | 5 | 0 |

The Second Iteration

# Solution of the Assignment Model (6 of 7)

- At least four lines are required to cross out all zeros in table 38.

- This indicates an optimal solution has been reached.

- Assignments and distances:

| Assignment | Distance | Assignment | Distance |
|---|---|---|---|
| Team A → Atlanta | 90 | Team A → Clemson | 160 |
| Team B → Raleigh | 100 | Team B → Atlanta | 70 |
| Team C → Durham | 140 | Team C → Durham | 140 |
| Team D → Clemson | 120 | Team D → Raleigh | 80 |
| | Total 450 miles | | Total 450 miles |

- If in initial assignment team A went to Clemson, result is the same; resulting assignments represent multiple optimal solutions.

# Solution of the Assignment Model (7 of 7)

- When supply exceeds demand, a dummy column is added to the tableau.

- When demand exceeds supply, a dummy row is added to the tableau.

- The addition of a dummy row or column does not affect the solution method.

- A prohibited assignment is given a large relative cost of *M* so that it will never be selected.

| | Game Sites | | | | |
|---|---|---|---|---|---|
| **Officials** | Raleigh | Atlanta | Durham | Clemson | Dummy |
| A | 210 | 90 | 180 | 160 | 0 |
| B | 100 | 70 | 130 | 200 | 0 |
| C | 175 | 105 | 140 | 170 | 0 |
| D | 80 | 65 | 105 | 120 | 0 |
| E | 95 | 115 | 120 | 100 | 0 |

An Unbalanced Assignment Tableau with a Dummy Column

# Solution of the Assignment Model
# Summary of Solution Steps

1. Perform row reductions.

2. Perform column reductions.

3 In the completed opportunity cost table, cross out all zeros using the minimum number of horizontal and/or vertical lines.

4. If fewer than *m* lines are required, subtract the minimum uncrossed value from all other uncrossed values, and add the same value to all cells where two lines intersect.

5. Leave all other values unchanged and repeat step 3.

6. If *m* lines are required, the tableau contains the optimal solution. If fewer than *m* lines are required, repeat step 4.

**Exercise:** Make the optimal assignment of the following typists to the appropriate jobs.

| Typist | Job | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| A | 10 | 11 | 6 |
| B | 11 | 9 | 6 |
| C | 8 | 8 | 7 |

| | | JOB | | |
|---|---|---|---|---|
| TYPIST | 1 | 2 | 3 | Row min |
| A | 10-6=4 | 11-6=5 | 6-6=0 | 6 |
| B | 11-6=5 | 9-6=3 | 6-6=0 | 6 |
| C | 8-7=1 | 8-7=1 | 7-7=0 | 7 |

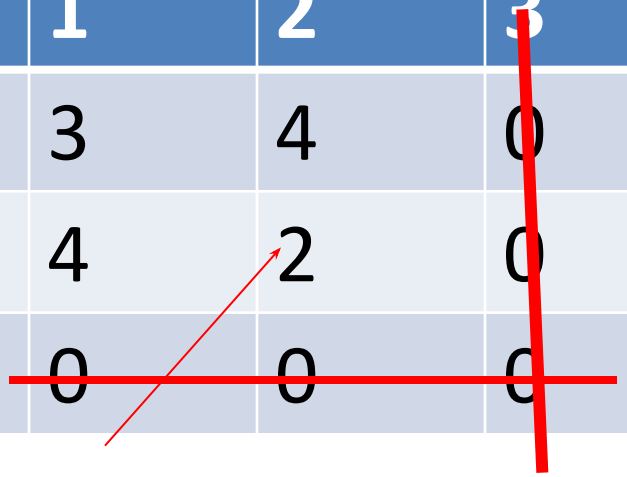
ROW REDUCED MATRIX

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 4 | 5 | 0 |
| B | 5 | 3 | 0 |
| C | 1 | 1 | 0 |

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 4 | 5 | 0 |
| B | 5 | 3 | 0 |
| C | 1 | 1 | 0 |
| **column min** | **1** | **1** | **0** |

Column reduced matrix

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 3 | 4 | 0 |
| B | 4 | 2 | 0 |
| C | 0 | 0 | 0 |



Draw minimum number of lines covering zeros

Identify the smallest value that is not covered by the line and subtract that value from all un covered values and add the same value on the intersection of the two lines.

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 3-2=1 | 4-2=2 | 0 |
| B | 4-2=2 | 2-2=0 | 0 |
| C | 0 | 0 | +2 |

Draw min number of lines again

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 1 | 2 | 0 |
| B | ~~2~~ | ~~0~~ | ~~0~~ |
| C | ~~0~~ | ~~0~~ | ~~2~~ |

- Make an assignment

| | JOB | | |
|---|---|---|---|
| TYPIST | 1 | 2 | 3 |
| A | 1 | 2 | 0 |
| B | 2 | 0 | 0 |
| C | 0 | 0 | 2 |

- Total assignment cost=6+9+8=23