This is a copy of Homework 7, released on myCourses. You can essentially work out all of the problems and simply go on myCourses to plug in some numbers and submit your work. This homework is based on some general principles of the thermodynamics of ideal gases. You will require an understanding of the ideas covered in Chapters 19 and 20 of Mazur.

Note: Numerical quantities in curly braces, such as {10} may be randomized on the actual assignment. You may use the value of the gas constant $R = 8.314\ \frac{J}{K\ mol}$ where necessary.

SHORT PROBLEMS

Problem S1: Isentropic Work

1 mol of a diatomic ideal gas undergoes an isentropic process that takes it from a state (P_1, V_1, T_1) to a state (P_2, V_2, T_2). If $P_1 = \{1000\}$ Pa, $V_1 = \{1000\}$ L and $V_2 = \{500\}$ L, then, in the form xxx.xx, $T_1 =$ — K, $T_2 =$ — K, and the work done by the gas in the process is (include the correct sign) $W =$ — J.

Problem S2: Triatomic Gas

The value of γ (also called the "adiabatic constant" or the "ratio of specific heats") for a triatomic gas is equal to (a) — at low temperatures and (b) — at high temperatures.

Problem S3: Isothermal Expansion

Two identical thermally insulated spherical tanks, A and B, are connected by a closed valve. Initially tank A contains {10} mol of an ideal diatomic gas, tank B is evacuated. If the valve is then opened and the gas expands isothermally from A to B, what is the change in the entropy of the gas $\left(\text{in units of } \frac{J}{K}\right)$?

Problem S4: Temperature-dependent Specific Heat

A cryogenic substance is found to have a specific heat capacity (at constant volume) c_V that varies with temperature according to $c_V = AT^2$, where A is an empirically derived constant with units $\frac{J}{K^3\ kg}$. If {231} J of energy must be transferred thermally (at constant volume) to an {8000} mg sample of this substance to raise the temperature of the sample from {1.00} to {6.00} K, what is the value of A?

LONG PROBLEMS:

Problem L1: Extensivity

You are told that $S(N, V, T)$ (Eq 20.39 in Mazur) is supposed to be an extensive function of N and V, i.e., $S(\lambda N, \lambda V, T) = \lambda S(N, V, T)$, for any λ. Based on this information, what term (that depends only on N and is independent of V and T) *must* the "constant" in Eq 20.39 in Mazur contain? What other kinds of terms *can* it contain?

Be sure to check that the previously known result: $(S, N) = \text{constants} \implies T^{\frac{d}{2}} V = \text{constant}$ is still valid.

Once you have written these terms down, you will be (almost) fully equipped to deal with problems where N itself can change during a process. You can calculate changes in entropy due to changes in number of particles at fixed volume and temperature, say.

Problem L2: Compressibility

One usually defines the isothermal and adiabatic compressibilities of a thermodynamic system as:

$$\text{Isothermal Compressibility} = \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N}$$
$$\text{Adiabatic Compressibility} = \kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{S,N}$$

For a monoatomic ideal gas, show that:

$$\frac{\kappa_T}{\kappa_S} = \frac{C_P}{C_V}$$

This explains, quantitatively, the discrepancy in the speed of sound in air that you explored in an earlier assignment. As it turns out, this is a universal theorem that holds for arbitrary systems, although proving it requires some gymnastics involving the mathematics of partial derivatives that have not been introduced to you yet.

Notation: If you are given a function $A(X, Y, Z)$, then $\left(\frac{\partial A}{\partial Y}\right)_{X,Z}$ means "take the derivative of A with respect to Y while holding X and Z fixed (i.e., pretend as though X and Z are constants)".