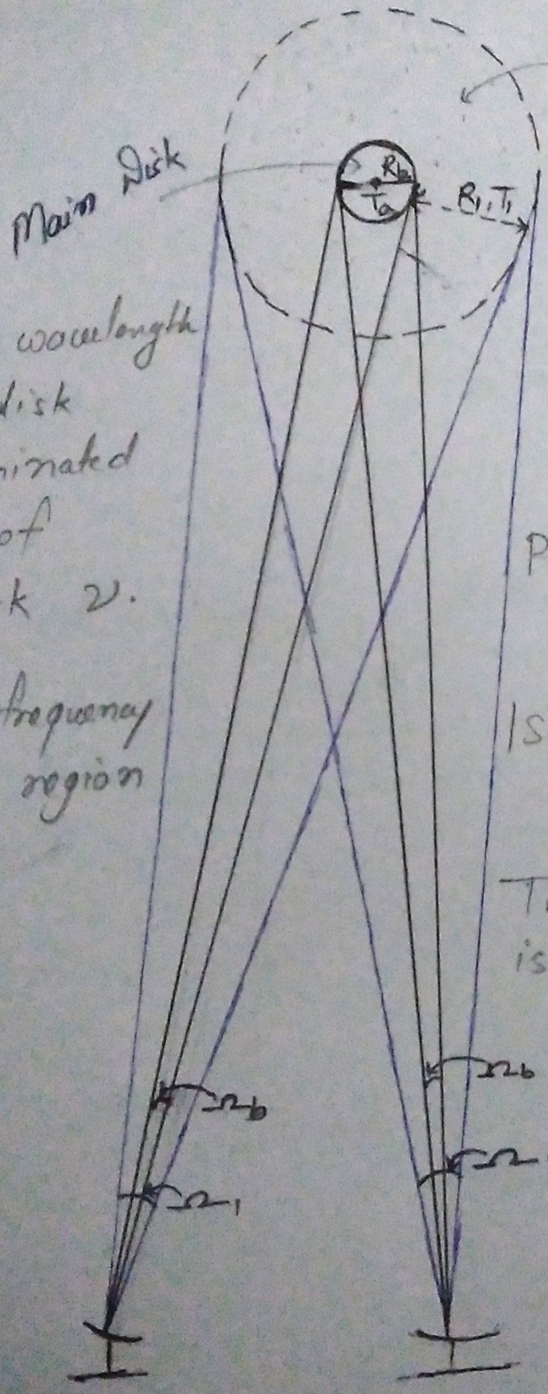
Squared Visibility Of A WR Source (Single System)

let's assume that we have a WR star in absent of any external force (like, other stellar object)



observation wavelength of main disk i.e. the dominated frequency of main disk ν.

Dominated frequency of emission region ν_1.

photon flux from main disk

$$|S(\Omega)|_b^2 = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \quad (1)$$

Photon flux from Emission Region

$$|S(\Omega)|_1^2 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \quad (2)$$

The radius of main disk is R_b & temparature T_b.

The radius of emission Region from the center is $(R_b + R_1)$ & temperature of the region R_1 is T_1

The correlation of intensity from two telescope

$$\phi_1 \phi_2^* \propto \int_{\Omega_1} e^{2\pi i \left(\frac{\nu}{c}\right)(x_1 - x_2)\cdot\Omega_1} |S(\Omega_1)|^2 \, d^2\Omega_1$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_1} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_1} \left[|S(\Omega)|_b^2 + |S(\Omega)|_1^2 \right] d^2\Omega_1$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_b} |S(\Omega)|_b^2 \, d^2\Omega_b$$

$$+ \int_{\Omega_1} e^{2\pi i \left(\frac{\nu_1}{c}\right) x \cdot \Omega_1} |S(\Omega)|_1^2 \, d^2\Omega_1$$

$$\phi_1 \phi_2^* \propto A_1 + A_2 \qquad (3)$$

A_1 is dominated with ν frequency
A_2 is dominated with ν_1 frequency

$\star$ star

baseline

projection of
$x \frac{r}{D} = x \frac{r}{D} \cos\phi$

$\frac{r}{D} = \Omega$

$$A_1 = \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_b} |S(\Omega)|_b^2 \, d^2\Omega_b$$

$$A_1 = \int_0^{r_b} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \frac{r}{D} \cos\phi} \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \cdot \frac{1}{D^2} \int_0^{R_b} r\,dr \int_0^{2\pi} e^{2\pi i\left(\frac{\nu}{c}\right)x \cdot \frac{r}{D}\cos\phi}\, d\phi$$

$$A_1 = f(\nu, T_b) \cdot \frac{1}{D^2} \int_0^{R_b} r\,dr \cdot 2\pi\, J_0\left[\frac{2\pi\nu x r}{cD}\right]$$

$$A_1 = f(\nu, T_b)\, \frac{2\pi}{D^2} \int_0^{R_b} r\, J_0\left[\frac{2\pi\nu x r}{cD}\right] dr$$

$$A_1 = f(\nu, T_b)\, \frac{2\pi R_b^2}{D^2} \cdot \frac{J_1\left[\frac{2\pi\nu x R_b}{cD}\right]}{\left[\frac{2\pi\nu x R_b}{cD}\right]}$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1}\, \frac{2\pi R_b^2}{D^2} \cdot \frac{J_1\left[\frac{2\pi\nu x R_b}{cD}\right]}{\left[\frac{2\pi\nu x R_b}{cD}\right]} \tag{4}$$

$$A_2 = \int_{\Omega_1} e^{2\pi i\left(\frac{\nu_1}{c}\right)x \cdot \Omega_1} \,|S(\Omega)|_1^2\, d^2\Omega_1$$

$$A_2 = \int_{R_b}^{R_1} \int_0^{2\pi} e^{2\pi i\left(\frac{\nu_1}{c}\right)x \cdot \frac{r}{D}\cos\phi}\, \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1}\, \frac{r\,dr\,d\phi}{D^2}$$

$$A_2 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{1}{D^2} \int_{R_b}^{R_1} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu_1}{c}\right) x \cdot \frac{r}{D} \cos\phi} \, r \, dr \, d\phi$$

$$A_2 = f(\nu_1, T_1) \frac{1}{D^2} \int_{R_b}^{R_1} r \, dr \int_0^{2\pi} e^{2\pi i \left(\frac{\nu_1}{c}\right) x \cdot \frac{r}{D} \cos\phi} \, d\phi$$

$$A_2 = f(\nu_1, T_1) \frac{1}{D^2} \int_{R_b}^{R_1} r \, dr \; 2\pi \, J_0\left[\frac{2\pi \nu_1 x r}{CD}\right]$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \int_{R_b}^{R_1} r \, J_0\left[\frac{2\pi \nu_1 x r}{CD}\right] dr$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \int_{\frac{2\pi \nu_1 x R_b}{CD}}^{\frac{2\pi \nu_1 x R_1}{CD}} \left(\frac{CD}{2\pi \nu_1 x}\right) y \, J_0[y] \frac{CD}{2\pi \nu_1 x} \, dy$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \cdot \left(\frac{CD}{2\pi \nu_1 x}\right)^2 \int_{2\pi \nu_1 x R_b/CD = B_1}^{2\pi \nu_1 x R_1/CD = B_2} y \, J_0[y] \, dy$$

$$A_2 = f(\nu_1, T_1)\,\frac{2\pi}{D^2}\left(\frac{CD}{2\pi\nu x}\right)^2\left[\varepsilon_2 J_1[\varepsilon_2] - \varepsilon_1 J_1[\varepsilon_1]\right]$$

$$A_2 = f(\nu_1, T_1)\,\frac{2\pi}{D^2}\left(\frac{CD}{2\pi\nu x}\right)^2\left[\left(\frac{2\pi\nu_1 x R_1}{CD}\right) J_1\left(\frac{2\pi\nu x R_1}{CD}\right) - \left(\frac{2\pi\nu x R_b}{CD}\right) J_1\left(\frac{2\pi\nu x R_b}{CD}\right)\right]$$

$$A_2 = f(\nu_1, T_1)\,\frac{2\pi}{D^2}\left[R_1^2 \cdot \frac{J_1\left(\frac{2\pi\nu x R_1}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_1}{CD}\right)} - R_b^2\,\frac{J_1\left(\frac{2\pi\nu x R_b}{CD}\right)}{\left(\frac{2\pi\nu x R_b}{CD}\right)}\right]$$

$$A_2 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1}\,\frac{2\pi}{D^2}\left[R_1^2 \cdot \frac{J_1\left(\frac{2\pi\nu_1 x R_1}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_1}{CD}\right)} - R_b^2 \cdot \frac{J_1\left(\frac{2\pi\nu x R_b}{CD}\right)}{\left(\frac{2\pi\nu x R_b}{CD}\right)}\right] \quad \text{—— (5)}$$

From equation (4) & (5)

$$\phi_1\phi_2^* \propto \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1}\,\frac{2\pi R_b^2}{D^2}\,\frac{J_1\left(\frac{2\pi\nu x R_b}{CD}\right)}{\left(\frac{2\pi\nu x R_b}{CD}\right)} + \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1}\,\frac{2\pi R_1^2}{D^2}\,\frac{J_1\left(\frac{2\pi\nu_1 x R_1}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_1}{CD}\right)} - \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1}\,\frac{2\pi R_b^2}{D^2}\,\frac{J_1\left(\frac{2\pi\nu_1 x R_b}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_b}{CD}\right)} \quad \text{—— (6)}$$

let, $f(R, \nu, T) = \dfrac{\nu^2/c^2}{e^{h\nu/kT} - 1} \cdot \dfrac{2\pi R^2}{D^2}$

And, $\quad \rho = \dfrac{2\pi x}{cD}$

$$\Phi_1 \Phi_2^* \Big|_b \propto f(R_b, \nu, T)\, \frac{J_1(\rho \nu R_b)}{(\rho \nu R_b)} + f(R_1, \nu_1, T_1)\, \frac{J_1(\rho \nu_1 R_1)}{(\rho \nu_1 R_1)}$$
$$- f(R_b, \nu_1, T_1)\, \frac{J_1(\rho \nu_1 R_b)}{(\rho \nu_1 R_b)} \qquad \text{—— (7)}$$

As $x \longrightarrow 0$

$$\frac{J_1(x)}{x} \longrightarrow \frac{1}{2}$$

$$|\Phi_1|^2 = |\Phi_2|^2 = f(R_b, \nu, T)\cdot \frac{1}{2} + f(R_1, \nu_1, T_1)\frac{1}{2}$$
$$- f(R_b, \nu_1, T_1)\frac{1}{2}$$

$$|\Phi_1|^2 = |\Phi_2|^2 = \frac{1}{2}\Big[f(R_b, \nu, T) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1)\Big] \qquad \text{—— (8)}$$

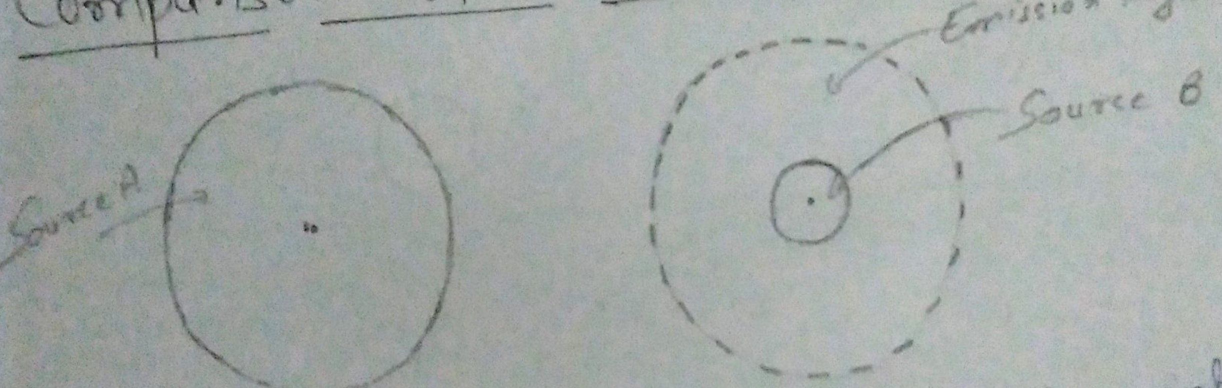
Visibility of A WR Star for baseline x

$$V(x) = \frac{\langle \Phi_1 \Phi_2^* \rangle}{\langle |\Phi_1|^2 \rangle \langle |\Phi_2|^2 \rangle}$$

$$V_b(x) = 2\frac{\left[f(R_b,\nu,T)\frac{J_1(\rho\nu R_b)}{(\rho\nu R_b)} + f(R_1,\nu_1,T_1)\frac{J_1(\rho\nu_1 R_1)}{(\rho\nu_1 R_1)} - f(R_b,\nu_1,T_1)\frac{J_1(\rho\nu_1 R_b)}{(\rho\nu_1 R_b)}\right]}{\left[f(R_b,\nu,T) + f(R_1,\nu_1,T_1) - f(R_b,\nu_1,T_1)\right]}$$

(8)

What If The WR Star Is In The Companison Of Blue Supergaint



Let's assume that in presence of blue supergaint also the shape of Roche lobe is spherical

The correlation of intensity for source A

$$\phi_1\phi_2^*\Big|_a = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1}\ \frac{2\pi R_a^2}{D^2}\ \frac{J_1\left[\frac{2\pi\nu \times R_a}{cD}\right]}{\left[\frac{2\pi\nu \times R_a}{cD}\right]}$$ (9)

The correlation of intensity for binary system

$$\phi_1\phi_2^* \propto \int_\Omega e^{2\pi i\left(\frac{\nu}{c}\right)x.\Omega}\, |S(\Omega)|^2\, d^2\Omega$$

$$\phi_1\phi_2^* \propto \int e^{2\pi i\left(\frac{\nu}{c}\right)x.(\Omega_a + \Delta\Omega + \Omega_b)}\, |S(\Omega)|^2\, d^2\Omega$$

$$\phi_1\phi_2^* \propto \phi_1\phi_2^*\Big|_a + \phi_1\phi_2^*\Big|_b$$

$$\phi_1\phi_2^* \propto \frac{\nu^2/c^2}{e^{h\nu/kT_a}-1}\,\frac{2\pi R_a^2}{D^2}\,\frac{J_1\left(\frac{2\pi\nu x R_a}{CD}\right)}{\left(\frac{2\pi\nu x R_a}{CD}\right)}$$

$$+\left\{f(R_b,\nu,T_b)\frac{J_1(e\nu R_b)}{(e\nu R_b)} + f(R_1,\nu_1,T_1)\frac{J_1(e\nu_1R_1)}{(e\nu_1R_1)} - f(R_b,\nu_1,T_1)\frac{J_1(e\nu_1R_b)}{(e\nu_1R_b)}\right\}\exp\left(\frac{2\pi i\nu x d}{CD}\right)$$

$$d=\sqrt{x^2+y^2}$$

$$\phi_1\phi_2^* \propto f(R_a,\nu,T_a)\frac{J_1(e\nu R_a)}{(e\nu R_a)} + \left\{f(R_b,\nu,T_b)\frac{J_1(e\nu R_b)}{(e\nu R_b)} + f(R_1,\nu_1,T_1)\frac{J_1(e\nu_1R_1)}{(e\nu_1R_1)} - f(R_b,\nu_1,T_1)\frac{J_1(e\nu_1R_b)}{(e\nu_1R_b)}\right\}\exp\left(\frac{2\pi i\nu x d}{CD}\right)$$

at $x=0$

$$|\phi_1|^2 = |\phi_2|^2 = f(R_a,\nu,T_a)\times\frac{1}{2} + f(R_b,\nu,T_b)\times\frac{1}{2} + f(R_1,\nu_1,T_1)\times\frac{1}{2} - f(R_b,\nu_1,T_1)\times\frac{1}{2}$$

$$|\phi_1|^2 = |\phi_2|^2 = \frac{1}{2}\Big\{ f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1) \Big\}$$

$$V = \frac{\langle \phi_1 \phi_2^* \rangle}{\sqrt{\langle |\phi_1|^2 \rangle \langle |\phi_2|^2 \rangle}}$$

$$V = 2\,\frac{\left[f(R_a, \nu, T_a) \frac{J_1(\ell \nu R_a)}{(\ell \nu R_a)} + \exp\left(\frac{2\pi i \nu x d}{cD}\right) \Big\{ f(R_b, \nu, T_b) \frac{J_1(\ell \nu R_b)}{(\ell \nu R_b)} + f(R_1, \nu_1, T_1) \frac{J_1(\ell \nu_1 R_1)}{(\ell \nu_1 R_1)} - f(R_b, \nu_1, T_1) \frac{J_1(\ell \nu_1 R_b)}{(\ell \nu_1 R_b)} \Big\} \right]}{\left[f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1) \right]}$$

$$V = \frac{\Big\{ V_1(x) + V_2(x) \exp\left(\frac{2\pi i \nu x d}{cD}\right) \Big\}}{\left[f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1) \right]}$$

$$V_1(x) = 2 f(R_a, \nu, T_a) \frac{J_1(\ell \nu R_a)}{(\ell \nu R_a)}$$

$$V_2(x) = 2 \Big\{ f(R_b, \nu, T_b) \frac{J_1(\ell \nu R_b)}{(\ell \nu R_b)} + f(R_1, \nu_1, T_1) \frac{J_1(\ell \nu_1 R_1)}{(\ell \nu_1 R_1)} - f(R_b, \nu_1, T_1) \frac{J_1(\ell \nu_1 R_b)}{(\ell \nu_1 R_b)} \Big\}$$

$$|V|^2 = \frac{|V_a(x)|^2 + |V_b(x)|^2 + 2\, V_a(x)\, V_b(x) \cos\left(\frac{2\pi \nu x d}{c D}\right)}{\left[f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1)\right]^2}$$

$$f(R, \nu, T) = \frac{\nu^2/c^2}{e^{h\nu/kT} - 1} \cdot \frac{2\pi R^2}{D^2}$$

$$\rho = \frac{2\pi x}{c D}$$

$$V_a(x) = 2 f(R_a, \nu, T_a) \frac{J_1(\rho \nu R_a)}{(\rho \nu R_a)}$$

$$V_b(x) = 2 \left\{ f(R_b, \nu, T_b) \frac{J_1(\rho \nu R_b)}{(\rho \nu R_b)} + f(R_1, \nu_1, T_1) \frac{J_1(\rho \nu_1 R_1)}{(\rho \nu_1 R_1)} - f(R_b, \nu_1, T_1) \frac{J_1(\rho \nu_1 R_b)}{(\rho \nu_1 R_b)} \right\}$$