Recalculations

① photon flux from binary system with & without Roche lobe = ?

② Squared Visibility for binary system with & without Roche lobe = ?

① Photon flux from source A

$$\Phi_a = \int_{\Omega_a} |s(\Omega)|_a^2 \, d^2\Omega$$

$$\Phi_a = \int_0^{R_a}\int_0^{2\pi} \frac{\nu^2/c^2}{e^{h\nu/kT_a}-1} \frac{r\,dr\,d\phi}{D^2}$$

$$\Phi_a = \frac{\nu^2/c^2}{e^{h\nu/kT_a}-1} \cdot \frac{1}{D^2} \int_0^{R_a}\int_0^{2\pi} r\,dr\,d\phi$$

$$\Phi_a = \frac{\nu^2/c^2}{e^{h\nu/kT_a}-1} \frac{\cancel{2}\pi}{D^2} \cdot \frac{R_a^2}{\cancel{2}}$$

$$\boxed{\Phi_a = \frac{\nu^2/c^2}{e^{h\nu/kT_a}-1} \frac{\pi R_a^2}{D^2}} \quad (1)$$

Photon flux from Source B which is having a Roche lobe with radius $(R_b + R_1)$ from center of star B.

$$\Phi_b = \int_{\Omega_b} |S(\Omega)|_b^2 \, d^2\Omega$$

$$\Phi_b = \int_0^{R_b+R_1} \int_0^{2\pi} \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$\Phi_b = \int_0^{R_b} \int_0^{2\pi} \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{r\,dr\,d\phi}{D^2} + \int_{R_b}^{R_1} \int_0^{2\pi} \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$\Phi_b = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{\cancel{2}\pi}{D^2} \frac{R_b^2}{\cancel{2}} + \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{\cancel{2}\pi}{D^2} \left[\frac{R_1^2}{\cancel{2}} - \frac{R_b^2}{\cancel{2}}\right]$$

$$\boxed{\Phi_b = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{\pi R_b^2}{D^2} + \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{\pi}{D^2}\left(R_1^2 - R_b^2\right)}$$

Total flux from a binary having a WR star from equation (1) & (2)

$$\Phi = \Phi_a + \Phi_b$$

$$\boxed{\Phi = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{\pi R_a^2}{D^2} + \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{\pi R_b^2}{D^2} + \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{\pi}{D^2} (R_1^2 - R_b^2)}$$ (3)

Or

let, $$\boxed{f(R, \nu, T) = \frac{\nu^2/c^2}{e^{h\nu/kT} - 1} \frac{\pi R^2}{D^2}}$$ (4)

$$\boxed{\Phi = f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1)}$$ (5)

(2) Correlation of photons from two telescopes

$$\phi_1 \phi_2^* \propto \int_{\Omega} e^{2\pi i \left(\frac{\nu}{c}\right)(X_1 - X_2)\cdot\Omega} |S(\Omega, t)|^2 d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_a} e^{2\pi i \left(\frac{\nu}{c}\right) X\cdot\Omega_a} |S(\Omega)|_a^2 d^2\Omega + \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) X\cdot\Omega_b + \Delta\Omega} |S(\Omega)|_b^2 d^2\Omega$$

I have ignored other integration parameter t as it will cancel out during normalization

$$\phi_1 \phi_2^* \propto A_1 + A_2 \exp\left(\frac{2\pi i \nu x d}{cD}\right) \quad \text{—— (6)}$$

$$A_1 = \int_{\Omega_a} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_a} \, |S(\Omega)|_a^2 \, d^2\Omega$$

$$A_2 = \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_b} \, |S(\Omega)|_b^2 \, d^2\Omega$$

$$A_1 = \int_0^{R_a} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \frac{r}{D} \cos\phi} \, \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \, \frac{r \, dr \, d\phi}{D^2}$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \cdot \frac{1}{D^2} \int_0^{R_a} r \, dr \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \frac{r}{D} \cos\phi} \, d\phi$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{1}{D^2} \int_0^{R_a} 2\pi r \, dr \, J_0\left[\frac{2\pi \nu x r}{cD}\right]$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{2\pi}{D^2} \int^{R_a} r \, J_0\left(\frac{2\pi \nu x r}{cD}\right) dr$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{2\pi}{D^2} \int_0^{R_a} r\, J_0\left(\frac{2\pi\nu x r}{cD}\right) dr$$

$$\text{let} \quad \frac{2\pi\nu x r}{cD} = y$$

$$r = \left(\frac{cD}{2\pi\nu x}\right) y$$

$$dr = \left(\frac{cD}{2\pi\nu x}\right) dy$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{2\pi}{D^2} \left(\frac{cD}{2\pi\nu x}\right)^2 \int_0^{2\pi\nu x R_a/cD} y\, J_0(y)\, dy$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{2\pi}{D^2} \cdot \left(\frac{cD}{2\pi\nu x}\right)^2 \left(\frac{2\pi\nu x R_a}{cD}\right) J_1\left(\frac{2\pi\nu x R_a}{cD}\right)$$

$$A_1 = \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{2\pi R_a^2}{D^2} \frac{J_1\left(\frac{2\pi\nu x R_a}{cD}\right)}{\left(\frac{2\pi\nu x R_a}{cD}\right)}$$

$$A_1 = 2 f(R_a, \nu, T_a) \frac{J_1\left(\frac{2\pi\nu x R_a}{cD}\right)}{\left(\frac{2\pi\nu x R_a}{cD}\right)} \quad \text{--- (7)}$$

$$\text{let} \quad \boxed{\frac{2\pi x}{cD} = \rho} \quad \text{--- (8)}$$

$$\boxed{A_1 = 2 f(R_a, \nu, T_a) \frac{J_1(\rho\nu R_a)}{(\rho\nu R_a)}} \quad \text{--- (9)}$$

$$A_2 = \int_{-\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_b} \left| S(-\Omega_b) \right|^2 d^2\Omega$$

$$A_2 = \int_0^{R_b + R_1} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \Omega_b} \left| S(\Omega_b) \right|^2 d^2\Omega$$

$$A_2 = \int_0^{R_b} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \cdot \frac{r}{D} \cos\phi} \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$+ \int_{R_b}^{R_1} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu_1}{c}\right) x \cdot \frac{r}{D} \cos\phi} \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$A_2 = 2 f(R_b, \nu, T_b) \frac{J_1(e \nu R_b)}{(e \nu R_b)} + A_3 \qquad (10)$$

$$A_3 = \int_{R_b}^{R_1} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu_1}{c}\right) x \cdot \frac{r}{D} \cos\phi} \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{1}{D^2} \int_{R_b}^{R_1} r\,dr \int_0^{2\pi} e^{2\pi i\left(\frac{\nu_1}{c}\right) x \frac{r}{D} \cos\phi} d\phi$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{1}{D^2} \int_{R_b}^{R_1} 2\pi r\,dr\, J_0\left(\frac{2\pi\nu_1 x r}{cD}\right)$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{2\pi}{D^2} \int_{R_b}^{R_1} r\, J_0\left(\frac{2\pi\nu_1 x r}{cD}\right) dr$$

let, $\frac{2\pi\nu_1 x r}{cD} = y$

$$r = \left(\frac{cD}{2\pi\nu_1 x}\right) y$$

$$dr = \left(\frac{cD}{2\pi\nu_1 x}\right) dy$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{2\pi}{D^2} \left(\frac{cD}{2\pi\nu_1 x}\right)^2 \int_{y_b}^{y_1} y\, J_0(y)\, dy$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{2\pi}{D^2} \left(\frac{cD}{2\pi\nu_1 x}\right)^2 \left[\left(\frac{2\pi\nu_1 x R_1}{cD}\right) J_1\left(\frac{2\pi\nu_1 x R_1}{cD}\right) - \left(\frac{2\pi\nu_1 x R_b}{cD}\right) J_1\left(\frac{2\pi\nu x R_b}{cD}\right)\right]$$

$$A_3 = \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{2\pi}{D^2} \left[R_1^2 \frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - R_b^2 \frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)} \right]$$

$$A_3 = 2 \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{\pi R_1^2}{D^2} \frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)}$$

$$- 2 \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1} - 1} \frac{\pi R_b^2}{D^2} \frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}$$

$$A_3 = 2 f(R_1, \nu_1, T_1) \frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)}$$

$$- 2 f(R_b, \nu_1, T_1) \frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)} \qquad (11)$$

from equ. (10) & (11)

$$\boxed{\begin{aligned} A_2 = {} & 2 f(R_b, \nu, T_b) \frac{J_1(e\nu R_b)}{(e\nu R_b)} + \\ & 2 f(R_1, \nu_1, T_1) \frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} \\ & - 2 f(R_b, \nu_1, T_1) \frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)} \end{aligned}}$$

$$(12)$$

the correlation of photons, from 6, 9 & 12

$$\phi_1 \phi_2^* \propto 2f(R_a, \nu, T_a)\frac{J_1(e\nu R_a)}{(e\nu R_a)}$$

$$+\Big\{2f(R_b, \nu, T_b)\frac{J_1(e\nu R_b)}{(e\nu R_b)}$$

$$+2f(R_1, \nu_1, T_1)\frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)}$$

$$-2f(R_b, \nu_1, T_1)\frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}\Big\}\exp\left(\frac{i\nu x d}{cD}\right) \quad \text{(13)}$$

for baseline zero, $x=0$

$$|\phi_1|^2 \propto f(R_a, \nu_1, T_a) + f(R_b, \nu, T_b) +$$

$$f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1)$$

$$|\phi_2|^2 = |\phi_1|^2 \quad \text{(14)}$$

A the Visibility of binary source system

$$V = \frac{V_a(x) + V_b(x)\exp\left(\frac{2\pi i \nu x d}{CD}\right)}{\Phi}$$

$$A\ |V|^2 = \frac{|V_a(x)|^2 + |V_b(x)|^2 + 2V_a(x)V_b(x)\cos\left(\frac{2\pi\nu x d}{CD}\right)}{\Phi^2}$$

$$|V_a(x)|^2 = \left[2f(R_a, \nu, T_a)\frac{J_1(e\nu R_a)}{(e\nu R_a)}\right]^2$$

$$|V_b(x)|^2 = \left[2f(R_b, \nu, T_b)\frac{J_1(e\nu R_b)}{(e\nu R_b)} + 2f(R_1, \nu_1, T_1)\frac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - 2f(R_b, \nu_1, T_1)\frac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}\right]^2$$

$$\Phi = f(R_a, \nu_1 T_a) + f(R_b, \nu, T_b) + f(R_1, \nu_1, T_1)$$
$$- f(R_b, \nu_1, T_1)$$

$$f(R, \nu, T) = \frac{\nu^2/c^2}{e^{h\nu/kT} - 1} \frac{\pi R^2}{D^2}$$

$$\ell = \frac{2\pi x}{cD}$$