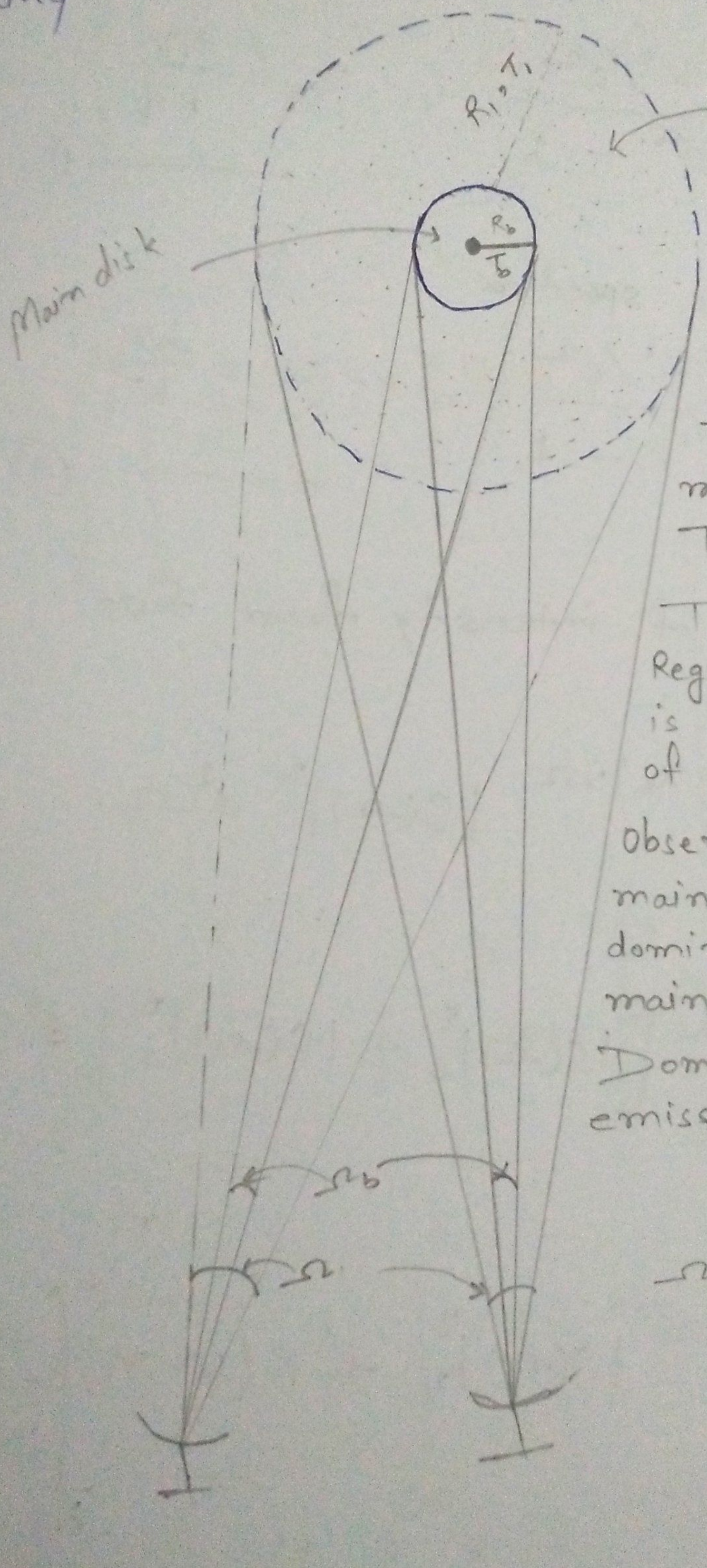
Squared Visibility Of A WR Source
(Single System)

Let's assume that we have a WR star in absent of any external force (like, other ~~st~~ stellar object)



The radius of the main disk is R_b & Temperature T_b.

The radius of emission Region from the center is $(R_b + R_1)$ & temperature of the region R_1 is T_1

Observing wavelength of main disk i.e. the dominated frequency of main disk ν.

Dominated frequency of emission region ν_1.

$$\Omega = \Omega_1 + \Omega_b$$

Let's assume that the main disk is a uniform disk (does not follow limb-darkening law) for now.

→ Photon flux for continuum spectra

$$|S(\Omega)|_b^2 = \frac{\nu^2/c^2}{e^{h\nu/kT_b} - 1}$$

$$|S(\Omega)|_b^2 = \frac{\nu^2/c^2}{e^{z_b} - 1}, \quad z_b = \frac{h\nu}{kT_b} \qquad (1)$$

→ Photon flux for line spectra

$$|S(\Omega)|_1^2 = \frac{\nu_1^2/c^2}{e^{z_1} - 1}, \quad z_1 = \frac{h\nu_1}{kT_1} \qquad (2)$$

→ The correlation of intensity from two telescope

$$\phi_1 \phi_2^* \propto \int_\Omega e^{2\pi i \left(\frac{\nu}{c}\right)(x_1 - x_2)\cdot\Omega} \, |S(\Omega)|^2 \, d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_\Omega e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\Omega} \left[|S(\Omega)|_b^2 + |S(\Omega)|_1^2 \right] d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\Omega} \, |S(\Omega)|_b^2 + \delta(\nu - \nu_1) \int_{\Omega_1} e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\Omega} \, |S(\Omega)|_1^2 \, d^2\Omega$$

$$\phi_1\phi_2^* \propto A_1 + \delta(\nu-\nu_1)\, A_2 \quad \text{---} \quad (3)$$

$$A_1 = \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\Omega}\, |S(\Omega)|_b^2\, d^2\Omega \quad \text{---} \quad (4)$$

$$A_2 = \int_{\Omega_1} e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\Omega}\, |S(\Omega)|_1^2\, d^2\Omega \quad \text{---} \quad (5)$$

from equation (4)

$$A_1 = \int_0^{R_b}\int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x \frac{r}{D}\cos\phi}\, \frac{\nu^2/c^2}{e^{z_b}-1}\, \frac{r\,dr\,d\phi}{D^2}$$

ϕ

baseline

projection of $x\cdot\frac{r}{D}$

$= x\frac{r}{D}\cos\phi$

$\Omega = \frac{r}{D}$

$$A_1 = \frac{\nu^2/c^2}{e^{z_b}-1}\cdot\frac{1}{D^2}\int_0^{R_b} r\,dr \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) x\cdot\frac{r}{D}\cos\phi}\, d\phi$$

$$A_1 = f(\nu, T_b)\cdot\frac{1}{D^2}\int_0^{R_b} r\,dr\; 2\pi\, J_0\left[\frac{2\pi\nu x r}{cD}\right]$$

$$A_1 = f(\nu, T_b)\,\frac{2\pi}{D^2}\int_0^{R_b} r\, J_0\left[\frac{2\pi\nu x r}{cD}\right] dr$$

$$A_1 = f(\nu, T_b) \frac{2\pi R_b^2}{D^2} \frac{J_1\left[\frac{2\pi\nu x R_b}{CD}\right]}{\left[\frac{2\pi\nu x R_b}{CD}\right]}$$

$$A_1 = \frac{\nu^2/c^2}{e^{z_b}-1} \frac{2\pi R_b^2}{D^2} \frac{J_1(2\pi\nu x R_b/CD)}{(2\pi\nu x R_b/CD)} \qquad (6)$$

$$A_2 = \int_{R_b}^{R_1}\int_0^{2\pi} e^{2\pi i\left(\frac{\nu_1}{c}\right)x\cdot\frac{r}{D}\cos\phi} \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1}-1} \frac{r\,dr\,d\phi}{D^2}$$

$$A_2 = \frac{\nu_1^2/c^2}{e^{z_1}-1} \frac{1}{D^2} \cdot \int_{R_b}^{R_1}\int_0^{2\pi} e^{2\pi i\left(\frac{\nu}{c}\right)x\cdot\frac{r}{D}\cos\phi}\, r\,dr\,d\phi$$

$$A_2 = f(\nu_1, T_1)\frac{1}{D^2}\int_{R_b}^{R_1} r\,dr \int_0^{2\pi} e^{2\pi i\left(\frac{\nu_1}{c}\right)x\frac{r}{D}\cos\phi}\, d\phi$$

$$A_2 = f(\nu_1, T_1)\frac{1}{D^2}\int_{R_b}^{R_1} r\,dr\; 2\pi\, J_0\left[\frac{2\pi\nu_1 x r}{CD}\right]$$

$$A_2 = f(\nu_1, T_1)\frac{2\pi}{D^2}\int_{R_b}^{R_1} r\, J_0(2\pi\nu x r/CD)\, dr$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \int_{(2\pi\nu_1 x R_b/CD)}^{(2\pi\nu_1 x R_1/CD)} (CD/2\pi\nu_1 x)\, y\, J_0[y]\, (CD/2\pi\nu_1 x)\, dy$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \left(\frac{CD}{2\pi\nu_1 x}\right)^2 \int_{2\pi\nu_1 x R_b/CD = B_1}^{2\pi\nu_1 x R_1/CD = B_2} y\, J_0[y]\, dy$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \left(\frac{CD}{2\pi\nu_1 x}\right)^2 \left[B_2\, J_1[B_2] - B_1\, J_1[B_1] \right]$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \left(\frac{CD}{2\pi\nu_1 x}\right)^2 \left[\frac{2\pi\nu_1 x R_1}{CD}\, J_1\left(\frac{2\pi\nu_1 x R_1}{CD}\right) - \left(\frac{2\pi\nu_1 x R_b}{CD}\right) J_1\left(\frac{2\pi\nu_1 x R_b}{CD}\right) \right]$$

$$A_2 = f(\nu_1, T_1) \frac{2\pi}{D^2} \left[R_1^2 \cdot \frac{J_1(2\pi\nu_1 x R_1/CD)}{(2\pi\nu_1 x R_1/CD)} - R_b^2 \frac{J_1(2\pi\nu_1 x R_b/CD)}{(2\pi\nu_1 x R_b/CD)} \right]$$

$$A_2 = \frac{\nu_1^2/c^2}{e^{z_1} - 1} \frac{2\pi}{D^2} \left[R_1^2 \cdot \frac{J_1\left(\frac{2\pi\nu_1 x R_1}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_1}{CD}\right)} - R_b^2 \frac{J_1\left(\frac{2\pi\nu_1 x R_b}{CD}\right)}{\left(\frac{2\pi\nu_1 x R_b}{CD}\right)} \right] \qquad (7)$$

from equation (3), (6) & (7)

$$\phi_1\phi_2^* \propto \frac{\nu^2/c^2}{e^{h\nu/kT_b}-1}\frac{2\pi R_b^2}{D^2}\frac{J_1(2\pi\nu x R_b/cD)}{(2\pi\nu x R_b/cD)} +$$

$$\delta(\nu-\nu_1)\left[\frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1}-1}\frac{2\pi R_1^2}{D^2}\frac{J_1(2\pi\nu_1 x R_1/cD)}{(2\pi\nu_1 x R_1/cD)} - \frac{\nu_1^2/c^2}{e^{h\nu_1/kT_1}-1}\frac{2\pi R_b^2}{D^2}\frac{J_1(2\pi\nu_1 x R_b/cD)}{(2\pi\nu_1 x R_b/cD)}\right] \quad (8)$$

let, $f(R,\nu,T) = \dfrac{\nu^2/c^2}{e^{h\nu/kT}-1}\dfrac{\pi R^2}{D^2}$

$$\rho = \frac{2\pi x}{cD}, \quad x = x_1 - x_2$$

$$\phi_1\phi_2^* \propto f(R_b,\nu,T_b)\frac{2J_1(\rho\nu R_b)}{(\rho\nu R_b)} + \delta(\nu-\nu_1)\left[f(R_1,\nu_1,T_1)\frac{2J_1(\rho\nu_1 R_1)}{(\rho\nu_1 R_1)} - f(R_b,\nu_1,T_1)\frac{2J_1(\rho\nu_1 R_b)}{(\rho\nu_1 R_b)}\right] \quad (9)$$

As, $x \longrightarrow 0$

$$\frac{J_1(x)}{x} \longrightarrow \frac{1}{2}$$

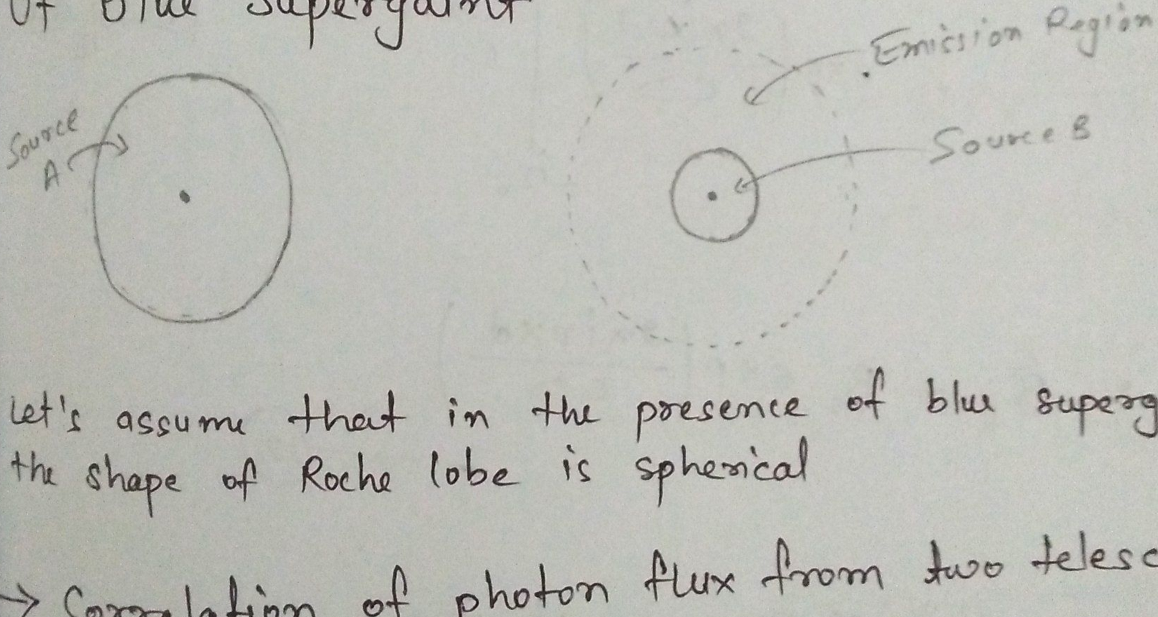
$$|\phi_1|^2 = |\phi_2|^2 = f(R_b,\nu_1,T_b) + \left[f(R_1,\nu_1,T_1) - f(R_b,\nu_1,T_1)\right]\delta(\nu-\nu_1) \quad (10)$$

So, visibility of a WR star for baseline x

$$V_b(x) = \frac{\langle \phi_1 \phi_2^* \rangle}{\sqrt{\langle |\phi_1|^2 \rangle \langle |\phi_2|^2 \rangle}}$$

$$\boxed{V_b(x) = 2\,\frac{\left[f(R_b, \nu, T_b)\dfrac{J_1(e\nu R_b)}{(e\nu R_b)} + \delta(\nu-\nu_1)\left\{f(R_1, \nu_1, T_1)\dfrac{J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - f(R_b, \nu_1, T_1)\dfrac{J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}\right\}\right]}{\left[f(R_b, \nu, T_b) + \delta(\nu-\nu_1)\left\{f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1)\right\}\right]}} \quad (11)$$

What If A WR star Is In the Companion Of Blue Supergaint



Let's assume that in the presence of blue supergaint, the shape of Roche lobe is spherical

→ Correlation of photon flux from two telescopes

$$\phi_1 \phi_2^* \propto \int_{\Omega} e^{2\pi i\left(\frac{\nu}{c}\right) x \cdot \Omega} |S(\Omega)|^2 \, d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_a} e^{2\pi i\left(\frac{\nu}{c}\right) x \cdot \Omega} |S(\Omega)|_a^2 \, d^2\Omega + \int_{\Omega_b + \Delta\Omega} e^{2\pi i\left(\frac{\nu}{c}\right) x \cdot \Omega} |S(\Omega)|_b^2 \, d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_a} e^{2\pi i \left(\frac{\nu}{c}\right) X.\Omega} |S(\Omega)|_a^2 \, d^2\Omega$$

$$\int_{\Omega_b + \Delta\Omega} e^{2\pi i \left(\frac{\nu}{c}\right) X.\Omega} |S(\Omega)|_b^2 \, d^2\Omega$$

$$\phi_1 \phi_2^* \propto \int_{\Omega_a} e^{2\pi i \left(\frac{\nu}{c}\right) X.\Omega} |S(\Omega)|_a^2 \, d^2\Omega \; +$$

$$\exp\left(\frac{2\pi i x \nu d}{cD}\right) \int_{\Omega_b} e^{2\pi i \left(\frac{\nu}{c}\right) X.\Omega} |S(\Omega)|_b^2 \, d^2\Omega \,.$$

$$\phi_1 \phi_2^* \propto B_1 + B_2 \exp\left(\frac{2\pi i \nu x d}{cD}\right) \qquad (12)$$

$$B_1 = \int_{\Omega_a} e^{2\pi i \left(\frac{\nu}{c}\right) X.\Omega} |S(\Omega)|_a^2 \, d^2\Omega$$

$$B_1 = \int_0^{R_a} \int_0^{2\pi} e^{2\pi i \left(\frac{\nu}{c}\right) X.\frac{r}{D} \cos\phi} \frac{\nu^2/c^2}{e^{h\nu/kT_a} - 1} \frac{r\,dr\,d\phi}{D^2}$$

$$B_1 = f(R_a, \nu, T_a)\,\frac{2J_1(e\nu R_a)}{(e\nu R_a)} \quad \text{——— (13)}$$

$$B_2 = \int_{\Omega_b} e^{2\pi i\left(\frac{\nu}{c}\right)x_1\cdot\Omega}\,|S(\Omega)|_b^2\,d^2\Omega$$

As source B is a WR star, so from equ. (9)

$$B_2 = f(R_b, \nu, T_b)\,\frac{2J_1(e\nu R_b)}{(e\nu R_b)} + \delta(\nu-\nu_1)\left\{f(R_1,\nu_1,T_1)\,\frac{2J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - f(R_b,\nu_1,T_1)\,\frac{2J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}\right\} \quad \text{——— (14)}$$

So, from equ. (12), (13) & (14)

$$\phi_1\phi_2^* \propto f(R_a,\nu,T_a)\,\frac{2J_1(e\nu R_a)}{(e\nu R_a)} + \exp\left(\frac{2\pi i\nu x d}{cD}\right)\left[f(R_b,\nu,T_b)\,\frac{2J_1(e\nu R_b)}{(e\nu R_b)} + \delta(\nu-\nu_1)\left\{f(R_1,\nu_1,T_1)\,\frac{2J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - f(R_b,\nu_1,T_1)\,\frac{2J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}\right\}\right] \quad \text{——— (15)}$$

for baseline zero, $x \longrightarrow 0$

$$|\phi_1|^2 = |\phi_2|^2 \propto f(R_a,\nu,T_a) + f(R_b,\nu,T_b) + \delta(\nu-\nu_1)\left\{f(R_1,\nu_1,T_1) - f(R_b,\nu_1,T_1)\right\} \quad \text{——— (16)}$$

So, the visibility of binary system which contains a blue supergaint & WR star

$$V(x) = \frac{\langle \phi_1 \phi_2^* \rangle}{\sqrt{\langle |\phi_1|^2 \rangle \langle |\phi_2|^2 \rangle}}$$

$$V(x) = \frac{f(R_a, \nu, T_a) \frac{2 J_1(e\nu R_a)}{(e\nu R_a)} + \exp\left(\frac{2\pi i \nu x d}{CD}\right) \left[f(R_b, \nu, T_b) \, 2 \frac{J_1(e\nu R_b)}{(e\nu R_b)} + \delta(\nu - \nu_1) \left\{ f(R_1, \nu_1, T_1) \frac{2 J_1(e\nu_1 R_1)}{(e\nu_1 R_1)} - f(R_b, \nu_1, T_1) \frac{2 J_1(e\nu_1 R_b)}{(e\nu_1 R_b)} \right\} \right]}{\left[f(R_a, \nu, T_a) + f(R_b, \nu, T_b) + \delta(\nu - \nu_1) \left\{ f(R_1, \nu_1, T_1) - f(R_b, \nu_1, T_1) \right\} \right]}$$

$$V(x) = \frac{f_a V_a(x) + \exp\left(\frac{2\pi i \nu x d}{CD}\right) \left[f_b V_b(x) + \delta(\nu - \nu_1) \left\{ f_1 V_1(x) - f_{b1} V_{b1}(x) \right\} \right]}{\left[f_a + f_b + \delta(\nu - \nu_1) \left\{ f_1 - f_{b1} \right\} \right]}$$

$f_a = f(R_a, \nu, T_a)$, $f_b = f(R_b, \nu, T_b)$

$f_1 = f(R_1, \nu_1, T_1)$, $f_{b1} = f(R_b, \nu_1, T_1)$

$$V_a(x) = \frac{2J_1(e\nu R_a)}{(e\nu R_a)}$$

$$V_b(x) = \frac{2J_1(e\nu R_b)}{(e\nu R_b)}$$

$$V_1(x) = \frac{2J_1(e\nu_1 R_1)}{(e\nu_1 R_1)}$$

$$V_{b1}(x) = \frac{2J_1(e\nu_1 R_b)}{(e\nu_1 R_b)}$$

$$V(x) = \frac{f_a V_a + \exp\left(\frac{2\pi i \nu x d}{cD}\right)\left[f_b V_b + \delta(\nu-\nu_1)\{f_1 V_1 - f_{b1} V_{b1}\}\right]}{\left[f_a + f_b + \delta(\nu-\nu_1)\{f_1 - f_{b1}\}\right]}$$

(17)

Squared Visibility

$$V(x)\, V^*(x) = |V(x)|^2$$

$$\boxed{|V(x)|^2 = \frac{\left[f_a^2 V_a^2 + (f_b V_b + \delta(\nu-\nu_1)\{f_1 V_1 - f_{b1} V_{b1}\})^2 + 2 f_a V_a (f_b V_b + \delta(\nu-\nu_1)\{f_1 V_1 - f_{b1} V_{b1}\}) \cos(2\pi \nu x d/cD)\right]}{\left[f_a + f_b + \delta(\nu-\nu_1)\{f_1 - f_{b1}\}\right]^2}}$$

(18)

$$V(x) = \frac{\left[f_a^2 V_a^2 + \left(f_b V_b + \delta(\nu - \nu_1)\{ f_1 V_1 - f_1 V_{b1} \} \right)^2 + 2 f_a V_a \left(f_b V_b + \delta(\nu - \nu_1)\{ f_1 V_1 - f_1 V_{b1} \} \right) \cos(2\pi\nu x d / cD) \right]}{\left[f_a + f_b + \delta(\nu - \nu_1)\{ f_1 - f_{b1} \} \right]^2}$$