$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

**Definition 1** *A function $f \colon [a, b] \to \mathbb{R}$ is Sabek integrable if there $\exists \mathcal{P}$ such that the upper sum of circle area - lower sum of circle area is less than any given $\varepsilon < 0$*

**Theorem 1** *For any give $S_{abek} < 0$*

# 1 Fundamental Theorems of Calculus

## 1.1 FTC1

$$F(x) = \int_a^x f \implies F'(x_0) = f(x_0)$$

- Preconditions:

  - $f : [a, b] \to \mathbb{R}$ is **bounded**, **integrable**, and **continuous** at $x_0 \in (a, b)$
  - $F : [a, b] \to \mathbb{R}$ is defined as $\int_a^x f$

- Implications:

  - $F$ is differentiable at $x_0 \in (a, b)$
  - $F'(x_0) = f(x_0)$
  - Loosely:

## 1.2 FTC2

$$\int_a^b f = F(b) - F(a)$$

- Preconditions:

  - $f : [a, b] \to \mathbb{R}$ is **bounded** and **integrable**
  - $F : [a, b] \to \mathbb{R}$ is **continuous** and **differentiable** on $(a, b)$
  - $F' = f$ on $(a, b)$

- Implications:

  - Stated above.

- Remarks:

  - If $f$ is continuous, we can define $F = \int_a^x f$. Then from FTC1, $F'$ is differentiable.

- Warnins:

  - We need that $f = F'$ is integrable. If we have $F$ continuous and differentiable and $F'$ bounded, then $F'$ integrable **does not necessarily follow**. See Volterra's function.

# 2 Integration

- The integral

$$\int_0^1 t^\lambda dt$$

converges **if and only if** $\lambda > -1$. $\lambda = -1$ is the borderline case.

- The integral

$$\int_1^\infty t^\lambda dt$$

converges **if and only if** $\lambda < -1$.