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## 1. Introduction

This report's goal is to assess the out-of-sample performance of three strategies for portfolio optimisation: a naïve equal-weighted model, a minimum-variance model, and a mean-variance model. Ken French's data library's historical equity return data is used to evaluate the effectiveness of these tactics. The purpose of the analysis is to analyse the relative performances of each strategy when input parameters are estimated using historical data.

## 2. Selection of Dataset

For this analysis, I have selected the Fama/French European 5 Factors from Ken French's data library. The dataset includes monthly returns for multiple equity factors, making it suitable for the mean-variance and minimum-variance optimization analyses. These factors are constructed to help explain the returns of stocks and identify various sources of risk. Although the risk-free asset considered in this dataset is the one-month US Treasury bill, which may not perfectly align with the European market, the dataset was chosen to study the returns of the European market specifically.

### Data Usage

- Monthly Returns: From July 1990 to March 2024 (all the data is in percentage points)

### Factor Construction

The dataset, has the following five factors:

1. Market ( $R_m - R_f$ ): This is the value-weighted market portfolio return for an area less the rate on US one-month T-bills. After accounting for the risk-free rate, it shows the total excess market return.
2. Small Minus Big, or SMB: The size premium, or the variation in returns between small-cap stocks and large-cap companies, is captured by SMB. It is created by grouping equities into three categories: investment (INV), operating profitability (OP), book-to-market equity (B/M), and big and small market size. The returns are then averaged.
3. HML (High Minus Low): HML measures the return differential between value (high book-to-market) and growth (low book-to-market) companies, or the value premium. The average return on the two value portfolios (small and large) is subtracted from the average return on the two growth portfolios (small and large) to create it.
4. RMW (Robust Minus Weak): RMW measures the return difference between equities with strong (high) and weak (low) operating profitability. This is known as the profitability premium. It is calculated by deducting the average return from the two large, robust portfolios from the average return from the two small, weak portfolios.

5. Aggressive Minus Conservative (CMA): The difference in returns between businesses that invest aggressively (high investment) and those that invest cautiously (low investment) is known as the investment premium, and CMA measures it. The average return on the two small and big conservative portfolios is subtracted from the average return on the two small and big aggressive portfolios to create it.

## **Region**

Europe: Contains nations including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

The selection of this dataset was based on its extensive coverage of developed market equity returns, which offers a strong foundation for estimating return mean and covariance matrices. With monthly returns for several equity components, the dataset is appropriate for optimisation analyses involving mean-variance and minimum-variance.

## **3. Selection of Estimate Windows M**

Selecting the right estimation windows is crucial for assessing portfolio optimisation techniques' out-of-sample performance. Three different estimating windows—60 months, 120 months, and 240 months—have been used for this investigation.

Three distinct estimation windows (M) :

1.  $M = 60$  months: The 60 months window provides a more current and trend-sensitive view of the market. This shorter-term perspective is more sensitive to recent shifts in market conditions since it covers the most recent market dynamics and trends (COVID-19). In markets that are erratic or that are changing quickly, this window can be very helpful for identifying and responding to recent changes in market behaviour. Though this window may be more sensitive to recent trends, it may also be more vulnerable to short-term noise and anomalies.
2.  $M = 120$  months: The 120 months window is designed to capture long-term trends while allowing for short-term fluctuations. It takes into account enough past data to yield reliable projections while remaining aware of recent changes in the market. This medium-term window is a solid option for the majority of portfolio optimisation techniques since it is good at seeing recurring patterns and linkages in the data. It strikes a compromise between the requirement to maintain relevance to the state of the market and the demand for historical background.
3.  $M = 240$  months: By offering a long-term view, the 240 months window lessens the impact of short-term volatility. It uses a significant amount of historical data as

leverage, which helps to capture long-term market trends and smooth out short-term volatility. By averaging out short-term anomalies and market cycles, it provides estimates of mean and covariance that are more trustworthy and stable. Though this window offers stability, it could not be as sensitive to recent shifts in the market, thus it might miss short-term patterns and adjustments that could be important given the state of the market.

#### **4. Estimation of mean and covariance matrix**

To effectively construct a mean-variance optimized portfolio, it is crucial to accurately estimate the mean and covariance matrix of returns. This involves using historical return data to derive these statistical measures, which serve as the foundation for the optimization process. At every possible time t, the data from the previous M periods (months) is used to estimate the mean and covariance matrix of returns for three distinct estimation windows 60 Months, 120 Months, 240 Months. The unbiased sample estimator is employed to estimate the mean and covariance matrix. The unbiased estimators for mean ( $\mu$ ) and covariance matrix ( $\Sigma$ ) are calculated as follows:

- **Mean (Expected Return):**

$$\hat{\mu} = \frac{1}{M} \sum_{i=1}^M R_i$$

Where  $R_i$  represents the return in period i.

- **Covariance Matrix:**

$$\hat{\Sigma} = \frac{1}{M-1} \sum_{i=1}^M (R_i - \hat{\mu})(R_i - \hat{\mu})^T$$

Here,  $R_i$  is the return vector in period i, and  $\hat{\mu}$  is the estimated mean return vector.

#### **5. Target Expected Return**

The target expected return is a critical input for the optimization process. For this analysis, the monthly expected target return is determined by averaging the returns implied by the Capital Asset Pricing Model (CAPM) with beta of 0.6 (<1) considering market being more volatile than the assets and the historical average returns of the portfolio. This dual approach provides a balanced and realistic expectation, integrating both forward-looking market estimates and historical performance.

## Target Expected Return

Using CAPM method to determine the returns

```
# Calculate the means of the last period for Mkt-RF and RF
mean_mkt_rf = df_months['Mkt-RF'].mean()
mean_rf = df_months['RF'].mean()

# On researching the market is more volatile than the assets the beta for the investment is 0.6
beta_a = 0.6 # Example beta value, replace with actual beta

# Calculate the expected return using the formula
expected_return = mean_rf + beta_a * (mean_mkt_rf)
print(f"Mean Market Risk Premium - Mean Risk Free Rate : {mean_mkt_rf}")
print(f"Mean Risk-Free Rate (RF): {mean_rf}")
print(f"Expected Return: {expected_return}")

✓ 0.0s
```

Python

```
Mean Market Risk Premium - Mean Risk Free Rate : 0.4985432098765432
Mean Risk-Free Rate (RF): 0.20925925925925926
Expected Return: 0.5083851851851852
```

Historical average return

```
# Calculate historical mean returns for each factor
historical_mean_returns = df_months.mean()

# Calculate the historical mean return of the portfolio
# Assuming equal weight for each factor
num_factors = len(historical_mean_returns) - 1 # Exclude RF
equal_weights = np.array([1. / num_factors] * num_factors)
historical_portfolio_mean_return = np.dot(equal_weights, historical_mean_returns[:-1]) # Exclude RF

print(f"\nHistorical Mean Returns:\n{historical_mean_returns}")
print(f"\nHistorical Portfolio Mean Return: {historical_portfolio_mean_return}")

✓ 0.0s
```

Python

```
Historical Mean Returns:
Mkt-RF      0.498543
SMB        0.017136
HML        0.278667
RMW        0.357852
CMA        0.125556
RF         0.209259
dtype: float64

Historical Portfolio Mean Return: 0.2555506172839507
```

$$\hat{r}_t = \frac{1}{2} (CAPM \text{ implied return} + \text{Historical average return})$$

By averaging the CAPM implied return and the historical average return, a more balanced and realistic target return is achieved. This approach ensures that the target return incorporates both the current market environment and the historical performance of the assets, providing a solid foundation for the mean-variance portfolio optimization process. This blended target return aims to optimize the portfolio's performance while managing risk, enhancing the overall robustness of the investment strategy.

```
Average of CAPM and historical means
```

```
expected_return_final = (historical_portfolio_mean_return + expected_return) / 2
expected_return_final = expected_return_final.round(4)
print(f'Final expected returns: {expected_return_final}')
```

```
✓ 0.0s
```

```
Python
```

```
Final expected returns: 0.382
```

The expected monthly target return came out to be 0.382%.

## 6. Mean-Variance Portfolio Optimization

This method seeks to construct an investment portfolio that minimizes risk for a given level of expected return. The goal is to achieve an optimal balance between risk and return, thereby enhancing portfolio performance.

The mean-variance optimization problem can be formulated as follows:

$$\begin{aligned} & \text{minimise}_w w^T \hat{\Sigma} w \\ & \text{subject to: } w^T \hat{\mu} = \hat{r}_t \\ & \sum_{i=1}^n w_i = 1 \\ & w \geq 0 \end{aligned}$$

Where:

- $w$  is the vector of portfolio weights.
- $\hat{\Sigma}$  is the estimated covariance matrix.
- $\hat{\mu}$  is the estimated mean return vector.
- $\hat{r}_t$  is the target expected return.
- The constraints ensure that the portfolio weights sum to 1 and that short-selling is not allowed (weights are non-negative).

```
def portfolio_optimization(mean_returns, cov_matrix, target_return):
    num_assets = len(mean_returns)
    args = (mean_returns, cov_matrix)

    def portfolio_variance(weights, mean_returns, cov_matrix):
        return np.dot(weights.T, np.dot(cov_matrix, weights))

    constraints = (
        {'type': 'eq', 'fun': lambda x: np.sum(x) - 1},
        {'type': 'eq', 'fun': lambda x: np.dot(x, mean_returns) - target_return}
    )
    bounds = tuple((0, 1) for asset in range(num_assets))
    result = minimize(portfolio_variance,
                      num_assets * [1. / num_assets],
                      args=args,
                      method='SLSQP',
                      bounds=bounds,
                      constraints=constraints)
    return result.x
```

The optimization problem is solved at every possible time  $t$ , using the data from the previous  $M$  periods to estimate the parameters. This yields the optimal portfolio weights  $w_t$  for time  $t$ . Once the optimal portfolio weights are determined for time  $t$ , the portfolio's return in the subsequent investment period (from  $t$  to  $t+1$ ) is computed. (Appendix A.4)

## 7. Minimum Variance Portfolio

The goal of this section is to evaluate the performance of a minimum-variance portfolio optimization strategy. This approach aims to minimize the portfolio's overall risk, represented by the variance of returns, without imposing any constraint on the expected return. By using historical data to estimate the covariance matrix of returns, the minimum-variance portfolio can be constructed to achieve the lowest possible risk for a given set of assets.

```
def minimum_variance_portfolio(cov_matrix):
    num_assets = len(cov_matrix)
    args = (cov_matrix,)

    def portfolio_variance(weights, cov_matrix):
        return np.dot(weights.T, np.dot(cov_matrix, weights))

    constraints = (
        {'type': 'eq', 'fun': lambda x: np.sum(x) - 1}
    )
    bounds = tuple((0, 1) for asset in range(num_assets))
    result = minimize(portfolio_variance,
                      num_assets * [1. / num_assets],
                      args=args,
                      method='SLSQP',
                      bounds=bounds,
                      constraints=constraints)

    return result.x

optimization_results = {}
```

The optimization problem is formulated to minimize the portfolio variance without any constraint on the mean return. The optimization is performed using the SLSQP method from the `scipy.optimize.minimize` function. The expected return, variance, and standard deviation of the optimized portfolio are calculated for each window size. (Appendix A.5)

## 8. Naïve 1/N Portfolio Strategy

In this step, we compute the returns for a naïve portfolio strategy where equal weights are assigned to each of the  $N$  available assets. This strategy, often referred to as the 1/ $N$  strategy, involves simply dividing the total investment equally among all assets, without any optimization based on historical data or statistical estimates and calculating the returns.

```

# Calculate naive portfolio
def naive_portfolio(mean_returns, cov_matrix, assets):
    num_assets = len(assets)
    equal_weights = np.array([1. / num_assets] * num_assets)

    expected_return = np.dot(equal_weights, mean_returns)
    portfolio_variance = np.dot(equal_weights.T, np.dot(cov_matrix, equal_weights))

    portfolio_std_dev = np.sqrt(portfolio_variance)

    return equal_weights, expected_return, portfolio_variance, portfolio_std_dev

```

The naive portfolio strategy is often used as a benchmark or starting point for more sophisticated portfolio optimization techniques. While it may not always lead to the most optimal risk-return trade-off, it serves as a simple and transparent method for portfolio allocation.

(Appendix A.6)

## 9. Conclusion and Interpretation of Results

Method	Expected Return	Variance	Standard Deviation
<b>Mean-Variance Portfolio (60 Months)</b>	0.3819	5.9966	2.4488
<b>Mean-Variance Portfolio (120 Months)</b>	0.374	22.6235	4.7564
<b>Mean-Variance Portfolio (240 Months)</b>	0.3819	2.0169	1.4202
<b>Minimum Variance Portfolio (60 Months)</b>	0.0146	0.3041	0.5514
<b>Minimum Variance Portfolio (120 Months)</b>	0.0644	0.2622	0.5121
<b>Minimum Variance Portfolio (240 Months)</b>	0.1745	0.2528	0.5028
<b>Equal Weights Portfolio / Naive Model</b>	0.2556	1.3713	1.171

### 1. Mean-Variance Portfolio:

The Mean-Variance Portfolio with a 240-month window size achieved the highest expected return (0.3819) with a relatively low standard deviation (1.4202). This suggests that using a longer historical period for covariance estimation can provide more stable and favourable results. The 60-month and 120-month window sizes also yielded high expected returns but with significantly higher standard deviations, indicating increased risk.

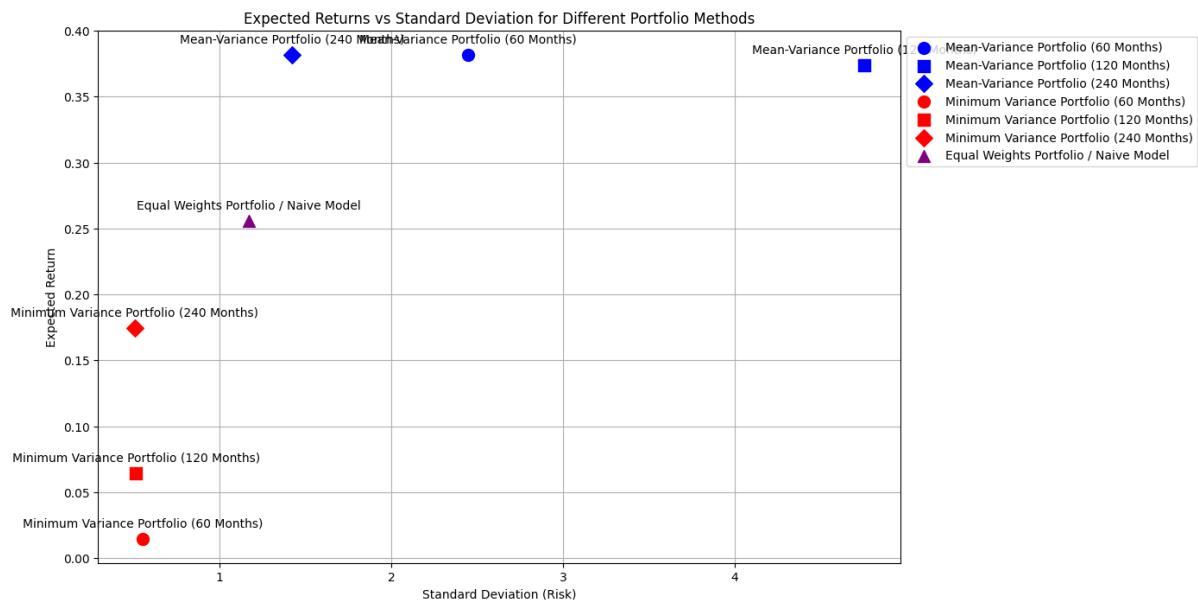
### 2. Minimum Variance Portfolio:

The Minimum Variance Portfolio with a 240-month window size achieved a moderate expected return (0.1745) with the lowest standard deviation (0.5028). This portfolio is suitable for risk-averse investors seeking to minimize risk. The 60-month and 120-month window sizes resulted in lower expected returns but also maintained low standard deviations, making them viable options for conservative investment strategies.

### 3. Equal Weights Portfolio (Naive Model):

The Equal Weights Portfolio achieved a reasonable expected return (0.2556) with a standard deviation (1.1710) that is higher than the Minimum Variance Portfolio but lower than the Mean-Variance Portfolio for shorter window sizes. This method provides a simple and balanced approach without the need for complex optimization.

The analysis demonstrates that the choice of window size for rolling covariance estimation significantly impacts the performance of optimized portfolios. The Mean-Variance Portfolio with a 240-month window size offers the highest expected return with moderate risk, making it an attractive option for investors seeking higher returns. The Minimum Variance Portfolio with a 240-month window size provides the lowest risk, suitable for conservative investors. The Equal Weights Portfolio offers a balanced approach with reasonable returns and risk. Investors should consider their risk tolerance and investment horizon when selecting the appropriate portfolio optimization method.



## 10. Recommendations

For High Returns: Consider the Mean-Variance Portfolio with a 240-month window size.

For Low Risk: opt for the Minimum Variance Portfolio with a 240-month window size.

For Simplicity and Balance: The Equal Weights Portfolio provides a straightforward and balanced approach.

By carefully selecting the appropriate portfolio optimization method, investors can better align their investment strategies with their financial goals and risk preferences.

## **References**

### Books and Articles

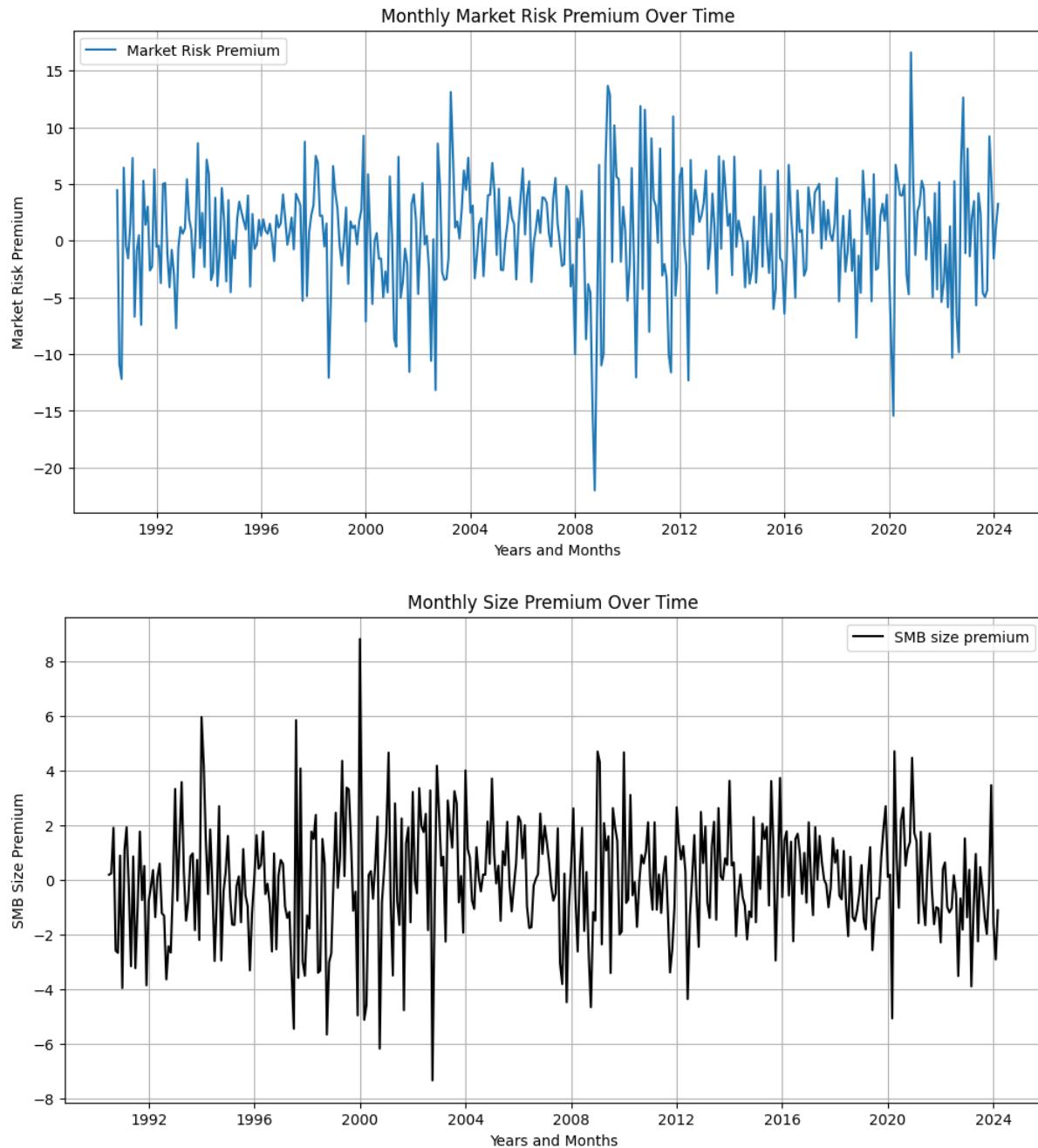
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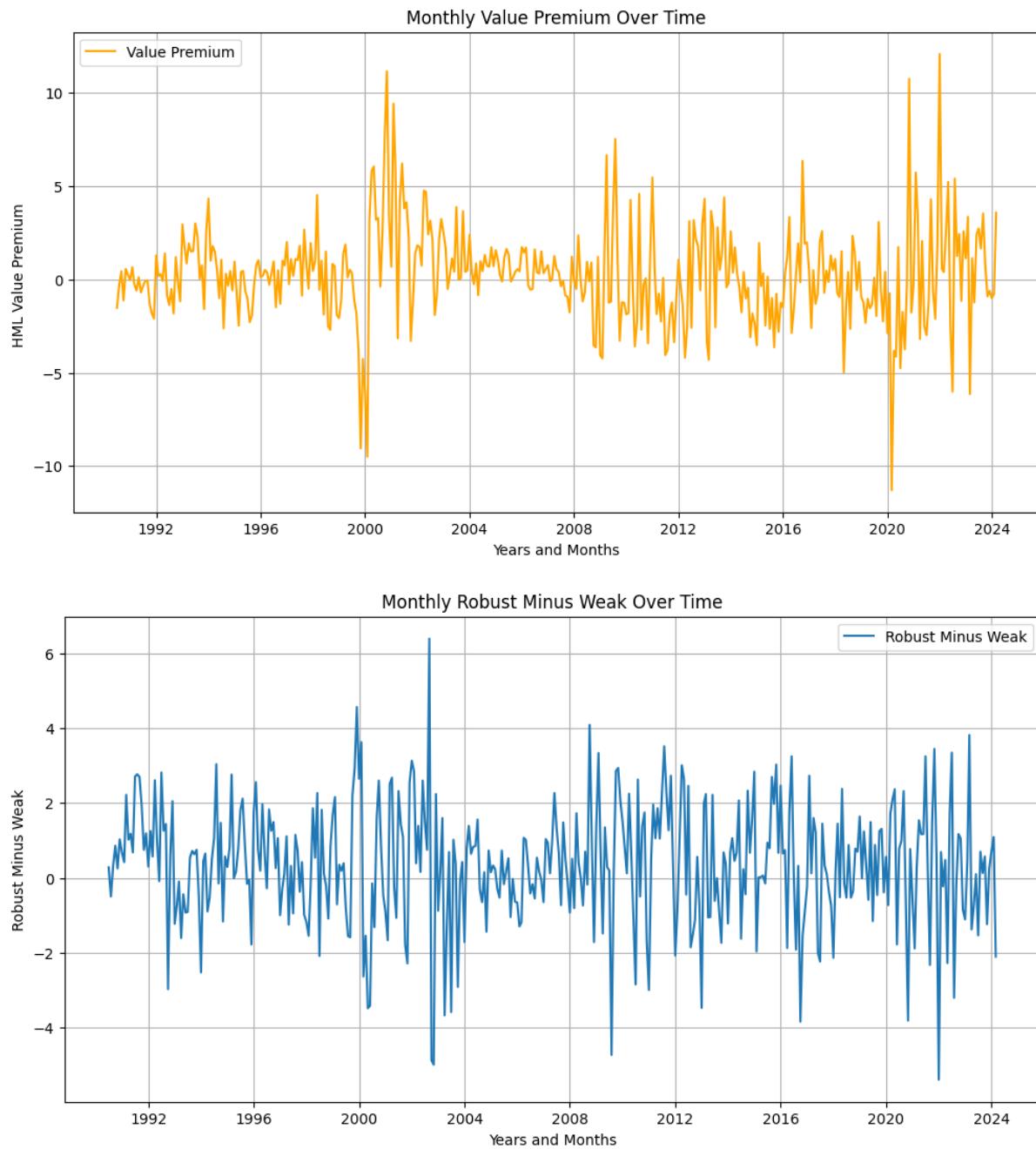
### Online Resources

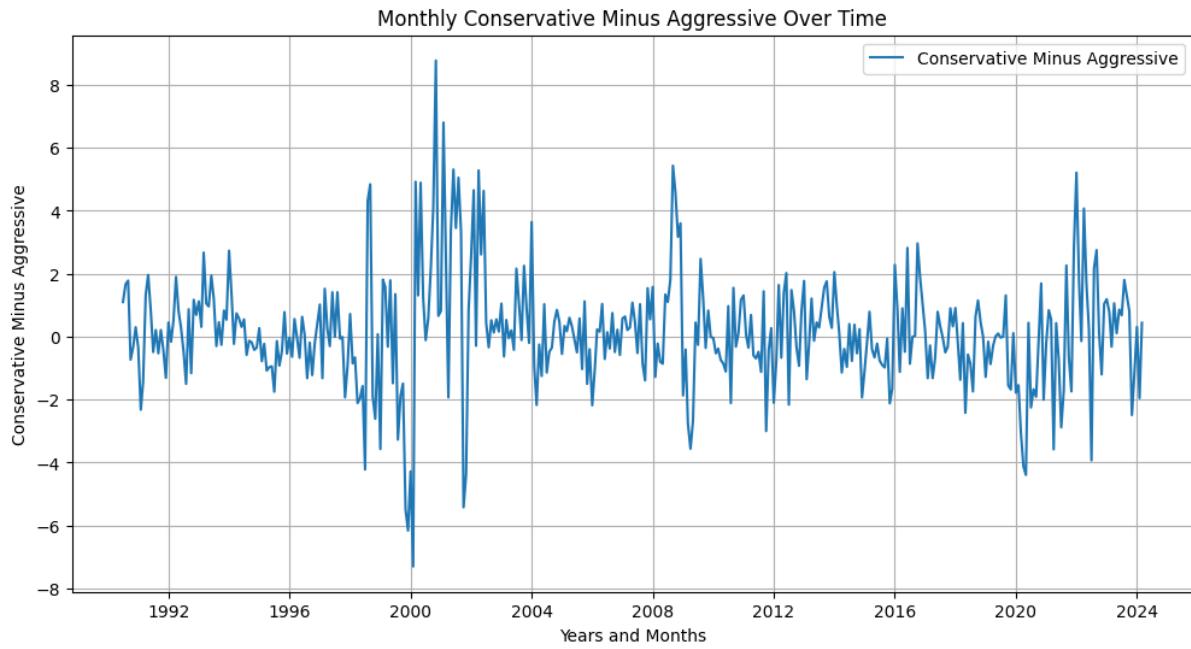
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## Appendix

### A.0 Data Plotting







### A.1 Mean and Covariance Matrix : 60 months estimation window (head)

... Rolling Means for Window Size 60 Months:						
	Mkt-RF	SMB	HML	RMW	CMA	RF
Years and Months						
1995-06-01	0.218833	-0.234167	0.205333	0.579833	0.266333	0.369000
1995-07-01	0.211000	-0.263167	0.213500	0.605333	0.218833	0.365167
1995-08-01	0.324833	-0.248333	0.180333	0.649000	0.188833	0.362000
1995-09-01	0.567500	-0.289500	0.140833	0.658333	0.143833	0.359167
1995-10-01	0.448167	-0.262333	0.154667	0.641333	0.148333	0.355667
Rolling Covariances for Window Size {M1_window_size} Months:						
	Mkt-RF	SMB	HML	RMW	CMA	\
Years and Months						
1995-06-01	Mkt-RF 18.663617	-1.699127	0.925145	0.749140	-0.496133	
	SMB -1.699127	4.092025	1.085621	-0.817138	0.578374	
	HML 0.925145	1.085621	1.931907	-1.165562	0.486566	
	RMW 0.749140	-0.817138	-1.165562	1.587063	-0.352962	
	CMA -0.496133	0.578374	0.486566	-0.352962	1.071037	
RF						
Years and Months						
1995-06-01	Mkt-RF -0.056037					
	SMB -0.037248					
	HML -0.074123					
	RMW 0.044751					
	CMA -0.027573					

## rolling\_mean\_60

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
1995-06-01	0.218833333333330	-0.2341666666666670	0.205333333333330	0.579833333333330	0.2663333333333300	0.369
1995-07-01	0.2110000000000000	-0.26316666666666700	0.2135000000000000	0.605333333333330	0.2188333333333300	0.3651666666666670
1995-08-01	0.324833333333330	-0.2483333333333300	0.1803333333333300	0.649000000000000	0.1888333333333300	0.362
1995-09-01	0.5675	-0.2895000000000000	0.140833333333330	0.658333333333330	0.1438333333333300	0.3591666666666670
1995-10-01	0.44816666666666700	-0.2623333333333300	0.15466666666666700	0.641333333333330	0.1483333333333300	0.3556666666666670
1995-11-01	0.449833333333330	-0.273	0.15816666666666700	0.636333333333330	0.1668333333333300	0.35316666666666700
1995-12-01	0.506333333333330	-0.30783333333330	0.1710000000000000	0.589500000000000	0.1528333333333300	0.351333333333330
1996-01-01	0.4885000000000000	-0.2410000000000000	0.1735000000000000	0.607333333333330	0.158	0.349833333333330
1996-02-01	0.398166666666666700	-0.23216666666666700	0.166	0.643	0.186	0.348333333333330
1996-03-01	0.5246666666666670	-0.2575	0.179	0.618833333333330	0.21916666666666700	0.3475000000000000
1996-04-01	0.5495	-0.24316666666666700	0.1950000000000000	0.605	0.1965	0.3463333333333300
1996-05-01	0.566333333333330	-0.161	0.1883333333333300	0.6181666666666670	0.15266666666666700	0.3455000000000000
1996-06-01	0.6935	-0.1840000000000000	0.2053333333333300	0.617833333333330	0.14916666666666700	0.3451666666666670
1996-07-01	0.575333333333330	-0.1325	0.227333333333330	0.568	0.1583333333333300	0.3445
1996-08-01	0.589333333333330	-0.13066666666666700	0.2038333333333300	0.552333333333330	0.1328333333333300	0.3436666666666670
1996-09-01	0.5585	-0.2038333333333300	0.212833333333330	0.528333333333330	0.1385	0.343333333333330
1996-10-01	0.6228	-0.1753333333333300	0.2128333333333300	0.502833333333330	0.11466666666666700	0.343333333333330
1996-11-01	0.733833333333330	-0.22616666666666700	0.259833333333330	0.5126666666666670	0.1188333333333300	0.3436666666666670
1996-12-01	0.662333333333330	-0.15916666666666700	0.307333333333330	0.5105	0.14766666666666700	0.345
1997-01-01	0.6651666666666670	-0.13416666666666700	0.319333333333330	0.4888333333333300	0.15716666666666700	0.346833333333330
1997-02-01	0.683833333333330	-0.1203333333333300	0.3121666666666670	0.4631666666666670	0.1378333333333300	0.34866666666666670
1997-03-01	0.7801666666666670	-0.1425000000000000	0.3255	0.4555	0.1548333333333300	0.3501666666666670
1997-04-01	0.6840000000000000	-0.143	0.329833333333330	0.4305	0.1275	0.352
1997-05-01	0.6681666666666670	-0.16416666666666700	0.324833333333330	0.392833333333330	0.10866666666666700	0.3555000000000000
1997-06-01	0.7486666666666670	-0.2318333333333300	0.3563333333333300	0.39966666666666700	0.12766666666666700	0.356333333333330
1997-07-01	0.869	-0.3023333333333300	0.409333333333330	0.336833333333330	0.13516666666666700	0.358333333333330
1997-08-01	0.7941666666666670	-0.183	0.403333333333330	0.335	0.18366666666666700	0.360833333333330
1997-09-01	0.9896666666666670	-0.182	0.47816666666666700	0.16816666666666700	0.363833333333330	
1997-10-01	1.036833333333300	-0.0735	0.473	0.36666666666666700	0.1873333333333300	0.367
1997-11-01	1.06016666666666700	-0.07916666666666700	0.4670000000000000	0.36416666666666700	0.13566666666666700	0.36966666666666700

## rolling\_cov\_60

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF	
1995-06-01	Mkt-RF	18.663617259887000	-1.6991269774011300	0.9251453107344630	0.7491404802259890	-0.496131638418080	-0.056036779661016900
1995-06-01	SMB	-1.6991269774011300	4.092024717514120	1.0856209039548000	-0.8171379943502820	0.5783742937853110	-0.037248305084745800
1995-06-01	HML	0.9251453107344630	1.0856209039548000	1.93190666666666700	-1.1655618079096000	0.48656564971751400	-0.07412338983050850
1995-06-01	RMW	0.7491404802259890	-0.8171379943502820	-1.1655618079096000	1.587062683615820	-0.3529616384180790	0.04475067796610170
1995-06-01	CMA	-0.496131638418080	0.5783742937853110	0.48656564971751400	-0.3529616384180790	0.10710371751412400	-0.027573220338983100
1995-06-01	RF	-0.056036779661016900	-0.037248305084745800	-0.07412338983050850	0.04475067796610170	-0.027573220338983100	0.016568474576271200
1995-07-01	Mkt-RF	18.59727796610200	-1.81403406779661	0.9702744067796610	0.849526779661017	-0.685319152542373	-0.07324593220338980
1995-07-01	SMB	-1.81403406779661	4.116876242937850	1.1258994067796610	-0.814065536723160	0.6154657344632770	-0.04144268361581920
1995-07-01	HML	0.9702744067796610	1.1258994067796610	1.907250254237290	-1.2002986440678000	0.5535566949152540	-0.0666929661016949
1995-07-01	RMW	0.849526779661017	-0.8414065536723160	-1.2002986440678000	1.6105270056497200	-0.3895343502824860	0.04811943502824860
1995-07-01	CMA	-0.685319152542373	0.6154657344632770	0.5535566949152540	-0.3895343502824860	1.1258714971751400	-0.034920988700565000
1995-07-01	RF	-0.07324593220338980	-0.04144268361581920	-0.0666929661016949	0.04811943502824860	-0.034920988700565000	0.015025395480225900
1995-08-01	Mkt-RF	16.8093609887007060	-1.82062151424300	1.0583831920904000	0.5272981355932210	-0.38501629943502800	-0.025026779661016900
1995-08-01	SMB	-1.82062151424300	4.1455988700565	1.0713265536723200	-0.7964288115593220	0.594849350282490	-0.04146610169491520
1995-08-01	HML	1.0583881920904000	1.0713265536723200	2.007891412428380	-1.2727166101694900	0.580319548022600	-0.06868203389830510
1995-08-01	RMW	0.5272981355932210	-0.7964288115593220	1.6267650847457600	-0.3704147457627120	0.56474915254237400	
1995-08-01	CMA	-0.38501629943502800	0.594849350282490	0.580319548022600	-0.3704147457627120	1.091935903954800	-0.042856949152542400
1995-08-01	RF	-0.025026779661016900	-0.04146610169491520	-0.06868203389830510	0.056474915254237400	0.01372813559322040	
1995-09-01	Mkt-RF	14.165775	-1.360981779661020	1.0500631355932200	0.4656838983050850	-0.07483601694915260	0.028513135593220300
1995-09-01	SMB	-1.360981779661020	4.065689576271190	1.070919915242400	-0.7855466101694920	0.5401573728813560	-0.05067923728813560
1995-09-01	HML	1.0500631355932200	1.070919915242400	2.080645056497180	-1.2795579096045200	0.6111645480225990	-0.07227556497175140
1995-09-01	RMW	0.46568893050850	-0.7855466101694920	-1.2795579096045200	1.6259361581920900	-0.3659138418079100	0.05806638418079090
1995-09-01	CMA	-0.07483601694915260	0.5401573728813560	0.6111645480225990	-0.3659138418079100	1.0678037005649700	-0.05068319209039550
1995-09-01	RF	0.028513135593220300	-0.05067923728813560	-0.07227556497175140	0.05806638418079090	-0.05068319209039550	0.012838276836158200
1995-10-01	Mkt-RF	13.592449124293800	-1.115012842857860	1.1867798870056500	0.46123299435028300	0.025908757062146900	-0.006229960451977980
1995-10-01	SMB	-1.115012842857860	3.983411412429380	1.0264127683615800	-0.7680866666666670	0.5128689265536720	-0.03936790960451980
1995-10-01	HML	1.1867798870056500	1.0264127683615800	2.056652429378530	-1.2690334463276800	0.5968367231638420	-0.06617943502824860
1995-10-01	RMW	0.46123299435028300	-0.7680866666666670	-1.2690334463276800	1.636303276836160	-0.354474011299435	0.05537197740112990
1995-10-01	CMA	0.025908757062146900	0.5128689265536720	0.5968367231638420	-0.354474011299435	1.0610209039548000	-0.04704971751412430
1995-10-01	RF	-0.006229960451977980	-0.03936790960451980	-0.06617943502824860	0.05537197740112990	-0.04704971751412430	0.01128937851073400
1995-11-01	Mkt-RF	13.589672853107300	-1.1107259322033900	1.1847437570621500	0.4644841242937850	0.01061471751412430	-0.003997118644067800
1995-11-01	SMB	-1.1107259322033900	4.04225524237290	1.0111333898305100	-0.7484501694915250	0.46110898305084700	-0.033997118644067800

## A.2 Mean and Covariance Matrix : 120 months estimation window

... Rolling Means for Window Size 120 Months:						
	Mkt-RF	SMB	HML	RMW	CMA	RF
<b>Years and Months</b>						
2000-06-01	0.612667	-0.302667	0.047167	0.468917	-0.091083	0.392417
2000-07-01	0.562333	-0.310000	0.087333	0.465333	-0.101167	0.390750
2000-08-01	0.639917	-0.306833	0.086667	0.458500	-0.110250	0.389417
2000-09-01	0.699917	-0.303417	0.105000	0.469083	-0.107083	0.388667
2000-10-01	0.623667	-0.333333	0.177917	0.483583	-0.066750	0.387667
Rolling Covariances for Window Size {M2_window_size} Months:						
	Mkt-RF	SMB	HML	RMW	CMA	\
<b>Years and Months</b>						
2000-06-01	Mkt-RF	16.245704	-3.381837	0.731468	0.349451	-2.261729
	SMB	-3.381837	5.842663	-0.446972	0.016687	0.339273
	HML	0.731468	-0.446972	4.666224	-2.163216	2.154984
	RMW	0.349451	0.016687	-2.163216	2.137027	-1.001985
	CMA	-2.261729	0.339273	2.154984	-1.001985	3.414190
RF						
<b>Years and Months</b>						
2000-06-01	Mkt-RF	-0.028651				
	SMB	-0.026765				
	HML	-0.028937				
	RMW	0.014081				
	CMA	-0.008097				

### 120\_rolling\_means

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2000-06-01	0.61266666666666670	-0.30266666666666670	0.047166666666666700	0.46891666666666670	-0.09108333333333330	0.3924166666666670
2000-07-01	0.5623333333333330	-0.31	0.0873333333333333	0.4653333333333330	-0.1011666666666670	0.39075
2000-08-01	0.63991666666666670	-0.3068333333333330	0.08666666666666670		0.4585	-0.11025
2000-09-01	0.69991666666666670	-0.30341666666666670	0.105	0.4690833333333330	-0.1070833333333330	0.388666666666666700
2000-10-01	0.62366666666666670	-0.3333333333333330	0.177916666666666700	0.4835833333333330	-0.0667500000000000	0.38766666666666670
2000-11-01	0.5892500000000000	-0.3178333333333330	0.266166666666666700		0.48825	0.00908333333333320
2000-12-01	0.6495	-0.32241666666666670	0.29166666666666670		0.47575	0.0120833333333330
2001-01-01	0.64016666666666670	-0.27491666666666670	0.297666666666666700		0.4625	0.02175000000000000
2001-02-01	0.50691666666666670	-0.2453333333333330	0.370666666666666700	0.4450833333333330	0.0977500000000000	0.385666666666666700
2001-03-01	0.484916666666666700	-0.267166666666666700		0.4215	0.4475	0.131916666666666700
2001-04-01	0.5538333333333330	-0.29375	0.40016666666666670	0.4613333333333330	0.1048333333333330	0.3843333333333330
2001-05-01	0.5078333333333330	-0.2440833333333330	0.43441666666666670	0.44941666666666670	0.1165000000000000	0.3830833333333330
2001-06-01	0.53916666666666670	-0.2577500000000000	0.49191666666666670	0.4348333333333330	0.1537500000000000	0.38191666666666670
2001-07-01	0.489166666666666700	-0.2445833333333330	0.5265833333333330	0.4315833333333330	0.1865833333333330	0.3803333333333330
2001-08-01	0.460416666666666700	-0.2177500000000000		0.56175	0.42041666666666670	0.226916666666666700
2001-09-01	0.33891666666666670	-0.27225	0.58166666666666670		0.407	0.258916666666666700
2001-10-01	0.38775	-0.255	0.5650833333333330	0.3760000000000000		0.212
2001-11-01	0.4407500000000000	-0.2432500000000000		0.571	0.35066666666666670	0.17925
2001-12-01	0.40341666666666670	-0.2240000000000000	0.60016666666666670	0.36191666666666670	0.1973333333333300	0.37216666666666670
2002-01-01	0.3685833333333330	-0.1907500000000000	0.60466666666666670		0.3855	0.2150833333333330
2002-02-01	0.37566666666666670	-0.1890000000000000	0.61766666666666670	0.3988333333333330	0.255166666666666700	0.36925
2002-03-01	0.449166666666666700	-0.1960833333333330	0.62141666666666670	0.3973333333333330	0.2485833333333330	0.3675
2002-04-01	0.40466666666666670	-0.1567500000000000	0.66166666666666670	0.387166666666666700		0.27675
2002-05-01	0.3658333333333330	-0.1410833333333330	0.68916666666666670	0.3808333333333330	0.2915833333333330	0.364916666666666700
2002-06-01	0.35491666666666670	-0.1315	0.71641666666666670	0.4025000000000000	0.327916666666666700	0.3633333333333300
2002-07-01	0.3008333333333330	-0.101166666666666700	0.7540833333333330	0.3915833333333330	0.3354166666666666700	0.3620000000000000
2002-08-01	0.308666666666666700	-0.1055833333333330	0.77591666666666670	0.3873333333333330	0.3450833333333330	0.361
2002-09-01	0.2238333333333330	-0.0479166666666666700		0.77525	0.42866666666666670	0.34216666666666670
2002-10-01	0.3595833333333330	-0.08883333333333340		0.75825	0.4128333333333330	0.35291666666666670
						0.35925

### 120\_rolling\_covariances

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2000-06-01	Mkt-RF	16.24570373501400	-3.3818365266106400	0.7314681232493000	0.3494508123249300	-2.2617290196078400
2000-06-01	SMB	-3.3818365266106400	5.842663417366950	-0.446972324929972	0.01668702801120500	0.3392727170868350
2000-06-01	HML	0.7314681232493000	-0.446972324929972	4.666223837535010	-2.1632157002801100	2.1549842997198900
2000-06-01	RMW	0.3494508123249300	0.016687002801120500	-2.1632157002801100	2.1370265476190500	-1.0019852170868300
2000-06-01	CMA	-2.2617290196078400	0.3392727170868350	2.1549842997198900	-1.0019852170868300	3.4141895728291300
2000-06-01	RF	-0.028651036414565800	-0.02676492997198800	-0.028936792717086800	0.014080791316526600	-0.008097359943977600
2000-07-01	Mkt-RF	16.159164257703100	-3.39150756302521	0.7242382072829130	0.3667807282913170	-2.3004006162465000
2000-07-01	SMB	-3.39150756302521	5.841682352941180	-0.4503697478991600	0.01942100840336140	0.3422689075630200
2000-07-01	HML	0.7242382072829130	-0.4503697478991600	4.732873501400560	-2.1824764145658300	2.170561568627450
2000-07-01	RMW	0.3667807282913170	0.01942100840336140	-2.1824764145658300	2.139932661064430	-1.0000323809523800
2000-07-01	CMA	-2.3004006162465000	0.33422689075630200	2.170561568627450	-1.0000323809523800	3.4021683753501400
2000-07-01	RF	-0.03964714285714290	-0.028269747899159600	-0.022687899159663900	0.014075798319327700	-0.01100668067226890
2000-08-01	Mkt-RF	15.091706463585400	-3.3547526750700300	0.6954207282913170	0.30648558823529400	-2.142371310251000
2000-08-01	SMB	-3.3547526750700300	5.846462156862750	-0.45223641456582600	0.009882941176470670	0.3312697058823530
2000-08-01	HML	0.6954207282913170	-0.45223641456582600	4.7334476190476200	-2.1786117647058800	2.1736521008403400
2000-08-01	RMW	0.30648558823529400	0.009882941176470670	-2.1786117647058800	2.1588397478991600	-0.9958776890756300
2000-08-01	CMA	-2.142371310251000	0.3312697058823530	2.1736521008403400	-0.9958776890756300	3.379805819327730
2000-08-01	RF	-0.01561097338935700	-0.028669565826330500	-0.02224145658263300	0.014611722689075600	-0.014387542016806700
2000-09-01	Mkt-RF	13.970548732493000	-3.2396238165266100	0.6116071428571430	0.23798647759103600	-2.0461745448179300
2000-09-01	SMB	-3.2396238165266100	5.863207556022410	-0.40233403361344500	0.037544740896358500	0.34619156162465000
2000-09-01	HML	0.6116071428571430	-0.40233403361344500	4.78684537815126	-2.1539331932773100	2.2166928571428600
2000-09-01	RMW	0.23798647759103600	0.037544740896358500	-2.1539331932773100	2.169537808123250	-0.9720931022408960
2000-09-01	CMA	-2.0461745448179300	0.34619156162465000	2.2166928571428600	-0.9720931022408960	3.3930813375350100
2000-09-01	RF	0.0014335014005602300	-0.02993568627450980	-0.020265546218487400	0.016003809523809500	-0.015429691876750700
2000-10-01	Mkt-RF	13.783397971988000	-2.96430700280112	0.4614110644257700	0.15933044817927171	-2.1334590756302500
2000-10-01	SMB	-2.96430700280112	6.107968627450980	-0.794544817927171	-0.05959131652661070	0.1275932773109240
2000-10-01	HML	0.4614110644257700	-0.794544817927171	5.244719152661070	-2.016226085431700	2.473983298319330
2000-10-01	RMW	0.1593304481792720	-0.05959131652661070	-2.016226085431700	2.206199656862750	-0.8951218277310930
2000-10-01	CMA	-2.1334590756302500	0.1275932773109240	2.473983298319330	-0.8951218277310930	3.5376238025210100
2000-10-01	RF	-0.017615742296918800	-0.03283893557422970	-0.006358683473389340	0.018129439775910300	-0.00779235294117645

### A.3 Mean and Covariance Matrix : 240 months estimation window

... Rolling Means for Window Size 240 Months:						
	Mkt-RF	SMB	HML	RMW	CMA	\
<b>Years and Months</b>						
2010-06-01	0.372625	9.291667e-03	0.509458	0.389583	0.255667	
2010-07-01	0.403542	1.333333e-03	0.534917	0.376542	0.255125	
2010-08-01	0.431125	-5.551115e-18	0.524958	0.389583	0.239417	
2010-09-01	0.530083	-4.166667e-03	0.522042	0.386125	0.238458	
2010-10-01	0.523958	9.125000e-03	0.527000	0.388333	0.240167	
 RF						
<b>Years and Months</b>						
2010-06-01	0.298583					
2010-07-01	0.295792					
2010-08-01	0.293083					
2010-09-01	0.290625					
2010-10-01	0.287833					
 Rolling Covariances for Window Size {M3_window_size} Months:						
	Mkt-RF	SMB	HML	RMW	CMA	\
<b>Years and Months</b>						
2010-06-01	Mkt-RF 24.470939	-2.182876	1.067980	-1.882339	-3.691918	
	SMB -2.182876	5.501366	0.073176	0.078094	0.126481	
	HML 1.067980	0.073176	5.749904	-1.633546	2.669247	
	RMW -1.882339	0.078094	-1.633546	2.501930	-0.380334	
	CMA -3.691918	0.126481	2.669247	-0.380334	4.021457	
 ...						
	SMB -0.067378					
	HML -0.008889					
	RMW 0.013729					
	CMA -0.009099					

240\_rolling\_means

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2010-06-01	0.372625	0.009291666666666660	0.5094583333333330	0.3895833333333300	0.25566666666666700	0.2985833333333330
2010-07-01	0.4035416666666670	0.00133333333333300	0.5349166666666670	0.37654166666666700	0.255125	0.2957916666666670
2010-08-01	0.431125	-5.55111512312578e-18	0.5249583333333330	0.3895833333333300	0.23941666666666700	0.2930833333333300
2010-09-01	0.5300833333333330	-0.00416666666666670	0.5220416666666670	0.386125	0.2384583333333300	0.290625
2010-10-01	0.523958333333330	0.00912499999999990	0.527	0.3883333333333300	0.24016666666666700	0.2878333333333300
2010-11-01	0.49225	0.024625000000000000	0.510375	0.3945833333333300	0.24204166666666700	0.2855
2010-12-01	0.5363750000000000	0.0297083333333330	0.5141666666666670	0.3833333333333300	0.24566666666666700	0.28304166666666700
2011-01-01	0.5454166666666670	0.0455	0.5370833333333300	0.368	0.25254166666666700	0.28091666666666700
2011-02-01	0.52775	0.0363333333333300	0.53725	0.3683333333333300	0.2624583333333300	0.2789583333333300
2011-03-01	0.554958333333330	0.0370833333333300	0.530666666666670	0.3672500000000000	0.2669583333333300	0.2771666666666670
2011-04-01	0.5925	0.0338333333333300	0.5299583333333300	0.3674166666666670	0.2643333333333300	0.2749583333333300
2011-05-01	0.577708333333330	0.0478333333333300	0.5200833333333300	0.3702500000000000	0.25366666666666700	0.2730000000000000
2011-06-01	0.6000416666666670	0.0392083333333300	0.5232916666666670	0.3717916666666670	0.24729166666666700	0.27125
2011-07-01	0.564083333333330	0.0535416666666670	0.507875	0.3698333333333300	0.2473333333333300	0.2692083333333300
2011-08-01	0.516083333333330	0.06116666666666670	0.49241666666666700	0.3729583333333300	0.24179166666666700	0.2673333333333300
2011-09-01	0.455125	0.049041666666666700	0.4847916666666670	0.3711666666666670	0.25	0.26541666666666700
2011-10-01	0.5117500000000000	0.038	0.4850833333333300	0.3683750000000000	0.2366625	0.26366666666666700
2011-11-01	0.5010416666666670	0.0253333333333300	0.478625	0.376625	0.2367083333333300	0.2620416666666670
2011-12-01	0.465625	0.0372916666666670	0.482958333333330	0.3732500000000000	0.24329166666666700	0.2604583333333300
2012-01-01	0.4914583333333300	0.0515833333333300	0.4820000000000000	0.3633333333333300	0.23266666666666700	0.25904166666666700
2012-02-01	0.52	0.05829166666666700	0.47975	0.3548333333333300	0.23016666666666700	0.257875
2012-03-01	0.5356666666666670	0.0599166666666670	0.47266666666666700	0.356625	0.2349166666666670	0.2564583333333300
2012-04-01	0.505458333333330	0.0707500000000000	0.4555000000000000	0.35829166666666700	0.2242083333333300	0.255125
2012-05-01	0.43291666666666700	0.0715833333333300	0.4389166666666670	0.3650833333333300	0.22654166666666700	0.254
2012-06-01	0.4675	0.05091666666666700	0.4555	0.3635833333333300	0.2338333333333300	0.25266666666666700
2012-07-01	0.48691666666666700	0.0503750000000000	0.4504583333333300	0.3620833333333300	0.22675	0.251375
2012-08-01	0.509	0.0560416666666670	0.465875	0.3490833333333300	0.23916666666666700	0.2503333333333300
2012-09-01	0.536	0.0780416666666670	0.482458333333330	0.33675	0.2389583333333300	0.24929166666666700
2012-10-01	0.5750416666666670	0.0857916666666670	0.484875	0.3444583333333300	0.24241666666666700	0.2483750000000000

240\_rolling\_covariances

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2010-06-01	Mkt-RF	24.470939105648500	-2.182876375523010	1.067980088912130	-1.882339069037660	-3.6919182845188300
2010-06-01	SMB	-2.182876375523010	5.501366441771270	0.07317618375174330	0.0780938458856346	0.12648143654114400
2010-06-01	HML	1.067980088912130	0.07317618375174330	5.749903889470020	-1.6335462517433800	2.6692474337517400
2010-06-01	RMW	-1.882339069037660	0.0780938458856346	-1.6335462517433800	2.501930369595540	-0.38033361227336100
2010-06-01	CMA	-3.6919182845188300	0.12648143654114400	2.6692474337517400	-0.38033361227336100	0.013729114365411400
2010-06-01	RF	-0.011659445606694500	-0.06737757670850770	-0.008888636680613660	0.013729114365411400	-0.00909947001394699
2010-07-01	Mkt-RF	24.954134266039100	-2.268671269177130	1.298367451185500	-2.0360395833333300	-3.6719475156903800
2010-07-01	SMB	-2.268671269177130	5.513518716875870	0.0456448814504881	0.1013816178521620	0.12066468619246900
2010-07-01	HML	1.298367451185500	0.0456448814504881	5.801688695955370	-1.6894540550906600	2.6886269979079500
2010-07-01	RMW	-2.0360395833333300	0.1013816178521620	-1.6894540550906600	2.545621044281730	-0.3896361767782430
2010-07-01	CMA	-3.6719475156903800	0.12066468619246900	2.6886269979079500	-0.3896361767782430	4.020609189330540
2010-07-01	RF	-0.03199047245467220	-0.06562825662482570	-0.01050558228730820	0.017779109135285900	-0.011310978033472800
2010-08-01	Mkt-RF	24.51165605125520	-2.2555025104602500	1.3221525575313800	-2.121754759414230	-3.5590361610878700
2010-08-01	SMB	-2.2555025104602500	5.513279497907950	0.04746276150627610	0.10163849372384900	0.11988786610878700
2010-08-01	HML	1.3221525575313800	0.04746276150627610	5.842187445955370	-1.7227924860530000	2.7252911889818700
2010-08-01	RMW	-2.121754759414230	0.10163849372384900	-1.7227924860530000	2.563482670850770	-0.4065780683403068300
2010-08-01	CMA	-3.5590361610878700	0.11988786610878700	2.7252911889818700	-0.4065780683403068300	0.016455683403068300
2010-08-01	RF	-0.009144069037656920	-0.0659255230125523	-0.005404055090655540	0.016455683403068300	-0.010666394700139500
2010-09-01	Mkt-RF	24.353533884239900	-2.110857810320780	1.281404850069740	-2.165980428870290	-3.416559285216180
2010-09-01	SMB	-2.110857810320780	5.501379218967920	0.0451114714086471	0.09867834728033480	0.11255254532775400
2010-09-01	HML	1.281404850069740	0.0451114714086471	5.844726776499300	-1.7199020972803300	2.7215316126220400
2010-09-01	RMW	-2.165980428870290	0.09867834728033480	-1.7199020972803300	2.5667669299163200	-0.41107545502092000
2010-09-01	CMA	-3.416559285216180	0.11255254532775400	2.7215316126220400	-0.41107545502092000	4.032763722106000
2010-09-01	RF	-0.005873692468619270	-0.06949110878661090	-0.0043724110878661200	0.017577327405857700	-0.014199450836820100
2010-10-01	Mkt-RF	24.289715228382100	-2.0355387813807500	1.313691422594140	-2.159013458856350	-3.4029584030683400
2010-10-01	SMB	-2.0355387813807500	5.475018896443510	0.026186485355648500	0.10633577405857700	0.10065705020920500
2010-10-01	HML	1.313691422594140	0.026186485355648500	5.834275481171550	-1.7185560669456100	2.7159256066945600
2010-10-01	RMW	-2.159013458856350	0.10633577405857700	-1.7185560669456100	2.570039051603910	-0.4115047419804740
2010-10-01	CMA	-3.4029584030683400	0.10065705020920500	2.7159256066945600	-0.4115047419804740	4.030141394700140
2010-10-01	RF	-0.020760425383542500	-0.06597847280334730	-0.001152552301255230	0.015632775453277500	-0.011961143654114400
2010-11-01	Mkt-RF	24.591128807531400	-2.0831987343096200	1.4545974790795000	-2.2080957112970700	-3.40084980125523
2010-11-01	SMB	-2.0831987343096200	5.449589816945610	0.00957064330543929	0.11068080543933100	0.09373194037656900
						-0.06401257322175730

#### A.4 Mean Variance Portfolio Optimisation Results

Optimal Weights for Window Size 60 Months:

Mkt-RF: 0.3786

SMB: 0.0000

HML: 0.0000

RMW: 0.6214

CMA: 0.0000

Target Expected Return: 0.3820

Expected Return of Optimized Portfolio: 0.3819679005442957

Variance of Optimized Portfolio: 5.9966

Standard Deviation of Optimized Portfolio: 2.4488

Portfolio Returns: [0.11516119705391747, 0.1934696263944934, -0.4209480614726232, -0.2675902142151783, 0.30095775236131306]

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Optimal Weights for Window Size 120 Months:

Mkt-RF: 1.0000

SMB: 0.0000

HML: 0.0000

RMW: 0.0000

CMA: 0.0000

Target Expected Return: 0.3820  
 Expected Return of Optimized Portfolio: 0.3740833333328675  
 Variance of Optimized Portfolio: 22.6235  
 Standard Deviation of Optimized Portfolio: 4.7564  
 Portfolio Returns: [0.5439977569131346, -1.1132041545595963, 1.3396638582193794, 3.655003916930271, 3.6296930944609347]

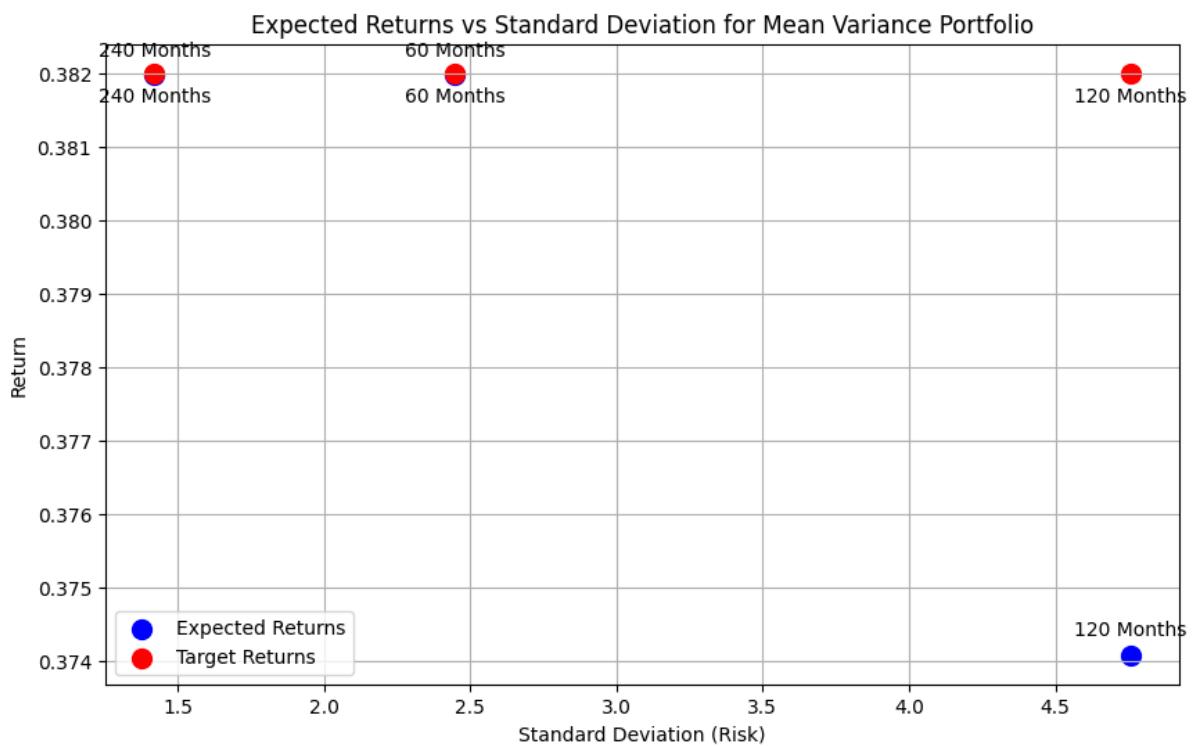
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Optimal Weights for Window Size 240 Months:

Mkt-RF: 0.2173  
 SMB: 0.0000  
 HML: 0.0000  
 RMW: 0.7827  
 CMA: 0.0000

Target Expected Return: 0.3820  
 Expected Return of Optimized Portfolio: 0.3819679012073063  
 Variance of Optimized Portfolio: 2.0169  
 Standard Deviation of Optimized Portfolio: 1.4202  
 Portfolio Returns: [0.5486951379242772, 0.15724363611205983, 0.8718274693184429, 1.1744575618355988, -0.47203851379199757]

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#### A.4 Minimum Variance Portfolio Optimisation

Optimal Weights for Window Size 60 Months:

Mkt-RF: 0.0076  
 SMB: 0.2042  
 HML: 0.0394  
 RMW: 0.4171

CMA: 0.3317  
Expected Return of Optimized Portfolio: 0.0146  
Variance of Optimized Portfolio: 0.3041  
Standard Deviation of Optimized Portfolio: 0.5514  
Portfolio Returns: [0.07082996262993377, 0.19861347193821968, -0.41875818211948107, -0.2849634801617105, 0.23136049798905955]

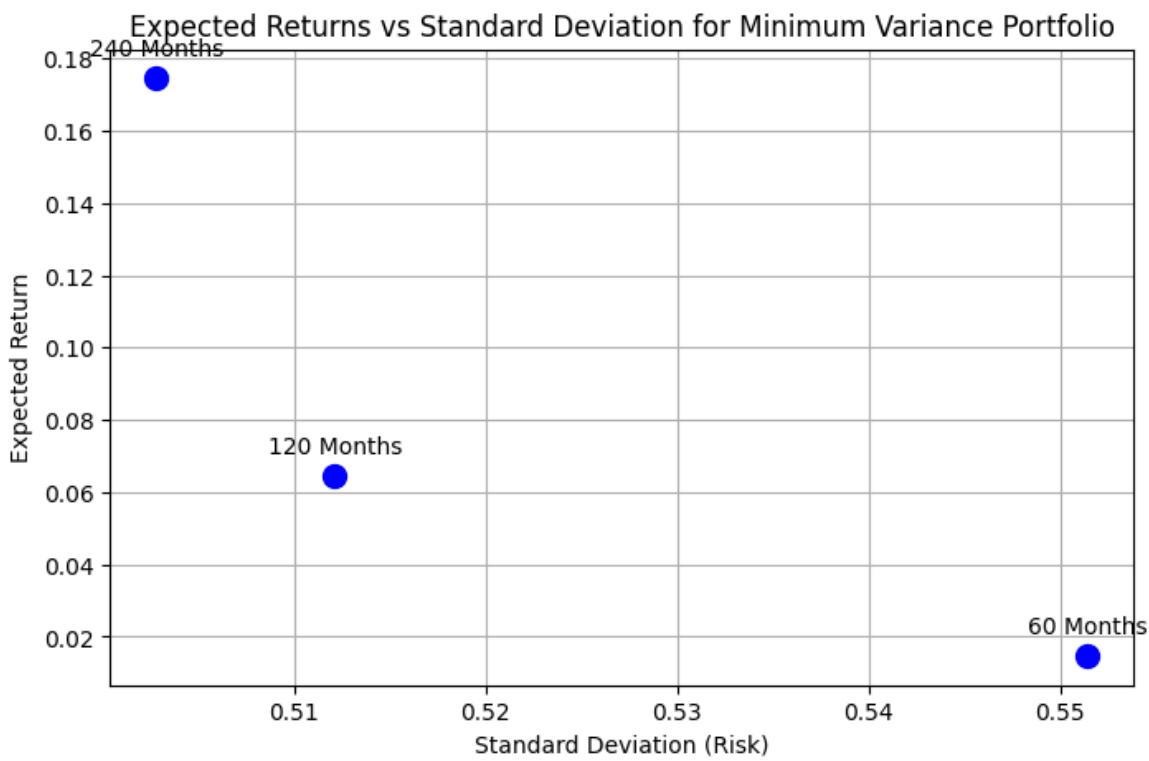
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Optimal Weights for Window Size 120 Months:  
Mkt-RF: 0.0225  
SMB: 0.1952  
HML: 0.0624  
RMW: 0.3896  
CMA: 0.3303  
Expected Return of Optimized Portfolio: 0.0644  
Variance of Optimized Portfolio: 0.2622  
Standard Deviation of Optimized Portfolio: 0.5121  
Portfolio Returns: [0.6729247219894053, -0.6862641665642788, 1.7276838707094888, 2.8405920640791344, 3.4477176671783356]

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Optimal Weights for Window Size 240 Months:  
Mkt-RF: 0.0289  
SMB: 0.1451  
HML: 0.1520  
RMW: 0.4412  
CMA: 0.2328  
Expected Return of Optimized Portfolio: 0.1745  
Variance of Optimized Portfolio: 0.2528  
Standard Deviation of Optimized Portfolio: 0.5028  
Portfolio Returns: [0.4313745357785271, -0.044268753999390016, 1.3596116773290003, 1.1329421305094953, -0.26892765723594864]

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### A.5 Naive Portfolio Optimisation

Equal Weights Portfolio:

Mkt-RF: 0.2000

SMB: 0.2000

HML: 0.2000

RMW: 0.2000

CMA: 0.2000

Expected Return of Equal Weighted Portfolio: 0.2556

Variance of Equal Weighted Portfolio: 1.3713

Standard Deviation of Equal Weighted Portfolio: 1.1710

Expected Returns vs Standard Deviation for Naive Model

