Master's Programmes: Assignment Cover Sheet

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1. Introduction

This report's goal is to assess the out-of-sample performance of three strategies for portfolio optimisation: a naïve equal-weighted model, a minimum-variance model, and a mean-variance model. Ken French's data library's historical equity return data is used to evaluate the effectiveness of these tactics. The purpose of the analysis is to analyse the relative performances of each strategy when input parameters are estimated using historical data.

2. Selection of Dataset

For this analysis, I have selected the Fama/French European 5 Factors from Ken French's data library. The dataset includes monthly returns for multiple equity factors, making it suitable for the mean-variance and minimum-variance optimization analyses. These factors are constructed to help explain the returns of stocks and identify various sources of risk. Although the risk-free asset considered in this dataset is the one-month US Treasury bill, which may not perfectly align with the European market, the dataset was chosen to study the returns of the European market specifically.

Data Usage

- Monthly Returns: From July 1990 to March 2024 (all the data is in percentage points)

Factor Construction

The dataset, has the following five factors:

1. Market ($R_m - R_f$): This is the value-weighted market portfolio return for an area less the rate on US one-month T-bills. After accounting for the risk-free rate, it shows the total excess market return.
2. Small Minus Big, or SMB: The size premium, or the variation in returns between small-cap stocks and large-cap companies, is captured by SMB. It is created by grouping equities into three categories: investment (INV), operating profitability (OP), book-to-market equity (B/M), and big and small market size. The returns are then averaged.
3. HML (High Minus Low): HML measures the return differential between value (high book-to-market) and growth (low book-to-market) companies, or the value premium. The average return on the two value portfolios (small and large) is subtracted from the average return on the two growth portfolios (small and large) to create it.
4. RMW (Robust Minus Weak): RMW measures the return difference between equities with strong (high) and weak (low) operating profitability. This is known as the profitability premium. It is calculated by deducting the average return from the two large, robust portfolios from the average return from the two small, weak portfolios.

5. Aggressive Minus Conservative (CMA): The difference in returns between businesses that invest aggressively (high investment) and those that invest cautiously (low investment) is known as the investment premium, and CMA measures it. The average return on the two small and big conservative portfolios is subtracted from the average return on the two small and big aggressive portfolios to create it.

Region

Europe: Contains nations including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

The selection of this dataset was based on its extensive coverage of developed market equity returns, which offers a strong foundation for estimating return mean and covariance matrices. With monthly returns for several equity components, the dataset is appropriate for optimisation analyses involving mean-variance and minimum-variance.

3. Selection of Estimate Windows M

Selecting the right estimation windows is crucial for assessing portfolio optimisation techniques' out-of-sample performance. Three different estimating windows—60 months, 120 months, and 240 months—have been used for this investigation.

Three distinct estimation windows (M) :

1. M = 60 months: The 60 months window provides a more current and trend-sensitive view of the market. This shorter-term perspective is more sensitive to recent shifts in market conditions since it covers the most recent market dynamics and trends (COVID-19). In markets that are erratic or that are changing quickly, this window can be very helpful for identifying and responding to recent changes in market behaviour. Though this window may be more sensitive to recent trends, it may also be more vulnerable to short-term noise and anomalies.
2. M = 120 months: The 120 months window is designed to capture long-term trends while allowing for short-term fluctuations. It takes into account enough past data to yield reliable projections while remaining aware of recent changes in the market. This medium-term window is a solid option for the majority of portfolio optimisation techniques since it is good at seeing recurring patterns and linkages in the data. It strikes a compromise between the requirement to maintain relevance to the state of the market and the demand for historical background.
3. M = 240 months: By offering a long-term view, the 240 months window lessens the impact of short-term volatility. It uses a significant amount of historical data as

leverage, which helps to capture long-term market trends and smooth out short-term volatility. By averaging out short-term anomalies and market cycles, it provides estimates of mean and covariance that are more trustworthy and stable. Though this window offers stability, it could not be as sensitive to recent shifts in the market, thus it might miss short-term patterns and adjustments that could be important given the state of the market.

4. Estimation of mean and covariance matrix

To effectively construct a mean-variance optimized portfolio, it is crucial to accurately estimate the mean and covariance matrix of returns. This involves using historical return data to derive these statistical measures, which serve as the foundation for the optimization process. At every possible time t, the data from the previous M periods (months) is used to estimate the mean and covariance matrix of returns for three distinct estimation windows 60 Months, 120 Months, 240 Months. The unbiased sample estimator is employed to estimate the mean and covariance matrix. The unbiased estimators for mean (μ) and covariance matrix (Σ) are calculated as follows:

- **Mean (Expected Return)**:

$$\hat{\mu} = \frac{1}{M}\sum_{i=1}^{M} Ri$$

  Where Ri represents the return in period i.

- **Covariance Matrix**:

$$\hat{\Sigma} = \frac{1}{M-1}\sum_{i=1}^{M}(Ri - \hat{\mu})(Ri - \hat{\mu})^T$$

  Here, Ri is the return vector in period i, and $\hat{\mu}$ is the estimated mean return vector.

5. Target Expected Return

The target expected return is a critical input for the optimization process. For this analysis, the monthly expected target return is determined by averaging the returns implied by the Capital Asset Pricing Model (CAPM) with beta of 0.6 (<1) considering market being more volatile than the assets and the historical average returns of the portfolio. This dual approach provides a balanced and realistic expectation, integrating both forward-looking market estimates and historical performance.

Target Expected Return

Using CAPM method to determine the returns

```python
# Calculate the means of the last period for Mkt-RF and RF
mean_mkt_rf = df_months['Mkt-RF'].mean()
mean_rf = df_months['RF'].mean()

# On researching the market is more volatile than the assets the beta for the investment is 0.6
beta_a = 0.6  # Example beta value, replace with actual beta

# Calculate the expected return using the formula
expected_return = mean_rf + beta_a * (mean_mkt_rf)
print(f"Mean Market Risk Premium - Mean Risk Free Rate : {mean_mkt_rf}")
print(f"Mean Risk-Free Rate (RF): {mean_rf}")
print(f"Expected Return: {expected_return}")
```

✓ 0.0s Python

```
Mean Market Risk Premium - Mean Risk Free Rate : 0.4985432098765432
Mean Risk-Free Rate (RF): 0.20925925925925926
Expected Return: 0.5083851851851852
```

Historical average return

```python
# Calculate historical mean returns for each factor
historical_mean_returns = df_months.mean()

# Calculate the historical mean return of the portfolio
# Assuming equal weight for each factor
num_factors = len(historical_mean_returns) - 1  # Exclude RF
equal_weights = np.array([1. / num_factors] * num_factors)
historical_portfolio_mean_return = np.dot(equal_weights, historical_mean_returns[:-1])  # Exclude RF

print(f"Historical Mean Returns:\n{historical_mean_returns}")
print(f"\nHistorical Portfolio Mean Return: {historical_portfolio_mean_return}")
```

✓ 0.0s Python

```
Historical Mean Returns:
Mkt-RF    0.498543
SMB       0.017136
HML       0.278667
RMW       0.357852
CMA       0.125556
RF        0.209259
dtype: float64

Historical Portfolio Mean Return: 0.2555506172839507
```

$$\widehat{r_t} = \frac{1}{2}(CAPM\ implied\ return + Historical\ average\ return)$$

By averaging the CAPM implied return and the historical average return, a more balanced and realistic target return is achieved. This approach ensures that the target return incorporates both the current market environment and the historical performance of the assets, providing a solid foundation for the mean-variance portfolio optimization process. This blended target return aims to optimize the portfolio's performance while managing risk, enhancing the overall robustness of the investment strategy.

Average of CAPM and historical means

```
expected_return_final = (historical_portfolio_mean_return + expected_return ) / 2
expected_return_final = expected_return_final.round(4)
print(f'Final expected returns: {expected_return_final}')
```

✓ 0.0s Python

```
Final expected returns: 0.382
```

The expected monthly target return came out to be 0.382%.

6. Mean-Variance Portfolio Optimization

This method seeks to construct an investment portfolio that This method seeks to construct an investment portfolio that minimizes risk for a given level of expected return. The goal is to achieve an optimal balance between risk and return, thereby enhancing portfolio performance.

The mean-variance optimization problem can be formulated as follows:

$$\text{minimise}_{w}\ w^{T}\hat{\Sigma}w$$

$$\text{subject to: } w^{T}\hat{\mu} = \hat{r_t}$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w \geq 0$$

Where:

- w is the vector of portfolio weights.
- $\hat{\Sigma}$ is the estimated covariance matrix.
- $\hat{\mu}$ is the estimated mean return vector.
- $\hat{r_t}$ is the target expected return.
- The constraints ensure that the portfolio weights sum to 1 and that short-selling is not allowed (weights are non-negative).

```
def portfolio_optimization(mean_returns, cov_matrix, target_return):
    num_assets = len(mean_returns)
    args = (mean_returns, cov_matrix)

    def portfolio_variance(weights, mean_returns, cov_matrix):
        return np.dot(weights.T, np.dot(cov_matrix, weights))

    constraints = (
        {'type': 'eq', 'fun': lambda x: np.sum(x) - 1},
        {'type': 'eq', 'fun': lambda x: np.dot(x, mean_returns) - target_return}
    )
    bounds = tuple((0, 1) for asset in range(num_assets))
    result = minimize(portfolio_variance,
                      num_assets * [1. / num_assets],
                      args=args,
                      method='SLSQP',
                      bounds=bounds,
                      constraints=constraints)
    return result.x
```

The optimization problem is solved at every possible time t, using the data from the previous M periods to estimate the parameters. This yields the optimal portfolio weights w_t for time t. Once the optimal portfolio weights are determined for time t, the portfolio's return in the subsequent investment period (from t to t+1) is computed. (Appendix A.4)

7. Minimum Variance Portfolio

The goal of this section is to evaluate the performance of a minimum-variance portfolio optimization strategy. This approach aims to minimize the portfolio's overall risk, represented by the variance of returns, without imposing any constraint on the expected return. By using historical data to estimate the covariance matrix of returns, the minimum-variance portfolio can be constructed to achieve the lowest possible risk for a given set of assets.

```
def minimum_variance_portfolio(cov_matrix):
    num_assets = len(cov_matrix)
    args = (cov_matrix,)

    def portfolio_variance(weights, cov_matrix):
        return np.dot(weights.T, np.dot(cov_matrix, weights))

    constraints = (
        {'type': 'eq', 'fun': lambda x: np.sum(x) - 1}
    )
    bounds = tuple((0, 1) for asset in range(num_assets))
    result = minimize(portfolio_variance,
                      num_assets * [1. / num_assets],
                      args=args,
                      method='SLSQP',
                      bounds=bounds,
                      constraints=constraints)
    return result.x

optimization_results = {}
```

The optimization problem is formulated to minimize the portfolio variance without any constraint on the mean return. The optimization is performed using the SLSQP method from the scipy.optimize.minimize function. The expected return, variance, and standard deviation of the optimized portfolio are calculated for each window size. (Appendix A.5)

8. Naïve 1/N Portfolio Strategy

In this step, we compute the returns for a naïve portfolio strategy where equal weights are assigned to each of the N available assets. This strategy, often referred to as the 1/N strategy, involves simply dividing the total investment equally among all assets, without any optimization based on historical data or statistical estimates and calculating the returns.

```python
# Calculate naive portfolio
def naive_portfolio(mean_returns, cov_matrix, assets):
    num_assets = len(assets)
    equal_weights = np.array([1. / num_assets] * num_assets)

    expected_return = np.dot(equal_weights, mean_returns)
    portfolio_variance = np.dot(equal_weights.T, np.dot(cov_matrix, equal_weights))
    portfolio_std_dev = np.sqrt(portfolio_variance)

    return equal_weights, expected_return, portfolio_variance, portfolio_std_dev
```

The naive portfolio strategy is often used as a benchmark or starting point for more sophistic ated portfolio optimization techniques. While it may not always lead to the most optimal risk-return trade-off, it serves as a simple and transparent method for portfolio allocation. (Appendix A.6)

9. Conclusion and Interpretation of Results

Method	Expected Return	Variance	Standard Deviation
Mean-Variance Portfolio (60 Months)	0.3819	5.9966	2.4488
Mean-Variance Portfolio (120 Months)	0.374	22.6235	4.7564
Mean-Variance Portfolio (240 Months)	0.3819	2.0169	1.4202
Minimum Variance Portfolio (60 Months)	0.0146	0.3041	0.5514
Minimum Variance Portfolio (120 Months)	0.0644	0.2622	0.5121
Minimum Variance Portfolio (240 Months)	0.1745	0.2528	0.5028
Equal Weights Portfolio / Naive Model	0.2556	1.3713	1.171

1. Mean-Variance Portfolio:

The Mean-Variance Portfolio with a 240-month window size achieved the highest expected return (0.3819) with a relatively low standard deviation (1.4202). This suggests that using a longer historical period for covariance estimation can provide more stable and favourable results. The 60-month and 120-month window sizes also yielded high expected returns but with significantly higher standard deviations, indicating increased risk.

2. Minimum Variance Portfolio:

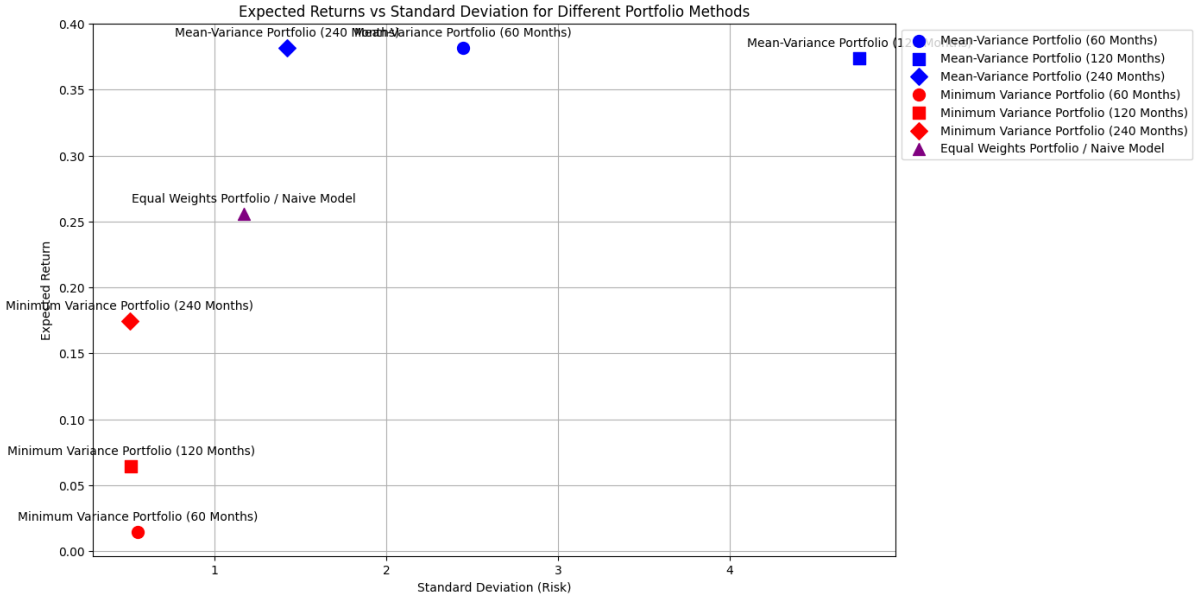
The Minimum Variance Portfolio with a 240-month window size achieved a moderate expected return (0.1745) with the lowest standard deviation (0.5028). This portfolio is suitable for risk-averse investors seeking to minimize risk. The 60-month and 120-month window sizes resulted in lower expected returns but also maintained low standard deviations, making them viable options for conservative investment strategies.

3. Equal Weights Portfolio (Naive Model):

The Equal Weights Portfolio achieved a reasonable expected return (0.2556) with a standard deviation (1.1710) that is higher than the Minimum Variance Portfolio but lower than the Mean-Variance Portfolio for shorter window sizes. This method provides a simple and balanced approach without the need for complex optimization.

The analysis demonstrates that the choice of window size for rolling covariance estimation significantly impacts the performance of optimized portfolios. The Mean-Variance Portfolio with a 240-month window size offers the highest expected return with moderate risk, making it an attractive option for investors seeking higher returns. The Minimum Variance Portfolio with a 240-month window size provides the lowest risk, suitable for conservative investors. The Equal Weights Portfolio offers a balanced approach with reasonable returns and risk.

Investors should consider their risk tolerance and investment horizon when selecting the appropriate portfolio optimization method.



10. Recommendations

For High Returns: Consider the Mean-Variance Portfolio with a 240-month window size.

For Low Risk: opt for the Minimum Variance Portfolio with a 240-month window size.

For Simplicity and Balance: The Equal Weights Portfolio provides a straightforward and balanced approach.

By carefully selecting the appropriate portfolio optimization method, investors can better align their investment strategies with their financial goals and risk preferences.

References

Books and Articles

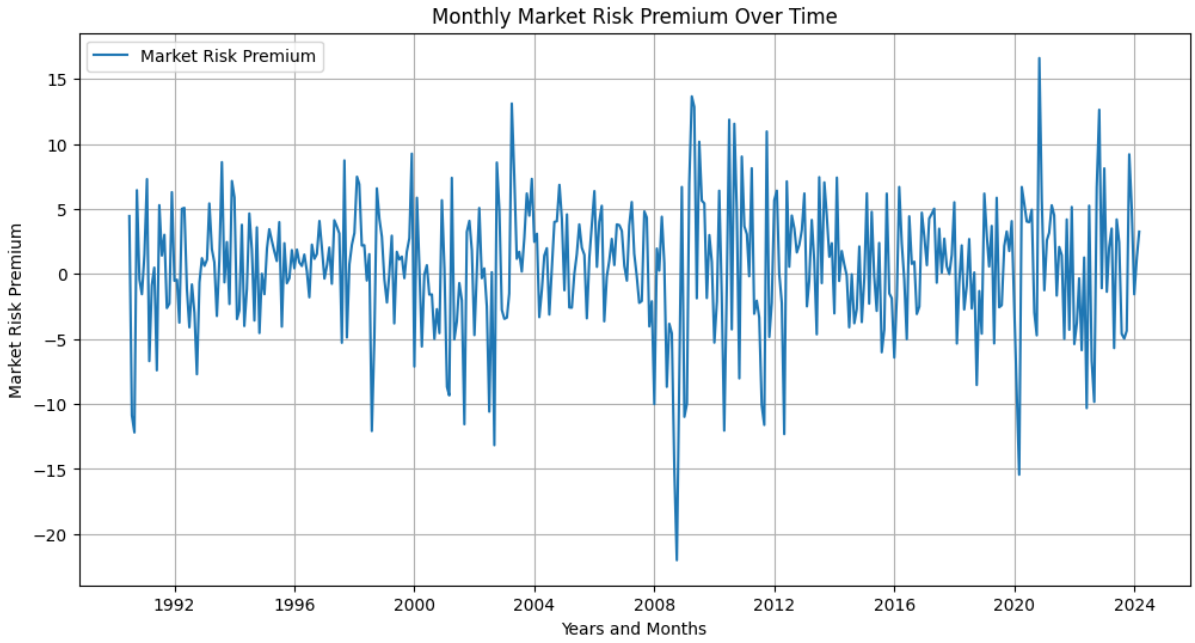
1. Black, F. and Litterman, R. (1992) 'Global Portfolio Optimization', Financial Analysts Journal, 48(5), pp. 28-43.
2. Bodie, Z., Kane, A. and Marcus, A.J. (2014) Investments. 10th edn. New York: McGraw-Hill Education.
3. Fabozzi, F.J., Gupta, F. and Markowitz, H.M. (2002) 'The Legacy of Modern Portfolio Theory', The Journal of Investing, 11(3), pp. 7-22.
4. Markowitz, H. (1952) 'Portfolio Selection', The Journal of Finance, 7(1), pp. 77-91.

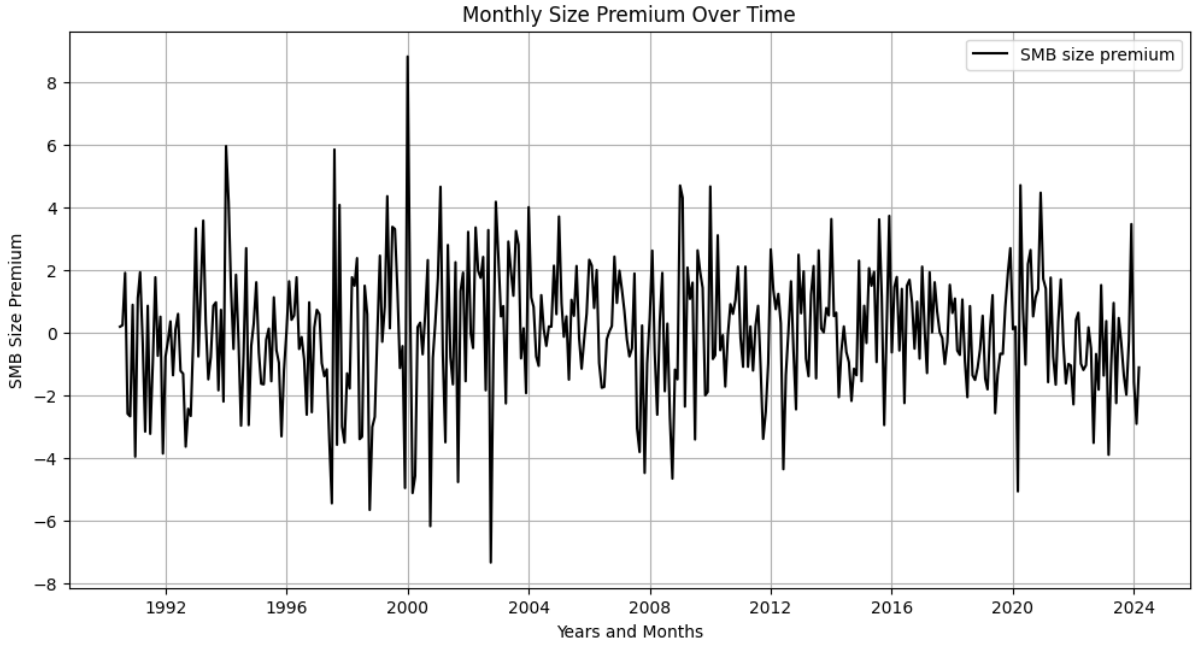
Online Resources

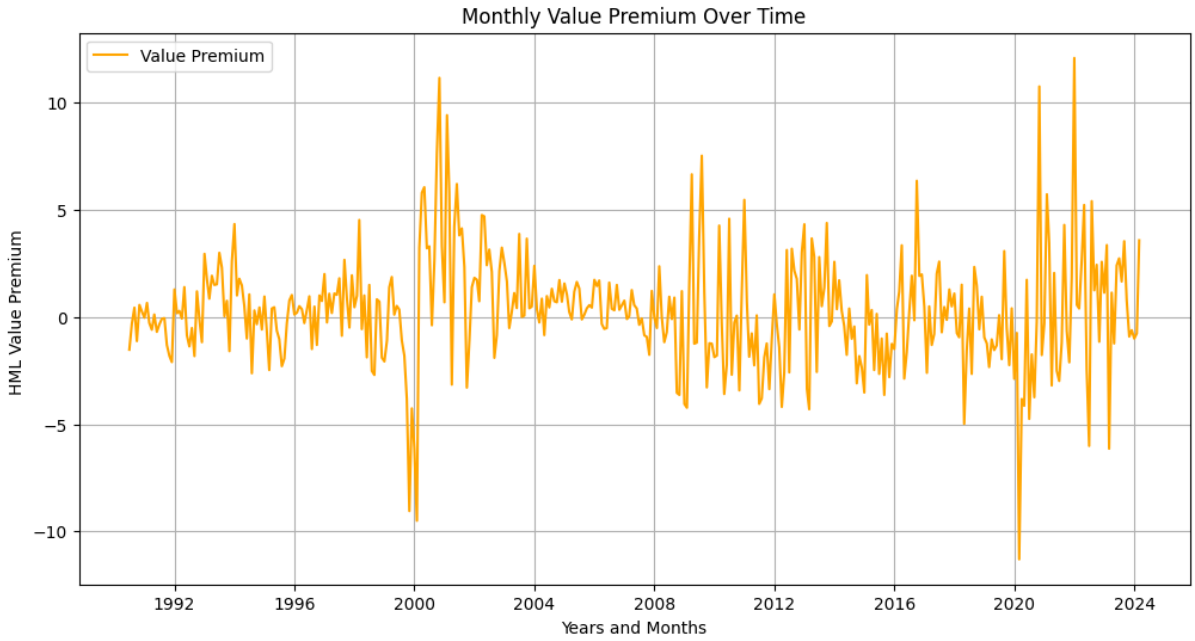
5. Matplotlib Documentation (no date) Available at: https://matplotlib.org/stable/contents.html (Accessed: 7 June 2024).
6. NumPy Documentation (no date) Available at: https://numpy.org/doc/stable/ (Accessed: 7 June 2024).
7. Pandas Documentation (no date) Available at: https://pandas.pydata.org/pandas-docs/stable/ (Accessed: 7 June 2024).
8. SciPy Documentation (no date) Available at: https://docs.scipy.org/doc/scipy/reference/ (Accessed: 7 June 2024).

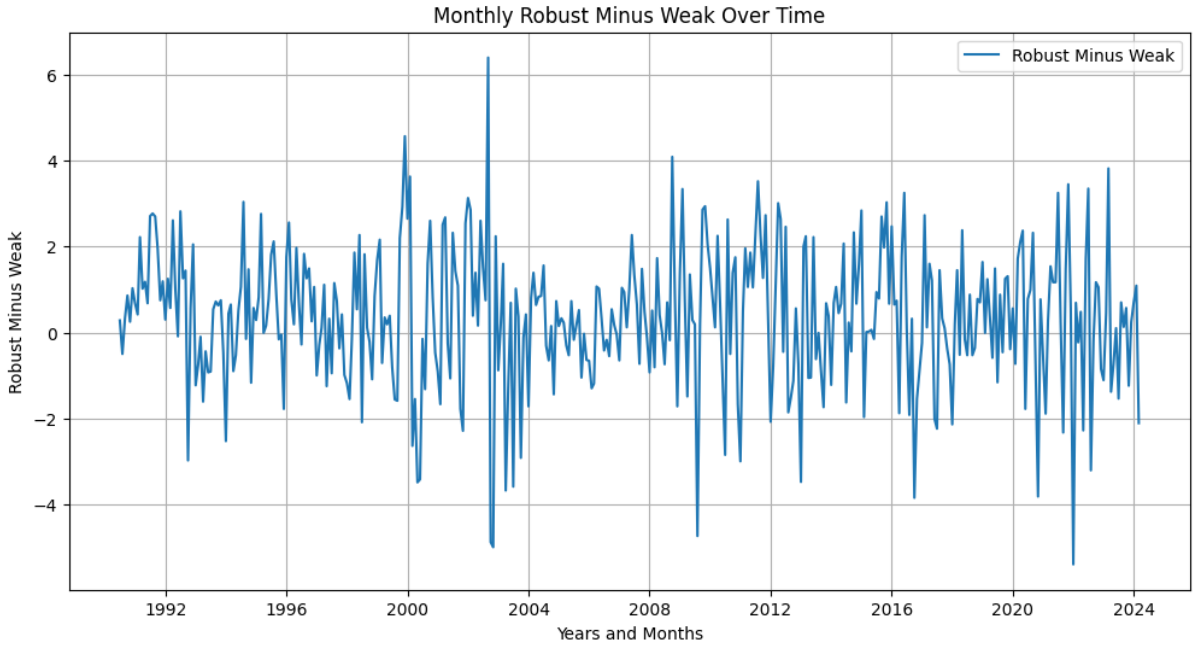
Appendix

A.0 Data Plotting






Monthly Value Premium Over Time
Value Premium
HML Value Premium
10
5
0
−5
−10
1992
1996
2000
2004
2008
2012
2016
2020
2024
Years and Months


Monthly Robust Minus Weak Over Time
Robust Minus Weak
Robust Minus Weak
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4
2
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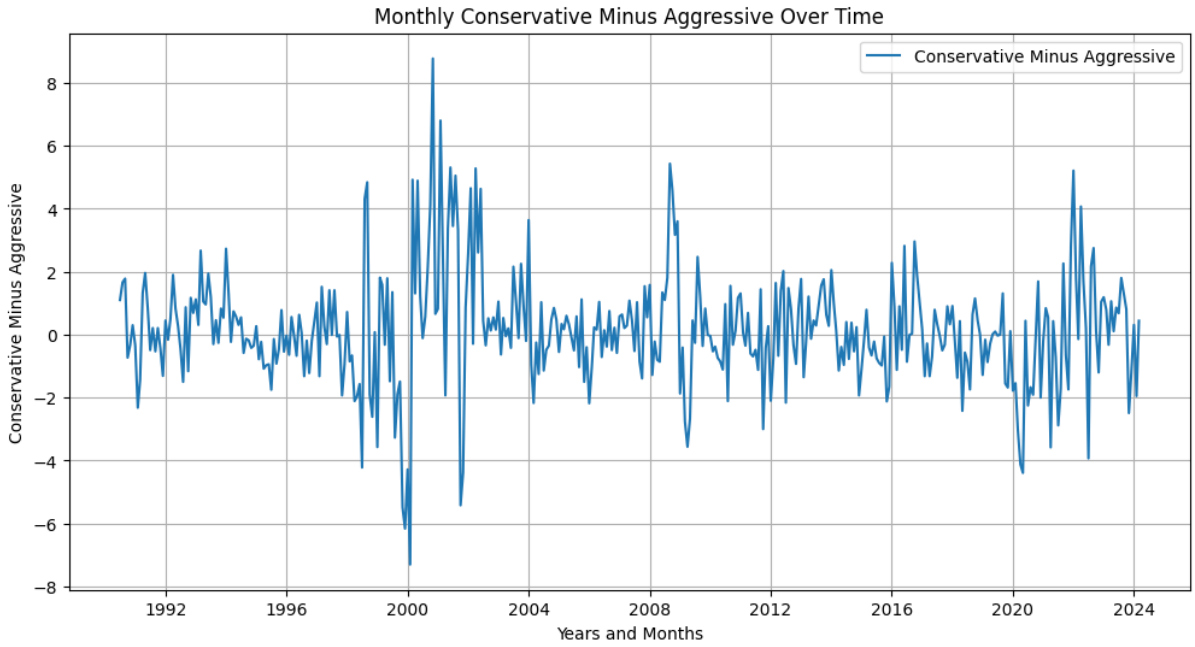
1992
1996
2000
2004
2008
2012
2016
2020
2024
Years and Months



A.1 Mean and Covariance Matrix : 60 months estimation window (head)

```
...   Rolling Means for Window Size 60 Months:
                        Mkt-RF       SMB       HML       RMW       CMA        RF
      Years and Months
      1995-06-01        0.218833 -0.234167  0.205333  0.579833  0.266333  0.369000
      1995-07-01        0.211000 -0.263167  0.213500  0.605333  0.218833  0.365167
      1995-08-01        0.324833 -0.248333  0.180333  0.649000  0.188833  0.362000
      1995-09-01        0.567500 -0.289500  0.140833  0.658333  0.143833  0.359167
      1995-10-01        0.448167 -0.262333  0.154667  0.641333  0.148333  0.355667

      Rolling Covariances for Window Size {M1_window_size} Months:
                                Mkt-RF       SMB       HML       RMW       CMA  \
      Years and Months
      1995-06-01       Mkt-RF  18.663617 -1.699127  0.925145  0.749140 -0.496133
                       SMB     -1.699127  4.092025  1.085621 -0.817138  0.578374
                       HML      0.925145  1.085621  1.931907 -1.165562  0.486566
                       RMW      0.749140 -0.817138 -1.165562  1.587063 -0.352962
                       CMA     -0.496133  0.578374  0.486566 -0.352962  1.071037

                                    RF
      Years and Months
      1995-06-01       Mkt-RF -0.056037
                       SMB    -0.037248
                       HML    -0.074123
                       RMW     0.044751
                       CMA    -0.027573
```

rolling_mean_60

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
1995-06-01	0.2188333333333330	-0.2341666666666670	0.2053333333333330	0.5798333333333330	0.26633333333333300	0.369
1995-07-01	0.2110000000000000	-0.2631666666666700	0.2135000000000000	0.6053333333333330	0.21883333333333300	0.3651666666666670
1995-08-01	0.3248333333333300	-0.2483333333333300	0.1803333333333300	0.6490000000000000	0.18883333333333300	0.362
1995-09-01	0.5675	-0.2895000000000000	0.1408333333333330	0.6583333333333330	0.14383333333333300	0.3591666666666670
1995-10-01	0.44816666666666700	-0.2623333333333300	0.1546666666666700	0.6413333333333330	0.14833333333333300	0.3556666666666670
1995-11-01	0.4498333333333330	-0.273	0.1581666666666700	0.6363333333333330	0.16683333333333300	0.3531666666666700
1995-12-01	0.5063333333333330	-0.3078333333333330	0.1710000000000000	0.5895000000000000	0.15283333333333300	0.3513333333333330
1996-01-01	0.4885000000000000	-0.2410000000000000	0.1735000000000000	0.6073333333333330	0.158	0.3498333333333330
1996-02-01	0.3981666666666700	-0.2321666666666700	0.166	0.643	0.186	0.3483333333333330
1996-03-01	0.5246666666666670	-0.2575	0.179	0.6188333333333330	0.21916666666666700	0.34750000000000000
1996-04-01	0.5495	-0.2431666666666700	0.1950000000000000	0.605	0.1965	0.34633333333333300
1996-05-01	0.5663333333333330	-0.161	0.1883333333333300	0.6181666666666670	0.15266666666666700	0.34550000000000000
1996-06-01	0.6935	-0.1840000000000000	0.2053333333333300	0.6178333333333330	0.14916666666666700	0.3451666666666670
1996-07-01	0.5753333333333330	-0.1325	0.2273333333333330	0.568	0.15833333333333300	0.3445
1996-08-01	0.5893333333333330	-0.1306666666666700	0.2038333333333300	0.5523333333333330	0.13283333333333300	0.3436666666666670
1996-09-01	0.5585	-0.2038333333333300	0.2128333333333330	0.5283333333333330	0.1385	0.3433333333333330
1996-10-01	0.628	-0.1753333333333330	0.2128333333333300	0.5208333333333330	0.11466666666666700	0.3433333333333330
1996-11-01	0.7338333333333330	-0.2261666666666700	0.2598333333333330	0.5126666666666670	0.11883333333333300	0.3436666666666670
1996-12-01	0.6623333333333330	-0.1591666666666700	0.3073333333333330	0.5105	0.14766666666666700	0.345
1997-01-01	0.6651666666666670	-0.1341666666666700	0.3193333333333330	0.4888333333333300	0.15716666666666700	0.3468333333333330
1997-02-01	0.6838333333333330	-0.1203333333333300	0.3121666666666670	0.4631666666666670	0.13783333333333300	0.3486666666666670
1997-03-01	0.7801666666666670	-0.1425000000000000	0.3255	0.4555	0.15483333333333300	0.3501666666666670
1997-04-01	0.6840000000000000	-0.143	0.3298333333333330	0.4305	0.1275	0.352
1997-05-01	0.6681666666666670	-0.1641666666666700	0.3248333333333300	0.3928333333333330	0.10866666666666700	0.3555000000000000
1997-06-01	0.7486666666666670	-0.2318333333333300	0.3563333333333300	0.3996666666666700	0.12766666666666700	0.3563333333333330
1997-07-01	0.869	-0.3023333333333300	0.4093333333333330	0.3368333333333330	0.13516666666666700	0.3583333333333300
1997-08-01	0.7941666666666670	-0.183	0.4033333333333330	0.335	0.18366666666666700	0.3608333333333330
1997-09-01	0.9896666666666670	-0.182	0.4781666666666700	0.3231666666666700	0.16816666666666700	0.3638333333333330
1997-10-01	1.0368333333333300	-0.0735	0.473	0.3666666666666700	0.18733333333333300	0.367
1997-11-01	1.0601666666666700	-0.07916666666666670	0.4670000000000000	0.3641666666666670	0.13566666666666700	0.36966666666666700

rolling_cov_60

Years and Months		Mkt-RF	SMB	HML	RMW	CMA	RF
1995-06-01	Mkt-RF	18.663617259887000	-1.6991269774011300	0.9251453107344630	0.7491404802259890	-0.4961331638418080	-0.056036779661016900
1995-06-01	SMB	-1.6991269774011300	4.092024717514120	1.0856209039548000	-0.8171379943502820	0.5783742937853110	-0.037248305084745800
1995-06-01	HML	0.9251453107344630	1.0856209039548000	1.9319066666666700	-1.1655618079096000	0.4865656497151400	-0.074123389830508500
1995-06-01	RMW	0.7491404802259890	-0.8171379943502820	-1.1655618079096000	1.587062683615820	-0.3529616384180790	0.04475067796610170
1995-06-01	CMA	-0.4961331638418080	0.5783742937853110	0.4865656497151400	-0.3529616384180790	1.0710371751412400	-0.027573220338983100
1995-06-01	RF	-0.056036779661016900	-0.037248305084745800	-0.074123389830508500	0.04475067796610170	-0.027573220338983100	0.016568474576271200
1995-07-01	Mkt-RF	18.599727796610200	-1.81403406779661	0.9702744067796610	0.849526779661017	-0.685319152542373	-0.07324593220338980
1995-07-01	SMB	-1.81403406779661	4.116876242937850	1.125899406779660	-0.8414065536723160	0.6154657344632770	-0.04144268361581920
1995-07-01	HML	0.9702744067796610	1.125899406779660	1.907250254237290	-1.2002986440678000	0.5535566949152540	-0.0666929661016949
1995-07-01	RMW	0.849526779661017	-0.8414065536723160	-1.2002986440678000	1.6105270056497200	-0.3895343502848600	0.04811943502824860
1995-07-01	CMA	-0.685319152542373	0.6154657344632770	0.5535566949152540	-0.38953435028248600	1.1258714971751400	-0.034920988700565000
1995-07-01	RF	-0.07324593220338980	-0.04144268361581920	-0.0666929661016949	0.04811943502824860	-0.034920988700565000	0.015025395480225900
1995-08-01	Mkt-RF	16.80936098870060	-1.8206217514124300	1.0583881920904000	0.5272981355932210	-0.3850162994350280	-0.025026779661016900
1995-08-01	SMB	-1.8206217514124300	4.14555988700565	1.0713265536723200	-0.7964288135593220	0.5948494350282490	-0.04146610169491520
1995-08-01	HML	1.0583881920904000	1.0713265536723200	2.007891412429380	-1.2727166101694900	0.5803139548022600	-0.06868203389830510
1995-08-01	RMW	0.5272981355932210	-0.7964288135593220	-1.2727166101694900	1.6267650847457600	-0.3704147457627120	0.05647491525423740
1995-08-01	CMA	-0.385016299435028000	0.5948494350282490	0.5803139548022600	-0.3704147457627120	1.091935903954800	-0.042856949152542400
1995-08-01	RF	-0.025026779661016900	-0.04146610169491520	-0.06868203389830510	0.05647491525423740	-0.042856949152542400	0.01372813559322040
1995-09-01	Mkt-RF	14.165775	-1.360981779661020	1.0500631355932200	0.4656838983050850	-0.07483601694915260	0.028513135593220300
1995-09-01	SMB	-1.360981779661020	4.065689576271190	1.0709199152542400	-0.7855466101694920	0.5401573728813560	-0.050679237288135600
1995-09-01	HML	1.0500631355932200	1.0709199152542400	2.080645056497180	-1.2795579096045200	0.6111645480225990	-0.07227556497175140
1995-09-01	RMW	0.4656838983050850	-0.7855466101694920	-1.2795579096045200	1.6259361581920900	-0.3659138418079100	0.05806638418079090
1995-09-01	CMA	-0.07483601694915260	0.5401573728813560	0.6111645480225990	-0.3659138418079100	1.0678037005649700	-0.05068319209039550
1995-09-01	RF	0.028513135593220300	-0.050679237288135600	-0.07227556497175140	0.05806638418079090	-0.05068319209039550	0.012838276836158200
1995-10-01	Mkt-RF	13.592449124293800	-1.115012824858760	1.1867798870056500	0.46123299435028300	0.02590875062146900	-0.006299604519773980
1995-10-01	SMB	-1.115012824858760	3.983411412429380	1.0264127683615800	-0.7680866666666670	0.5128689265536720	-0.03936790960451980
1995-10-01	HML	1.1867798870056500	1.0264127683615800	2.056652429378530	-1.2690334463276800	0.5968367231638420	-0.06617943502824860
1995-10-01	RMW	0.46123299435028300	-0.7680866666666670	-1.2690334463276800	1.636303276836160	-0.354474011299435	0.05537197740112990
1995-10-01	CMA	0.025908757062146900	0.5128689265536720	0.5968367231638420	-0.354474011299435	1.0610209039548000	-0.04704971751412430
1995-10-01	RF	-0.006299604519773980	-0.03936790960451980	-0.06617943502824860	0.05537197740112990	-0.04704971751412430	0.011289378531073400
1995-11-01	Mkt-RF	13.589672853107300	-1.1107259322033900	1.1847437570621500	0.4644841242937850	0.010614717514124300	-0.003979124293785260
1995-11-01	SMB	-1.1107259322033900	4.042255254237290	1.0111333898305100	-0.7484501694915250	0.46110898305084700	-0.033997118644067800

A.2 Mean and Covariance Matrix : 120 months estimation window

```
...   Rolling Means for Window Size 120 Months:
                        Mkt-RF       SMB       HML       RMW       CMA        RF
      Years and Months
      2000-06-01        0.612667 -0.302667  0.047167  0.468917 -0.091083  0.392417
      2000-07-01        0.562333 -0.310000  0.087333  0.465333 -0.101167  0.390750
      2000-08-01        0.639917 -0.306833  0.086667  0.458500 -0.110250  0.389417
      2000-09-01        0.699917 -0.303417  0.105000  0.469083 -0.107083  0.388667
      2000-10-01        0.623667 -0.333333  0.177917  0.483583 -0.066750  0.387667

      Rolling Covariances for Window Size {M2_window_size} Months:
                                  Mkt-RF       SMB       HML       RMW       CMA  \
      Years and Months
      2000-06-01       Mkt-RF  16.245704 -3.381837  0.731468  0.349451 -2.261729
                       SMB     -3.381837  5.842663 -0.446972  0.016687  0.339273
                       HML      0.731468 -0.446972  4.666224 -2.163216  2.154984
                       RMW      0.349451  0.016687 -2.163216  2.137027 -1.001985
                       CMA     -2.261729  0.339273  2.154984 -1.001985  3.414190

                                       RF
      Years and Months
      2000-06-01       Mkt-RF -0.028651
                       SMB    -0.026765
                       HML    -0.028937
                       RMW     0.014081
                       CMA    -0.008097
```

120_rolling_means

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2000-06-01	0.6126666666666670	-0.3026666666666670	0.047166666666666700	0.4689166666666670	-0.09108333333333330	0.3924166666666670
2000-07-01	0.5623333333333330	-0.31	0.0873333333333333	0.4653333333333330	-0.10116666666666700	0.39075
2000-08-01	0.6399166666666670	-0.3068333333333330	0.08666666666666670	0.4585	-0.11025	0.38941666666666700
2000-09-01	0.6999166666666670	-0.3034166666666670	0.105	0.46908333333333300	-0.10708333333333300	0.38866666666666700
2000-10-01	0.6236666666666670	-0.3333333333333330	0.17791666666666700	0.48358333333333300	-0.06675000000000000	0.3876666666666670
2000-11-01	0.5892500000000000	-0.3178333333333330	0.26616666666666700	0.48825	0.009083333333333320	0.38716666666666700
2000-12-01	0.6495	-0.3224166666666670	0.2916666666666670	0.47575	0.01208333333333330	0.3863333333333330
2001-01-01	0.6401666666666670	-0.2749166666666670	0.29766666666666700	0.4625	0.021750000000000000	0.38650000000000000
2001-02-01	0.5069166666666670	-0.24533333333333300	0.37066666666666700	0.4450833333333330	0.09775000000000000	0.38566666666666700
2001-03-01	0.48491666666666700	-0.26716666666666700	0.4215	0.4475	0.13191666666666700	0.38550000000000000
2001-04-01	0.5538333333333330	-0.29375	0.4001666666666670	0.4613333333333330	0.1048333333333330	0.38433333333333300
2001-05-01	0.5078333333333330	-0.24408333333333300	0.4344166666666670	0.4494166666666670	0.11650000000000000	0.3830833333333330
2001-06-01	0.5391666666666670	-0.25775000000000000	0.4919166666666670	0.43483333333333300	0.15375000000000000	0.3819166666666670
2001-07-01	0.48916666666666700	-0.24458333333333300	0.5265833333333330	0.4315833333333330	0.1865833333333330	0.38033333333333300
2001-08-01	0.46041666666666700	-0.21775000000000000	0.56175	0.4204166666666670	0.22691666666666700	0.3790833333333330
2001-09-01	0.3389166666666670	-0.27225	0.5816666666666670	0.407	0.25891666666666700	0.37758333333333300
2001-10-01	0.38775	-0.255	0.5650833333333330	0.37600000000000000	0.212	0.37591666666666700
2001-11-01	0.44075000000000000	-0.24325000000000000	0.571	0.3506666666666670	0.17925	0.3740833333333330
2001-12-01	0.4034166666666670	-0.22400000000000000	0.6001666666666670	0.3619166666666670	0.19733333333333300	0.3721666666666670
2002-01-01	0.3685833333333330	-0.19075000000000000	0.6046666666666670	0.3855	0.2150833333333330	0.37050000000000000
2002-02-01	0.3756666666666670	-0.18900000000000000	0.6176666666666670	0.3988333333333330	0.25516666666666700	0.36925
2002-03-01	0.44916666666666700	-0.19608333333333300	0.6214166666666670	0.3973333333333330	0.24858333333333300	0.3675
2002-04-01	0.4046666666666670	-0.15675000000000000	0.6616666666666670	0.38716666666666700	0.27675	0.3660833333333330
2002-05-01	0.36583333333333300	-0.14108333333333300	0.6891666666666670	0.38008333333333300	0.29158333333333300	0.36491666666666700
2002-06-01	0.3549166666666670	-0.1315	0.7164166666666670	0.40250000000000000	0.32791666666666700	0.36333333333333300
2002-07-01	0.30083333333333300	-0.10116666666666700	0.7540833333333330	0.39158333333333300	0.33541666666666700	0.36200000000000000
2002-08-01	0.30866666666666700	-0.10558333333333300	0.7759166666666670	0.3873333333333330	0.3450833333333330	0.361
2002-09-01	0.22383333333333300	-0.047916666666666700	0.77525	0.4286666666666670	0.3421666666666670	0.36000000000000000
2002-10-01	0.35958333333333300	-0.08883333333333340	0.75825	0.41283333333333300	0.3529166666666670	0.35925

120_rolling_covariances

Years and Months		Mkt-RF	SMB	HML	RMW	CMA	RF
2000-06-01	Mkt-RF	16.245703753501400	-3.3818365266106400	0.7314681232493000	0.3494508123249300	-2.2617290196078400	-0.028651036414565800
2000-06-01	SMB	-3.3818365266106400	5.842663417366950	-0.446972324929972	0.016687002801120500	0.3392727170868350	-0.02676492997198880
2000-06-01	HML	0.7314681232493000	-0.446972324929972	4.666223837535010	-2.1632157002801100	2.1549842997198900	-0.02893679271708680
2000-06-01	RMW	0.3494508123249300	0.016687002801120500	-2.1632157002801100	2.1370265476190500	-1.0019852170868300	0.014080791316526600
2000-06-01	CMA	-2.2617290196078400	0.3392727170868350	2.1549842997198900	-1.0019852170868300	3.4141895728291300	-0.008097359943977600
2000-06-01	RF	-0.028651036414565800	-0.02676492997198880	-0.02893679271708680	0.014080791316526600	-0.008097359943977600	0.009593270308123240
2000-07-01	Mkt-RF	16.159164257703100	-3.39150756302521	0.7242382072829130	0.3667807282913170	-2.300400616246500	-0.03964714285714290
2000-07-01	SMB	-3.39150756302521	5.841682352941180	-0.4503697478991600	0.01942100840336140	0.33422689075630200	-0.028269747899159600
2000-07-01	HML	0.7242382072829130	-0.4503697478991600	4.732873501400560	-2.1824764145658300	2.170561568627450	-0.022687899159663900
2000-07-01	RMW	0.3667807282913170	0.01942100840336140	-2.1824764145658300	2.139932661064430	-1.0000323809523800	0.014075798319327700
2000-07-01	CMA	-2.300400616246500	0.33422689075630200	2.170561568627450	-1.0000323809523800	3.4021683753501400	-0.01100668067226890
2000-07-01	RF	-0.03964714285714290	-0.028269747899159600	-0.022687899159663900	0.014075798319327700	-0.01100668067226890	0.00895993697478993
2000-08-01	Mkt-RF	15.091076463585400	-3.3547526750700300	0.6954207282913170	0.30648558823529400	-2.1423731302521000	-0.015610973389355700
2000-08-01	SMB	-3.3547526750700300	5.846462156862750	-0.4522364145658260	0.00988294117647067	0.3312697058823530	-0.028669565826330500
2000-08-01	HML	0.6954207282913170	-0.4522364145658260	4.7334476190476200	-2.1786117647058800	2.1736521008403400	-0.022241456582633000
2000-08-01	RMW	0.30648558823529400	0.00988294117647067	-2.1786117647058800	2.1588397478991600	-0.9958776890756300	0.014611722689075600
2000-08-01	CMA	-2.1423731302521000	0.3312697058823530	2.1736521008403400	-0.9958776890756300	3.379805819327730	-0.014387542016806700
2000-08-01	RF	-0.015610973389355700	-0.028669565826330500	-0.022241456582633000	0.014611722689075600	-0.014387542016806700	0.008449236694677880
2000-09-01	Mkt-RF	13.970548732493000	-3.2396238165266100	0.611607142857143O	0.23798647759103600	-2.0461745448179300	0.0014335014005602300
2000-09-01	SMB	-3.2396238165266100	5.863207556022410	-0.40233403361344500	0.037544740896358500	0.34619156162465000	-0.02993568627450980
2000-09-01	HML	0.6116071428571430	-0.40233403361344500	4.78684537815126	-2.1539331932773100	2.2166928571428600	-0.020265546218487400
2000-09-01	RMW	0.23798647759103600	0.037544740896358500	-2.1539331932773100	2.169537808123250	-0.9720931022408960	0.016003809523809500
2000-09-01	CMA	-2.0461745448179300	0.34619156162465000	2.2166928571428600	-0.9720931022408960	3.3930813375350100	-0.015429691876750700
2000-09-01	RF	0.0014335014005602300	-0.02993568627450980	-0.020265546218487400	0.016003809523809500	-0.015429691876750700	0.008198207282913190
2000-10-01	Mkt-RF	13.783979719888000	-2.96430700280112	0.4614110644257700	0.1593304481792720	-2.1334590756302500	-0.017615742296918800
2000-10-01	SMB	-2.96430700280112	6.107968627450980	-0.794544817927171	-0.05959131652661070	0.1275932773109240	-0.03283893557422970
2000-10-01	HML	0.4614110644257700	-0.794544817927171	5.244719152661070	-2.0162260854341700	2.473983298319330	-0.006358683473389340
2000-10-01	RMW	0.1593304481792720	-0.05959131652661070	-2.0162260854341700	2.206199656862750	-0.8951218277310930	0.018129439775910300
2000-10-01	CMA	-2.1334590756302500	0.1275932773109240	2.473983298319330	-0.8951218277310930	3.5376238025210100	-0.00779235294117645
2000-10-01	RF	-0.017615742296918800	-0.03283893557422970	-0.006358683473389340	0.018129439775910300	-0.00779235294117645	0.007730644257703040

A.3 Mean and Covariance Matrix : 240 months estimation window

```
...   Rolling Means for Window Size 240 Months:
                        Mkt-RF           SMB       HML       RMW       CMA  \
      Years and Months
      2010-06-01        0.372625  9.291667e-03  0.509458  0.389583  0.255667
      2010-07-01        0.403542  1.333333e-03  0.534917  0.376542  0.255125
      2010-08-01        0.431125 -5.551115e-18  0.524958  0.389583  0.239417
      2010-09-01        0.530083 -4.166667e-03  0.522042  0.386125  0.238458
      2010-10-01        0.523958  9.125000e-03  0.527000  0.388333  0.240167

                            RF
      Years and Months
      2010-06-01        0.298583
      2010-07-01        0.295792
      2010-08-01        0.293083
      2010-09-01        0.290625
      2010-10-01        0.287833

      Rolling Covariances for Window Size {M3_window_size} Months:
                                Mkt-RF       SMB       HML       RMW       CMA  \
      Years and Months
      2010-06-01       Mkt-RF  24.470939 -2.182876  1.067980 -1.882339 -3.691918
                       SMB     -2.182876  5.501366  0.073176  0.078094  0.126481
                       HML      1.067980  0.073176  5.749904 -1.633546  2.669247
                       RMW     -1.882339  0.078094 -1.633546  2.501930 -0.380334
                       CMA     -3.691918  0.126481  2.669247 -0.380334  4.021457
      ...
                       SMB    -0.067378
                       HML    -0.008889
                       RMW     0.013729
                       CMA    -0.009099
```

240_rolling_means

Years and Months	Mkt-RF	SMB	HML	RMW	CMA	RF
2010-06-01	0.372625	0.009291666666666660	0.5094583333333330	0.38958333333333300	0.25566666666666700	0.2985833333333330
2010-07-01	0.4035416666666670	0.0013333333333333300	0.5349166666666670	0.37654166666666700	0.255125	0.2957916666666670
2010-08-01	0.431125	-5.55111512312578E-18	0.5249583333333330	0.38958333333333300	0.23941666666666700	0.29308333333333300
2010-09-01	0.5300833333333330	-0.004166666666666670	0.5220416666666670	0.386125	0.23845833333333300	0.290625
2010-10-01	0.5239583333333330	0.009124999999999990	0.527	0.38833333333333300	0.24016666666666700	0.28783333333333300
2010-11-01	0.49225	0.024625000000000000	0.510375	0.39458333333333300	0.24204166666666700	0.2855
2010-12-01	0.5363750000000000	0.02970833333333330	0.5141666666666670	0.38333333333333300	0.24566666666666700	0.28304166666666700
2011-01-01	0.5454166666666670	0.0455	0.5370833333333330	0.368	0.2525416666666670	0.28091666666666700
2011-02-01	0.52775	0.03633333333333330	0.53725	0.36833333333333300	0.26245833333333300	0.2789583333333330
2011-03-01	0.5549583333333330	0.03708333333333330	0.5306666666666670	0.3672500000000000	0.26695833333333300	0.2771666666666670
2011-04-01	0.5925	0.03383333333333330	0.5299583333333330	0.3674166666666670	0.26433333333333300	0.2749583333333330
2011-05-01	0.5777083333333330	0.047833333333333300	0.5200833333333330	0.3702500000000000	0.25366666666666700	0.27300000000000000
2011-06-01	0.6000416666666670	0.03920833333333330	0.5232916666666670	0.3717916666666670	0.24729166666666700	0.27125
2011-07-01	0.5640833333333330	0.05354166666666670	0.507875	0.36983333333333300	0.24733333333333300	0.2692083333333330
2011-08-01	0.5160833333333330	0.06116666666666670	0.49241666666666700	0.3729583333333330	0.24179166666666700	0.26733333333333300
2011-09-01	0.455125	0.04904166666666700	0.4847916666666670	0.3711666666666670	0.25	0.26541666666666700
2011-10-01	0.5117500000000000	0.038	0.48508333333333300	0.36837500000000000	0.236625	0.26366666666666700
2011-11-01	0.5010416666666670	0.02533333333333330	0.478625	0.376625	0.23670833333333300	0.2620416666666670
2011-12-01	0.465625	0.03729166666666670	0.4829583333333330	0.37325000000000000	0.24329166666666700	0.26045833333333300
2012-01-01	0.49145833333333300	0.05158333333333330	0.48200000000000000	0.36333333333333300	0.2326666666666670	0.25904166666666700
2012-02-01	0.52	0.058291666666666700	0.47975	0.35483333333333300	0.23016666666666700	0.257875
2012-03-01	0.5356666666666670	0.05991666666666670	0.47266666666666700	0.356625	0.2349166666666670	0.25645833333333300
2012-04-01	0.5054583333333330	0.07075000000000000	0.45550000000000000	0.35829166666666700	0.22420833333333300	0.255125
2012-05-01	0.43291666666666700	0.07158333333333330	0.4389166666666670	0.36508333333333300	0.2265416666666670	0.254
2012-06-01	0.4675	0.05091666666666670	0.4555	0.36358333333333300	0.23383333333333300	0.25266666666666700
2012-07-01	0.48691666666666700	0.050375000000000000	0.45045833333333300	0.36208333333333300	0.22675	0.251375
2012-08-01	0.509	0.05604166666666670	0.465875	0.34908333333333300	0.23916666666666700	0.25033333333333300
2012-09-01	0.536	0.07804166666666670	0.4824583333333330	0.33675	0.23895833333333300	0.24929166666666700
2012-10-01	0.5750416666666670	0.08579166666666670	0.484875	0.34445833333333300	0.24241666666666700	0.24837500000000000

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Years and Months		Mkt-RF	SMB	HML	RMW	CMA	RF
2010-06-01	Mkt-RF	24.470939105648500	-2.182876375523010	1.067980088912130	-1.882339069037660	-3.6919182845188300	-0.011659445606694500
2010-06-01	SMB	-2.182876375523010	5.501366441771270	0.07317618375174330	0.0780938458856346	0.12648143654114400	-0.06737757670850770
2010-06-01	HML	1.067980088912130	0.07317618375174330	5.749903889470020	-1.6335462517433800	2.6692474337517400	-0.008888636680613660
2010-06-01	RMW	-1.882339069037660	0.0780938458856346	-1.6335462517433800	2.501930369595540	-0.38033361227336100	0.013729114365411400
2010-06-01	CMA	-3.6919182845188300	0.12648143654114400	2.6692474337517400	-0.38033361227336100	4.021457294281730	-0.00909947001394699
2010-06-01	RF	-0.011659445606694500	-0.06737757670850770	-0.008888636680613660	0.013729114365411400	-0.00909947001394699	0.025833131101813100
2010-07-01	Mkt-RF	24.954134266039100	-2.268671269177130	1.298367451185500	-2.0360395833333300	-3.6719475156903800	-0.03199047245467220
2010-07-01	SMB	-2.268671269177130	5.513518716875870	0.0456448814504881	0.1013816178521620	0.12066468619246900	-0.06562825662482570
2010-07-01	HML	1.298367451185500	0.0456448814504881	5.801688695955370	-1.6894540550906600	2.6886269979079500	-0.01050558228730820
2010-07-01	RMW	-2.0360395833333300	0.1013816178521620	-1.6894540550906600	2.545621044281730	-0.3896361767782430	0.017779109135285900
2010-07-01	CMA	-3.6719475156903800	0.12066468619246900	2.6886269979079500	-0.3896361767782430	4.020609189330540	-0.011310978033472800
2010-07-01	RF	-0.03199047245467220	-0.06562825662482570	-0.01050558228730820	0.017779109135285900	-0.011310978033472800	0.0255650610181311
2010-08-01	Mkt-RF	24.51165605125520	-2.255502510460250	1.3221525575313800	-2.121754759414230	-3.5590361610878700	-0.009144069037656920
2010-08-01	SMB	-2.255502510460250	5.513279497907950	0.04746276150627610	0.10163849372384900	0.11988786610878700	-0.0659255230125523
2010-08-01	HML	1.3221525575313800	0.04746276150627610	5.842187445955370	-1.7227924860530000	2.7252911889818700	-0.005404055090655540
2010-08-01	RMW	-2.121754759414230	0.10163849372384900	-1.7227924860530000	2.563482670850770	-0.40657806834030700	0.016455683403068300
2010-08-01	CMA	-3.5590361610878700	0.11988786610878700	2.7252911889818700	-0.40657806834030700	4.035508444909340	-0.010666394700139500
2010-08-01	RF	-0.009144069037656920	-0.0659255230125523	-0.005404055090655540	0.016455683403068300	-0.010666394700139500	0.025344428172942800
2010-09-01	Mkt-RF	24.353533884239900	-2.110857810320780	1.281404850069740	-2.165980428870290	-3.416559285216180	-0.005873692468619270
2010-09-01	SMB	-2.110857810320780	5.501379218967920	0.0451114714086471	0.09867834728033480	0.11255254532775400	-0.06949110878661090
2010-09-01	HML	1.281404850069740	0.0451114714086471	5.844726776499300	-1.7199020972803300	2.7215316126220400	-0.0043724110878661200
2010-09-01	RMW	-2.165980428870290	0.09867834728033480	-1.7199020972803300	2.5667669299163200	-0.41107545502092000	0.017577327405857700
2010-09-01	CMA	-3.416559285216180	0.11255254532775400	2.7215316126220400	-0.41107545502092000	4.032763722106000	-0.014199450836820100
2010-09-01	RF	-0.005873692468619270	-0.06949110878661090	-0.0043724110878661200	0.017577327405857700	-0.014199450836820100	0.025279524058577400
2010-10-01	Mkt-RF	24.289715228382100	-2.0355387813807500	1.313691422594140	-2.159013458856350	-3.4029584030683400	-0.020760425383542500
2010-10-01	SMB	-2.0355387813807500	5.475018896443510	0.026186485355648500	0.10633577405857700	0.10065705020920500	-0.06597847280334730
2010-10-01	HML	1.313691422594140	0.026186485355648500	5.834275481171550	-1.7185560669456100	2.7159256066945600	-0.001152552301255230
2010-10-01	RMW	-2.159013458856350	0.10633577405857700	-1.7185560669456100	2.570039051603910	-0.4115047419804740	0.015632775453277500
2010-10-01	CMA	-3.4029584030683400	0.10065705020920500	2.7159256066945600	-0.4115047419804740	4.030141394700140	-0.011961143654114400
2010-10-01	RF	-0.020760425383542500	-0.06597847280334730	-0.001152552301255230	0.015632775453277500	-0.011961143654114400	0.024966834030683400
2010-11-01	Mkt-RF	24.591128807531400	-2.0831987343096200	1.4545974790795000	-2.2080957112970700	-3.40084980125523	-0.009776443514644330
2010-11-01	SMB	-2.0831987343096200	5.449589816945610	0.00957064330543929	0.11068080543933100	0.09373194037656900	-0.06401257322175730

A.4 Mean Variance Portfolio Optimisation Results

Optimal Weights for Window Size 60 Months:
Mkt-RF: 0.3786
SMB: 0.0000
HML: 0.0000
RMW: 0.6214
CMA: 0.0000

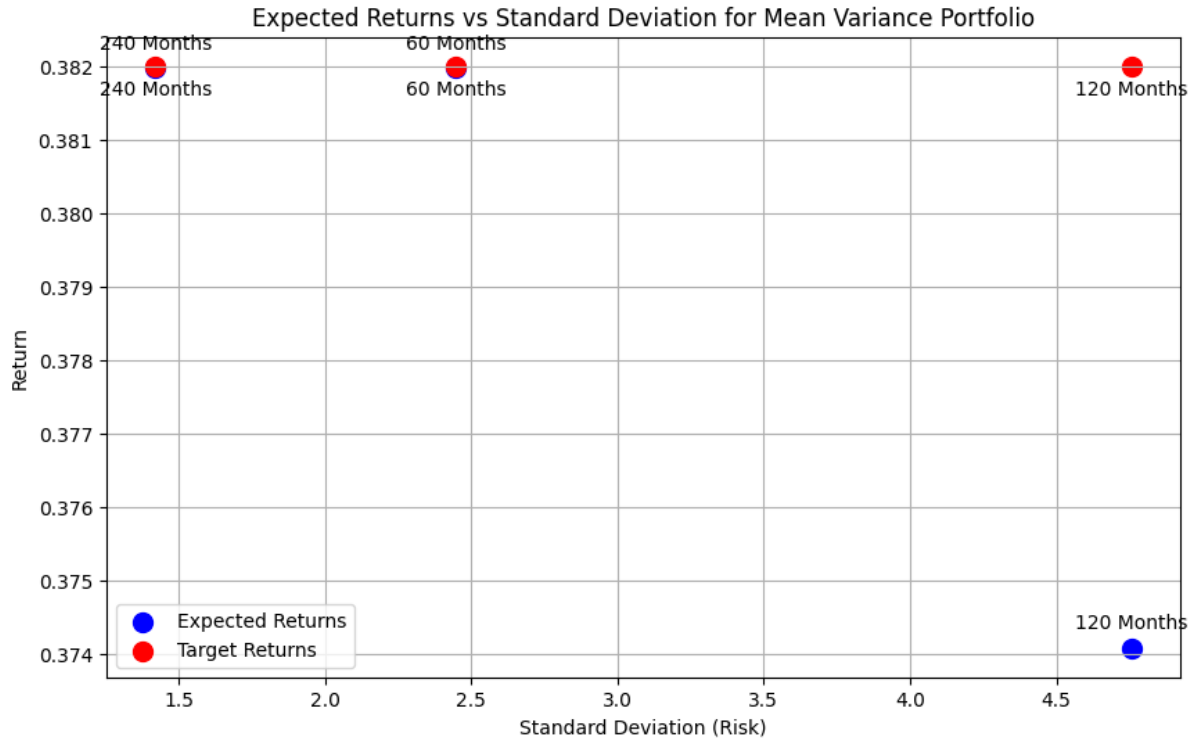
Target Expected Return: 0.3820
Expected Return of Optimized Portfolio: 0.38196790054429 57
Variance of Optimized Portfolio: 5.9966
Standard Deviation of Optimized Portfolio: 2.4488
Portfolio Returns: [0.11516119705391747, 0.1934696263944934, -0.4209480614726232, -0.2675902142151783, 0.30095775236131306]

Optimal Weights for Window Size 120 Months:
Mkt-RF: 1.0000
SMB: 0.0000
HML: 0.0000
RMW: 0.0000
CMA: 0.0000

Target Expected Return: 0.3820
Expected Return of Optimized Portfolio: 0.37408333333328675
Variance of Optimized Portfolio: 22.6235
Standard Deviation of Optimized Portfolio: 4.7564
Portfolio Returns: [0.5439977569131346, -1.1132041545595963, 1.3396638582193794, 3.655003916930271, 3.6296930944609347]

Optimal Weights for Window Size 240 Months:
Mkt-RF: 0.2173
SMB: 0.0000
HML: 0.0000
RMW: 0.7827
CMA: 0.0000

Target Expected Return: 0.3820
Expected Return of Optimized Portfolio: 0.3819679012073063
Variance of Optimized Portfolio: 2.0169
Standard Deviation of Optimized Portfolio: 1.4202
Portfolio Returns: [0.5486951379242772, 0.15724363611205983, 0.8718274693184429, 1.1744575618355988, -0.47203851379199757]



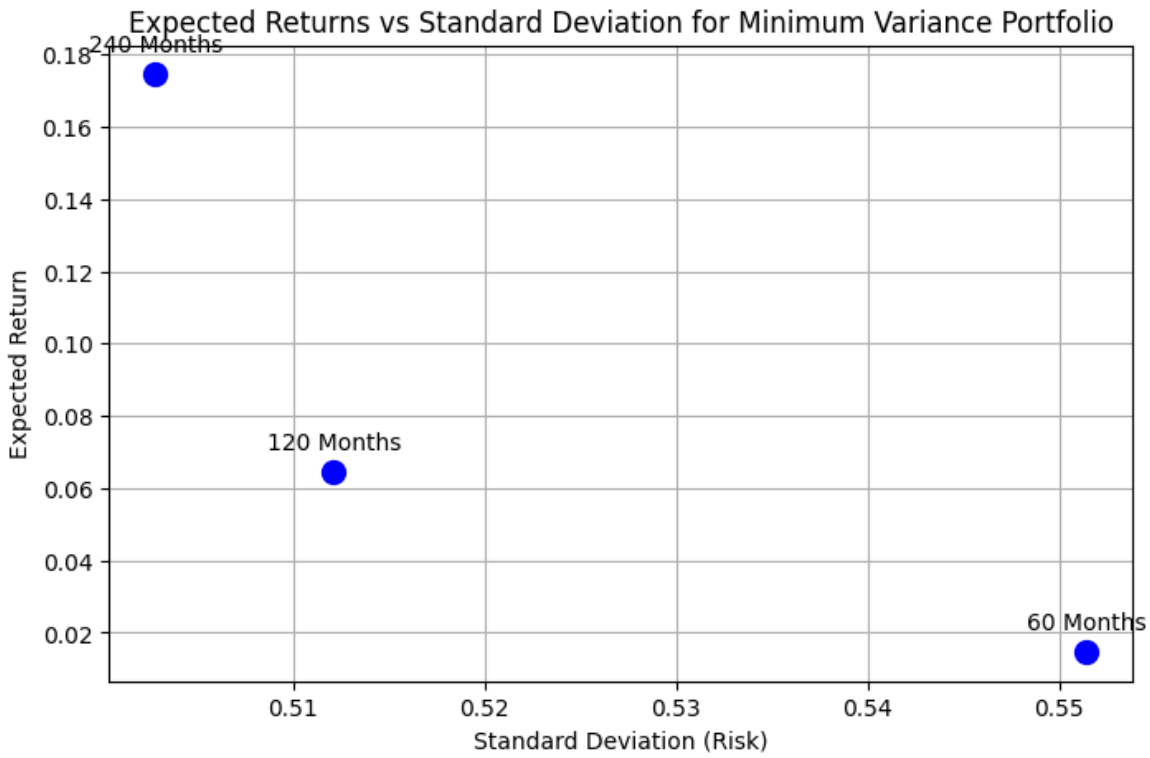
A.4 Minimum Variance Portfolio Optimisation

Optimal Weights for Window Size 60 Months:
Mkt-RF: 0.0076
SMB: 0.2042
HML: 0.0394
RMW: 0.4171

CMA: 0.3317
Expected Return of Optimized Portfolio: 0.0146
Variance of Optimized Portfolio: 0.3041
Standard Deviation of Optimized Portfolio: 0.5514
Portfolio Returns: [0.07082996262993377, 0.19861347193821968, -0.41875818211948107, -0.2849634801617105, 0.23136049798905955]

Optimal Weights for Window Size 120 Months:
Mkt-RF: 0.0225
SMB: 0.1952
HML: 0.0624
RMW: 0.3896
CMA: 0.3303
Expected Return of Optimized Portfolio: 0.0644
Variance of Optimized Portfolio: 0.2622
Standard Deviation of Optimized Portfolio: 0.5121
Portfolio Returns: [0.6729247219894053, -0.6862641665642788, 1.7276838707094888, 2.8405920640791344, 3.4477176671783356]

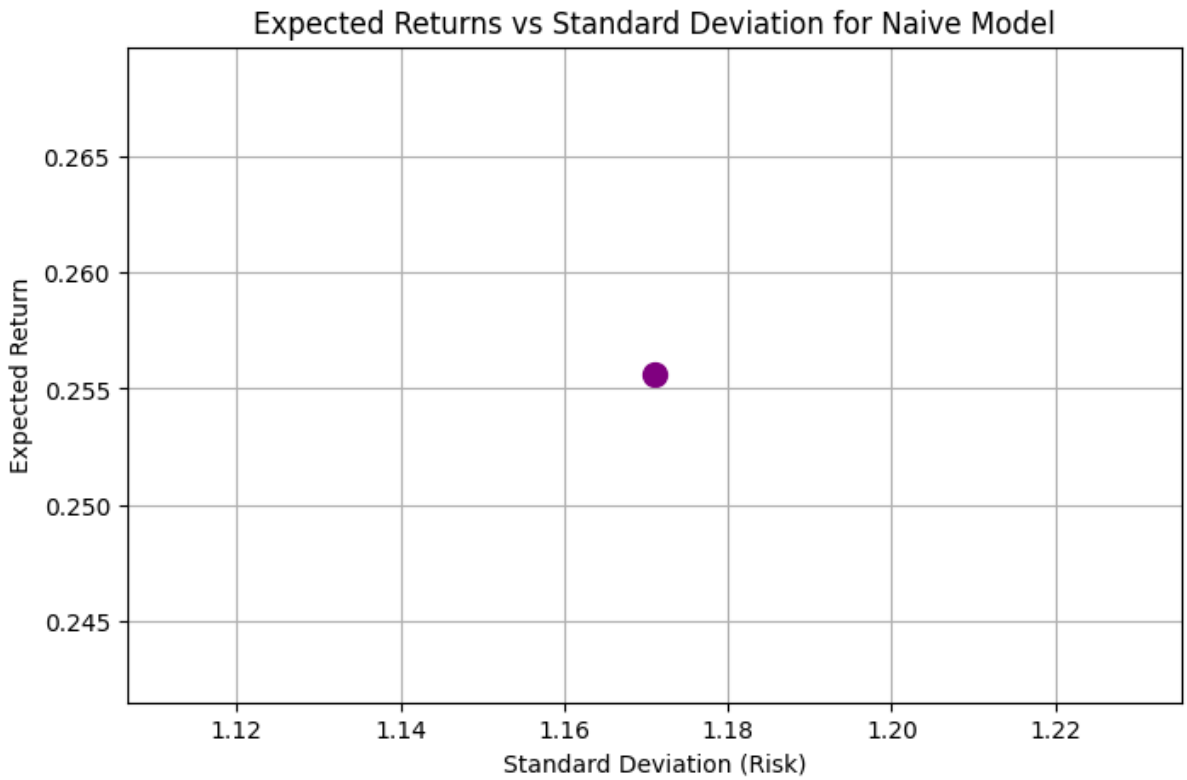
Optimal Weights for Window Size 240 Months:
Mkt-RF: 0.0289
SMB: 0.1451
HML: 0.1520
RMW: 0.4412
CMA: 0.2328
Expected Return of Optimized Portfolio: 0.1745
Variance of Optimized Portfolio: 0.2528
Standard Deviation of Optimized Portfolio: 0.5028
Portfolio Returns: [0.4313745357785271, -0.044268753999390016, 1.3596116773290003, 1.1329421305094953, -0.26892765723594864]



A.5 Naive Portfolio Optimisation

Equal Weights Portfolio:
Mkt-RF: 0.2000
SMB: 0.2000
HML: 0.2000
RMW: 0.2000
CMA: 0.2000

Expected Return of Equal Weighted Portfolio: 0.2556
Variance of Equal Weighted Portfolio: 1.3713
Standard Deviation of Equal Weighted Portfolio: 1.1710


Expected Returns vs Standard Deviation for Naive Model
Expected Return
0.265
0.260
0.255
0.250
0.245
1.12
1.14
1.16
1.18
1.20
1.22
Standard Deviation (Risk)