Unit - IV

LOGIC CIRCUITS

[Book: Digital Logic & Computer Design, by Morris Mano]

It is easy if you focus & understand,

$1 + 1 = 0$, (Binary addition)

$= 1$, (Logical - OR)

$= 2$, (Decimal addition)

Number systems :

→ To quantify something, the numbers are used.

- Decimal number system is the widely used in our Day to Day life.
- Binary number system is used by Computers.

Widely used number systems,

Binary: $\{0, 1\}_2$ ← Base (Indicates the no of digits available in the number system)

Octal: $\{0, 1, 2, 3, 4, 5, 6, 7\}_8$

Decimal: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}_{10}$

Hexa decimal: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}_{16}$.

Q Identify the invalid number system from the following?

(a) $(5,8,9)_{10}$ (b) $(8,1,5)_9$ (c) ✓ $(3,7,0)_7$ (d) $(2,0,7)_8$

Conversion from "Any system" to "Decimal" :—

Q $(10101)_2 \longrightarrow (\ ?\ )_{10}$

LSB → Least significant Bit

MSB → most significant Bit.

Ans:

$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 1 \\ \times & \times & \times & \times & \times \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

MSB (first digit), LSB (last digit)

$$= 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 = \underline{\underline{21}}.$$

Q $(536)_8 \longrightarrow (\ ?\ )_{10}$

Ans:

$$= 5\times 8^2 + 3\times 8^1 + 6\times 8^0$$

$$= \underline{350}$$

Q $(90B)_{16} \longrightarrow (\ ?\ )_{10}$

Ans:

$$= 9\times 16^2 + 0\times 16^1 + 11\times 16^0$$

$$= \underline{2315}$$

Q $(1010.101)_2 \longrightarrow (\quad)_{10}$

Ans:

$$\underset{2^3}{1}\ \underset{2^2}{0}\ \underset{2^1}{1}\ \underset{2^0}{0}\ .\ \underset{2^{-1}}{1}\ \underset{2^{-2}}{0}\ \underset{2^{-3}}{1}$$

$$= (1\times 8 + 0\times 4 + 1\times 2 + 0\times 1) . (1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3})$$

$$= 10 . \left(\frac{1}{2} + 0 + \frac{1}{8}\right)$$

$$= \underline{10.625}$$

(Valid for any fractional system to Decimal)

Conversion from "Decimal" to "Any System" :—

Q $(46)_{10} \rightarrow (?)_2$

Ans:

$$
\begin{array}{r|ll}
2 & 46 & \\
\hline
2 & 23 & - \ 0 \\
\hline
2 & 11 & - \ 1 \\
\hline
2 & 5 & - \ 1 \\
\hline
2 & 2 & - \ 1 \\
\hline
 & 1 & - \ 0
\end{array}
$$

$= (101110)_2$

Q $(615)_{10} \longrightarrow (\quad)_8$

Ans:

$$\begin{array}{r|l} 8 & 615 \\ \hline 8 & \\ \hline 8 & \\ \hline \end{array}$$

$$= (1147)_8$$

Q $(13.625)_{10} \rightarrow (\quad)_2$

Ans:

$$
\begin{array}{r|l}
2 & 13 \\ \hline
2 & 6 - 1 \\ \hline
2 & 3 - 0 \\ \hline
  & 1 - 1
\end{array}
$$

$= (1101.101)_2$

$0.625 \times 2 = 1.250 \quad 1$

$0.250 \times 2 = 0.5 \quad 0$

$0.5 \times 2 = 1.0 \quad 1$

0.0

(Valid for fractional Decimal to any system)

Octal ↔ Binary :-

Q $(712)_8 \rightarrow (?)_2$

Ans: $(111 \quad 001 \quad 010)_2$

Q $(576)_8 \rightarrow (101 \quad 111 \quad 110)_2$

Q $(1110101001)_2 \rightarrow (?)_8$

Ans: $001 \; 110 \; 101 \; 001 = 1651$

octal	Binary		
	2^2	2^1	2^0 ← weights
0 →	0	0	0
1 →	0	0	1
2 →	0	1	0
3 →	0	1	1
4 →	1	0	0
5 →	1	0	1
6 →	1	1	0
7 →	1	1	1

Hexa-Decimal $\longleftrightarrow$ Binary :—

Q $(9B12)_{16} \rightarrow (1001\ 1011\ 0001\ 0010)_2$

Q $(1011011100 10)_2 \rightarrow (\quad)_{16}$

$$\underbrace{0101\,1}_{5}\ \underbrace{1011}_{B}\ \underbrace{0010}_{2}$$

$0\,1011\ 011\ 0010$

5 B 2

$=(5B2)_{16}$

Hexa Decimal	Binary 2^3	2^2	2^1	2^0
0 $\rightarrow$	0	0	0	0
1 $\rightarrow$	0	0	0	1
2 $\rightarrow$	0	0	1	0
3 $\rightarrow$	0	0	1	1
4 $\rightarrow$	0	1	0	0
5 $\rightarrow$	0	1	0	1
6 $\rightarrow$	0	1	1	0
7 $\rightarrow$	0	1	1	1
8 $\rightarrow$	1	0	0	0
9 $\rightarrow$	1	0	0	1
A $\rightarrow$	1	0	1	0
B $\rightarrow$	1	0	1	1
C $\rightarrow$	1	1	0	0
D $\rightarrow$	1	1	0	1
E $\rightarrow$	1	1	1	0
F $\rightarrow$	1	1	1	1

$$(\quad)_8 \longrightarrow (\quad)_{16}$$

Decimal
(or) Binary

Design your own Number system :—

Let us design a penta (5) number system

$(0, 1, 2, 3, 4)_5$ → Base.

Decimal | penta (5)

Decimal		5^2	5^1	5^0
0	→			0
1	→			1
2	→			2
3	→			3
4	→			4
5	→		1	0

		5^2	5^1	5^0
6	→		1	1
7	→		1	2
8	→		1	3
9	→		1	4
10	→		2	0
11	→		2	1

		5^2	5^1	5^0
12	→		2	2
13	→		2	3
14	→		2	4
15	→		3	0
⋮				⋮

Signed (Negative) Number Representation :—

3 methods are available to represent negative Binary numbers,

① Signed - Magnitude form ⟶ An extra bit is used at MSB to represent the sign.

at MSB	
0 ⟶	+ve number
1 ⟶	−ve number.

② 1's Complement form ⟶ The Negative numbers are represented with 1's Complement of their postive number.

+43 ⟶ (10 - - - 1)

↓   ↓ 1's Complent

−43 ⟶ (   )

③ 2's Complement form $\rightarrow$ The negative numbers are represented using 2's Complement of the positive number.

$$+43 \rightarrow (10\cdots1)$$

2's Complement

$$-43 \rightarrow (\quad\quad)$$

<u>Note</u>:

<u>Binary</u>

1's Complement $\Longrightarrow$ Toggle (Change) 0 with 1 & 1 with 0.

2's Complement $\Longrightarrow$ 1's Complement + 1

(or) From LSB wait till 1, thereafter toggle the bits.

$101011 \xrightarrow{\text{1's comp.}} 010100$

101011

010100

$101011 \xrightarrow{\text{2's comp.}} 010101$ ✓

101011

010101

Check: 1's comp + 1

$$\begin{array}{r} 010100 \\ +\quad 1 \\ \hline 010101 \end{array}$$ ✓

$111001100 \xrightarrow{\text{1's comp.}}$

000110011 1's Complement.

$111001100 \xrightarrow{\text{2's comp.}}$

000110100 ✓ 2's Complement.

Check: 1's complement + 1 = 2's.

$$\begin{array}{r} 000110011 \\ +\quad 1 \\ \hline 000110100 \end{array}$$ ✓

	Binary		
Octal	✗ Signed-Magnitude form	✗ 1's complement form	✓ 2's complement form
0	* 0000 (+0)	0000 (+0)	0000 (+0)
1	0001	0001	0001 (+1)
2	0010	0010	0010 (+2)
3	0011	0011	0011 (+3)
4	0100	0100	0100 (+4)
5	0101	0101	0101 (+5)
6	0110	0110	0110 (+6)
7	0111	0111	0111 (+7)
−0	* 1000 (−0)	1111 (−0)	0000 (−0)
−1	1001	1110	1111 (−1)
−2	1010	1101	1110 (−2)
−3	1011	1100	1101 (−3)
−4	1100	1011	1100 (−4)
−5	1101	1010	1011 (−5)
−6	1110	1001	1010 (−6)
−7	1111	1000	1001 (−7)

−ve

$$+0 = -0 = 0$$

Signed magnitude form $\Rightarrow$ 0 has 2 representations.

1's complement form $\Rightarrow$ " .

2's complement form $\Rightarrow$ 0 has only one representation.

Hence, 2's complement form is preferred to represent -ve numbers.

$(\qquad)_{10} \longrightarrow$ 2's Complement form.

Binary

$(\qquad)_{16} \longrightarrow$ 2's Complement form.

Binary

→ Procedure to get 2's complement from other number systems :—

Step-1: Convert the given number into Binary.

Step-2: (i) If given number is +ve, then place 0 at MSB of the binary got in step-1. This is your 2's complement representation of the +ve number.

(ii) If given number is -ve, then place 0 at MSB of the binary got in step-1. Then convert this binary into 2's complement. This will give the representation of -ve number.

Q $(+43)_{10} \longrightarrow$ 2's Complement

Q $(-64)_{10} \longrightarrow$ 2's Complement

C1 Answers Discussion

4 $(80)_K = (224)_{K-3}$

Convert both of them to any one number system, then you should get the same value. Let us convert these into Decimal:

$$8 \times K^1 + 0 \times K^0 = 2 \times (K-3)^2 + 2 \times (K-3)^1 + 4 \times (K-3)^0$$

$$\Rightarrow 8K = 2K^2 - 12K + 18 + 2K - 6 + 4$$

$$\Rightarrow 2K^2 - 18K + 16 = 0$$

$$\Rightarrow K^2 - 9K + 8 = 0$$

$$\Rightarrow K^2 - 8K - K + 8 = 0 \Rightarrow K(K-1) - 8(K-1) = 0$$

$K = 1$ ✗ & $K = 8$ ✗

Given, $(80)_K = (224)_{K+3}$

$\Downarrow$

For no value of K the given expression is valid.

$K = 1 \Rightarrow \{0\}$

$K = 8 \Rightarrow \{0, 1, 2, 3, 4, 5, 6, 7\}$

3 Given, Si diode $\rightarrow$ $\eta = 2$

$T = 300\,K$

$I = 15\ mA$

$l_n = l_p = 20\ \AA$

Diffusion capacitance $(C_D) = \dfrac{\tau I}{\eta V_T}$

$I = 15\,mA$
$\eta = 2$
$V_T = 26\,mV$
$\tau = ?$

Life time, $\tau = \tau_n + \tau_h$

$= \dfrac{l_p^2}{D_p} + \dfrac{l_n^2}{D_n}$

$\dfrac{D_p}{\mu_p} = \dfrac{D_n}{\mu_n} = 26\,mV$

$C = \frac{Q}{V}$

$= \frac{It}{V}$

$F = \left(\frac{A \times S}{V}\right)$

$$\tau = \frac{l_p^2}{500\ \frac{cm^2}{\cancel{VS}} \times 26\,m\cancel{V}} + \frac{l_n^2}{1300\ \frac{cm^2}{\cancel{VS}} \times 26\,m\cancel{V}}$$

$$= \frac{4 \times 10^{-14}\ \cancel{cm^2}}{500\ \frac{\cancel{cm^2}}{s} \times 26\,m} + \frac{4 \times 10^{-14}\ \cancel{cm^2}}{1300\ \frac{\cancel{cm^2}}{s} \times 26\,m}$$

$$= 4.25 \times 10^{-15}\ s \text{ (seconds)}$$

$$= 4.25\ fs.$$

μ_p = mobility of hole in si

$= 500\ cm^2/V\text{-}s.$

μ_n = mobility of $\bar{e}$ in si

$= 1300\ cm^2/V\text{-}S.$

$20\ \text{Å} = 2\,nm$

$= 2 \times 10^{-9}\ m$

$= 2 \times 10^{-9} \times 100\ cm$

$= 2 \times 10^{-7}\ cm$

$$C_D = \frac{\tau I}{\eta V_T} = \frac{4.25\ fs \times 15\,\cancel{m}A}{2 \times 26\ \cancel{m}V}$$

$$\simeq 1.22\ f\ \frac{S \times A}{V} \rightarrow \text{Farads}$$

2

Various methods to distinguish extrinsic S.C.

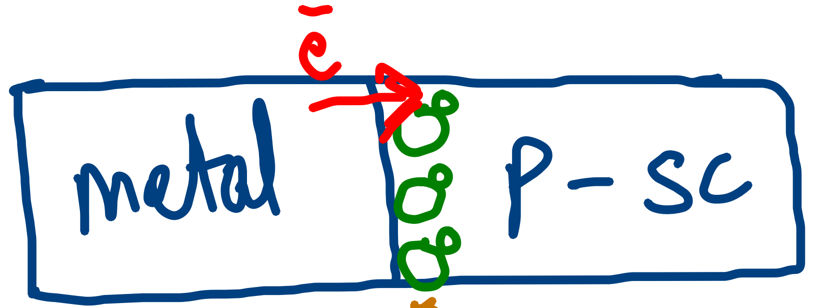
- Hall effect ✓
- Hot probe

etc.

1

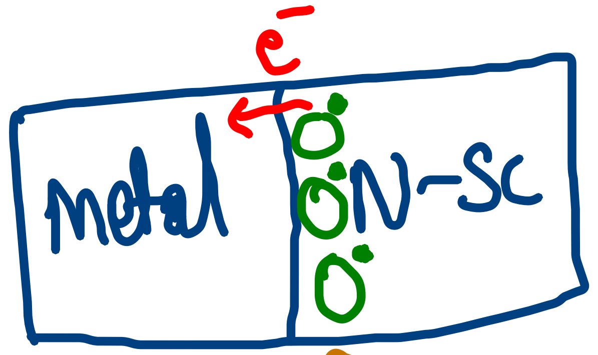
Rectifying Junction

Schottky Junction. (diode)



Dopant atoms should gain $\bar{e}$s to form depletion Region

$\phi_m < \phi_{P\text{-}SC}$

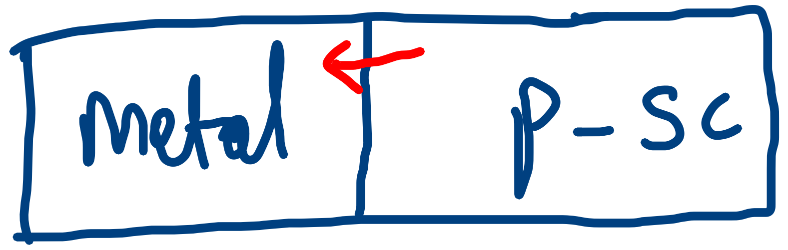


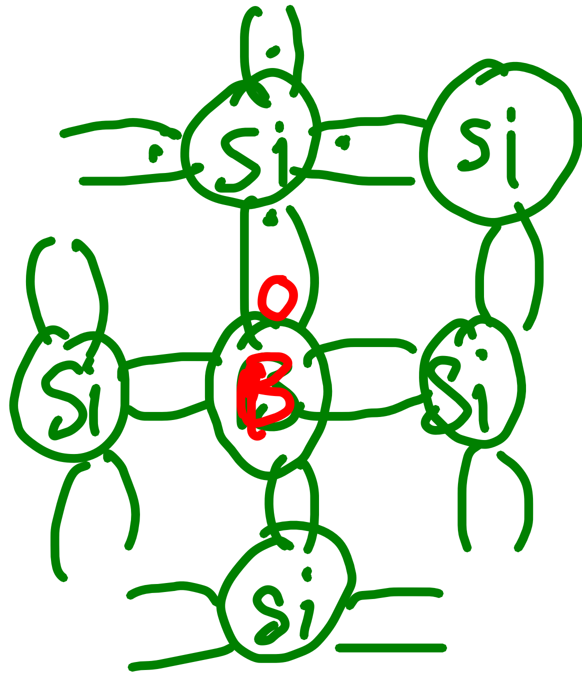
Dopant atoms should lose $\bar{e}$s to form depletion Region

$\phi_m > \phi_{N\text{-}SC}$

Non-Rectifying Junction

Ohmic Contact.

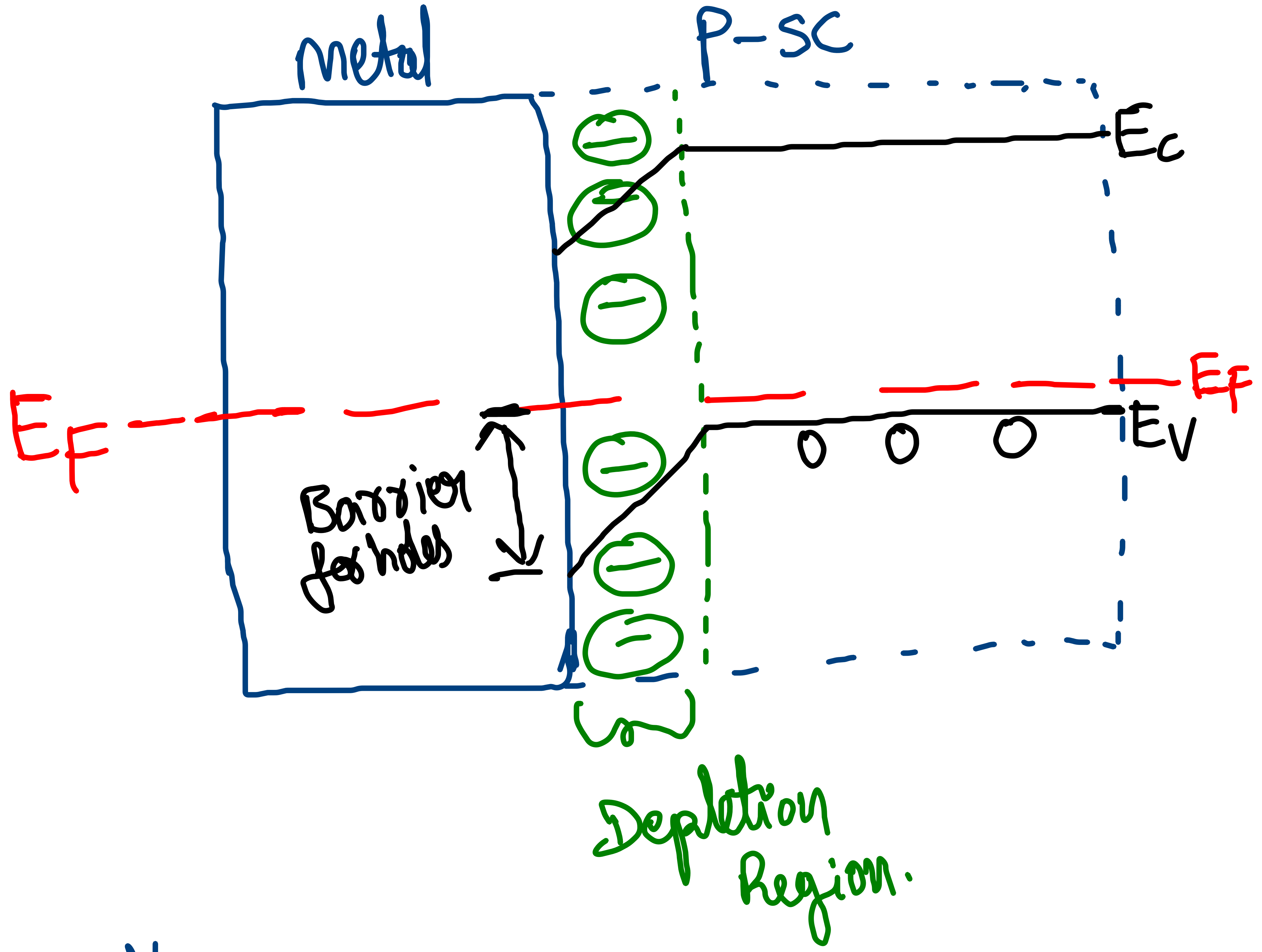




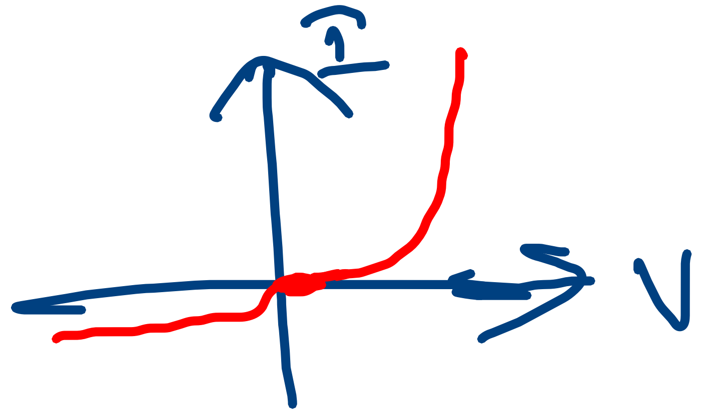
$\phi_m > \phi_{P\text{-}SC}$

(No depletion Region)

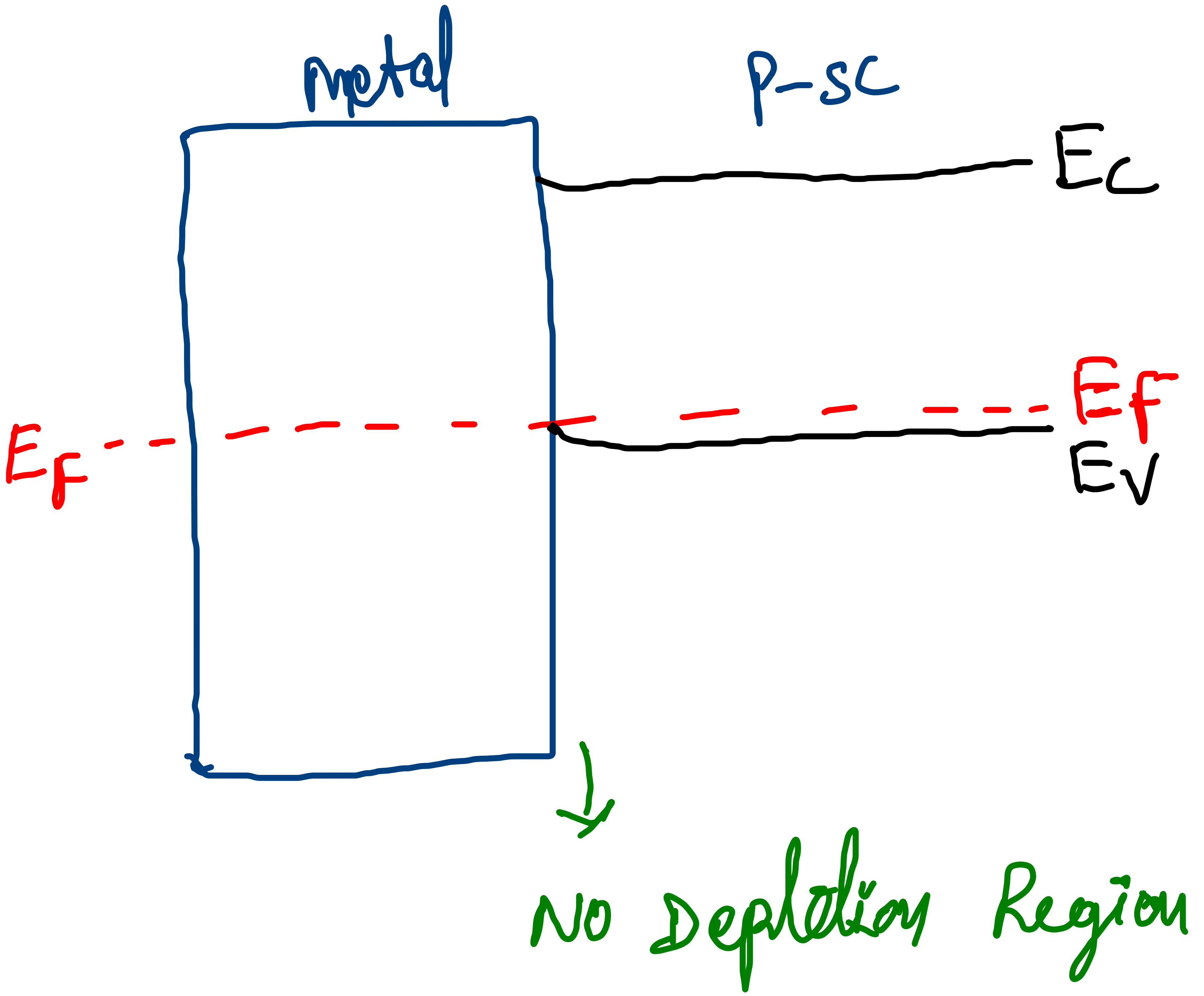
Energy Band Diagram (schottky)



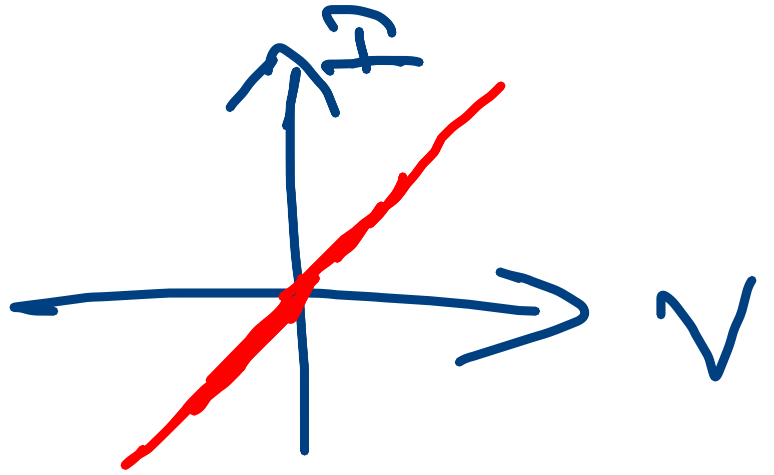
IV



Energy Band Diagram (ohmic)



IV



Sum of products (SOP) & product of sum (POS) :-

[Refer to Lab notes for basic information before this.]

SOP expressions ⟶ use NAND gates for implementation.

POS " ⟶ use NOR " " .

In case, if SOP is given but asked to implement using NOR gates, then convert SOP into POS.

Vice-Versa.

SOP to POS & POS to SOP Conversion:

Q) **Example:** $F = AB + \bar{A}B$ (SOP)

To Convert this to POS?

⇓

procedure:

understand the min-terms in the given boolean expression.

$$F = \underset{\substack{1\,1 \\ m_3}}{AB} + \underset{\substack{0\,1 \\ m_1}}{\bar{A}B}$$

⇒ All other terms will be there in POS as max terms.

Now, Assess all the possible combinations using A&B.

			A	B			
✓ (M_0)	$(A+B)$	$\Leftarrow$	0	0	$\Rightarrow$	$\bar{A}\bar{B}$	(m_0)
(M_1)	$(A+\bar{B})$	$\Leftarrow$	0	1	$\Rightarrow$	$\bar{A}B$	(m_1) ✓
✓ (M_2)	$(\bar{A}+B)$	$\Leftarrow$	1	0	$\Rightarrow$	$A\bar{B}$	(m_2)
(M_3)	$(\bar{A}+\bar{B})$	$\Leftarrow$	1	1	$\Rightarrow$	AB	(m_3) ✓

Max Terms (used in POS expressions.)

Min-terms (used in SOPs)

$(m_1 \,\&\, m_3)$ are in the given sop.

$\Downarrow$

Then M_0 & M_2 will be there in its POS.

so, if $sop = AB + \bar{A}B = m_3 + m_1 = \Sigma m(1,3)$ — 1

then its $pos = M_0 \cdot M_2 = \Pi M(0,2)$

$= (A+B)\cdot(\bar{A}+B)$ — 2

check if 1 = 2,

Let us expand 2,

$(A+B)\cdot(\bar{A}+B) = \cancel{A\bar{A}} + AB + \bar{A}B + B\cdot B$

$\xrightarrow{} 0$

$= AB + \bar{A}B + B$

$= \bar{A}B + B(\cancel{A+1})$ $\rightarrow 1$

$= \bar{A}B + B$

$$= B(\overline{A} + \cancel{1})$$

$$= \underline{\underline{B}}$$

Simplify 1,

$$AB + \overline{A}B = B(\cancel{A + \overline{A}})$$

$$= \underline{\underline{B}}$$

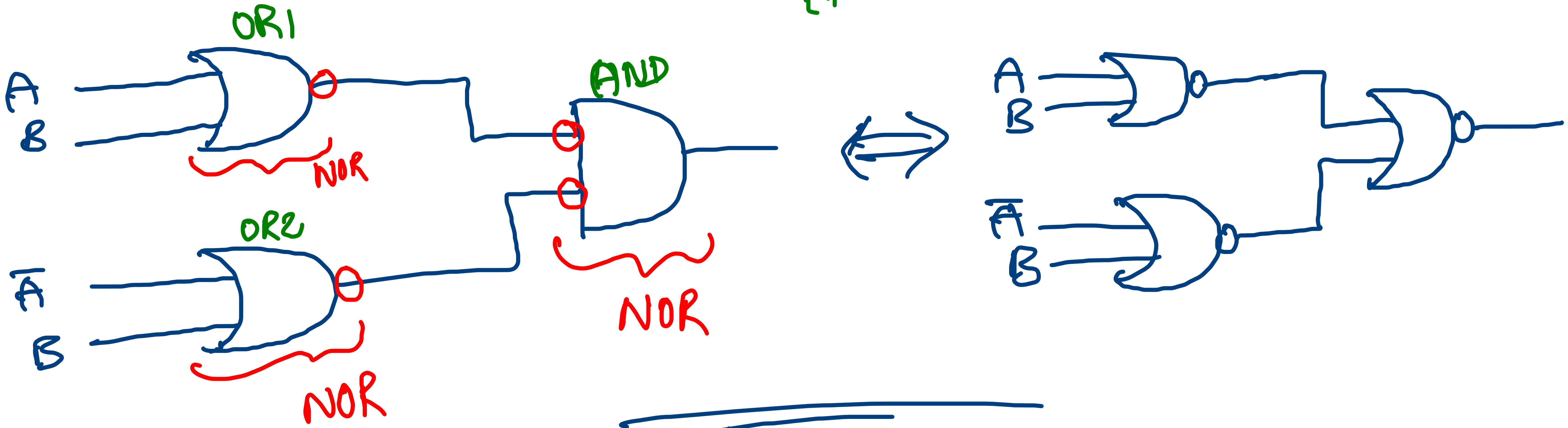
1 = 2, so SOP $\boxed{\underbrace{AB + \overline{A}B}}$ = POS $\boxed{\underbrace{(A+B)\cdot(\overline{A}+B)}}$

Now, represent $AB + \bar{A}B$ using NOR gates ?

Needs POS

$$\underbrace{(A+B)}_{OR1} \cdot \underbrace{(\bar{A}+B)}_{OR2}$$

AND



Example:

$$F = A\overline{B} + \overline{A}B \quad (SOP)$$

$\Updownarrow$

$$F = (A+B)\cdot(\overline{A}+\overline{B}) \quad POS$$

Example:

$$F = A\overline{B} + \overline{A}\,\overline{B} \quad (SOP)$$

$\Updownarrow$

$$F = (\overline{A}+\overline{B})\cdot(A+\overline{B}) \quad POS$$

3-Variables (A, B, C) case :—

				A	B	C			
M_0	$(A+B+C)$	$\Leftarrow$	(0)	0	0	0	$\Rightarrow$	$\bar{A}\bar{B}\bar{C}$	(m_0)
M_1	$(A+B+\bar{C})$	$\Leftarrow$	(1)	0	0	1	$\Rightarrow$	$\bar{A}\bar{B}C$	(m_1)
M_2	$(A+\bar{B}+C)$	$\Leftarrow$	(2)	0	1	0	$\Rightarrow$	$\bar{A}B\bar{C}$	(m_2)
M_3	$(A+\bar{B}+\bar{C})$	$\Leftarrow$	(3)	0	1	1	$\Rightarrow$	$\bar{A}BC$	(m_3)
M_4	$(\bar{A}+B+C)$	$\Leftarrow$	(4)	1	0	0	$\Rightarrow$	$A\bar{B}\bar{C}$	(m_4)
M_5	$(\bar{A}+B+\bar{C})$	$\Leftarrow$	(5)	1	0	1	$\Rightarrow$	$A\bar{B}C$	(m_5)
M_6	$(\bar{A}+\bar{B}+C)$	$\Leftarrow$	(6)	1	1	0	$\Rightarrow$	$AB\bar{C}$	(m_6)
M_7	$(\bar{A}+\bar{B}+\bar{C})$	$\Leftarrow$	(6)	1	1	1	$\Rightarrow$	ABC	(m_7)

(used for POS) $\leftarrow$ Max terms

Min-Terms (used for SOP expressions.)

Example:

$$F(A,B,C) = AB\overline{C} + A\overline{B}C + ABC \quad (SOP)$$

$$\begin{array}{ccc} 110 & 101 & 111 \\ m_6 & m_5 & m_7 \end{array}$$

$$\Sigma m(5,6,7)$$

$$\Pi M(0,1,2,3,4) \quad POS$$

$$= (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+C)$$

Example: $F(A,B,C) = AB\overline{C} + \overline{B}$ (SOP)

To get min-terms convert this to standard SOP.

***

$= AB\overline{C} + (A+\overline{A})\cdot\overline{B}\cdot(C+\overline{C})$

$= AB\overline{C} + A\overline{B}C + \overline{A}\overline{B}C + A\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}$

$AB\overline{C}$	$A\overline{B}C$	$\overline{A}\overline{B}C$	$A\overline{B}\overline{C}$	$\overline{A}\overline{B}\overline{C}$
110	101	001	100	000
m_6	m_5	m_1	m_4	m_0

$= \Sigma m(0,1,4,5,6)$

so, its POS will be $= \Pi M(2,3,7)$

$= (A+\overline{B}+C)\cdot(A+\overline{B}+\overline{C})\cdot(\overline{A}+\overline{B}+\overline{C})$

Simplification of Boolean Expressions :

why simplify Boolean expressions ?

$\Downarrow$

No simplification will force us to use more logic gates.

$\downarrow$

More transistors in IC.

$\downarrow$

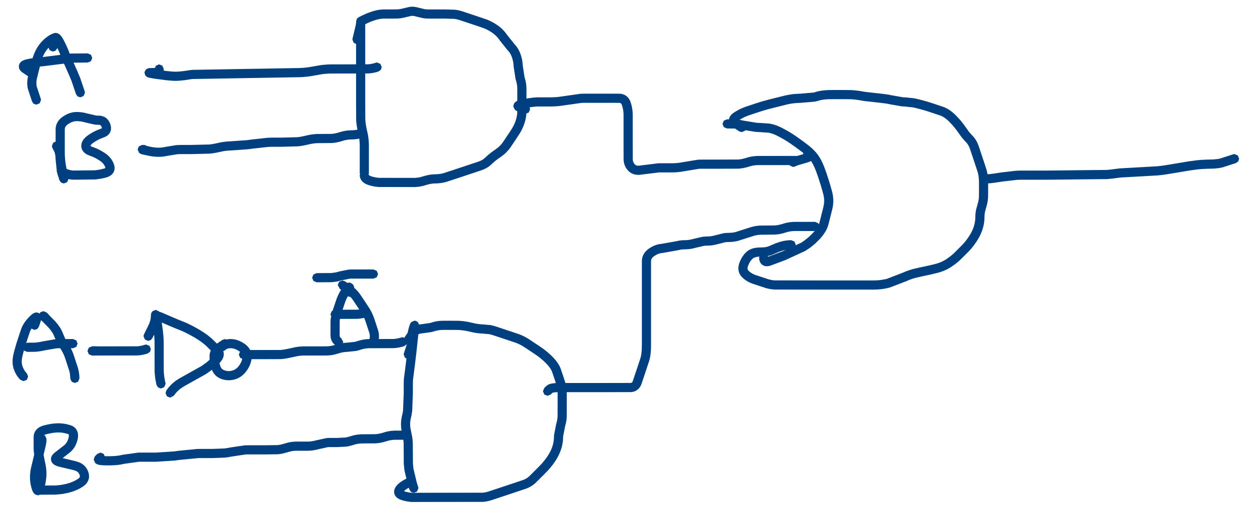
Large IC size.

To avoid this issue, the boolean expressions simplified first.

2 widely used methods for simplification.

- Using theorems & postulates of boolean Algebra.
- Using Karnaugh Maps (K-Maps) ✓

Ex: $F(A,B) = AB + \bar{A}B$ ⟹ 2 AND, 1 OR, 1 NOT gate are required (5 gates)



Simplify,

$$F = B(\underbrace{A+\bar{A}}_{1})$$

$$= B$$

B —— F No gate required.

Ex: $F(A,B,C) = ABC + \bar{A}BC + \bar{A}B\bar{C}$

$\downarrow$

Simplify using boolean Algebra theorems & postulates.

$$F(A,B,C) = ABC + \bar{A}B(\underbrace{C+\bar{C}}_{1})$$

$$= ABC + \bar{A}B$$

$$= B(AC + \bar{A})$$

$$= B(C + \bar{A})$$

(or)

$$= \bar{A}B + BC$$

C or $\bar{C} = 1$

$A + \bar{A}B$

$= (\underbrace{A+\bar{A}}_{1})(A+B)$

$= A+B$

Simplify above expression using K-map,

BCD (Binary coded decimal)

$$F(A,B,C) = \underset{\substack{1\,1\,1 \\ m_7}}{ABC} + \underset{\substack{0\,1\,1 \\ m_3}}{\bar{A}BC} + \underset{\substack{0\,1\,0 \\ m_2}}{\bar{A}B\bar{C}}$$

$$= \Sigma m(2,3,7)$$

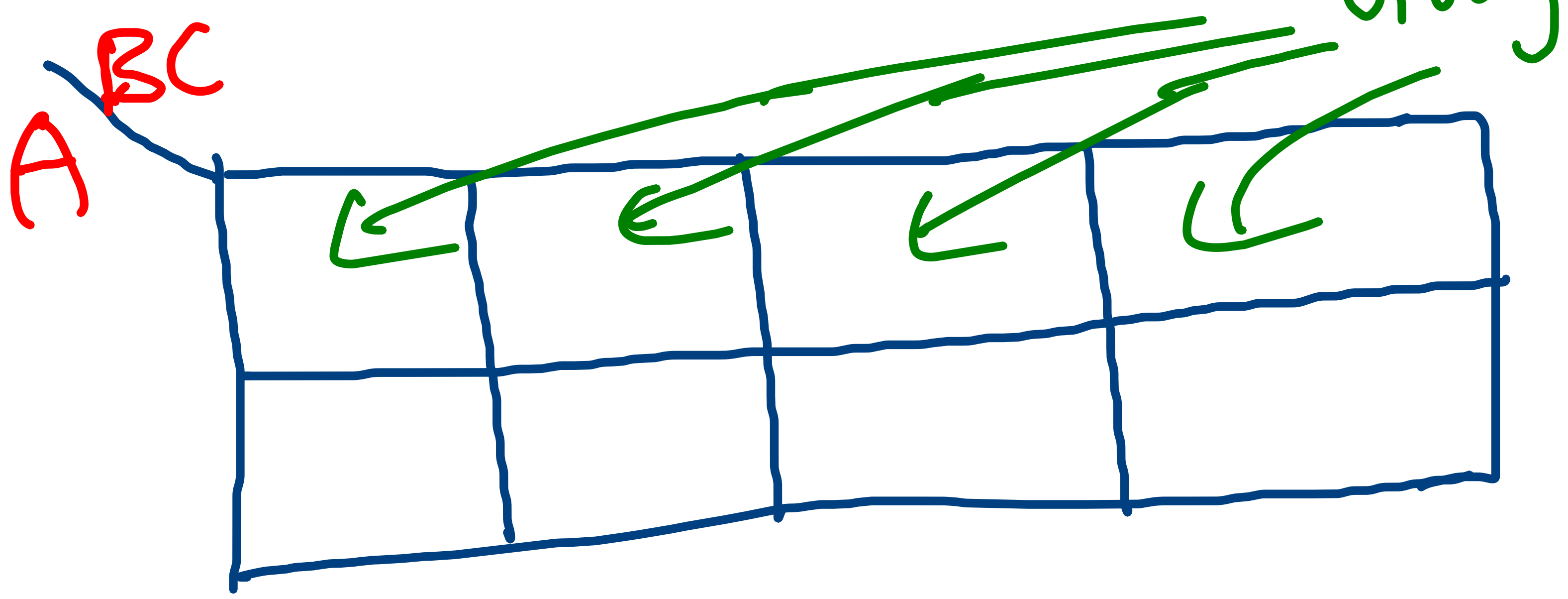
m_0
m_1
$\vdots$
m_7

8 min terms

$\Downarrow$

8 blocks in K-map

Use 3-variable K-map here,



Gray code for addressing.

To assign an address to each of these blocks use "Gray Code".

Gray code:

Also called Mirror Code.

It maintains Adjacency property.

only 1 bit changes between two Consequetive code.

produce Gray code using mirror,

put zeros above

1's below the mirror

0	0	0	→ 0
0	0	1	→ 1
mirror here			
0	1	1	→ 2
0	1	0	→ 3
mirror here			
1	1	0	→ 4
1	1	1	→ 5
1	0	1	→ 6
1	0	0	→ 7

Gray	Decimal
0 0 0	→ 0
0 0 1	→ 1
Mirror	
0 1 1	→ 2
0 1 0	→ 3
Mirror	
1 1 0	→ 4
1 1 1	→ 5
1 0 1	→ 6
1 0 0	→ 7
Mirror	

1 bit change

1 bit change

1 bit change

1 bit change

⋮

Binary, 2 → 10

Gray, 2 → 11

A \ BC	00	01	11	10
0	000 (0)	001 (1)	011 (3)	010 (2)
1	100 (4)	101 (5)	111 (7)	110 (6)

← Gray code for columns

Gray code for Rows

A \ BC	00	01	11	10
0	0 (0)	0 (1)	1 (3)	1 (2)
1	0 (4)	0 (5)	1 (7)	0 (6)

$$F = \Sigma m(2,3,7)$$
$$= \Pi M(0,1,4,5,6)$$
$$= \bar{A}B + BC.$$

same

Map 2^n blocks $(n = 0,1,2,3 \ldots)$

$2^0 = 1,\ 2^1 = 2,\ 2^2 = 4,\ 2^3 = 8 \ldots$

Procedure for K-Map :—

S-1 Get the min-terms or max-terms of given expression.

S-2 Draw the suitable K-map & give the block numbering with help of gray code.

S-3 put 1's in the min-term locations.
(or) 0's in the max-term locations.

S-4 Map the 1's in 2^n blocks together $(n = 0, 1, 2 \dots)$
i.e 1 or 2 or 4 or 8 ....

Always go for highest possible block mapping.

S-5 get the simplified term of each mapping.

$F = ABC + \bar{A}BC + \bar{A}B\bar{C}$

$= \Sigma m(2, 3, 7)$

A \ BC	00	01	11	10
0	0	1	1 (3)	1 (2)
1	4	5	1 (7)	6

$= \bar{A} \cdot B$

A \ BC	00	01	11	10
0	0	1	1 (3)	1 (2)
1	4	5	1 (7)	6

$= BC$

Answer $= \bar{A}B + BC.$

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$\vdots$

11 10

$= BC + B\bar{C}$

$= B(C + \bar{C})$

$= B$

Ex:

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + AB\bar{C} + \bar{A}\bar{B}C + ABC + A\bar{B}C$$

$$= \Sigma m(0,6,1,7,5)$$

$$= \Sigma m(0,1,5,6,7)$$

A \ BC	00	01	11	10
0	1 (0)	1 (1)	(3)	(2)
1	(4)	1 (5)	1 (7)	1 (6)

(or)

A \ BC		01	11	
0	1	1		
1		1	1	1

$$= \bar{A}\bar{B} + AB + \bar{B}C \quad \text{(or)} \quad = \bar{A}\bar{B} + AB + AC$$

Ex:

$$F(A,B,C) = \underset{101}{A\bar{B}C} + \underset{110}{AB\bar{C}} + \underset{001}{\bar{A}\bar{B}C} + \underset{111}{ABC} + \underset{100}{A\bar{B}\bar{C}}$$

$$= \Sigma m(5, 6, 1, 7, 4)$$

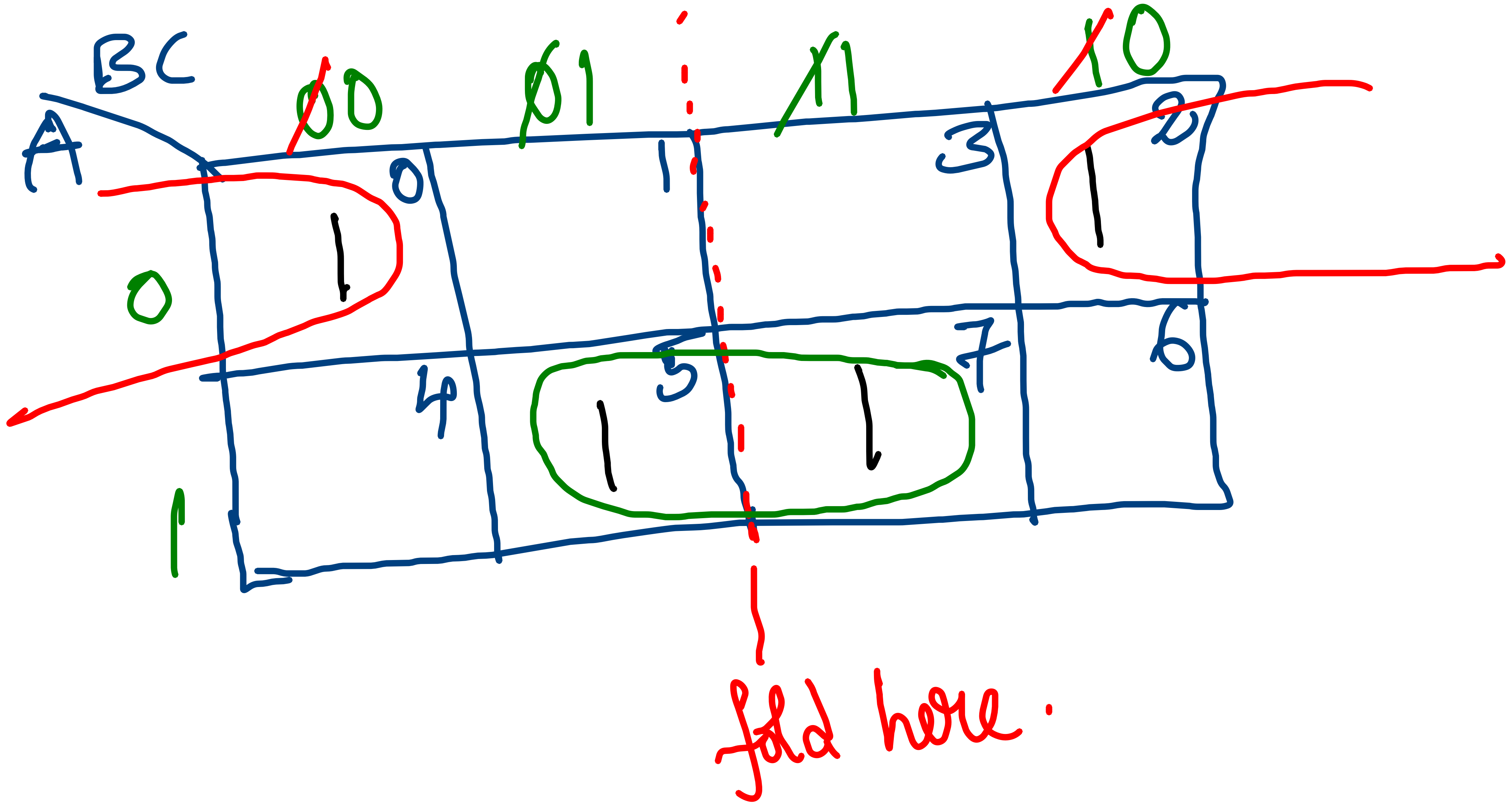
$$= \Sigma m(1, 4, 5, 6, 7)$$

$A \backslash BC$	00	01 ($\bar{B}C$)	11	10
0	(0)	1 (1)	(3)	(2)
1	1 (4)	1 (5)	1 (7)	1 (6)

$2^n \quad n = 0, 1, 2, \ldots$

$$= A + \bar{B}C$$

Ex: $F(A,B,C) = \sum m(0, 2, 5, 7)$



$= \bar{A}\bar{C} + AC$

Note: K-map is like a paper & you can fold it to map the 1's.

Ex:

$$F(A,B,C) = \sum m(0,1,2,3,4,5,6,7)$$

A \ BC	00	01	11	10
0	1 0	1 1	1 3	1 2
1	1 4	1 5	1 7	1 6

$$= \underline{\underline{1}}$$

$$F(A,B,C) = \prod M(0,1,2,3,4,5,6,7)$$

$$= \underline{\underline{0}}$$

Ex: $F(A,B,C) = \Pi M(0,1,4,6) = \Sigma m(2,3,5,7)$

maxterms

A \ BC	00	01	11	10
0	0 (m_0)	0 (m_1)	(m_3)	(m_2)
1	0 (m_4)	(m_5)	(m_7)	0 (m_6)

$$= (A+B)\cdot(\overline{A}+C)$$

4-Variable K-map:

$2^4 = 16$ blocks

AB \ CD	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0100 4	0101 5	0111 7	0110 6
11	1100 12	1101 13	1111 15	1110 14
10	1000 8	1001 9	1011 11	1010 10

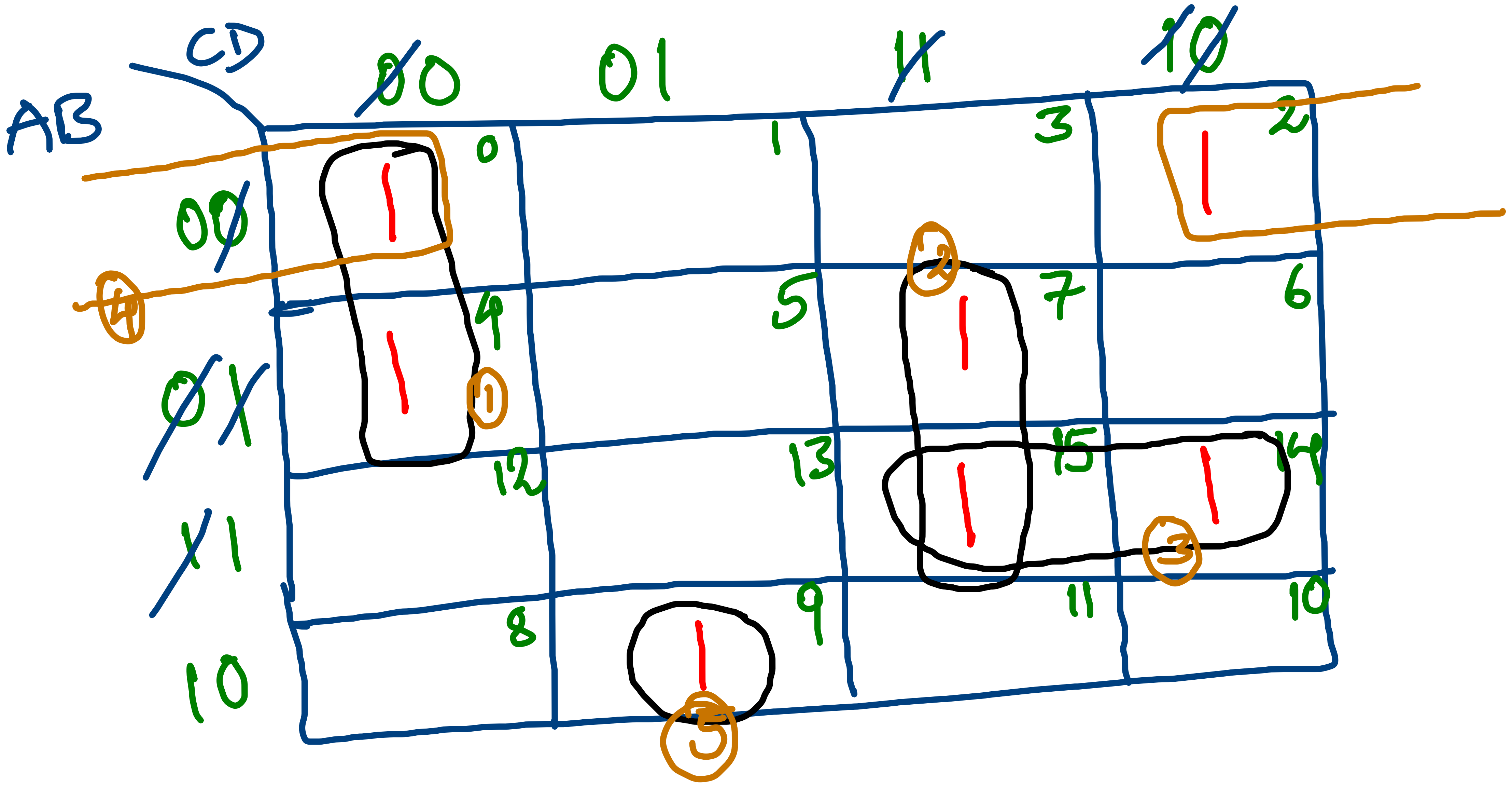
Gray code (column headings CD)

Gray code (row headings AB)

These are binary (or) BCD, so write the decimal value.

Ex:

$$F(A,B,C,D) = \Sigma m(0,2,4,7,9,14,15)$$

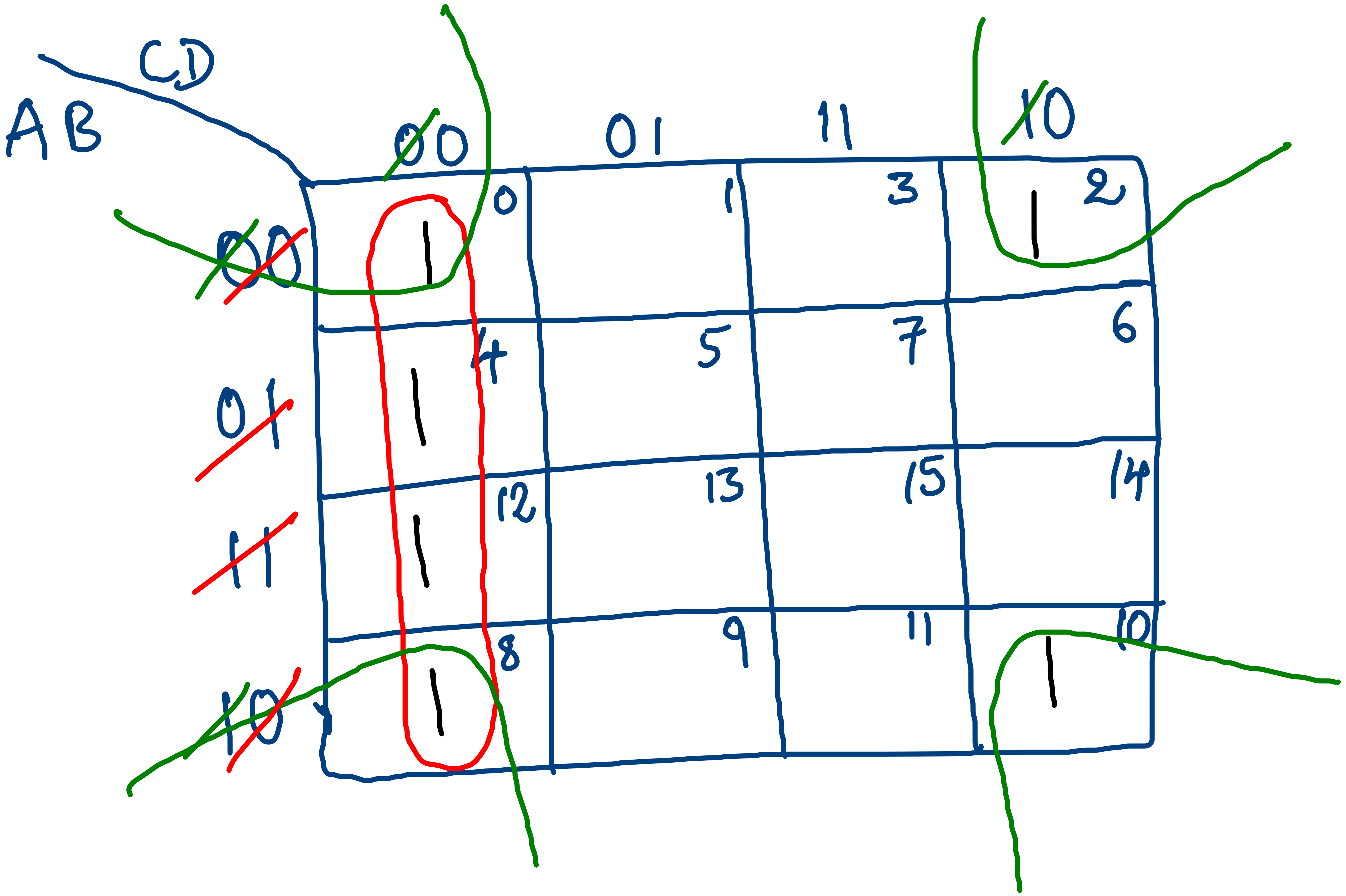


$$F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + BCD + ABC + \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}D$$

1 2 3 4 5

Ex:

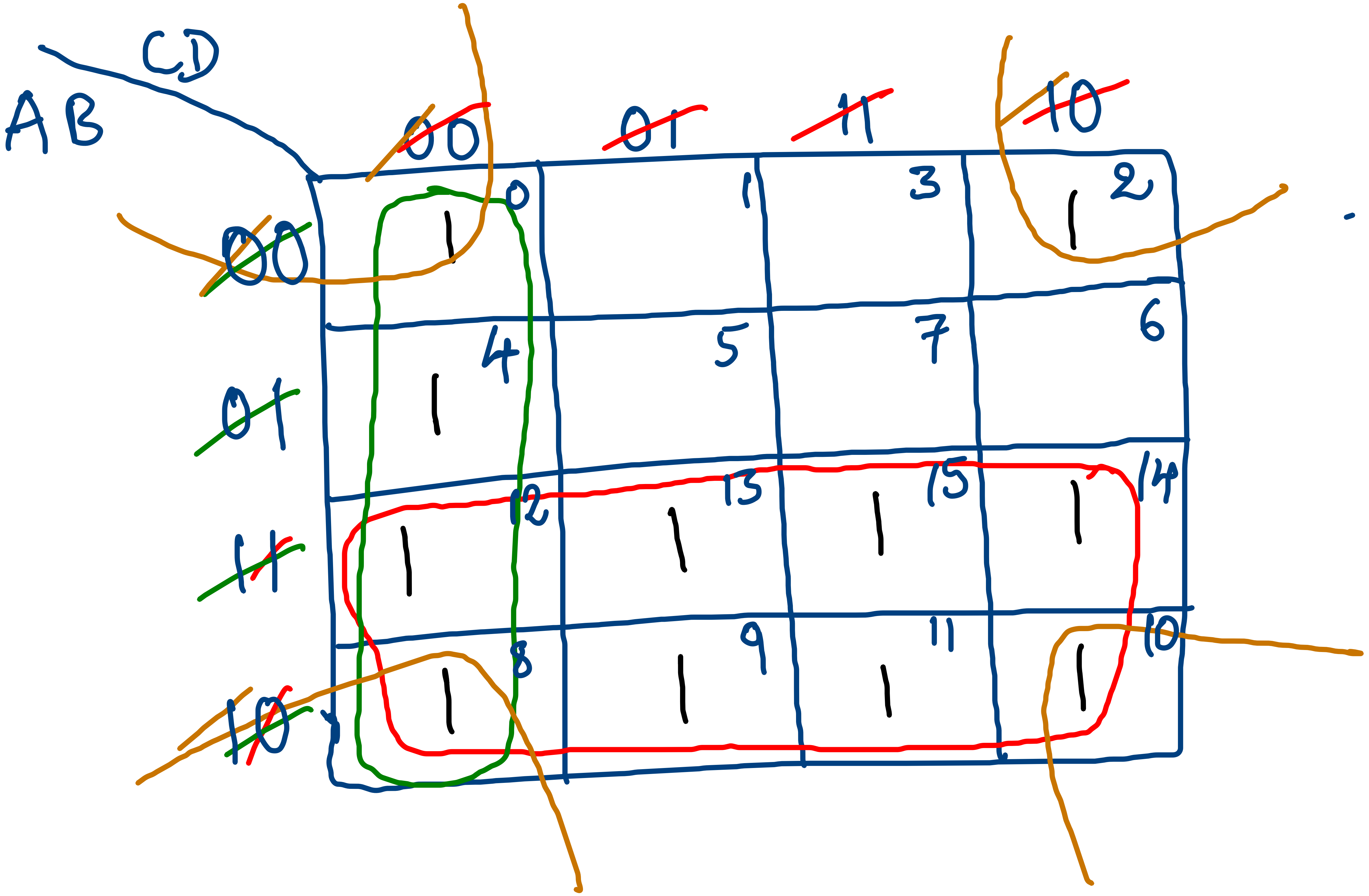
$$F(A,B,C,D) = \Sigma m(0,2,4,8,10,12)$$



$$F = \bar{C}\bar{D} + \bar{B}\bar{D}$$

Ex:

$$F(A,B,C,D) = \Sigma m(0,2,4,8,9,10,11,12,13,14,15)$$



$2^0, 2^1, 2^2, 2^3 \ldots$

$$F = A + \bar{C}\bar{D} + \bar{B}\bar{D}$$

Ex:

$$F(A,B,C,D) = ABC + \bar{A}\bar{B}D$$

$$= ABC(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C})D$$

$$= \underset{\substack{1111 \\ m_{15}}}{ABCD} + \underset{\substack{1110 \\ m_{14}}}{ABC\bar{D}} + \underset{\substack{0011 \\ m_{3}}}{\bar{A}\bar{B}CD} + \underset{\substack{0001 \\ m_{1}}}{\bar{A}\bar{B}\bar{C}D}$$

$$= \Sigma m(1,3,14,15)$$

AB \ CD	00	01	11	10
00	0	1 ¹	1 ³	2
01	4	5	7	6
11	12	13	1 ¹⁵	1 ¹⁴
10	8	9	11	10

$$F = \bar{A}\bar{B}D + ABC$$

Non-reducible

Ex:

$$F(A,B,C,D) = \Sigma m(1,3,5,7,9) + \underbrace{DC}(10,11)$$

↓

Don't care

↓

can be used as either a minterm (or) a Maxterm if required.

You can also leave them un-mapped if not required.

AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11				
10		1	X	X

$$F = \bar{A}D + \bar{B}D$$

Ex: $F(A,B,C,D) = \Sigma m(1,5,8,9,13) + DC(6,14)$

AB \ CD	00	01	11	10
00	$_{0}$	1_{1}	$_{3}$	$_{2}$
01	$_{4}$	1_{5}	$_{7}$	X_{6}
11	$_{12}$	1_{13}	$_{15}$	X_{14}
10	1_{8}	1_{9}	$_{11}$	$_{10}$

$F = \bar{C}D + A\bar{B}\bar{C}$

→ Binary to Gray code :-

XOR	
00	→ 0
01	→ 1
10	→ 1
11	→ 0

(1)

$1 \overset{\oplus}{\leftrightarrow} 1 \overset{\oplus}{\leftrightarrow} 1 \overset{\oplus}{\leftrightarrow} 1$ (Binary)

$1 \quad 0 \quad 0 \quad 0$ (Gray)

	Binary		Gray
(15)	1111	⟺	1000

procedure for Binary to Gray —

(S1) Take MSB of Binary as it is.

(S-2) perform XOR operation on consequetive Binary bits.

Gray code

Gray code	
0 0 0 0	→ 0
0 0 0 1	→ 1
0 0 1 1	→ 2
0 0 1 0	→ 3
0 1 1 0	→ 4
0 1 1 1	→ 5
0 1 0 1	→ 6
0 1 0 0	→ 7
1 1 0 0	→ 8
1 1 0 1	→ 9
1 1 1 1	→ 10
1 1 1 0	→ 11
1 0 1 0	→ 12
1 0 1 1	→ 13
1 0 0 1	→ 14
1 0 0 0	→ 15

② 1 0 1 0 (Binary)

1 1 1 1 (Gray)

Binary 1010 → 10

⇕

Gray 1111

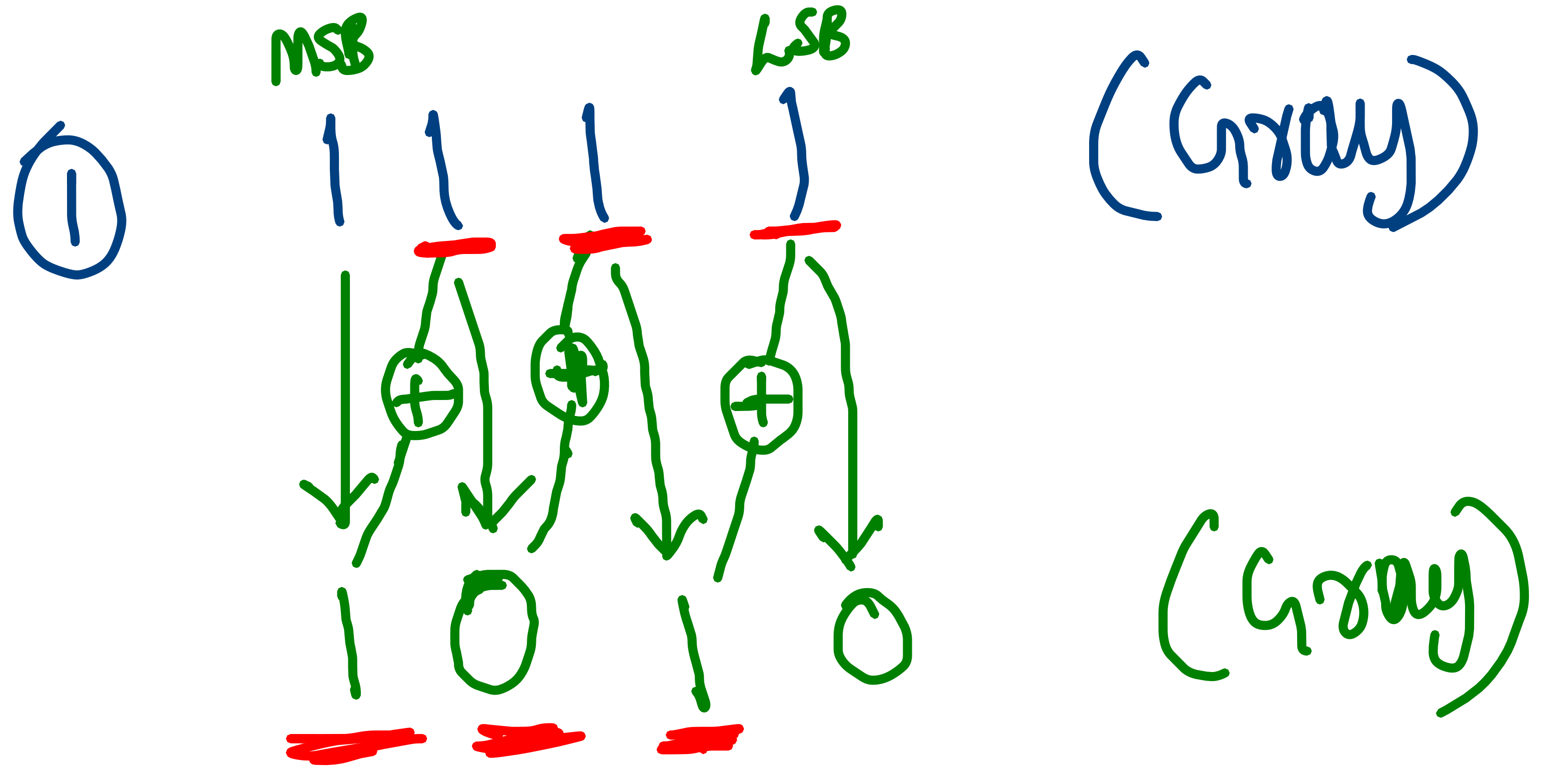
③ 1 1 0 0 1 1 0 1 0 (Binary)

1 0 1 0 1 0 1 1 1 (Gray)

④ 1 0 1 0 1 1 1 0 1 0 0 1 (Binary)

1 1 1 1 1 0 0 1 1 1 0 1

→ Gray code to Binary code conversion :-

① 

1111 (Gray)
⇓
1010 (Binary)

Gray to Binary procedure -

(S1) Take MSB as it is.

(S2) perform XOR operation between the fresh Gray bit you got & the next binary bit.

2

1 1 0 0 1 1 1 0 1 0 1

1 0 0 0 1 0 1 1 0 0 1