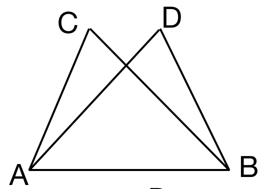
Lesson 13: Proposition 1.7 & 1.8

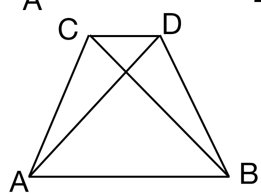
Proposition 1.7 (Theorem): *If two straight lines are constructed from the ends of a straight line meeting at a point, then there* ***cannot*** *be two other straight lines of equal length as the others, constructed on the same side that meet at a* ***different*** *point.*

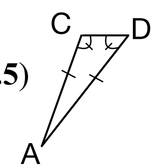
Proof

1. Let $\overline{AC} = \overline{AD}$ and $\overline{BC} = \overline{BD}$.

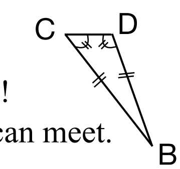


2. Connect points C and D (**P1**)



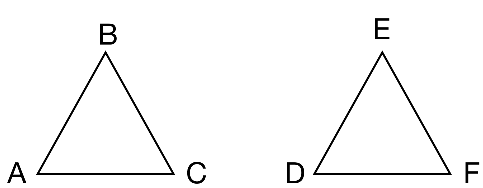


3. Since, $\overline{AC} = \overline{AD}$ then $\angle ACD = \angle ADC$ (**Prop 1.5**)
4. $\angle ACD = \angle ACB + \angle BCD$, so
   $\angle ACD > \angle BCD$ (**A5**).
   Then, $\angle ADC > \angle BCD$ (because $\angle ACD = \angle ADC$).
5. $\angle BDC = \angle BDA + \angle ADC$, so
   $\angle BDC > \angle ADC$ (**A5**).
6. $\angle ADC > \angle BCD$ and $\angle BDC > \angle ADC$, therefore,
   $\angle BDC > \angle ADC > \angle BCD$, which means $\angle BDC >> \angle BCD$.
7. But $\overline{BC} = \overline{BD}$, therefore $\angle BCD = \angle BDC$ (**Prop 1.5**), WHICH IS ABSURD!
8. In conclusion, $\overline{AC} \neq \overline{AD}$ and $\overline{BC} \neq \overline{BD}$ and there is only one point that they can meet.



Q.E.D.

Proposition 1.8 (Theorem): If two triangles have two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal, which are contained by the equal straight lines.



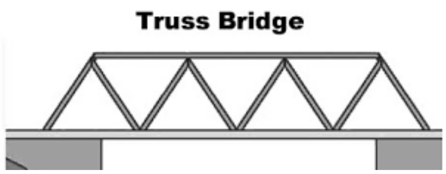
Proof

1. Let point A be superimposed on point D, and $\overline{AC}$ be placed on $\overline{DF}$. Since $\overline{AC} = \overline{DF}$, point C and point F will also coincide.
2. Since $\overline{AB} = \overline{DE}$ and $\overline{BC} = \overline{EF}$, point B and point E will also coincide (**Prop 1.7** - there is only one point at which two lines equal to two lines at the ends of a base can meet on the same side).
3. Then $\angle ABC$ coincides with $\angle DEF$, therefore $\angle ABC = \angle DEF$.

Q.E.D.

Two conditions are required for triangles to be congruent.

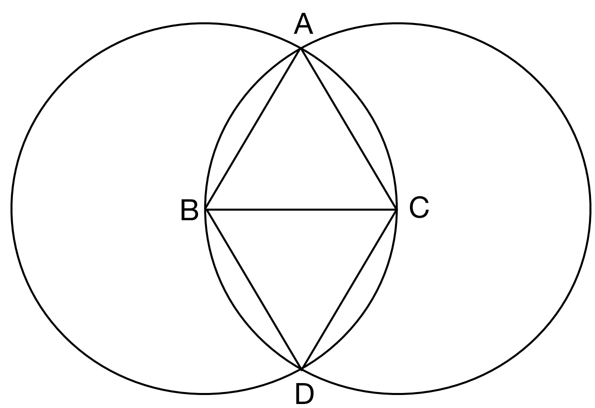
1. Two sides equal to two sides and equal angles between them
2. If all sides of the triangle are equal to all sides



Why do most bridges use triangles?

Rigidity! A triangle has three sides and three angles, and each angle is held solidly in place by the side opposite it. This means that a triangle's angles are fixed, and that if pressure is placed anywhere on a triangle, its angles, unlike those of other shapes, will not change.

Practice: If point B is the center of circle ACD and point C is the center of circle ABD, then $\triangle ABC = \triangle DBC$.



Proposition 1.9 (Problem): Bisect a given rectilineal angle.

Bisect: divide into two equal parts

Given:

Construct:

Prove: