Name: **Answer Key**

Math 101: Euclidean Geometry
Midterm

Directions

1. You have 90 mins to complete the midterm. If you finish early, you may leave *quietly*.
2. You can only use *your* notes. Sharing notes between students is not allowed.
3. Read each problem carefully to understand what you need to do. If you don't understand, ask Ms. Nitya.
4. Show all your work. Write clearly. Be as specific as you can. Don't assume I know what you mean.
5. *A test doesn't define who you are, the way you live your life does.* So be honest, do your own work, and don't cheat. If you're stuck on any question, ask the Holy Spirit for help. God wants you to succeed. Don't worry, and do the best you can!

"I can do all things through Christ who strengthens me."

Philippians 4:13

Section	Points Earned	Possible Points
Part A		26
Part B		13
Part C		30
Part D		31
Extra Credit		(10)
Total		100

Midterm Grade:

Part A: Definitions, Axioms, Postulates (26 points)

Directions: Match the following. Terms can be used more than once.

1. __**D**__: The boundary of a figure.
2. __**C**__: Assumptions used throughout mathematics
3. __**J**__: Has length and breadth (width) only.
4. __**H**__: $\angle$
5. __**A**__: The boundary of a line.
6. __**K**__: Trilateral figure with equal sides.
7. __**F**__: Straight lines that never intersect.
8. __**D**__: Only has length.
9. __**L**__: Type of angle made by two straight lines
10. __**E**__: A plane figure contained by one line such that all straight lines falling upon it from the center are equal to one another.
11. __**B**__: The straight line that creates adjacent right angles with another straight line.
12. __**A**__: Has no part.
13. __**B**__: $\perp$
14. __**G**__: Assumptions specific to geometry.
15. __**I**__: Triangle with only two of its sides equal.

a. Point
b. Perpendicular
c. Axioms
d. Line
e. Circle
f. Parallel
g. Postulates
h. Angle
i. Isosceles
j. Surface
k. Equilateral $\triangle$
l. Rectilineal

Directions: Fill in the blanks. In the parentheses, write the corresponding axiom or postulate (ex. Axiom 1).

16. A straight line can be made with any two ___**points**___ (_____**Postulate 1**____).

17. If $\overline{AB}$ coincides with $\overline{CD}$, then _____$\overline{AB} = \overline{CD}$_____ (___**Axiom 1**______).

18. If $\angle ABC$ and $\angle DEF$ are right angles, then ___$\angle ABC = \angle DEF$__ (__**Postulate 4**___).

19. If $z = a$ and $z = y$, then ____$a = y$______ (__**Axiom 1**_____).

20. Circles can be constructed with any __**center**___ and __**distance**__ (_**Postulate 3**_).

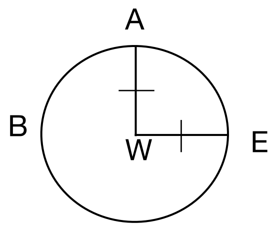
Part B: Labeling (13 points)

Directions: Labeling the following.

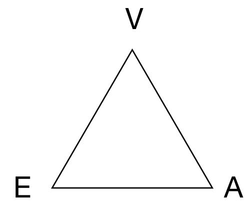
21. Center: __Point W_

Radii: _$\overline{AW}, \overline{EW}$____

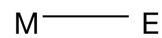
Circle: __ABE____



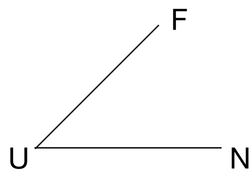
24. __ $\triangle AVE$___



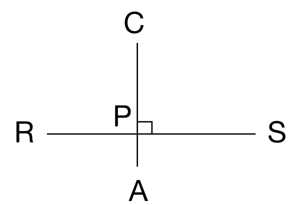
25. __$\overline{ME}$____



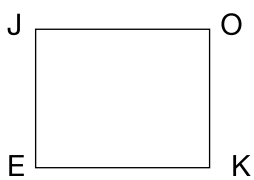
22. __$\angle FUN$_____



26. _$\overline{CA} \perp \overline{RS}$@$P$__



23. _Square JOKE_



Directions: Answer the following questions.

27. What is the circle in Figure 1?

Circle ABC

28. What are the triangles in Figure 1?

$\triangle$ABE, $\triangle$AEC, $\triangle$ABC

29. What are the rectilineal angles in Figure 1?

$\angle$ABC, $\angle$BAC, $\angle$ACB, $\angle$AEB, $\angle$AEC,
$\angle$BED, $\angle$CED, $\angle$BAE, $\angle$EAC

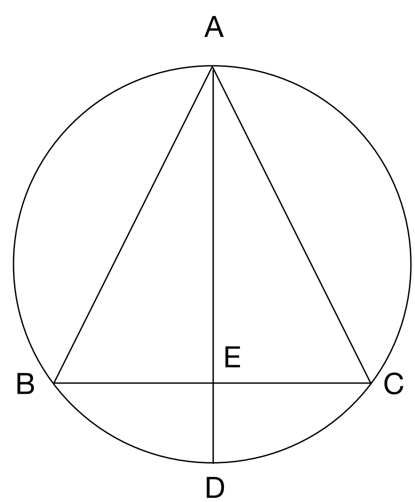


Figure 1

30. What angle can be written as a sum of two angles in Figure 1?
$\angle$BAC=$\angle$BAE+$\angle$EAC

31. What are the lines segments in Figure 1? $\overline{AB}, \overline{AE}, \overline{AC}, \overline{AD}, \overline{BC}, \overline{BE}, \overline{EC}, \overline{ED}$

Part C: Propositions (30 points)

Directions: Construct and prove the following. Use the space below for the construction. On the next page, use sentences to explain what you did in the construction steps, but proofs can be done using sentences or using the 2-column method (Statement/Reason).

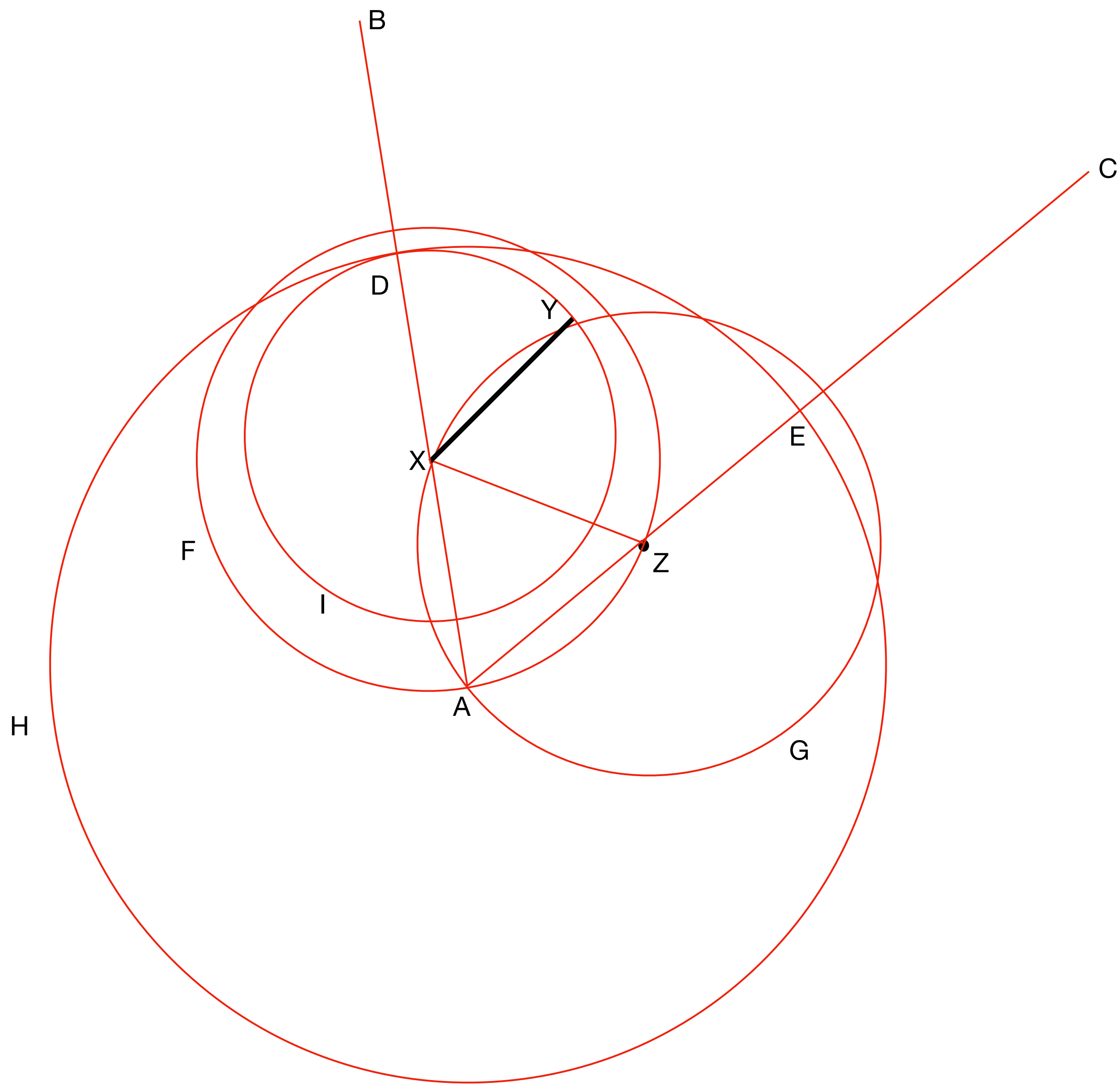
Construct a straight line equal to the given finite straight line $\overline{XY}$ on the given point Z.

Given: finite straight line $\overline{XY}$ and point Z

Construct: a straight line on point Z

Prove: Both straight lines are equal

Construction (4 points)



Construction Steps (10 points)

1. Connect points X and Z (Postulate 1)
2. Create an equilateral triangle XZA (Prop 1.1)
3. Extend lines $\overline{AX}$ and $\overline{AZ}$ to make lines $\overline{AB}$ and $\overline{AC}$ (Postulate 2)
4. Using point X as the center and XY as the distance, create circle DIY (Postulate 4)
5. Using point A as the center and AD as the distance, create circle DHE (Postulate 4)
6. We have now create line $\overline{ZE}$

Proof (13 points)

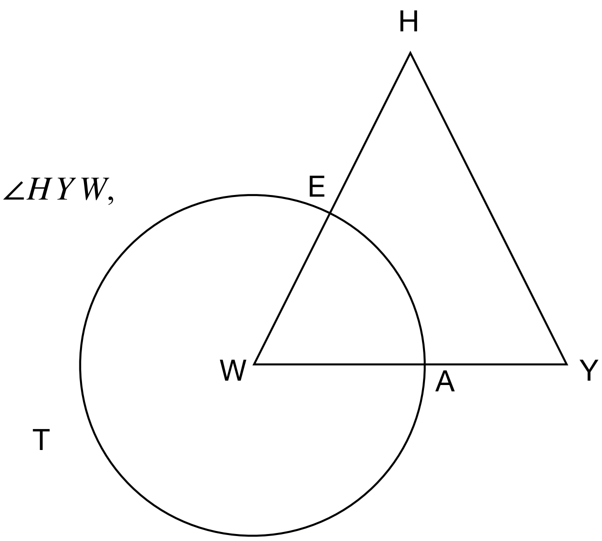
Statement	**Proof**
$\overline{AE} = \overline{AD}$	$\overline{AE}, \overline{AD}$ are radii of circle DHE (Def 15)
$\overline{AZ} + \overline{ZE} = \overline{AX} + \overline{XD}$	Since $\overline{AE} = \overline{AZ} + \overline{ZE}$ and $\overline{AD} = \overline{AX} + \overline{XD}$ (A1)
$\overline{AZ} + \overline{ZE} = \overline{AZ} + \overline{XD}$	$\overline{AZ} = \overline{AX}$ because $\triangle XZA$ is equilateral (Def 20)
$\overline{ZE} = \overline{XD}$	Subtract $\overline{AZ}$ (A3)
$\overline{ZE} = \overline{XY}$	(A1) $\overline{XD} = \overline{XY}$ since they are both radii of circle DIY (Def 15).

Q.E.F.

Part D: Proofs (31 points)

Directions: Prove the following.

7. If point W is the center of circle EAT and $\angle WHY = \angle HYW$, then $\overline{HE} = \overline{YA}$. (10 points)



Given:

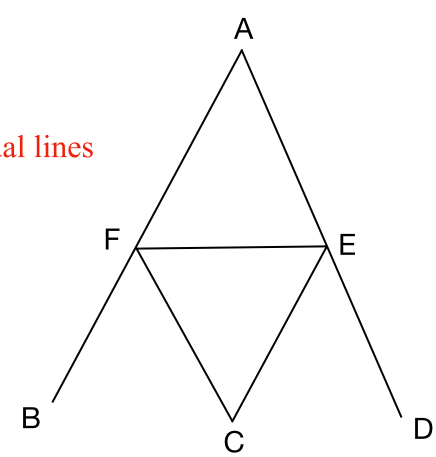
- point W is the center of circle $EAT \rightarrow \overline{WE} = \overline{WA}$
- $\angle WHY = \angle HYW$

Prove: $\overline{HE} = \overline{YA}$

Statement	Reason
$\overline{WH} = \overline{WY}$	$\overline{WH}$, $\overline{WY}$ are opposite equal angles, $\angle WHY = \angle HYW$, of a triangle (Prop 1.6)
$\overline{WE} + \overline{EH} = \overline{WA} + \overline{AY}$	$\overline{WH} = \overline{WE} + \overline{EH}$ and $\overline{WY} = \overline{WA} + \overline{AY}$
$\overline{WE} + \overline{EH} = \overline{WE} + \overline{AY}$	(A1) $\overline{WE} = \overline{WA}$ since they are radii of the circle EAT (Def 20)
$\overline{EH} = \overline{AY}$	Subtract $\overline{WE}$ (A3)

Q.E.D

8. If $\triangle AFE$ is isosceles with $\overline{AF} = \overline{AE}$ and $\overline{AF}$ extends to form $\overline{FB}$ and $\overline{AE}$ extends to form $\overline{ED}$ and $\triangle FEC$ is equilateral, then $\angle BFC = \angle CED$. (10 points)



Given:

- $\triangle AFE$ is isosceles, and $\overline{FB}$ and $\overline{ED}$ are extended from its equal lines
- $\triangle FEC$ is equilateral

Prove: $\angle BFC = \angle CED$

Statement	Reason
$\angle BFE = \angle DEF$	They are the angles below the base of the isosceles triangle AFE with extended lines (Prop 1.5)
$\angle BFC + \angle CFE = \angle FEC + \angle CED$	$\angle BFE = \angle BFC + \angle CFE$ and $\angle DEF = \angle FEC + \angle CED$ (A1)
$\angle BFC + \angle CFE = \angle CFE + \angle CED$	(A1) $\angle CFE = \angle CEF$ since they are at the base of a triangle with equal sides (Prop 1.5)
$\angle BFC = \angle CED$	Subtract $\angle CFE$ (A3)

Q.E.D.

9. Roelli and Troylin live in France. Roelli stays in the city of Antran and Troylin has an apartment in Burgundy. Shandy also wants to move to France, but they all want to live the same distance away from each other. Where should Shandy look for a place to live? Why?

Shandy should stay in Paris!

Using Proposition 1.1, we know that the distance between Antran and Burgundy is the same as Antran and Paris. The distance between Burgundy and Antran is the same as Burgundy and Paris. Therefore, the distance between Antran and Paris is the same as Burgundy and Paris.
In conclusion, the distance between all three cities are equal.

Extra Credit

10. What does Q.E.D and Q.E.F mean? (2 points)

Q.E.D. mean that which was to be demonstrated.

Q.E.F. means that which was to be done.

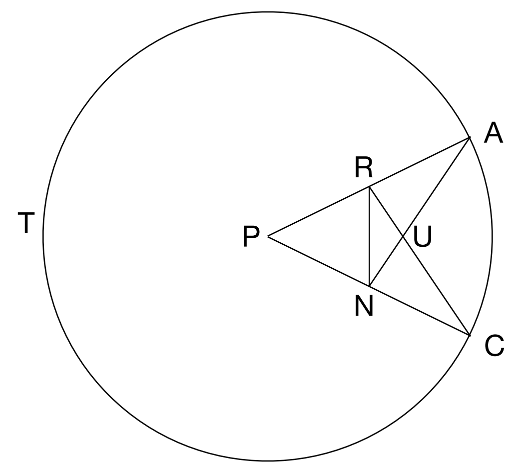
11. What is the difference between Postulate 3 and Definition 15? (2 points)

The difference is that Postulate 3 creates a circle with a center and distance and Definition 15 describes a circle.

12. Prove the following: If $\triangle PRN$ is an isosceles triangle and point P is the center of circle CAT, then $\triangle RAN = \triangle NCR$. (6 points)

Given:
- $\triangle PRN$ is an isosceles triangle
- point P is the center of circle CAT

Prove: $\triangle RAN = \triangle NCR$



Statement	Reason
$\overline{PA} = \overline{PC}$	$\overline{PA}, \overline{PC}$ are radii of circle CAT (Def 15)
$\overline{PR} + \overline{RA} = \overline{PN} + \overline{NC}$	$\overline{PA} = \overline{PR} + \overline{RA}$ and $\overline{PC} = \overline{PN} + \overline{NC}$ (A1)
$\overline{PR} + \overline{RA} = \overline{PR} + \overline{NC}$	$\overline{PR} = \overline{PN}$ since $\triangle PRN$ is an isosceles
$\overline{RA} = \overline{NC}$	Subtract $\overline{PR}$ (A3)
$\overline{NR} = \overline{NR}$	They are the same line
$\angle NRA = \angle ARN$	Since they are angles below the base on an isosceles triangle with extended lines (Prop 1.5)
$\triangle RAN = \triangle NCR$	Since $\overline{RA} = \overline{NC}$, $\overline{NR} = \overline{NR}$, and the angles contained between them $\angle NRA = \angle ARN$ (Prop 1.4)

Q.E.D.