Lesson 8: Proposition 1.4

Propositions

Proposition 1.1: We can make an equilateral triangle on a given finite straight line.
Proposition 1.2: We can make a finite line equal to a given finite line on a given point.
Proposition 1.3: Given two unequal straight lines, cut off from the greater, a straight line equal to the less.

Benque Beauty
There are two electrical towers of different lengths in the city of Benque. Jackie, the mayor of the city, doesn't like unequal lines. So she issued an order that all electrical poles should be of equal length. Where should Salvador, the electrical engineer, cut the bigger pole so they will be equal? Why?

Proposition 1.4 (Theorem): If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.

Given:

Prove:

<u>Dictionary</u>
Superimpose: place or lay (one thing) over another
Coincide: match in position
Congruent: identical or equal, except in position

Proof
Let triangle ABC and triangle DEF be two triangles.
Let $\overline{AB} = \overline{DE}$ and $\overline{AC} = \overline{DF}$, and let $\angle BAC = \angle EDF$.
If we superimpose triangles ABC and DEF so that point A coincides with point D and the line $\overline{AB}$ coincides with line $\overline{DE}$, then point B coincides with point E because $\overline{AB} = \overline{DE}$, and line $\overline{AC}$ coincides with line $\overline{DE}$ because $\angle BAC = \angle EDF$.
Point C coincides with point F because $\overline{AC} = \overline{DF}$.
Since point B coincides with point E and point C coincides with point F, then $\overline{BC}$ coincides with $\overline{EF}$, so $\overline{BC} = \overline{EF}$ (P4).
$\triangle ABC$ coincides with $\triangle DEF$, therefore $\triangle ABC = \triangle DEF$ (P4).
And the remaining angles coincide with each other, so $\angle ABC = \angle DEF$ and $\angle ACB = \angle DFE$.

In conclusion, if two triangles have two sides equal to two sides respectively and the angles contained by them are equal, then their bases are equal, the triangles are equal, and their remaining angles are equal.

Q.E.D.