# Lesson 6: Proposition 1.1 & 1.2

## Review

### Lines

- Naming a line: $\overline{AB}$ (only use 2 points)
- Lines can be added and subtracted
  - $\overline{AB} + \overline{BC} = \overline{AC}$
  - $\overline{AC} - \overline{AB} = \overline{BC}$
- Lines can be made of up of smaller line segments
  - $\overline{AC} = \overline{AB} + \overline{BC}$
- Perpendicular (⊥) lines form right angles
- Parallel (||) lines never intersect (meet)

### Angles

- Naming an angle:
  - $\angle BAC$ when referring to the points of the lines and the vertex, the point where the angle is located. The letter at the vertex is in the middle.
  - $\angle\theta$ when the angle is named
- Rectilineal angles are made of straight lines

### Circles

- Naming a circle: Circle ABC
- Three points on the boundary of the circle
- If D is the center, lines from the center point to the boundary are equal (ex. $\overline{DA} = \overline{DB}$)
- Definition 15 says that lines from the center to the boundary are equal
- Postulate 3 says that I can make a circle and all I need is a point, which will be the center and a distance, which will be the length of the radii

### Triangles

- Naming a triangle: $\Delta ABC$
- The boundary of a triangle (sides) are 3 line segments, $\overline{AB}, \overline{AC}, \overline{BC}$
- Equilateral triangles have 3 sides that are equal, $\overline{AB} = \overline{AC} = \overline{BC}$

### Squares

- Naming a square: square ABCD
- The points at the 4 corners - clockwise or counter clockwise

**Postulates**

1. A straight line can be drawn from any point to any point.
2. Straight lines can be extended (infinitely).
3. Circles can be drawn with any center and distance.
4. All right angles are equal to one another.
5. If the interior angles on the same side of two straight lines intersected by another straight line are less than two right angles, the two straight lines, if produced indefinitely, will meet on the side where the angles are less than the right angles.

**Axioms**

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

**Theorem:** formal statement that may be demonstrated from known propositions. It consists of two parts, the hypothesis, that which is assumed, and the conclusion, that which is asserted to follow.

*Ex. If X is Y, then Z is W.*
Given: X=Y
Prove: Z=W

*Ex. If X and Y, then Z is W.*
Given: X, Y
Prove: Z=W

**Q.E.D. (quod erat demonstrandum):** which was to be demonstrated.

**Problem:** a proposition in which something is proposed to be done under some given conditions.

*Ex. Construct an equilateral triangle on a given line.*
Given: A line.
Construct: A triangle.
Prove: The sides of the triangle are equal.

*Ex. Construct a line equal to a given line on a given point.*
Given: A line and a point.
Construct: Another line on the given point.
Prove: Both lines are equal.

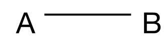
**Q.E.F. (quod erat faciendum):** which was to be done.

# Propositions

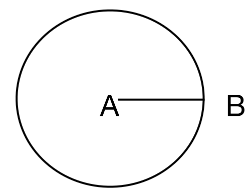
**Proposition 1.1 (Problem):** On a given finite straight line, construct an equilateral triangle.

*Given:* Finite straight line.
*Construct:* A triangle on the given line.
*Prove:* The triangle is equilateral, meaning the sides of the triangle are equal to each other.

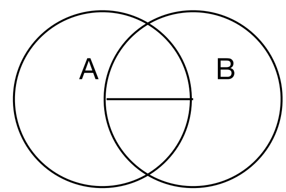
*Construct*



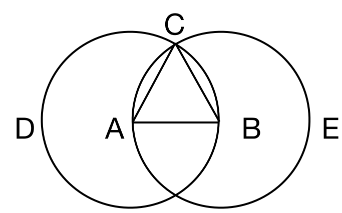

1. We are given a finite straight line, so let's draw one and call it $\overline{AB}$.
2. The ends of finite straight lines are points, and straight lines have distance, so we can use the straight line to make a circle. Draw a circle and let point A be the center and $\overline{AB}$ the radius (**Postulate 3**: Circles can be made with a center and a distance).




3. We can draw another circle with point B as the center and $\overline{AB}$ as the radius (**Postulate 3**: Circle can be made with a center and a distance).




4. There is a point where both the circles intersect (where they cross one another). Let's call that point C. Now that we have two points A and C, we can make a straight line $\overline{AC}$. We can do the same with B and C to create the line $\overline{BC}$. (**Postulate 1**: A straight line can be draw from any point to any point).
5. We have constructed a triangle!




*Prove*

1. Since point A is the center of the circle CDB and the radii are equal, then $\overline{AC} = \overline{AB}$ (**Def 15**: A circle is a plane figure contained by one line such that all straight lines falling upon it from one point (the center) among those lying within the figure are equal to one another).
2. Since the point B is the center of the circle CAE, $\overline{BC} = \overline{BA}$ (**Def 15**).
3. $\overline{AB} = \overline{AC}$ and $\overline{AB} = \overline{BC}$. Therefore, $\overline{AC} = \overline{AB} = \overline{BC}$ (**Axiom 1**: Things which are equal to the same thing are also equal to one another).
4. In conclusion, the triangle ABC is equilateral and it has been constructed on the given finite straight line $\overline{AB}$.

Q.E.F.

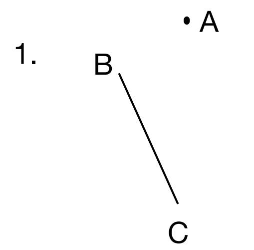
***Conclusion: We can make an equilateral triangle on a given finite straight line!***

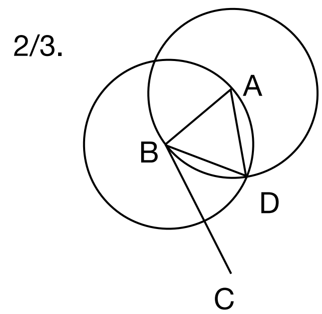
***WOW!***

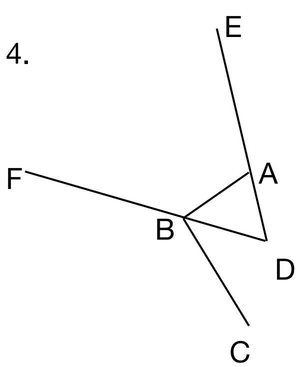
**Proposition 1.2 (Problem):** From a given point, draw a straight line equal to a given finite straight line.
*Given:* A finite straight line and a point.
*Construct:* A straight line on the given point.
*Prove:* The given straight line and the constructed straight line on the given point are equal.

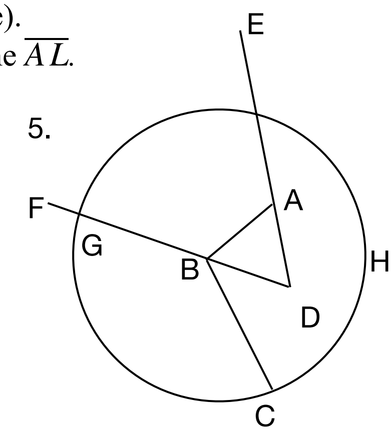
*Construct*

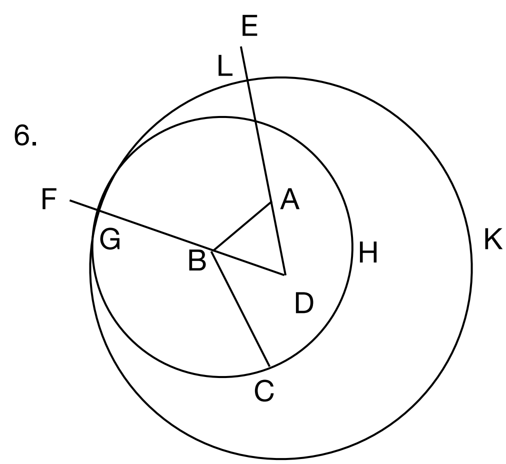
1. We are given a point and a finite straight line. So let's call the point, A, and the line, $\overline{BC}$.
2. Now we'll connect point A and point B to make the line $\overline{AB}$.
3. From **Proposition 1.1**, we know that we can make an equilateral triangle on a given finite straight line. So let's make an equilateral triangle on line $\overline{AB}$, ΔDAB.
4. Now we'll extend straight lines $\overline{DA}$ and $\overline{DB}$ to make $\overline{DE}$ and $\overline{DF}$ (**Postulate 2**: Straight lines can be extended).
5. With center at point *B* and distance *BC*, we can draw a circle, and let's called it circle *CGH*. (**Postulate 3**: Circles can be made with a center and a distance).
6. With center at point *B* and distance *BC*, we can draw a circle, and let's called it circle *GKL*. (**Postulate 3**: Circles can be made with a center and a distance).
7. We have constructed the line $\overline{AL}$.




*Prove*

1. Point D is the center of circle GKL, and points L and G are on the boundary so the lines $\overline{DL}$, $\overline{DG}$ are the radii, therefore $\overline{DL} = \overline{DG}$ (**Def 15**).
2. $\overline{DL} = \overline{DA} + \overline{AL}$ and $\overline{DG} = \overline{DB} + \overline{BG}$, so $\overline{DA} + \overline{AL} = \overline{DB} + \overline{BG}$ because $\overline{DL} = \overline{DG}$ (**Axiom 1**: Things which are equal to the same thing are also equal to one another).
3. But $\overline{DA} = \overline{DB}$ because ΔDAB is equilateral (**Def 20**: An *equilateral triangle* is that which has its three sides equal), so $\overline{DA} + \overline{AL} = \overline{DA} + \overline{BG}$ (**Axiom 1**).
4. $\overline{DA} + \overline{AL} - \overline{DA} = \overline{DA} + \overline{BG} - \overline{DA}$, therefore $\overline{AL} = \overline{BG}$ (**Axiom 3**: If equals be subtracted from equals, the remainders are equal).
5. Since point B is the center of circle GKL, and points G and C are on the boundary so the lines $\overline{BC}$, $\overline{BG}$ are the radii, therefore $\overline{BC} = \overline{BG}$ (Def 15).
6. $\overline{BC} = \overline{BG}$ and $\overline{AL} = \overline{BG}$, therefore $\overline{BC} = \overline{BG} = \overline{AL}$ (**Axiom 1**).

Q.E.F.

***Conclusion: We can make a finite line equal to a given finite line on a given point!***