# Lesson 2: Definitions, Postulates, and Common Notions

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*"There is no royal road to geometry." - Euclid*

**Definitions:** list of terms that will be used. Defined intuitively or explained with the help of "physical models". Helps orient the reader.

## Definitions

**Def 1:** A *point* is that which as no part.
- A point will be "something" with no width, length, or breadth, but an indivisible (unable to be divided or separated) location

**Def 2:** A *line* is breadthless length.
- Breathless length means it has one dimension, length, but no breadth (width)
- This could be any line (straight or curved)

**Def 3:** The ends of a line are points.
- Presents a relation between lines and points
- But it doesn't indicate how many ends a line can have
    - Circumference of a circle has no ends, but a finite line has two end points

**Def 4:** A *straight lin*e is a line which lies evenly with the points on itself.
- Refers to a kind of line

**Def 5:** A *surface* is that which has length and breadth only.
- Surface has two dimension

**Def 6:** The extremities of a surface are lines.

**Def 7:** A *plane surface* is a surface which lies evenly with the straight lines on itself.
- More frequently called a plane
- Something that is flat and level

**Def 8:** A *plane angle* is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

- Could be the meeting of curved or straight lines
- A broad definition of angles, more specific in the next ones

**Def 9:** And when the lines containing the angle are straight, the angle is called *rectilineal*.

- This definition is more specific
- When several angles meet at a point, each of them is indicated by three letters, of which the one at the *vertex* of the angle is read between other two
- A single letter is sufficient when there is only one angle at a point
- Angles are unaffected by the length of the lines

**Def 10:** When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right,* and the straight line standing on the other is called a *perpendicular* to that on which it stands.

**Def 11:** An *obtuse angle* is an angle greater than a right angle.

**Def 12:** An *acute angle* is an angle less than a right angle.

**Def 13:** A *boundary* is that which is an extremity of anything.

**Def 14:** A *figure* is that which is contained by any boundary or boundaries.

**Def 15:** A *circle* is a plane figure contained by one line such that all straight lines falling upon it from one point among those lying within the figure are equal to one another;

**Def 16:** And the point is called the *center* of the circle.

**Def 17:** A *diameter* of a circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

**Def 18:** A *semicircle* is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of a circle.

**Def 19:** *Rectilineal figures* are those which are contained by straight lines.
*Trilateral figures* being those contained by thee.
*Quadrilateral* those contained by four
*Multilateral* those contained by more than four straight lines.

**Def 20:** Of trilateral figures, an *equilateral triangle* is that which has its three sides equal.
An *isosceles triangle* that which has two of its sides alone equal.
A *scalene triangle* that which has its three sides unequal.

**Def 21:** Further, of trilateral figures, a *right-angled triangle* is that which has a right angle.
An *obtuse angled triangle* that which has an obtuse angle.
An *acute-angle triangle* that which has its three angles acute.

**Def 22:** Of quadrilateral figures, a *square* is that which is both equilateral and right-angled.
An *oblong* that which is right angled but not equilateral.
A *rhombus* that which is equilateral but not right-angled.
A *rhomboid* that which has its opposite sides and angles equal to one another but is neither equilateral nor right angled. And let quadrilaterals other than these be called *trapezia*.

**Def 23:** *Parallel* straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Starting with his definitions, Euclid assumed certain properties, which were not to be proved – obvious universal truths, divided into two types: axioms and postulates.
**Postulate:** assumptions that were specific to geometry.

- They are self-evident and reasonable assumptions

**Axioms:** common notions, assumptions used throughout mathematics and not specifically linked to geometry.

## Postulates

Let the following be postulated:

1. *To draw a straight line from any point to any point.*
    - A straight line can be draw between points. We can test to observe that a straight line can in fact be made between points, which are its ends
2. *To produce finite straight line continuously in a straight line.*
    - Every straight line can extend without limits in either direction or both. So an infinite straight line can be produced from a fine one.
3. *To describe a circle with any center and distance.*
    - This is determined from the definition of a circle that has a equal distance from the center to the boundaries. Additional information does not give more knowledge about a circle.
4. *That all right angles are equal to one another.*
    - This also follows from the definition of right angles being equal, therefore all right angles must be equal
5. *That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the right angles.*

## Axioms/Common Notions

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
    - Placing one geometrical magnitude on another, such as a line on a line, a triangle on a triangle, a circle on a circle, etc, without changing the form or size, is called superimposition. If they coincide, match exactly, they are equal.
5. The whole is greater than the part.