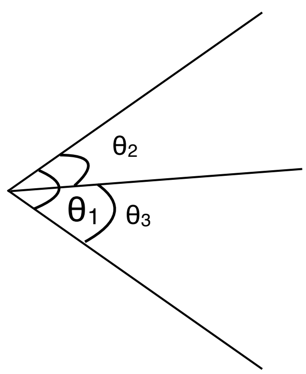
Lesson 9: Proposition 1.5

Propositions

Proposition 1.1: We can make an equilateral triangle on a given finite straight line.
Proposition 1.2: We can make a finite line equal to a given finite line on a given point.
Proposition 1.3: Given two unequal straight lines, cut off from the greater, a straight line equal to the less.
Proposition 1.4: If two triangles have two sides equal to two sides respectively and the angles contained by them are equal, then their bases are equal, the triangles are equal, and their remaining angles are equal.

Adding/Subtracting Angles



$$\angle\theta_1 - \angle\theta_2 = \angle\theta_3$$
$$\angle\theta_1 - \angle\theta_3 = \angle\theta_2$$
$$\angle\theta_2 + \angle\theta_3 = \angle\theta_1$$

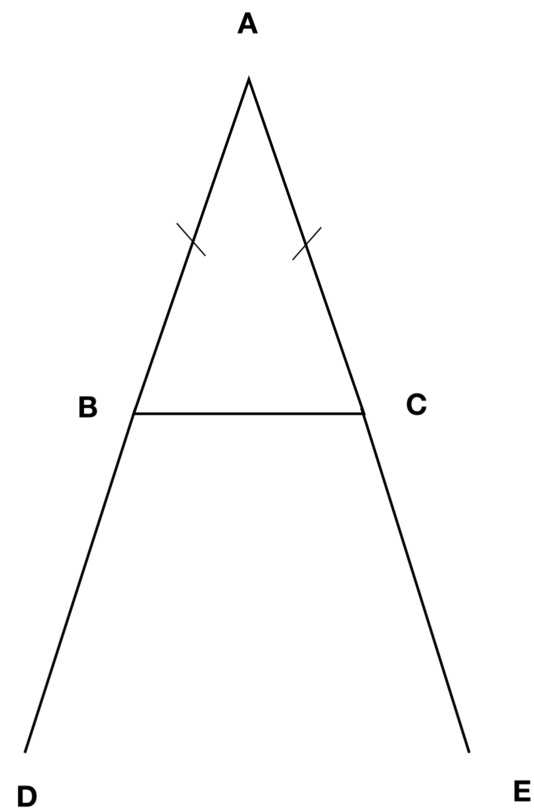
Proposition 1.5 (Theorem): The angles at the base of an isosceles triangle are equal. And if the equal sides be extended, the angles under the base will be equal.

Given:
- Isosceles triangle
- Extended equal sides of the isosceles triangle

Prove:
- Angles at the base are equal
- Angles under the base are equal

<u>Proof</u>:

Let ABC be an isosceles triangle with $\overline{AB} = \overline{AC}$ (**Def 20**), and let them be extended further to produce lines $\overline{BD}$ and $\overline{CE}$ (**P2**). We need to show that $\angle ABC = \angle ACB$ and $\angle DBC = \angle ECB$.



Given:

- Isosceles $\triangle ABC$
- Extended equal sides of the isosceles triangle: $\overline{BD}$ and $\overline{CE}$

Prove:

- Angles at the base are equal: $\angle ABC = \angle ACB$
- Angles under the base are equal: $\angle DBC = \angle ECB$

Let point F be on line $\overline{BD}$. Cut off from the larger line $\overline{AE}$ a smaller line equal to $\overline{AF}$ and let point G mark the cut so that $\overline{AF} = \overline{AG}$ (**Proposition 1.3**).
Join points F and C to produce line $\overline{FC}$, and point G and B to produce line $\overline{GB}$ (**P1**).
With respect to $\triangle AFC$ and $\triangle AGB$, $\overline{AF} = \overline{AG}$ and $\overline{AB} = \overline{AC}$ and the angle contained by them is the same, $\angle FAC = \angle GAB$, therefore $\overline{FC} = \overline{GB}$, $\triangle AFC = \triangle AGB$, and the remaining angles are equal, $\angle AFC = \angle AGB$ and $\angle ACF = \angle ABG$ (**Proposition 1.4**).
In addition, since $\overline{AF} = \overline{AG}$ and $\overline{AB} = \overline{AC}$, then

$$\overline{AB} + \overline{BF} = \overline{AC} + \overline{CG} \qquad (\textbf{A1})$$
$$\overline{AB} + \overline{BF} = \overline{AB} + \overline{CG} \qquad (\textbf{A1})$$
$$\overline{FB} = \overline{GC} \qquad (\textbf{A3}).$$

With respect to $\triangle FBC$ and $\triangle GCB$, $\overline{FB} = \overline{GC}$ and $\overline{FC} = \overline{GB}$ and $\angle BFC = \angle CGB$. Therefore, the base, $\overline{BC} = \overline{BC}$ (they share the same line), $\triangle FBC = \triangle GCB$, and the remaining angles are equal , $\angle FCB = \angle GBC$ and $\angle FBC = \angle GCB$ (**Proposition 1.4**).
Then $\angle FCB = \angle GBC = \theta_1$ and $\angle ACF = \angle ABG = \theta_3$.

$\angle\theta_3 = \angle\theta_3$

$\angle\theta_3 - \angle\theta_1 = \angle\theta_3 - \angle\theta_1$ (**A3**)

$\angle\theta_4 = \angle\theta_4$

Therefore, $\angle ABC = \angle ACB$.
It has been shown that $\angle FBC = \angle GCB$, therefore, $\angle DBC = \angle ECB$.
In conclusion, the angles at the base of an isosceles triangle are equal, and if the equal sides be extended, the angles under the base are equal.

Q.E.D.