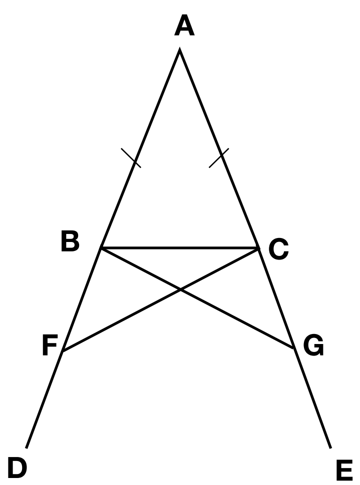
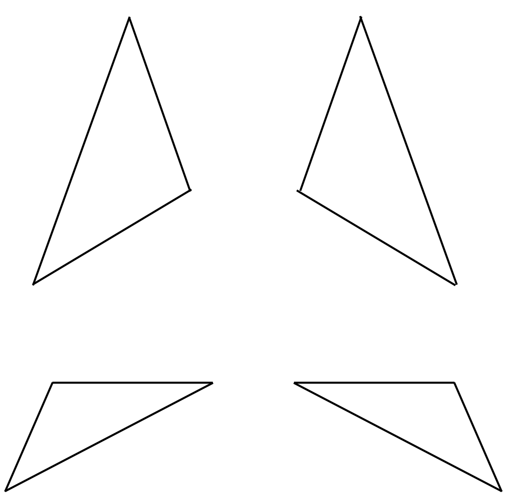
Lesson 10: Proposition 1.5

<u>Propositions</u>

Proposition 1.1: We can make an equilateral triangle on a given finite straight line.
Proposition 1.2: We can make a finite line equal to a given finite line on a given point.
Proposition 1.3: Given two unequal straight lines, cut off from the greater, a straight line equal to the less.
Proposition 1.4: If two triangles have two sides equal to two sides respectively and the angles contained by them are equal, then their bases are equal, the triangles are equal, and their remaining angles are equal.
Proposition 1.5: The angles at the base of an isosceles triangle are equal. And if the equal sides be extended, the angles under the base will be equal.





Logical Reasoning

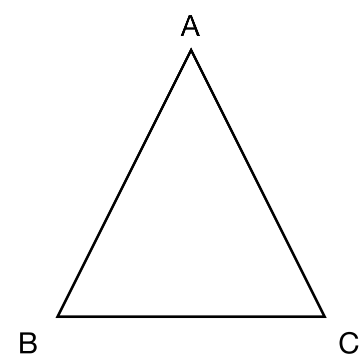
Contradiction: consists of a logical incompatibility between two or more statements

- If two statements taken to be true contradict each other, then the conclusion is false.

Ex. I know I promised to show up today, but I don't see why I should come if I don't feel like it.
Ex. The restaurant opens at five o'clock and it begins serving between four and nine.

Proof by Contradiction: a form of indirect proof by assuming the opposite is true, and then showing that such an assumption leads to a contradiction.

Proposition 1.6 (Theorem): If two angles of a triangle are equal, the sides opposite the equal angles will also be equal.



Given:

Prove:

If $\overline{AB} \neq \overline{AC}$, then one must be greater than the other.
Let $\overline{AB}$ be the greater. Let point D be on line $\overline{AB}$ such that $\overline{BD} = \overline{AC}$ (**Proposition 1.3**).
Join point D and C to make $\overline{DC}$ (**P1**).
Looking at the two triangles, $\triangle DBC$ and $\triangle ACB$,
$\overline{BD} = \overline{AC}$, $\overline{BC}$ is common to both, and $\angle DBC = \angle ABC$,
therefore the bases are equal, $\overline{DC} = \overline{AB}$,
$\triangle DBC = \triangle ACB$, and the remaining angles are equal (**Proposition 1.4**), which is absurd!

Hence $\overline{AB} = \overline{AC}$.
In conclusion, if two angles of a triangle are equal, the sides opposite the equal angles will also be equal.