Lesson 16: Proposition 1.13, 1.14, & 1.15

Proposition 1.13 (Theorem): If a straight line stands on a straight line, then it makes either two right angles or angles whose sum equals two right angles.

Given: A straight line and another straight line that stands on it.
Prove: The angles are two right angles or their sum is the sum of two right angles.
Proof

1. Let $\overline{AB}$ be a straight line standing on straight line $\overline{CD}$, making $\angle ABC$ and $\angle ABD$.
2. If $\angle CBA = \angle ABD$, then they are right angles (**Def 10**).
3. If not, then construct $\overline{BE}$ on point B at right angles to $\overline{CD}$ (**Prop 1.11**). Therefore, $\angle CBE$ and $\angle DBE$ are right angles.
4. $\angle ABD + \angle ABC = \angle DBE + \angle EBA + \angle ABC$ (**A1** since $\angle ABD = \angle DBE + \angle EBA$)
   $\angle ABD + \angle ABC = \angle DBE + \angle CBE - \angle ABC + \angle ABC$ (**A1** since $\angle EBA = \angle CBE - \angle ABC$)
   $\angle ABD + \angle ABC = \angle DBE + \angle CBE$ Q.E.D.

Proposition 1.14 (Theorem): If, with any straight line and at point on it, two straight lines not lying on the same side make the sum of the adjacent angles equal to two right angles, then the two straight lines are in a straight line with one another.

Given: two straight lines that meet at a point on another straight line (and are not on the same side) and make adjacent angles equal to two right angles
Prove: the two straight lines are in a straight line with one another
Proof

1. Let $\overline{AB}$ be a straight line and point B be the point on it at which two straight lines, $\overline{BC}$ and $\overline{BD}$ are not lying on the same side make the sum of adjacent angles, $\angle ABC$ and $\angle ABD$, equal to two right angles.

Proof by contradiction

2. If $\overline{BD}$ is not in a straight line with $\overline{BC}$, then extend $\overline{BC}$ to make $\overline{BE}$ (**P2**).
3. Since $\overline{AB}$ stands on line $\overline{CE}$, then $\angle ABC + \angle ABE$ = two right angles (**Prop 1.13**).
4. But $\angle ABC + \angle ABD$ = two right angles, therefore $\angle ABC + \angle ABE = \angle ABC + \angle ABD$ (**P4**).
5. $\angle ABC + \angle ABE - \angle ABC = \angle ABC + \angle ABD - \angle ABC$ (**A3**)
   $\angle ABE = \angle ABD$ **WHICH IS ABSURD!**
6. Therefore, $\overline{BD}$ is in a straight line with $\overline{BC}$.

Q.E.D.

Proposition 1.15 (Theorem): If two straight lines cut one another, then they make the vertical angles equal to one another.

Given:

Prove:

Proof

1. Let $\overline{AB}$ and $\overline{CD}$ intersect at point E.
2. Since $\overline{AE}$ stands on $\overline{CD}$ making angles $\angle CEA$ and $\angle AED$, therefore $\angle CEA + \angle AED$=two right angles (**Prop 1.13**).
3. Since $\overline{DE}$ stands on $\overline{AB}$ making angles $\angle AED$ and $\angle DEB$, therefore $\angle AED + \angle DEB$=two right angles (**Prop 1.13**).
4. $\angle CEA + \angle AED = \angle AED + \angle DEB$ (**P4 & A1**)
   $\angle CEA + \angle AED - \angle AED = \angle AED + \angle DEB - \angle AED$ (**A3**)
   $\angle CEA = \angle DEB$

Q.E.D.

Practice: Prove that $\angle BEC = \angle AED$.