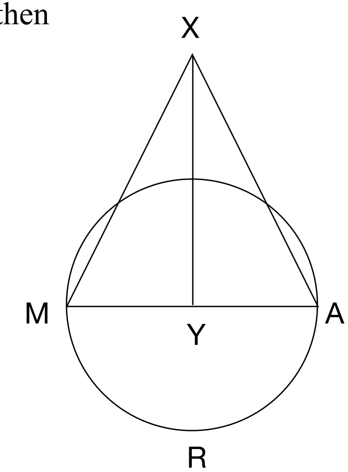
Lesson 11: Midterm Study Guide Solutions

- Definitions, axioms, postulates
- Labeling
- Propositions
  - Proposition 1.2 - know the construction and proof.
  - Remember to end proofs with Q.E.D. or Q.E.F.
  - Conclusions of proofs - using them in other poofs
- Proofs and Word Problems

Practice Problems

1. Name the triangles, line segments, angles, and circle. What angles are sums of others?
2. Prove the following: If point Y is the center of circle MAR and $\overline{XY} \perp \overline{MA}$, then $\triangle MXY = \triangle AXY$.



1. $\triangle MXA, \triangle MXY, \triangle AXY$

$\overline{MX}, \overline{XY}, \overline{XA}, \overline{MA}, \overline{MY}, \overline{YA}$

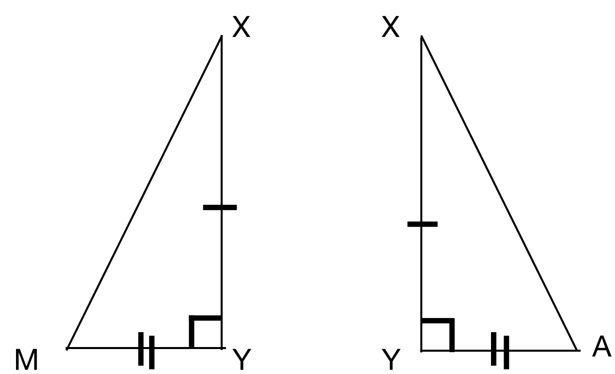
$\angle XMY, \angle XAY, \angle XYM, \angle XYA, \angle YXM, \angle YXA, \angle MXA$

Circle MAR

$\angle YXM + \angle YXA = \angle MXA$

2.
Given:
- Point Y is the center $\rightarrow \overline{MY} = \overline{YA}$
- $\overline{XY} \perp \overline{MA} \rightarrow \angle XYM = \angle XYA$

Prove: $\triangle MXY = \triangle AXY$

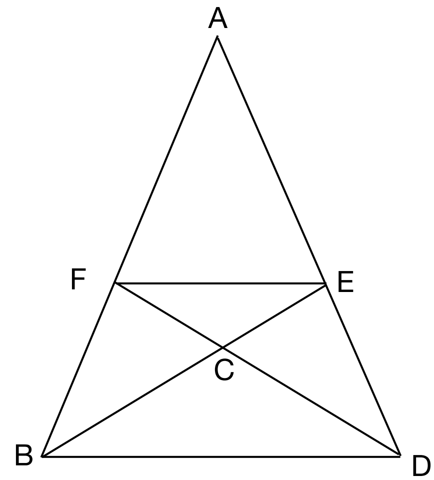


$\overline{XY} = \overline{XY}$ (its the same line)

Since $\overline{XY} = \overline{XY}$, $\overline{MY} = \overline{YA}$, and $\angle XYM = \angle XYA$, we can use **Proposition 1.4**. Therefore $\triangle MXY = \triangle AXY$.

Q.E.D.

2. Name the triangles, line segments, and angles. What angles are sums of others?
3. If $\triangle AFE$ is an isosceles triangle and $\overline{BF} = \overline{DE}$ then
   a. $\triangle FCE$ is an isosceles triangle
   b. $\angle BFC = \angle DEC$



1.
$\triangle ABD, \triangle AFE, \triangle BCD,$
$\triangle FEC, \triangle FCB, \triangle ECD,$
$\triangle FDB, \triangle EBD, \triangle FBE,$
$\triangle FDE, \triangle AFD, \triangle AEB$

$\overline{AB}, \overline{BD}, \overline{DA}$
$\overline{AF}, \overline{FE}, \overline{EA}$
$\overline{FB}, \overline{BC}, \overline{CF}$
$\overline{CD}, \overline{ED}, \overline{CE}$

$\angle ABD, \angle ABE, \angle EBD$
$\angle ADB, \angle ADF, \angle FDB$
$\angle BAD$
$\angle EFB, \angle EFD, \angle DFB, \angle AFE, \angle AFD$
$\angle FED, \angle FEB, \angle BED, \angle AEF, \angle AEB$
$\angle FCB, \angle FCE, \angle ECD, \angle DCB$

$\angle EFB = \angle EFD + \angle DFB$
$\angle AFD = \angle AFE + \angle EFD$
$\angle FED = \angle FEB + \angle BED$
$\angle AEB = \angle AEF + \angle FEB$
$\angle FBD = \angle FBE + \angle EBD$
$\angle EDB = \angle EDF + \angle FDB$

Given:

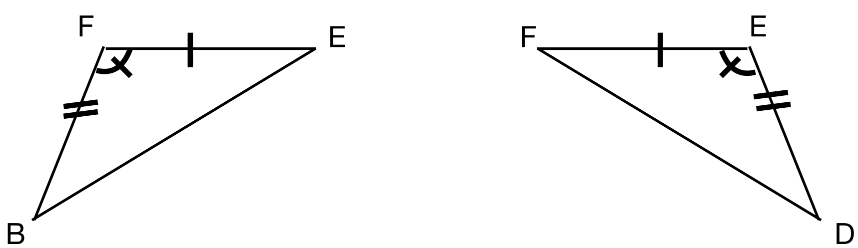
- $\triangle AFE$ is an isosceles triangle $\rightarrow \overline{AF} = \overline{AE}$
- $\overline{BF} = \overline{DE}$

Prove:

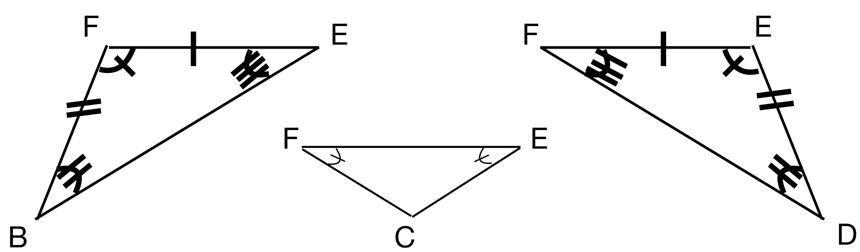
- $\triangle FCE$ is an isosceles triangle $\rightarrow \overline{CF} = \overline{CE}$
- $\angle BFC = \angle DEC$

Proof ***(Please note there is more than one way to prove this theorem.)***

1. Since $\triangle AFE$ is an isosceles triangle and lines $\overline{BF}$, $\overline{DE}$ are extended from the equal lines of the triangle, $\overline{AF}$ and $\overline{AE}$, then $\angle EFB = \angle FED$, because they are the angles below the base of the triangle (**Prop 1.5**).



2. With respect with the two triangles, $\triangle EFB$ and $\triangle FED$, $\overline{FE} = \overline{FE}$ because they are the same line, $\overline{BF} = \overline{DE}$ is given, and $\angle EFB = \angle FED$, therefore $\overline{BE} = \overline{DF}$, $\triangle EFB = \triangle FED$, and $\angle FED = \angle EFB$ and $\angle EBF = \angle FDE$ (**Prop 1.4**).



3. Since $\angle FEB = \angle EFD$ and $\angle FEB = \angle FEC$ and $\angle EFD = \angle EFC$ then $\angle FEC = \angle EFC$ (**A1**). Therefore, $\overline{CE} = \overline{CF}$ (**Prop 1.6**) and $\triangle FCE$ is an isosceles triangle (**Def 20**).
4. $\angle EFB = \angle FED$ (From #1).
   $\angle EFD + \angle DFB = \angle FEB + \angle BED$ because $\angle EFB = \angle EFD + \angle DFB$ and $\angle FED = \angle FEB + \angle BED$ (**A1**).
   $\angle EFD + \angle DFB = \angle EFD + \angle BED$ since $\angle FEB = \angle EFD$ (**A1**). Then $\angle DFB = \angle BED$ after subtracting $\angle EFD$ (**A3**).
   But $\angle DFB = \angle BFC$ and $\angle BED = \angle DEC$, therefore, $\angle BFC = \angle DEC$ (**A1**).
5. In conclusion, if $\triangle AFE$ is an isosceles triangle and $\overline{BF} = \overline{DE}$, then $\triangle FCE$ is an isosceles triangle and $\angle BFC = \angle DEC$.

Q.E.D.