

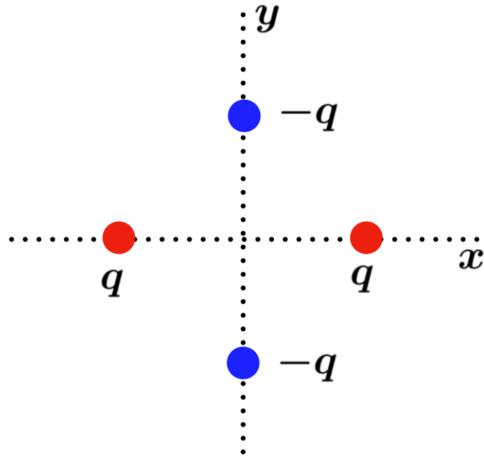
PHYS UN1602 Recitation Week 1 Worksheet

TA: Nitya Nigam

January 29-30, 2025

Problem 1

Four charged particles are located in the $x - y$ plane, each a distance d from the origin. The two charges on the x axis have charge q , and the two on the y axis have charge $-q$, as shown in the diagram below.



- Determine the electric field at the origin, $(x, y, z) = (0, 0, 0)$.
- Determine the electric field at any point on the z axis, i.e. at $(0, 0, z)$.
- Find an exact expression for the electric field along the x axis, $\vec{E}(x, 0, 0)$, for:
 - $x < -d$
 - $-d < x < d$
 - $x > d$
- Find an approximate expression for your result in part c) in the limit $x \gg d$, using the binomial approximation:

$$(1 + \delta)^a \approx 1 + a\delta + \frac{a(a-1)}{2}\delta^2$$

Make sure to evaluate all terms to the same order in d/x .

Solution.

- a) $\vec{E}(0, 0, 0) = 0$ due to the symmetry of the system - the charges on each axis cancel out.
- b) $\vec{E}(0, 0, z) = 0$ as well, since the total charge in the $x - y$ plane is zero.
- c) $\vec{E}(x, 0, 0) = k \left(\frac{q(x-d)\hat{i}}{|x-d|^3} + \frac{q(x+d)\hat{i}}{|x+d|^3} + \frac{-q(x\hat{i}-d\hat{j})}{(x^2+d^2)^{3/2}} + \frac{-q(x\hat{i}+d\hat{j})}{(x^2+d^2)^{3/2}} \right) = kq \left(\frac{x-d}{|x-d|^3} + \frac{x+d}{|x+d|^3} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$
 - i) For $x < -d$:
$$\vec{E}(x, 0, 0) = kq \left(-\frac{1}{(x-d)^2} - \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$
 - ii) For $-d < x < d$:
$$\vec{E}(x, 0, 0) = kq \left(-\frac{1}{(x-d)^2} + \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$
 - iii) For $x > d$:
$$\vec{E}(x, 0, 0) = kq \left(\frac{1}{(x-d)^2} + \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$
- d) We begin by factorizing out x from the denominators:

$$\vec{E}(x, 0, 0) = kq \left(\frac{1}{x^2 (1 - \frac{d}{x})^2} + \frac{1}{x^2 (1 + \frac{d}{x})^2} - \frac{2x}{|x|^3 (1 + \frac{d^2}{x^2})^{3/2}} \right) \hat{i}$$

Since $x > d$, this simplifies to:

$$\frac{kq}{x^2} \left(\frac{1}{(1 - \frac{d}{x})^2} + \frac{1}{(1 + \frac{d}{x})^2} - \frac{2}{(1 + \frac{d^2}{x^2})^{3/2}} \right) \hat{i}$$

Applying the binomial approximation (to the second order for the first two terms, and the first order for the last one) gives:

$$\begin{aligned} \vec{E}(x, 0, 0) &= \frac{kq}{x^2} \left[\left(1 + 2\frac{d}{x} + 3\frac{d^2}{x^2} \right) + \left(1 - 2\frac{d}{x} + 3\frac{d^2}{x^2} \right) - 2 \left(1 - \frac{3}{2}\frac{d^2}{x^2} \right) \right] \hat{i} \\ &= \frac{kq}{x^2} \left[9\frac{d^2}{x^2} \right] \hat{i} \\ &= \frac{9kqd^2}{x^4} \hat{i} \end{aligned}$$