PHYS UN1602 Recitation Week 7 Worksheet

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Problem 1

a) Find the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current I, as shown in Figure 1.

b) Find the magnetic field at point P on the axis of a tightly wound solenoid (helical coil) consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I, as shown in Figure 2. Express your answer in terms of θ_1 and θ_2. Consider the turns to be essentially circular, and use the result of part a). What is the field on the axis of an infinite solenoid (infinite in both directions)?

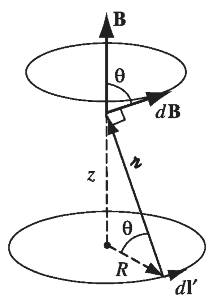

Figure 1

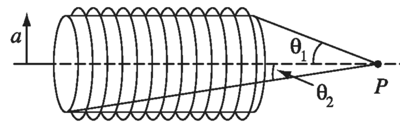

Figure 2

Solution.

a) The field $d\vec{B}$ attributable to the segment $d\vec{l'}$ points as shown. As we integrate $d\vec{l'}$ around the loop, $d\vec{B}$ sweeps out a cone. The horizontal components cancel, and the vertical components combine, giving:

$$B(z) = \frac{\mu_0}{4\pi} I \int \frac{dl'}{R^2+z^2} \cos\theta$$

The factor of $\cos\theta$ projects out the vertical component. Since $\cos\theta$ and R^2+z^2 are constants, and $\int dl'$ is just the circumference $2\pi R$, we have:

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{R^2+z^2}\right) 2\pi R = \frac{\mu_0 I}{2} \frac{R}{\sqrt{R^2+z^2}} \frac{R}{R^2+z^2} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$$

b) Using our answer to part a) for a ring of width dz and replacing I with $nIdz$, we have:

$$B = \frac{\mu_0 n I}{2} \int \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}} dz$$

But:

$$z = \frac{a}{\tan\theta} \implies dz = -\frac{a}{\sin^2\theta} d\theta$$

So:

$$B = \frac{\mu_0 n I}{2} \int \frac{a^2 \sin^3\theta}{a^3 \sin^2\theta}(-a) d\theta = \frac{\mu_0 n I}{2} \int -\sin\theta d\theta = \frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)$$

For an infinite solenoid, $\theta_1 = \pi$ and $\theta_2 = 0$, so $\cos\theta_2 - \cos\theta_1 = 1 - (-1) = 2$ and $B = \mu_0 n I$.

Problem 2

A current I runs along an arbitrarily shaped wire that connects two given points, as shown in Figure 3 (it need not lie in a plane). Show that the magnetic field at distant locations is essentially the same as the field due to a straight wire with current I running between the two points.

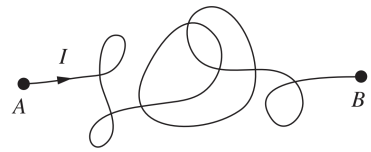

Figure 3

Solution.

The Biot-Savart law gives the field contribution from piece $d\vec{l}$ of the wire as:

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} d\vec{l} \times \hat{r}$$

At a point very far from the wire, the $\hat{r}$ vector and r distance are essentially the same for all points in the wire. So when we integrate over the entire wire, we can take these quantities outside the integral. The field due to the wire is therefore:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi r^2} \int \left(d\vec{l}\right) \times \hat{r} = \frac{\mu_0 I}{4\pi r^2} \vec{l} \times \hat{r}$$

where $\vec{l}$ is the vector from one point to the other. This is the field due to a straight wire between the two points, as desired.