PHYS UN1601 Recitation Worksheet 1

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Problem 1

a) In unit-vector notation, what is $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ if $\vec{a} = 5.0\hat{i} + 4.0\hat{j} - 6.0\hat{k}, \vec{b} = -2.0\hat{i} + 2.0\hat{j} + 3.0\hat{k}$, and $\vec{c} = 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}$?

$\vec{a} - \vec{b} + \vec{c} = (5 - (-2) + 4)\hat{i} + (4 - 2 + 3)\hat{j} + (-6 - 3 + 2)\hat{k}$

$= 11\hat{i} + 5\hat{j} - 7\hat{k}$

b) Calculate the angle between $\vec{r}$ and the positive z axis.

$\hat{z} = 0\hat{i} + 0\hat{j} + 1\hat{k} \Rightarrow \vec{r} \cdot \hat{z} = 11 \cdot 0 + 5 \cdot 0 + -7 \cdot 1 = -7$

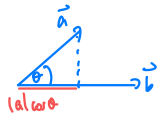
$\vec{r} \cdot \hat{z} = |\vec{r}||\hat{z}| \cos\theta \Rightarrow \cos\theta = \frac{-7}{\sqrt{11^2+5^2+7^2}} = \frac{-7}{\sqrt{121+25+49}} = \frac{-7}{\sqrt{195}} \simeq -0.501$

$\Rightarrow \theta = \cos^{-1}(-0.501) = 2.10 \text{ rad} \simeq 120°$

c) What is the component of $\vec{a}$ along the direction of $\vec{b}$?

$\vec{a} \cdot \vec{b} = 5 \cdot (-2) + 4 \cdot 2 + (-6) \cdot 3 = -10 + 8 - 18 = -20 = |a||b| \cos\theta$

we need $|a|\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|b|} = \frac{-20}{\sqrt{2^2+2^2+7^2}} = \frac{-20}{\sqrt{17}} = -4.85$



d) What is the component of $\vec{a}$ perpendicular to the direction of $\vec{b}$ but in the plane of $\vec{a}$ and $\vec{b}$?

subtract $\vec{b}_{\parallel}$ component: $\vec{a} - (-\frac{20}{17}\hat{b}) = \vec{a} + \frac{20}{17}(-2\hat{i} + 2\hat{j} + 3\hat{k})$

$= 5\hat{i} + 4\hat{j} - 6\hat{k} + (-\frac{40}{17}\hat{i} + \frac{40}{17}\hat{j} + \frac{60}{17}\hat{k})$

$= \frac{45}{17}\hat{i} + \frac{108}{17}\hat{j} - \frac{42}{17}\hat{k}$

$|\vec{a} - |a|\cos\theta\,\hat{b}| = \sqrt{2.65^2 + 6.35^2 + 2.47^2} \simeq 7.31$