

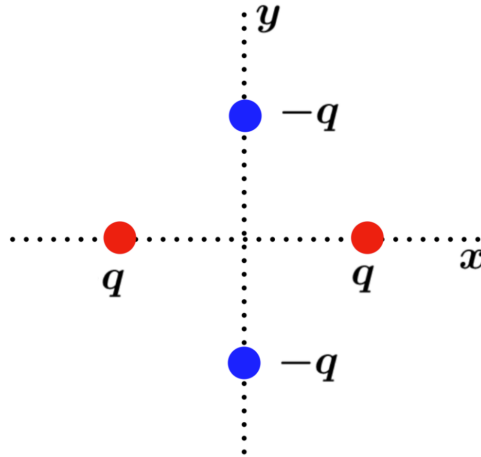
# PHYS UN1602 Recitation Week 1 Worksheet

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## Problem 1

Four charged particles are located in the  $x - y$  plane, each a distance  $d$  from the origin. The two charges on the  $x$  axis have charge  $q$ , and the two on the  $y$  axis have charge  $-q$ , as shown in the diagram below.



- a) Determine the electric field at the origin,  $(x, y, z) = (0, 0, 0)$ .
- b) Determine the electric field at any point on the  $z$  axis, i.e. at  $(0, 0, z)$ .
- c) Find an exact expression for the electric field along the  $x$  axis,  $\vec{E}(x, 0, 0)$ , for:
  - i)  $x < -d$
  - ii)  $-d < x < d$
  - iii)  $x > d$
- d) Find an approximate expression for your result in part c) in the limit  $x \gg d$ , using the binomial approximation:

$$(1 + \delta)^a \approx 1 + a\delta + \frac{a(a-1)}{2}\delta^2$$

Make sure to evaluate all terms to the same order in  $d/x$ .

*Solution.*

a)  $\vec{E}(0,0,0) = 0$  due to the symmetry of the system - the charges on each axis cancel out.

b)  $\vec{E}(0,0,z) = 0$  as well, since the total charge in the  $x - y$  plane is zero.

$$c) \vec{E}(x,0,0) = k \left( \frac{q(x-d)\hat{i}}{|x-d|^3} + \frac{q(x+d)\hat{i}}{|x+d|^3} + \frac{-q(x\hat{i}-d\hat{j})}{(x^2+d^2)^{3/2}} + \frac{-q(x\hat{i}+d\hat{j})}{(x^2+d^2)^{3/2}} \right) = kq \left( \frac{x-d}{|x-d|^3} + \frac{x+d}{|x+d|^3} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$

i) For  $x < -d$ :

$$\vec{E}(x,0,0) = kq \left( -\frac{1}{(x-d)^2} - \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$

ii) For  $-d < x < d$ :

$$\vec{E}(x,0,0) = kq \left( -\frac{1}{(x-d)^2} + \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$

iii) For  $x > d$ :

$$\vec{E}(x,0,0) = kq \left( \frac{1}{(x-d)^2} + \frac{1}{(x+d)^2} - \frac{2x}{(x^2+d^2)^{3/2}} \right) \hat{i}$$

d) We begin by factorizing out  $x$  from the denominators:

$$\vec{E}(x,0,0) = kq \left( \frac{1}{x^2 \left(1 - \frac{d}{x}\right)^2} + \frac{1}{x^2 \left(1 + \frac{d}{x}\right)^2} - \frac{2x}{|x|^3 \left(1 + \frac{d^2}{x^2}\right)^{3/2}} \right) \hat{i}$$

Since  $x > d$ , this simplifies to:

$$\frac{kq}{x^2} \left( \frac{1}{\left(1 - \frac{d}{x}\right)^2} + \frac{1}{\left(1 + \frac{d}{x}\right)^2} - \frac{2}{\left(1 + \frac{d^2}{x^2}\right)^{3/2}} \right) \hat{i}$$

Applying the binomial approximation (to the second order for the first two terms, and the first order for the last one) gives:

$$\begin{aligned} \vec{E}(x,0,0) &= \frac{kq}{x^2} \left[ \left(1 + 2\frac{d}{x} + 3\frac{d^2}{x^2}\right) + \left(1 - 2\frac{d}{x} + 3\frac{d^2}{x^2}\right) - 2\left(1 - \frac{3}{2}\frac{d^2}{x^2}\right) \right] \hat{i} \\ &= \frac{kq}{x^2} \left[ 9\frac{d^2}{x^2} \right] \hat{i} \\ &= \frac{9kqd^2}{x^4} \hat{i} \end{aligned}$$