PHYS UN1601 Recitation Worksheet 7

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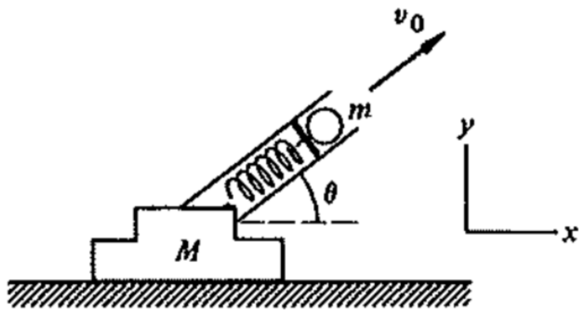
Problem 1

A loaded spring gun, initially at rest on a horizontal frictionless surface, fires a marble at angle θ. The mass of the gun is M, the mass of the marble is m, and the muzzle velocity of the marble is v_0. What is the final motion of the gun?



Solution.

We will use momentum conservation to solve this problem. We are looking for the gun's final recoil speed relative to the table, which we will call v_r. The horizontal component of the marble's velocity in the muzzle is $v_0 \cos\theta$, so its total horizontal velocity with respect to the table is $v_0 \cos\theta - v_r$. Then by momentum conservation:

$$
\begin{aligned}
Mv_r &= m(v_0 \cos\theta - v_r) \\
v_r &= \frac{mv_0 \cos\theta}{M+m}
\end{aligned}
$$

Problem 2

A circus acrobat of mass M leaps straight up with initial velocity v_0 from a trampoline. As he rises up, he takes a trained monkey of mass m off a perch at a heigh h above the trampoline. What is the maximum height attained by the pair?

Solution.

The acrobat's vertical position and velocity are

$$y(t) = v_0 t - \frac{1}{2} g t^2 \tag{1}$$

$$y'(t) = v_0 - gt \tag{2}$$

. At $y = h$, we have $\frac{1}{2} g t^2 - v_0 t + h = 0$ so $t = \frac{v_0^2 \pm \sqrt{v_0 - 2gh}}{g}$ and hence $y' = v_i = \pm\sqrt{v_0^2 - 2gh}$ at h. Now we apply momentum conservation:

$$M v_i = (M+m) v_f$$

$$v_f = \frac{M v_i}{M+m}$$

The time taken to get from h to the maximum height can be obtained by plugging into Eq. 1:

$$0 = v_f - gt \implies t = \frac{v_f}{g}$$

Then plugging this into Eq. 2, the distance travelled from h to the maximum height is:

$$\begin{aligned} \Delta y &= \frac{v_f^2}{g} - \frac{1}{2}\frac{v_f^2}{g} \\ &= \frac{v_f^2}{2g} \\ &= \frac{1}{2g}\left(\frac{M v_i}{M+m}\right)^2 \\ &= \frac{v_0^2 - 2gh}{2g}\left(\frac{M}{M+m}\right)^2 \end{aligned}$$

So the maximum height is $h + \Delta y$:

$$y_{\max} = \left(\frac{v_0^2}{2g} - h\right)\left(\frac{M}{M+m}\right)^2 + h$$