PHYS UN1602 Recitation Week 11 Worksheet

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Problem 1

Consider the two oppositely traveling electric-field waves:

$$\vec{E}_1 = E_0 \cos(kz - \omega t)\hat{x} \text{ and } \vec{E}_2 = E_0 \cos(kz + \omega t)\hat{x}$$

The sum of these two waves is the standing wave $2E_0 \cos(kz)\cos(\omega t)\hat{x}$.

a) Find the magnetic field associated with this standing electric wave by finding the $\vec{B}$ fields associated with each of the above traveling $\vec{E}$ fields, and then adding them.

b) Find the magnetic field by instead using Maxwell's equations to find the $\vec{B}$ field associated with the standing electric wave $2E_0 \cos(kz)\cos(\omega t)\hat{x}$.

Problem 2

We are familiar with the Lorentz boost in the following form (in the x direction):

$$\begin{aligned} ct' &= \gamma\,(ct - \beta x) \\ x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \end{aligned}$$

where $\beta = v/c$. In this problem, we will write the Lorentz boost in a different form.

a) Define a spacetime vector for an event as:

$$\mathbf{X} = \begin{pmatrix} ct \\ x \end{pmatrix}$$

Write the Lorentz transformation in matrix form using this 2-vector, i.e., express the transformation as:

$$\mathbf{X}' = \mathbf{\Lambda}\mathbf{X}$$

Give the 2×2 matrix $\mathbf{\Lambda}$ explicitly in terms of γ and β.

b) Recall that $\beta = v/c$. Taylor expand $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ for small β (i.e. $v \ll c$) and rewrite $\mathbf{\Lambda}$ for this case. Show that this recovers the Galilean transformation, except for a small correction to t.

c) Let's define a new variable called *rapidity*, ζ, such that:

$$\beta = \tanh\zeta \text{ and } \gamma = \cosh\zeta$$

Show that $\gamma\beta = \sinh\zeta$.

d) Using these identities, rewrite the Lorentz transformation matrix $\mathbf{\Lambda}$ in terms of hyperbolic functions $\cosh\zeta$ and $\sinh\zeta$.

e) Compare the matrix you obtained to a standard 2D rotation matrix:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

How are they similar? How are they different?

f) Show that under this transformation, the spacetime interval

$$s^2 = -(ct)^2 + x^2$$

is preserved, i.e. $s'^2 = s^2$. Compare this to the way distances are preserved under circular rotations in Euclidean space. What's the analogy?