PHYS UN1601 Recitation Week 8 Demonstration Problems

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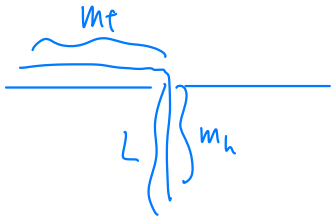
Problem 1

A rope of length l and mass m lies at rest on a horizontal frictionless table. A length l_0 of the rope hangs through a hole in the table. Find an expression for the hanging length of rope, L, in terms of time, l, and l_0.



$$m_t = \left(\frac{l-L}{l}\right)m$$

$$m_h = \frac{L}{l}m$$

$$\Rightarrow \quad T = m_t a_t = m_t \ddot{L} \quad \Rightarrow \quad T = \left(\frac{l-L}{l}\right)m\ddot{L}$$

$$T - m_h g = -m_h \ddot{L} \quad \Rightarrow \quad T - \frac{L}{l}mg = -\frac{L}{l}m\ddot{L}$$

$$\Rightarrow \left(\frac{l-L}{l}\right)m\ddot{L} - \frac{L}{l}mg = -\frac{L}{l}m\ddot{L}$$

$$l\ddot{L} - L\ddot{L} - Lg = -L\ddot{L}$$

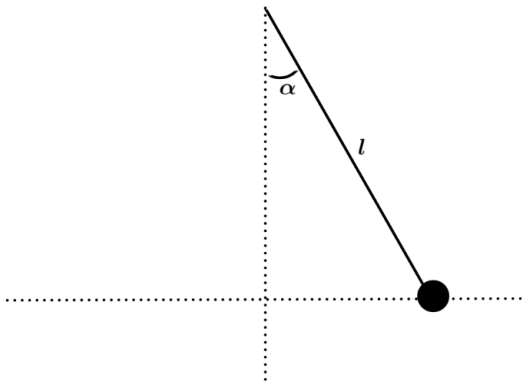
$$\ddot{L} = \frac{g}{l}L \quad \Rightarrow \quad L(t) = Ae^{\sqrt{\frac{g}{l}}t} + Be^{-\sqrt{\frac{g}{l}}t}$$

$$L(0) = A + B = l_0 \; ; \quad \dot{L}(0) = A\sqrt{\frac{g}{l}} - B\sqrt{\frac{g}{l}} = 0 \Rightarrow A = B$$

$$\Rightarrow A = B = \frac{l_0}{2} \quad \Rightarrow \quad \boxed{L(t) = \frac{l_0}{2}\left(e^{\sqrt{\frac{g}{l}}t} + e^{-\sqrt{\frac{g}{l}}t}\right)}$$

Problem 2

A conical pendulum is formed by a mass m hanging from a rope of length l. The mass moves in a circle in the horizontal plane with angular frequency ω.



a) Find α, the angle the rope makes with the vertical, in terms of m, l, and ω.

b) Find the tension of the rope.

a)

$T\cos\alpha = mg \Rightarrow T = \frac{mg}{\cos\alpha}$

$T\sin\alpha = m\omega^2 l\sin\alpha$

$\Rightarrow \frac{mg}{\cos\alpha} = m\omega^2 l \Rightarrow \boxed{\alpha = \cos^{-1}\left(\frac{g}{l\omega^2}\right)}$

b) $T = \frac{mg}{\cos\alpha} = \frac{mg}{g/l\omega^2} = ml\omega^2$

OR $T\sin\alpha = m\omega^2 l\sin\alpha \Rightarrow T = m\omega^2 l$ directly