

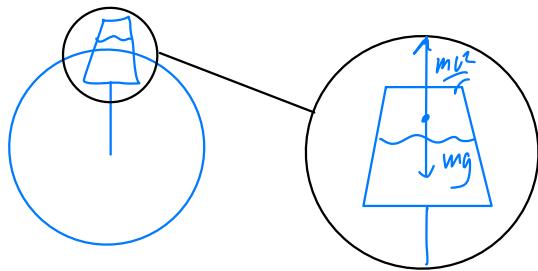
PHYS UN1601 Recitation Worksheet 4

TA: Nitya Nigam

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Problem 1

You hold the handle of a bucket of water and swing it around in a vertical circle, keeping your arm straight. If you swing it around fast enough, the water will stay inside the bucket, even at the highest point where the bucket is upside down. What, roughly, does “fast enough” mean here? (You can specify the maximum time of each revolution.) Make whatever reasonable assumptions you want to make for the various parameters involved. You can work in the approximation where the speed of the bucket is roughly constant throughout the motion.



$$\Rightarrow \frac{mv^2}{R} = mg$$

$$v = \sqrt{gR}$$

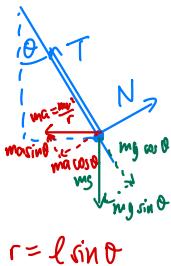
$$\text{for } R \sim 1\text{m}, \quad v \approx 3\text{ms}^{-1}$$

$$\Rightarrow T = \frac{2\pi R}{v} \approx 2\text{s}$$

Problem 2

A mass m is attached by a massless string of length l to the tip of a frictionless cone. The half-angle at the vertex of the cone is θ . If the mass moves around in a horizontal circle at speed v on the cone, find:

- the tension in the string
- the normal force from the cone
- the maximum speed v for which the mass stays in contact with the cone.



a) direction parallel to cone

$$a = \frac{v^2}{r}, \quad r = l \sin \theta$$

$$ma \sin \theta = T - mg \cos \theta$$

$$\Rightarrow T = mg \cos \theta + \frac{mv^2}{l \sin \theta} \sin \theta = m(g \cos \theta + \frac{v^2}{l})$$

note that as $\theta \rightarrow 0$, $T \rightarrow mg$ (since $v \rightarrow 0$)

as $v \uparrow$, $T \uparrow$ as expected

b) $ma \cos \theta = mg \sin \theta - N$

$$\Rightarrow N = mg \sin \theta - \frac{mv^2}{l \sin \theta} \cos \theta = m(g \sin \theta - \frac{v^2}{l \tan \theta})$$

as $\theta \rightarrow 0$, $N \rightarrow 0$; $\theta \rightarrow \frac{\pi}{2}$, $N \rightarrow mg$

$v \uparrow$, $N \downarrow$ all as expected

c) stays in contact for $N \geq 0$

$$\Rightarrow g \sin \theta \geq \frac{v^2}{l \tan \theta} \Rightarrow v \leq \sqrt{g l \sin \theta \tan \theta} = v_{\max}$$

$\theta \rightarrow 0$, $v_{\max} \rightarrow 0$; $\theta \rightarrow \frac{\pi}{2}$, $v_{\max} \rightarrow \infty$