PHYS UN1601 Recitation Week 9

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Rocket motion

A rocket moves by burning fuel that is continuously ejected backwards with velocity u_e relative to the rocket. Let the velocity of the rocket as function of time be $u(t)$, and let the amount of mass expelled in an infinitesimal time increment dt be dm. We also define the mass of the rocket and the fuel it still carries to be $m(t)$. We can use momentum conservation to find, and then solve, a differential equation for $u(t)$.

The momentum of the rocket before expelling dm of fuel is $p_i = m \cdot u$. After an increment dt, the momentum is $p_f = (m - dm)(u + du) + dm(u + u_e)$, where the first term is the new momentum of the rocket, and the second term is the momentum of the ejected fuel (note the velocity change is $u + u_e$, since u_e is a velocity relative the rocket that is moving at u). We can expand this:

$$\begin{aligned} p_f &= (m - dm)(u + du) + dm(u + u_e) \\ &= mu \cancel{- udm} + mdu - du\,dm \cancel{+ udm} + u_e dm \qquad = mu + mdu + u_e dm \end{aligned}$$

We drop the $du\,dm$ term since it is a product of infinitesimal quantities.

We now apply the impulse equation:

$$\begin{aligned} \Delta p = F dt &= p_f - p_i \\ &= \cancel{mu} + mdu + u_e dm - \cancel{mu} \end{aligned}$$

$$\implies F_{net} = \frac{\Delta p}{dt} = m(t)\frac{du}{dt} + u_e \frac{dm}{dt} \tag{1}$$

Equation 1 above is the **rocket equation**, relating the net external forces on the rocket to its acceleration and rate of mass loss. We can use it to solve problems involving rocket motion, such as those in Dourmashkin 12.3.