PHYS UN1601 Recitation Worksheet 2

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Problem 1

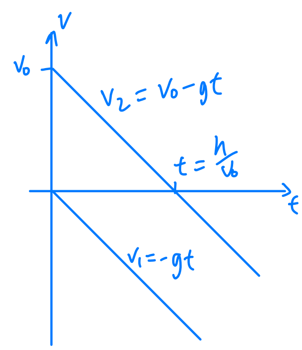
A ball is dropped from rest at height h. Directly below on the ground, a second ball is simultaneously thrown upward with speed v_0. If the two balls collide at the moment the second ball is instantaneously at rest, what is the height of the collision? What is the relative speed of the balls when they collide? Draw the v vs. t plots for both balls.

$y_1 = h - \frac{1}{2}gt^2$; $y_2 = v_0 t - \frac{1}{2}gt^2$ $\Rightarrow$ $y_1 = y_2$ when $h = v_0 t$ $\Rightarrow$ $t = h/v_0$

$y_1' = -gt$; $y_2' = v_2 = v_0 - gt = 0$ for at rest $\Rightarrow$ $t = \frac{v_0}{g}$; $v_0 = gt = \frac{gh}{v_0}$ $\Rightarrow$ $v_0^2 = gh$

$y_c = h - \frac{1}{2}g\frac{h^2}{v_0^2} = h - \frac{1}{2}\cancel{g}\frac{h^{\cancel{2}}}{\cancel{gh}} = h - \frac{h}{2} = \boxed{\frac{h}{2} = y_c}$

relative vel. $= y_2' - y_1' = v_0 - \cancel{gt - (-gt)} = \boxed{v_0}$



Problem 2

A ball is thrown at an angle θ up to the top of a cliff of height L, from a point a distance L from the base, as shown in Fig. 1.

a) As a function of θ, what initial speed causes the ball to land right at the edge of the cliff?

$L = v_0 \cos\theta \, t$; $L = v_0 \sin\theta \, t - \frac{1}{2}gt^2$

↳ $t = \frac{L}{v_0 \cos\theta}$ $\Rightarrow$ $\cancel{L} = \cancel{v_0}\sin\theta \frac{\cancel{L}}{\cancel{v_0}\cos\theta} - \frac{1}{2}g\frac{L^{\cancel{2}}}{v_0^2\cos^2\theta}$

$\Rightarrow$ $\frac{gL}{2v_0^2\cos^2\theta} = \tan\theta - 1$ $\Rightarrow$ $\boxed{\sqrt{\frac{gL}{2\cos^2\theta(\tan\theta - 1)}} = v_0}$

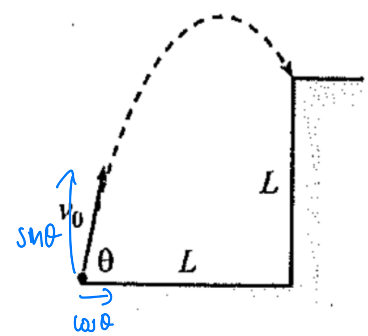


Figure 1

b) There are two special values of θ for which you can check your result. Check these.

$\theta = 90°$ (straight up): $\cos\theta = 0$ $\Rightarrow$ denominator $\to 0$, $v_0 \to \infty$

$\theta = 45°$ (straight at corner): $\tan\theta = 1$ $\Rightarrow$ denominator $\to 0$, $v_0 \to \infty$ (gravity pulls ball down)

can consider this from formula $v_0 = \sqrt{\frac{gL}{2\cos^2\theta(\tan\theta - 1)}}$

↳ if $\tan\theta - 1 < 0$ $\Rightarrow$ $\tan\theta < 1$ $\Rightarrow$ $\theta < 45°$, we have $-$ sign inside sqrt $\to$ problem

if $\cos^2\theta = 0$ $\Rightarrow$ $\cos\theta = 0$ $\Rightarrow$ $\theta = 90°$, denominator blows up $\to$ problem

Problem 3

You throw a ball from a plane inclined at angle θ. The initial velocity is perpendicular to the plane, as shown in Fig. 2. Consider the point P on the trajectory that is farthest from the plane. For what angle θ does P have the same height as the starting point? (For the case shown in the figure, P is higher.) Answer this in two steps:

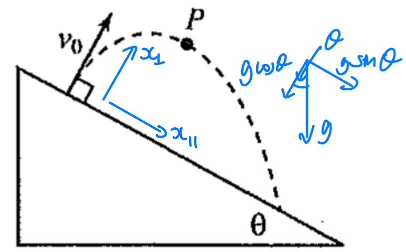


Figure 2

a) Give a continuity argument that explains why such a θ should in fact exist.

θ small: plane flat, v_0 straight up $\Rightarrow$ P above starting point

θ large: plane vertical, v_0 horizontal $\Rightarrow$ ball falls, P below starting point

so there must be some θ btw. 0 and 90° such that P is the same height as the starting point

b) Find θ. In getting a handle on where (and when) P is, it is helpful to use a tilted coordinate system and to isolate what is happening in the direction perpendicular to the plane.

$x_\perp = x_0 + v_0 t - \frac{g\cos\theta}{2} t^2 \Rightarrow$ maximized when $x_\perp' = 0$

$x_\perp' = v_0 - g\cos\theta\, t = 0 \Rightarrow t_p = \frac{v_0}{g\cos\theta}$

vertical component of $v_0 = v_0\cos\theta \Rightarrow t$ to peak $= \frac{v_0\cos\theta}{g} \Rightarrow t_{return} = \frac{2v_0\cos\theta}{g}$

$t_p = t_r \Rightarrow \frac{\cancel{v_0}}{\cancel{g}\cos\theta} = \frac{2\cancel{v_0}\cos\theta}{\cancel{g}} \Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \boxed{\theta = 45°}$