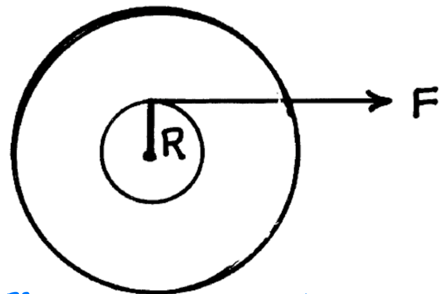
PHYS UN1601 Recitation Week 12 Worksheet

TA: Nitya Nigam

December 4, 2024

Problem 1

A wheel is attached to a fixed shaft, and the system is free to rotate without friction. To measure the moment of inertia of the wheel-shaft system, a tape of negligible mass wrapped around the shaft is pulled by a known constant force F. When a length L of tape has unwound, the system is rolling with angular speed ω_0. Find the moment of inertia of the system, I_0.



$$\tau = FR = I_0 \frac{d\omega}{dt} \Rightarrow \frac{d\omega}{dt} = \frac{FR}{I_0} \Rightarrow d\omega = \frac{FR}{I_0}\, dt' \Rightarrow \omega(t) = \frac{FR}{I_0} t$$

$$\theta = \int \omega\, dt = \frac{FR}{I_0} \int t\, dt = \frac{FR}{2I_0} t^2$$

at $t = t_0$, $L = R\theta = \frac{FR^2}{2I_0} t_0^2 \Rightarrow t_0 = \sqrt{\frac{2I_0 L}{FR^2}}$

$\omega = \omega_0$ at $t = t_0 \Rightarrow \omega_0 = \frac{FR}{I} t_0 = \sqrt{\frac{2\cancel{I_0}LF^{\cancel{2}}\cancel{R^2}}{\cancel{FR^2}I_0^{\cancel{2}}}} = \sqrt{\frac{2LF}{I_0}} \Rightarrow \boxed{I_0 = \frac{2LF}{\omega_0^2}}$

OR $\tau = FR = I_0 \alpha$

$$v^2 = \cancel{v_0^2} + 2aL \underset{v=\omega R}{\Rightarrow} \omega_0^2 R^{\cancel{2}} = 2\alpha \cancel{R} L \Rightarrow \alpha = \frac{\omega_0^2 R}{2L}$$

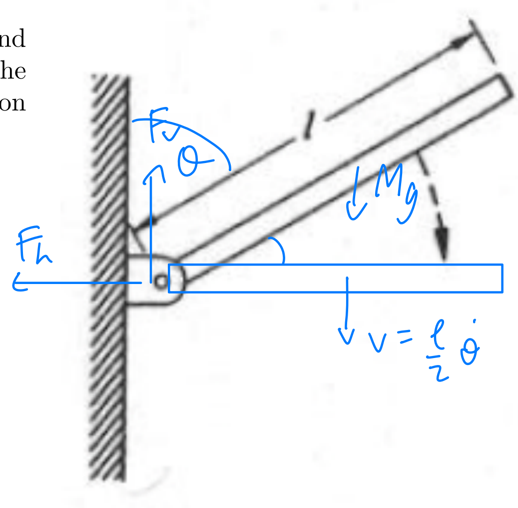
$$\Rightarrow I_0 = \frac{FR}{\alpha} = F\cancel{R} \cdot \frac{2L}{\omega_0^2 \cancel{R}} = \frac{2FL}{\omega_0^2}$$

OR work $= \Delta E \Rightarrow W = FL = \frac{1}{2} I_0 \omega_0^2$

$$\Rightarrow I_0 = \frac{2FL}{\omega_0^2}$$

Problem 2

A thin plank of mass M and length l is pivoted at one end (see figure to the right). The plank is released at 60° from the vertical. What is the magnitude and direction of the force on the pivot when the plank is horizontal?



$$F_h = \frac{Mv^2}{(\frac{l}{2})} = \frac{2M}{l}\left(\frac{l}{2}\dot{\theta}\right)^2 = \frac{Ml\dot{\theta}^2}{2}$$

$$Mg - F_v = Ma = M\frac{l}{2}\ddot{\theta}$$

$$\Rightarrow \quad I\ddot{\theta} = \tau = Mg\frac{l}{2}\sin\theta$$

$$\ddot{\theta}(90°) = \frac{\cancel{M}g\frac{\cancel{l}}{2}\cdot 1}{\frac{1}{3}\cancel{M}l^{\cancel{2}}} = \frac{3}{2}\frac{g}{l}$$

energy conservation: $E(60°) = Mg\frac{l}{2}\cos(60°) = \frac{Mgl}{4}$

$$E(90°) = \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}\frac{1}{3}Ml^2\dot{\theta}^2 = \frac{\dot{\theta}^2}{6}Ml^2$$

$$\frac{\cancel{M}g\cancel{l}}{\cancel{4}\,2} = \frac{1}{\cancel{6}\,3}\cancel{M}l^{\cancel{2}}\dot{\theta}^2 \Rightarrow \dot{\theta}(90°) = \sqrt{\frac{3g}{2l}}$$

$$\Rightarrow \quad F_h = \frac{M\cancel{l}}{2}\frac{3g}{2\cancel{l}} = \frac{3Mg}{4}, \quad F_v = Mg - \frac{Ml}{2}\ddot{\theta}$$

$$= Mg - \frac{M\cancel{l}}{2}\frac{3}{2}\frac{g}{\cancel{l}} = \frac{Mg}{4}$$