PHYS UN1602 Recitation Week 6 Worksheet

TA: Nitya Nigam

March 5-6, 2025

Problem 1

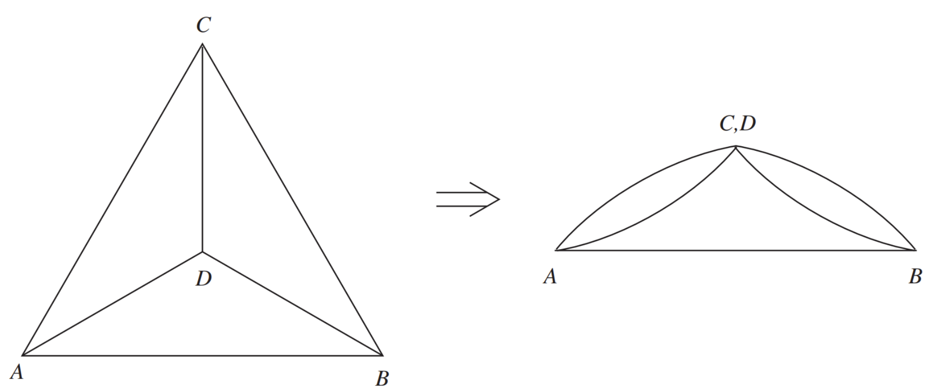
A tetrahedron has equal resistors R along each of its six edges. Find the equivalent resistance between any two vertices. Do this by:

a) using the symmetry of the tetrahedron to reduce it to an equivalent resistor;

b) laying the tetrahedron flat on a table, hooking up a battery with an emf $\mathcal{E}$ to two vertices, and writing down the four loop equations.

Solution.

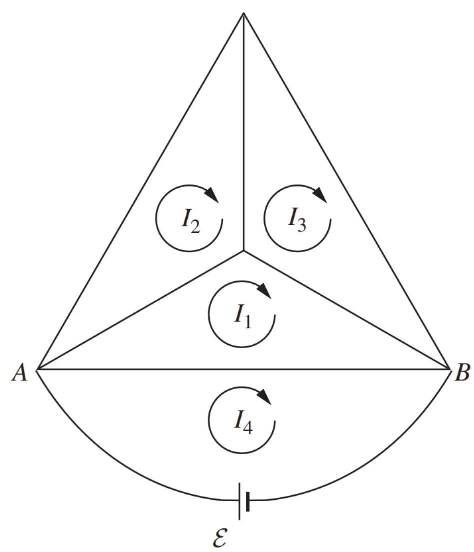
a) Let the two vertices be A and B. By symmetry, the other two vertices are at the same potential, so we can collapse them to one point (because no current will flow in the resistor connecting them). We then have the equivalent network in the figure below, and the effective resistance quickly comes out to be $R/2$.



b) The four loop equations for the figure below are (dividing through by R in all of them)

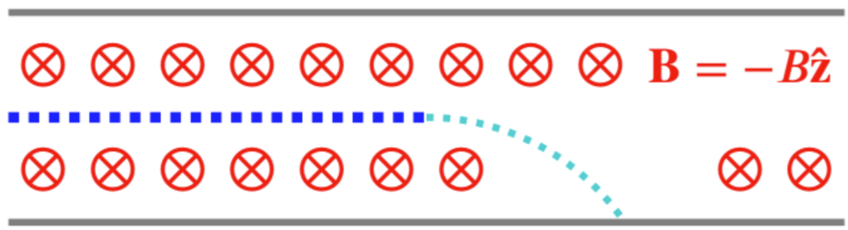
$$\begin{aligned} \mathcal{E}/R - (I_4 - I_1) = 0 \\ -(I_1 - I_4) - (I_1 - I_2) - (I_1 - I_3) = 0 \\ -I_2 - (I_2 - I_3) = (I_2 - I_1) = 0 \\ -I_3 - (I_3 - I_2) - (I_3 - I_1) = 0 \end{aligned}$$

The last two equations are symmetric in I_2 and I_3. Taking their difference yields $I_2 = I_3$, as expected from the symmetry. The third equation then gives $I_1 = 2I_2$, which gives us $I_4 = 2I_1$ from the second equation. Finally, the first equation gives $\mathcal{E}/R - (I_4 - I_4/2) = 0 \implies \mathcal{E} = I_4(R/2)$. Since I_4 is the current through the battery, this implies that the effective resistance between A and B is $R/2$, in agreement with part a).



Problem 2

Consider a parallel plate capacitor of distance d with a beam of electrons each with velocity v travelling through the capacitor arrangement and a magnetic field, B, coming into the page, as shown below, where the smaller dotted line shows where the electrons initially go in the presence of the magnetic field. You can assume that the charge carrier density is n and the drift velocity is the same as our velocity v.



a) Compute the magnetic force (including its direction) on the beam of charges.

b) Eventually, enough negative charges will gather upon the plate to produce a uniform electric field, imparting an opposing force on the electron beam realigning it into a straight path. What is that electric field?

c) Given that the electric field is constant, what is the potential difference?

d) Write the potential difference as a product involving the current. What is the expression for the current of charges? The remaining term is called the *Hall resistance*. What is the Hall resistance?

Solution.

a) The charge is $-e$ for each charge. The velocity is $\vec{v} - v\hat{x}$. The magnetic field is $\vec{B} = -B\hat{z}$. So:

$$\vec{F} = (-e)(v\hat{x}) \times (-B\hat{z}) = -evB\hat{y}$$

b) The total force when an electric field is involved is $q(\vec{E} + \vec{v} \times \vec{B})$. The electric force must be equal and opposite the above, so $-e\vec{E} = evB\hat{y}$. The electric field is hence $\vec{E} = -vB\vec{y}$.

c) The electric potential for a constant electric field is just $|E|d$, where d is the distance along the direction of the electric field, which is just the distance between the two capacitor plates. This gives us $V = vBd$.

d) We know that the current is $I = JA = nev(xd)$, where xd is the cross-sectional area and nev is the current density of charges. As such:

$$V = vBd = \frac{B}{nex}I$$

The remaining term is the Hall resistance:

$$R = \frac{B}{nex}$$