PHYS UN1602 Recitation Week 11 Worksheet

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April 23-24, 2025

Problem 1

The star Gliese 832 is located 16 lightyears away from the Earth. Suppose we send a spacecraft to investigate the recently discovered exoplanet in that star system with velocity $v = \frac{4}{5}c$ and $\gamma = \frac{5}{3}$.

a) According to the occupants of the spacecraft, how long does the trip take?

b) According to the occupants of the spacecraft, how far did they travel?

c) Once they arrive, they send a light-based signal back to Earth. According to the people on Earth, how much time passed between the spacecraft leaving and the signal being received?

Solution.

a) According to someone in the Earth's rest frame, traveling 16 lightyears at a velocity of $v = \frac{4}{5}c\ \mathrm{ms}^{-1}$ will take:

$$\Delta t_E = \frac{16\ \mathrm{ly}}{\frac{4}{5}\ \mathrm{ly\ /\ yr}} = \left(\frac{5}{4}\ \mathrm{yr\ /\ ly}\right)(16\ \mathrm{ly}) = 20\ \mathrm{yr}$$

Someone on the spacecraft moving at a relative velocity v will experience time dilation compared to the observer on Earth and measure an elapsed time of:

$$\Delta t_{SC} = \frac{1}{\gamma}\Delta t_E = \frac{3}{5}(20\ \mathrm{yr}) = 12\ \mathrm{yr}$$

b) According to a stationary observer in Earth's rest frame, the spacecraft traveled 16 lightyears. However, due to length contraction the occupants of the spacecraft measure the distance to be:

$$\Delta L_{SC} = \frac{1}{\gamma}\Delta L_E = \frac{3}{5}(16\ \mathrm{ly}) = 9.6\ \mathrm{ly}$$

c) According to the Earth-based observer, it took them 20 yr to get to Gliese 832, and then 16 yr for the light to return. So 36 years in total.

Problem 2

High-energy photons propagating through space can convert into electron-positron pairs by scattering with cosmic microwave background (CMB) photons. Taking the average CMB temperature of 2.8 K, a typical CMB photon will have an energy of roughly 7×10^{-4} eV. Calculate the minimum energy required for the high-energy photon to produce an electron-positron pair ($m_e = 511$ keV) if:

a) The CMB photon momentum is perpendicular to that of the high-energy photon.

b) The CMB photon propagates in the direction opposite the high-energy photon.

c) Suppose the CMB photon propagates in the same direction as the high-energy photon. Is it ever possible for the two photons to collide and produce an electron-positron pair?

Solution.

a) The ability to produce an e^-e^+ pair is constrained by the system mass of the scattering, which is given by:

$$m_{\text{total}}^2 = \mathbf{p}_{\text{total}}^2 = \mathbf{p}_\gamma^2 + \mathbf{p}_{\text{CMB}}^2 + 2\mathbf{p}_\gamma \cdot \mathbf{p}_{\text{CMB}} = 2\mathbf{p}_\gamma \cdot \mathbf{p}_{\text{CMB}}$$

where we used the fact that photons have zero mass. Further simplifying:

$$m_{\text{total}}^2 = 2(E_\gamma E_{\text{CMB}} - \vec{p}_\gamma \cdot \vec{p}_{\text{CMB}}) = 2E_\gamma E_{\text{CMB}}(1 - \cos\theta)$$

Solving for E_γ gives:

$$E_\gamma = \frac{m_{\text{total}}^2}{2E_{\text{CMB}}(1 - \cos\theta)}$$

So at an angle of θ between the high-energy photon and the CMB photon, the minimum energy required to produce an electron-positron pair is:

$$E_\gamma(\theta) = \frac{(2 \cdot 511 \times 10^3)^2}{2 \cdot 7 \times 10^{-4} \cdot (1 - \cos\theta)}$$

Using this formula:

$$E_\gamma\left(\theta = \frac{\pi}{2}\right) = 746 \text{ TeV}$$

b)

$$E_\gamma(\theta = \pi) = 373 \text{ TeV}$$

c) At $\theta = 0$, the formula for E_γ becomes singular. Looking again at the expression for the system mass, we see that this is because in this case m_{total} is always 0. This tells us that there is no energy available for interactions, so the photons cannot collide to produce an electron-positron pair. This is in line with our intuition, as both photons must always move colinearly at the speed of light in all inertial frames.