PHYS UN1601 Recitation Week 11 Worksheet

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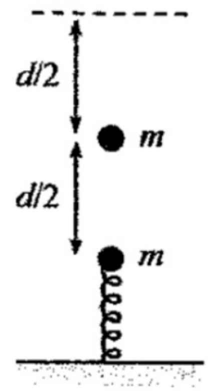
Problem 1



A vertical spring is compressed a distance d relative to its relaxed length. A mass m is attached to the top end and held at rest. Another mass m is suspended a distance $\frac{d}{2}$ above it, as shown in the figure. The spring is released. The bottom mass rises up, smashes and sticks to the upper mass, and resulting blob continues to rise up. What must the spring constant k be, so that the blob reaches its maximum height at the dotted line shown (where the spring is at its relaxed length)?

energy conservation btw. initial time & immediately before collision:

$$\underbrace{0}_{KE} + \underbrace{0}_{\text{grav PE}} + \underbrace{\frac{1}{2}kd^2}_{\text{spring PE}} = \underbrace{\frac{1}{2}mv^2}_{KE} + \underbrace{mg\frac{d}{2}}_{\text{grav PE}} + \underbrace{\frac{1}{2}k\left(\frac{d}{2}\right)^2}_{\text{spring PE}}$$

$$\Rightarrow \quad v^2 = \frac{3}{4m}kd^2 - gd$$

momentum conservation accross collision: $mv = 2mv_f \Rightarrow v_f = \frac{v}{2} \Rightarrow v_f^2 = \frac{v^2}{4}$

energy conservation btw. immediately after collision & max height:

$$\cancel{\tfrac{1}{2}}\cancel{2}m v_f^2 + \cancel{2}mg\frac{d}{\cancel{2}} + \frac{1}{2}k\left(\frac{d}{2}\right)^2 = 0 + 2mgd + 0$$

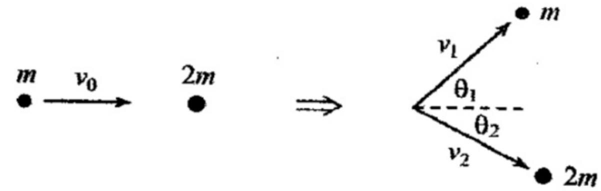
$$m\frac{v^2}{4} = mgd - \frac{1}{2}k\frac{d^2}{4}$$

$$v^2 = 4gd - \frac{kd^2}{2m} = \frac{3}{4m}kd^2 - gd$$

$$\frac{5}{4m}kd^2 = 5gd \quad \Rightarrow \quad \boxed{k = \frac{4mg}{d}}$$

Problem 2

A mass m moving with speed v_0 collides elastically with a stationary mass $2m$, as shown in the figure. Assuming that the final energies of the masses turn out to be equal, find the two final speeds v_1 and v_2. Also, find the two angles of deflection, θ_1 and θ_2. *Hint:* The best way to solve for angles is usually to square equations and use the fact that $\sin^2\theta + \cos^2\theta = 1$.



elastic $\Rightarrow E_0 = E_1 + E_2$, but $E_1 = E_2 \Rightarrow E_1 = E_2 = \frac{E_0}{2}$

$$\Rightarrow \cancel{\tfrac{1}{2}}\cancel{m} v_1^2 = \frac{1}{2}\left(\cancel{\tfrac{1}{2}}\cancel{m} v_0^2\right), \quad \frac{1}{2} 2m v_2^2 = \frac{1}{2}\left(\frac{1}{2} m v_0^2\right) \Rightarrow \boxed{v_1 = \frac{v_0}{\sqrt{2}}, \quad v_2 = \frac{v_0}{2}}$$

horizontal p-conservation: $\cancel{m}\cancel{v_0} = \cancel{m}\frac{\cancel{v_0}}{\sqrt{2}}\cos\theta_1 + \cancel{2m}\frac{\cancel{v_0}}{\cancel{2}}\cos\theta_2 \Rightarrow \cos\theta_2 = 1 - \frac{\cos\theta_1}{\sqrt{2}}$

vertical p-conservation: $0 = \cancel{m}\frac{\cancel{v_0}}{\sqrt{2}}\sin\theta_1 - \cancel{2m}\frac{\cancel{v_0}}{\cancel{2}}\sin\theta_2 \Rightarrow \sin\theta_2 = \frac{\sin\theta_1}{\sqrt{2}}$

$$\Rightarrow \cancel{\sin^2\theta_2 + \cos^2\theta_2}^{\,1} = \cancel{\frac{\sin^2\theta_1}{2} + \frac{\cos^2\theta_1}{2}}^{\,\frac{1}{2}} - \frac{2\cos\theta_1}{\sqrt{2}} + \cancel{1}$$

$$\Rightarrow \frac{2\cos\theta_1}{\sqrt{2}} = \frac{1}{2} \Rightarrow \cos\theta_1 = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} \Rightarrow \boxed{\theta_1 \simeq 69.3^\circ}$$

$$\cos\theta_2 = 1 - \frac{\cos\theta_1}{\sqrt{2}} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \boxed{\theta_2 \simeq 41.4^\circ}$$