PHYS UN1602 Recitation Week 5 Worksheet

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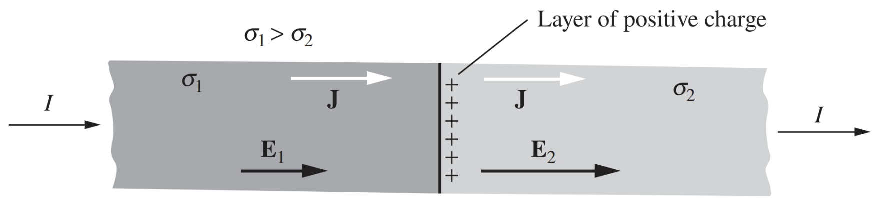
Problem 1

Show that the total amount of charge at the junction of the two materials in the figure below is $\varepsilon_0 I \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$, where I is the current flowing through the junction, and σ_1 and σ_2 are the conductivities of the two conductors.



Solution.

Since J is the same in both materials, we have:

$$J = \sigma_1 E_1 = \sigma_2 E_2$$

By Gauss' law, the "charge enclosed" on the boundary is given by:

$$\frac{Q}{\varepsilon_0} = \Delta E \cdot A$$
$$\frac{Q}{\varepsilon_0} = \implies \left(\frac{J}{\sigma_2} - \frac{J}{\sigma_1}\right) \cdot A$$
$$Q = \varepsilon_0 J A \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$
$$Q = \varepsilon_0 I \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

as required.

Problem 2

A laminated conductor is made by depositing, alternately, layers of silver 100 angstroms thick and layers of tin 200 angstroms thick (1 angstrom $= 10^{-10}$ m). The composite material, considered on a larger scale, may be considered a homogeneous but anisotropic material with an electrical conductivity $\sigma_\perp$ for currents perpendicular to the planes of the layers, and a different conductivity $\sigma_\parallel$ for currents parallel to that plane. Given that the conductivity of silver is 7.2 times that of tin, find the ratio $\sigma_\perp/\sigma_\parallel$.

Solution.

The ratio of conductivities, 7.2/1, and the ratio of layer thicknesses, 1/2, are the only two things that matter. Let us arbitrarily pick $\sigma_s = 1$ and $\sigma_t = 1/7.2$ (ignoring the units). The resistance of an object with length L and cross-sectional area A can be written as $\frac{L}{\sigma A}$.

For perpendicular currents, the layers are in series, so the resistances add:

$$\begin{aligned} R_\perp = R_s + R_t \implies \frac{L}{\sigma_\perp A} &= \frac{L_s}{\sigma_s A} + \frac{L_t}{\sigma_t A} \\ \frac{1}{\sigma_\perp} &= \frac{1}{\sigma_s}\frac{L_s}{L} + \frac{1}{\sigma_t}\frac{L_t}{L} \\ &= 1 \cdot \frac{1}{3} + 7.2 \cdot \frac{2}{3} = 5.133 \end{aligned}$$

For parallel currents, the layers add via inverses:

$$\begin{aligned} R_\parallel = \frac{1}{R_s} + \frac{1}{R_t} \implies \frac{\sigma_\parallel A}{L} &= \frac{\sigma_s A_s}{L} + \frac{\sigma_t A_t}{L} \\ \sigma_\parallel &= \sigma_s \frac{A_s}{A} + \sigma_t \frac{A_t}{A} \\ &= 1 \cdot \frac{1}{3} + \frac{1}{7.2} \cdot \frac{2}{3} = 0.426 \end{aligned}$$

Then $\frac{\sigma_\perp}{\sigma_\parallel} = \frac{1}{5.133} \cdot \frac{1}{0.426} = 0.457$.