PHYS UN1602 Recitation Week 8 Worksheet

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Problem 1

a) Consider an infinite straight wire carrying current I. We know that the magnetic field outside the wire is $\vec{B} = (\mu_0 I/2\pi r)\hat{\theta}$. There are no currents outside the wire, so $\nabla \times \vec{B} = 0$. Verify this by explicitly calculating the curl.

b) Since $\nabla \times \vec{B} = 0$, we should be able to write $\vec{B}$ as the gradient of a function, $\vec{B} = \nabla\psi$. Find ψ, but then explain why the usefulness of ψ as a potential function is limited.

Solution.

a) Since $\vec{B}$ only has a $\hat{\theta}$ component, the only term in the curl (in cylindrical coordinates) that might be nonzero is:

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}\hat{z}$$

But $B_\theta \propto \frac{1}{r}$, so this term is zero as desired.
If we were to use Cartesian coordinates, we would notice that the z-axis os a symmetry, and the tangential $\vec{B}$ field is proportional to $\pm(-y, x, 0)$. Since the magnitude of $\vec{B}$ must be $\mu_0 I/2\pi r$, we need:

$$\vec{B} = \frac{\mu_0 I}{2\pi(x^2+y^2)}(-y, x, 0)$$

Then the only possibly nonzero component of the curl would be

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$$

which we can easily check equals zero.

b) In cylindrical coordinates, the $\hat{\theta}$ component of the gradient of a function ψ is

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\theta}$$

So we need:

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\mu_0 I}{2\pi r} \implies \frac{\partial\psi}{\partial\theta} = \frac{\mu_0 I}{2\pi} \implies \psi = \frac{\mu_0 I\theta}{2\pi}$$

The problem is that ψ is multivalued: for a given r and z, the values $\theta = 0, 2\pi, 4\pi\cdots$ all correspond to the same point in space, but have different values of ψ. Therefore, ψ cannot be used as a potential that uniquely labels each point in space.

Problem 2

A thin ring of radius a carries a static charge q. This ring is in a magnetic field of strength B_0, parallel to the ring's axis, and is supported so that it is free to rotate about that axis. If the field is switched off, how much angular momentum will be added to the ring? Suppose the mass of the ring is m. Show that the ring, if initially at rest, will acquire an angular velocity $\omega = qB_0/2mc$.

Solution.

The force applied to the ring is

$$\vec{F} = q\vec{E}d \implies d\vec{F} = \vec{E}\, dq = \vec{E}\, \lambda\, d\vec{l}$$

Applying the right-hand-rule, we can calculate the torque:

$$d\vec{\tau} = \vec{a} \times d\vec{F} = a\lambda\vec{E}d\vec{l} \implies \vec{\tau} = a\lambda \int \vec{E}d\vec{l} = a\lambda\Delta V$$

Recalling that torque is rate of change of angular momentum L:

$$\frac{dL}{dt} = a\lambda\left(-\frac{1}{c}\frac{d\Phi_B}{dt}\right) \implies L = \frac{qB_0a^2}{2c}$$

Then, since $L = I\omega$ and the momentum of inertia for a ring is $I = ma^2$:

$$ma^2\omega = \frac{qB_0a^2}{2c} \implies \omega = \frac{qB_0}{2mc}$$