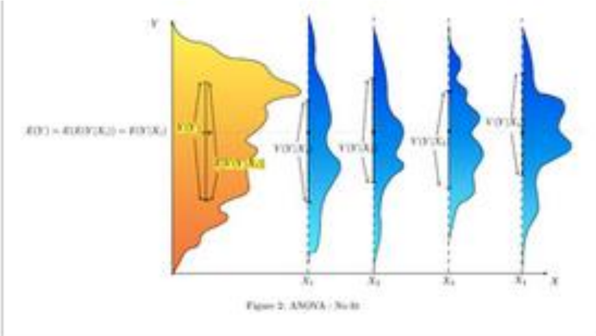
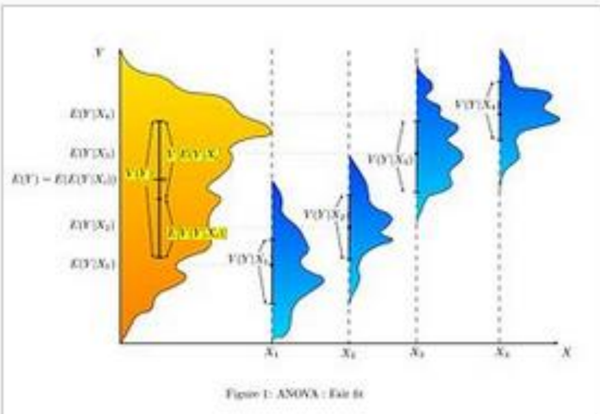
Analysis of variance

Analysis of variance (ANOVA) is a collection of statistical models used to analyze the differences among group means and their associated procedures (such as "variation" among and between groups).

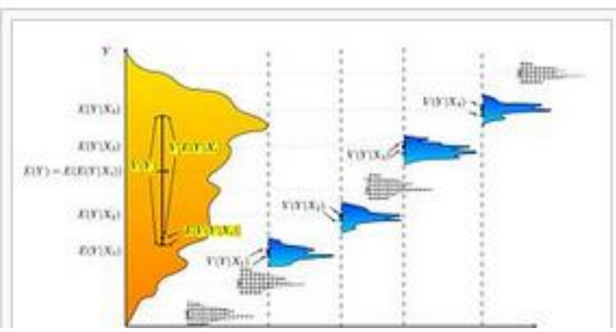
In the ANOVA setting, the observed variance in a particular variable is partitioned into components attributable to different sources of variation. In its simplest form, ANOVA provides a statistical test of whether or not the means of several groups are equal, and therefore generalizes the t-test to more than two groups. ANOVAs are useful for comparing (testing) three or more means (groups or variables) for statistical significance. It is conceptually similar to multiple two-sample t-tests, but is less conservative (results in less type I error) and is therefore suited to a wide range of practical problems.



No fit.



Fair fit



Good fit

The calculations of ANOVA can be characterized as computing a number of means and variances, dividing two variances and
Calculating a treatment effect is then trivial, "the effect of any treatment is estimated by taking the difference between the mea

Partitioning of the sum of squares [edit]

Main article: Partition of sums of squares

ANOVA uses traditional standardized terminology. The definitional equation of sample variance is $s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2$, w
sum of squares (SS), the result is called the mean square (MS) and the squared terms are deviations from the sample mean.
deviations from the grand mean, an error variance based on all the observation deviations from their appropriate treatment m
of treatment means from the grand mean, the result being multiplied by the number of observations in each treatment to acco
means.

The fundamental technique is a partitioning of the total sum of squares *SS* into components related to the effects used in the
different levels.

$$SS_{\text{Total}} = SS_{\text{Error}} + SS_{\text{Treatments}}$$

The number of degrees of freedom *DF* can be partitioned in a similar way: one of these components (that for error) specifies
same is true for "treatments" if there is no treatment effect.

$$DF_{\text{Total}} = DF_{\text{Error}} + DF_{\text{Treatments}}$$

See also Lack-of-fit sum of squares.

The F-test [edit]

Main article: F-test

The F-test is used for comparing the factors of the total deviation. For example, in one-way, or single-factor ANOVA, statistica

$$F = \frac{\text{variance between treatments}}{\text{variance within treatments}}$$

$$F = \frac{MS_{\text{Treatments}}}{MS_{\text{Error}}} = \frac{SS_{\text{Treatments}}/(I-1)}{SS_{\text{Error}}/(n_T - I)}$$

where *MS* is mean square, I = number of treatments and n_T = total number of cases

to the F-distribution with $I-1$, $n_T - I$ degrees of freedom. Using the F-distribution is a natural candidate because the test
chi-squared distribution.

```
> anova(lm(folate~ventilation))
Analysis of Variance Table

Response: folate
            Df Sum Sq Mean Sq F value  Pr(>F)
ventilation  2  15516    7758  3.7113 0.04359 *
Residuals   19  39716    2090
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Chi-Square Goodness of Fit Test

Suppose that you manage an academic support service for high school students. The district leadership has been skeptical about adopting your model because they claim that your students (we'll call them tutored students) do not represent the general population of students in the district high schools. To challenge their claim, you need to compare your students with the general characteristics of students in the district. You could conduct a number of independent samples *t* tests on performance measures (e.g., CAHSEE scores), but how do you compare characteristics, such as gender, SES, ethnicity, and mobility?

Let's start with gender. The district high school student gender composition is 65% female and 35% male. You have been helping 14 female students and 10 male students. How similar or different are these two groups regarding gender composition? To answer this question, you should conduct a chi-square goodness of fit test (or one-sample chi-square test). The specifics for this test are as follows. First, you are interested in a nominal variable (i.e., gender). Second, you have both a current group and a comparison group. In this example, the current group is the 24 students using your service (14 girls, 10 boys), and the comparison group is the total district high school student population, which you know to be 65% female and 35% male. Third, based on these two groups, you compare the two sets of frequencies. Two very important concepts are involved: observed frequency and expected frequency. Observed frequency is the count of observations for the current group - in this case, the 24 students. Expected frequency is the count of frequencies in the comparison group - in this case, all high school students. The comparison of observed frequencies and expected frequencies is called chi-square analysis. This type of analysis is often displayed using a cross-tabulation (or crosstab) table. Here is the start of the table for this analysis.

	Female	**Male**	**Total**
Tutored Students	14	10	24
High School Students			

The numbers in the row for Tutored Students are the **observed** frequencies. What are the expected frequencies? Using the percentages 65% and 35% we can construct expected frequencies to use for comparison. For the girls, 65% of 24 is 15.6 and for the boys 35% of 24 is 8.4. These numbers are used in the row for High School Students.

	Female	**Male**	**Total**
Tutored Students	14	10	24
High School Students	15.6	8.4	24

Obviously, the frequencies are different, but how important is the difference? To answer this question, we use a familiar technique - we find the difference between the Observed and Expected frequencies (called a residual), square each of those differences, divided each squared difference by the Expected frequency, and then add up the results. This technique should sound vaguely similar to the calculation of the variance. Residual is another synonym for deviation.

For this example, the residuals are shown below:

	Female	Male
Residuals	-1.6	1.6

Squaring these residuals and dividing by 15.6 and 8.4, respectively, results in the following:

	Female	Male	Total
Squared Residuals/ Expected Frequency	.164	.305	.469

The value in the **Total** column is called X^2 (chi-square, pronounced ki as in kite). This is the test statistic for the comparison of the observed and expected frequencies. As with other test statistics, we compare the obtained value with the critical value to determine whether to reject or retain the null hypothesis.

For a chi-square test, the null hypothesis is that the two sets of frequencies (i.e., observed and expected) are equal. The alternative hypothesis is that they are unequal.

The closer the obtained chi-square is to zero, the more similar the two sets of frequencies are - or, stated another way, the better the observed data fit the expected pattern. This interpretation is where the term "goodness of fit" originates.

Because .494 is greater than .05, we fail to reject the null hypothesis, and instead, determine that the two patterns of genders are similar.