PROCRASTINATION UNDER UNCERTAINTY

MINGZI NIU*

ABSTRACT: I propose a tractable model of procrastination. A present-biased agent has a task of uncertain workload by a fixed deadline. I characterize the agent's effort over time and study comparative statics with respect to present bias and workload uncertainty. The analysis reveals that workload uncertainty can alleviate procrastination, but it entails a welfare loss that increases in present bias. I provide two applications of the model. First, I consider a natural commitment device for a long-term task, namely, committing to a series of short-term goals. I show that short-term goals weakly impair a present-biased agent's welfare. This provides a cautionary counterpoint to the bulk of literature on time inconsistency, where commitment can strictly enhance welfare for present-biased agents. Second, I show that present bias and naivete are observationally equivalent; however, time preference and workload uncertainty can be disentangled.

KEYWORDS: Procrastination, Quasi-Hyperbolic Discounting, Limited Commitment, Belief Elicitation, Present Bias, Naivete

JEL CLASSIFICATION: D81, D83, D91

*DEPARTMENT OF ECONOMICS, RICE UNIVERSITY. MINGZI.NIU@RICE.EDU
I am particularly indebted to Mallesh Pai for his continued guidance, and I am grateful to Nina Bobkova, Hülya Eraslan, and Isabelle Perrigne for their valuable suggestions. I also thank Attila Ambrus, Charles Becker, Rahul Deb, Jeffrey Ely, Botond Kőszegi, Yusufcan Masatlioglu, Matt Mitchell, Phillip Sadowski, Todd Sarver, Leeat Yariv, Huseyin Yildirim as well as conference participants at SEA, Midwest, NASMES, AMES, EGSC and TETC for helpful comments.

1. Introduction

Procrastination prevails among people. From students staying up late before assignment due dates to business analysts working overtime before quarterly financial reports, from households postponing profitable mortgage refinance to politicians cramming for agenda deadlines, similar behavioral patterns of self-defeating delay arise across scenarios.[1] While individuals may envision an ideal work schedule that smooths the workload over time before they begin a task, they often prioritize immediate gratification over long-term consequences, putting off work to the future.[2] Due to this inconsistency in dynamic choices, people have to catch up with an overwhelming backlog of work as the due date approaches. Intense work at the last minute is usually stressful, error-prone, and thus undesirable ex ante.[3]

In the study of procrastination, there are two puzzles that require further investigation. The first puzzle centers on the insufficient demand for commitment. Theories (e.g., Gul and Pesendorfer (2001)) suggest that commitment devices can be used to counter dynamic inconsistency and ultimately improve individual well-being. However, in both experimental and market settings, individuals do not purchase commitment devices as much as predicted by theory (e.g., Laibson (2015)).

The second puzzle is: how potential driving forces for procrastination can be disentangled? The existing literature relies primarily on present bias and naivete to account for procrastination; however, workload uncertainty may well be another plausible source for procrastination.[4] I then ask: to what degree does an agent procrastinate due to her innate inclination to put off work; and to what degree is the procrastination simply caused by her uncertainty about the task workload? The separation of preference and belief has always been an intriguing problem in empirical studies and has accrued growing attention.

[1] I use the term "procrastination" to mean the detrimental tendency of delaying work and subsequently rushing to finish the task. The precise definition of procrastination (and worse procrastination) is provided by *Definition 1* in *Section 2*. Anecdotal and experimental evidence abounds for procrastination; see *Section 5.1* for a detailed literature review. Many universities have built special programs to combat academic procrastination, e.g., `https://mcgraw.princeton.edu/understanding-and-overcoming-procrastination`, `https://www.lib.sfu.ca/about/branches-depts/slc/learning/procrastination`. For political procrastination, see, e.g., `https://www.nytimes.com/2021/10/01/us/politics/infrastructure-bill-last-minute.html?searchResultPosition=3`.

[2] In Augenblick, Niederle and Sprenger (2015)'s experiments on time inconsistency regarding real effort tasks, subjects arranged around 9% less work to the present compared to their original plan.

[3] Empirical studies (e.g., Tice and Baumeister (1997), Burns et al. (2000), Sirois and Pychyl (2016)) show that procrastinators, on average, report lower achievements, self-esteem and poorer health. More broadly, household financial reluctance affects macroeconomic policy-making (Laibson, Maxted and Moll (2021)), and a surging work intensity to meet deadline can breed careless mistakes and ruin collective goal pursuits.

[4] Loosely speaking, "present bias" refers to a tendency to attach a distinctively high weight to the present well-being in the intertemporal tradeoff; "naivete" refers to a mistake in self-perception that underestimates present bias in the future. For a more precise definition, see *Section 2*.

To this end of identifying the source of procrastination, we need a model that incorporates present bias, naivete, and workload uncertainty.

In this paper, I provide a tractable model in which a potentially present-biased and naive agent needs to complete a long-term task before a fixed deadline but she is uncertain about the task's difficulty. By a long-term task, I mean the task cannot be completed in one shot (such as paying a traffic fine); instead, the agent must exert effort over time to complete the task (such as working on a project). A difficult task requires more effort (i.e., workload) to complete. I allow the task features of the workload, the time available, and the workload uncertainty to be flexible. I also include an option to insert intermediate deadlines for the task as a commitment device. In this rich task environment, I then investigate what task features exacerbate procrastination, and how the effects of task features vary with present bias and naivete.

The current paper departs from the literature that identifies *individual behavioral frictions* (e.g., present bias, naivete; see *Section 5.1* for a literature review) as the cause of time-inconsistent behaviors. Rather, the main focus of this paper is to identify the *task features* that contribute to procrastination, given individual behavioral frictions. It speaks to a major paradigm in social psychology that a nudge in situational factors can overwhelm personal trait differences in affecting people's behaviors (e.g., Ross (1977)). Such a concern motivates O'Donoghue and Rabin (2001) as well. In their paper, an increased value of task options or an expanded menu of tasks may induce procrastination. Following this line of inquiry, I investigate how the workload, the time available, the workload uncertainty and intermediate deadlines shape an agent's work schedule. The analysis thus informs long-term task design. For example, it can guide project managers in choosing how much information about task workload to reveal, and whether or how to insert intermediate deadlines for projects.

This paper sheds light on the two puzzles above regarding procrastination. Researchers have approached the enigma of insufficient commitment demand through the lenses of:

(a) naivete — people underestimate their future present bias;
(b) preference for flexibility — payoff-relevant information may arrive later;
(c) the direct cost — the price of commitment exceeds the benefit from it.

In this paper, I find an alternative mechanism that is particularly involved in the long-term task completion, such that absent these three channels, a present-biased agent is still better off *not* committing herself. For the puzzle of identifying driving forces for procrastination, the analytical solution of the model provides a way to distinguish the influence of behavioral frictions from that of workload uncertainty.

1.1. *Overview of Model and Main Results*

In the model, an agent exerts effort in continuous time to complete a task with a fixed deadline. Once the task is completed by the deadline, she gets a reward. The problem is that she is uncertain about the total workload — the amount of accumulated effort that yields task completion. The workload can be either low (w_L) or high (w_H). The agent has a prior belief that the workload is low with probability $\mu \in [0, 1]$. The flow cost of effort is convex, and the agent evaluates future payoffs using quasi-hyperbolic discounting. She optimizes her work schedule *dynamically* to complete the task on time and minimize her (perceived) continuation effort cost for the task completion. For welfare assessment, I distinguish two overall effort cost measures, both evaluated at the start of the task: the ex-ante perceived cost and the long-run cost. The ex-ante perceived cost is evaluated by the agent, measuring the expected total effort cost of her *perceived* work schedule. In contrast, the long-run cost is evaluated by a time-consistent advisor, measuring the expected total effort cost of the agent's *actual* work schedule.[5]

My methodological contribution is to characterize dynamic effort choices under quasi-hyperbolic discounting in a finite horizon. Building on Harris and Laibson (2013), the key to solving dynamic problems under quasi-hyperbolic discounting is a pair of value functions: the value function using quasi-hyperbolic discounting, denoted by $W(\cdot)$, and the continuation value function using exponential discounting, denoted by $V(\cdot)$. Harris and Laibson (2013) study dynamic consumption choices in an infinite horizon; without the need to keep track of time, they characterize a pair of *stationary* value functions. By contrast, in a finite horizon, the value function pair (W, V) varies over time and needs to be calculated recursively at every moment before the deadline, which is usually technically challenging. The continuous-time framework and the assumption of CRRA flow payoff grant analytical tractability.[6] In *Proposition 1*, I characterize the *time-varying* value functions and the agent's work schedule in closed form.

Based on *Proposition 1*, I examine how the work schedule and the overall effort costs vary with behavioral types (namely, present bias and naivete) and task features (namely, the workload, the time available, and workload uncertainty). I show that, for a given task, present bias and naivete trigger and worsen procrastination; for a given agent, increased

[5] These two cost measures differ in two aspects. First, they aggregate flow effort costs using different discounting fashions: a present-biased agent adopts quasi-hyperbolic discounting, whereas a time-consistent advisor adopts exponential discounting. Second, in the case when the agent is naive, the two cost measures calculate cost from different action paths: a naive agent misperceives her future work trajectory, whereas a time-consistent advisor measures the agent's actual work trajectory.

[6] Laibson and Maxted (2020) look at psychologically relevant duration (from minutes to days), after which choices are considered to be made by future selves, for a present-biased agent. They demonstrate that the Instantaneous Gratification (IG) specification of Harris and Laibson (2013), which I adopt in the current paper, closely approximates dynamic choices in discrete time with psychologically relevant duration.

uncertainty around the workload alleviates procrastination. Both behavioral frictions (of present bias and naivete) and environmental frictions (of a heavy workload, little time available, and uncertain workload) make the agent strictly worse off. More importantly, these frictions reinforce each other in undermining the agent's welfare (*Corollary 1* and *Corollary 2*). For example, an increase in workload impairs welfare for any agent since it raises the average effort across time. However, a stronger present bias results in a more disproportionate distribution of this additional workload, thus inflicting an even greater cost in exerting effort. As such, a behaviorally flawed agent suffers more from an adverse task environment compared to a perfectly rational agent. This finding challenges a convenient assumption in behavioral studies that the total cost of action is the sum of the monetary cost and the psychological cost (e.g., Andersen et al. (2020)). Instead, it suggests an interactive term of behavioral frictions and environmental frictions when specifying their overall effects on an agent.

Application 1. The inefficiency arising from present bias and naivete calls for regulation. To overcome procrastination for long-term tasks, one common strategy in practice is to set several intermediate deadlines with short-term goals that break down the total workload. The majority of the existing studies emphasize the positive value of commitment for a present-biased agent. Interestingly, this paper uncovers an unintended negative side effect of commitment that is rarely discussed in the literature.[7] I show that, while committing to short-term goals can mitigate procrastination, it inflicts both a higher long-run cost and a higher ex-ante perceived cost (*Proposition 2*).

One may imagine this adverse effect results from the fact that commitment shrinks the set of alternatives. This is indeed true for a time-consistent agent. However, commitment has no negative welfare impact as long as it does not preclude the most cost-efficient alternative. Furthermore, for an agent experiencing self-control problems (such as a present-biased agent), tying her hands enables the agent to select an ex-ante better option and thereby improve welfare. So the real question posed here is: what is the lurking opposing effect of commitment on a present-biased agent that offsets this positive effect, ultimately resulting in a net value of zero or even a negative value of commitment?

The logic of my result is as follows. If a present-biased agent can commit her choices at every moment, then by committing exactly to a time-consistent agent's work schedule, she can properly induce early efforts and strictly enhance her welfare. However, when commitment is limited — in the sense that choices are made in continuous time while short-term goals are only finite — the agent can only guarantee the amount of accumulated effort before each intermediate deadline, but cannot guarantee how she will

[7]A similar idea is raised in Mullainathan and Shafir (2013). In their book, people with limited resources tend to be captured by a "scarcity mindset". Under this mindset, people focus on their urgent problems, ignoring potential risks and costs involved in their current actions.

arrange her workload over time to achieve that. Actually, driven by present bias, the agent is bound to procrastinate and rush for every short-term goal. I show that the high effort required at the last moment in the repeated procrastination process is extremely costly, and it outweighs the regulatory benefit of committing to short-term goals.

The unique feature of this application is a combination of (i) the agent is present-biased, and (ii) the frequency of dynamic choices exceeds that of commitments. As the preceding arguments suggest, these two conditions combined give rise to the negative commitment value. Studies on commitment usually discuss cases wherein the agent can (fully or partially) commit her actions whenever she gets to make a move. Nevertheless, progress checks in reality are usually less frequent than action choices. For example, an employee can choose whether to work or shirk at any moment, whereas her work is only checked by the manager periodically. As I show in *Section 4.1.3*, the value of commitment dinimishes as the frequency of dynamic choices increases relative to the frequency of commitments. In the limiting case where choices can be made in continuous time under only finite short-term goals, the negative effect of limited commitment is significant enough to completely neutralize its positive regulatory effect, even under optimal short-term goals.

Application 2. If the workload is high, there exist three potential forces that contribute to procrastination: (i) present bias, (ii) naivete, and (iii) a belief that the workload can be low. I discuss in *Section 4.2* the possibility of disentangling these three forces when an agent's work trajectory is observed. Here, the underlying parameters of interest are present bias, naivete, and workload prior. I show that a measure of time preferences and a measure of the prior belief on workload can be jointly identified, whereas present bias and naivete are observationally equivalent (*Proposition 3*).

Outline. The rest of the paper unfolds as follows. I set up the baseline model and a static benchmark in *Section 2*, and work backwards to characterize the agent's dynamic choices in *Section 3*. With the work schedule and individual welfare characterization, I then examine the value of short-term goals and the empirical content of the model in *Section 4*. *Section 5* concludes the paper with a discussion about related literature, a summary of this paper, and avenues for future work. Further results, such as scenarios where the agent has the option to quit the task halfway, can be found on my website for readers who are interested, and are omitted in the current paper for brevity.[8]

2. MODEL

I consider a continuous time, finite horizon framework in which an agent with quasi-hyperbolic discounting optimizes her work schedule dynamically to complete a task. The task requires a total workload $w > 0$ by a deadline at time $T > 0$. The agent is uncertain

[8] `https://drive.google.com/file/d/1RX3df-ELcKxi1UmzoQk7b8R-eJ996bwv/view?usp=drive_link`

about the workload w, and she gets a reward when the task is completed. Effort into the task completion is costly to the agent, so her objective is to complete the task on time with the minimal expected overall effort cost.

Information Structure. The workload can take two values, i.e., $w \in \{w_L, w_H\}$ with $w_H > w_L > 0$. The agent has the prior that $w = w_L$ with probability $\mu \in [0, 1]$. Since the agent gets the reward once she completes the task, the workload uncertainty is naturally resolved after she finishes the workload w_L: if she attains the reward, she learns $w = w_L$; otherwise, she learns $w = w_H$ and continues to complete the work left $w_H - w_L$ within the remaining time. Thus, completing w_L reveals the workload information to the agent.

Payoff. The agent gets a reward $v > 0$ once her total effort into the task reaches w by the deadline T. Denote the effort at time $t \in [0, T]$ by $y_t \geq 0$, and denote the work finished (or the accumulated effort) before time t by $x_t \equiv \int_0^t y_s \mathrm{d}s$. For tractability, I assume the flow cost of effort to be $c(y_t) = \gamma y_t^\alpha$ where $\gamma > 0$ and $\alpha > 1$. The convex effort cost captures the idea that smoothing the workload over time is more cost-efficient for the agent.

Time Preference. To study choices that have influence over time, we need to settle two issues. The first is how the agent evaluates future utility flows at present, which is characterized by her discounting function. The second is how the agent anticipates her future choices, which is captured by the concept of sophistication.

For the first issue, I adopt the instantaneous gratification (IG) model proposed by Harris and Laibson (2013) and specify the discount factor at time t for utility at time $s \geq t$ as

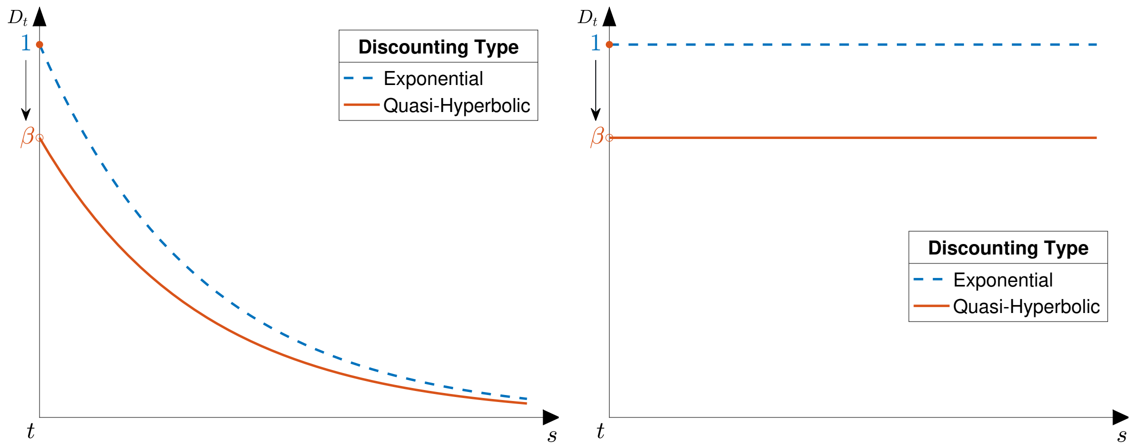
$$D_t(s; \beta, \delta) = \begin{cases} 1 & \text{for } s = t, \\ \beta e^{-\delta(s-t)} & \text{for } s > t, \end{cases} \tag{1}$$

where $\delta \geq 0$ and $\beta \in (0, 1]$. It nests the classic exponential discounting when $\beta = 1$. When $\beta < 1$, all future flow payoffs are further discounted by β in addition to the exponential discounting. Hence, the present flow payoff is weighed distinctively higher in the overall utility, capturing the present bias. As illustrated in *Figure 1*, a smaller β corresponds to a sharper immediate drop in the discount factor and thus a larger present bias.

Fix an effort trajectory $\boldsymbol{y} = \{y_s : 0 \leq s \leq T\}$. Denote $\boldsymbol{y}^t \equiv \{y_s : t \leq s \leq T\}$ as a continuation path of $\boldsymbol{y}$ starting from time $t \in [0, T]$. The agent's effort cost derived from $\boldsymbol{y}$ evaluated at time t is given by

$$C_t(\boldsymbol{y}^t; \beta) = \int_t^T D_t(s; \beta, \delta) c(y_s) \mathrm{d}s. \tag{2}$$

For the second issue, following O'Donoghue and Rabin (2001), I allow the agent to misperceive $\hat{\beta} \in (0, 1]$ as her present bias in the future. Therefore, at time $t \in [0, T)$, the



(a) $\beta \in (0,1), \delta > 0$

(b) $\beta \in (0,1), \delta = 0$

FIGURE 1. Discount Factors Evaluated at Time t

agent anticipates that her effort cost derived from the effort trajectory $\boldsymbol{y}^t$, evaluated at any future moment $r \in (t, T]$, would be $C_r(\boldsymbol{y}^r; \hat{\beta})$.

In the baseline model, I assume the agent discounts all future payoffs equally (i.e., $\delta = 0$),[9] and characterize the agent's time preference by a pair of the actual present bias and the perceived present bias $(\beta, \hat{\beta}) \in (0,1]^2$. We say an agent is sophisticated if she precisely perceives her present bias in the future (i.e., $\hat{\beta} = \beta$), and an agent is naive if she underestimates her present bias in the future (i.e., $\hat{\beta} > \beta$). In particular, a time-consistent agent bears no present bias (i.e., $\hat{\beta} = \beta = 1$), and a fully naive agent is subject to present bias but misperceives that she would be time consistent in the future (i.e., $\beta < \hat{\beta} = 1$).

Work Schedule. Under workload uncertainty, the agent exerts effort over time to complete the task on time and minimize the expected overall effort cost. I allow the agent to adjust her work schedule dynamically. That is, she has the flexibility to revise her plan at every moment.

I denote the work schedule by $(\boldsymbol{x}, \boldsymbol{y}) \equiv \{(x_t, y_t) : t \in [0, T]\}$, where x_t denotes the work that the agent has finished by time $t \in [0, T]$, and y_t denotes her effort at t. At any moment, the agent cares about future payoffs but cannot directly control future choices; she anticipates the choices made by future selves and optimizes her current effort choice accordingly. Therefore, the dynamic work schedule is generated from an intrapersonal

[9]The general case when $\delta \geq 0$ is provided in the online appendix available from the author's webpage. The case $\delta = 0$ is of particular interest. First, due to convex flow effort cost, the most cost-efficient work schedule at any time is to distribute the work left evenly within the remaining time. With this simple benchmark, we can easily detect procrastination by an increasing effort trajectory. Second, the exponential discount rate δ is calibrated at approximately 0 in empirical studies. For example, the daily discount rate is estimated to fall in the range of $(0.001, 0.003)$ in Andreoni and Sprenger (2012), and the weekly discount factor estimated in Augenblick, Niederle and Sprenger (2015) for real effort work is 0.999 (i.e., the weekly discount rate is around 0.001).

sequential game between the current self and future selves. Following Harris and Laibson (2013) and the intergenerational game literature (e.g., Bernheim and Ray (1987)), I use Markov-perfect equilibrium as the solution concept for this game. In a Markov-perfect equilibrium, players (i.e., the current self and future selves) base their decisions on the directly payoff-relevant information — (i) the work finished and (ii) the time spent.

Formally, a Markov pure strategy $\mathbf{e} : [0, w] \times [0, T] \to \mathbb{R}_+$ maps the state of the work finished and the time spent to the effort. Denote by $\boldsymbol{y}(x, t|\mathbf{e})$ an effort trajectory such that the agent uses the effort strategy $\mathbf{e}$ starting from the state (x, t). A Markov-perfect equilibrium consists of a pair of the perceived effort strategy $\hat{\mathbf{e}}$ and the actual effort strategy $\mathbf{e}$ such that for any $x \in [0, w]$,

(i) the perceived effort $\hat{\mathbf{e}}(x, t)$ minimizes the effort cost under the perceived present bias $\hat{\beta}$, $C_t(\boldsymbol{y}(x, t|\hat{\mathbf{e}}); \hat{\beta})$, at any time $t \in (0, T]$, and

(ii) given the perceived future effort strategy $\hat{\mathbf{e}}$, the actual effort $\mathbf{e}(x, t)$ minimizes the effort cost under the actual present bias β, $C_t(\boldsymbol{y}(x, t|\mathbf{e}'); \beta)$, at any time $t \in [0, T]$ where $\mathbf{e}' = \mathbf{e}$ at the current time t and $\mathbf{e}' = \hat{\mathbf{e}}$ at any future time $s > t$.

Here $C_t(\cdot)$ is given by (2). The work schedule $(\boldsymbol{x}, \boldsymbol{y})$ derived from a Markov-perfect equilibrium $(\mathbf{e}, \hat{\mathbf{e}})$ is thus given by: for any $t \in [0, T]$, $x_0 = 0$, $y_t = \mathbf{e}(x_t, t)$, and $x_t = \int_0^t y_s \mathrm{d}s$.

2.1. *Preliminary Results*

Before solving the model, I provide the first-best benchmark in which the agent can fully commit to her initial plan, and she knows the workload and her present bias in the future. Against this benchmark, I define procrastination in the long-term task completion and give a measure for the degree of procrastination.

In this benchmark case, the agent commits to an effort trajectory $\boldsymbol{y}$ that minimizes

$$C_0(\boldsymbol{y}; \beta) = \int_0^T D_0(t; \beta) c(y_t) \mathrm{d}t \quad \text{subject to} \quad \int_0^T y_t \mathrm{d}t = w,$$

which yields the most cost-efficient work schedule to complete the task on time. By the variational approach, it is easy to see that, regardless of the magnitude of present bias, the first-best work schedule $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ is always to spread out the workload evenly over the time available, that is, for $t \in [0, T]$,

$$x_t^* = \frac{w}{T} t, \quad y_t^* = \frac{w}{T}. \tag{3}$$

Intuitively, since the flow cost of effort is strictly convex, any deviation from an even-distributed workload over time would incur a higher overall effort cost.

With this first-best benchmark, I say an agent procrastinates if she falls behind her first-best work schedule $(\boldsymbol{x}^*, \boldsymbol{y}^*)$; an agent procrastinates more than another agent if she finishes strictly less work at any time before the deadline. Formally, I provide the following measure of procrastination.

DEFINITION 1 (Procrastination). *Given any task (w, T),*

(i) an agent with the work schedule $(\boldsymbol{x}, \boldsymbol{y})$ exhibits procrastination if

$$x_t \leq x_t^* \text{ for any } t \in [0, T], \text{ and } x_t < x_t^* \text{ for some } t \in (0, T),$$

where x_t^* is given by (3);

(ii) an agent with the work schedule $(\boldsymbol{x}, \boldsymbol{y})$ procrastinates more than an agent with the work schedule $(\boldsymbol{x}', \boldsymbol{y}')$ if $x_t \leq x_t'$ for any $t \in [0, T]$, and $x_t < x_t'$ for some $t \in (0, T)$.

3. DYNAMIC WORK SCHEDULE

Now we turn to the case where the agent has the discretion about her effort at any moment. Let $\mathcal{T} = \langle w_L, w_H, \mu, T \rangle$ represent features of the task and let $\mathcal{B} = \langle \beta, \hat{\beta} \rangle$ represent behavioral frictions of the agent. *Proposition 1* presents the main result of this paper, which characterizes the agent's work schedule and overall effort costs to complete a task of uncertain workload before the deadline.[10]

PROPOSITION 1 (Work Schedule and Overall Costs for an Uncertain Long-Term Task).

Let $B = (\beta/\hat{\beta})^{\frac{1}{\alpha-1}}(\alpha - 1)/(\alpha - \hat{\beta})$, and $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$. The time when an agent $(\beta, \hat{\beta})$ finishes the low workload w_L is:

$$\tau = \left[1 - (1 - w_L/\lambda)^{\frac{1}{B}}\right] T. \tag{4}$$

The unique work schedule for the agent is:

$$x_t(\mathcal{T}, \mathcal{B}) = \begin{cases} \lambda\left[1 - \left(1 - \frac{t}{T}\right)^B\right] & \text{if } 0 \leq t < \tau, \\ w_L & \text{if } \tau \leq t < T \text{ and } w = w_L, \\ w_L + (w_H - w_L)\left[1 - \left(1 - \frac{t-\tau}{T-\tau}\right)^B\right] & \text{if } \tau \leq t < T \text{ and } w = w_H; \end{cases}$$

$$y_t(\mathcal{T}, \mathcal{B}) = \begin{cases} \frac{\lambda B}{T}\left(1 - \frac{t}{T}\right)^{B-1} & \text{if } 0 \leq t < \tau, \\ 0 & \text{if } \tau \leq t < T \text{ and } w = w_L, \\ \frac{(w_H - w_L)B}{T-\tau}\left(1 - \frac{t-\tau}{T-\tau}\right)^{B-1} & \text{if } \tau \leq t < T \text{ and } w = w_H. \end{cases}$$

The cost function (or the ex-ante perceived cost) is

$$C(\mathcal{T}, \mathcal{B}) = \frac{\gamma B^{\alpha-1}\lambda^\alpha}{T^{\alpha-1}}. \tag{5}$$

[10]All proofs are omitted in the main text and can be found in the appendix.

The long-run cost associated with the work schedule is

$$LC(\mathcal{T},\mathcal{B}) = \begin{cases} \frac{\gamma B^{\alpha}\lambda^{\alpha}}{[1-\alpha(1-B)]T^{\alpha-1}} & \text{if } \alpha(1-B) < 1, \\ \infty & \text{if } \alpha(1-B) \geq 1. \end{cases} \tag{6}$$

Here, $B \in (0,1]$ indicates behavioral frictions borne by the agent, and λ can be interpreted as the agent's initial workload target under workload uncertainty. *Proposition 1* implies that the first-best benchmark is attained by dynamic choices if and only if the agent is time consistent ($B = 1$) and there is no workload uncertainty ($\lambda = w$). If $B < 1$, the agent bunches effort near the deadline, and the long-run cost of the agent's work schedule exceeds the total effort cost perceived by the agent ex ante. Besides, if the workload is uncertain ($\mu \in (0,1)$), task completion involves two phases: completing w_L by time $\tau < T$, followed by a divergence in the work schedule based on whether the reward is granted.

Note that B decreases in present bias and naivete. For a sophisticated agent, $B = (\alpha - 1)/(\alpha - \beta) > 1 - 1/\alpha$ for any $\beta \in (0,1]$.[11] Only for a highly naive agent, B could possibly go below $1 - 1/\alpha$. In this case, too much work is concentrated around the deadline, producing an extremely high long-run cost such that no task is worth taking in the first place. In the main body of the paper, I avoid the discussion about this extreme case and restrict attention to the situation when $\alpha(1 - B) < 1$.[12]

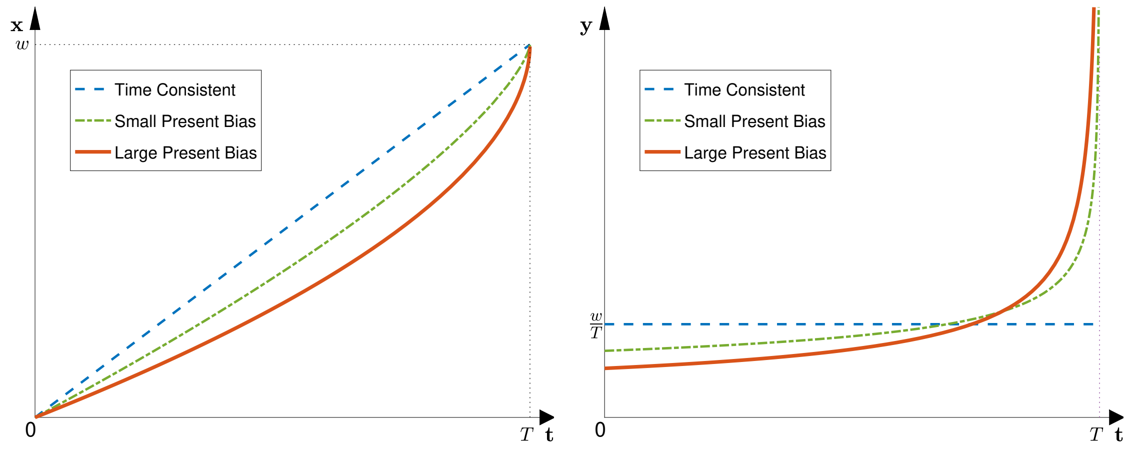
Drawing on *Proposition 1*, I first illustrate the key findings regarding the interactive impacts of behavioral frictions and task features, and demonstrate how workload uncertainty distorts the agent's dynamic decision-making. At the end of this section, I provide the techniques to solve finite-horizon dynamic optimization under quasi-hyperbolic discounting and workload uncertainty.

3.1. *The Interactive Impacts of Behavioral Frictions and Task Features*

We start with the case when the task workload w is known. *Figure 2* illustrates how present bias affects the agent's work schedule. Observe that the work schedule for a time-consistent agent is to spread the total workload evenly over the time available, which achieves the first best for the agent. When present bias comes in, procrastination emerges.

[11] The parameter $\alpha > 1$ captures the agent's aversion to variation in the work intensity (or flow effort); I take it as a measure of "rush aversion". By *Proposition 1*, when α goes to 1, B goes to 0 and a present-biased agent tends to leave all the work to the last moment ($\lim_{\alpha \downarrow 1} x_t = 0$ for all $t < T$, $\beta < 1$). On the other extreme, when α goes to infinity, B goes to 1 and a present-biased agent tends to distribute the workload evenly, acting like a time-consistent agent. This is because, with an extremely high rush aversion, bunching effort near the deadline is too costly. In general, procrastination worsens as rush aversion decreases.

[12] This paper assumes that the agent has to complete the task on time. In the online appendix, I allow the agent to quit if the continuation cost overwhelms the reward value. In this case, I show that a naive agent may too readily undertake a task and then fail to complete it despite her foregone effort. Additionally, short-term goals can secure the completion of a task whose overall cost for the agent exceeds the reward.



(a) Work Trajectory

(b) Effort Trajectory

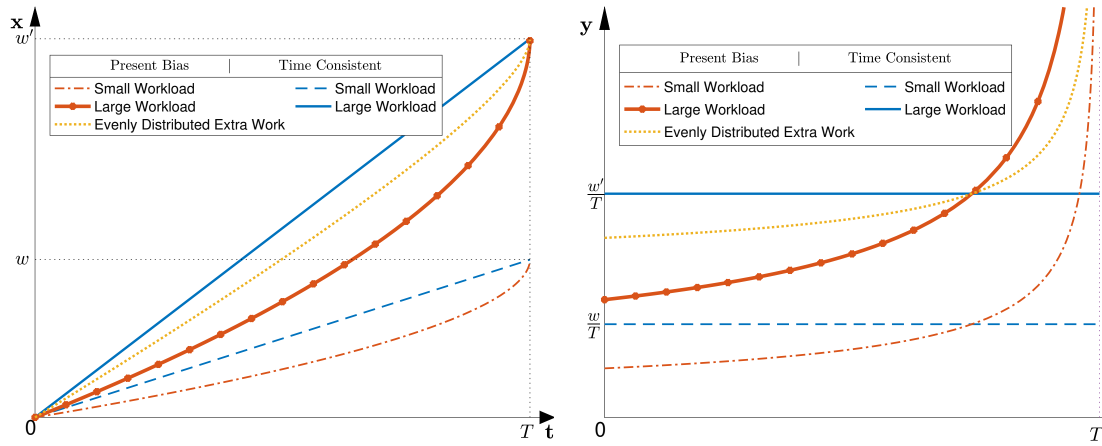
FIGURE 2. Work Schedule: Sophisticated Agent & Certain Workload

The agent's effort into the task is initially low and then rises over time. As the present bias increases, the agent puts off more of the workload toward the deadline and exhibits more severe procrastination. Similarly, a larger naivete corresponds to a work trajectory that tilted more towards deadline — I show later in *Section 4.2* that present bias and naivete are observationally equivalent for a fixed task.

To measure individual welfare, I use the long-run cost LC as in O'Donoghue and Rabin (2001). It represents the overall cost of the agent's work schedule from the perspective of an outsider (or the agent herself in reflection), exempt from present bias. Intuitively, a higher long-run cost is inflicted by a more demanding task (with a heavier workload w and shorter time available T) and more intense behavioral frictions (with a larger present bias and naivete). Furthermore, $\partial LC/\partial w$ and $|\partial LC/\partial T|$ strictly decrease in B. This implies that present bias and naivete magnify the welfare impact of the adverse task features to the agent. Conversely, $\partial LC/\partial B$ grows as w/T gets larger. Therefore, the harm of behavioral friction to the agent is intensified by a harsher task requirement.

COROLLARY 1. *The welfare impact of the workload and of the time available is amplified by her present bias. Additionally, the welfare loss due to present bias is also aggravated by more demanding task requirements (i.e., a larger w or a smaller T).*

The interaction between behavioral frictions and task features in affecting the agent's welfare is a testable prediction of this model. It is caused by the fact that these two factors are intertwined with each other in shaping the agent's work schedule. For example, a higher workload raises the average effort across time for both a time-consistent agent and a present-biased agent, thus impairing welfare to both. However, in contrast to the time-consistent agent's even distribution of the additional workload, the present-biased agent



(a) Work Trajectory

(b) Effort Trajectory

FIGURE 3. The Interactive Impacts: Present Bias & Increased Workload

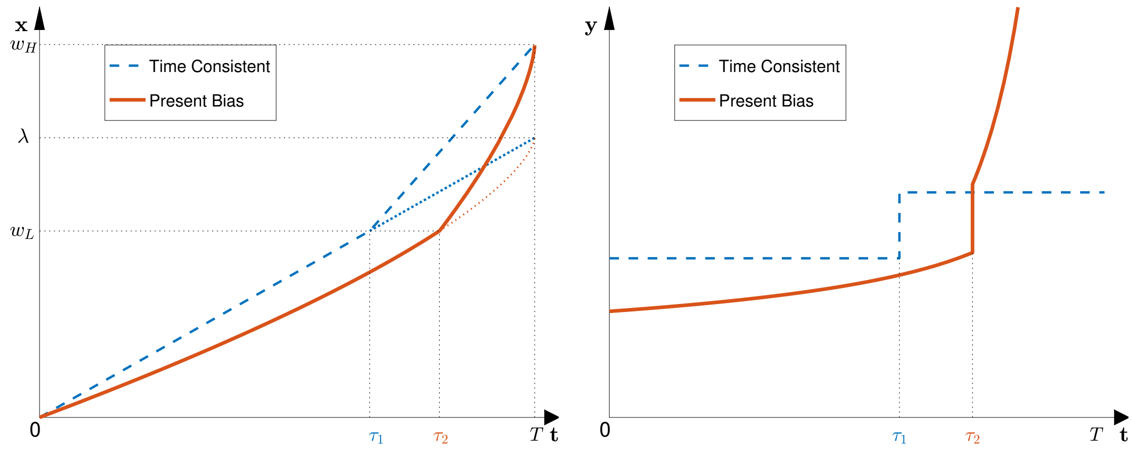
tends to postpone more of the additional workload to the future, engendering an extra cost (see *Figure 3* for an illustration).

3.2. *The Effect of Workload Uncertainty*

Let τ be the time when the agent finishes the low workload w_L. Recall that, once she completes w_L, the agent knows the task workload — based on whether or not she obtains the task completion reward. It follows that work schedule after τ can be characterized as in *Section 3.1*. Therefore, to examine the impact of workload uncertainty, we only need to study: (i) how the agent should exert effort over time before she completes w_L, and (ii) how τ is determined.

Proposition 1 suggests that, before she finishes w_L, the agent acts as if her workload is $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L) \in (\mu w_L + (1-\mu)w_H, w_H)$.The time to learn the actual task workload, τ, is determined accordingly as the accumulated effort reaches w_L, which is given by (4). As depicted in *Figure 4*, the agent's learning-by-doing process features an endogenous "moving-the-goalpost" pattern. Suppose the actual workload is high and the agent is uncertain about that. The agent would initially aim at an intermediate target λ until she completes the workload w_L. Without receiving the reward for task completion, she then raises her target to the actual workload w_H.[13] Uncertainty affects the agent's work schedule through this misspecified target, $\lambda \notin \{w_L, w_H\}$. It depends on task features (w_H, w_L, μ), and is independent of the behavioral frictions of present bias and naivete.

[13]This dynamic pattern manifests itself more clearly when the workload uncertainty is resolved gradually; check the online appendix available from the author's webpage for an illustration. Moreover, as shown clearly in *Figure 4(b)*, an upsurge in effort occurs at time τ. The effort jump strictly increases in the prior μ, capturing the surprise at a high workload.



(a) Work Trajectory

(b) Effort Trajectory

FIGURE 4. Work Schedule: Sophisticated Agent & Uncertain Workload

To examine the impact of workload uncertainty, I take a mean-preserving spread of the workload prior and let it represent an increased workload uncertainty. Formally, denote the prior mean of workload as $\bar{w} = \mu w_L + (1-\mu) w_H$, and let

$$w_H = (1+\varepsilon)\bar{w}, \tag{7}$$

$$w_L = [1-(1-\mu)\varepsilon/\mu]\,\bar{w}. \tag{8}$$

Fixing $\mu \in (0,1)$, I use $\varepsilon \in [0, \mu/(1-\mu)]$ to measure the magnitude of the workload uncertainty. If $\varepsilon = 0$, then $w_H = w_L = \bar{w}$ and there is no workload uncertainty in the prior. As ε rises, the dispersion of the workload prior expands, capturing the idea that the agent is less certain about the task difficulty. Plugging (7) and (8) in the expression of λ, we can obtain that the initial workload target $\lambda = \left\{1 + \left[(1-\mu)^{\frac{1}{\alpha}} - (1-\mu)\right]\varepsilon/\mu\right\}\bar{w} \geq \bar{w}$. The equality holds if and only if there is no workload uncertainty (i.e., $\varepsilon = 0$), and the initial target is boosted up as the workload uncertainty ε increases. This implies that an increased workload uncertainty alleviates procrastination and accelerates the learning of the workload.

So how does the workload uncertainty affect the agent's welfare? As a benchmark, I first calculate the expected cost when uncertainty is resolved *before* the agent chooses her work schedule:

$$\begin{aligned}\Pi(\mathcal{T},\mathcal{B}) &= \mu \cdot LC(w_L, T, \mathcal{B}) + (1-\mu)\cdot LC(w_H, T, \mathcal{B}) \\ &= \frac{\gamma B^{\alpha}[\mu w_L^{\alpha} + (1-\mu) w_H^{\alpha}]}{[1-\alpha(1-B)]T^{\alpha-1}}.\end{aligned}$$

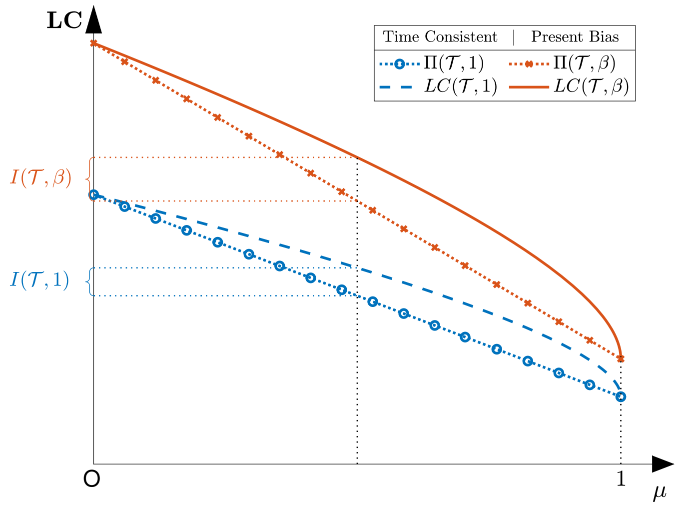


FIGURE 5. The Welfare Effect of Uncertainty

The gap between $LC(\mathcal{T},\mathcal{B})$ and $\Pi(\mathcal{T},\mathcal{B})$, denoted by $I(\mathcal{T},\mathcal{B})$, captures the value of learning workload before starting to work for a present-biased agent:

$$
\begin{aligned}
I(\mathcal{T},\mathcal{B}) &\equiv LC(\mathcal{T},\mathcal{B}) - \Pi(\mathcal{T},\mathcal{B}) \\
&= \frac{\gamma B^{\alpha}}{[1-\alpha(1-B)]T^{\alpha-1}}\left[\lambda^{\alpha} - \mu w_L^{\alpha} - (1-\mu)w_H^{\alpha}\right] \geq 0, \qquad (9)
\end{aligned}
$$

where the equality holds if and only if there is no workload uncertainty (see *Figure 5* for an illustration).[14]

Here, the interactive impact of behavioral frictions and task features rearises: for a given task $\mathcal{T}$, the welfare loss inflicted by workload uncertainty (i.e., the value of information) grows with the agent's present bias and naivete. Intuitively, a present-biased agent, without commitment, cannot overcome the innate inclination to put off what she can do today until tomorrow, and thus fail to finish w_L as early and ideally as a time-consistent agent. (4) formally confirms this intuition since τ strictly decreases in B. Therefore, under workload uncertainty, apart from the effort misdistribution that affects agents of all time preferences, behavioral frictions additionally induce a delay in discovering the actual workload.

COROLLARY 2. *Consider an agent who is uncertain about the workload.*

(i) The agent acts as if her workload is higher than the expected workload until she completes the low workload w_L.

[14]One may wonder: whether it is possible to improve the agent's *ex post* welfare by raising the agent's initial target λ. The answer is positive, as long as $\lambda \in (w, \bar{\lambda})$ in which w is the actual workload, and $w/\bar{\lambda}$ is the unique fixed point in $(0,1)$ for the function $f(\varphi) = 1-(1-\varphi^{\alpha})^{B/[1-\alpha(1-B)]}$.

(ii) An increased workload uncertainty leads to a higher initial workload target, thus promoting more early effort and alleviating procrastination.

(iii) Workload uncertainty undermines the agent's welfare, and behavioral friction amplifies the welfare loss from workload uncertainty (i.e., the value of information).

3.3. *Dynamic Optimization under Quasi-Hyperbolic Discounting*

In this section, I present the main steps in deriving *Proposition 1*. The procedure below can be used to solve dynamic optimization under continuous-time quasi-hyperbolic discounting in finite horizon, and it also allows for workload uncertainty and (partial/full) naivete.

3.3.1. *Sophiscated Agent: When the Workload is Certain.* The dynamic programming problem is formulated as follows. Let $W^S(x,t;w,T,\beta)$ be the perceived cost function at any state $(x,t) \in [0,w] \times [0,T]$, and let $\mathbf{e}^S(x,t;w,T,\beta)$ be a corresponding policy function.[15] Here, the superscript S indicates that the agent is sophisticated; the work state x denotes the work finished, and the time state t denotes the time spent. In what follows, I suppress the dependence of W^S and $\mathbf{e}^S$ on the fixed parameter (w,T,β) for convenience of exposition. Let $\langle \boldsymbol{x}(x,t), \boldsymbol{y}(x,t)\rangle$ be the corresponding work and effort trajectories starting from the state (x,t). The cost function at state (x,t) is thus given by

$$W^S(x,t) \equiv C_t(\boldsymbol{y}(x,t);\beta) = \int_t^T D_t(s;\beta)c(y_s(x,t))\mathrm{d}s. \tag{10}$$

Step 1. Define the sophisticated agent's continuation cost in the state (x,t) as

$$V^S(x,t) \equiv C_t(\boldsymbol{y}(x,t);1) = \int_t^T c(y_s(x,t))\mathrm{d}s. \tag{11}$$

In the classic dynamic programming under exponential discounting, there is no present bias (i.e., $\beta = 1$); thus the continuation cost function $V^S(\cdot)$ amounts to the cost function $W^S(\cdot)$. Under present bias β, we can still relate $W^S(\cdot)$ to $V^S(\cdot)$ using (10) and (11) as follows,

$$W^S(x,t) = \beta V^S(x,t), \tag{12}$$

for all $x \in [0,w], t \in [0,T]$. Intuitively, (12) holds because (i) the IG discounting factor (1) coincides with exponential discount factor only at the current instant and the current effort cost is negligible in the integral of the overall cost; (ii) all future payoffs are further discounted by β in the IG discounting, as opposed to the exponential discounting.

Step 2. Derive the first-order condition for the policy function.

[15] Solving the dynamic programming is equivalent to characterizing the Markov-Perfect Equilibrium defined in *Section 2*. Here, $\mathbf{e}^S$ is the actual effort strategy in equilibrium. Since a sophisticated agent correctly perceives her effort strategy, $(\mathbf{e}^S, \mathbf{e}^S)$ constitutes a Markov-percept equilibrium.

The agent optimizes effort, trading off the cost of the current effort against the benefit from workload reduction in the future. By (12), the F.O.C. for the optimal effort $\hat{y} > 0$ in any state (x, t) gives us

$$c'(\hat{y}) = -\beta V_x^S(x, t),$$

where $V_x^S(\cdot)$ stands for the partial derivative of $V^S(\cdot)$ with respect to x. The F.O.C. equalizes the marginal cost of effort with the marginal reduction in the perceived cost to go. Thus, for all $(x, t) \in [0, w] \times [0, T]$, the policy function is given by

$$\mathbf{e}^S(x, t) = \begin{cases} \hat{y} = \left[-\frac{\beta}{\alpha\gamma} V_x^S(x, t) \right]^{\frac{1}{\alpha - 1}} & \text{if } V_x^S(x, t) < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Step 3. Derive Hamilton–Jacobi–Bellman (HJB) Equation to characterize the dynamics of continuation cost $V^S(\cdot)$.

For any $x \in [0, w], t \in [0, T]$,

$$c(\mathbf{e}^S(x, t)) + V_t^S(x, t) + V_x^S(x, t)\mathbf{e}^S(x, t) = 0, \tag{14}$$

where $V_t^S(\cdot)$ stands for the partial derivative of $V^S(\cdot)$ with respect to t. The HJB equation of $V^S(\cdot)$ indicates that at the optimal effort level, the flow cost of effort equals the instantaneous reduction in the continuation cost.

Step 4. Use boundary conditions on the work trajectory and value matching condition to pin down the solution to the dynamic optimization.

Boundary conditions on the initial and terminal states are as follows:

$$x_t(x, t) = x, \quad x_T(x, t) = w, \tag{15}$$

for any $x \in [0, w], t \in [0, T]$. Additionally, we have the value matching condition

$$V^S(w, t) = 0, \tag{16}$$

for any $t \in (0, T]$, since the agent incurs no further effort cost once the task is completed.

LEMMA 1 (Value and Policy Characterization)**.** *If $(W^S, V^S, \mathbf{e}^S)$ satisfies (12), (13), (14), (15) and (16), then $C(w, T, \beta) = W^S(0, 0; w, T, \beta)$ is the (perceived) cost function for the dynamic optimization, $LC(w, T, \beta) = V^S(0, 0; w, T, \beta)$ is the long-run cost function, and $\boldsymbol{y}(w, T, \beta) = \boldsymbol{y}(0, 0; w, T, \beta)$ is the **unique** optimal effort trajectory (up to Lebesgue measure 0).*

Lemma 1 asserts that the above conditions fully characterize the dynamic work schedule for a sophisticated agent. Using the *"Specify and Verify"* technique, I then solve the dynamic programming analytically. In particular, I obtain for any $(x,t) \in [0,w] \times [0,T)$,

$$V^S(x,t;w,T,\beta) = \frac{\gamma}{\beta}\left[\frac{\alpha-1}{(\alpha-\beta)(T-t)}\right]^{\alpha-1}(w-x)^{\alpha}. \tag{17}$$

3.3.2. *Sophisticated Agent: When the Workload is Uncertain.* Since the agent gets to know her task workload as soon as she finishes w_L, the agent's objective function of the dynamic optimization before she finishes w_L is formulated as follows:

$$\min_{\tau\in[0,T]}\left\{\min_{\{\boldsymbol{y}:\int_0^\tau y_s \mathrm{d}s = w_L\}}\int_0^\tau D_0(t)c(y_t)\mathrm{d}t + (1-\mu)\beta V^S(w_L,\tau;w_H,T,\beta)\right\},$$

where $V^S(\cdot)$ is the continuation cost given by (17). This objective function is the sum of two terms. The first term is the perceived effort cost to finish w_L. The second term is the expected cost when w_L is finished: with probability μ, the agent completes the task and incurs no more cost; with probability $1-\mu$, she still has $w_H - w_L$ to do. Following the four-step procedure above to solve the dynamic programming, I then obtain the continuation value for a sophisticated agent under workload uncertainty: for $(x,t) \in [0,w_L) \times [0,T)$,

$$\tilde{V}^S(x,t;\mathcal{T},\beta) = \frac{\gamma}{\beta}\left(\frac{\alpha-1}{\alpha-\beta}\right)^{\alpha-1}\frac{(\lambda-x)^{\alpha}}{(T-t)^{\alpha-1}}. \tag{18}$$

3.3.3. *Allowing for Naivete.* For a naive agent $(\beta,\hat{\beta})$, she anticipates the future self would act as a sophisticated agent with less present bias, i.e., $\hat{\beta} \in (\beta,1]$. This misperception links her cost function $\tilde{W}(\cdot)$ to the continuation cost $\tilde{V}^S(\cdot)$ for a sophisticated agent $\hat{\beta}$ as

$$\tilde{W}(x,t;\mathcal{T},\mathcal{B}) = \beta\tilde{V}^S(x,t;\mathcal{T},\hat{\beta}),$$

for $x < w_L$, where $\tilde{V}^S(\cdot)$ is given by (18). Following the procedure in *Section 3.3.1*, we finally characterize the work schedule and overall effort costs for a potentially present-biased and naive agent under workload uncertainty, as in *Proposition 1*.

4. Applications

4.1. *Committing to Short-Term Goals*

Welfare loss due to procrastination calls for regulation — either from self-discipline or from external supervision. A popular commitment device used to combat procrastination for a long-term project is to insert a series of short-term goals. For example, Ph.D. students must achieve milestones throughout their graduate study to earn the doctorate degree; an employer can set down a timetable at the start of a project, requiring workers to report their work progress. The basic idea in these scenarios is to smooth the workload over time, preventing the last-minute rush right before the deadline. Now, a big task breaks down into several small tasks by intermediate deadlines; one long-term goal becomes a series of successive short-term goals.

In this section, I examine the value of such short-term goals using the schedule and welfare characterized in *Proposition 1*. The analysis here focuses on the case when the agent knows the workload, and the main results hold under uncertain workload as well. I proceed by asking: (i) How should a present-biased agent set short-term goals optimally? (ii) Will a time-consistent advisor set different short-term goals from the agent's self-imposed goals to minimize the agent's long-run cost? (iii) Could short-term goals strictly enhance the agent's welfare?

Formally, fix the task (w, T). A commitment device is available at the start of the project $t = 0$ to set $k \in \mathbb{N}_+$ goals

$$\boldsymbol{G}^k = \{(\hat{w}_1, \hat{\tau}_1), (\hat{w}_2, \hat{\tau}_2), \cdots, (\hat{w}_k, \hat{\tau}_k)\}.$$

For any $1 \le i \le k$, $G_i = (\hat{w}_i, \hat{\tau}_i)$ is a short-term goal to finish at least $\hat{w}_i$ by time $\hat{\tau}_i$, and $0 \le \hat{w}_{i-1} \le \hat{w}_i \le w, 0 < \hat{\tau}_{i-1} < \hat{\tau}_i \le T$ with the convention that $(\hat{w}_0, \hat{\tau}_0) = (0, 0)$, $(\hat{w}_k, \hat{\tau}_k) = (w, T)$. In particular, when $k = 1$, the agent only commits to the final goal (w, T) as in our baseline model.

If a short-term goal is lower than what the agent would have finished without it (i.e., $\hat{w}_i \le \hat{w}_{i-1} + x_{\hat{\tau}_i - \hat{\tau}_{i-1}}(w - \hat{w}_{i-1}, T - \hat{\tau}_{i-1}, \mathcal{B})$ for some $1 \le i < k$), the goal does not affect the original work schedule, which is a trivial case. In what follows, I focus on *effective goals* such that the agent completes more work at each goal than the case without short-term goals. In this case, every short-term goal becomes urgent to the agent, and the work schedule is now characterized by phase-wise task completion.

4.1.1. *Optimal Short-Term Goals.* In each goal phase i spanning from $\hat{\tau}_{i-1}$ to $\hat{\tau}_i$, the agent completes the work margin $\hat{w}_i - \hat{w}_{i-1}$. Therefore, the overall perceived cost and long-run cost of task completion with short-term goals $\boldsymbol{G}^k$ become

$$\hat{C}(\boldsymbol{G}^k) = \sum_{i=1}^{k} C(\hat{w}_i - \hat{w}_{i-1}, \hat{\tau}_i - \hat{\tau}_{i-1}, \mathcal{B}) = \gamma B^{\alpha-1} \sum_{i=1}^{k} \frac{(\hat{w}_i - \hat{w}_{i-1})^\alpha}{(\hat{\tau}_i - \hat{\tau}_{i-1})^{\alpha-1}},$$

$$\hat{LC}(\boldsymbol{G}^k) = \sum_{i=1}^{k} LC(\hat{w}_i - \hat{w}_{i-1}, \hat{\tau}_i - \hat{\tau}_{i-1}, \mathcal{B}) = \frac{\gamma B^\alpha}{\alpha B + 1 - \alpha} \sum_{i=1}^{k} \frac{(\hat{w}_i - \hat{w}_{i-1})^\alpha}{(\hat{\tau}_i - \hat{\tau}_{i-1})^{\alpha-1}},$$

where $C(\cdot)$ and $LC(\cdot)$ are given by *Proposition 1*. From here, I then derive the optimal k short-term goals, selected by both the agent $(\beta, \hat{\beta})$ and her advisor. For all $1 \le i \le k-1$, F.O.C. with regard to $\hat{\tau}_i$ gives us

$$-(\alpha - 1)\frac{(\hat{w}_i - \hat{w}_{i-1})^\alpha}{(\hat{\tau}_i - \hat{\tau}_{i-1})^\alpha} + (\alpha - 1)\frac{(\hat{w}_{i+1} - \hat{w}_i)^\alpha}{(\hat{\tau}_{i+1} - \hat{\tau}_i)^\alpha} = 0 \quad \Rightarrow \quad \frac{\hat{\tau}_{i+1} - \hat{\tau}_i}{\hat{\tau}_i - \hat{\tau}_{i-1}} = \frac{\hat{w}_{i+1} - \hat{w}_i}{\hat{w}_i - \hat{w}_{i-1}}.$$

Since $(\hat{w}_0, \hat{\tau}_0) = (0, 0)$, $(\hat{w}_k, \hat{\tau}_k) = (w, T)$, we have $\hat{w}_i = \frac{w}{T}\hat{\tau}_i = x^*_{\hat{\tau}_i}$. This implies that if the agent or her advisor can choose the short-term goals at the start of the project, they would

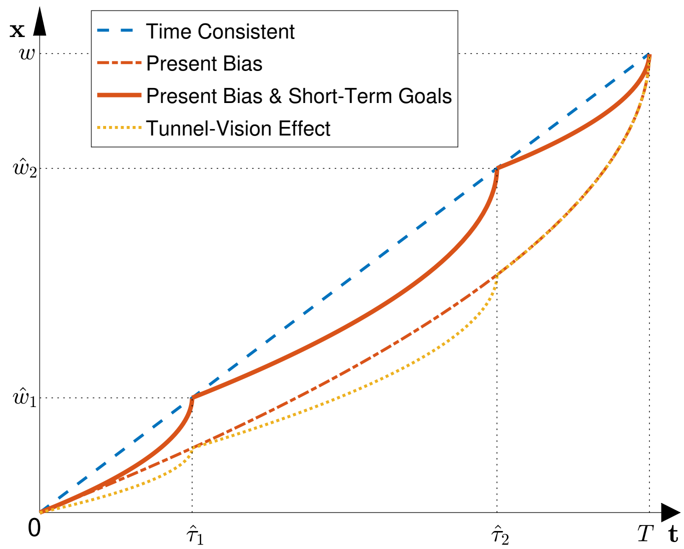


FIGURE 6. Work Trajectory under Certain Workload with Optimal short-term Goals

choose the same as a time-consistent agent, trying to distribute the workload evenly over time, as illustrated in *Figure 6*. It holds true regardless of the agent's time preferences.

Next, I discuss the value of these optimal short-term goals. Indeed, the optimal short-term goals make the present-biased agent procrastinate less. Formally, the agent finishes more of the work before any time $t \in (0, T)$; that is, for any $1 \le i \le k$, $t \in [\hat{\tau}_{i-1}, \hat{\tau}_i]$, $\beta < 1$ and $\hat{\beta} \in [\beta, 1]$, we have

$$\hat{x}_t(\boldsymbol{G}^k) = \hat{w}_{i-1} + x_{t-\hat{\tau}_{i-1}}(\hat{w}_i - \hat{w}_{i-1}, \hat{\tau}_i - \hat{\tau}_{i-1}, \mathcal{B}) > x_t(w, T, \mathcal{B}).$$

However, less procrastination caused by short-term goals does not reduce the overall (perceived/long-run) effort cost. To see this, I first calculate the minimal perceived cost with short-term goals as follows:

$$\begin{aligned}\hat{C}^*(k) &\equiv \min_{\boldsymbol{G}^k} \hat{C}(\boldsymbol{G}^k) \\ &= \gamma B^{\alpha-1}\left(\frac{w}{T}\right)^{\alpha-1} \sum_{i=1}^{i=k} (\hat{w}_i - \hat{w}_{i-1}) = \gamma B^{\alpha-1} \frac{w^\alpha}{T^{\alpha-1}} = C(w, T, \mathcal{B}).\end{aligned}$$

Likewise, the minimal long-run cost with short-term goals is

$$\begin{aligned}\hat{LC}^*(k) &\equiv \min_{\boldsymbol{G}^k} \hat{LC}(\boldsymbol{G}^k) \\ &= \frac{B}{\alpha B + 1 - \alpha} \hat{C}^*(k) = \frac{B}{\alpha B + 1 - \alpha} C(w, T, \mathcal{B}) = LC(w, T, \mathcal{B}).\end{aligned}$$

Therefore, although full commitment to the first-best schedule restores efficiency, committing to finite short-term goals cannot lower either the perceived cost or the long-run cost for a present-biased agent. Worse still, if not set optimally, the intervention of short-term goals would strictly increase these overall effort costs. Hence, both the agent and her advisor would rather *not* set short-term goals for the agent, regardless of the task features and the agent's time preferences. Note also that the value of short-term goals does not vary with the number of goals. Hence, even arbitrarily many short-term goals cannot improve individual welfare.

4.1.2. *Two Effects Generated by Short-Term Goals.* To understand why the value for finite commitments is weakly negative, I isolate the following two opposite forces at play:

(i) *Keep on Track* $(+)$: smooth out the workload over phases so that work is not accumulated near the final deadline.

(ii) *Tunnel Vision* $(-)$: rush to finish the short-term goals in every phase.

The *Keep-on-Track Effect* is easy to understand; after all, this is the main purpose of arranging these short-term goals in the first place. However, the *Tunnel-Vision Effect* is an unintended side effect when applying short-term goals and requires more explanation. To influence the agent's work schedule, short-term goals urge the agent to finish more work by intermediate deadlines compared to the case when she is not subject to any short-term goals. As a result, the agent is prompted to be myopic and focus exclusively on the urgent short-term goal at each phase. The *Tunnel-Vision Effect* precisely captures the adverse effect arising from being myopic in achieving short-term goals.

To see this negative effect, consider an agent who sets short-term goals aligned with what she would have done without these goals (i.e., $\hat{w}_i = x_{\hat{\tau}_i}(w, T, \mathcal{B})$) but narrows her attention solely to the upcoming short-term goals in each phase; see *Figure 6* for an illustration. We can calculate her ex-ante perceived cost and long-run cost as follows:

$$\begin{aligned}\hat{C}(\boldsymbol{G}^k) &= \gamma B^{\alpha-1} w^{\alpha} \left\{ \frac{[1-(1-\frac{\hat{\tau}_1}{T})^B]^{\alpha}}{\hat{\tau}_1^{\alpha-1}} + \frac{[(1-\frac{\hat{\tau}_1}{T})^B - (1-\frac{\hat{\tau}_2}{T})^B]^{\alpha}}{(\hat{\tau}_2-\hat{\tau}_1)^{\alpha-1}} + \cdots + \frac{[(1-\frac{\hat{\tau}_{k-1}}{T})^B]^{\alpha}}{(T-\hat{\tau}_{k-1})^{\alpha-1}} \right\} \\ &\geq \gamma B^{\alpha-1} w^{\alpha} / T^{\alpha-1} = C(w, T, \mathcal{B}), \qquad (19)\\ \hat{LC}(\boldsymbol{G}^k) &= \frac{B}{\alpha B + 1 - \alpha} \hat{C}(\boldsymbol{G}^k) \geq \frac{B}{\alpha B + 1 - \alpha} C(w, T, \mathcal{B}) = LC(w, T, \mathcal{B}),\end{aligned}$$

where the equality holds if and only if the agent is time consistent (i.e., $B = 1$).[16] In other words, the adverse *Tunnel-Vision Effect* only exists for a present-biased agent. A time-consistent agent constantly exerts effort w/T over time, so her overall cost remains the same regardless of her consideration horizon for dynamic decisions. However, if a present-biased agent focuses solely on the most urgent short-term goal, the repeated procrastination for each short-term goal would entail higher overall costs to her.

[16] One way to see (19) is by observing that $\hat{C}(\boldsymbol{G}^k) \geq \hat{C}^*(k) = C(w, T, \mathcal{B})$. The online appendix provides an alternative proof for (19) and shows that the equality holds if and only if $B = 1$.

On the whole, the *Keep-on-Track Effect* smooths the workload over time by setting short-term goals as checkpoints; however, the *Tunnel-Vision Effect* increases overall costs by inducing repeated rushing for deadlines. If the short-term goals are chosen optimally, these two effects exactly eliminate each other; otherwise, the *Tunnel-Vision Effect* dominates the *Keep-on-Track Effect* for a present-biased agent. As effective short-term goals can lead to a higher overall effort cost for the agent, the agent and her advisor prefer to work only under an end goal rather than a series of short-term goals.

4.1.3. *Limited Commitment in Discrete-Time Case.* I further examine the nature of the *Tunnel-Vision Effect* by viewing continuous-time choices as a limiting case of discrete-time choices when the choice frequency goes to infinity. I will show that the value of short-term goals to a present-biased agent can be positive in discrete time. However, the commitment value is attenuated as the frequency of actions relative to the frequency of goals grows. It eventually dissipates when dynamic choices are made in continuous time under only finite short-term goals.

Suppose $t = 1, 2, \cdots, T$, and a sophisticated agent with present bias $\beta \in [0, 1]$ has a task (w, T) to complete. Denote $y_t > 0$ as the effort at the period t, and denote $x_t = \sum_{\tau=1}^{t} y_\tau$ as the accumulated effort (or the work finished) up to period t. The quasi-hyperbolic discounting factor at time t for utility at time $s \geq t$ is specified by $D_t(s, \beta, \delta) = \beta\delta^{s-t}$. As in the baseline model, I further assume $\delta = 1$ so that future payoffs are uniformly discounted by $\beta \in (0, 1]$.

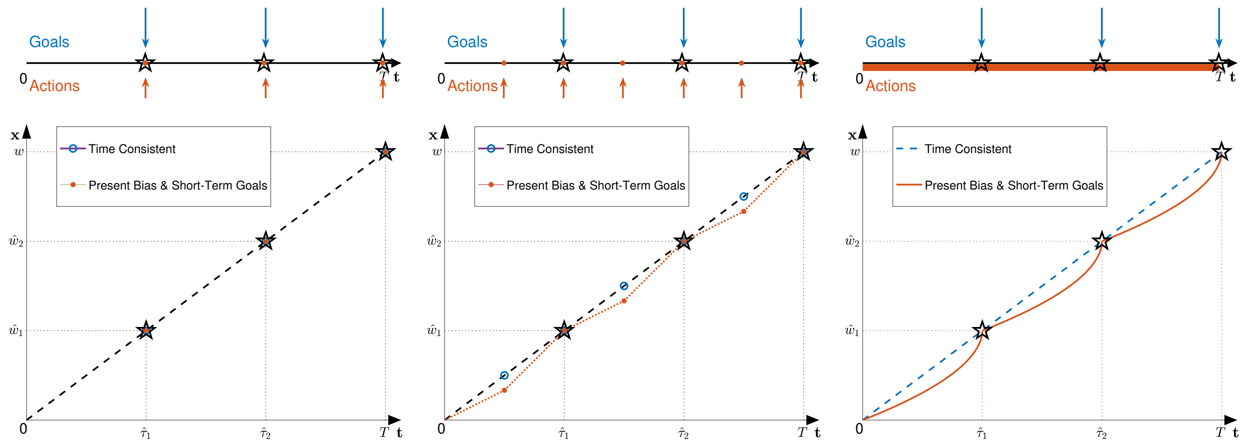
Let $\mathbf{e}(x, t)$ be the agent's optimal effort when work finished is x in the period t. By backward induction, I solve the dynamic optimization that minimizes the overall perceived effort cost, subject to the boundary constraints $x_0 = 0, x_T = w$, and characterize

$$\mathbf{e}(x, T-k) = A_k(w - x)\mathbb{1}\{x < w\},$$

where $\mathbb{1}\{\cdot\}$ is an indicator function, $A_0 = 1, A_k = 1/\left\{1 + [A_{k-1}^{\alpha-1} - (1-\beta)A_{k-1}^{\alpha}]^{-\frac{1}{\alpha-1}}\right\}$. In particular, $A_k = 1/(k+1)$ for $\beta = 1$, indicating that a time-consistent agent distributes the remaining workload evenly among the remaining periods. Moreover, A_k is strictly increasing in β for $k \geq 1$. Therefore, given the remaining work and time, an agent with a larger present bias exerts strictly less effort in the current period.

Figure 7 shows how the agent's work trajectory varies with the frequency of actions. *Figure 7(a)* depicts the full commitment case when the frequency of actions matches the frequency of goals. Here short-term goals help the agent attain her first best by completely binding her effort choices. It manifests the positive *Keep-on-Track* effect of commitment.

Now fix the number of short-term goals and increase the frequency of dynamic choices. The commitment becomes inadequate to bind the agent's every action. As depicted in *Figure 7(b)*, the agent procrastinates in achieving each goal whenever she is left unchecked



(a) Full Commitment
$\frac{\#(\text{actions})}{\#(\text{goals})} = 1$

(b) Limited Commitment
$\frac{\#(\text{actions})}{\#(\text{goals})} > 1$

(c) Continous Time
$\frac{\#(\text{actions})}{\#(\text{goals})} = \infty$

FIGURE 7. Variation in Relative Frequency of Actions to Goals

— this is where the *Tunnel-Vision Effect* comes in. Unlike the continuous-time case, short-term goals in discrete time can indeed bring down the long-run cost. However, the commitment value vanishes as T gets sufficiently large. For example, take $T = 52$ (weeks), $\beta = 0.8$ (estimated present bias in unpleasant task completion by Augenblick and Rabin (2019)). Although full commitment (weekly checks) can fully recover the welfare loss caused by present bias and reduce the long-run cost by 8.9%, quarterly checks ($k = 4$) merely result in a savings of 0.54% in the long-run cost. If the agent works daily instead ($T = 365$), then the value of quarterly checks becomes even more negligible, only reducing the long-run cost by roughly 0.12%. In this light, the baseline continuous-time model approximates the discrete-time case when the dynamic choices are far more frequent than short-term goals.

Intuitively, increasing the relative frequency of dynamic choices gives the present-biased agent more freedom to procrastinate before each short-term goal. This freedom amplifies the *Tunnel-Vision Effect* — rushing more towards the end of each phase. In the limiting continuous-time case when the relative frequency of dynamic choices to short-term goals goes to infinity, the *Tunnel-Vision Effect* completely offsets the *Keep-on-Track Effect*, causing a weakly negative value of short-term goals.

Proposition 2 summarizes our findings about the impact of short-term goals.

PROPOSITION 2 (The Value of Short-Term Goals).

(i) The optimal short-term goals are chosen along the first-best work trajectory.

(ii) Finite short-term goals alleviate procrastination for a present-biased agent.

(iii) In terms of both the ex-ante perceived cost and the long-run cost, finite short-term goals are at best of no value to the agent, if not harmful.

(iv) Present bias and limited commitment combined give rise to the Tunnel-Vision Effect. This negative effect grows as the ratio of the action frequency to the goal frequency increases, and it completely neutralizes the positive Keep-on-Track Effect when the choices are made in continuous time under finite short-term goals.

REMARK 1. *The result that intermediate deadlines entail weakly higher effort costs is consistent with experimental findings documented in Ariely and Wertenbroch (2002).*[17] *They assigned participants to complete unpleasant tasks in different deadline settings, and asked them to evaluate their overall experience afterward. The group with the mandatory evenly-spaced deadlines was reported to suffer the most from the task, whereas the group with only one end deadline was reported to dislike the task the least.*

REMARK 2. *In Appendix D, I investigate the reason why the long-run cost under the optimal short-term goals is precisely the same as without any short-term goal. The key observation is that, the accumulated time when the flow effort is no more than any effort level y is homogeneous of degree 1 in the task requirement (w, T). This implies that the total time the agent spends on each flow effort level remains unchanged when short-term goals are proportionally inserted. Consequently, the overall effort cost remains the same under the optimal short-term goals.*

4.2. *Identifying the Sources of Procrastination*

People may procrastinate because: (i) they particularly value immediate gratification; (ii) they think naively about future choices; (iii) they make a bad work plan as they are unaware of the actual task difficulty; (iv) they are not very averse to rushing at the last moment after all. A natural question to ask is: can we disentangle these four potential driving forces empirically from an observed work trajectory? This inquiry can concern researchers who seek to structurally pin down people's task completion patterns and design the optimal long-term task accordingly. It can also be relevant when an agent (with imperfect memory) reflects on her previous work schedule and tries to identify the cause of her undesirable procrastination.

In the model, these four forces correspond to (i) present bias $\beta < 1$, (ii) naivete $\beta/\hat{\beta} < 1$, (iii) workload uncertainty $\mu \in (0, 1)$, and (iv) rush aversion $\alpha < \infty$. Thus we can reformulate our identification issue more explicitly: can we recover unobserved parameters of

[17] I further show in the online appendix that a present-biased agent incurs strictly higher effort costs under all short-term goals when $\delta > 0$.

interest $(\beta, \hat{\beta}, \alpha, \mu)$ from observed data of a work trajectory $\boldsymbol{x} = \{x_t : t \in [0, T]\}$?[18] Using the model, I show that a measure of time preferences $B = (\beta/\hat{\beta})^{\frac{1}{\alpha-1}}(\alpha-1)/(\alpha-\hat{\beta})$ and a measure of the agent's belief on workload $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$ can be jointly identified. However, present bias and naivete are observationally equivalent.[19]

Joint Identification of B and λ. Our first goal is to separate time preference from workload belief in explaining an agent's procrastination. By the closed-form work trajectory in *Proposition 1*, time preference parameters $(\beta, \hat{\beta})$ are entirely summarized in B, whereas prior belief factors, namely the possible binary workloads (w_H and w_L) and the likelihood (μ) are entirely summarized in λ. More importantly, B and λ affect the work trajectory in a distinguishable way: time preferences control how much the work trajectory is tilted towards the deadline, whereas the prior belief on workload determines the initial target under the workload uncertainty. We can thus exploit this variation to jointly identify time preference and belief on workload.

Formally, fix any vector of (B, λ). Assume a researcher observe an agent's work trajectory $\{x_t : t \in [0, T]\}$. If the work trajectory is smooth and has no kink, then one can conclude immediately that the dynamic choices are made under workload certainty ($w_L = x_T, \mu = 1$). Moreover, we can identify B by looking at work finished at any two distinct time $0 < s < t < T$. Since

$$\frac{1-(1-s/T)^B}{1-(1-t/T)^B} = \frac{x_s}{x_t} \in (0,1),$$

and the function $\varphi(B) \equiv \frac{1-(1-s/T)^B}{1-(1-t/T)^B}$ is strictly increasing in B, we obtain $B = \varphi^{-1}(x_s/x_t)$.

If, otherwise, the work trajectory has one kink at τ, we can first recover B by

$$B = \varphi^{-1}(x_s/x_t),$$

where $0 < s < t \leq \tau$. Then recover λ by

$$\lambda = \frac{x_t}{1-(1-t/T)^B}.$$

Observational Equivalence of Present Bias β and naivete $\hat{\beta}$. Given that B can be identified from an observed work trajectory, I show that present bias β and naivete $\hat{\beta}$, nevertheless, cannot be identified separately. This is true even if all the other contributing factors, namely the rush aversion α and the prior belief (w_L, w_H, μ), are known.

[18]Here I focus on the binary workload case. The discussion can be easily generalized to the case with any arbitrary number of possible workloads using characterization in the online appendix.

[19]I show in the online appendix that rush aversion α cannot be identified from any observed work trajectory. In particular, any rush aversion is observationally equivalent to a larger rush aversion with a larger present bias or with a larger prior likelihood of a low workload. This result echoes the general finding that time preferences cannot be identified without knowing or assuming the payoff function (see, e.g., Andersen et al. (2008), Heidhues and Strack (2021)).

To see this, for any time preference $(\beta, \hat{\beta})$ with $0 < \beta \leq \hat{\beta} < 1$, there always exists an alternative time preference $(\beta', 1)$ that yields the same work trajectory, in which $\beta' = \beta/\hat{\beta}\left[(\alpha-1)/(\alpha-\hat{\beta})\right]^{\alpha-1} \in (\beta, 1)$. Therefore, the work trajectory for a sophisticated or partially naive agent is the same as that for a fully naive agent with a smaller present bias.

Besides, the work trajectory for any naive agent is the same as that for a sophisticated agent with a larger present bias. To be more specific, for any time preference $(\beta, \hat{\beta})$ such that $0 < \beta < \hat{\beta} \leq 1$, there always exists an alternative time preference (β', β') that yields the same work schedule, in which $\beta' = \alpha - (\alpha - \hat{\beta})(\hat{\beta}/\beta)^{\frac{1}{\alpha-1}} \in (0, \beta)$.

PROPOSITION 3 (Identification)**.**

(i) A measure of time preference $B = \left(\beta/\hat{\beta}\right)^{\frac{1}{\alpha-1}}(\alpha-1)/(\alpha-\hat{\beta})$ *and a measure of prior belief* $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$ *can be jointly identified from a work trajectory.*

(ii) The present bias parameter β *and the naive parameter* $\hat{\beta}$ *cannot be jointly identified from any observed work trajectory. In particular, partial naivete is observationally equivalent to full sophistication with a larger present bias, and is also observationally equivalent to full naivete with a smaller present bias.*

5. Discussion

5.1. *Related Literature*

***Experimental and Empirical Evidence*.** The classic exponential discounting model implies time consistency: preferences over alternatives are independent of the time of evaluation; thus, the best plan for the future remains optimal when the future arrives. However, both laboratory experiments and empirical work suggest that this implication about human behaviors is often descriptively inaccurate (e.g., Mischel (1974), Loewenstein and Prelec (1992), Frederick, Loewenstein and O'Donoghue (2002), DellaVigna and Malmendier (2006), Steel (2007), DellaVigna (2009), Mullainathan and Shafir (2013), Thaler (2015), Sirois and Pychyl (2016), Cohen et al. (2020)). Augenblick, Niederle and Sprenger (2015) show that, as opposed to the insignificant present bias exhibited in intertemporal monetary choices, subjects exhibit substantial present bias in real effort tasks.

***Time Inconsistency Models*.** Procrastination manifests the time inconsistency between ex-ante plans and ex-post choices. Early economic work on time inconsistency is pioneered by Strotz (1956) and Thaler and Shefrin (1981). Then, a growing body of theoretical models arises to account for time-inconsistent choices.This literature has three main paradigms: (quasi-)hyperbolic discounting (e.g., Laibson (1997), O'Donoghue and Rabin (1999), O'Donoghue and Rabin (2001), Ahn, Iijima and Sarver (2020)), temptation (e.g., Gul and Pesendorfer (2001), Ahn et al. (2019), Banerjee and Mullainathan (2010)) and multi-self models (e.g., Fudenberg and Levine (2006)), each explaining time-inconsistent choices based on different assumptions about dynamic preferences. In hyperbolic discounting models, the short-term discount rate is greater than the long-run discount rate, capturing the diminishing impatience over time. For tractability, Laibson (1997) proposes the quasi-hyperbolic discounting model in discrete time and characterizes time preferences with only two parameters: the exponential discount factor δ as in exponential discounting, and the present bias parameter β, attaching the additional weight to the present. Harris and Laibson (2013) then extend the quasi-hyperbolic discounting model from discrete time to continuous time. To further explain the planning fallacy, this strand of literature accommodates bounded rationality such as naivete, perfectionism and imperfect memory (e.g., O'Donoghue and Rabin (2001), Kopylov (2012), Ericson (2017); see Ericson and Laibson (2019) for a survey).

The most closely related paper is Maxted (2023), who studies consumption-saving choices for a household under present bias in the infinite horizon. Using the IG model, the author also finds that self-imposed financial commitment devices (e.g., asset illiquidity and high-cost borrowing) cannot reduce the welfare loss arising from present bias. My paper differs from it in the source of uncertainty and the time horizon. Shocks (in income and asset returns) are instantaneous in Maxted (2023); in contrast, my model features uncertainty (in workload) that is resolved only when the accumulated effort reaches some threshold. More importantly, my model features a time limit on the dynamic choices. It is more suited for many task completion scenarios and enables us to study the effect of the deadline, especially the key finding of the *Tunnel-Vision Effect*.

Commitment Contracts. A general lesson from the time inconsistency literature is that commitment can alleviate self-control problems and thus enhance a time-inconsistent agent's long-run welfare (e.g., Himmler, Jäckle and Weinschenk (2019), Schilbach (2019)). A market has arisen to provide desired commitment devices, which motivates a burgeoning strand of literature (e.g., Amador, Werning and Angeletos (2006), Ambrus and Egorov (2013), Laibson (2015), Bond and Sigurdsson (2018); see Bryan, Karlan and Nelson (2010) for a survey). The key tension under study is between the demand for commitment (due to present bias) and the demand for flexibility (due to taste shocks). Another concern in this literature is around exploitative contract design (DellaVigna and Malmendier (2004), Heidhues and Kőszegi (2010), Kőszegi (2014), Galperti (2015)). These papers point out that firms can design contracts to screen and exploit naive consumers, creating market inefficiency — although Gottlieb and Zhang (2021) show that the inefficiency asymptotically disappears for a long-term contract.

I augment to this literature a negative effect of commitment even in the absence of taste shocks, naivete, and costs incurred by device purchases — the *Tunnel-Vision Effect*. Committed to a series of short-term goals, a present-biased agent needs to prioritize the most urgent short-term goal. In this light, one may resort to short-term goals because she thinks she successfully arranges the workload more evenly across periods. Welfare-enhancing as this commitment device appears at first glance, yet it backfires and impairs her welfare. Now, the agent would repeatedly rush to meet every short-term goal, (weakly) prolonging the total duration of high effort.

Task Completion under Uncertainty. The current model of task completion in continuous time resembles Ely and Szydlowski (2020) in that task difficulty is unknown to the agent and is defined as the accumulated effort needed for success. They fully characterize the optimal information disclosure about the workload to induce effort. Instead, this paper highlights time inconsistency in dynamic choices and assumes the flow effort cost is convex in the work intensity, capturing the aversion to rushed work. Saez-Marti and Sjögren (2008) study the optimal deadline choice for a principal of a task. In their model, a time-consistent agent needs two periods to complete the task. The opportunity cost of effort at each period is random, with a high effort cost interpreted as distractions. Heidhues and Strack (2021) study a quasi-hyperbolic discounting model for task completion with a deadline and taste shocks over time. In their model, only the timing of task completion is observed. Logically, their results share some common features with my work: the task completion rate in their paper and the effort level in my paper are both increasing before the deadline. They establish that present bias is pinned down by payoff distribution for any observed data with an increasing task-completion rate. Without knowing payoff distribution, time preference parameters cannot be identified. I further show that, even if the entire path of effort is observed, time preference parameters cannot be identified when the payoff function is unknown.

5.2. *Concluding Remarks*

I propose a tractable dynamic model in which a potentially present-biased and naive agent works on a long-term task with a fixed deadline and uncertain workload. The paper yields three key insights into the study of procrastination. First, behavioral frictions (of present bias and naivete) intensify the welfare loss due to task features (of a demanding task and uncertain workload), and vice versa. Second, present bias and naivete induce procrastination by adding curvature to the work trajectory, whereas workload uncertainty distorts the work schedule by raising the initial workload target. This variation can be exploited to distinguish the agent's workload belief from her time preference. Third, commitment to short-term goals, even if optimally chosen (by the agent or by her advisor), cannot reduce the welfare loss caused by present bias.

The last point is particularly noteworthy and novel to the literature. The intervention of short-term goals is designed to correct behaviors and benefit the agent. However, the unchanged motive of procrastination (e.g., present bias and naivete) finds its way to rebound during unregulated times; the agent would focus on achieving the most urgent short-term goal and undergo repeated procrastination. This side effect arises when (i) the agent has present bias, and (ii) the commitment is inadequate to bind her every choice. The negative effect counteracts the benefit from keeping the agent on track, and it looms as dynamic choices become increasingly more frequent relative to the commitments. In the limit as the choices are made in continuous time while the commitments are finite, the negative effect eliminates the positive effect under the optimal short-term goals, and it strictly dominates the positive effect if the short-term goals are not chosen optimally. Therefore, although full commitment strictly enhances the welfare of a present-biased agent, the limited commitment of short-term goals leaves the agent at best as well-off as she would be without any commitment.

I conclude the paper by listing three avenues for future investigation into procrastination. The first is to examine alternative commitment device (e.g., random checking on progress instead of explicit intermediate deadlines), and further characterize the properties of limited commitment that improves a present-biased agent's welfare; The second is to study the task design for teamwork with agents of different time preference types, and analyze how the agents should be screened and grouped for different tasks. The third is to incorporate insights from recent psychological studies (e.g., Fee and Tangney (2000), Tice, Bratslavsky and Baumeister (2001), Sirois and Pychyl (2016)), viewing procrastination as more of a self-protection strategy for coping with anxiety than a time management problem. The psychological cost incurred by anxiety works differently than the effort cost modeled in this paper. It may suggest that chunking, breaking down a big task into several manageable pieces, could promote steady progress and shorten the costly self-defeating process.

APPENDIX A. PROOF OF *Lemma 1*

Fix the initial state $(x, t) \in [0, w) \times [0, T)$, and let $(\bar{\boldsymbol{x}}(x, t), \bar{\boldsymbol{y}}(x, t))$ be an admissible work schedule for a sophisticated agent starting from (x, t). Denote the perceived cost

$$\bar{W}(x, t) = \int_t^T D_t(s; \beta, \delta) c(\bar{y}_s(x, t)) \mathrm{d}s,$$

and the long-run cost

$$\bar{V}(x, t) = \int_t^T D_t(s; 1, \delta) c(\bar{y}_s(x, t)) \mathrm{d}s.$$

First, I show that $(\bar{\boldsymbol{x}}(x, t), \bar{\boldsymbol{y}}(x, t))$ must satisfy (12), (15) and (16). (12) relates the perceived cost $\bar{W}(\cdot)$ to the long-run cost $\bar{V}(\cdot)$ by definition. Intuitively, the long-run cost $\bar{V}(\cdot)$ evaluates continuation effort costs of the work schedule $(\bar{\boldsymbol{x}}(x, t), \bar{\boldsymbol{y}}(x, t))$ by exponential discounting. In contrast, the perceived cost $\bar{W}(\cdot)$ discounts all future payoffs further by β (besides exponential discounting). Since the impact of the current payoff only lasts for an instant and is thus negligible, we can obtain $\bar{W}(\cdot) = \beta \bar{V}(\cdot)$. The boundary conditions (15) and value matching condition (16) describe the initial and terminal state of the dynamic problem. They guarantee that the agent finish the remaining workload $w - x$ within the time available $T - t$, and she does not need to incur any effort cost once she finishes the workload w.

Next I prove that an optimal work schedule $(\boldsymbol{x}^S, \boldsymbol{y}^S)$ for a sophisticated agent must satisfy (14). Since a sophisticated agent correctly anticipates her present bias in the future, all her predictions about future choices are realized. Therefore, the continuation cost along the sophisticated agent's work schedule $(\boldsymbol{x}^S, \boldsymbol{y}^S)$ is given by

$$V(x_t^S, t) = \int_t^T c(y_\tau^S) \mathrm{d}\tau \tag{A.1}$$

for all $t \in [0, T]$. Differentiating both sides of (A.1) with regard to t, we have

$$V_x(x_t^S, t) y_t^S + V_t(x_t^S, t) = -c(y_t^S),$$

which is the HJB equation (14) for the initial state $(0, 0)$. Since the preceding argument can be repeated for any initial state $(x, t) \in [0, w) \times [0, T)$, we can establish that (14) holds for any sophisticated agent's continuation cost $V^S(x, t)$ and the policy function $\mathbf{e}^S(x, t)$.

Then it suffices to show that the F.O.C. (13) characterizes an optimal work schedule for a sophisticated agent. To see this, note that the flow cost function $c(\cdot)$ is strictly convex and the continuation cost $V^S(\cdot)$ is differentiable. Therefore, the F.O.C. (13) gives us the optimal solution to the corresponding unconstrained cost minimization problem in scheduling.

Combining these pieces, if a work schedule $(\boldsymbol{x}^S, \boldsymbol{y}^S)$ satisfies the F.O.C. (13) and also satisfies the constraints (12), (14), (15) and (16), it is the most cost-efficient work schedule that is implementable for a sophisticated agent.

To prove the uniqueness of the optimal work schedule $(\boldsymbol{x}^S, \boldsymbol{y}^S)$, first note that the cost function $W^S(\cdot)$, which is the minimized perceived cost subject to constraints (12), (14), (15) and (16), must be unique. Then the continuation cost function $V^S(\cdot)$ is uniquely pinned down by (12), which in turn uniquely pin down the policy function $\mathbf{e}^S(\cdot)$ by the F.O.C. (13). Since the optimal work schedule $(\boldsymbol{x}^S, \boldsymbol{y}^S)$ is generated by the unique policy function $\mathbf{e}^S(\cdot)$ with the initial state at $(0,0)$, the optimal work schedule that satisfies (12), (13),(14), (15) and (16), if it ever exists, must be unique.

APPENDIX B. PROOF OF *Proposition 1*

As outlined in *Section 3.3*, to derive *Proposition 1*, I first characterize a sophisticated agent's work schedule and welfare when she knows the workload (*Lemma B.1*). I then trace back her work schedule when the sophisticated agent is uncertain about the workload (*Lemma B.2*). Finally, I allow the agent to be naive, (mis)perceiving that her future selves are sophisticated and bear a smaller present bias.

LEMMA B.1. *The unique work schedule for a sophisticated agent with present bias $\beta \in (0,1]$ and a certain task (w, T) is: for all $t \in [0, T]$,*

$$x_t(w,T,\beta) = w\left[1 - (1 - t/T)^{\frac{\alpha-1}{\alpha-\beta}}\right],$$

$$y_t(w,T,\beta) = [(\alpha-1)w/(\alpha-\beta)T]\,(1-t/T)^{\frac{\beta-1}{\alpha-\beta}}.$$

The cost function (or ex-ante perceived cost) is

$$C(w,T,\beta) = \gamma w^\alpha/(T^{\alpha-1})\,[(\alpha-1)/(\alpha-\beta)]^{\alpha-1}.$$

The continuation cost (or long-run cost) is

$$LC(w,T,\beta) = \gamma w^\alpha/(\beta T^{\alpha-1})\,[(\alpha-1)/(\alpha-\beta)]^{\alpha-1}.$$

PROOF OF LEMMA B.1. The dynamic programming problem is formulated as follows. Fix the initial state $(x,t) \in [0,w] \times [0,T]$. Let $W^S(x,t)$ be the perceived cost function at any state (x,t); let $\mathbf{e}^S(x,t)$ be the corresponding policy function; and let $\langle \boldsymbol{x}^S(x,t), \boldsymbol{y}^S(x,t)\rangle$ be the corresponding work and effort trajectories starting from the state (x,t).

Suppose $V^S(x,t) = f(t)g(x)$ where $f(\cdot), g(\cdot)$ are two continuously differentiable functions, and $f' > 0, g' < 0$. Then equation (13) yields

$$\mathbf{e}^S(x,t) = \left[-\frac{\beta}{\alpha\gamma} f(t) g'(x)\right]^{\frac{1}{\alpha-1}}.$$

Plugging in the HJB equation (14) and rearranging terms, we have

$$\frac{f'(t)}{[f(t)]^{\frac{\alpha}{\alpha-1}}} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha-1}} - \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right] \frac{(-g'(x))^{\frac{\alpha}{\alpha-1}}}{g(x)}. \tag{B.1}$$

Note that the left hand side of (B.1) is unrelated to x, and the right hand side is unrelated to t. Therefore, there exists a constant $H > 0$ such that

$$H = \frac{f'(t)}{f(t)^{\frac{\alpha}{\alpha-1}}},$$

$$H = \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha-1}} - \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right] \frac{(-g'(x))^{\frac{\alpha}{\alpha-1}}}{g(x)}.$$

Solving these two first-order differential equations, we can obtain

$$f(t) = \left(\frac{\alpha-1}{-Ht+J}\right)^{\alpha-1},$$

$$g(x) = (Ax+B)^{\alpha},$$

where $A = -\frac{\gamma^{\frac{1}{\alpha}}}{\beta}\left(\frac{H\beta}{\alpha-\beta}\right)^{\frac{\alpha-1}{\alpha}}$, and $B, J \in \mathbb{R}$ are two constants. Therefore,

$$V^S(x,t) = f(t)g(x) = \left(\frac{\alpha-1}{-Ht+J}\right)^{\alpha-1}(Ax+B)^{\alpha},$$

$$\mathbf{e}^S(x,t) = \left[-\frac{\beta}{\alpha\gamma}f(t)g'(x)\right]^{\frac{1}{\alpha-1}} = \frac{\alpha-1}{-Ht+J}(Ax+B)\left(-\frac{\beta A}{\gamma}\right)^{\frac{1}{\alpha-1}}.$$

Then the value matching condition $V^S(w,t) = 0$, implies that

$$V^S(w,t) = \left(\frac{\alpha-1}{-Ht+J}\right)^{\alpha-1}(Aw+B)^{\alpha}$$

for all $t \in [0,T]$. Thus, we have $B = -Aw$.

Since for any $s \in [t,T]$, $\dot{x}_s^S(x,t) = y_s^S(x,t) = \mathbf{e}^S(x_s(x,t),s) = \frac{\alpha-1}{-Hs+J}(Ax_s^S(x,t) + B)(-\frac{\beta A}{\gamma})^{\frac{1}{\alpha-1}}$, we have

$$x_s^S(x,t) = E\left[(-Hs+J)/A\right]^{\frac{\alpha-1}{\alpha-\beta}} + w,$$

where E is a function of the initial state variables (x,t) and is invariant with s. Then by the boundary conditions $x_T^S(x,t) = w$ and $x_t^S(x,t) = x$,

$$x_T^S(x,t) = E\left[(-HT+J)/A\right]^{\frac{\alpha-1}{\alpha-\beta}} + w = w,$$

$$x_t^S(x,t) = E\left[(-Ht+J)/A\right]^{\frac{\alpha-1}{\alpha-\beta}} + w = x.$$

Therefore, $J = HT, E = (x-w)/[H(T-t)/A]^{\frac{\alpha-1}{\alpha-\beta}}$.

Finally, we can derive the cost function, the continuation cost, the policy function, and the corresponding optimal state and control trajectories in any state $(x,t) \in [0,w] \times [0,T]$ as follows:

$$
\begin{aligned}
W^S(x,t) &= \beta V^S(x,t), \\
V^S(x,t) &= \frac{\gamma}{\beta}\left[\frac{\alpha-1}{(\alpha-\beta)(T-t)}\right]^{\alpha-1}(w-x)^{\alpha}, \\
x_s^S(x,t) &= w-(w-x)\left(\frac{T-s}{T-t}\right)^{\frac{\alpha-1}{\alpha-\beta}} \text{ for any } s \in [t,T], \\
y_s^S(x,t) &= \dot{x}_s(x,t) = \frac{\alpha-1}{\alpha-\beta}(w-x)\frac{(T-s)^{\frac{\alpha-1}{\alpha-\beta}-1}}{(T-t)^{\frac{\alpha-1}{\alpha-\beta}}} \text{ for any } s \in [t,T].
\end{aligned}
$$

Taking the initial state (x,t) as $(0,0)$, we obtain *Lemma B.1*.

■

LEMMA B.2. *Let $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L), \tau = \left[1-(1-w_L/\lambda)^{\frac{\alpha-\beta}{\alpha-1}}\right]T$. For $t \in [0,\tau)$, the unique work schedule for a sophisticated agent with present bias $\beta \in (0,1]$ and prior $\mu \in [0,1]$ is*

$$
\begin{aligned}
x_t(\mathcal{T},\beta) &= \lambda[1-(1-t/T)^{\frac{\alpha-1}{\alpha-\beta}}], \\
y_t(\mathcal{T},\beta) &= [(\alpha-1)\lambda/(\alpha-\beta)T]\,(1-t/T)^{-\frac{1-\beta}{\alpha-\beta}}.
\end{aligned}
$$

For $t \in [\tau, T)$, if $w = w_H$, then the unique work schedule is given by Lemma B.1 with the workload $w_H - w_L$ and the time available $T - \tau$; otherwise, the agent stops working.

The cost function (or ex-ante perceived cost) is

$$C(\mathcal{T},\beta) = \gamma\lambda^{\alpha}/T^{\alpha-1}[(\alpha-1)/(\alpha-\beta)]^{\alpha-1}.$$

The continuation cost (or long-run cost) is

$$LC(\mathcal{T},\beta) = \gamma\lambda^{\alpha}/\left(\beta T^{\alpha-1}\right)[(\alpha-1)/(\alpha-\beta)]^{\alpha-1}. \tag{B.2}$$

PROOF OF LEMMA B.2. The dynamic programming problem is formulated as follows. Fix the initial state $(x,t) \in [0,w_L) \times [0,T]$. Let $\langle \tilde{\boldsymbol{x}}^S(x,t), \tilde{\boldsymbol{y}}^S(x,t)\rangle$, $\tilde{W}^S(x,t)$ and $\tilde{V}^S(x,t)$ be the work schedule, cost function, and continuation cost function, respectively, for a sophisticated agent starting from the state (x,t), and let $\tau(x,t)$ be the corresponding optimal time to finish w_L.

We can follow the procedure described in *Section 3.3.1* to solve this dynamic programming. The only twists here consist in the long-run cost function and boundary conditions:

$$\tilde{V}^S(x,t) = \int_t^{\tau(x,t)} c(\tilde{y}^S_s(x,t))\mathrm{d}s + (1-\mu)V^S(w_L, \tau(x,t); w_H, T, \beta),$$
$$\tilde{x}^S_{\tau(x,t)}(x,t) = w_L,$$
$$\lim_{x'\uparrow w_L,\, t'\uparrow\tau(x,t)} \tilde{V}^S(x',t') = (1-\mu)V^S(w_L,\tau(x,t); w_H, T, \beta), \tag{B.3}$$

for any $(x,t) \in [0, w_L) \times [0, T]$, where $V^S(\cdot)$ is the continuation cost given by (17).

Suppose $\tilde{V}^S(x,t) = f(t)g(x)$ where $f(\cdot), g(\cdot)$ are two continuously differentiable functions, and $f'(t) > 0, g'(x) < 0$. Again, the optimal effort equation (13) gives us

$$\tilde{\mathbf{e}}^S(x,t) = \left[-\frac{\beta}{\alpha\gamma} f(t)g'(x)\right]^{\frac{1}{\alpha-1}}.$$

Plugging in the HJB equation (14) and rearranging terms, we have

$$\frac{f'(t)}{[f(t)]^{\frac{\alpha}{\alpha-1}}} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{\beta}{\alpha}\right)^{\frac{1}{\alpha-1}} - \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right] \frac{[-g'(x)]^{\frac{\alpha}{\alpha-1}}}{g(x)}.$$

Following the proof steps for *Lemma B.1* and solving the first-order ordinary differential equations, we obtain:

$$\begin{aligned}
f(t) &= \left(\frac{\alpha-1}{-Ht+J}\right)^{\alpha-1}, \\
g(x) &= (Ax+B)^\alpha, \\
\tilde{V}^S(x,t) &= f(t)g(x) = \left(\frac{\alpha-1}{-Ht+J}\right)^{\alpha-1}(Ax+B)^\alpha, \\
\tilde{\mathbf{e}}^S(x,t) &= \left(-\frac{\beta}{\alpha\gamma} f(t)g'(x)\right)^{\frac{1}{\alpha-1}} = \frac{\alpha-1}{-Ht+J}(Ax+B)\left(-\frac{\beta A}{\gamma}\right)^{\frac{1}{\alpha-1}},
\end{aligned} \tag{B.4}$$

where $A = -\gamma^{\frac{1}{\alpha}}/\beta\,[H\beta/(\alpha-\beta)]^{\frac{\alpha-1}{\alpha}}$, and $B, H, J \in \mathbb{R}$ are three constants.

Then using the boundary condition (B.3), we have for any $(x,t) \in [0, w_L) \times [0, T]$,

$$\tilde{V}^S(w_L, \tau(x,t)) = \lim_{x'\uparrow w_L, t'\uparrow\tau(x,t)} \tilde{V}^S(x',t') = (1-\mu)V^S(w_L, \tau(x,t); w_H, T, \beta). \tag{B.5}$$

Combining (B.4) and (B.5), we have,

$$\begin{aligned}
\left[\frac{\alpha-1}{-H\tau(x,t)+J}\right]^{\alpha-1}(Aw_L+B)^\alpha &= \frac{\gamma}{\beta}\left[\frac{\alpha-1}{(\alpha-\beta)(\frac{J}{H}-\tau(x,t))}\right]^{\alpha-1}\left(-\frac{B}{A}-w_L\right)^\alpha \\
&= (1-\mu)\frac{\gamma}{\beta}\left[\frac{\alpha-1}{(\alpha-\beta)(T-\tau(x,t))}\right]^{\alpha-1}(w_H-w_L)^\alpha.
\end{aligned}$$

Thus, $J = HT$ and $B = -\lambda A$, where $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$.

Since

$$\dot{\tilde{x}}_s^S(x,t) = \tilde{y}_s^S(x,t) = \tilde{\mathbf{e}}^S(s, \tilde{x}_s^S(x,t)) = \frac{\alpha-1}{\alpha-\beta}\frac{\lambda - \tilde{x}_s(x,t)}{T-s},$$

for any $s \in [t,T]$, we have

$$\tilde{x}_s^S(x,t) = E\left[H(T-s)/A\right]^{\frac{\alpha-1}{\alpha-\beta}} + \lambda,$$

where E is a function of the initial state variables (x,t) and is invariant with s. Then the boundary conditions $\tilde{x}_\tau^S(x,t) = w_L$ and $\tilde{x}_t^S(x,t) = x$ imply that

$$E\left[H(T-\tau(x,t))/A\right]^{\frac{\alpha-1}{\alpha-\beta}} = w_L - \lambda,$$
$$E\left[H(T-t)/A\right]^{\frac{\alpha-1}{\alpha-\beta}} = x - \lambda.$$

Hence, we obtain

$$\left[\frac{T-\tau(x,t)}{T-t}\right]^{\frac{\alpha-1}{\alpha-\beta}} = \frac{\lambda - w_L}{\lambda - x} \Rightarrow \tau(x,t) = (T-t) - (T-t)\left(\frac{\lambda - w_L}{\lambda - x}\right)^{\frac{\alpha-\beta}{\alpha-1}},$$
$$E\left(H/A\right)^{\frac{\alpha-1}{\alpha-\beta}} = (x-\lambda)(T-t)^{\frac{\alpha-\beta}{\alpha-1}} \Rightarrow \tilde{x}_s(x,t) = (x-\lambda)\left[(T-s)/(T-t)\right]^{\frac{\alpha-1}{\alpha-\beta}} + \lambda.$$

Finally, we can derive the cost function, the continuation cost, the policy function, and the corresponding optimal state and control trajectories in any state $(x,t) \in [0,w_L) \times [0,T]$ as follows:

$$\begin{aligned}
\tilde{W}^S(x,t) &= \beta \tilde{V}^S(x,t), \\
\tilde{V}^S(x,t) &= \frac{\gamma}{\beta}\left(\frac{\alpha-1}{\alpha-\beta}\right)^{\alpha-1} \frac{(\lambda-x)^\alpha}{(T-t)^{\alpha-1}}, \\
\tilde{x}_s^S(x,t) &= \lambda - (\lambda - x)\left(\frac{T-s}{T-t}\right)^{\frac{\alpha-1}{\alpha-\beta}} \text{ for any } s \in [t,T], \\
\tilde{y}_s^S(x,t) &= \dot{x}_s(x,t) = \frac{\alpha-1}{\alpha-\beta}(\lambda - x)\frac{(T-s)^{\frac{\alpha-1}{\alpha-\beta}-1}}{(T-t)^{\frac{\alpha-1}{\alpha-\beta}}} \text{ for any } s \in [t,T], \\
\tau(x,t) &= (T-t) - (T-t)\left(\frac{\lambda - w_L}{\lambda - x}\right)^{\frac{\alpha-\beta}{\alpha-1}},
\end{aligned}$$

where $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$.

Taking the initial state (x,t) as $(0,0)$, we obtain *Lemma B.2*.

■

Now, we are in position to characterize dynamic work schedule and welfare for a potentially naive agent under workload uncertainty. Fix the initial state $(x,t) \in [0,w_L) \times [0,T]$. Let $\langle \tilde{\boldsymbol{x}}(x,t), \tilde{\boldsymbol{y}}(x,t) \rangle$, $\tilde{W}(x,t)$ and $\tilde{V}(x,t)$ be the work schedule, cost function, and continuation cost function starting from the state (x,t), respectively.

I first relate the cost function for a naive agent $(\beta, \hat{\beta})$, $\tilde{W}(\cdot)$, to the continuation cost for a sophisticated agent $\hat{\beta}$, $\tilde{V}^S(\cdot)$:

$$\tilde{W}(x,t;\mathcal{T},\mathcal{B}) = \beta \tilde{V}^S(x,t;\mathcal{T},\hat{\beta}) = \gamma\beta\left[(\alpha-1)/(\alpha-\hat{\beta})\right]^{\alpha-1}(\lambda-x)^{\alpha}/\left[\hat{\beta}(T-t)^{\alpha-1}\right]$$

for $x < w_L$, where $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$. Thus, the cost function at the start is

$$C(\mathcal{T},\mathcal{B}) = \tilde{W}(0,0;\mathcal{T},\mathcal{B}) = \frac{\gamma\beta}{\hat{\beta}}\left(\frac{\alpha-1}{\alpha-\hat{\beta}}\right)^{\alpha-1}\frac{\lambda^{\alpha}}{T^{\alpha-1}}.$$

We can then obtain the optimal current effort by the F.O.C. as

$$\tilde{\mathbf{e}}(x,t;\mathcal{T},\mathcal{B}) = \left[-\frac{\beta}{\alpha\gamma}\tilde{V}^S_x(x,t;\mathcal{T},\hat{\beta})\right]^{\frac{1}{\alpha-1}} = B\frac{\lambda-x}{T-t},$$

where $B = (\beta/\hat{\beta})^{\frac{1}{\alpha-1}}(\alpha-1)/(\alpha-\hat{\beta})$. Therefore,

$$\dot{\tilde{x}}_s(x,t) = \tilde{\mathbf{e}}(\tilde{x}_s(x,t),s;\mathcal{T},\mathcal{B}) = B\,(\lambda-\tilde{x}_s(x,t))/\,(T-s).$$

Solving this first-order differential equation with the boundary condition $\tilde{x}_t(x,t) = x$, we obtain the work trajectory starting from the state $(x,t) \in [0,w_L) \times [0,T]$ as follows:

$$\tilde{x}_s(x,t) = \lambda - (\lambda - x)\left(1 - \frac{s-t}{T-t}\right)^B.$$

Finally taking the initial state $(x,t) = (0,0)$, we get:

(i) A potentially naive agent's work and effort trajectories before she finishes w_L are

$$x_t = \tilde{x}_t(0,0) = \lambda - \lambda\,(1-t/T)^B,$$
$$y_t = \dot{x}_t = B\,(1-t/T)^{B-1}\,\lambda/T.$$

In particular, when the agent knows the workload w, her effort trajectory is

$$y_t(w,T,\mathcal{B}) = B\,(1-t/T)^{B-1}\,w/T. \tag{B.6}$$

(ii) The time when the naive agent finishes the low workload is given by

$$x_\tau = \lambda - \lambda\,(1-\tau/T)^B = w_L$$
$$\Rightarrow \quad \tau = \left[1-(1-w_L/\lambda)^{\frac{1}{B}}\right]T.$$

(iii) The expected long-run cost associated with this work schedule,

$$\begin{aligned}LC =& \int_0^{\tau} c\left(y_t\right) \mathrm{d}t + (1-\mu) \int_0^{T-\tau} c\left(y_t(w_H - w_L, T-\tau, \mathcal{B})\right) \mathrm{d}t \\ =& \frac{\gamma T}{\alpha(B-1)+1}\left(\frac{B\lambda}{T}\right)^{\alpha}\left[1-\left(1-\frac{\tau}{T}\right)^{\alpha(B-1)+1}\right] + \frac{\gamma(1-\mu)(T-\tau)}{\alpha(B-1)+1}\left[\frac{B(w_H-w_L)}{T-\tau}\right]^{\alpha} \\ =& \frac{\gamma B^{\alpha}\lambda^{\alpha}}{(\alpha B+1-\alpha)T^{\alpha-1}},\end{aligned}$$

where $y_t(\cdot)$ is given by (B.6).

Appendix C. Dynamic Effort Choices in the Discrete Time

Denote $y_k(x)$ as the effort at period $T-k$ when the remaining work is x. Fix the remaining work $x \in (0, w]$. In this section, I prove when $x > 0$, $y_k(x) \equiv y(w-x, T-k) = A_k x$ where $A_0 = 1$ and

$$A_k = \frac{1}{1 + [A_{k-1}^{\alpha-1} - (1-\beta)A_{k-1}^{\alpha}]^{-\frac{1}{\alpha-1}}}$$

for $k = 1, \cdots, T$.

Lemma C.1. *when $x > 0$, $y_k(x) = A_k x$ where $A_0 = 1$ and $A_0 = 1$ and for $k = 1, \cdots, T$,*

$$\frac{1}{\beta}(A_k^{-1} - 1)^{1-\alpha} = \sum_{i=0}^{k-1} A_i^{\alpha} \prod_{j=i+1}^{k-1} (1 - A_j)^{\alpha}, \tag{C.1}$$

with the convention that $\Pi_{i=k}^{k-1}(1 - A_i)^{\alpha} = 1$.

Proof of Lemma C.1. The proof is conducted by mathematical induction.

(i) For $k = 0$ (the last period), the agent needs to finish all the remaining work $y_0(x) = x$, and therefore $A_0 = 1$.

(ii) For $k = 1$ (the second-to-last period), the agent minimizes $c(y) + \beta c(y_0(x-y))$ over $y \in [0, w]$. By F.O.C, we have

$$c'(y^*) - \beta c'(x - y^*) = 0 \;\Rightarrow\; y_1(x) = y^* = x/\left(1 + \beta^{-\frac{1}{\alpha-1}}\right).$$

Therefore, $A_1 = 1/\left(1 + \beta^{-\frac{1}{\alpha-1}}\right)$ and thus $\frac{1}{\beta}(A_1^{-1} - 1)^{1-\alpha} = 1 = A_0^{\alpha}$, showing that $y_k(x) = A_k x$ and (C.1) holds for $k = 1$.

(iii) Take any $m = 1, 2, \cdots, T$. Suppose $y_k(x) = A_k x$ and (C.1) holds for all $k = 1, \cdots, m-1$. I show in this step that $y_m(x) = A_m x$ and (C.1) holds for $k = m$. At this period, the agent minimizes $c(y) + \sum_{i=0}^{m-1} \beta c(y_i)$ over $y \in [0, w]$, where

$$\begin{aligned}y_{m-1} &= A_{m-1}(x-y),\\ y_{m-2} &= A_{m-2}(x - y - y_{m-1}) = A_{m-2}(1 - A_{m-1})(x-y),\end{aligned}$$

$$y_{m-3} = A_{m-3}(x - y - y_{m-1} - y_{m-2}) = A_{m-3}(1 - A_{m-2})(1 - A_{m-1})(x - y),$$

and I can then prove by induction that $y_i = A_i \prod_{j=i+1}^{m-1}(1 - A_j)(x - y)$ for $i = 0, 1, \cdots, m-1$. Then by F.O.C. of the cost minimization, we have

$$\begin{aligned} 0 &= c'(y^*) + \sum_{i=0}^{m-1} \beta c'(y_i) \left. \frac{dy_i}{dy} \right|_{y=y^*} \\ &= \gamma\alpha (y^*)^{\alpha-1} - \sum_{i=0}^{m-1} \beta\gamma\alpha \left[A_i \prod_{j=i+1}^{m-1} (1 - A_j)(x - y^*) \right]^{\alpha-1} A_i \prod_{j=i+1}^{m-1} (1 - A_j) \\ &= \gamma\alpha \left\{ (y^*)^{\alpha-1} - \sum_{i=0}^{m-1} \beta \left[A_i \prod_{j=i+1}^{m-1} (1 - A_j) \right]^{\alpha} (x - y^*)^{\alpha-1} \right\}. \end{aligned}$$

Therefore, $y_m(x) = y^* = x / \left\{ 1 + \beta^{-\frac{1}{\alpha-1}} \sum_{i=0}^{m-1} \left[A_i \prod_{j=i+1}^{m-1} (1 - A_j) \right]^{-\frac{\alpha}{\alpha-1}} \right\}$. So we have $y_m(x) = A_m x$ where

$$A_m = \frac{1}{1 + \beta^{-\frac{1}{\alpha-1}} \sum_{i=0}^{m-1} \left[A_i \prod_{j=i+1}^{m-1} (1 - A_j) \right]^{-\frac{\alpha}{\alpha-1}}},$$

and (C.1) holds for $k = m$.

In sum, we establish that $y_k(x) = A_k x$, $A_0 = 1$ and (C.1) holds for all $k = 1, \cdots, T$.

■

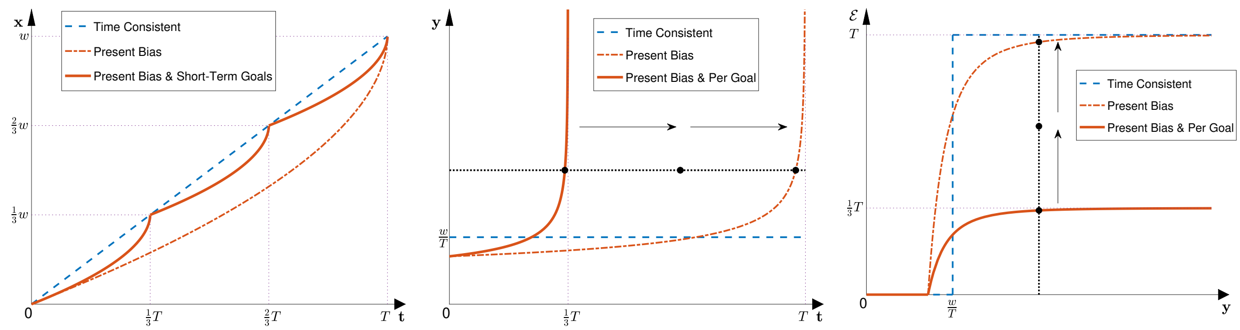
Lemma C.1 above implies that

$$\begin{aligned} (A_k^{-1} - 1)^{1-\alpha} &= \beta \sum_{i=0}^{k-1} A_i^{\alpha} \prod_{j=i+1}^{k-1} (1 - A_j)^{\alpha} = \beta A_{k-1}^{\alpha} + \beta (1 - A_{k-1})^{\alpha} \sum_{i=0}^{k-2} A_i^{\alpha} \prod_{j=i+1}^{k-2} (1 - A_j)^{\alpha} \\ &= \beta A_{k-1}^{\alpha} + (1 - A_{k-1})^{\alpha} (A_{k-1}^{-1} - 1)^{1-\alpha} = A_{k-1}^{\alpha-1} - (1 - \beta) A_{k-1}^{\alpha}, \end{aligned}$$

where the second equality follows from the convention $\Pi_{i=k}^{k-1}(1 - A_i)^{\alpha} = 1$, and the third equality holds using (C.1) for A_{k-1}. Therefore, $A_k = 1 / \left\{ 1 + [A_{k-1}^{\alpha-1} - (1 - \beta) A_{k-1}^{\alpha}]^{-\frac{1}{\alpha-1}} \right\}$.

Appendix D. Constant Return to (Task) Scale

Consider the case when the workload is known to be w. By *Proposition 1*, both cost functions, $C(\cdot)$ and $LC(\cdot)$, are homogeneous of degree 1 in the task requirement (w, T). In other words, ex-ante perceived cost and long-run cost are both constant return to (task) scale. This implies that optimal short-term goals, which distribute the total workload proportionally to the duration of each short-term task phase, do not affect the overall effort costs. In this appendix, I will demonstrate the reason behind this result.



(a) Level Comparison (b) Slope Aggregation (c) Effort Time Distribution

FIGURE D.1. Constant Effort Cost to Task Scale

I define the "effort time distribution" function $\mathcal{E}(\cdot)$ as follows: for any $y \geq Bw/T$,

$$\mathcal{E}(y; w, T, \mathcal{B}) \equiv \psi^{-1}(t; w, T, \mathcal{B}) = T - T\left[Ty/(Bw)\right]^{-\frac{1}{1-B}}, \tag{D.1}$$

where $B < 1$ and $\psi(t; w, T, \mathcal{B}) = y_t(w, T, \mathcal{B})$.[20] The effort time distribution is given by the inverse function of y_t characterized in (B.6). It represents the accumulated time when the flow effort is no more than y.

Observe that the effort time distribution function $\mathcal{E}(y; w, T, \beta)$ is homogeneous of degree 1 in the task requirement (w, T). This implies that the time spent below any flow effort level y is prolonged proportionally to the task requirement (w, T). For example, suppose the workload and the time available to complete the task are both tripled; then the time spent below any flow effort, and thus the aggregated effort cost, are also tripled (see *Figure D.1* for an illustration).

Under the optimal short-term goals, workload is distributed proportionally to the duration of each short-term task phase. Therefore, although less work is left to finish near the final due date compared to working without these goals, the total time that the agent spends on every flow effort level, measured by the effort time distribution function $\mathcal{E}$, remains unchanged.

Given that $\mathcal{E}$ is homogeneous of degree 1 in the task requirement (w, T), now I will show that the same holds true for C and LC, which in turn implies that the value of the optimal short-term goals is null. By the definition of the long-run cost (11) and the definition of the effort time distribution function (D.1), we obtain

$$LC(w, T, \mathcal{B}) = \int_0^T c(y_t(w, T, \mathcal{B}))\mathrm{d}t = \int_{\frac{Bw}{T}}^{\infty} c(y) d\mathcal{E}(y; w, T, \mathcal{B}),$$

[20]Since ψ is strictly increasing in $t \in [0, T)$ for a present-biased agent, $\mathcal{E}$ is well defined for $y \in [y_0(w, T, \mathcal{B}), \infty) = [Bw/T, \infty)$.

where the second equality holds by the change of integrated variable. Therefore, when $\mathcal{E}(y; w, T, \beta)$ is homogeneous of degree 1 in (w, T), for any $\lambda > 0$, we have

$$\begin{aligned} LC(\lambda w, \lambda T, \mathcal{B}) &= \int_{\frac{Bw}{T}}^{\infty} c(y) d\mathcal{E}(y; \lambda w, \lambda T, \mathcal{B}) = \int_{\frac{Bw}{T}}^{\infty} c(y) d\left(\lambda \mathcal{E}(y; w, T, \mathcal{B})\right) \\ &= \lambda LC(w, T, \mathcal{B}), \end{aligned}$$

which implies the constant return to task scale of LC (and thus $C = \frac{\alpha B + 1 - \alpha}{B} LC$) as desired.

References

Ahn, David S., Ryota Iijima, and Todd Sarver. 2020. "Naivete about Temptation and Self-Control: Foundations for Recursive Naive Quasi-Hyperbolic Discounting." *Journal of Economic Theory*, 189: 105087.

Ahn, David S., Ryota Iijima, Yves Le, and Todd Sarver. 2019. "Behavioral Characterizations of Naiveté for Time-Inconsistent Preferences." *Review of Economic Studies*, 86: 2319–2355.

Amador, Manuel, Iván Werning, and George-Marios Angeletos. 2006. "Commitment vs. Flexibility." *Econometrica*, 74(2): 365–396.

Ambrus, Attila, and Georgy Egorov. 2013. "Comment on "Commitment vs. Flexibility"." *Econometrica*, 81(5): 2113–2124.

Andersen, Steffen, Glenn W Harrison, Morten I Lau, and E Elisabet Rutström. 2008. "Eliciting Risk and Time Preferences." *Econometrica*, 76(3): 583–618.

Andersen, Steffen, John Y Campbell, Kasper Meisner Nielsen, and Tarun Ramadorai. 2020. "Sources of Inaction in Household Finance: Evidence from the Danish Mortgage Market." *American Economic Review*, 110(10): 3184–3230.

Andreoni, James, and Charles Sprenger. 2012. "Estimating Time Preferences from Convex Budgets." *American Economic Review*, 102(7): 3333–56.

Ariely, Dan, and Klaus Wertenbroch. 2002. "Procrastination, Deadlines, and Performance: Self-Control by Precommitment." *Psychological Science*, 13(3): 219–224.

Augenblick, Ned, and Matthew Rabin. 2019. "An Experiment on Time Preference and Misprediction in Unpleasant Tasks." *Review of Economic Studies*, 86(3): 941–975.

Augenblick, Ned, Muriel Niederle, and Charles Sprenger. 2015. "Working over Time: Dynamic Inconsistency in Real Effort Tasks." *The Quarterly Journal of Economics*, 130(3): 1067–1115.

Banerjee, Abhijit, and Sendhil Mullainathan. 2010. "The Shape of Temptation: Implications for the Economic Lives of the Poor." National Bureau of Economic Research.

Bernheim, B Douglas, and Debraj Ray. 1987. "Economic Growth with Intergenerational Altruism." *The Review of Economic Studies*, 54(2): 227–243.

Bertsekas, Dimitri P. 2005. *Dynamic Programming and Optimal Control.* Vol. 1 of *Athena Scientific Optimization and Computation Series*. 3 ed., Athena Scientific Nashua, NH.

Bond, Philip, and Gustav Sigurdsson. 2018. "Commitment Contracts." *The Review of Economic Studies*, 85(1): 194–222.

Bryan, Gharad, Dean Karlan, and Scott Nelson. 2010. "Commitment Devices." *Annual Review of Economics*, 2(1): 671–698.

Burns, Lawrence R., Katherine Dittmann, Ngoc-Loan Nguyen, and Jacqueline K. Mitchelson. 2000. "Academic Procrastination, Perfectionism, and Control: Associations with Vigilant and Avoidant Coping." *Journal of Social Behavior and Personality*, 15(5): 35–46.

Cohen, Jonathan, Keith Marzilli Ericson, David Laibson, and John Myles White. 2020. "Measuring Time Preferences." *Journal of Economic Literature*, 58(2): 299–347.

DellaVigna, Stefano. 2009. "Psychology and Economics: Evidence from the Field." *Journal of Economic Literature*, 47(2): 315–72.

DellaVigna, Stefano, and Ulrike Malmendier. 2004. "Contract Design and Self-Control: Theory and Evidence." *The Quarterly Journal of Economics*, 119(2): 353–402.

DellaVigna, Stefano, and Ulrike Malmendier. 2006. "Paying Not to Go to the Gym." *The American Economic Review*, 96(3): 694–719.

Ely, Jeffrey C, and Martin Szydlowski. 2020. "Moving the Goalposts." *Journal of Political Economy*, 128(2): 468–506.

Ericson, Keith M. 2017. "On the Interaction of Memory and Procrastination: Implications for Reminders, Deadlines, and Empirical Estimation." *Journal of the European Economic Association*, 15(3): 692–719.

Ericson, Keith M., and David Laibson. 2019. "Intertemporal Choice." In *Handbook of Behavioral Economics: Applications and Foundations*. Vol. 2, , ed. B. Douglas Bernheim, Stefano DellaVigna and David Laibson, Chapter 1, 1–67. Elsevier.

Fee, Ronda L, and June P Tangney. 2000. "Procrastination: A Means of Avoiding Shame or Guilt?" *Journal of Social Behavior and Personality*, 15(5): 167.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue. 2002. "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature*, 40(2): 351–401.

Fudenberg, Drew, and David K. Levine. 2006. "A Dual-Self Model of Impulse Control." *American Economic Review*, 96(5): 1449–1476.

Galperti, Simone. 2015. "Commitment, Flexibility, and Optimal Screening of Time Inconsistency." *Econometrica*, 83(4): 1425–1465.

Gottlieb, Daniel, and Xingtan Zhang. 2021. "Long-Term Contracting with Time-Inconsistent Agents." *Econometrica*, 89(2): 793–824.

Gul, Faruk, and Wolfgang Pesendorfer. 2001. "Temptation and Self-Control." *Econometrica*, 69(6): 1403–1435.

Harris, Christopher, and David Laibson. 2013. "Instantaneous Gratification." *The Quarterly Journal of Economics*, 128(1): 205–248.

Heidhues, Paul, and Botond Kőszegi. 2010. "Exploiting Naivete about Self-control in the Credit Market." *American Economic Review*, 100(5): 2279–2303.

Heidhues, Paul, and Philipp Strack. 2021. "Identifying Present Bias from the Timing of Choices." *American Economic Review*, 111(8): 2594–2622.

Himmler, Oliver, Robert Jäckle, and Philipp Weinschenk. 2019. "Soft Commitments, Reminders, and Academic Performance." *American Economic Journal: Applied Economics*, 11(2): 114–42.

Kopylov, Igor. 2012. "Perfectionism and Choice." *Econometrica*, 80(5): 1819–1843.

Kőszegi, Botond. 2014. "Behavioral Contract Theory." *Journal of Economic Literature*, 52(4): 1075–1118.

Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting." *The Quarterly Journal of Economics*, 112(2): 443–478.

Laibson, David. 2015. "Why Don't Present-Biased Agents Make Commitments?" *American Economic Review*, 105(5): 267–72.

Laibson, David, and Peter Maxted. 2020. "The β–δ–Δ Sweet Spot." *working paper*.

Laibson, David, Peter Maxted, and Benjamin Moll. 2021. "Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy." Working Paper.

Loewenstein, George F., and Drazen Prelec. 1992. "Anomalies in Intertemporal Choice: Evidence and an Interpretation." *The Quarterly Journal of Economics*, 107(2): 573–597.

Maxted, Peter. 2023. "Present Bias Unconstrained: Consumption, Welfare, and the Present-Bias Dilemma." Working Paper.

Mischel, Walter. 1974. "Processes in Delay of Gratification." In *Advances in Experimental Social Psychology*. Vol. 7, 249–292. Elsevier.

Mullainathan, Sendhil, and Eldar Shafir. 2013. *Scarcity: Why Having Too Little Means So Much.* Macmillan.

O'Donoghue, Ted, and Matthew Rabin. 1999. "Doing It Now or Later." *American Economic Review*, 89(1): 103–124.

O'Donoghue, Ted, and Matthew Rabin. 2001. "Choice and Procrastination." *The Quarterly Journal of Economics*, 116(1): 121–160.

Ross, Lee. 1977. "The Intuitive Psychologist and His Shortcomings: Distortions in the Attribution Process." In *Advances in Experimental Social Psychology*. Vol. 10, 173–220. Elsevier.

Saez-Marti, Maria, and Anna Sjögren. 2008. "Deadlines and Distractions." *Journal of Economic Theory*, 143(1): 153–176.

Schilbach, Frank. 2019. "Alcohol and Self-Control: A Field Experiment in India." *American Economic Review*, 109(4): 1290–1322.

Sirois, Fuschia M., and Timothy A. Pychyl. 2016. *Procrastination, Health, and Well-being.* Academic Press.

Steel, Piers. 2007. "The Nature of Procrastination: a Meta-Analytic and Theoretical Review of Quintessential Self-Regulatory Failure." *Psychological Bulletin*, 133(1): 65.

Strotz, Robert Henry. 1956. "Myopia and Inconsistency in Dynamic Utility Maximization." *The Review of Economic Studies*, 23(3): 165–180.

Thaler, Richard H. 2015. *Misbehaving: The Making of Behavioral Economics.* WW Norton New York.

Thaler, Richard H., and Hersh M. Shefrin. 1981. "An Economic Theory of Self-Control." *Journal of Political Economy*, 89(2): 392–406.

Tice, Dianne M., and Roy F. Baumeister. 1997. "Longitudinal Study of Procrastination, Performance, Stress, and Health: The Costs and Benefits of Dawdling." *Psychological Science*, 8(6): 454–458.

Tice, Dianne M., Ellen Bratslavsky, and Roy F. Baumeister. 2001. "Emotional Distress Regulation Takes Precedence over Impulse Control: If You Feel Bad, Do It!" *Journal of Personality and Social Psychology*, 80(1): 53.