Competing Signaling Intermediaries

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Motivation

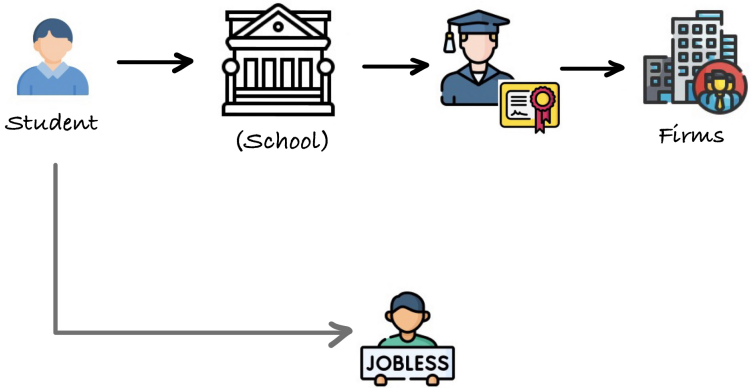
Spence (1973): people use education to costly signal their ability to firms

- exogenous signaling technology

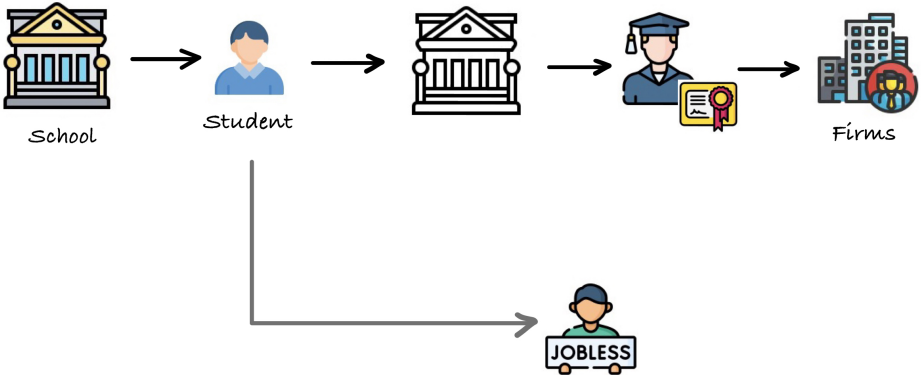
Our contribution: take signaling intermediaries as active players

- intermediaries choose signaling technology and charge fees for service
- equilibrium outcomes vary with market structure of intermediaries

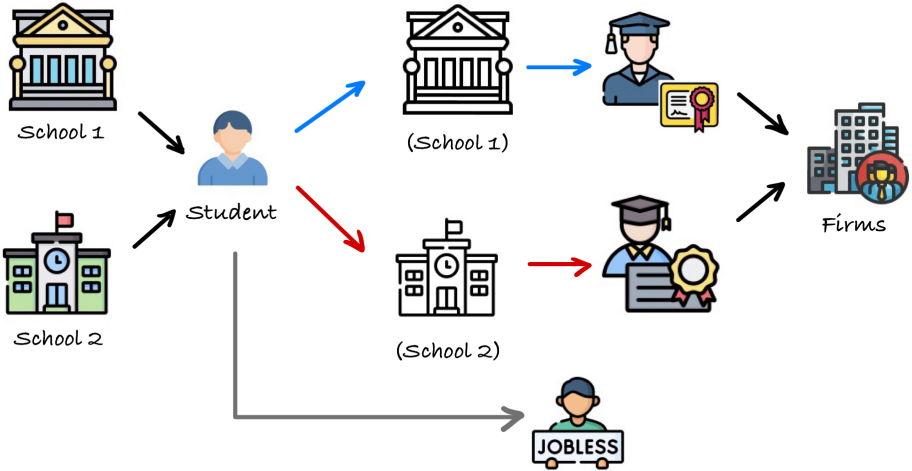
Spence: Exogenous Signaling Technology



Departure 1: Intermediary as an Active Player



Departure 2: Market Structure of Intermediaries Matters



Questions

Intermediaries control the signaling opportunities available to senders

- What outcome will emerge under endogenous signaling technology and costs?
- What is the impact of competition among intermediaries?

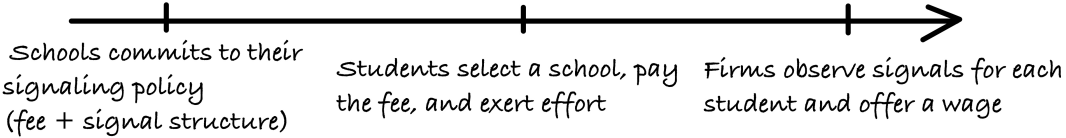
Model

- Players: a mass of students, a mass of firms, a finite set I of schools
  - monopolistic school: $I = \{1\}$
  - competitive schools: $I = \{1, 2\}$
- Students privately know their ability type $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L, \theta_H > 0$
  - θ represents the market value of hiring the student
  - prior: $\theta = \theta_H$ with probability $\lambda \in (0, 1)$

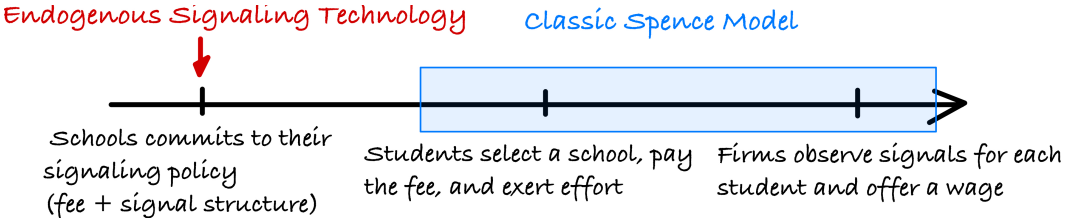
Actions

- Students: signal ability type through schooling choices
  - select school $i \in I$, or do not attend any school
  - exert effort $e \geq 0$: marginal cost of effort $c_e(\theta, e)$ decreases in θ
- Schools: commit a signaling policy (f_i, M_i)
  - charge a fee to its students: $f_i \geq 0$
  - send a message as a function of effort: $M_i : \mathbb{R}_+ \longrightarrow \Delta\mathcal{M}$
    - $\mathcal{M}$: the set of pure messages that schools can send to firms
- Firms: hire employees in a competitive labor market
  - offer wage = the candidate's expected productivity conditional on observed signals (the school attended & the message generated)

Timeline


Schools commits to their signaling policy (fee + signal structure)
Students select a school, pay the fee, and exert effort
Firms observe signals for each student and offer a wage

Timeline


Endogenous Signaling Technology
Classic Spence Model
Schools commits to their signaling policy (fee + signal structure)
Students select a school, pay the fee, and exert effort
Firms observe signals for each student and offer a wage

Information Structure and Payoffs

- Students:
  - observe their ability types and schools' policies
  - payoff: wage net of schooling costs $w - f_i - c(\theta, e)$ or the outside option 0
- Schools:
  - observe the enrolled students' effort
  - payoff: total fees $\alpha_i f_i$
    - $\alpha_i \in [0, 1]$: the mass of students enrolled in school i
- Firms:
  - observe schools' monitoring structure and candidate's schooling signals (i, m)
  - payoff: employees' productivity net of their wage $\theta - w$

Perfect Bayesian Equilibrium under D-1 Refinement

On top of the usual requirements for PBE, we further restrict the off-path belief for firms to satisfy the D-1 criterion (Cho and Kreps, 1987)

- Discipline off-path play to reduce multiplicity of PBE and sharpen prediction
- Basic idea: for a given off-path message, if it is "easier" for type θ' to deviate from their equilibrium strategy to sending this message compared to type θ, then firms should conclude that this message cannot be sent by type θ

D-1 Refinement on Off-Path Beliefs

- Basic idea: if a smaller range of wages triggers a profitable deviation to sending this message for type θ than the other type, then it cannot be sent by type θ

- Consider an off-path message (i, m) where, for some $e^m \geq 0$,

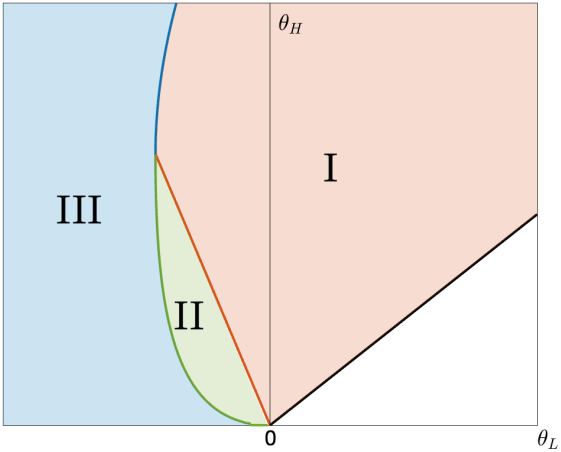
$$M_i(m|e) = \begin{cases} 1 & \text{if } e = e^m \\ 0 & \text{otherwise} \end{cases}$$

- D-1 criterion: $\mu(\theta|i, m) = 0$ if for $\theta' \neq \theta$,

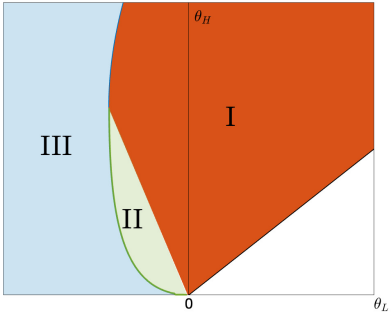
$$w - f_i - c(\theta, e^m) \geq U_\theta^* \quad \Rightarrow \quad w - f_i - c(\theta', e^m) > U_{\theta'}^*$$

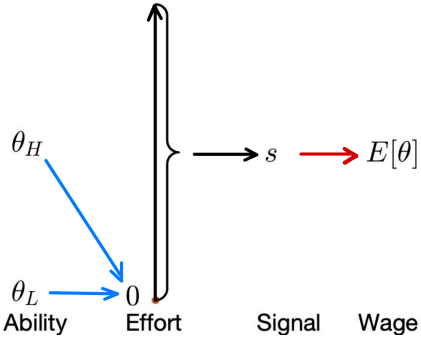
  - U_θ^*: type-θ student's equilibrium utility

Benchmark: Monopolistic School

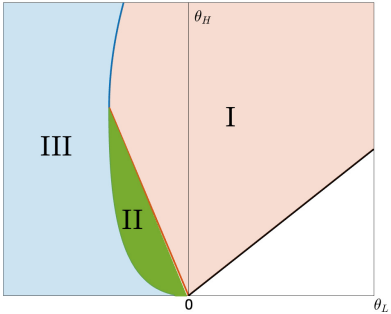


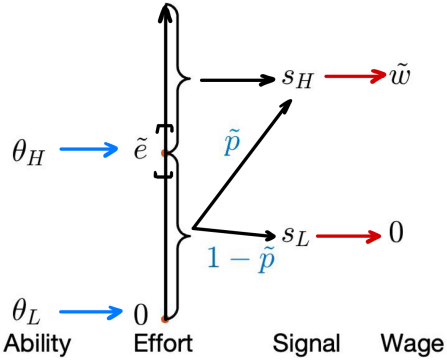
Monopoly Region I: Full Pooling



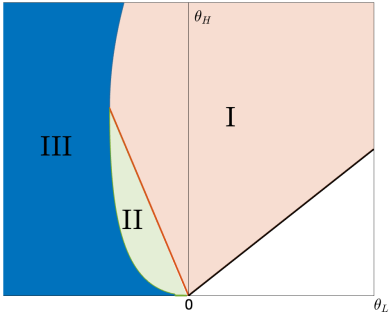


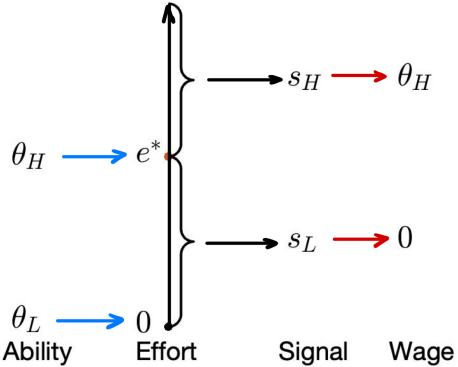
Monopoly Region II: Semi-Pooling



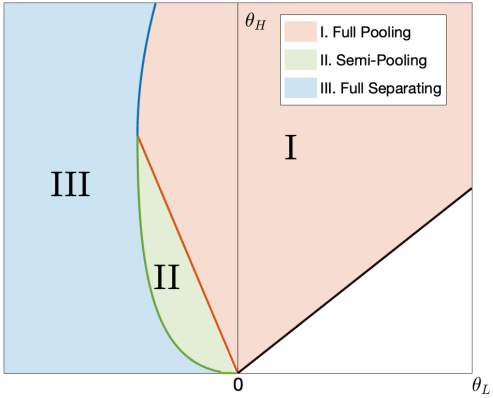


Monopoly Region III: Full Separating



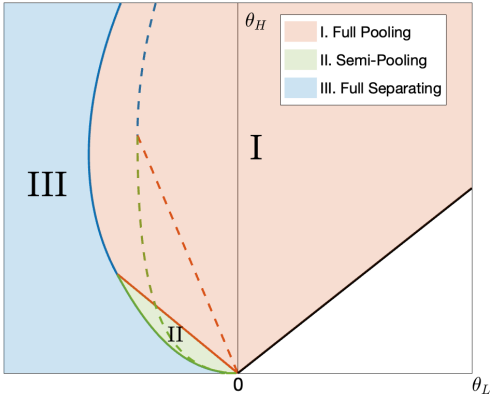


Monopolistic School Tends to Separate for Lower θ_L



- Key tension: coverage (include low-type) vs fee (separate high-type)
- As θ_L decreases, the signaling policy shifts from full pooling to full separating

Monopolistic School Tends to Pool for Higher λ

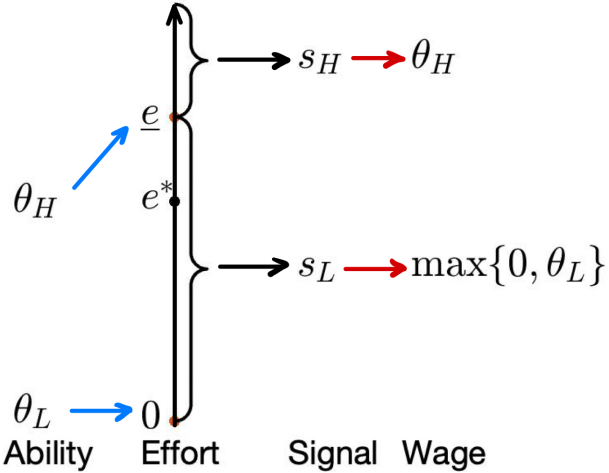


- Key tension: coverage (include low-type) vs fee (separate high-type)
- As λ increases, the school chooses to pool students in more cases

Competition among Signaling Intermediaries

Unique symmetric equilibrium outcome: Riley outcome

- replicates what happens when effort is directly observable to firms [*different logic*]



Welfare Comparison: Monopolistic School

Regions	**(I) Full Pooling**	**(II) Semi-Pooling**	**(III) Full Separating**
School	$E[\theta]$	$\tilde{p}\tilde{w}$	$\lambda\theta_H$
High Type	0	$(1-\tilde{p})\tilde{w} - c(\theta_H, \tilde{e})$	0
Low Type	0	0	0

- $\tilde{p}\tilde{w} = \frac{\lambda}{1-\lambda}(\sqrt{\theta_H - \theta_L} - \sqrt{-\theta_L})^2 > \max\{E[\theta], \lambda\theta_H\}$ in Region II
- $\tilde{e} \in [\tilde{e}_1, \tilde{e}_2]$ where

$$c(\theta_L, \tilde{e}_1) = c(\theta_H, \tilde{e}_2) = (1-\tilde{p})\tilde{w}$$

Welfare Comparison: Competitive Schools

Regions	**(I) Full Pooling**	**(II) Semi-Pooling**	**(III) Full Separating**
School	~~$E[\theta]$~~	~~$\tilde{p}\tilde{w}$~~	~~$\lambda\theta_H$~~ 0
High Type	~~0~~	~~$(1-\tilde{p})\tilde{w} - c(\theta_H, \tilde{e})$~~	~~0~~ $\theta_H - c(\theta_H, \underline{e})$
Low Type	~~0~~ $\max\{0, \theta_L\}$	0	0

A Shift in Surplus Distribution

Regions	**(I) Full Pooling**	**(II) Semi-Pooling**	**(III) Full Separating**
School		$\downarrow$	
High Type		$\uparrow$	
Low Type		$\uparrow$ or constant	

Competition Forces:

1. to undercut the competitor $\Rightarrow$ zero fee
   $\Rightarrow$ a shift in surplus from school to students

A Reduction in Total Surplus

Regions	**(I) Full Pooling**	**(II) Semi-Pooling**	**(III) Full Separating**
School		$\downarrow$	
High Type		$\uparrow$	
Low Type		$\uparrow$ or constant	
Total Surplus		$\downarrow$	

Competition Forces:

1. to undercut the competitor $\Rightarrow$ zero fee
   $\Rightarrow$ a shift in surplus from school to students
2. to uphold a high standard $\Rightarrow$ a higher effort threshold to send a good signal
   $\Rightarrow$ further efficiency loss since effort is socially wasteful

Conclusion

Under D1 Refinement

Exogenous signaling technology / observable effort: Riley outcome

If intermediaries control the signaling technology:

- Monopolistic school: full separating, semi-pooling, or full pooling
  - the school tends to pool students if θ_L or λ increases
- Competitive schools: Riley outcome with no fee
  - surplus shifts from school to students
  - tradeoff btw efficiency and info revelation: competition enhances efficiency only when the externality of productivity is sufficiently high

Bibliography (Incomplete)

- **Costly Signaling**: Spence (1973), Spence (1974), Lu (2018), Lu (2021)
- **Disclosure**: Dranove & Jin (2010)
- **Certification**: Lizzeri (1999), Albano & Lizzeri (2001), Ostrovsky & Schwarz (2010), Popov & Bernhardt (2013), Boleslavsky & Cotton (2015), Zubrickas (2015), Harbaugh & Rasmusen (2018), Boleslavsky & Kim (2020), Bizzotto & Vigier (2021), Ali & Haghpanah & Lin & Siegel (2022)
- **Equilibrium Selection**: Riley (1975), Riley (1979), Banks & Sobel (1987), Cho & Kreps (1987)