Procrastination under Uncertainty

Mingzi Niu

Department of Economics
Rice University

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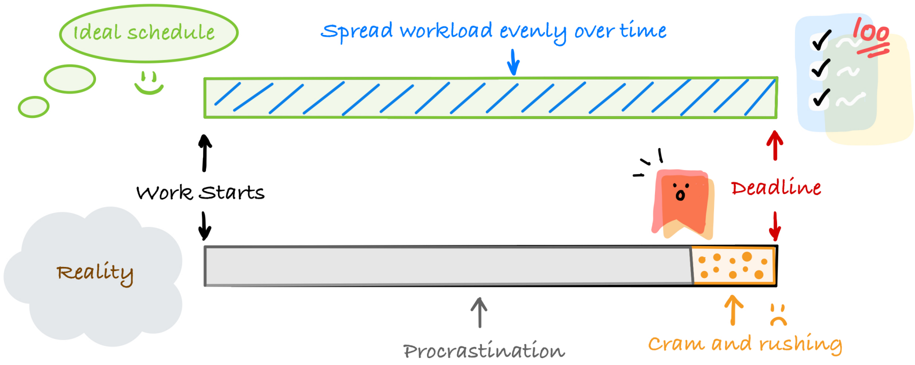
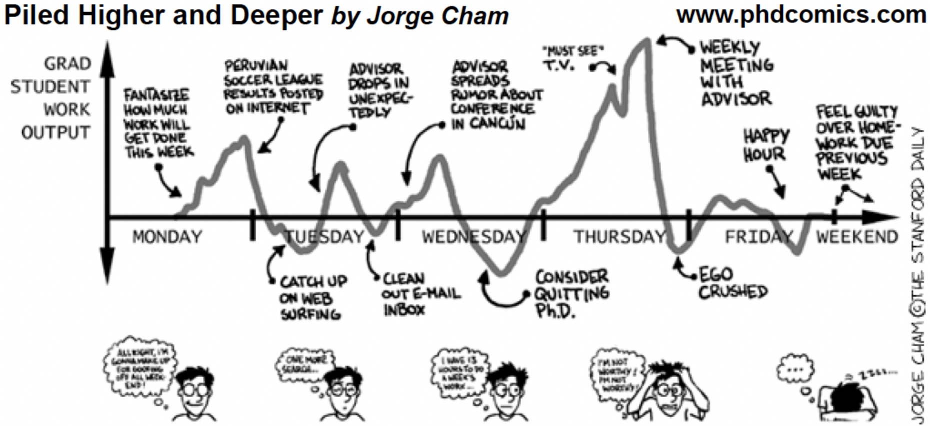
Motivation

People procrastinate, e.g., students finish assignments near due date



Motivation



title: "Graph - Work output" - originally published 5/3/1999

Motivation: Scenario

- A present-biased agent is assigned a long-term task with a deadline
- Once the task is completed, she gets a reward
- She is uncertain about the total effort needed to complete the task

Questions

Two types of contributing factors to procrastination:

1. personal factors/behavioral frictions: present bias, naivete
2. environmental factors/task features: workload, deadline, workload uncertainty

- How do behavioral frictions and task features interact in shaping procrastination?
- Are intermediate short-term goals valuable to a present-biased agent?

Main Results

- I develop a tractable framework to study *continuous-time* dynamic optimization with quasi-hyperbolic discounting in finite horizon
  - An increased workload uncertainty leads to a higher initial workload target, and thus induces more early work
  - Behavioral and environmental frictions reinforce each other in impairing welfare
    - an agent with larger present bias and/or naivete is more sensitive to adverse task features (i.e., heavier workload, shorter time available, more workload uncertainty)

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- Intermediate goals, as a commitment device, can make people procrastinate less; but in terms of individual welfare, they make the agent weakly worse off
- Time preferences and workload beliefs can be disentangled empirically from work schedule data, whereas present bias and naivete are observationally equivalent

Model

- Task
  - Requirement: complete total workload $w > 0$ within the deadline $T > 0$
  - Reward: upon task completion, the agent gets $v > 0$
  - Workload can be either low or high, i.e., $w \in \{w_L, w_H\}$
  - Prior: $w = w_L$ with probability $\mu \in [0, 1]$
- Effort
  - Effort (or work intensity) at time $t \in [0, T]$: $y_t \geq 0$
  - Workload finished by time $t \in [0, T]$: $x_t \equiv \int_0^t y_\tau d\tau$
  - Flow cost of effort: $c(y) = \gamma y^\alpha$ where $\gamma > 0, \alpha > 1$
- Uncertainty is resolved once the agent completes the lower workload w_L

Model: Time Preference

- **Discounting Function**: how to evaluate future utility flows at present
- **Sophistication**: how to anticipate future choices

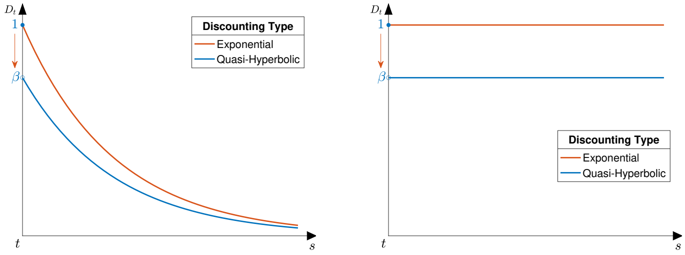
Discounting Function

Continuous-time quasi-hyperbolic discounting (*Harris & Laibson, 2013*)

- The discount factor evaluated at time t for utility at time $s \geq t$ as

$$D_t(s;\beta,\delta) = \begin{cases} 1 & \text{for } s = t, \\ \beta e^{-\delta(s-t)} & \text{for } s > t, \end{cases}$$

where $\beta \in (0,1]$ denotes present bias, $\delta \geq 0$ denotes the exponential discount rate



(a) $\beta \in (0,1)$, $\delta > 0$ (b) $\beta \in (0,1)$, $\delta = 0$

Sophistication

Allow for mistakes in self-perception

- Perceived present bias $\hat{\beta} \in [\beta, 1]$: sophisticated ($\hat{\beta} = \beta$); naive ($\hat{\beta} > \beta$)

Model: Dynamic Optimization

- The agent chooses effort inputs over time under workload uncertainty so as to minimize the expected overall effort cost:

$$\mathbf{y}^* \in \underset{\substack{\{y_t:\, t\in[0,T]\} \\ \tau\in(0,T]}}{\operatorname{argmin}} \int_0^\tau D_0(t)c(y_t)dt + (1-\mu)\beta \int_\tau^T c(y_t)dt$$

$$\text{subject to} \int_0^\tau y_t dt = w_L, \int_0^T y_t dt = w_H$$

- Intrapersonal game btw current self and future selves: Markov-Perfect Equilibrium
  - directly payoff-relevant info: remaining work and remaining time
  - at any time $t \in [0, T)$, given her perception about future selves' choices, the current self chooses the optimal effort input under the **actual** present bias β
  - the perceived future selves' choices are consistent with the **perceived** present bias $\hat{\beta}$

Characterize Work Schedule and Individual Welfare

Proposition (Potentially Naive Agent & Uncertain Workload)

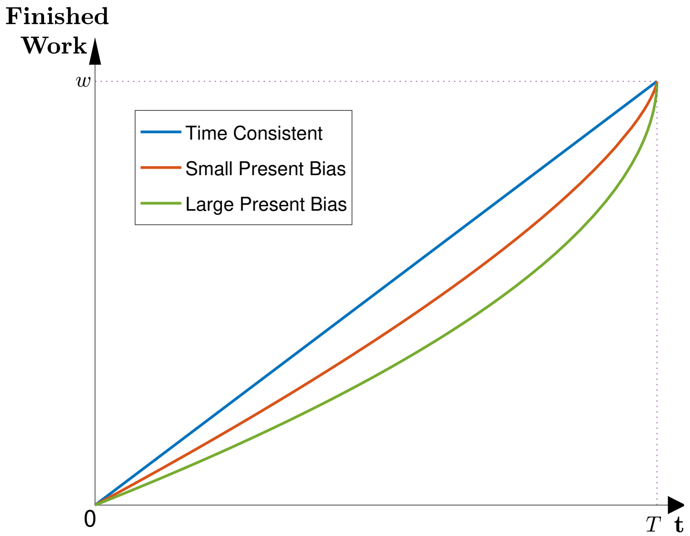
Let $B = (\beta/\hat{\beta})^{\frac{1}{\alpha-1}}(\alpha-1)/(\alpha-\hat{\beta})$, and $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$. The time when an agent $(\beta, \hat{\beta})$ finishes the low workload w_L is $\tau = \left[1-(1-w_L/\lambda)^{\frac{1}{B}}\right]T$. The unique work schedule for the agent is:

$$x_t(\mu,\beta,\hat{\beta}) = \begin{cases} \lambda\left[1-(1-t/T)^B\right] & \text{if } 0 \le t < \tau, \\ w_L & \text{if } \tau \le t < T \text{ and } w = w_L, \\ w_H - (w_H - w_L)\left[(T-t)/(T-\tau)\right]^B & \text{if } \tau \le t < T \text{ and } w = w_H; \end{cases}$$

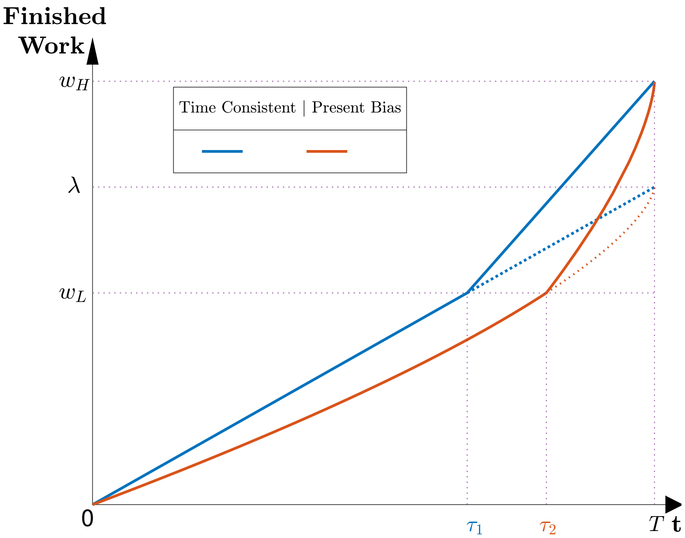
$$y_t(\mu,\beta,\hat{\beta}) = \begin{cases} \frac{\lambda B}{T}(1-t/T)^{B-1} & \text{if } 0 \le t < \tau, \\ 0 & \text{if } \tau \le t < T \text{ and } w = w_L, \\ (w_H - w_L)B\left[(T-t)/(T-\tau)\right]^{B-1}/(T-\tau) & \text{if } \tau \le t < T \text{ and } w = w_H. \end{cases}$$

The ex-ante perceived cost is $C(\mu,\beta,\hat{\beta}) = \gamma B^{\alpha-1}\lambda^{\alpha}/T^{\alpha-1}$.
The long-run cost is $LC(\mu,\beta,\hat{\beta}) = \gamma B^{\alpha}\lambda^{\alpha}/\left[(\alpha B + 1 - \alpha)T^{\alpha-1}\right]$.

When the Workload is Certain: $\mu = 0$ or 1



When the Workload is Uncertain: $\mu \in (0, 1)$

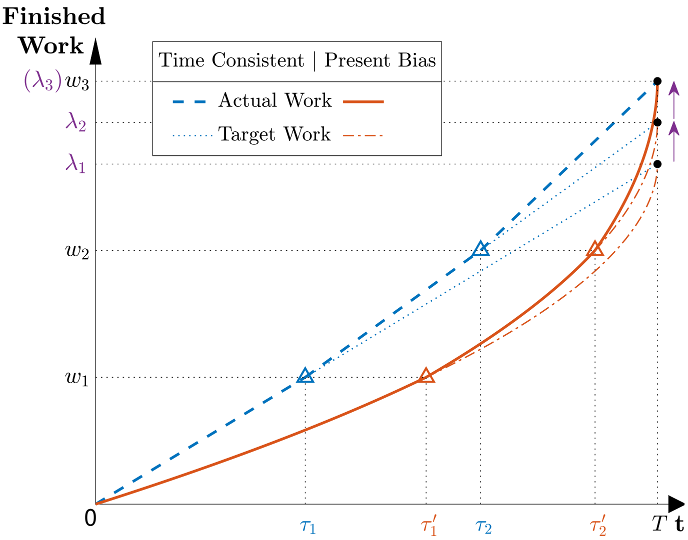


Workload Uncertainty Alleviates Procrastination

- Target under workload uncertainty: $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$

- An increased uncertainty about workload (i.e., a mean-preserving spread of the workload prior) leads to a higher λ

(Extension) Gradual Learning by Doing: Moving Goalposts



Workload Uncertainty Entails Welfare Loss

- Benchmark: Expected cost when uncertainty is resolved before work

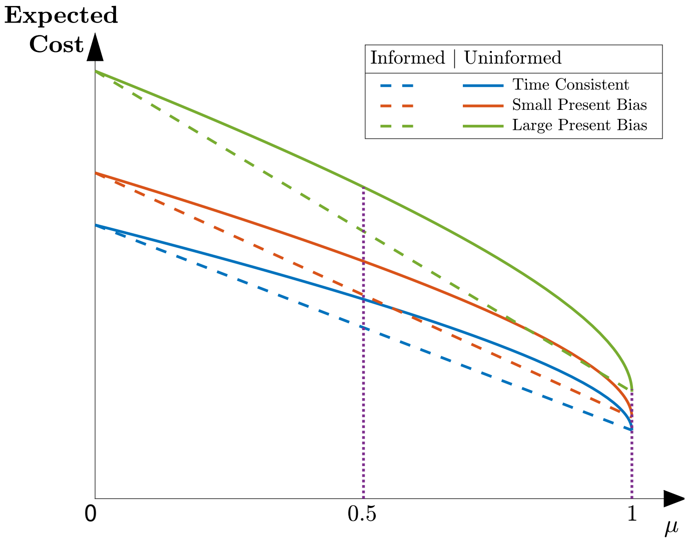
$$\Pi(\mu, \beta, \hat{\beta}) = \mu \cdot LC(w_L, T, \beta, \hat{\beta}) + (1-\mu) \cdot LC(w_H, T, \beta, \hat{\beta}) = \frac{\gamma B^{\alpha}[\mu w_L^{\alpha} + (1-\mu) w_H^{\alpha}]}{[1-\alpha(1-B)]\, T^{\alpha-1}}.$$

- The value of information:

$$I(\mu, \beta, \hat{\beta}) \equiv LC(\mu, \beta, \hat{\beta}) - \Pi(\mu, \beta, \hat{\beta}) = \frac{\gamma B^{\alpha}\,[\lambda^{\alpha} - \mu w_L^{\alpha} - (1-\mu) w_H^{\alpha}]}{[1-\alpha(1-B)]\, T^{\alpha-1}} \geq 0,$$

- Given the magnitude of uncertainty μ, $I(\mu, \beta, \hat{\beta})$ decreases in B
    - uncertainty affects more present-biased/naive agent more adversely

The Value of Information



Committing to Intermediate Goals

- A natural class of commitment devices to regulate a long-term task: committed to a successive series of short-term goals
  - e.g., milestones for graduate studies, weekly report on work progress

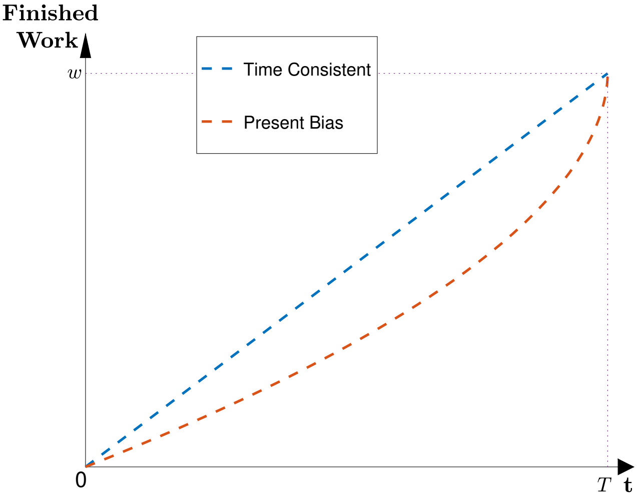
- Suppose the agent can commit to some intermediate goals:

$$G^k = \{(w_1, \tau_1), (w_2, \tau_2), \cdots, (w_k, \tau_k)\}$$

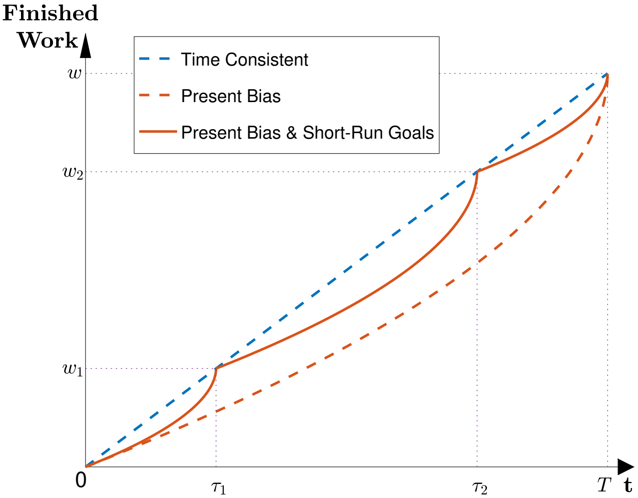
  with $(w_k, \tau_k) = (w, T)$

- Lesson from time inconsistency literature
  - Commitment devices can strictly enhance long-run welfare for a present-biased agent

Certain Workload: Work Schedule



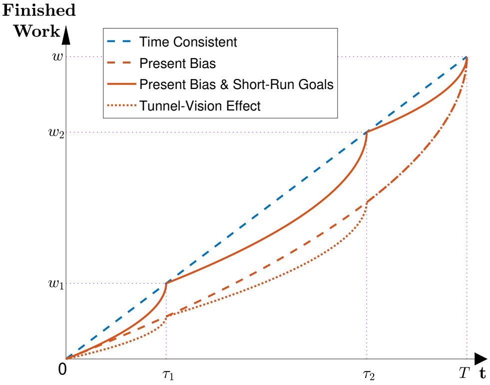
Certain Workload: Work Schedule under Optimal Short-Term Goals



The Value of Commitment Device

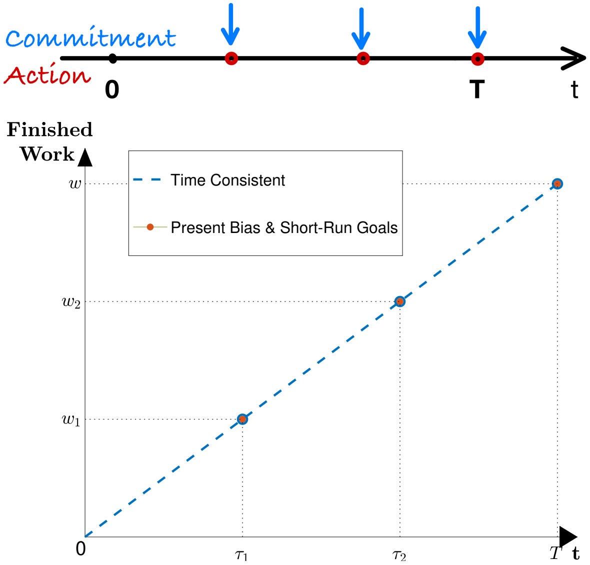
- Compare overall effort cost for task completion with and without short-term goals
  - The ex-ante perceived cost: $\hat{C}(G^k) \geq C(w, T, \beta, \hat{\beta})$
  - The long-run cost: $L\hat{C}(G^k) \geq LC(w, T, \beta, \hat{\beta})$
- No intermediate goals decrease the overall effort cost for any agent

Two Forces at Play

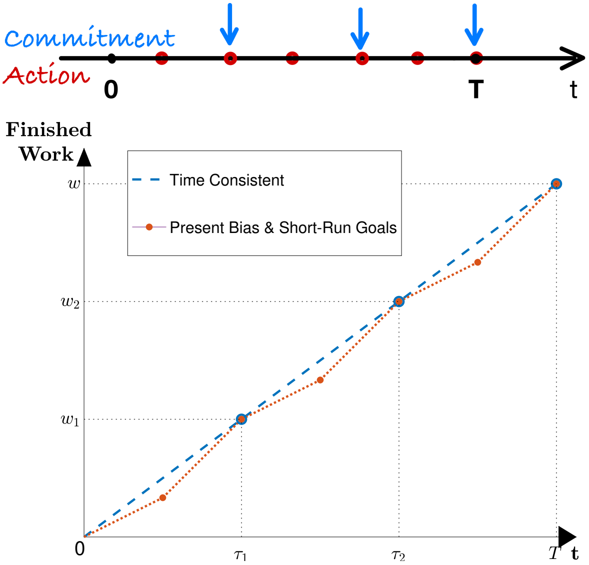


1. *Keeping on Track* $(+)$: induce early work so less work left near the final deadline
2. *Tunnel Vision* $(-)$: focus on and rush for the urgent short-term goal at each phase

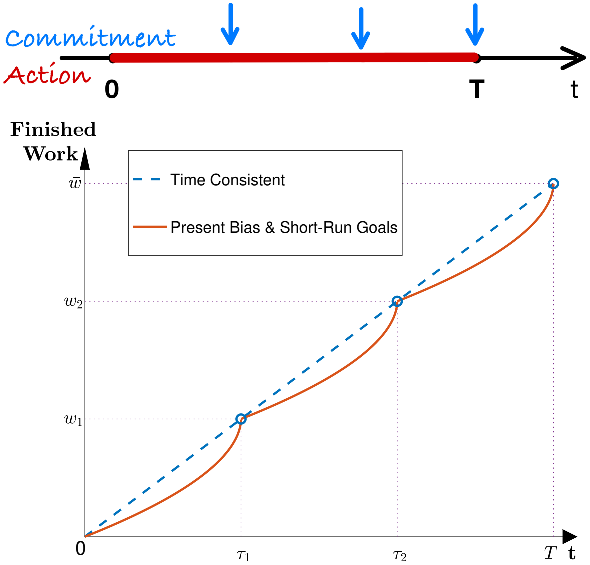
Full Commitment



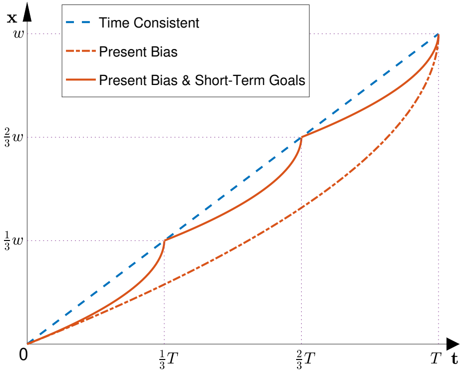
Inadequate Commitment



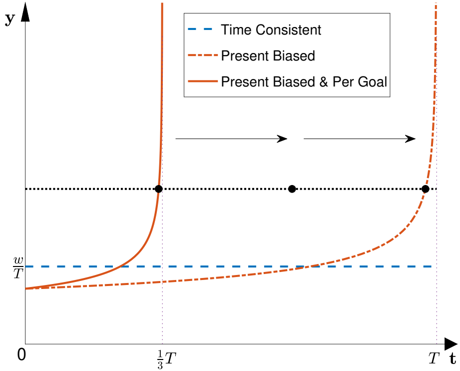
Relative Frequency of Actions and Commitments Matters



Constant Overall Effort Cost to Task Scale



(a) Level Comparison



(b) Slope Aggregation

Empirical Content of the Model

- People procrastinate because:
  - present bias β
  - naivete $\beta/\hat{\beta}$
  - workload uncertainty μ
  - rushing aversion α
- Can we disentangle these driving forces empirically?
  - Data: work trajectory $\{x_t : t \in [0, T]\}$
  - Parameters of interest: $(\beta, \hat{\beta}, \mu, \alpha)$

Joint Identification of Time Preference and Belief

- We can jointly identify a measure of time preference B and a measure of prior belief λ using (partial) data on work schedule with a two-step procedure:

1. To identify B: $f(B) \equiv \frac{1-(1-\frac{s}{T})^B}{1-(1-\frac{t}{T})^B} = \frac{x_s}{x_t} \Rightarrow B = f^{-1}(\frac{x_s}{x_t})$
2. To identify λ: $\lambda = \frac{x_t}{1-\left(1-\frac{t}{T}\right)^B}$

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  2. To identify λ: $\lambda = \frac{x_t}{1-\left(1-\frac{t}{T}\right)^B}$

- B and λ affect the work trajectory in a distinguishable way:
  - time preferences control how much the work trajectory is tilted towards the deadline
  - the prior belief on workload decides the initial target under the workload uncertainty
- We can exploit this variation to jointly identify time preference and workload belief

Observational Equivalence of Present Bias and Naivete

- Since we have recovered $B = \left(\frac{\beta}{\hat{\beta}}\right)^{\frac{1}{\alpha-1}} \frac{\alpha-1}{\alpha-\hat{\beta}}$ from work schedule data, can we further identify β and $\hat{\beta}$ separately?

- The answer is: **NO** — even with a complete record of the agent's work schedule

- Work schedule for any (partially) naive agent is *observationally equivalent* to that for a fully naive agent with a smaller present bias, or for a sophisticated agent with a larger present bias

Observational Equivalence of Present Bias and Naivete

Work schedule for any (partially) naive agent is *observationally equivalent* to that for a fully naive agent with a smaller present bias, or for a sophisticated agent with a larger present bias

- For any time preference $(\beta, \hat{\beta})$ such that $0 < \beta \leq \hat{\beta} < 1$, there always exists an alternative time preference $(\beta', 1)$ that replicates the work schedule, in which $\beta' = \frac{\beta}{\hat{\beta}} \left(\frac{\alpha-1}{\alpha-\hat{\beta}} \right)^{\alpha-1} \in (\beta, 1)$
- For any time preference $(\beta, \hat{\beta})$ such that $0 < \beta < \hat{\beta} \leq 1$, there always exists an alternative time preference (β', β') that replicates the work schedule, in which $\beta' = \alpha - (\alpha - \hat{\beta}) \left(\frac{\hat{\beta}}{\beta} \right)^{\frac{1}{\alpha-1}} < 1$

Non-Identification of Rushing Aversion α

- Rushing aversion $\alpha > 1$ affects both the time preference measure $B = \left(\frac{\beta}{\hat{\beta}}\right)^{\frac{1}{\alpha-1}} \frac{\alpha-1}{\alpha-\hat{\beta}}$ and the prior belief measure $\lambda = w_L + (1-\mu)^{\frac{1}{\alpha}}(w_H - w_L)$
- Given (w_H, w_L), (B, λ) summarizes what we can learn from a work trajectory
- We cannot recover α from complete work trajectory data, or rather, from (B, λ)
- Any level of rush aversion is observationally equivalent to a larger rushing aversion with a larger present bias or with a larger prior likelihood of a low workload

Conclusion

- I develop a model in which a present-biased person chooses how to distribute workload over time by a deadline under workload uncertainty
- I provide closed-form solutions for individual work schedule and welfare
  - present bias and naivete add curvature to the work trajectory
  - workload uncertainty raises the initial workload target
- Commitment device can make people procrastinate less; but in terms of individual welfare, it is at best of no value, if not harmful.
  - A negative effect arises under present bias and limited commitment
  - It grows as the frequency of actions relative to the frequency of short-term goals increases, and it can be significant enough to completely neutralize and even strictly dominate the positive disciplining effect of commitment

Bibliography

- **Experimental and Empirical Evidence**: Thaler & Shefrin (1981), Loewenstein & Prelec (1992), **Ariely & Wertenbroch (2002),** DellaVigna & Malmendier (2006), Choi, Laibson & Madrian (2009), DellaVigna (2009), Mullainathan & Shafir (2013), Pychyl (2013), Thaler (2015), Agarwal, Rosen & Yao (2016)
- **Time Inconsistency Models**: Strotz (1956), Thaler & Shefrin (1981), Laibson (1997), **O'Donoghue & Rabin (1999,2001)**, Gul & Pesendorfer (2001), Fudenberg & Levine (2006), Sarver (2008), Banerjee & Mullainathan (2010), Ericson (2011), **Harris & Laibson (2013),** Bernheim, Ray & Yeltekin (2015), Ericson & Laibson (2019), Ahn, Iijima, Le & Sarver (2019), Ahn, Iijima, Le & Sarver (2020)
- **Commitment Contract**: DellaVigna & Malmendier (2004), Amador, Werning & Angeletos (2006), Bryan, Karlan & Nelson (2010), Heidhues & Kőszegi (2010), Ambrus & Egorov (2013), Kőszegi (2014), Galperti (2015), Laibson (2015), Bond & Sigurdsson (2018), Gottlieb & Zhang (2021), Laibson (2015)
- **Task Completion under Uncertainty**: **Ely & Szydlowski (2020), Heidhues & Strack (2021)**