

## 7: Appendix

**Figure 1: Example of a simple random network**

```
ER[n_, p_] :=
  Table[If[j == k, 0, RandomChoice[{p, 1 - p} → {1, 0}]], {j, 1, n}, {k, 1, n}];
A0 = ER[20, 0.1];
GraphPlot[A0, DirectedEdges → True]
```

**Figure 3: Plot showing the gradient-learning model and the scenarios**

```
U[J_] := m J -  $\frac{c}{2}$  J2;
D[U[J], J];
Solve[-c J + m == 0, J];
DSolve[{∂t J[t] == α U[J[t]] - β J[t], J[0] == J0}, J[t], t];
Plot[
  {



$$\frac{2 e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} (m \alpha - \beta)}{-e^{t \beta + \frac{i \beta (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} + c e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} \alpha} /. m \rightarrow 5 /. c \rightarrow 0.1 /. \alpha \rightarrow 0.9 /. \beta \rightarrow 0.9 /. J0 \rightarrow 0.001,$$


$$\frac{2 e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} (m \alpha - \beta)}{-e^{t \beta + \frac{i \beta (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} + c e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} \alpha} /. m \rightarrow 5 /. c \rightarrow 0.1 /. \alpha \rightarrow 0.2 /. \beta \rightarrow 0.2 /. J0 \rightarrow 0.001,$$


$$\frac{2 e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} (m \alpha - \beta)}{-e^{t \beta + \frac{i \beta (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} + c e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} \alpha} /. m \rightarrow 5 /. c \rightarrow 0.1 /. \alpha \rightarrow 0.9 /. \beta \rightarrow 0.2 /. J0 \rightarrow 0.001,$$


$$\frac{2 e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} (m \alpha - \beta)}{-e^{t \beta + \frac{i \beta (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} + c e^{m t \alpha + \frac{i m \alpha (\pi - i \text{Log}[J0] + i \text{Log}[-c J0 \alpha + 2 m \alpha - 2 \beta])}{m \alpha - \beta}} \alpha} /. m \rightarrow 5 /. c \rightarrow 0.1 /. \alpha \rightarrow 0.2 /. \beta \rightarrow 0.9 /. J0 \rightarrow 0.001\}, \{t, 0, 100\},
  FrameLabel → {"t (Time)", "J[t] (Number of job applications over time)"},
  PlotLabel →
    Style["Gradient-learning model showing AI impact on job applications",
      18, Bold, Black],
  PlotTheme → "Scientific",
  ImageSize → Large,
  LabelStyle → Directive[Black, Bold],
  PlotLegends → {"Scenario 1", "Scenario 2", "Scenario 3", "Scenario 4"}]$$

```

 **Solve**: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. 

**Figure 4: TFP over time for a developed and developing country**

```

U[x_] :=  $\alpha (A x + B x^2)$ ;
 $\partial_t x[t] = L \partial_{x[t]} U[x[t]]$ ;
Plot[Floor[t], {t, 2023, 2100}, Frame → True]
F[ $\lambda$ _] := RandomVariate[PoissonDistribution[ $\lambda$ ]];
InnovationList0 = Table[F[1], {j, 1, 1000}];
AISchum[t_, InnovationList_] :=  $AI0 \gamma^{\sum_{k=1}^{Floor[t]} InnovationList[[k]]}$ ;
SolSchum1[L $\alpha$ _, AI1_,  $\gamma$ 1_, x0_, T_, InnovationList_] := x[t] /.
  NDSolve[{x'[t] == L $\alpha$  ((AISchum[t, InnovationList] /. AI0 → AI1 /.  $\gamma$  →  $\gamma$ 1) - x[t]),
    x[0] == x0}, x[t], {t, 0, T}][[1]];
Solx1 = SolSchum1[0.8, 100, 0.9, 1, 100, InnovationList0];
Solx2 = SolSchum1[0.2, 100, 0.9, 1, 100, InnovationList0];
Plot[{Solx1, Solx2}, {t, 0, 100},
  Frame → True,
  FrameLabel → {"Time (years from now)", "Total Factor Productivity"},
  PlotRange → Full,
  PlotLabel → Style["Total Factor Productivity over time", 20, Bold, Black],
  PlotTheme → "Scientific",
  PlotLegends → {"Developed country", "Developing country"},
  LabelStyle → Directive[Black, Bold],
  ImageSize → Large]

```

**Figure 5: Development gap in technological progress over time for a developed and developing country**

```

Plot[{Solx1 - (AISchum[t, InnovationList0] /. AI0 → 100 /.  $\gamma$  → 0.9),
  Solx2 - (AISchum[t, InnovationList0] /. AI0 → 100 /.  $\gamma$  → 0.9)}, {t, 0, 100},
  Frame → True,
  FrameLabel → {"Time (years from now)", "Development-gap"},
  PlotRange → Full,
  PlotLabel → Style["Development gap over time", 20, Bold, Black],
  PlotTheme → "Scientific",
  PlotLegends → {"Developed country", "Developing country"},
  LabelStyle → Directive[Black, Bold],
  ImageSize → Large]

```