Module 3 Assignment Solution

Individual Assignment Part A

We are to integrate two actions and two measurements with the Bayes filter algorithm. To shorten the notation, we will write *living* for $x = living\,room$ and *bed* for $x = bedroom$. Since we have only one action "switch-room", we will just denote that action with u. Further, we will denote the belief of state x after t steps of the algorithm as $Bel_t(x)$, where $Bel_0(x)$ is the prior probability of that state.

We are given the following probability values:

- $P(bed \mid u, living) = 0.7$
- $P(living \mid u, bed) = 0.8$
- $P(z = living \mid bed) = 0.3$
- $P(z = living \mid living) = 0.9$

From the rule of total probability, we deduce the values

- $P(living \mid u, living) = 0.3$
- $P(bed \mid u, bed) = 0.2$
- $P(z = bed \mid bed) = 0.7$
- $P(z = bed \mid living) = 0.1$

We are further given the sequence of actions and observations $d = [u_1, z_1 = bed, u_2, z_2 = bed]$, thus we have to run the algorithm four times. After each run, we will state the belief for the two possible states.

As we are not given any information about the prior location of the robot, we assume that the prior probability of the robot being in one of the rooms is equal, i.e., $Bel_0(living) = 0.5$ and $Bel_0(bed) = 0.5$.

Step 1

We first integrate the action item u_1.

$$
\begin{aligned}
Bel_1(living) &= \sum_{x'} P(living \mid u_1, x')Bel_0(x') \\
&= P(living \mid u_1, living)Bel_0(living) + P(living \mid u_1, bed)Bel_0(bed) \\
&= \frac{3}{10}\frac{5}{10} + \frac{8}{10}\frac{5}{10} \\
&= \frac{55}{100} = 0.55
\end{aligned}
$$

$$
\begin{aligned}
Bel_1(bed) &= \sum_{x'} P(bed \mid u_1, x')Bel_0(x') \\
&= P(bed \mid u_1, living)Bel_0(living) + P(bed \mid u_1, bed)Bel_0(bed) \\
&= \frac{7}{10}\frac{5}{10} + \frac{2}{10}\frac{5}{10} \\
&= \frac{45}{100} = 0.45
\end{aligned}
$$

Step 2

Next, we integrate the observation $z_1 = bed$. To that end, we first compute the non-normalized beliefs Bel' and then apply normalization to obtain the final beliefs.

$$
\begin{aligned}
Bel'_2(living) &= P(z_1 = bed \mid living)Bel_1(living) \\
&= \frac{1}{10}\frac{55}{100} = \frac{55}{1000}
\end{aligned}
$$

$$
\begin{aligned}
Bel'_2(bed) &= P(z_1 = bed \mid bed)Bel_1(bed) \\
&= \frac{7}{10}\frac{45}{100} = \frac{315}{1000}
\end{aligned}
$$

The normalizer in this step is

$$
\eta_2 = Bel'_2(living) + Bel'_2(bed) = \frac{55}{1000} + \frac{315}{1000} = \frac{370}{1000}
$$

With that, the final beliefs are

$$
\begin{aligned}
Bel_2(living) &= \frac{Bel_2'(living)}{\eta_2} = \frac{55}{370} = \frac{11}{74} \approx 0.149 \\
Bel_2(bed) &= \frac{Bel_2'(bed)}{\eta_2} = \frac{315}{370} = \frac{63}{74} \approx 0.851
\end{aligned}
$$

Step 3

We integrate the second action item u_2. Since it is the same action as in Step 1, we just replace the belief values Bel_0 with Bel_2.

$$
\begin{aligned}
Bel_3(living) &= \sum_{x'} P(living \mid u_1, x') Bel_2(x') \\
&= P(living \mid u_1, living) Bel_2(living) + P(living \mid u_2, bed) Bel_2(bed) \\
&= \frac{3}{10}\frac{11}{74} + \frac{8}{10}\frac{63}{74} = \frac{33 + 504}{740} = \frac{537}{740} \approx 0.726
\end{aligned}
$$

$$
\begin{aligned}
Bel_3(bed) &= \sum_{x'} P(bed \mid u_1, x') Bel_2(x') \\
&= P(bed \mid u_1, living) Bel_2(living) + P(bed \mid u_1, bed) Bel_2(bed) \\
&= \frac{7}{10}\frac{11}{74} + \frac{2}{10}\frac{63}{74} = \frac{77 + 126}{740} = \frac{203}{740} \approx 0.274
\end{aligned}
$$

Step 4

Finally, we integrate the second observation $z_2 = bed$ analogously to Step 2.

$$
\begin{aligned}
Bel_4'(living) &= P(z_1 = bed \mid living) Bel_3(living) \\
&= \frac{1}{10}\frac{537}{740} = \frac{537}{7400} \approx 0.073
\end{aligned}
$$

$$
\begin{aligned}
Bel_4'(bed) &= P(z_1 = bed \mid bed) Bel_3(bed) \\
&= \frac{7}{10}\frac{203}{740} = \frac{1421}{7400} \approx 0.192
\end{aligned}
$$

$$
\eta_4 = Bel_4'(living) + Bel_4'(bed) = \frac{537}{7400} + \frac{1421}{7400} = \frac{1958}{7400} \approx 0.265
$$

$$Bel_4(living) = \frac{Bel'_4(living)}{\eta_4} = \frac{537}{1958} \approx 0.274$$
$$Bel_4(bed) = \frac{Bel'_4(bed)}{\eta_4} = \frac{1421}{1958} \approx 0.726$$

The final beliefs are thus $Bel_4(living) = \frac{537}{1958} \approx 0.274$ and $Bel_4(bed) = \frac{1421}{1958} \approx 0.726$

Individual Assignment Part B

The goal is to find the prior distribution Bel_0 in such a way that $Bel_4(bed)$ is minimized, i.e., we want the robot to end up in the living room after the two switch actions. Since $Bel_0(living) = 1 - Bel_0(bed)$, we could express $Bel_4(bed)$ as a function of $Bel_0(bed)$ and analyze the derivative to find the requested value of $Bel_0(bed)$ analytically, with the result $Bel_0^{min}(bed) = 0$ and $Bel_0^{min}(living) = 1$. In other words, we deterministically set the initial location of the robot to be the living room.

Here, however, it is sufficient to provide some intuition about why this must be the case. First, we notice that after executing the action, the probability of the robot actually switching the room is higher than staying in the same room. That means that if we want the robot to be in the living room after the second action, we need to maximize the probability $Bel_2(bed)$ of the robot being in the bedroom after the first action. In this example, $x = bed$ after the first action is also in agreement with $z_1 = bed$. To maximize $Bel_2(bed)$, we use the same logic and conclude that $Bel_0(living)$ should be maximized (or $Bel_0(bed)$ minimized).

Group Assignment

The major assumption of the Bayes filter is the Markov property. Informally, it means that the knowledge ot the current state is sufficient to describe the system and makes all prior knowledge obsolete. In particular, we assume that given the current state, the current measurement is independent of the previous ones.

Assume that we have perfect knowledge of the environment geometry and a robot equipped with a sonar range sensor. Since the speed of sound is relatively low, some sound waves send out at time step t might arrive at the sensor at a later time, say, $t + 2$. This might happen because the sound is reflected several times by the environment. In this case, the measurement z_{t+2} might be not completely independent of z_t. (However, here one could argue that extending the state by modeling the sound waves would solve the problem).

Generally, the Markov property almost never holds in physical systems, since in practice it is very difficult to model the world perfectly. There are two reasons: First, modeling all physical properties even of a small scene (in reasonable time) would by far exceed the computational power of modern computers. Second, even if the modeling was possible, it would be very difficult to acquire the state of the system at the necessary level of precision due to sensor hardware limitations.

Using an imperfect model breaks the Markov assumption whenever a phenomenon occurs that is not captured by the model. One major challenge here are dynamic environments. For example, people near the robot can certainly influence sensor measurements, but are inherently difficult to model.