SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Homework 9

1. Suppose a player plays the following gambling games which is known as the wheel of fortune. The player bets on one of the numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appears i times, $i = 1, 2, 3$, then the player wins i units; on the other hand, if the number bet by the player does not appear on any of the dies, then the player loses 1 unit. Is this game fair to the player?

2. Suppose the random variable X takes non-negative integer values only. Show that

$$\mathrm{E(X)} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{P(X > n)} = \sum_{\mathrm{n}=1}^{\infty} \mathrm{P(X \geq n)}.$$

3. (a) Suppose the random variable X obeys the uniformly distribution over interval $[a, b]$. Find $E(X)$.
   (b) Suppose the random variable X obeys the general Γ distribution with parameters λ and α where $\lambda > 0$ and $\alpha > 0$. Write down the p.d.f of this general Γ random variable and the analytic form of the Γ function $\Gamma(\alpha)$ for $\alpha > 0$ and hence find the E(X) of this general Γ random variable.
   (c) Suppose $Y = X^2$ where X is normally distributed with parameters μ and σ^2. Obtain the p.d.f of Y and then find E(Y).

4. (a) Suppose that the two discrete random variables X and Y have joint p.m.f given by

X \ Y	$Y = 1$	$Y = 2$	$Y = 3$	$Y = 4$
$X = 1$	2/32	3/32	4/32	5/32
$X = 2$	3/32	4/32	5/32	6/32

   Obtain E(X) and E(Y).
   (b) Suppose that the two continuous random variables X and Y have joint p.d.f

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Find E(X) and E(Y).

5、叙述马尔科夫不等式，并证明连续情形。

6、7 如下:

1. 设随机变量X的概率密度为

$$f(x)=\begin{cases} e^{-x}, & x>0, \\ 0, & x \leqslant 0. \end{cases}$$

求 (1) $Y=2X$; (2) $Y=e^{-2X}$ 的数学期望.

2. 设随机变量(X,Y)的概率密度为

$$f(x,y)=\begin{cases} 12y^2, & 0 \leqslant y \leqslant x \leqslant 1, \\ 0, & \text{其他}. \end{cases}$$

求 $E(X)$, $E(Y)$, $E(XY)$, $E(X^2+Y^2)$.