Statistical Linear Model

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Assignment 6

Problem 1.

(a). From problem, $\Rightarrow$ for months after murder in city A, $\Rightarrow X_1 = X_2 = 1$.

Hence, we have: $E(y) = \beta_0 + \beta_1 + \beta_2 + \beta_3$

(b) for months before the murder in city B, $\Rightarrow X_1 = X_2 = 0$.

Hence, we have: $E(y) = \beta_0$.

(c). From problem, we have

Full model: i.e. model G, Reduced model: $E(y) = \beta_0 + \beta_1 X_1$

Null Hypothesis: $H_0: \beta_2 = \beta_3 = 0$

Alternative Hypothesis: there at least exist one $\beta_i \neq 0$ for $i = 2, 3$

The test statistics: $F = \frac{SSR(x_2, x_1x_2 | x_1)}{2} \Big/ MSE(x_1, x_2, x_1x_2)$

Rejection rule: We will reject the H_0 if $F > F_{(0.05, 2, n-4)}$ (n is # sample)
(F is test statistic)

Problem 2.

(a). This model has an unreasonable assumption: ↓

The salary difference between pilot and mechanic and difference between mechanic and filght attendance are same.

(b) Consider using the dummy variables.

Suppose $x_1 = \begin{cases} 1 & \text{if pilot} \\ 0 & \text{otherwise} \end{cases}$ $x_2 = \begin{cases} 1 & \text{if mechanic} \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow$ then we have the new model: $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Problem 3.

(a). From problem, we have $\beta_0 = \mu_1$, $\beta_1 = \mu_2 - \mu_1$, $\beta_3 = \mu_3 - \mu_1$

(b) i.e. estimated $\mu_3 - \mu_1 = \hat{\beta}_2 = 198.20$

(c) i.e. estimated $\mu_3 - \mu_2 = (\mu_3 - \mu_1) - (\mu_2 - \mu_1) = \hat{\beta}_2 - \hat{\beta}_1 = 198.20 - 80.3 = 117.9$

(d). Null hypothesis H_0: $\beta_1 = \beta_2 = 0$

Alternative hypothesis H_1: at least one of $\beta_i \neq 0$. $i = 1.2$

The test statistic: $F = \frac{MSR}{MSE} = \frac{99386}{28543} \approx 3.48$

The rejection decision: We will reject the null hypothesis if $F > F_{(0.05, 2, 27)}$ (F is test statistic)

Note that $F_{0.05, 2, 27} = 3.35 < F = 3.48$.

$\Rightarrow$ Conclusion: We reject the null hypothesis at $\alpha = 0.05$ $\Rightarrow$ there at least exists two of the population mean delinquent amounts of three group are significantly different.