THE SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Homework 1

1. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.
   (a) List the sample space Ω.
   (b) List the elements that make up the following events:
      (1) $A =$ the sum of the two values is at least 5;
      (2) $B =$ the value for the first die is higher than the value of the second;
      (3) $C =$ the first value is 4.
   (c) List the elements of the following events:
      (1) $A \cap C$;
      (2) $B \cup C$;
      (3) $A \cap (B \cup C)$.

2. Let A and B be two arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection, and complement.

3. Suppose A and B are two events such that $A \subset B$. Show that

$$P(B \setminus A) = P(B) - P(A).$$

4. Verify the following extension of the addition rule (a) by an appropriate Venn diagram and (b) by a formal argument using the axioms of probability and the propositions proved in the Lecture Notes.

$$\begin{aligned} P(A \cup B \cup C) = &P(A) + P(B) + P(C) \\ &- P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &+ P(A \cap B \cap C) \end{aligned}$$

5. Suppose that $\{A_n; n \geq 1\}$ is a sequence of events which may not be disjoint. Show that the following sub-additive property is true:

$$P(\bigcup_{n=1}^{\infty} A_n) \leq \sum_{n=1}^{\infty} P(A_n).$$

Also, for any $k \geq 2$, we have

$$P(\bigcup_{n=1}^{k} A_n) \leq \sum_{n=1}^{k} P(A_n).$$

In particular, for any two events A and B, we have $P(A \cup B) \leq P(A) + P(B)$.

6. Suppose $\{A_i; 1 \leq i \leq n\}$ are events.

   (i) Show that the following inclusion-exclusion formula is true.

$$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i<j\leq n} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k\leq n} P(A_i \cap A_j \cap A_k)$$
$$- \ldots + (-1)^{n-1} P(A_1 \cap A_2 \ldots A_n).$$

   (ii) Write down this formula for cases of $n = 2, n = 3, n = 4$, and $n = 5$ clearly.

7. (i) If $\{A_n; n \geq 1\}$ is an increasing sequence of events, i.e. for all $n \geq 1, A_n \subset A_{n+1}$, show that $\lim_{n\to\infty} P(A_n) = P(\bigcup_{n=1}^{\infty} A_n)$.

   (ii) If $\{A_n; n \geq 1\}$ is a decreasing sequence of events, i.e. for all $n \geq 1, A_n \supset A_{n+1}$, show that $\lim_{n\to\infty} P(A_n) = P(\bigcap_{n=1}^{\infty} A_n)$.

一．事件及其概率

1. 设 A, B, C 为三个事件，试写出下列事件的表达式：

   (1) A, B, C 都不发生；

   (2) A, B, C 不都发生；

   (3) A, B, C 至少有一个发生；

   (4) A, B, C 至多有一个发生。

2. 设 A, B 为两相互独立的随机事件, $P(A)=0.4$, $P(B)=0.6$, 求 $P(A\cup B), P(A-B)$ 。

3. 设 A, B 互斥， $P(A)=0.5$， $P(A\cup B)=0.9$，求 $P(B), P(A-B)$ 。

4. 设 $P(A)=0.5, P(B)=0.6, P(A|B)=0.5$，求 $P(A\cup B), P(A\bar{B})$ 。

5. 袋中有 4 个黄球， 6 个白球，在袋中任取两球，求
   (1) 取到两个黄球的概率；
   (2) 取到一个黄球、一个白球的概率。

6. 从 0 ~ 9 十个数字中任意选出三个不同的数字，求三个数字中最大数为 5 的概率。

7. 甲袋中装有 5 只红球，15 只白球，乙袋中装有 4 只红球，5 只白球，现从甲袋中任取一球放入乙袋中，再从乙袋中任取一球，问从乙袋中取出红球的概率为多少？