SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Homework 8

1. Suppose that the continuous random variables X and Y have joint p.d.f

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1, 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the joint cumulative distribution function (joint c.d.f) of the random vector $(X,\ Y)$.
   (b) Find the marginal p.d.fs of X and Y.
   (c) Find P(X > Y),
   (d) Find P(X ≤ 0.5).

2. Find the joint probability density function of the two random variables X and Y whose joint (cumulative) distribution function (joint c.d.f)is given by

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & x > 0, y > 0; \\ 0 & \text{otherwise.} \end{cases}$$

   Also use the joint probability density to determine P(1 < X < 3, 1 < Y < 2).

3. Consider a circle of radius R and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point. (In other words, the point is uniformly distributed within the circle.) If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen, it follows, since (X,Y) is equally likely to be near each point in the circle, that the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} c & \text{if } x^2+y^2 \le R^2, \\ 0 & \text{if } x^2+y^2 > R^2. \end{cases}$$

   for some value of c.

   (a) Determine the constant c.
   (b) Find the marginal density functions of X and Y.
   (c) Compute the probability that the distance from the origin of the point selected is not greater that a. $(0 \le a \le R.)$
   (d) Are X and Y independent? Specify your reasons clearly.

4. A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 p.m., find the probability that the first to arrive has to wait longer than 10 minutes.