

Example-Ch10: One-way ANOVA

The example dataset comes from a study of blood coagulation times: 24 animals were randomly assigned to four different diets and the samples were taken in a random order.

```
>library(faraway)
```

```
>data(coagulation)
```

```
>coagulation
```

```
coag diet
```

```
1 62 A
```

```
2 60 A
```

```
3 63 A
```

```
4 59 A
```

```
5 63 B
```

```
6 67 B
```

```
.....
```

```
21 63 D
```

```
22 64 D
```

```
23 63 D
```

```
24 59 D
```

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

$$i = 1, 2, 3, \dots, K$$

Subjective impression (guess)

exploratory data analysis

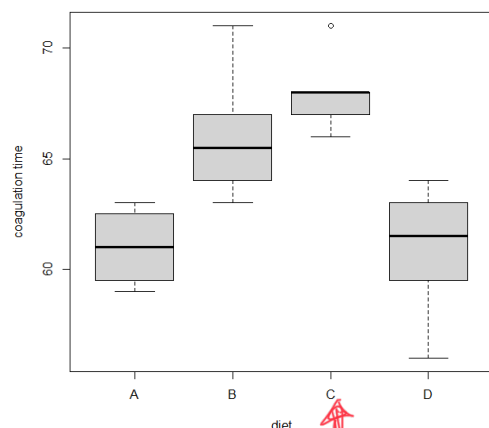
summary statistics
— min 2. L2 L3 max
mean - s.d.

— box-plot, scatter plot

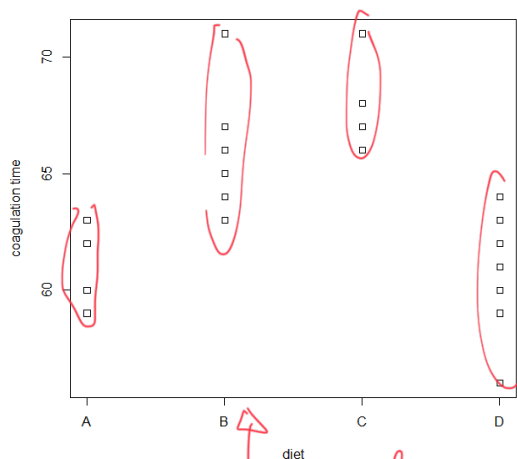
The first step is to plot the data by comparing boxplots and strip plots:

```
>plot(coag~diet,coagulation,ylab="coagulation time")
```

```
>with(coagulation,stripchart(coag~diet,vertical=TRUE, methods="stack",  
xlab="diet", ylab="coagulation time" ))
```



n_{ij} — large



n_{ij} — small

Fit the model using a linear regression model (the default set)

```
>g<-lm(coag~diet,coagulation)
```

```
>summary(g)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	6.100e+01	1.183e+00	51.554	< 2e-16 ***
dietB	5.000e+00	1.528e+00	3.273	0.003803 **
dietC	7.000e+00	1.528e+00	4.583	0.000181 ***
dietD	2.991e-15	1.449e+00	0.000	1.000000

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.366 on 20 degrees of freedom

Multiple R-squared: 0.6706, Adjusted R-squared: 0.6212

F-statistic: 13.57 on 3 and 20 DF, p-value: 4.658e-05

We can fit the model without an intercept term as

```
> gi<-lm(coag~diet -1,coagulation)
```

```
> summary(gi)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
dietA	61.0000	1.1832	51.55	<2e-16 ***
dietB	66.0000	0.9661	68.32	<2e-16 ***
dietC	68.0000	0.9661	70.39	<2e-16 ***
dietD	61.0000	0.8367	72.91	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.366 on 20 degrees of freedom

Multiple R-squared: 0.9989, Adjusted R-squared: 0.9986

F-statistic: 4399 on 4 and 20 DF, p-value: < 2.2e-16

Note that the R-squared and F-test are not correctly calculated. To generate usual test, we may use the following way:

$$y_{ij} = \mu + \alpha_i + \delta_{ij}$$

$$\alpha_i = 0$$

treatment effect

$$A \quad 61$$

$$B \quad 61 + 5 = 66$$

$$C \quad 61 + 7 = 68$$

$$D \quad 61 + 0 = 61$$

$$\mu = 0 \quad y_{ij} = \alpha_i + \delta_{ij}$$

treatment effects

$$R^2 = \frac{SSR}{SST}$$

$$y_i = (\beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \epsilon_i)$$

meaningless

```
> gnull<-lm(coag~1,coagulation)
```

```
> anova(gnull, gi)
```

Analysis of Variance Table

Model 1: coag ~ 1

Model 2: coag ~ diet - 1

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	23	340				
2	20	112	3	228	13.571	4.658e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

We can also use a sum coding:

```
> options(contrasts=c("contr.sum","contr.poly"))
```

```
> gs<-lm(coag~diet,coagulation)
```

```
> summary(gs)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	64.0000	0.4979	128.537	< 2e-16 ***
diet1	-3.0000	0.9736	-3.081	0.005889 **
diet2	2.0000	0.8453	2.366	0.028195 *
diet3	4.0000	0.8453	4.732	0.000128 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.366 on 20 degrees of freedom

Multiple R-squared: 0.6706, Adjusted R-squared: 0.6212

F-statistic: 13.57 on 3 and 20 DF, p-value: 4.658e-05

$$y_{ij} = \mu + \epsilon_{ij}$$

$$y_{ij} = \alpha_i + \epsilon_{ij}$$

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

$$\sum \alpha_i = 0$$

treatment effects

$$A \quad 64 - 3 = 61$$

$$B \quad 64 + 2 = 66$$

$$C \quad 64 + 4 = 68$$

$$D \quad 64 + \alpha_4 = 64 - 3 = 61$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0$$

$$\Rightarrow \alpha_4 = -3$$