THE SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Tutorial 01

Set: Thursday 14th September 2017.

Note: You do not need to hand in your solutions for this tutorial. However, please try some questions, particularly Questions 1, 5, and 6.

1. Provide a strict proof for the following set relations.

   (i) $B \setminus A = B \cap A^c$;

   (ii) $(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$;

   (iii) $(\bigcup_{k=1}^{\infty} A_k)^c = \bigcap_{k=1}^{\infty} A_k^c$;

   (iv) $(\bigcap_{k=1}^{\infty} A_k)^c = \bigcup_{k=1}^{\infty} A_k^c$;

   (v) $A \cup (\bigcap_{k=1}^{\infty} B_k) = \bigcap_{k=1}^{\infty} (A \cup B_k)$;

   (vi) $A \cap (\bigcup_{k=1}^{\infty} B_k) = \bigcup_{k=1}^{\infty} (A \cap B_k)$.

   As the generalizations of (iii) to (vi) we have the following general De Morgan's Laws and Distributive laws: For any index set I, we have

   (vii) $(\bigcup_{i \in I} A_i)^c = \bigcap_{i \in I} (A_i)^c$;

   (viii) $(\bigcap_{i \in I} A_i)^c = \bigcup_{i \in I} (A_i)^c$;

   (ix) $A \cup (\bigcap_{i \in I} B_i) = \bigcap_{i \in I} (A \cup B_i)$;

   (x) $A \cap (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cap B_i)$.

2. A sequence of sets $\{A_1, A_2, \ldots, A_n, \ldots\}$ is called increasing if

   $$A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_n \subset A_{n+1} \subset \cdots .$$

   Similarly, a sequence of sets $\{A_1, A_2, \ldots, A_n, \ldots\}$ is called decreasing if

   $$A_1 \supset A_2 \supset A_3 \supset \cdots \supset A_n \supset A_{n+1} \supset \cdots .$$

   Show that

   (i) If $\{A_n; n \geq 1\}$ is an increasing set sequence, then for any $n \geq 1$, $\bigcup_{k=1}^{n} A_k = A_n$ and $\lim_{n\to\infty} A_n = \bigcup_{k=1}^{\infty} A_k = \bigcup_{n=1}^{\infty} A_n$.

(ii) If $\{A_n; n \geq 1\}$ is a decreasing set sequence, then for any $n \geq 1$, $\bigcap_{k=1}^{n} A_k = A_n$ and $\lim_{n\to\infty} A_n = \bigcap_{k=1}^{\infty} A_k = \bigcap_{n=1}^{\infty} A_n$.

3. Show that if $A_1, A_2, \ldots, A_n$ are all countable sets, then so is the n-tuple Cartesian product
$$A_1 \times A_2 \times \cdots \times A_n.$$
In particular, if A is a countable set, then so is A^n.

4. Suppose that the three sets A,B and C have the relationship $A \subset B \subset C$ and that $Card(A) = Card(C)$, then
$$Card(A) = Card(B) = Card(C),$$
where $Card(A)$ denotes the cardinal number of the set A etc.

5. Show that the set $[0, 1]$ is not countable.

6. Show that the Cardinal number of the real number $\mathbb{R}$ is equal to the cardinal number of the open unit internal $(0, 1)$.

7. Suppose $\{A_n; n = 1, 2, \ldots\}$ is an increasing sequence of sets.

   Define $B_1 = A_1$, $B_2 = A_2 \setminus A_1$, and in general, $B_n = A_n \setminus A_{n-1} (n \geq 2)$. Show that

   (i) $\{B_n; n \geq 1\}$ are disjoint.

   (ii) For any $k \geq 1$, $\bigcup_{n=1}^{k} B_n = A_k$;

   (iii) $\bigcup_{n=1}^{\infty} B_n = \bigcup_{n=1}^{\infty} A_n$.

8. Let S be the set of all the sequences with elements 0 and 1 only.

   Is S countable or not? Prove your conclusion.