MA329 Statistical linear models 20-21

Assignment 3 (Due date: Oct 29, 11pm. For late submission, each day costs 10 percent. The solution will be released at 6pm Oct 30 since midterm test is on Nov 1. This assignment will not be accepted once the solution is released.)

1. (10 marks) Consider a nonsingular $n \times n$ matrix $\boldsymbol{A}$ whose elements are functions of the scalar x. Also consider the full-rank $p \times n$ matrix $\boldsymbol{B}$. Let $\boldsymbol{H} = \boldsymbol{B}'(\boldsymbol{B}\boldsymbol{A}\boldsymbol{B}')^{-1}\boldsymbol{B}$. Show that
$$\frac{\partial \boldsymbol{H}}{\partial x} = -\boldsymbol{H}\frac{\partial \boldsymbol{A}}{\partial x}\boldsymbol{H}$$

2. (20 marks) Let $\boldsymbol{X} \sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{X}' = (X_1, X_2, X_3)$, $\boldsymbol{\mu}' = (3, -2, 0)$ and
$$\boldsymbol{\Sigma} = \begin{pmatrix} 5 & 0 & -3 \\ 0 & 9 & 0 \\ -3 & 0 & 2 \end{pmatrix}$$
   (a) Are X_2 and $2X_1 - X_3$ independent? Explain.
   (b) Find the distribution of $\begin{pmatrix} 2X_1 - 5X_3 \\ X_1 + X_2 \end{pmatrix}$.
   (c) Find the conditional distribution of X_3, given that $X_1 = 1$ and $X_2 = -2$.

3. (10 marks) Let $\mathbf{Y} = (Y_1,\ \ Y_2,\ \ Y_3)'$. $E(\mathbf{Y}) = (2,\ \ 3,\ \ 4)'$ and the covariance matrix of $\mathbf{Y}$ is
$$\boldsymbol{\Sigma} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{pmatrix}.$$
Let $U = \sum_{i=1}^{3}(Y_i - \overline{Y})^2$. Find the expected value of U.

4. (10 marks) Let $\mathbf{Y} = (Y_1, ..., Y_n)'$. $E(\mathbf{Y}) = \mu\mathbf{1}$ and the covariance matrix of $\mathbf{Y}$ is $\sigma^2\mathbf{I}$. Let
$$U = \sum_{i<j}(Y_i - Y_j)^2.$$
Find the expected value of U, and find a constant k such that kU is an unbiased estimator of σ^2.