SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Homework 10

1. Suppose $Y = e^X$ where X is normally distributed with parameters μ and σ^2. Use the following two methods to obtain E(Y).

   (i) First obtain the p.d.f of Y, denoted by $f_Y(y)$ and then find E(Y) by using $f_Y(y)$.

   (ii) Find E(Y) directly by viewing Y as a function of X and then using the formula of getting the expected value of a function of the random variable X.

2. (a) Suppose the random variable X obeys the uniformly distribution over interval $[a, b]$. Find $\mathrm{E(X^2)}$ and then obtain the value of $\mathrm{E(X^2) - (E(X))^2}$.

   (b) Suppose X is normally distributed random variable with parameters μ and σ^2. Find $\mathrm{E(X^2)}$ and then obtain the value of $\mathrm{E(X^2) - (E(X))^2}$.

3. (a) If the probability density function of an (absolutely) continuous random variable X is given by

$$f_X(x) = \begin{cases} \dfrac{1}{x(\ln 3)} & 1 < x < 3, \\ 0 & otherwise. \end{cases}$$

   Find E(X), $\mathrm{E(X^2)}$ and $\mathrm{E(X^3)}$.

   (b) Use the results of part (a) to determine $\mathrm{E(X^3 + 2X^2 - 3X + 1)}$.

4. If the probability density function of an (absolutely) continuous random variable X is given by

$$f_X(x) = \begin{cases} \dfrac{x}{2} & 0 < x \le 1, \\ \dfrac{1}{2} & 1 < x \le 2, \\ \dfrac{3-x}{2} & 2 < x \le 3, \\ 0 & otherwise. \end{cases}$$

   Find the expectation of $g(X) = X^2 - 5X + 3$.

5. The two continuous random variables X and Y have joint p.d.f

$$f(x, y) = \begin{cases} x + y & 0 \le x \le 1, 0 \le y \le 1, \\ 0 & otherwise. \end{cases}$$

   Find $\mathrm{E[(X + Y)^2]}$.