SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Homework 2

1. Show that if the conditional probabilities exist, then

$$\mathrm{P}(\mathrm{A}_1 \cap \mathrm{A}_2 \cap \cdots \cap \mathrm{A}_\mathrm{n}) = \mathrm{P}(\mathrm{A}_1)\mathrm{P}(\mathrm{A}_2|\mathrm{A}_1)\mathrm{P}(\mathrm{A}_3|\mathrm{A}_1 \cap \mathrm{A}_2)\cdots\mathrm{P}(\mathrm{A}_\mathrm{n}|\mathrm{A}_1 \cap \mathrm{A}_2 \cap \cdots \cap \mathrm{A}_{\mathrm{n}-1}).$$

2. Urn A has three red balls and two white balls, and urn B has two red balls and five white balls. A fair coin is tossed; if it hands heads up, a ball is drawn from urn A and otherwise a ball is drawn from urn B.

   (a) What is the probability that a red ball is drawn ?

   (b) If a red ball is drawn, what is the probability that the coin landed heads up?

3. Urn A has four red, three blue and two green balls. Urns B has two red, three blue and four green balls. A ball is drawn from urn A and put into urn B and then a ball is drawn from urn B.

   (a) What is the probability that a red ball is drawn from urn B?

   (b) If a red ball is drawn from urn B, what is the probability that a red ball was drawn from urn A?

4. There are three cabinets A, B, C, each of which has two drawers. Each drawer contains one coin; A has two gold coins, B has two silver coins and C has one gold and one silver coin. Take a experiment as a cabinet is chosen at random, one drawer is opened

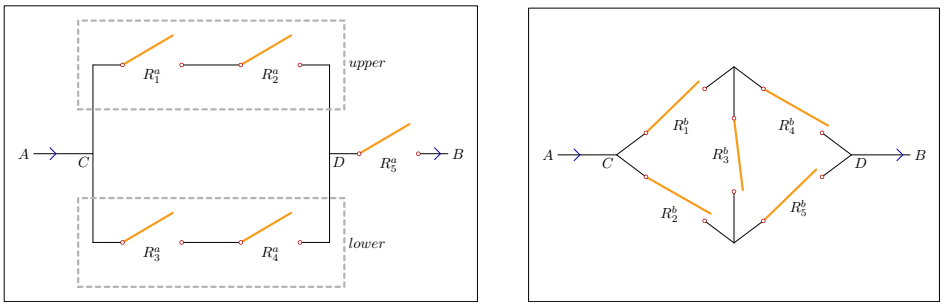
and a silver coin has found. What is the probability that the other drawer in that cabinet contains a silver coin?

5. If B is an event with $\mathrm{P(B)} > 0$, show that the set function $\mathrm{Q(A)} = \mathrm{P(A|B)}$ is a probability measure. Thus, we can use the following formulas:

$$\mathrm{P(A \cup C|B)} = \mathrm{P(A|B)} + \mathrm{P(C|B)} - \mathrm{P(A \cap C|B)}, \quad \mathrm{P(A^c|B)} = 1 - \mathrm{P(A|B)}.$$

6. Show that if A, B, C are mutually independent, then $A \cap B$ and C are independent and $A \cup B$ and C are independent.

7. The probability of the closing of the ith relay in the circuits shown is given by $p_i, i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits?



(a) circuit a

(b) circuit b

Figure 1: circuits