THE SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

1. Suppose a probability density function $f(x)$ takes the form of

$$f(x) \quad = \quad cx^3 \qquad (0 < x < 1)$$

   (a) find the value of the constant c;
   (b) sketch $f(x)$;
   (c) obtain the cumulative distribution function $F(x)$;
   (d) find $\Pr(0.25 < X < 0.75)$.

2. A continuous random variable X is said to have a memoryless property if $Pr(X > s + t | X > t) = Pr(X > s)$ is true for all $s > 0$ and $t > 0$. Show that any exponential random variable has the memoryless property.

3. For a certain type of electrical component, the lifetime X (in thousands of hours) has an Exponential distribution with rate parameter $\lambda = 0.5$. What is the probability that a new component will last longer than 1000 hours? If a component has already lasted 1000 hours, what is the probability that it will last at least 1000 hours more?

4. The number of phone calls received at a certain residence in any period of t hours is a Poisson random variable with parameter $\lambda = \mu t$ for some $\mu > 0$. What is the probability that no calls are received during a period of t hours? Denoting by T the time (in hours) at which the first call after time zero is received, write down an expression for $\Pr(T \leq t)$. What is the name of the distribution of the random variable T?

5. The Weibull distribution with parameters $\alpha > 0$ and $\beta > 0$ has cumulative distribution function

$$F(x) \quad = \quad 1 - \exp\left\{ - \left(\frac{x}{\alpha}\right)^{\beta} \right\} \qquad x > 0$$

   Find the median of the distribution in terms of the parameters α, β (The median of a random variable X is the value m such that $Pr(X \leq m) = 0.5$) .

   From the Weibull distribution function given above, derive an expression for the corresponding probability density function.