SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

<u>**MA215** Probability Theory</u>

Homework 3

1. For each of the following statements, say whether true or false. For false statements, give the correct version of the statement.

   (i) $P(A \cap B) = P(A) \times P(B)$ if A, B are independent.

   (ii) $P(A \cup B) = P(A) + P(B)$ if A, B are independent.

   (iii) In a sequence of n independent identical trials, each of which results in either "success" or "failure", with probability θ of success, the number of successes follows a Bernoulli distribution.

2. In five independent tosses of an unbiased coin, find

   (i) the probability that the total number of heads is even;

   (ii) the probability that there are exactly five heads.

   (Note: zero is also a even number.)

3. A discrete random variable X has possible values $-2, 1, 3, 4$ with probabilities satisfying

$$P(X = -2) = P(X = 1) = 2P(X = 3) = 3P(X = 4).$$

   Find the probability mass function and the (cumulative) distribution function of X, and graph them both.

4. The following table shows the probability mass function of a discrete random variable X. Plot the (cumulative)distribution function of this random variable.

Table 1: p.m.f of a discrete random variable X

k	1	2	3	4	5
P(X=k)	0.1	0.2	0.4	0.1	0.2

5. Suppose $F(x)$ is the cdf of a random variable X. Show that $F(x)$ has the following properties:

(i) $0 \le F(x) \le 1$;

(ii) $F(x)$ is an increasing function of x, i.e. for any $x < y$, then $F(x) \le F(y)$;

(iii) $\lim\limits_{x \to +\infty} F(x) = 1$; $\lim\limits_{x \to -\infty} F(x) = 0$;

(iv) Show that $F(x)$ is a right-continuous function of $x \in \mathbb{R}$.(Just show that if a sequence of real numbers $x_n \downarrow x$, then $\lim\limits_{n \to \infty} F(x_n) = F(\lim\limits_{x \to \infty} x_n) = F(x)$)