

# Top-down and Bottom-up Strategies for Generating Incremental Covering Arrays

Xintao Niu\*, Changhai Nie\*, Hareton Leung<sup>†</sup>, Jeff Y. Lei<sup>‡</sup>, Xiaoyin Wang<sup>§</sup>, Yan Wang<sup>¶</sup>, Mengfan Zeng\*, and Jiayi Xu<sup>¶</sup>

\*State Key Laboratory for Novel Software Technology

Nanjing University, China, 210023

Email: niuxintao@gmail.com, changhainie@nju.edu.cn, zengmengfan@foxmail.com

<sup>†</sup>Department of Computing Hong Kong Polytechnic University

Kowloon, Hong Kong

Email: cshleung@comp.polyu.edu.hk

<sup>‡</sup> Department of Computer Science and Engineering

The University of Texas at Arlington

Email: ylel@cse.uta.edu

<sup>§</sup> Department of Computer Science

University of Texas at San Antonio

Email: Xiaoyin.Wang@utsa.edu

<sup>¶</sup> School of Information Engineering

Nanjing Xiaozhuang University

Email: wangyan@njxzc.edu.cn, xujiayi@njxzc.edu.cn

**Abstract**—Combinatorial Testing (CT) is an effective technique for testing the interactions of factors in the Software Under Test (SUT). By designing an efficient set of test cases, i.e., a covering array, CT aims to check every possible valid interaction in SUT. Most existing covering array generation algorithms require a given *degree*  $t$  in prior, such that only the interactions with no more than  $t$  factors are to be checked. In practice, however, the value of  $t$  cannot be properly determined, especially for systems with complex interaction space. Hence, incremental covering arrays are preferred. In this paper, we propose two strategies (*Bottom-up* and *Top-down*) for generating incremental covering arrays which can increase the coverage strength when required. A comparative evaluation of these two strategies is performed on 55 software subjects (including 25 real-world software and 30 synthetic systems). The results mainly shows that strategy *Bottom-up* have an advantage over strategy *Top-down* at lower degrees with respect to the size of covering arrays, while strategy *Top-down* performs better at higher degrees. However, when concerning the fault detection rate, strategy *Top-down* gets higher scores at lower degrees, while *Bottom-up* is better at higher degrees.

**Index Terms**—Combinatorial Testing, Covering Array, Incremental Covering Array, Bottom-up and Top-down strategy

## I. INTRODUCTION

With the growing complexity and scale of modern software systems, various factors, such as input values and configure options, can affect the behaviour of a system. Even worse, some interactions between them can trigger unexpected negative effects on the software. To ensure the correctness and quality of the software system, it is desirable to detect and locate these *bad* interactions. The simplest way to solve this problem is to perform exhaustive testing for all the possible

valid interactions of the system. It is, however, not practical due to the combinatorial explosion. Therefore, the selection of a group of representative test cases from the whole testing space is required.

Combinatorial testing has been proven to be effective in sampling an effective set of test cases[1]. It works by generating a relatively small set of test cases, i.e., a covering array, to test the interactions involving a given number of factors that may affect the behavior of a system. The number of factors involved in those selected interactions is limited in a moderate range, which is usually from 2 to 6 [2].

Many algorithms have been proposed to generate covering arrays. Despite many differences between them, those works all assume that the degree  $t$  is a given priori. The degree  $t$  indicates the largest number of factors involved in the interactions to be covered, and the corresponding covering arrays which can satisfy this coverage criteria is called  $t$ -way covering arrays. In practice, however, the value of  $t$  is difficult to determine due to two reasons as follows. First, many software systems suffer from complex interaction space which make it challenging to estimate  $t$ . In such case, even experienced testers can make mistakes and estimate a wrong value for  $t$ , which can significantly affect the effectiveness and efficiency of CT. Specifically, if  $t$  is estimated to be larger than required, many redundant test cases will be generated, which is a waste of computing resource. And if  $t$  is estimated to be smaller than required, then the generated covering array is not sufficient to obtain an effective test set. Second, even though  $t$  has been properly determined, there may not be enough time to completely execute all the test cases in the covering array.

This is because testing software only makes sense before the next version is released. This time interval between two release versions may sometimes be too short for a complete testing of a high-strength covering array, especially in the scenario of continuous integration [3].

To address these shortcomings of traditional covering arrays, the notion of incremental covering array [3] has been proposed. Such object can be deemed as adaptive covering array, which can increase the degree  $t$  when required. As it can generate higher strength covering array based on lower strength covering array, it can reduce the cost when comparing with generating multiple ways of covering arrays. Additionally, it can be better applied on testing the software of which the released time is frequently changed and cannot be predicted. Another advantage for generating incremental covering array is that, testers can detect most faults in the software as soon as possible. This is because according to [2], most faults (about 70% to 80%) are caused by 2-degree interactions, and almost all faults can be covered by 6-way covering arrays. As incremental covering arrays first cover those lower-degree interactions, the faults caused by them will be detected sooner.

In consideration of the size of the total number of test cases, we argue that this approach of generating incremental covering array may produce too many test cases. This is obvious, as generating higher-strength covering array based on the lower-strength covering array (called *bottom-up* strategy later) does not aim to optimize the size of the higher-strength covering array. As a result, it may generate more test cases than those approaches that focus on generating a particular higher-strength covering array.

Then, a natural question, and also the motivation of this paper is, **is it possible to generate an incremental covering array with the same number of the overall test cases as those by the particular high-way covering array generation algorithms?** Due to the obvious conclusion that any high way covering array must cover all the lower way interactions, the answer for this question is *yes*, as we can just apply a particular covering array generation algorithm to generate the high-strength covering array, and then generate the lower-strength covering arrays by extracting some subset of the test cases which can cover all the lower degree interactions. We refer to this strategy as the *top-down* strategy.

In this paper, we give two different implementations for each strategy. Specifically, for *bottom-up* strategy, we first gives an approach which is based on directly seeding test cases of lower-way covering array. Another approach comes from the existing incremental covering array algorithm [3], which uses multiple covering arrays instead of one for seeding test cases. For *top-down* strategy, the key of the first implementation is to select the test cases with as much un-covered interactions as possible from higher-way covering arrays, while the second implementation is to select test cases with maximal hamming distances at each iteration.

We evaluated these four approaches by comparing them at constructing several incremental covering arrays on 55 wildly used subjects (including 25 real-world software and

30 synthetic systems). There are three metrics that are used in the empirical studies, i.e., the size of the incremental covering arrays for each degrees, the time cost when generating these test cases, and the fault detection rate of these covering arrays. The results mainly show that strategy *Bottom-up* has an advantage over strategy *Top-down* at lower degrees with respect to the size of covering arrays, while strategy *Top-down* performs better at higher degrees. However, when concerning the fault detection rate, strategy *Top-down* gets higher scores at lower degrees, while *Bottom-up* is better at higher degrees. Besides these observations, we also found that different implementations of these generation strategies also have impacts on the performance, especially for strategy *Top-down*, of which the implementation with considering un-covered interactions had a significant advantage over the one based on hamming distance.

Our contributions include:

- 1) We propose a formal description of incremental covering array, and give a proof of the existence of it.
- 2) We propose two strategies for generating incremental covering arrays, and for each strategy we give two different implementations.
- 3) We conduct a series of empirical experiments to evaluate the characteristics (test size, time cost, and fault detection) of these four approaches.
- 4) We offer a guideline for selecting which strategy when generating incremental covering array in practice.

## II. BACKGROUND

This section gives some formal definitions related to CT. Assume that the behaviour of SUT is influenced by  $k$  parameters, and each parameter  $p_i$  has  $a_i$  discrete values from the finite set  $V_i$ , i.e.,  $a_i = |V_i|$  ( $i = 1, 2, \dots, k$ ). Then a *test case* of the SUT is a group of values that are assigned to each parameter, which can be denoted as  $(v_1, v_2, \dots, v_k)$ . An  $t$ -degree interaction can be formally denoted as  $(-, \dots, v_{n_1}, -, \dots, v_{n_2}, -, \dots, v_{n_t}, -, \dots)$ , where some  $t$  parameters have fixed values and other irrelevant parameters are represented as "-". In fact, a test case can be regarded as a  $k$ -degree interaction.

### A. Covering array

**Definition 1.** A  $t$ -way covering array  $MCA(N; t, k, (a_1, a_2, \dots, a_k))$  is a test set in the form of  $N \times k$  table, where each row represents a *test case* and each column represents a parameter. For any  $t$  columns, each possible  $t$ -degree interaction of the  $t$  parameters must appear at least once. When  $a_1 = a_2 = \dots = a_k = v$ , a  $t$ -way covering array can be denoted as  $CA(N; t, k, v)$ .

For example, Table I (a) shows a 2-way covering array  $CA(5; 2, 4, 2)$  for the SUT with 4 boolean parameters. For any two columns, any 2-degree interaction is covered. Covering array has proven to be effective in detecting the failures caused by interactions of parameters of the SUT. Many existing algorithms focus on constructing covering arrays such that the number of test cases, i.e.,  $N$ , can be as small as possible.

### B. Incremental covering array

**Definition 2.** An incremental covering array  $ICA([N_{t_1}, N_{t_1+1}, \dots, N_{t_2}]; [t_1, t_2], k, (a_1, a_2, \dots, a_k))$  is a test set in the form of  $N_{t_2} \times k$  table, where  $t_1 < t_2$  and  $N_{t_1} < N_{t_1+1} < \dots < N_{t_2}$ . In this table, the first  $N_{t_1}$  lines ( $t_1 \leq t_i \leq t_2$ ) is a covering array  $MCA(N_{t_1}; t_1 + i, k, (a_1, a_2, \dots, a_k))$ .

When  $a_1 = a_2 = \dots = a_k = v$ , an incremental covering array can be denoted as  $ICA([N_{t_1}, N_{t_1+1}, \dots, N_{t_2}]; [t_1, t_2], k, v)$ .

Table I shows an example of incremental covering array, in which the two-way covering array  $CA(5; 2, 4, 2)$  is a subset of the three-way covering array  $CA(9; 3, 4, 2)$ , which is also an incremental covering array  $ICA([5, 9]; [2, 3], 4, 2)$ .

TABLE I: Experiment of Incremental covering array

(a) $CA(5; 2, 4, 2)$		(b) $CA(9; 3, 4, 2)$ && $ICA([5, 9]; [2, 3], 4, 2)$
0 0 1 0	→	0 0 1 0
1 0 0 0	→	1 0 0 0
1 1 1 0	→	1 1 1 0
0 1 0 1	→	0 1 0 1
1 0 1 1	→	1 0 1 1
		0 1 0 0
		0 0 0 1
		0 1 1 1
		1 1 0 1

**Theorem.** For each covering array  $MCA(N_{t_2}; t_2, k, (a_1, a_2, \dots, a_k))$ , we can find an  $ICA([N_{t_1}, N_{t_1+1}, \dots, N_{t_2}]; [t_1, t_2], k, (a_1, a_2, \dots, a_k))$ , s.t., their test cases are the same.

*Proof.* We just need to prove that for any covering array,  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$ , we can find a  $MCA(N_{t-1}; t-1, k, (a_1, a_2, \dots, a_k))$ , such that,  $N_{t-1} < N_t$  and for any test case  $f \in MCA(N_{t-1}; t-1, k, (a_1, a_2, \dots, a_k))$ , it has  $f \in MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$ .

First,  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$  itself must be an  $MCA(N_t; t-1, k, (a_1, a_2, \dots, a_k))$ , as it must cover all the  $(t-1)$ -degree interactions.

Then, assume to obtain a  $(t-1)$ -way covering array, any one test case in  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$  can not be reduced. With this assumption, any test case will cover at least one  $(t-1)$ -degree interaction that only appears in this test case. Without loss of generality, let test case  $(v_1, v_2, \dots, v_k)$  cover the  $(t-1)$ -degree interaction  $(-, \dots, v_{p_1}, \dots, -, \dots, v_{p_2}, \dots, -, \dots, v_{p_{t-1}}, \dots, -, \dots, v_{p_t}, \dots, -, \dots)$  which only appears in the test case. Then obviously the  $t$ -degree interaction  $(v'_1, \dots, v_{p_1}, \dots, -, \dots, v_{p_2}, \dots, -, \dots, v_{p_{t-1}}, \dots, -, \dots, v_{p_t}, \dots, -, \dots)$  ( $v'_1 \neq v_1$ ) will never be covered by any test case in  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$ , and hence  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$  is not a  $t$ -way covering array (Note this is based on that the parameter can take on more than one value).

This is a contradiction, and means that we can reduce at least one test case in  $MCA(N_t; t, k, (a_1, a_2, \dots, a_k))$ , so that

it is still a  $(t-1)$ -way covering array.  $\square$

This theorem shows the existence of the incremental covering arrays. As discussed before, generating the incremental covering arrays is of importance, as it supports adaptively increasing the coverage strength. According to this theorem, when testing a SUT, testers can firstly execute the lowest-strength covering array in the incremental covering arrays, and then execute additional test cases from those higher-strength covering arrays as required. The reuse of previously executed test cases will reduce the cost for generating multiple different-ways covering arrays.

### III. GENERATING INCREMENTAL COVERING ARRAYS

This section presents two strategies to generate the incremental covering arrays. The first strategy, i.e., the *bottom-up* strategy starts from generating the lowest-strength covering array and then the higher-strength ones. The second strategy, i.e., the *top-down* strategy firstly generated the highest-strength covering array, then the lower-strength covering array.

#### A. Bottom-up strategy

This strategy is listed as Strategy 1. The inputs for this

---

#### Strategy 1 Bottom-up strategy

---

**Input:** *Params* ▷ Parameters (and their values)  
 $t_1$  ▷ the lowest strength  
 $t_2$  ▷ the highest strength  
**Output:** *ICA* ▷ the incremental covering arrays  
1:  $ICA \leftarrow EmptySet$   
2: **for**  $t_i = t_1; t_i \leq t_2; t_i++$  **do**  
3:    $CA_i \leftarrow EmptySet$   
4:   **if**  $t_i == t_1$  **then**  
5:      $CA_i \leftarrow CA\_Gen(Params, t_i)$   
6:   **else**  
7:      $CA_i \leftarrow extend(Params, t_i, CA_{i-1})$   
8:   **end if**  
9:    $ICA.append(CA_i)$   
10: **end for**  
11: **return** *ICA*

---

strategy consists of the values for each parameter of the SUT  $-Params$ , the lowest strength  $t_1$  of the covering array in the incremental covering array, and the highest strength  $t_2$  of the covering array. The output of this strategy is an incremental covering array  $-ICA$ .

This strategy generates the covering array from lower-strength to higher-strength (line 2). If the current coverage strength  $t_i$  is equal to  $t_1$ , it just utilizes a covering array generation algorithm to generate the particular covering array (line 4 - 5). Otherwise, it will first take the previous generated covering array  $CA_{i-1}$  as seeds, and then utilize covering array generation algorithm to append additional test cases to satisfy higher coverage criteria (line 6 - 7).

Fig.1 presents an example for constructing  $ICA([6, 13, 24]; [2, 4], 5, 2)$  by this strategy. In this example, the covering array

c1	0	0	0	0	0
c2	1	1	1	1	0
c3	0	0	1	1	1
c4	1	1	0	0	1
c5	0	1	0	1	0
c6	1	0	1	0	0
c7	1	0	0	1	1
c8	0	1	1	0	1
c9	0	1	1	0	0
c10	0	0	0	0	1
c11	1	0	0	1	0
c12	0	1	0	1	1
c13	1	0	1	0	1
c14	1	1	0	0	0
c15	0	0	1	1	0
c16	1	1	1	1	1
c17	0	0	1	0	0
c18	1	0	0	0	0
c19	0	1	0	0	0
c20	0	0	0	1	0
c21	1	1	1	0	0
c22	0	1	1	1	0
c23	1	1	0	1	0
c24	1	0	1	1	0

Fig. 1: Bottom-up strategy example

generation algorithm used is AETG [4]. The two-way covering array (test cases  $c_1$  to  $c_6$ ) is directly generated, and the three-way covering array (test cases  $c_1$  to  $c_{13}$ ) is generated by adding additional test cases (test cases  $c_7$  to  $c_{13}$ ) based on the previous two-way covering array. The four-way covering array (test cases  $c_1$  to  $c_{24}$ ) is constructed on the previous three-way covering array. In total, to reach the 4-way covering array, 24 test cases are needed for this strategy.

#### B. Top-down strategy

This strategy is listed as Strategy 2, which generates covering arrays in the opposite order (line 2) of the bottom-up strategy. Similarly, if the current coverage strength  $t_i$  is equal to  $t_2$ , it directly generates the particular covering array (line 4 - 5). Otherwise, it will extract the covering array from a higher covering array ( $CA_{i+1}$ ) (line 6 - 7).

An example for this strategy is given in Fig.2. In this example, we first generated the 4-way covering array  $CA(16; 4, 5, 2)$ . Then we selected 14 test cases to form a three-covering array  $CA(14; 3, 5, 2)$ . Next the two-way covering array  $CA(8; 2, 5, 2)$  is extracted from  $CA(14; 3, 5, 2)$ . The rows with dark background from the higher-strength covering arrays represent those selected test cases.

From the two examples, an obvious observation is that for the *top-down* strategy, it has a significant advantage over the *bottom-up* strategy with respect to the size of the 4-way covering array (16 test cases generated by *top-down*, while 24 by *bottom-up*). But when considering the lower-strength covering arrays, the *bottom-up* strategy performs slightly better

#### Strategy 2 Top-down strategy

---

**Input:**  $Params$  ▷ Parameters (and their values)  
 $t_1$  ▷ the lowest strength  
 $t_2$  ▷ the highest strength  
**Output:**  $ICA$  ▷ the incremental covering arrays

---

```

1:  $ICA \leftarrow EmptySet$ 
2: for  $t_i = t_2; t_i \geq t_1; t_i --$  do
3:    $CA_i \leftarrow EmptySet$ 
4:   if  $t_i == t_2$  then
5:      $CA_i \leftarrow CA\_Gen(Params, t_i)$ 
6:   else
7:      $CA_i \leftarrow extract(Params, t_i, CA_{i+1})$ 
8:   end if
9:    $ICA.append(CA_i)$ 
10: end for
11: return  $ICA$ 

```

---

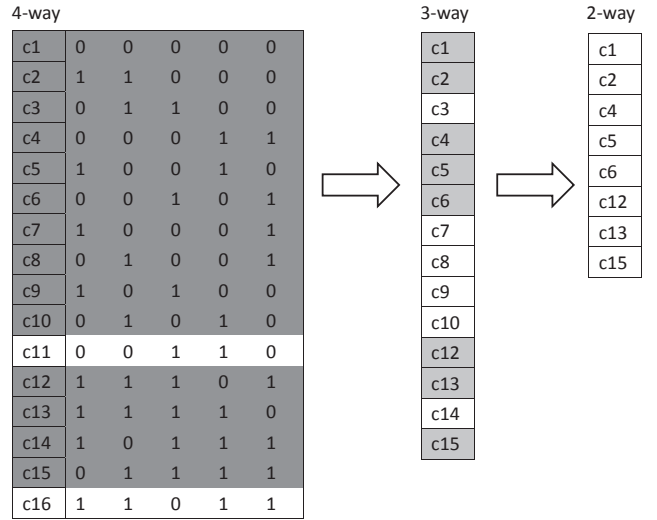


Fig. 2: Top-down strategy example

(13 and 6 by *bottom-up* for degree 3 and 2, respectively; while 14 and 8 by *top-down*).

#### IV. IMPLEMENTATION OF THE TWO STRATEGIES

In this section, we will describe a specific implementation of the two strategies proposed in the previous section.

##### A. Common parts of two strategies

1) *Covering array generation method:* Our covering array generation approach is IPOG [5], [6], [7], which firstly builds a  $t$ -way test set for the first  $t$  parameters, then extends the test set to build a  $t$ -way test set for the first  $t+1$  parameters, and then continues to extend the test set until it builds a  $t$ -way test set for all the parameters. We select this algorithm mainly because it is very efficient, that is, it can quickly generate a  $t$ -way covering array even though  $t$  is a relatively large number (up to 6). This is important, as in our experiments, some subjects contain an extremely large input space, which

is unfavorable to apply some time-consuming covering array generation approaches.

2) *Constraints handling*: Our constraints handling technique is based on Minimum Forbidden Tuples (MFTs) [8], [9], which is well supported by IPOG generation algorithm. A forbidden tuple is a tuple as the value assignment of some parameters that violates constraints. A minimum forbidden tuple is a forbidden tuple of minimum size that covers no other forbidden tuples. Given some constraints for one SUT, we should compute all the MFTs of the SUT, and then limit the test cases to the ones that do not contain any tuple in the MFTs.

### B. Implementation of strategy Bottom-up

The key part of strategy *Bottom-up* is seeding existing test cases for higher-way covering array. Specifically, when given a lower-way covering array as seed, we need to eliminate those higher-degree tuples that have already been covered by this covering array. After this, we need to generate additional test cases to cover the un-covered higher-degree tuples until all the higher-degree tuples are covered.

In this paper, we offer two approaches with different seeding process. The first one is simple: We directly take all the test cases in the lower covering array as the initial test set of higher covering array. We later call them the *Single Bottom-up* approach. The second approach comes from an existing study [3], of which the seeding test cases consist of two parts: one part is partial test cases (instead of the whole covering array) from the lower-way covering array, the other part comes from an existing incremental covering array which is generated along with this covering array. We later call this approach the *Collaborative bottom-up* approach. The main reason that *collaborative bottom-up* generates distinct incremental covering arrays for one SUT is for handling undetermined failures and fault localization issues.

More specifically, Figure 3 depicts the differences between these two approaches. In this figure, the left side simply shows how do approach *single bottom-up* generate incremental covering arrays, while the right side shows the *collaborative bottom-up* approach. From this figure, we can easily learn that the single bottom-up approach is very simple and straightforward, it only needs to build the  $t$ -way covering array based on the seed of  $t-1$ -way covering array. While for *collaborative bottom-up* approach, the process is slightly more complicated. First of all, its target is to generate several incremental covering arrays. For the first one in these incremental covering arrays, it follows the *single bottom-up* approach, i.e., take  $t-1$ -way covering array as the seed to build  $t$ -way covering array. For the remaining incremental covering arrays, the  $t$ -way covering arrays need seeding test cases from their own  $t-1$ -way covering arrays and some test cases from the initially generated  $t+1$  covering array (the first incremental covering array). For example, in the second incremental covering array (at the middle) generated by “collaborative bottom-up” approach in Figure 3, we can find some part ( $E$ ) of the 3-way covering array comes from

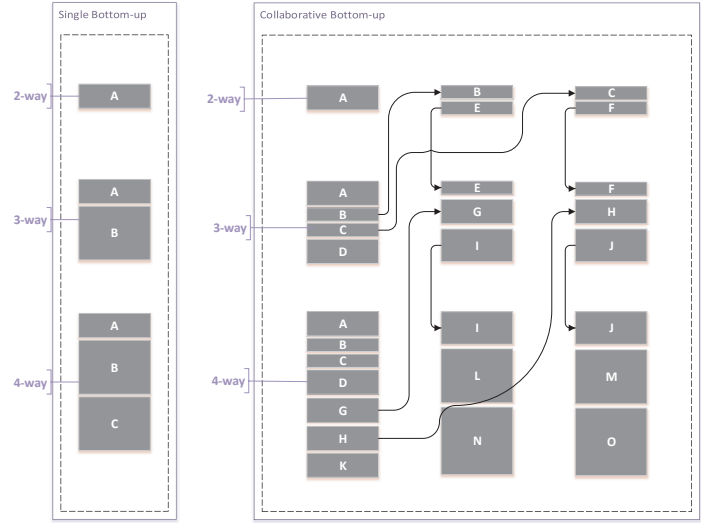


Fig. 3: Single Bottom-up approach and Collaborative bottom-up approach

the 2-way covering array at the middle, while the other part( $G$ ) comes from the 4-way covering array at the left.

### C. Implementation of strategy Top-down

The key part of the top-down strategy is extracting test cases from higher-way covering array. It is desirable to obtain a minimal  $t$ -way covering array  $CA_t$  from a  $t+1$ -way covering array  $CA_{t+1}$ . However, this is impractical if  $CA_{t+1}$  is too large, because to get the minimal  $CA_t$ , we need to exhaustively check every possible subset of the  $CA_{t+1}$ . Hence, in this paper, we offer two pragmatic approaches to resolve this issue.

The first approach selects the test cases from  $CA_{t+1}$  with the maximal un-covered  $t$ -degree interactions at each interaction until the  $CA_t$  is constructed. Note that after one test case is selected, the un-covered interactions of each of the remaining test cases should be re-computed. One simple method is to completely search each  $t$ -degree interactions of each test case and then judge whether it is already covered. However, it is very time consuming, especially considering the fact that there are more than 60 million 5-degree interactions of one test case for some subjects in our empirical studies. In fact, we do not need to re-compute all the interactions for all the remaining test cases when one test case is selected. Instead, we only need to record the newly covered interactions of the last selected test case, and then check each of the remaining test cases whether it contains some of these newly covered interactions; if so, the number of un-covered interactions should be updated by subtracting the number of these contained interactions. We later call this approach the *Coverage-based Top-down* approach, or *CB top-down* for short.

The second approach selects the test case from  $CA_{t+1}$  which has the longest hamming distance from the test cases that have already been selected. The hamming distance

between two test cases, say,  $c_1$  and  $c_2$ , is computed in the following formula:

$$\frac{\#the\ num\ of\ different\ elements\ between\ c_1\ and\ c_2}{\#the\ num\ of\ all\ the\ elements\ in\ c_1\ and\ c_2}$$

The selection process repeats until we obtain a complete  $t$ -way covering array. Note that computing the hamming distance of one test case is much easier than computing the uncovered interactions for the first approach, because the number of elements in one test case is normally much fewer than the number of interactions in one test case. The second approach is referred to as *Hamming distance-based Top-down* approach, or *HDB top-down* for short, in the remaining of the paper.

## V. RESEARCH QUESTIONS

We set up the following three research questions to investigate the effectiveness and the efficiency of the four approaches which belong to two strategies.

### RQ1.What about the size of the incremental covering array generated by the four approaches?

To evaluate these approaches, the size of the covering array is one important metric. In CT, the main goal is to make the size of the covering array as small as possible. With respect to the incremental covering array, instead of focusing on covering array with one specified strength, we should evaluate all the covering arrays with different strengths in it. How to apply the two strategies in practice depends on what strength of the covering arrays they are good at generating. For example, if one strategy can generate smaller size of lower-way covering array, it is a better choice at generating incremental covering array within a short testing time (current software under testing will be soon expired for the releasing of the next version). On the contrary, if one strategy can generate smaller size of higher-way covering array, it is better to be applied on testing the software within a long testing time.

### RQ2.How efficient are these approaches in terms of computation time cost?

The computation time that is needed to generate covering array is also a very important metric to evaluate the four approaches. Many approaches in CT is limited to a given time budgeted for searching the optimal covering arrays. But when apply covering array in practice, it is normally not possible to obtain a proper time budgeted in prior. Hence, it is necessary to optimize the covering array generation process to reduce the time cost. Similar to our first research question, we will investigate the time to generate the covering arrays with different test strengths in the incremental covering array.

### RQ3.How efficient are these approaches with respect to fault detection?

A  $\tau$ -way covering array guarantees all the  $\tau$ -degree (or less than  $\tau$ -degree) faulty interactions can be detected after execution. However, for different  $\tau$ -way covering arrays, the rate of faults detection varies. This reason is obvious: The test cases and test sequences are different for those  $\tau$ -way covering arrays, and hence, the number of tests that must be run varies

for detecting specific  $\tau$ -degree faulty interactions. In this paper, we firstly use the metric “Rate of Fault Detection”(RFD) [10] to evaluate the efficiency of covering arrays with respect to fault detection.

More formally [10], let  $T=\{t_1, t_2, \dots, t_m\}$  be a test suite for SUT, with test cases in  $T$  to be run in the order:  $t_1 \rightarrow t_2 \rightarrow \dots \rightarrow t_m$ . Let  $T_i = \{t_1, t_2, \dots, t_i\}$  for  $1 \leq i \leq m$ , and let  $T_i(\tau - degree)$  be the set of  $t$ -degree interactions that are covered by test set  $T_i$ . Then, the  $\tau$  way RFD of test suite  $T$  is:

$$\tau\text{-RFD}(T) = \sum_{i=1}^m |T_i(\tau - degree)|.$$

Note that, the  $\tau$ -RFD value of one test suite is influenced by the number of tests in this suite. In order to remove the influence of the number of tests, we compute the  $\tau$ -RFD of each approach by using the same number of tests (for each subject, we will take the minimal size of the covering array generated by the four approaches as the number of tests).

## VI. EXPERIMENT SET-UPS

### A. Benchmarks

We used 55 SUT as the subjects of our experiments, which are shown in Table II. These subjects are wildly used to evaluate the performance of the covering array generation approach. Among these subjects, the first 20 subjects (from *Banking1* to *Telecom*), are obtained from a recent benchmark created by Segall et al [11], which have already been used in several works [12], [13]. These 20 subjects cover a wide range of applications, including telecommunications, health care, storage and banking systems. The following five subjects (from *SPIN-S* to *Bugzilla*) were firstly introduced by Cohen et al. [14], [15]. Among them, SPIN-S and SPIN-V are two components for model simulation and verification, GCC is a compiler system, Apache is a web server application, and Bugzilla is a web-based bug tracking system. These five subjects have been wildly used in literatures [16], [14], [15], [17], [18], [19], [13]. The last 30 (from Syn1 to Syn30) are synthetic subject models that are generated by Cohen et al. [15]. These synthetic subject models are designed with similar characterization (input space, constraints, etc.) of those real subjects and have been used in following studies[17], [18], [19], [13].

In Table II, the model is presented in the abbreviated form  $\#values\#number\ of\ parameters$ , e.g.,  $7^36^2$  indicates the software has 3 parameters that can take 7 values and 2 parameters take 6 values. The constraint is presented in the form  $\#degree\#number\ of\ constraints$ , e.g.,  $2^33^{18}$  means that the SUT contains 3 constraints with 2-degree and 18 constraints with 3-degree.

### B. Set-ups

For each SUT listed in Table II, we adopted four approaches i.e., *single bottom-up*, *collaborative bottom-up*, *coverage-based top-down*, and *hamming distance-based top-down*, to generate incremental covering arrays from 2-way to 6-way. We recorded and compared the size of each covering array contained in these incremental covering arrays, generation

TABLE II: The models of the SUT

Name	Model	Constraint
Banking1	3 <sup>4</sup> 4 <sup>1</sup>	5 <sup>112</sup>
Banking2	2 <sup>14</sup> 4 <sup>1</sup>	2 <sup>3</sup>
CommonProtocol	2 <sup>10</sup> 7 <sup>1</sup>	2 <sup>103</sup> 3 <sup>10</sup> 4 <sup>12</sup> 5 <sup>96</sup>
Concurrency	2 <sup>5</sup>	2 <sup>4</sup> 3 <sup>15</sup> 2
Healthcare1	2 <sup>6</sup> 3 <sup>2</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>3</sup> 3 <sup>18</sup>
Healthcare2	2 <sup>5</sup> 3 <sup>6</sup> 4 <sup>1</sup>	2 <sup>1</sup> 3 <sup>6</sup> 5 <sup>18</sup>
Helathcare3	2 <sup>16</sup> 3 <sup>6</sup> 4 <sup>5</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>31</sup>
Helathcare4	2 <sup>13</sup> 3 <sup>12</sup> 4 <sup>6</sup> 5 <sup>2</sup> 6 <sup>1</sup> 7 <sup>1</sup>	2 <sup>22</sup>
Insurance	2 <sup>6</sup> 3 <sup>1</sup> 5 <sup>1</sup> 6 <sup>2</sup> 11 <sup>1</sup> 13 <sup>1</sup> 17 <sup>1</sup> 31 <sup>1</sup>	-
NetworkMgmt	2 <sup>2</sup> 4 <sup>1</sup> 5 <sup>3</sup> 10 <sup>2</sup> 11 <sup>1</sup>	2 <sup>20</sup>
ProcessorComm1	2 <sup>3</sup> 3 <sup>6</sup> 4 <sup>6</sup>	2 <sup>13</sup>
ProcessorComm2	2 <sup>3</sup> 3 <sup>12</sup> 4 <sup>8</sup> 5 <sup>2</sup>	1 <sup>4</sup> 2 <sup>121</sup>
Services	2 <sup>3</sup> 3 <sup>4</sup> 5 <sup>2</sup> 8 <sup>2</sup> 10 <sup>2</sup>	3 <sup>386</sup> 4 <sup>2</sup>
Storage1	2 <sup>1</sup> 3 <sup>1</sup> 4 <sup>1</sup> 5 <sup>1</sup>	4 <sup>95</sup>
Storage2	3 <sup>4</sup> 6 <sup>1</sup>	-
Storage3	2 <sup>9</sup> 3 <sup>1</sup> 5 <sup>3</sup> 6 <sup>1</sup> 8 <sup>1</sup>	2 <sup>38</sup> 3 <sup>10</sup>
Storage4	2 <sup>5</sup> 3 <sup>7</sup> 4 <sup>1</sup> 5 <sup>2</sup> 6 <sup>2</sup> 7 <sup>1</sup> 9 <sup>1</sup> 13 <sup>1</sup>	2 <sup>24</sup>
Storage5	2 <sup>5</sup> 3 <sup>8</sup> 5 <sup>3</sup> 6 <sup>2</sup> 8 <sup>1</sup> 9 <sup>1</sup> 10 <sup>2</sup> 11 <sup>1</sup>	2 <sup>151</sup>
SystemMgmt	2 <sup>5</sup> 3 <sup>4</sup> 5 <sup>1</sup>	2 <sup>13</sup> 3 <sup>4</sup>
Telecom	2 <sup>5</sup> 3 <sup>1</sup> 4 <sup>2</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>11</sup> 3 <sup>14</sup> 9
SPIN-S	2 <sup>13</sup> 4 <sup>5</sup>	2 <sup>13</sup>
SPIN-V	2 <sup>42</sup> 3 <sup>2</sup> 4 <sup>11</sup>	2 <sup>47</sup> 3 <sup>2</sup>
GCC	2 <sup>189</sup> 3 <sup>10</sup>	2 <sup>37</sup> 3 <sup>3</sup>
Apache	2 <sup>158</sup> 3 <sup>8</sup> 4 <sup>4</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>3</sup> 3 <sup>14</sup> 2 <sup>51</sup>
Bugzilla	2 <sup>49</sup> 3 <sup>1</sup> 4 <sup>2</sup>	2 <sup>4</sup> 3 <sup>1</sup>
Syn1	2 <sup>86</sup> 3 <sup>3</sup> 4 <sup>1</sup> 5 <sup>5</sup> 6 <sup>2</sup>	2 <sup>20</sup> 3 <sup>3</sup> 4 <sup>1</sup>
Syn2	2 <sup>86</sup> 3 <sup>3</sup> 4 <sup>3</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>19</sup> 3 <sup>3</sup>
Syn3	2 <sup>27</sup> 4 <sup>2</sup>	2 <sup>9</sup> 3 <sup>1</sup>
Syn4	2 <sup>51</sup> 3 <sup>4</sup> 4 <sup>2</sup> 5 <sup>1</sup>	2 <sup>15</sup> 3 <sup>2</sup>
Syn5	2 <sup>155</sup> 3 <sup>7</sup> 4 <sup>3</sup> 5 <sup>5</sup> 6 <sup>4</sup>	2 <sup>32</sup> 3 <sup>6</sup> 4 <sup>1</sup>
Syn6	2 <sup>73</sup> 4 <sup>3</sup> 6 <sup>1</sup>	2 <sup>26</sup> 3 <sup>4</sup>
Syn7	2 <sup>29</sup> 3 <sup>1</sup>	2 <sup>13</sup> 3 <sup>2</sup>
Syn8	2 <sup>109</sup> 3 <sup>2</sup> 4 <sup>2</sup> 5 <sup>3</sup> 6 <sup>3</sup>	2 <sup>32</sup> 3 <sup>4</sup> 4 <sup>1</sup>
Syn9	2 <sup>57</sup> 3 <sup>1</sup> 4 <sup>1</sup> 5 <sup>1</sup> 6 <sup>1</sup>	2 <sup>30</sup> 3 <sup>7</sup>
Syn10	2 <sup>130</sup> 3 <sup>6</sup> 4 <sup>5</sup> 5 <sup>2</sup> 6 <sup>4</sup>	2 <sup>40</sup> 3 <sup>7</sup>
Syn11	2 <sup>84</sup> 3 <sup>4</sup> 4 <sup>2</sup> 5 <sup>2</sup> 6 <sup>4</sup>	2 <sup>28</sup> 3 <sup>7</sup>
Syn12	2 <sup>136</sup> 3 <sup>4</sup> 4 <sup>3</sup> 5 <sup>1</sup> 6 <sup>3</sup>	2 <sup>23</sup> 3 <sup>4</sup>
Syn13	2 <sup>124</sup> 3 <sup>4</sup> 4 <sup>1</sup> 5 <sup>2</sup> 6 <sup>2</sup>	2 <sup>22</sup> 3 <sup>4</sup>
Syn14	2 <sup>81</sup> 3 <sup>5</sup> 4 <sup>3</sup> 6 <sup>3</sup>	2 <sup>13</sup> 3 <sup>2</sup>
Syn15	2 <sup>50</sup> 3 <sup>4</sup> 4 <sup>1</sup> 5 <sup>2</sup> 6 <sup>1</sup>	2 <sup>20</sup> 3 <sup>2</sup>
Syn16	2 <sup>81</sup> 3 <sup>3</sup> 4 <sup>2</sup> 6 <sup>1</sup>	2 <sup>30</sup> 3 <sup>4</sup>
Syn17	2 <sup>128</sup> 3 <sup>3</sup> 4 <sup>2</sup> 5 <sup>1</sup> 6 <sup>3</sup>	2 <sup>25</sup> 3 <sup>4</sup>
Syn18	2 <sup>127</sup> 3 <sup>2</sup> 4 <sup>4</sup> 5 <sup>6</sup> 6 <sup>2</sup>	2 <sup>23</sup> 3 <sup>4</sup> 4 <sup>1</sup>
Syn19	2 <sup>172</sup> 3 <sup>9</sup> 4 <sup>9</sup> 5 <sup>3</sup> 6 <sup>4</sup>	2 <sup>38</sup> 3 <sup>5</sup>
Syn20	2 <sup>138</sup> 3 <sup>4</sup> 4 <sup>5</sup> 5 <sup>4</sup> 6 <sup>7</sup>	2 <sup>42</sup> 3 <sup>6</sup>
Syn21	2 <sup>76</sup> 3 <sup>3</sup> 4 <sup>2</sup> 5 <sup>1</sup> 6 <sup>3</sup>	2 <sup>40</sup> 3 <sup>6</sup>
Syn22	2 <sup>72</sup> 3 <sup>4</sup> 4 <sup>1</sup> 6 <sup>2</sup>	2 <sup>31</sup> 3 <sup>4</sup>
Syn23	2 <sup>25</sup> 3 <sup>1</sup> 6 <sup>1</sup>	2 <sup>13</sup> 3 <sup>2</sup>
Syn24	2 <sup>110</sup> 3 <sup>2</sup> 5 <sup>1</sup> 6 <sup>4</sup>	2 <sup>25</sup> 3 <sup>4</sup>
Syn25	2 <sup>118</sup> 3 <sup>6</sup> 4 <sup>2</sup> 5 <sup>1</sup> 6 <sup>4</sup>	2 <sup>23</sup> 3 <sup>3</sup> 4 <sup>1</sup>
Syn26	2 <sup>87</sup> 3 <sup>1</sup> 4 <sup>3</sup> 5 <sup>4</sup>	2 <sup>28</sup> 3 <sup>4</sup>
Syn27	2 <sup>55</sup> 3 <sup>2</sup> 4 <sup>2</sup> 5 <sup>1</sup> 6 <sup>4</sup>	2 <sup>17</sup> 3 <sup>3</sup>
Syn28	2 <sup>167</sup> 3 <sup>16</sup> 4 <sup>2</sup> 5 <sup>3</sup> 6 <sup>6</sup>	2 <sup>31</sup> 3 <sup>6</sup>
Syn29	2 <sup>134</sup> 3 <sup>7</sup> 5 <sup>3</sup>	2 <sup>19</sup> 3 <sup>3</sup>
Syn30	2 <sup>73</sup> 3 <sup>3</sup> 4 <sup>3</sup>	2 <sup>20</sup> 3 <sup>2</sup>

time (seconds) that are consumed to generate these covering arrays, and the RFD of each covering array.

Note that, as we mentioned before, *collaborative bottom-up* generates multiple incremental covering arrays for one SUT to increase its abilities to handle the random or non-determined failures. Hence, to make a fair comparison, we recorded the average characteristics of these covering arrays, which include the size, corresponding time cost and RFD, and then compared

them with those of other approaches.

We ran all the experiments on a server which has a 6-core CPU of 2.0GHz (Intel Xeon E5-2640 v2), and a 16 GB memory. It took us four weeks in total to obtain all the results. Note that not all the covering arrays can run up to 6 way because of the memory limitation and computing time.

## VII. RESULTS AND DISCUSSION

### A. RQ1. The size of the incremental covering array

The results are listed in Fig.4. There are 5 sub figures in this figure, each of which focus on reporting the covering array with specific strength. For each sub-figure, there are four polygonal lines that represent the size of the generated covering array of the four approaches respectively. The x axis means the 55 subjects listed in Table II, while the y axis means the covering array size. These values are normalized between 0 to 1 for visualization purposes. Additionally, the smaller the value of y is, the better the corresponding approach performed. Note the value of "-1" in y axis means we cannot generate the specific covering array of the corresponding subject due to the limitation of memory and computing time.

There are several observations that can be made:

- 1) Approach *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 54 out of 55 subjects and all the 55 subjects, respectively, for 2-way covering arrays. Among these subjects, *single bottom-up* reduced the size of covering arrays by 22%, and 77% on average (not shown in this figure), respectively. Approach *collaborative bottom-up* outperformed these two top-down approaches at 53 and 55 subjects, respectively, for 2-way covering arrays. The reductions on the size of covering arrays made by *collaborative bottom-up* are around 18% and 76%, respectively. For 3-way covering strength, the advantages of *bottom-up* approaches over *top-down* approaches slightly decreased. Specifically, approach *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 45, and 47 subjects, respectively. The corresponding reductions on the size of covering arrays are around 16% and 69% on average. Approach *collaborative bottom-up* also outperformed *CB top-down* and *HDB top-down* at 45, and 47 subjects, respectively. The corresponding reductions on the size of covering arrays are around 2% and 69% on average.
- 2) For the highest covering strength, i.e., 6-way, approach *CB top-down* outperformed both *single bottom-up* and *HDB top-down* at all the 10 subjects (only 10 subjects can be tested under 6-way testing strength). Among these subjects, *CB top-down* reduced by 3%, and 4%, on average, respectively, on the size of the covering array. The *HDB top-down* obtained the same results as approach *CB top-down* at this testing strength. This is because they share the same covering array at the highest covering strength (the difference between the two top-down approaches is their different extracting methods for generating lower-way covering array).



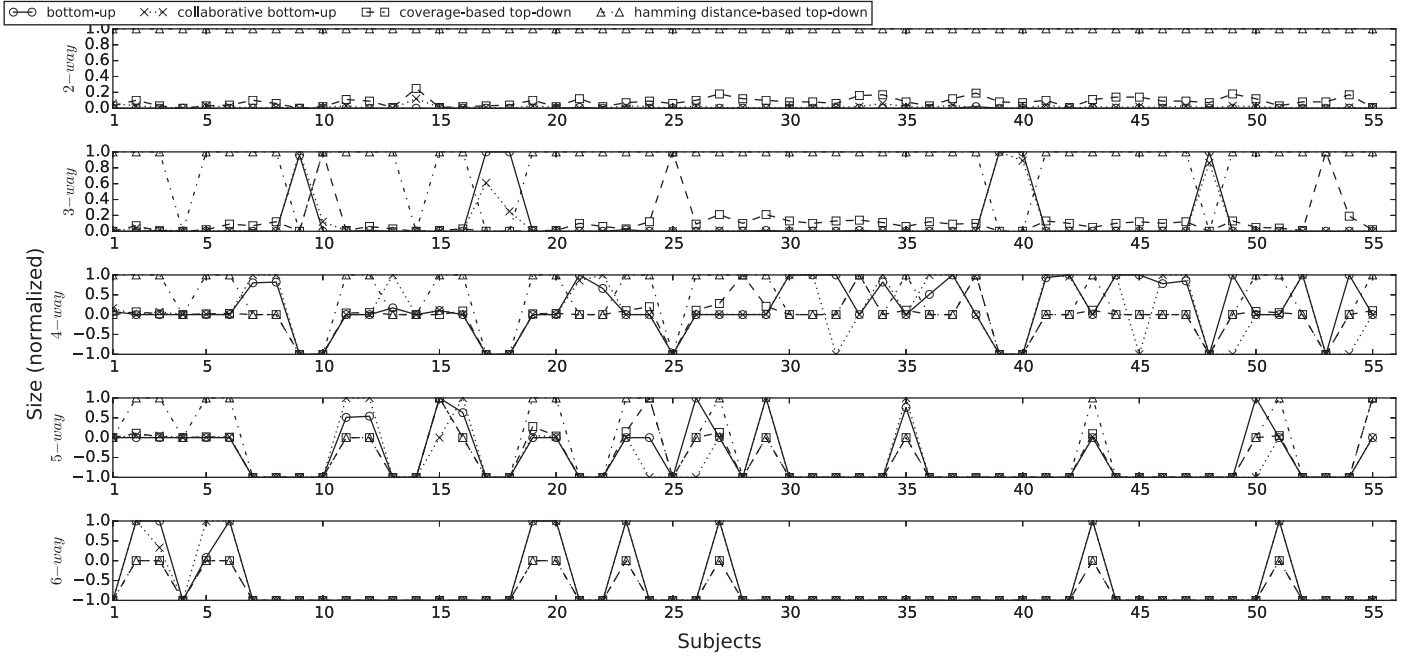


Fig. 4: The comparison of the four approaches with respect to the size of covering arrays

- 3) For the medium testing strength, i.e., 4 and 5, top-down approaches and bottom-up approaches performed neck and neck on the size of the covering array. Specifically, for 4-way covering arrays, *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 22 and 26 subjects, respectively, and *collaborative bottom-up* outperformed *CB top-down* and *HDB top-down* at 21 and 24 subjects. On the other hand, *CB top-down* outperformed *single bottom-up* and *collaborative bottom-up* at 22 and 21 subjects, respectively, and *HDB top-down* outperformed these two bottom-up approaches at 20 and 16 subjects, respectively. Among these subjects, the extent to which the reductions of the size of covering arrays made by these approaches against each other are also similar (around 2% to 5%). Approach *HDB top-down* was an exception, other 3 approaches reduced about by 30% to 50% on the size of the covering arrays when compared to its generated covering arrays. The results of 5-way covering strength matched well with those of 4-way covering strength.
- 4) When comparing the two approaches that both belong to *bottom-up* strategy, approach *single bottom-up* slightly performed better than *collaborative bottom-up* at almost all the testing strengths. Specifically, there are 45, 24, 21, 14, and 1 subjects, on which approach *single bottom-up* generated smaller size of covering array than than *collaborative bottom-up* for testing strength 2, 3, 4, 5, and 6, respectively, while there are 6, 21, 7, 1, and 1 subjects, on which *collaborative bottom-up* worked better. The specific reductions on the size of covering arrays made by these two approaches are 1% to 6%.

- 5) When comparing the two approaches that both belong to *top-down* strategy, approach *CB top-down* exhibited a significant advantage over *HDB top-down*. Specifically, there are 55, 44, 21, 10, and 0 subjects, on which approach *CB top-down* generated smaller size of covering arrays when compared to *HDB top-down* for testing strength 2, 3, 4, 5, and 6, respectively, while there are no subjects on which *HDB top-down* worked better than *CB top-down* for all the testing strengths. More specifically, among these subjects, approach *CB top-down* reduced the size of the covering arrays by 46% and 71% when compared to *HDB top-down*.

In summary, we can conclude that, with respect to the size of the covering array, *Bottom-up* strategy has a significant advantage over *Top-down* strategy at low testing strengths, while *Top-down* strategy works better at the high testing strength. Additionally, different implementations of the strategy also affect on the results. Specifically, *single bottom-up* performs slightly better than *collaborative bottom-up*, while *CB top-down* exhibits significant advantage over *HDB top-down*.

It is as expected that *Bottom-up* and *Top-down* strategies both have their ups and downs, considering that *Bottom-up* makes the lowest-way covering array generation a priority (using specialized covering array generation approach for the 2-way covering array), while *Top-down* makes the highest-way covering array generation a priority. With respect to *Bottom-up* strategy, the reason that approach *single bottom-up* slightly performs better than *collaborative bottom-up* is that the latter approach considers more aspects (un-deterministic failures reappearance) such that it cannot only focus on the size of the covering arrays. The observation that *CB top-down*



performs much better *HDB top-down* is interesting, which implies that the methods for extracting lower-way covering arrays from higher-way covering arrays are important for the size of tests.

#### B. RQ2. The time cost of the incremental covering array

The results are visualized in Fig.5. The organization of this figure is similar to Fig. 4. The only difference is that the y axis in this figure indicates the time consumed for generating these covering arrays (which is also normalized between 0 to 1 for visualization purposes).

We made the following observations:

- 1) Approach *CB top-down* was the best approach in terms of the rate of fault detection when compared with all the other three approaches at most cases. Specifically, *CB top-down* outperformed *single bottom-up* approach at 45, 50, 24, 10, and 0 subjects for the 5 testing strengths, respectively, while *single bottom-up* only performed better at 9, 5, 20, 9, and 10 subjects, respectively, and *CB top-down* outperformed *collaborative bottom-up* at 53, 50, 31, 11, and 0 subjects, while *collaborative bottom-up* only performed better at 1, 5, 10, 5, and 10 subjects, respectively.
- 2) The second best approach was *single bottom-up*. Among these subjects, *single bottom-up* reduced time cost by 85%, 84%, 59%, 62%, and 31% on average, respectively. Besides, *single bottom-up* outperformed another top-down approach, i.e., *HDB top-down* at 49, 53, 43, 18, and 7 subjects for the five different testing strengths, respectively. The time cost reduction made by *single bottom-up* is about 93%, 79%, 54%, 57%, and 31%, respectively, for these subjects when compared to approach *HDB top-down*. Approach *collaborative bottom-up* also outperformed these two top-down approaches. Specifically, *collaborative bottom-up* outperformed *CB top-down* at 27, 40, 30, 13, and 4 subjects, with corresponding time cost reduction of 60%, 81%, 57%, 67%, and 15%, respectively, for the 5 different testing strengths. It outperformed *HDB top-down* at 41, 43, 29, 13, and 4 subjects, with corresponding time cost reduction of 79%, 80%, 51%, 56%, and 15%, respectively, for the 5 different testing strengths.
- 3) With respect to the two approaches both belong to *bottom-up* strategy, *single bottom-up* exhibited a clear advantage in terms of time cost. Specifically, *single bottom-up* outperformed *collaborative bottom-up* at 54, 52, 39, 15, and 9 subjects, respectively, for these 5 testing strengths. Among these subjects, *single bottom-up* made reductions of 89%, 58%, 50%, 64%, and 58%, respectively, at the time cost.
- 4) With respect to *top-down* strategy, these two approaches performed neck and neck. Specifically, there are 50, 23, 12, 4, and 0 subjects at which *CB top-down* performed better, while there are 5, 23, 10, 6, and 0 subjects at which *HDB top-down* consumed less time. Besides, there are 0, 9, 24, 12, and 10 subjects that these two

approaches performed nearly equally well, of which the percentage is not low.

In summary, we can conclude that, with respect to the time cost of generating the covering arrays, *single bottom-up* approach exhibits a significant advantage over all the remaining three approaches at all the testing strengths. *Collaborative bottom-up* also performs better than the other two *top-down* approaches at most of the subjects, but the reduction of time costs is not as much as that of *single bottom-up*. As for the two *top-down* approaches, there is not significant difference between them at the time cost.

The observation that *bottom-up* approaches generally needed less time to compute the incremental covering arrays than *top-down* approaches is very interesting. It indicates that the process that extracting lower covering is time-consuming. With respect to *bottom-up* approaches, the reason why *single bottom-up* performs more efficiently than *collaborative bottom-up* is mainly because *single bottom-up* does not need to generate the higher-way covering arrays in prior, while *collaborative bottom-up* needs to take some part of the test cases from higher-way covering array as the seeding test cases of the lower-way covering arrays.

#### C. RQ3. The RFD of the incremental covering array

The results are listed in Fig.6. In this figure, the y axis indicates the RFD of the corresponding covering arrays (which is also normalized between 0 to 1). Another difference from the previous two experiments is that in this figure, the larger the value of the y axis is, the better the corresponding approach performed.

The following could be observed.

- 1) Approach *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 54 out of 55 subjects and all the 55 subjects, respectively, for 2-way covering arrays. Among these subjects, *single bottom-up* reduced the size of covering arrays by 22%, and 77% on average (not shown in this figure), respectively. Approach *collaborative bottom-up* outperformed these two top-down approaches at 53 and 55 subjects, respectively, for 2-way covering arrays. The reductions on the size of covering arrays made by *collaborative bottom-up* are around 18% and 76%, respectively. For 3-way covering strength, the advantages of *bottom-up* approaches over *top-down* approaches slightly decreased. Specifically, approach *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 45, and 47 subjects, respectively. The corresponding reductions on the size of covering arrays are around 16% and 69% on average. Approach *collaborative bottom-up* also outperformed *CB top-down* and *HDB top-down* at 45, and 47 subjects, respectively. The corresponding reductions on the size of covering arrays are around 2% and 69% on average.
- 2) For the highest covering strength, i.e., 6-way, approach *CB top-down* outperformed both *single bottom-up* and *HDB top-down* at all the 10 subjects (only 10 subjects can be tested under 6-way testing strength).

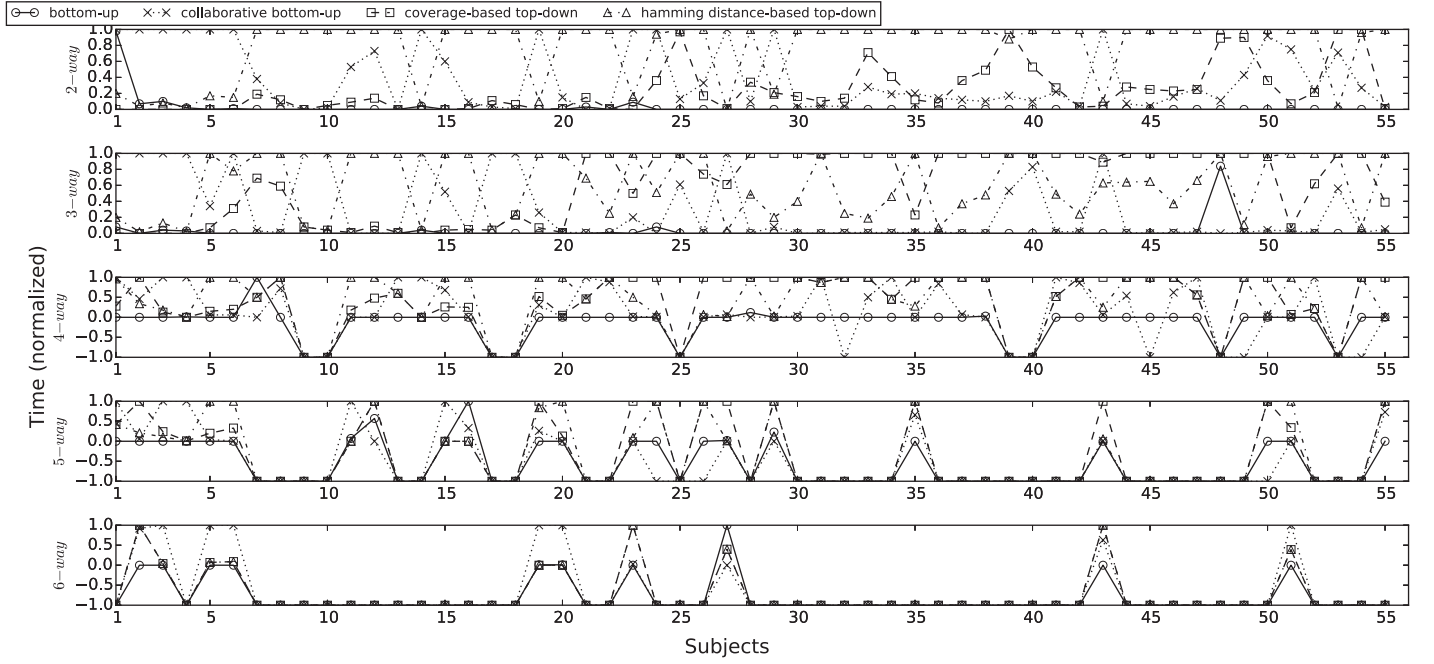


Fig. 5: The comparison of the four approaches with respect to the time cost

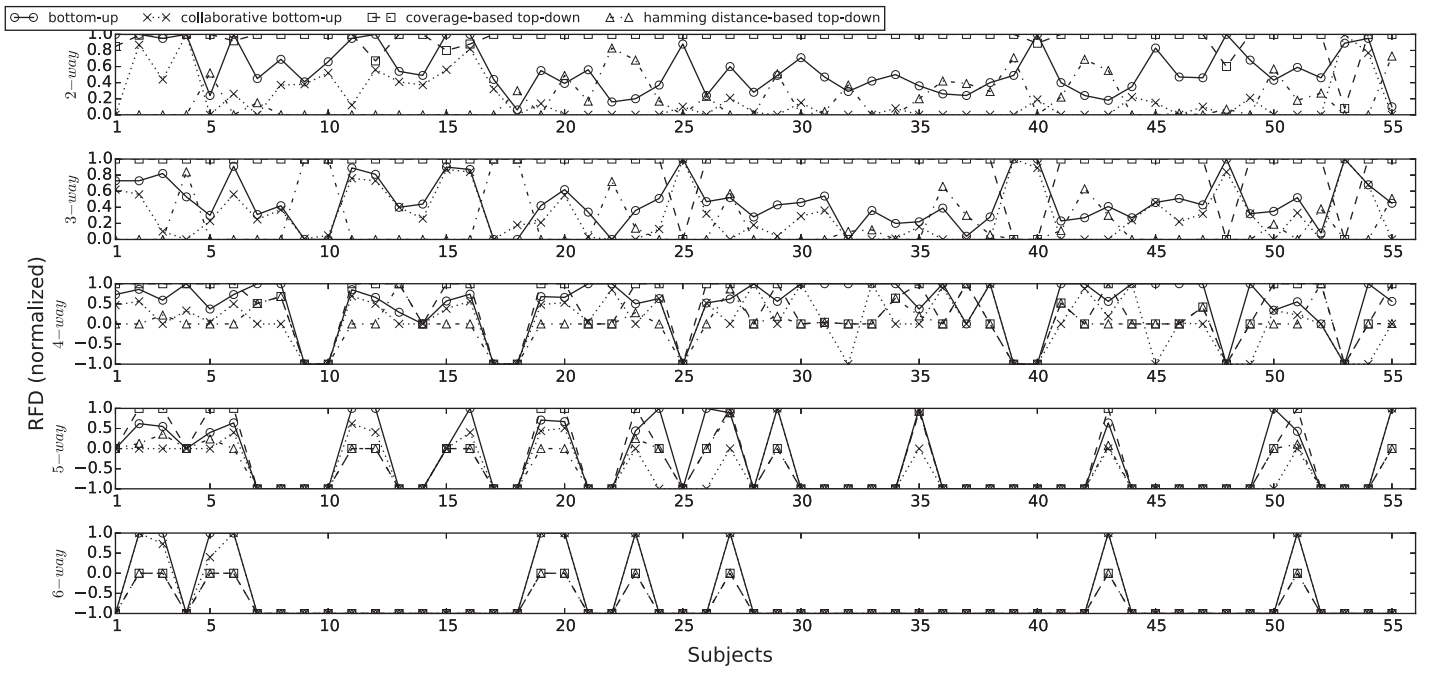


Fig. 6: The comparison of the four approaches with respect to the RFD

Among these subjects, *CB top-down* reduced by 3%, and 4%, on average, respectively, on the size of the covering array. The *HDB top-down* obtained the same results as approach *CB top-down* at this testing strength. This is because they share the same covering array at the highest covering strength (the difference between the two top-down approaches is their different extracting methods for generating lower-way covering array).

- 3) For the medium testing strength, i.e., 4 and 5, top-down approaches and bottom-up approaches performed neck and neck on the size of the covering array. Specifically, for 4-way covering arrays, *single bottom-up* outperformed *CB top-down* and *HDB top-down* at 22 and 26 subjects, respectively, and *collaborative bottom-up* outperformed *CB top-down* and *HDB top-down* at 21 and 24 subjects. On the other hand, *CB top-down* outperformed *single bottom-up* and *collaborative bottom-up* at 22 and 21 subjects, respectively, and *HDB top-down* outperformed these two bottom-up approaches at 20 and 16 subjects, respectively. Among these subjects, the extent to which the reductions of the size of covering arrays made by these approaches against each other are also similar (around 2% to 5%). Approach *HDB top-down* was an exception, other 3 approaches reduced about by 30% to 50% on the size of the covering arrays when compared to its generated covering arrays. The results of 5-way covering strength matched well with those of 4-way covering strength.
- 4) When comparing the two approaches that both belong to *bottom-up* strategy, approach *single bottom-up* slightly performed better than *collaborative bottom-up* at almost all the testing strengths. Specifically, there are 45, 24, 21, 14, and 1 subjects, on which approach *single bottom-up* generated smaller size of covering array than *collaborative bottom-up* for testing strength 2, 3, 4, 5, and 6, respectively, while there are 6, 21, 7, 1, and 1 subjects, on which *collaborative bottom-up* worked better. The specific reductions on the size of covering arrays made by these two approaches are 1% to 6%.
- 5) When comparing the two approaches that both belong to *top-down* strategy, approach *CB top-down* exhibited a significant advantage over *HDB top-down*. Specifically, there are 55, 44, 21, 10, and 0 subjects, on which approach *CB top-down* generated smaller size of covering arrays when compared to *HDB top-down* for testing strength 2, 3, 4, 5, and 6, respectively, while there are no subjects on which *HDB top-down* worked better than *CB top-down* for all the testing strengths. More specifically, among these subjects, approach *CB top-down* reduced the size of the covering arrays by 46% and 71% when compared to *HDB top-down*.

In summary, we can conclude that, with respect to the size of the covering array, *Bottom-up* strategy has a significant advantage over *Top-down* strategy at low testing strengths, while *Top-down* strategy works better at the high testing strength.

Additionally, different implementations of the strategy also affect on the results. Specifically, *single bottom-up* performs slightly better than *collaborative bottom-up*, while *CB top-down* exhibits significant advantage over *HDB top-down*.

It is as expected that *Bottom-up* and *Top-down* strategies both have their ups and downs, considering that *Bottom-up* makes the lowest-way covering array generation a priority (using specialized covering array generation approach for the 2-way covering array), while *Top-down* makes the highest-way covering array generation a priority. With respect to *Bottom-up* strategy, the reason that approach *single bottom-up* slightly performs better than *collaborative bottom-up* is that the latter approach considers more aspects (un-deterministic failures reappearance) such that it cannot only focus on the size of the covering arrays. The observation that *CB top-down* performs much better *HDB top-down* is interesting, which implies that the methods for extracting lower-way covering arrays from higher-way covering arrays are important for the size of tests.

## VIII. THREATS TO VALIDITY

There are several threats to validity in our studies.

First, our incremental covering array generation approach is implemented by using IPOG. Although IPOG can efficiently generate higher-way covering array for the SUT with large input space, it may generate too many test cases (due to the greedy strategy). It is necessary to try more covering array generation approaches to make the conclusion more general.

Second, the method to extract lower-way covering arrays from higher coverings is two greedy algorithms in this paper. This may result in that the size of the lower-way covering arrays generated by *Top-down* strategy may be larger than needed. If adopted some other methods, e.g., the post-optimal approach [20], [21], or alternatively heuristic searching methods, the number of test cases generated by *Top-down* strategy may be significantly decreased.

At last, our results are based on the 55 wildly used input models. That is, with other input models, the results may be different from this paper. It is appealing to further study the influence of the characteristics of the input models, e.g., the number of parameters, and the constraints, on the results of incremental covering array generation approach.

## IX. RELATED WORK

Combinatorial testing has been proven to a effective testing technique in practice [22], especially on domains like configuration testing [23], [24], [25] and software inputs testing [4], [26], [27]. The most important task in CT is to generate covering arrays, which support the checking of various valid interactions of factors that influence the system under testing.

Nie et al. [1] gave a survey for combinatorial testing, in which the methods for generating covering arrays are classified. However, traditional static covering arrays can be hardly directly applied in practice, hence many adaptive methods are proposed.

Cohen et al. [28], [15] studied the constraints that can render some test cases in the covering array invalid. They proposed a SAT-based approach to avoid the impact. Nie et al. [29] proposed a model for adaptive CT, in which both the failure-inducing interactions and constraints can be dynamically detected and removed in the early test cases generation iteration. Further, the coverage strength of covering array can be adaptively changed as required.

S.Fouché et al. [3] proposed the incremental covering array, and gave a method to generate it. The method can be deemed as a special case of *bottom-up* strategy, with the only difference being that it used multiple lower-strength covering arrays instead only one in this paper to construct the higher-strength covering array. This is because their work needs to characterize the failure-inducing interactions in the covering array, in which multiple covering arrays can support a better diagnosis.

Calvagna et al. [30] proposed another work to generate higher-way in an iterative way. Different than our work, its main task to generate a covering array with a specific given degree. It builds a relationship between higher-way covering array with lower-covering array. Based on this, it can reduce the higher-way covering array generation problem to multiple lower covering array generation problem.

#### X. CONCLUSIONS AND FUTURE WORKS

Incremental covering array is an important object to make CT used in practice, because it makes CT more flexible, such that we do not need to give the specific test strength in prior. This paper formally defined the incremental covering array and gave a proof to its existence. Furthermore, this paper proposed two strategies, i.e., strategy *Bottom-up* and *Top-down* to generate the incremental covering array. *Bottom-up* strategy generates incremental covering array from low-degree to high degree sequently, while *Top-down* strategy first generates the highest-way covering array and then extract lower-way covering arrays.

For each strategy we offer two different implements, which forms four generation approaches in total for generating incremental covering arrays. Experimental results showed that both strategies have their own advantages; *Bottom-up* has an advantage over strategy *Top-down* at lower degrees with respect to the size of covering arrays, while strategy *Top-down* performs better at higher degrees. However, when concerning the fault detection rate, strategy *Top-down* gets higher scores at lower degrees, while *Bottom-up* is better at higher degrees. Additionally, different implementations of the two strategies also have influences of the results, especially for strategy *Top-down*, of which the implementation with considering uncovered interactions had a significant advantage over the implementation based on hamming distance.

As a future work, we will apply more covering array generation algorithms, to compare their performance at generating incremental covering arrays. Another interesting work is to combine the two strategies, so that we can first select a median coverage strength  $t$  and generate incremental covering arrays by *top-down* strategy. Then if the maximal-way covering

array is generated, we can use *bottom-up* strategy to generate further higher-strength covering arrays. We believe such a combination strategy may offer a better performance.

#### REFERENCES

- [1] C. Nie and H. Leung, "A survey of combinatorial testing," *ACM Computing Surveys (CSUR)*, vol. 43, no. 2, p. 11, 2011.
- [2] D. R. Kuhn and M. J. Reilly, "An investigation of the applicability of design of experiments to software testing," in *Software Engineering Workshop, 2002. Proceedings. 27th Annual NASA Goddard/IEEE*. IEEE, 2002, pp. 91–95.
- [3] S. Fouché, M. B. Cohen, and A. Porter, "Incremental covering array failure characterization in large configuration spaces," in *Proceedings of the eighteenth international symposium on Software testing and analysis*. ACM, 2009, pp. 177–188.
- [4] D. M. Cohen, S. R. Dalal, M. L. Fredman, and G. C. Patton, "The aetg system: An approach to testing based on combinatorial design," *Software Engineering, IEEE Transactions on*, vol. 23, no. 7, pp. 437–444, 1997.
- [5] Y. Lei, R. Kacker, D. R. Kuhn, V. Okun, and J. Lawrence, "Ipog: A general strategy for t-way software testing," in *Engineering of Computer-Based Systems, 2007. ECBS'07. 14th Annual IEEE International Conference and Workshops on the*. IEEE, 2007, pp. 549–556.
- [6] —, "Ipog/ipog-d: efficient test generation for multi-way combinatorial testing," *Software Testing, Verification and Reliability*, vol. 18, no. 3, pp. 125–148, 2008.
- [7] L. Yu, Y. Lei, M. Nourozborazjany, R. N. Kacker, and D. R. Kuhn, "An efficient algorithm for constraint handling in combinatorial test generation," in *Software Testing, Verification and Validation (ICST), 2013 IEEE Sixth International Conference on*. IEEE, 2013, pp. 242–251.
- [8] L. Yu, F. Duan, Y. Lei, R. N. Kacker, and D. R. Kuhn, "Combinatorial test generation for software product lines using minimum invalid tuples," in *High-Assurance Systems Engineering (HASE), 2014 IEEE 15th International Symposium on*. IEEE, 2014, pp. 65–72.
- [9] —, "Constraint handling in combinatorial test generation using forbidden tuples," in *Software Testing, Verification and Validation Workshops (ICSTW), 2015 IEEE Eighth International Conference on*. IEEE, 2015, pp. 1–9.
- [10] C. Nie, H. Wu, X. Niu, F.-C. Kuo, H. Leung, and C. J. Colbourn, "Combinatorial testing, random testing, and adaptive random testing for detecting interaction triggered failures," *Information and Software Technology*, vol. 62, pp. 198–213, 2015.
- [11] I. Segall, R. Tzoref-Brill, and E. Farchi, "Using binary decision diagrams for combinatorial test design," in *Proceedings of the 2011 International Symposium on Software Testing and Analysis*. ACM, 2011, pp. 254–264.
- [12] Y. Jia, M. B. Cohen, M. Harman, and J. Petke, "Learning combinatorial interaction test generation strategies using hyperheuristic search," in *Software Engineering (ICSE), 2015 IEEE/ACM 37th IEEE International Conference on*, vol. 1. IEEE, 2015, pp. 540–550.
- [13] E.-H. Choi, C. Artho, T. Kitamura, O. Mizuno, and A. Yamada, "Distance-integrated combinatorial testing," in *Software Reliability Engineering (ISSRE), 2016 IEEE 27th International Symposium on*. IEEE, 2016, pp. 93–104.
- [14] M. B. Cohen, M. B. Dwyer, and J. Shi, "Interaction testing of highly-configurable systems in the presence of constraints," in *Proceedings of the 2007 international symposium on Software testing and analysis*. ACM, 2007, pp. 129–139.
- [15] —, "Constructing interaction test suites for highly-configurable systems in the presence of constraints: A greedy approach," *Software Engineering, IEEE Transactions on*, vol. 34, no. 5, pp. 633–650, 2008.
- [16] D. R. Kuhn and V. Okun, "Pseudo-exhaustive testing for software," in *Software Engineering Workshop, 2006. SEW'06. 30th Annual IEEE/NASA*. IEEE, 2006, pp. 153–158.
- [17] B. J. Garvin, M. B. Cohen, and M. B. Dwyer, "An improved meta-heuristic search for constrained interaction testing," in *Search Based Software Engineering, 2009 1st International Symposium on*. IEEE, 2009, pp. 13–22.
- [18] —, "Evaluating improvements to a meta-heuristic search for constrained interaction testing," *Empirical Software Engineering*, vol. 16, no. 1, pp. 61–102, 2011.

- [19] J. Lin, C. Luo, S. Cai, K. Su, D. Hao, and L. Zhang, "Tca: An efficient two-mode meta-heuristic algorithm for combinatorial test generation (t)," in *Automated Software Engineering (ASE), 2015 30th IEEE/ACM International Conference on*. IEEE, 2015, pp. 494–505.
- [20] X. Li, Z. Dong, H. Wu, C. Nie, and K.-Y. Cai, "Refining a randomized post-optimization method for covering arrays," in *Software Testing, Verification and Validation Workshops (ICSTW), 2014 IEEE Seventh International Conference on*. IEEE, 2014, pp. 143–152.
- [21] P. Nayeri, C. J. Colbourn, and G. Konjevod, "Randomized post-optimization of covering arrays," *European Journal of Combinatorics*, vol. 34, no. 1, pp. 91–103, 2013.
- [22] D. R. Kuhn, R. N. Kacker, and Y. Lei, "Practical combinatorial testing," *NIST Special Publication*, vol. 800, p. 142, 2010.
- [23] C. Yilmaz, M. B. Cohen, and A. A. Porter, "Covering arrays for efficient fault characterization in complex configuration spaces," *Software Engineering, IEEE Transactions on*, vol. 32, no. 1, pp. 20–34, 2006.
- [24] M. B. Cohen, J. Snyder, and G. Rothermel, "Testing across configurations: implications for combinatorial testing," *ACM SIGSOFT Software Engineering Notes*, vol. 31, no. 6, pp. 1–9, 2006.
- [25] X. Qu, M. B. Cohen, and G. Rothermel, "Configuration-aware regression testing: an empirical study of sampling and prioritization," in *Proceedings of the 2008 international symposium on Software testing and analysis*. ACM, 2008, pp. 75–86.
- [26] M. N. Borazjany, L. Yu, Y. Lei, R. Kacker, and R. Kuhn, "Combinatorial testing of acts: A case study," in *Software Testing, Verification and Validation (ICST), 2012 IEEE Fifth International Conference on*. IEEE, 2012, pp. 591–600.
- [27] L. S. G. Ghandehari, M. N. Bourazjany, Y. Lei, R. N. Kacker, and D. R. Kuhn, "Applying combinatorial testing to the siemens suite," in *Software Testing, Verification and Validation Workshops (ICSTW), 2013 IEEE Sixth International Conference on*. IEEE, 2013, pp. 362–371.
- [28] M. B. Cohen, M. B. Dwyer, and J. Shi, "Exploiting constraint solving history to construct interaction test suites," in *Testing: Academic and Industrial Conference Practice and Research Techniques-MUTATION, 2007*. IEEE, 2007, pp. 121–132.
- [29] C. Nie, H. Leung, and K.-Y. Cai, "Adaptive combinatorial testing," in *Quality Software (QSIC), 2013 13th International Conference on*. IEEE, 2013, pp. 284–287.
- [30] A. Calvagna and E. Tramontana, "Incrementally applicable t-wise combinatorial test suites for high-strength interaction testing," in *Computer Software and Applications Conference Workshops (COMPSACW), 2013 IEEE 37th Annual*. IEEE, 2013, pp. 77–82.