Identifying minimal failure-causing schemas for multiple failures

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Combinatorial testing(CT) has been proven to be effective to reveal the potential failures caused by the interaction of the inputs or options of the System Under Test (SUT). To extend and fully use CT, the theory of Minimal Failure-Causing Schema (MFS) has been proposed. The use of MFS helps to isolate the root cause of a failure after detected by CT. Most MFS-based algorithms focus on handling a single failure in the SUT. However, we argue that multiple failures are the more common testing scenario, and under which masking effects may be triggered so that some expected failures will not be observed. Traditional MFS theory, as well as the related identifying algorithms, lack a mechanism to handle such effects; hence, they may incorrectly isolate the MFS in the SUT. To address this problem, we propose a new MFS model that takes into account multiple failures. We first formally analyse the impact of the multiple failures on MFS isolating algorithms, especially in situations where masking effects are triggered by these multiple failures. Based on this, we then develop an approach that can assist traditional algorithms to better handle multiple failure testing scenarios. Empirical studies were conducted using several kinds of open-source software, which showed that multiple failures with masking effects do negatively affect traditional MFS identifying approaches and that our approach can help to alleviate these effects.

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1. INTRODUCTION

With the increasing complexity and size of modern software, many factors, such as input parameters and configuration options, can affect the behaviour of the SUT. The unexpected failures caused by the interaction of these factors can make software testing challenging, especially when the interaction space is large. In the worst case, we need to examine every possible combination of these factors as each combination can contain unique failure [Song et al. 2012]. While conducting exhaustive testing is ide-

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Table I. MS word example

id	*Highlight*	*Status bar*	*Bookmarks*	*Smart tags*	**Outcome**
1	On	On	On	On	PASS
2	Off	Off	On	On	PASS
3	Off	On	Off	Off	Fail
4	On	Off	Off	On	PASS
5	On	Off	On	Off	PASS

al and necessary in theory, it is impractical and uneconomical. One remedy for this problem is combinatorial testing, which systematically samples the interaction space and selects a relatively small set of test cases that cover all valid iterations, with the number of factors involved in the interaction no more than a prior fixed integer, i.e., the *strength* of the interaction. Many works in CT aim to construct the smallest set of efficient testing objects [Cohen et al. 1997; Bryce et al. 2005; Cohen et al. 2003], which is also called *covering array*.

Once failures are detected by the covering array, the failure-inducing combinations in these failing test cases must be isolated. This task is important in CT as it can facilitate debugging efforts by reducing the code scope that needed for inspection [Ghandehari et al. 2012]. However, information from the covering array sometimes does not clearly identify the location and magnitude of the failure-inducing combinations [Colbourn and McClary 2008]. Thus, deeper analysis is needed. Consider the following example [Bach and Schroeder 2004], Table I presents a two-way covering array for testing an MS-Word application in which we want to examine various combinations of options for the MS-Word 'Highlight', 'Status Bar', 'Bookmarks' and 'Smart tags'. Assume the third test case failed. We then can get five two-way suspicious combinations that may be responsible for this failure. They are respectively (Highlight: Off, Status Bar: On), (Highlight: Off, Bookmarks: Off), (Highlight: Off, Smart tags: Off), (Status Bar: On, Bookmarks: Off), (Status Bar: On, Smart tags: Off), and (Bookmarks: Off, Smart tags: Off). Without additional information, we cannot figure out the specific combinations in this suspicious set caused the failure. In fact, taking into account the higher strength combination, e.g., (Highlight: Off, Status Bar: On, Smart tags: Off), the problem becomes more complicated.

To address this problem, prior work [Nie and Leung 2011a] specifically studied the properties of the minimal failure-causing schemas in SUT, based on which a further diagnosis by generating additional test cases was applied that can identify the MFS in the test case. Other works have proposed ways to identify the MFS in SUT, which include approaches such as building a tree model [Yilmaz et al. 2006], generating additional test cases by mutating one factor of the original failing test case at a time [Nie and Leung 2011a], ranking suspicious combinations based on some rules [Ghandehari et al. 2012], using graphic-based deduction [Martínez et al. 2008], among others.

Nie's approach as well as other MFS-identifying approaches mainly focus on the ideal scenario in which SUT only contains one failure, i.e., the test case either fails or passes the testing. However, in this paper, we argue that SUT with multiple distinguished failures is the more common testing scenario in practice, and moreover, this impacts the Failure-inducing Combinations Identifying(FCI) approaches. One main impact of multiple failures on FCI approaches is the masking effect. A masking effect [Dumlu et al. 2011; Yilmaz et al. 2013] happens on the execution of a test case, such that it will be terminated unexpectedly with the removing part of the test case unchecked. Take the Linux command *Grep* for example. We noticed that there are two different failures reported in the bug tracker system. The first [1] claims that Grep

[1] http://savannah.gnu.org/bugs/?29537

incorrectly matches unicode patterns with '\<\>', while the second [2] claims an incompatibility between option '-c' and '-o'. When we put these two scenarios into one test case, only one failure will be observed, which means the other failure is masked by the observed failure. This effect will prevent test cases from executing normally, resulting in incorrect judgments of the correlation between the combinations checked in the test case and the failure that been masked and therefore not observed. This effect was firstly noted by Dumlu and Yilmaz in [Dumlu et al. 2011], in which they found that the masking effects in CT can make traditional covering array fail to detect some combinations and they proposed a feedback-driven approach to work around them. Their recent work [Yilmaz et al. 2013] further empirically studied the impacts on the Failure-inducing Combinations Identifying(FCI) approach : Classification Tree Approach(CTA) [Yilmaz et al. 2006], of which CTA has two versions, i.e., ternary-class and multiple-class.

As we know that masking effects negatively affect the performance of FCI approaches, a natural question is how this effect biases the results of these approaches. In this paper, we formalize the process of identifying the MFS under the circumstances in which masking effects exist in the SUT and try to answer this question. One insight from the formal analysis is that we cannot completely get away from the impact of masking effects even if we do exhaustive testing. Even worse, either ignore the masking effects or regard multiple failures as one failure is detrimental to the FCI process.

Based on this concern, we propose a strategy to alleviate this impact. This strategy adopts the divide and conquer framework, i.e., separately handles each failure in the SUT. For a particular failure under analysis, when applying traditional FCI approaches to identify the failure-inducing combinations, we pick the test cases generated by FCI approaches that trigger unexpected failures and replace them with newly regenerated test cases. These new test cases should either pass or trigger the same failure under analysis.

The key to our approach is to search for a test case that does not trigger unexpected failures which may introduce the masking effect. To guide the search process, i.e., to reduce the possibility that the extra generated test case will trigger an unexpected failure, a natural idea is to take some characteristics from the existing test cases and make the characteristics of the newly searched test case as different as possible from the existing test cases which triggered the unexpected failure. To reach this target, we define the *related strength* between the factor and the failure. The higher the *related strength* between a factor and a particular failure, the greater the likelihood that the factor will trigger this failure. We then use the integer linear programming (ILP) technique to find a test case which has the least *related strength* with the unexpected failure.

To evaluate the performance of our approach, we applied our strategy on the FCI approach FIC_BS [Zhang and Zhang 2011]. The subjects we used were several open-source software systems found in the developers' forum in the Source-Forge community. Through studying their bug reports in the bug tracker system as well as their user's manuals, we built a testing model which can reproduce the reported bugs with specific test cases. We then compared the FCI approach augmented with our strategy to the traditional FCI approach with these subjects. We further empirically studied the performance of the important component of our strategy – searching satisfied test cases. To conduct this study, we compare our approach with the augmented FCI approach by randomly searching satisfied test cases. We finally compared our approach with the existing masking handling technique – FDA-CIT [Yilmaz et al. 2013]. All of

[2]http://savannah.gnu.org/bugs/?33080

```
public float foo(int a, int b, int c, int d){
  //step 1 will cause an exception when b == c
  float x = (float)a / (b - c);

  //step 2 will cause an exception when c < d
  float y = Math.sqrt(c - d);

  return x+y;
  }
```

Fig. 1. A toy program with four input parameters

these empirical studies showed that our replacing strategy as well as the searching test case component achieved a better performance than these traditional approaches when the subject suffered multiple failures, especially when these failures can import masking effects.

The main contributions of this paper are:

— We studied the impact of the masking effects among multiple failures on the isolation of the failure-inducing combinations in SUT.
— We proposed a divide and conquer strategy of selecting test cases to alleviate the impact of these effects.
— We designed an efficient test case searching method which can efficiently find a test case that does not trigger an unexpected failure.
— We conducted several empirical studies and showed that our strategy can assist FCI approaches to achieve better performance in identifying failure-inducing combinations in SUT with masking effects.

2. MOTIVATING EXAMPLE

For convenience, we constructed a small program example to illustrate the motivation of our approach. Assume we have a method *foo* which has four input parameters: *a, b, c,* and d. The four parameter types are all integers and the values that they can take are: $v_a = \{7, 11\}, v_b = \{2, 4, 5\}, v_c = \{4, 6\}, v_d = \{3, 5\}$. The code detail of *foo* is shown in Figure 1.

Considering the simple code in Figure 1, we can find two potential failures: first, in step 1 we can get an *Arithmetic Exception* when b is equal to c, i.e., $b = 4$ and $c = 4$, that causes a division by zero. Second, another *Arithmetic Exception* will be triggered in step 2 when $c < d$, i.e., $c = 4$ and $d = 5$, which results in a square root of negative number. So the expected failure-inducing combinations in this example should be (-, 4, 4, -) and (-, -, 4, 5).

Traditional FCI algorithms do not consider the code detail; instead, they apply black-box testing, i.e., feed inputs to the programs and execute them to observe the result. The basic justification behind those approaches is that the failure-inducing combinations for a particular failure can only appear in those inputs that trigger this failure. Traditional FCI approaches aim at using as few inputs as possible to get the same (or approximate) result as exhaustive testing, so the results derived from an exhaustive testing set are the best that these FCI approaches can achieve. Next, we will show how exhaustive testing works to identify the failure-inducing combinations in the program.

We first generate every possible input listed in the column "test inputs" of Table II, and the execution results are listed in the result column of Table II. In this column, *PASS* means that the program runs without any exception under the input in the same row. *Ex 1* indicates that the program triggered an exception corresponding to

Table II. test inputs and their corresponding result

id	test inputs	results	id	test inputs	result
1	(7, 2, 4, 3)	PASS	13	(11, 2, 4, 3)	PASS
2	(7, 2, 4, 5)	Ex 2	14	(11, 2, 4, 5)	Ex 2
3	(7, 2, 6, 3)	PASS	15	(11, 2, 6, 3)	PASS
4	(7, 2, 6, 5)	PASS	16	(11, 2, 6, 5)	PASS
5	(7, 4, 4, 3)	Ex 1	17	(11, 4, 4, 3)	Ex 1
6	(7, 4, 4, 5)	Ex 1	18	(11, 4, 4, 5)	Ex 1
7	(7, 4, 6, 3)	PASS	19	(11, 4, 6, 3)	PASS
8	(7, 4, 6, 5)	PASS	20	(11, 4, 6, 5)	PASS
9	(7, 5, 4, 3)	PASS	21	(11, 5, 4, 3)	PASS
10	(7, 5, 4, 5)	Ex 2	22	(11, 5, 4, 5)	Ex 2
11	(7, 5, 6, 3)	PASS	23	(11, 5, 6, 3)	PASS
12	(7, 5, 6, 5)	PASS	24	(11, 5, 6, 5)	PASS

Table III. Identified failure-inducing combinations and their corresponding Exception

Failure-inducing combinations	Exception
(-, 4, 4, -)	Ex 1
(-, 2, 4, 5)	Ex 2
(-, 5, 4, 5)	Ex 2

step 1 and *Ex 2* indicates the program triggered an exception corresponding to step 2. From the data listed in Table II, we can determine that (-, 4 , 4, -) must be the failure-inducing combination of Ex 1 as all the inputs that triggered Ex 1 contain this combination. Similarly, the combination (-, 2, 4, 5) and (-, 5, 4, 5) must be the failure-inducing combinations of Ex 2. We list these combinations and the corresponding exceptions in Table III.

Note that in this case we did not get the expected result with traditional FCI approaches. The failure-inducing combinations we got for Ex 2 are (-,2,4,5) and (-,5,4,5), respectively, instead of the expected combination (-,-,4,5). So why did we fail to get the (-,-,4,5)? The reason lies in *input 6* (7,4,4,5) and *input 18* (11,4,4,5). These two inputs contain the combination (-,-,4,5), but they didn't trigger Ex 2; instead, Ex 1 was triggered.

Now let us get back to the source code of *foo*. We can find that if Ex 1 is triggered, it will stop executing the remaining code and report the exception information. In other word, Ex 1 has a higher failure level than Ex 2, so it may mask Ex 2. Let us re-examine the combination (-,-,4,5). If we suppose that *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1, then we can conclude that (-,-,4,5) should be the failure-inducing combination of the Ex 2, which is identical to the expected one.

Unless we fix the code that triggers Ex 1 and re-execute all the test cases, we cannot validate the supposition that *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1. So in practice, when we do not have enough resources to execute all the test cases repeatedly or can only do black-box testing, a more economical and efficient approach to alleviate the masking effect on FCI approaches is desired.

3. FORMAL MODEL

This section presents some definitions and propositions for a formal model to solve the FCI problem.

3.1. Failure-causing Schemas in CT

Assume that the SUT is influenced by k parameters, and each parameter p_i has a_i discrete values from the finite set V_i, i.e., $a_i = |V_i|$ (i = 1,2,..k). Some of the definitions below were originally defined in [Nie and Leung 2011b].

Definition 3.1. A *test case* of the SUT is an array of k values, one for each parameter of the SUT, which is denoted as a k-tuple (v_1, v_2,...,v_k), where $v_1 \in V_1$, $v_2 \in V_2$ ... $v_k \in V_k$.

In practice, these parameters in the test case can represent many factors, such as input variables, run-time options, building options, or various combinations of them. We need to execute the SUT with these test cases to ensure the correctness of the software behaviour.

Definition 3.2. A *failure* is the abnormal execution of a test case.

In CT, such a *failure* can be a thrown exception, compilation error, assertion failure or constraint violation. In this paper, we focus on studying the impact of the multiple *failures* on the CT fault diagnosis process, and to facilitate our discussion, we introduce the following assumptions that will be used throughout this paper:

ASSUMPTION 1. *The execution result of a given test case is deterministic.*

This assumption is the most common assumption of CT fault diagnosis. It indicates that outcome of executing a test case is reproducible and will not be affected by some unexpected random events.

ASSUMPTION 2. *Different failures in the SUT can be distinguished by various information such as exception traces, state conditions, or the like.*

This assumption indicates that the testers can detect different failures during testing. As different failures will complicate the testing task, distinguishing them is the first step to handle them.

In particle, these two assumptions will not always be satisfied. Elimination of them can introduce the probabilistic and clustering parts into our model, which will further complicate our analysis and so we will not discuss them in this paper. Now let us consider the condition that some failures are triggered by some test cases. It is then desired to determine the cause of these failures and hence some subsets of the test case must be analysed.

Definition 3.3. For the SUT, the t-tuple (-,v_{k_1},...,v_{k_t},...) is called a t-degree *schema* ($0 < t \leq k$) when some t parameters have fixed values and the others can take on their respective allowable values, represented as "-".

In effect a test case itself is a t-degree *schema*, when t = k. Furthermore, if every fixed value in a schema is in a test case, we say this test case *contains* the schema.

For example, (-, 4, 4, -) in Table III is a two-degree schema. And the test case (7, 4 ,4, 3) contains this schema.

Definition 3.4. Let c_l be a l-degree schema, c_m be an m-degree schema in SUT, and $l < m$. If all the fixed parameter values in c_l are also in c_m, then c_m *subsumes* c_l. In this case, we can also say that c_l is a *sub-schema* of c_m, and c_m is a *parent-schema* of c_l, which can be denoted as $c_l \prec c_m$.

For example, in the motivation example, the two-degree schema (-, 4, 4, -) is a sub-schema of the three-degree schema (-, 4, 4, 5), that is, (-,4,4,-) $\prec$ (-,4,4,5).

Definition 3.5. If all test cases contain a schema, say c, and trigger a particular failure, say F, then we call this schema c the *failure-causing schema* for F. Additionally, if none of the sub-schema of c is the *failure-causing schema* for F, we then call the schema c the *Minimal Failure-causing Schema*, i.e., the MFS for F.

In fact, MFS is identical to the failure-inducing combinations we discussed previously. Figuring this out can eliminate all details that are irrelevant to the cause of the failure and, hence, facilitate the debugging efforts.

Some notations used later are listed below for convenient reference:

— k : the number of parameters that influence the SUT.
— V_i : the set of discrete values that the ith parameter of the SUT can take.
— T^* : The exhaustive set of test cases for the SUT. For a SUT with k parameters, and each parameter can take $|V_i|$ values, the number of this set of test cases T^* is $\prod_{i=1}^{i<=k} |V_i|$.
— $A \backslash B$: the set of elements that belong to set A but not to B. For example $T_i \backslash T_j$ indicates the set of test cases that belong to set T_i, but not to T_j.
— L : the number of failures contained in the SUT.
— F_m : the mth failure in the SUT; for different failures, we can differentiate them from their exception traces or other buggy information.
— T_{F_m} : All the test cases that can trigger the failure F_m.
— $\mathcal{T}(c)$: All the test cases that can contain the schema c. Based on the definition of MFS, we know that if schema c is MFS for F_m, then $\mathcal{T}(c)$ must be subsumed in T_{F_m}.
— $\mathcal{I}(t)$: All the schemas that are contained in the test case t, e.g., $\mathcal{I}(\ (111)\ ) = \{(1--)(-1-)(--1)(11-)(1-1)(-11)(111)\}$.
— $\mathcal{I}(T)$: All the schemas that are contained in a set of test cases T, i.e., $\mathcal{I}(T) = \bigcup_{t \in T} \mathcal{I}(t)$.
— $\mathcal{S}(T)$: All the schemas that are only contained in the set of test cases. It is important to note that this set is different from $\mathcal{I}(T)$, as the schemas contained by the test cases in T can also be contained by other test cases that do not belong to this set. In fact, $\mathcal{S}(T)$ is computed by $\{c | c \in \mathcal{I}(T)\ and\ c\ \notin \mathcal{I}(T^* \backslash T\}$.
— $\mathcal{C}(T)$: the minimal schemas that are only contained by the set of test cases T. This set is the sub-set of $\mathcal{S}(T)$, which is defined as $\{c | c \in \mathcal{S}(T)\ and\ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$.

PROPOSITION 3.6 (SUB SCHEMAS HAVE A LARGER SET OF TEST CASES). *For l-degree schema c_l and m-degree schema c_m, if $c_l \prec c_m$, then all the test cases that contain c_m must also contain c_l, i.e., $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$.*

PROOF. For $\forall t \in \mathcal{T}(c_m)$, we have t contains c_m. Then as $c_l \prec c_m$, we must also have t contains c_l. This is because all the elements in c_l are also in c_m, which are contained in the test case t. Therefore, we get $t \in \mathcal{T}(c_l)$. Thus $t \in \mathcal{T}(c_m)$ implies $t \in \mathcal{T}(c_l)$, so it follows that $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$. □

Table IV illustrates an example of the SUT with four binary parameters (unless otherwise specified, the following examples also assume a SUT with binary parameters). The left column lists the schema (0,0,-,-) as well as all the test cases that contain this schema, while the right column lists the test cases for schema (0,-,-,-). We can observe that (0,-,-,-) $\prec$ (0,0,-,-), and the set of test cases which contain (0,-,-,-) includes the set of test cases that contain (0,0,-,-).

PROPOSITION 3.7 (ANY SET OF TEST CASES HAS ITS OWN MINIMAL SCHEMAS). *For any set T of test cases of a SUT, we can always get a set of minimal schemas $\mathcal{C}(T)$ = $\{c |\ \nexists c' \in \mathcal{C}(T), s.t. c' \prec c\}$, such that,*

$$T = \bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c)$$

PROOF. We prove this by producing this set of schemas.

Table IV. Example of Proposition 3.6

c
(0, 0, - , -)
$\mathcal{T}(c)$
(0, 0, 0 ,0)
(0, 0, 0 ,1)
(0, 0, 1 ,0)
(0, 0, 1 ,1)

c
(0, -, - , -)
$\mathcal{T}(c)$
(0, 0, 0 ,0)
(0, 0, 0 ,1)
(0, 0, 1 ,0)
(0, 0, 1 ,1)
(0, 1, 0 ,0)
(0, 1, 0 ,1)
(0, 1, 1 ,0)
(0, 1, 1 ,1)

We have denoted the exhaustive test cases for SUT as T^* and let $T^*\backslash T$ be the test cases that are in T^* but not in T. Obviously for $\forall t \in T$, we can always find at least one schema which is contained in t, i.e., we can find $c \in \mathcal{I}(t)$, such that $c \notin \mathcal{I}(T^*\backslash T)$. Specifically, at least the test case t itself as schema holds.

Then we collect all the satisfied schemas which are only contained by the test cases in the test cases of T, which can be denoted as: $\mathcal{S}(T) = \{c | c \in \mathcal{I}(T)\ and\ c \notin \mathcal{I}(T^*\backslash T)\}$.

For the schemas in $\mathcal{S}(T)$, we can have $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) = T$. This is because first, for $\forall t \in \mathcal{T}(c), c \in \mathcal{S}(T)$, it must have $t \in T$. This is because if not so, then $t \in T^*\backslash T$, which contradicts with the definition of $\mathcal{S}(T)$. So $t \in T$. Hence, $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset T$.

Then second, for any test case t in T, as we have learned at least one c' in $\mathcal{I}(t)$, such that c' in $\mathcal{S}(T)$ (The t itself as a schema holds). In another word, the test case t contains the schema c', which implies $t \in \mathcal{T}(c'), c' \in \mathcal{S}(T)$. And obviously $\mathcal{T}(c') \subset \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$, so $t \in \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$, therefore, $T \subset \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$.

Since $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset T$ and $T \subset \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$, so it follows $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) = T$.

Then we denote the minimal schemas of $\mathcal{S}(T)$ as $M(\mathcal{S}(T)) = \{c | c \in \mathcal{S}(T)\ and\ \ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$. For this set, we can still have $\bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c) = T$. We also prove this by two steps, first and obviously, $\bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c) \subset \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$. Then we just need to prove that $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$.

In fact by definition of $M(\mathcal{S}(T))$, for $\forall c' \in \mathcal{S}(T)\backslash M(\mathcal{S}(T))$, we can have some $c \in M(\mathcal{S}(T))$, such that $c \prec c'$. According to the Proposition 3.6, $\mathcal{T}(c') \subset \mathcal{T}(c)$. So for any test case $t \in \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$, we have either $\exists c' \in \mathcal{S}(T)\backslash M(\mathcal{S}(T)), s.t., t \in \mathcal{T}(c')$ or $\exists c \in M(\mathcal{S}(T)), s.t., t \in \mathcal{T}(c)$. Both cases can deduce $t \in \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$. So, $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$.

Hence, $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) = \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$, and $M(\mathcal{S}(T))$ is the set of schemas that holds this proposition. □

For example, Table V lists the $\mathcal{S}(T)$ and minimal schemas $\mathcal{C}(T)$ for the set of test cases T. We can see that for any other schema not in $\mathcal{C}(T)$, either we can find a test case not in T that contains the schema, e.g.,(0,0,-,-) with the test case (0,0,1,1) not in T, or that is the parent schema of one of the two minimal schemas, e.g.,(0,0,0,0) is the parent schema of both (0,0,0,-) and (0,0,-,0).

Let T_{F_m} denotes the set of all the test cases triggering failure F_m , then $\mathcal{C}(T_{F_m})$ actually is the set of MFS of F_m by definition of MFS.

From the construction process of $\mathcal{C}(T)$, one observation is that the minimal schema set $\mathcal{C}(T)$ is the subset of the schema set $\mathcal{S}(T)$, i.e., $\mathcal{C}(T) \subset \mathcal{S}(T)$, and for any schema in $\mathcal{S}(T)$, it either belongs to $\mathcal{C}(T)$, or is the parent schema of one element of $\mathcal{C}(T)$. Then, we can have the following proposition.

Table V. Example of the minimal schemas

T	$\mathcal{S}(T)$	$\mathcal{C}(T)$
(0, 0, 0 ,0)	(0, 0, 0, 0)	(0, 0, 0, -)
(0, 0, 0 ,1)	(0, 0, 0, 1)	(0, 0, -, 0)
(0, 0, 1 ,0)	(0, 0, 1, 0)	
	(0, 0, 0, -)	
	(0, 0, -, 0)	

PROPOSITION 3.8 (SCHEMAS WILL BELONG TO SET $\mathcal{S}(T)$ IF THE TEST CASES ARE IN T). *For any test case set T and schema c, if every test case contains c is in the set T, i.e., $\mathcal{T}(c) \subset T$, then it must be that $c \in \mathcal{S}(T)$.*

PROOF. We consider $c \in \mathcal{C}(\mathcal{T}(c))$, this is obviously and in fact the minimal schemas for the test cases set $\mathcal{T}(c)$ only contain one schema, which is exactly c itself. As discussed previously we have $\mathcal{C}(\mathcal{T}(c)) \subset \mathcal{S}(\mathcal{T}(c))$, so it must be $c \in \mathcal{S}(\mathcal{T}(c))$.

Then as $\mathcal{T}(c) \subset T$, it follows that $\mathcal{S}(\mathcal{T}(c)) \subset \mathcal{S}(T)$ by definition. In detail, $\mathcal{S}(\mathcal{T}(c)) = \{c|c \in \mathcal{I}(\mathcal{T}(c)) \; and \; c \notin \mathcal{I}(T^* \backslash \mathcal{T}(c))\}$, so $\mathcal{S}(\mathcal{T}(c)) \subset \{c|c \in \mathcal{I}(T) \; and \; c \notin \mathcal{I}(T^* \backslash T)\}$, which is exactly $\mathcal{S}(T)$.

So as $c \in \mathcal{S}(\mathcal{T}(c))$ and hence $c \in \mathcal{S}(T)$. □

For two different sets of test cases, there exist some relationships between the minimal schemas of these two sets that varies in relevancy with respect to the two different sets of test cases. In fact, there are three possible associations between two different sets of test cases: *inclusion*, *disjointed*, and *intersection*, as shown in Figure 2. We did not show the condition for two sets that are identical, because for this case the minimal schemas must also be identical. The relationship of the minimal schemas between two different sets of test cases is important as the masking effects between multiple failures will make the MFS identifying process work incorrectly, i.e., these FCI approaches may isolate the minimal schemas for the set of test cases which are different from the expected failing set of test cases. And these properties can help to figure out the impact of masking effects on the FCI approaches. Next, we will separately discuss the relationship between minimal schemas under the three conditions.

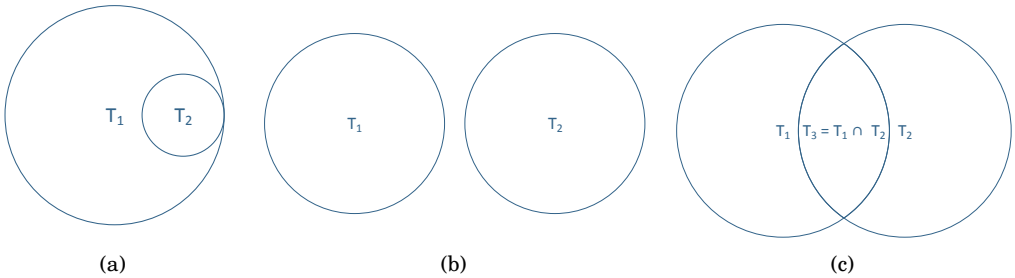


Fig. 2. Test suite relationships

3.2. Inclusion

It is the first relationship corresponding to Figure 2(a). We have the following proposition with two sets of test cases which have an inclusion relationship.

PROPOSITION 3.9 (SCHEMAS FROM THE SMALLER SET TEND TO BE PARENT SCHEMAS). *For two sets of test cases T_l and T_k, assume that $T_l \subset T_k$. Then for $\forall c_l \in \mathcal{C}(T_l)$ we have either $c_l \in \mathcal{C}(T_k)$ or $\exists c_k \in \mathcal{C}(T_k), s.t., c_k \prec c_l$.*

Table VI. Inclusion example

T_l	T_k
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(0, 1, 1)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, 0, -)	(0, -, -)
(0, -, 0)	

T_l	T_k
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(1, 1, 0)
	(1, 1, 1)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, 0, -)	(0, 0, -)
(0, -, 0)	(0, -, 0)
	(1, 1, -)

PROOF. Obviously $\forall c_l \in \mathcal{C}(T_l)$, we can get $\mathcal{T}(c_l) \subset T_l \subset T_k$. According to Proposition 3.8, we can have $c_l \in \mathcal{S}(T_k)$. So this proposition holds as the schema in $\mathcal{S}(T_k)$ either is also in $\mathcal{C}(T_k)$, or must be the parent of some schemas in $\mathcal{C}(T_k)$. □

Likewise, we can get the properties of the schemas identified for the larger set of test cases.

PROPOSITION 3.10 (SCHEMAS FROM THE LARGER SET TEND TO BE SUBSCHEMAS). *For two sets of test cases T_l and T_k, assume that $T_l \subset T_k$. Then for $\forall c_k \in \mathcal{C}(T_k)$ we have either (1) $c_k \in \mathcal{C}(T_l)$ or (2) $\exists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$ or (3) $\nexists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$ or $c_k = c_l$, or $c_l \prec c_k$.*

This proposition is exactly the antithesis of Proposition 3.9. We need to note the third condition, i.e., $\nexists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$ *or* $c_k = c_l$, *or* $c_l \prec c_k$. We refer to this condition as c_k is *irrelevant* to $\mathcal{C}(T_l)$. Furthermore, we can also say a schema is *irrelevant* to another schema if these two schemas are neither identical nor subsuming each other.

We illustrate these scenarios in Table VI. There are two parts in this table, with each part showing two sets of test cases: T_l and T_k, which have $T_l \subset T_k$. For the left part, we can see that in the schema in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0), both are the parent-schemas of the one in $\mathcal{C}(T_k)$:(0,-,-). While for the right part, the schemas in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0) are both also in $\mathcal{C}(T_k)$. Furthermore, one schema in $\mathcal{C}(T_k)$: (1,1,-) is irrelevant to $\mathcal{C}(T_l)$.

3.3. Disjoint

This relationship corresponds to Figure 2(b). For two different sets of test cases, one obvious property is listed as follows:

PROPOSITION 3.11 (NO RELATIONSHIPS BETWEEN THE SCHEMAS). *For two test cases set T_1 and T_2, if $T_1 \bigcap T_2 = \emptyset$, we have, $\mathcal{S}(T_1) \bigcap \mathcal{S}(T_2) = \emptyset$.*

PROOF. Assume that $\mathcal{S}(T_1) \bigcap \mathcal{S}(T_2) \neq \emptyset$. Without loss of generality, we let $c \in \mathcal{S}(T_1) \bigcap \mathcal{S}(T_2)$ we can show that $\mathcal{T}(c)$ must both in T_1 and T_2, which is contradiction. □

This property indicates that the minimal schemas of two disjointed test cases should be irrelevant to each other. Table VII shows an example of this scenario. We can learn from this table that for two different test case sets T_l and T_k, their minimal schemas, i.e., (0, - , 0) and (1, 0, -),(1, -, 0), respectively, are irrelevant to each other.

3.4. Intersect

This relationship corresponds to Figure 2(c). This scenario is the most common scenario for two given sets of test cases, but is also the most complicated scenario for analysis. To conveniently illustrate the properties of the minimal schemas of this scenario, we assume that $T_1 \bigcap T_2 = T_3$ as depicted in Figure 2(c). Then, we have the following properties:

Table VII. Disjoint example

T_l	T_k
(0, 0, 0)	(1, 0, 0)
(0, 1, 0)	(1, 0, 1)
	(1, 1, 0)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, -, 0)	(1, 0, -)
	(1, -, 0)

Table VIII. Example of Intersection by irrelevant examples

T_1	T_2
(1, 0, 0)	(1, 0, 0)
(1, 0, 1)	(1, 1, 0)
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$
(1, 0, -)	(1, -, 0)

PROPOSITION 3.12 (MUST HAVE SCHEMAS THAT ARE IRRELEVANT). *For two intersecting sets of test cases T_1 and T_2 (these two sets are neither identical nor have members subsume each other), $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$. s.t. c_1 and c_2 are irrelevant.*

PROOF. First, we can learn that $\mathcal{C}(T_1 \backslash T_3)$ are irrelevant to $\mathcal{C}(T_2 \backslash T_3)$, as $(T_1 \backslash T_3) \bigcap (T_2 \backslash T_3) = \emptyset$.

Given the schemas in $\mathcal{C}(T_1 \backslash T_3)$ are either identical to some schemas in $\mathcal{C}(T_1)$ or parent schemas of them. Now, if some of them are identical, i.e., $\exists c', s.t., c' \in \mathcal{C}(T_1 \backslash T_3)$ and $c' \in \mathcal{C}(T_1)$, then schemas c' must be irrelevant to these schemas in $\mathcal{C}(T_2)$ as $(T_1 \backslash T_3) \bigcap T_2 = \emptyset$. This also holds if $\mathcal{C}(T_2 \backslash T_3)$ is identical to some schemas in $\mathcal{C}(T_2)$.

Next, if both $\mathcal{C}(T_1 \backslash T_3)$ and $\mathcal{C}(T_2 \backslash T_3)$ are parent schemas of some of $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, respectively. Without loss of generality, we let $c_1 \prec c_{1-3}$, ($c_{1-3} \in \mathcal{C}(T_1 \backslash T_3)$ and $c_1 \in \mathcal{C}(T_1)$) and $c_2 \prec c_{2-3}$, ($c_{2-3} \in \mathcal{C}(T_2 \backslash T_3)$ and $c_2 \in \mathcal{C}(T_2)$). Then, these corresponding sub-schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, i.e., c_1 and c_2 respectively, must also be irrelevant to each other. This is because $\mathcal{T}(c_1) \supset \mathcal{T}(c_{1-3})$ and $\mathcal{T}(c_2) \supset \mathcal{T}(c_{1-3})$. And as $\mathcal{T}(c_{1-3}) \bigcap \mathcal{T}(c_{2-3}) = \emptyset$, so $\mathcal{T}(c_1)$ and $\mathcal{T}(c_2)$ are neither identical nor subsuming each other. This also implies that c_1 and c_2 are irrelevant to each other. □

For example, Table VIII shows two test cases that interact with each other at test case (1,0,0), but their minimal schemas, (1,0,-) and (1,-,0), respectively, are irrelevant to each other.

PROPOSITION 3.13 (CAN BE IDENTICAL). *For two intersecting sets of test cases T_1 and T_2, and let $T_3 = T_1 \bigcap T_2$, if $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$ which are identical, then $c_1 = c_2 \in \mathcal{C}(T_3)$*

PROOF. As we see that identical schemas must have the same set of test cases that contain them, then the only same set of test cases between T_1 and T_2 is $T_1 \bigcap T_2 = T_3$. So the only possible identical schema between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$ is in $\mathcal{C}(T_3)$. □

We must know that this proposition holds when some schemas in $\mathcal{C}(T_1 \bigcap T_2)$ are identical to some schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$.

For example, Table IX shows two test cases that interact with each other at test cases (1,1,0) and (1,1,1), and they have identical minimal schema, i.e., (1,1,-), which is also the minimal schema in $\mathcal{C}(T_3)$.

Table IX. Example of Intersection by identical examples

T_1	T_2	$T_3 = T_1 \bigcap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 1, 0)
(1, 1, 0)	(0, 0, 1)	(1, 1, 1)
(1, 1, 1)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(0, 0, -)	(1, 1, -)
(1, 1, -)	(1, 1, -)	

Table X. Example of Intersect by subsuming examples

T_1	T_2	$T_3 = T_1 \bigcap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 1)
(1, 0, 1)	(1, 0, 1)	(1, 1, 0)
(1, 1, 0)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(-, 0, 0)	(1, 0, -)
(1, 0, -)	(1, -, -)	(1, -, 0)
(1, -, 0)		

PROPOSITION 3.14 (CAN BE SUBSUMING EACH OTHER). *For two intersecting sets of test cases T_1 and T_2, let $T_3 = T_1 \bigcap T_2$, if $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, and if c_1 is the parent-schema of c_2, then $c_1 \in \mathcal{C}(T_3)$. (and vice versa).*

PROOF. We have proved previously if two schemas have a subsuming relationship, then their test cases must also have an inclusion relationship. And as the only inclusion relationship between T_1 and T_2 is that $T_3 \subset T_1$ and $T_3 \subset T_2$, the parent schemas must be in $\mathcal{C}(T_3)$. □

It is noted that this proposition holds when some schemas in $\mathcal{C}(T_3)$ are also in $\mathcal{C}(T_1)$ (or $\mathcal{C}(T_2)$), and simultaneously the same schemas in $\mathcal{C}(T_3)$ must be the parent-schema of the minimal schemas of another set of test cases, i.e., $\mathcal{C}(T_2)$ (or $\mathcal{C}(T_1)$).

Table X illustrates this scenario, in which, the minimal schemas of T_1: (1,0,-),(1,-,0),which are also the schemas in $\mathcal{C}(T_3)$, is the parent schema of the minimal schema of T_2: (1,-,-).

It is noted that these three conditions can simultaneously appears when two sets of test cases intersect with each other.

3.5. Identify the MFS

According to this analysis, we can determine that $\mathcal{C}(T_{F_m})$ actually is the set of failure-causing schemas of F_m. Then in theory, if we want to accurately figure out the MFS in the SUT, we need to exhaustively execute each possible test case, and collect the failing test cases T_{F_m}. This is impossible in practice, especially when the testing space is very large.

So for traditional FCI approaches, they can only select a subset of the exhaustive test cases to execute. In this case, in order to identify the MFS, each of the remaining test cases must be predicted to be failing or not. Or alternatively, a set of schemas is given as the candidate of the MFS [Ghandehari et al. 2012]. For this case, these schemas should be prioritized according to their likelihood to be the MFS. As giving a ranking of these candidate schemas can also be regard as a special case of making a prediction (with computing the possibility), so we next only formally describe the mechanism of FCI approaches belonging to the first type.

We refer to the observed failing test case as $T_{fail_{observed}}$, and refer to the remaining failing test cases based on prediction as $T_{fail_{predicted}}$. We also denote the actual entire failing test cases as T_{fail}. Then the MFS identified by FCI approaches can be depicted as:

MFS = $\mathcal{C}(T_{fail_{observed}} \bigcup T_{fail_{predicted}})$.

Each FCI approach applies different way to predict the $T_{fail_{predicted}}$ according to observed failing test cases; furthermore, as the test cases it generates are different, the failing test cases observed by different FCI approaches, i.e., $T_{fail_{observed}}$ also varies. We offer an example using the OFOT approach to identify the MFS.

Suppose that the SUT has 3 parameters, each of which can take on 2 values. And assume the test case (1, 1, 1) failed. Then, we can describe the FCI process as shown in Table XI. In this table, test case *t* failed, and OFOT mutated one parameter value of the test case *t* at a time to generate new test cases: $t_1; t_2; t_3$, It found the t_1 passed, which indicates that this test case breaks the MFS in the original test case *t*. So, the (1,-,-) should be one failure-causing factor, and as the other test cases (t_2, t_3) all failed , this means no other failure-inducing factors were broken; therefore, the MFS in *t* is (1,-,-).

Now let us explain this process with our formal model. Obviously the $T_{fail_{observed}}$ is {(1,1,1),(1,0,1),(1,1,0)}. And as having found (0,-,-) broke the MFS, hence by theory [Nie and Leung 2011a], all the test cases that contain (0,-,-) should pass the testing (This conclusion is built on the assumption that the SUT just contain one failure-causing schema). As a result, (0,1,1),(0,0,1),(0,1,0),(0,0,0) should pass the testing. Further, as obviously the test case either passes or fails (the condition that a test case skips testing, i.e., doesn't produce an output, is labeled as a special case of failing), so the remaining test case(1,0,0), will be predicted to be failing, i.e., $T_{fail_{predicted}}$ is {(1,0,0)}. Taken together, the MFS using the OFOT strategy can be described as: $\mathcal{C}(T_{fail_{observed}} \bigcup T_{fail_{predicted}})$ = $\mathcal{C}(\{(1,1,1),(1,0,1),(1,1,0),(1,0,0)\})$ = (1,-,-), which is identical to the one that was gotten previously.

Table XI. OFOT with our strategy

original test case				**Outcome**
t	1	1	1	Fail
observed				
t_1	0	1	1	Pass
t_2	1	0	1	Fail
t_3	1	1	0	Fail
predicted				
t_4	0	0	1	Pass
t_5	0	1	0	Pass
t_6	1	0	0	Fail
t_7	0	0	0	Pass

Similarly, other FCI approaches can also be modeled into this formal description. It is noted that the test cases FCI predicts to be failing are not always identical to the actually failing test cases. In fact, we can generally depict the process of FCI approaches as shown in Figure 3.

We can see in Figure 3 that area A denotes the test cases that should have passed testing but were predicted to be failing, area B depicts the test cases that the approach observed to be failing test cases, area C refers to the failing test cases that were not observed but were predicted to be failing test cases, and area D shows the failing test cases that are neither observed nor predicted. This figure actually represents the condition in which two sets of test cases intersect with each other; in specific, comparing to the Figure 2(c), we can learn $A \bigcup B \bigcup C = T_1$, $D \bigcup B \bigcup C = T_2$ and $B \bigcup C = T_1 \bigcap T_2 = T_3$.

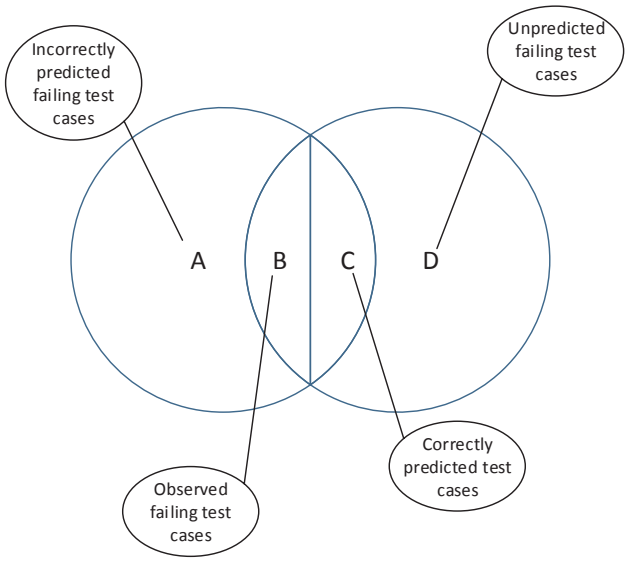


Fig. 3. Generally model of FCI

As we have known, in the case of Figure 2(c), there are various relationships between the schemas identified in T_1 and the schemas identified in T_2. According to Proposition 3.12, some schemas in $\mathcal{C}(T_1)$ and in $\mathcal{C}(T_2)$ must be irrelevant to each other. Mapping to Figure 3, we can get that some schemas in $\mathcal{C}(A \bigcup B \bigcup C)$ and in $\mathcal{C}(D \bigcup B \bigcup C)$ must be irrelevant to each other, which means that the FCI approach will identify some minimal schemas that are irrelevant to the actual MFS, and must ignore some actual MFS. Moreover, under the appropriate conditions listed in Propositions 3.13 and 3.14, FCI may identify the identical schemas or parent-schema or sub-schema of the actual MFS. In this case, the schemas (those identical ones, parent-schemas, or sub-schemas) are all depended on the area $B \bigcup C$, namely $T_1 \bigcap T_2$ in Figure 2(c). So to identify the schemas as accurately as possible, the FCI approach needs to make $A \bigcup B \bigcup C$ as similar as possible to $D \bigcup B \bigcup C$.

However, even though each FCI approach tries to identify the MFS as accurately as possible, masking effects arising from different test cases will reduce its effectiveness. We next discuss the masking problem and how it affects the FCI approaches.

4. MASKING EFFECT

Definition 4.1. A *masking effect* is an effect that when a test case t contains an MFS for a particular failure, but the test case t does not trigger the expected failure because another failure was triggered ahead of it that prevents t from being normally checked.

Taking the masking effects into account, when identifying the MFS for a specific failure, say, F_m, we should not ignore those test cases which did not trigger F_m but should have triggered it. We call these test cases $T_{mask(F_m)}$. Hence, the MFS for failure F_m should be $\mathcal{C}(T_{F_m} \bigcup T_{mask(F_m)})$.

As an example, in the motivation example in section 2, $F_{mask(F_{Ex\ 2})}$ is { (7,4,4,5),(11,4,4,5)}, So the MFS for $Ex2$ is $\mathcal{C}(T_{F_{Ex\ 2}} \bigcup T_{mask(F_{Ex\ 2})})$, which is (-,-,4,5).

Table XII. masking effects for exhaustive testing

T_1	$T_{mask(1)}$	T_*
(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 1, 0, 0)
(1, 1, 1, 0)	(0, 1, 1, 1)	(0, 0, 0, 0)
(1, 1, 0, 1)		(1, 0, 0, 0)
		(1, 0, 1, 1)
		(0, 0, 1, 1)
actual MFS for 1 $\mathcal{C}(T_1 \bigcup T_{mask(1)})$	regarded as one failure $\mathcal{C}(T_1 \bigcup T_{mask(1)} \bigcup T_*)$	distinguishing failures $\mathcal{C}(T_1)$
(1, 1, -, -)	(-, -, 0, 0)	(1, 1, -, 1)
(-, 1, 1, 1)	(1, 1, -, -)	(1, 1, 1, -)
	(-, -, 1, 1)	

In practice with masking effects, however, it is not possible to correctly identifying the MFS, unless we fix some bugs in the SUT and re-execute the test cases to figure out $T_{mask(F_m)}$.

For traditional FCI approaches, without the knowledge of $T_{mask(F_m)}$, only two strategies can be adopted when facing the multiple failures problem, i.e., *regarded as one failure* and *distinguishing failures*. The former strategy treats all types of failures as one failure–*failure*, and others as *pass*, while the latter distinguishes the failures but with no considering the masking effects, i.e., if a test case fails with a particular type of fault, this strategy presumes it does not contain other type of faults. We will separately analyse the two strategies under exhaustive testing condition and normal FCI testing condition.

4.1. Masking effects for exhaustive testing

4.1.1. Regarded as one failure strategy. The first is the most common strategy. With this strategy, the minimal schemas we identify are the set $\mathcal{C}(\bigcup_{i=1}^{L} T_{F_i})$, L is the number of all the failures in the SUT. Obviously, $T_{F_m} \bigcup T_{mask(F_m)} \subset \bigcup_{i=1}^{L} T_{F_i}$. So in this case, by Proposition 3.10, some schemas may be the sub-schemas of some of the actual MFS, or be irrelevant to the actual MFS.

As an example, consider the test cases in Table XII. In this example, assume we need to characterize the MFS for error *1*. All the test cases that triggered error 1 are listed in column T_1; similarly, we list the test cases that triggered other failures in column $T_{mask(1)}$ and T_*, respectively, in which the former masked the error 1, while the latter did not. Actually the MFS for error 1 should be (1,1,-,-) and (-,1,1,1) as we listed them in the column *actual MFS for 1*. However, when we use the *regarded as one failure* strategy, the minimal schemas we get will be (-,-,0,0), (1,1,-,-), (-,-,1,1), in which (-,-,0,0) is irrelevant to the actual MFS for error 1, and (-,-,1,1) is a sub-schema of the actual MFS (-,1,1,1).

4.1.2. Distinguishing failures strategy. Distinguishing the failures by the exception traces or error code can help make the MFS related to particular failure. Yilmaz [Yilmaz et al. 2013] proposed the *multiple-class* failure characterizing method instead of the *ternary-class* approach to make the characterizing process more accurate. Besides, other approaches can also be easily extended with this strategy for testing SUT with multiple failures.

This strategy focuses on identifying the set of $\mathcal{C}(T_{F_m})$, and as $T_{F_m} \bigcup T_{mask(F_m)} \supset T_{F_m}$, consequently, some schemas obtained using this strategy may be the parent-schema of some actual MFS. Moreover, some MFS may be irrelevant to the schemas we get with this strategy, which means this strategy will ignore these actual MFS.

For the simple example in Table XII, when we use this strategy, we will get the minimal schemas (1, 1, -, 1) and (1, 1, 1, -), which are both the parent schemas of the

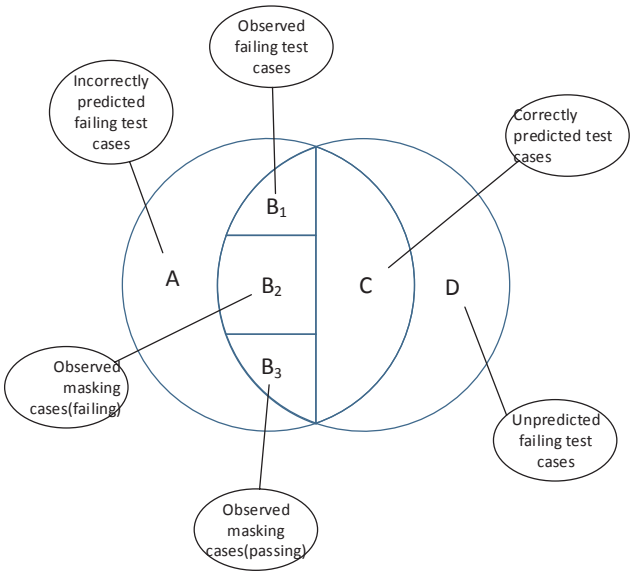


Fig. 4. FCI with masking effects

actual MFS (1,1,-,-), and we observe that no schemas gotten by this strategy have any relationship with the actual MFS (-,1,1,1), which means it was ignored.

It is noted that the motivation example in section 2 actually adopted this strategy, so we see that the schemas identified for Ex 2: (-,2,4,5), (-,3,4,5) are the parent-schemas of the correct MFS(-,-,4,5).

4.2. Masking effects for FCI approaches

With masking effects, the scenario of traditional FCI approaches is a bit more complicated than the previous two exhaustive testing scenarios, and is depicted in Figure 4. In this figure, areas A, C and D are the same as in Figure 3, and area B is further divided into three sub-areas in which B_1 still represents the observed failing test cases for the current analysed failure, area B_2 represents the test cases that triggered other failures which masked the current failure, and area B_3 represents the test cases that triggered other failures which did not mask the current failure. It can be found that the actual MFS set for the SUT is $\mathcal{C}(B_1 \bigcup B_2 \bigcup C \bigcup D)$.

With this model, if we know which test cases mask the expected failure, i.e., if we have figured out B2 and B3, then the schemas that the FCI approach will identify can be described as $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$. We next denote the condition that we have known the masking effects in prior as *knowing masking effects*. However, as discussed before, to get this result is not possible without human involvement. Correspondingly, when using the *regarded as one failure* strategy, the set of MFS traditional FCI identify is $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$. And for the *distinguishing failures* strategy, the MFS is $\mathcal{C}(A \bigcup B_1 \bigcup C)$. Next, we will discuss the influence of masking effects on the two strategies.

4.2.1. Using the regarded as one failure strategy. For the first strategy: *regarded as one failure*, the impact of masking effects on FCI approaches can be described as shown in Table XIII. To understand the content of this table, let us go back to the relationship between the minimal schemas of two different sets of test cases. For the *knowing masking effects* condition, the test cases that are used for identifying the

MFS, i.e., $A \bigcup B_1 \bigcup B_2 \bigcup C$, *intersect* the test cases that are used to compute the actual MFS($B_1 \bigcup B_2 \bigcup C \bigcup D$). It means that the obtained minimal schemas can be identical, parent-schema, sub-schema, and irrelevant to the actual MFS. And if we apply the *regarded as one failure* strategy, the minimal schemas we get are $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$. Obviously, we have $A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C \supset A \bigcup B_1 \bigcup B_2 \bigcup C$. So the minimal schema gotten by this strategy is either the sub-schema or identical to some schemas from the ones gotten by *known masking effects* strategy, or alternatively, existing some schemas that are irrelevant to all of them. Considering these two properties together, we determine the results shown in Table XIII.

Table XIII. Masking effects influence on FCI with regarded as one failure strategy

1	If $c_m = c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} = c'_m$
2	If $c_m = c_{origin}$ and $c_{new} \prec c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
3	If $c_m \prec c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
4a	If $c_m \prec c_{origin}$ and $c_{new} \prec c_{origin}$	Then either,$c'_m, s.t., c_{new} \prec c'_m$
4b		Or,c_{new} *irrelevant to all* c'_m
5	If $c_{origin} \prec c_m$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
6	If $c_{origin} \prec c_m$ and $c_{new} \prec c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
7	If c_{origin} *irrelevant all* c_m and $c_{new} = c_{origin}$	Then, c_{new} *irrelevant to all* c'_m
8a	If c_{origin} *irrelevant all* c_m and $c_{new} \prec c_{origin}$	Then either,$\exists c'_m, s.t., c_{new} \prec c'_m$
8b		Or,c_{new} *irrelevant to all* c'_m
9a	If c_{new} *irrelevant to all* c_{origin}	Then either,$\exists c'_m, s.t., c_{new} \prec c'_m$
9b		Or,c_{new} *irrelevant to all* c'_m

There are totally 9 rules for this strategy, which are labeled from 1 to 9 respectively. In these rules, c_m and c'_m are the actual MFSs, i.e., $c_m, c'_m \in C(B_1 \bigcup B_2 \bigcup C \bigcup D)$. c_{origin} is the minima schema that are gotten by *knowing masking effects* strategy, i.e., $c_{origin} \in \mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$. c_{new} is the schema that are gotten by *regarded as one failure* strategy, i.e., $c_{new} \in \mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$. c_{new} satisfies $c_{new} = c_{origin}$ or $c_{new} \prec c_{origin}$. Each rule in Table XIII describes one possible relationship between the schemas gotten by strategy *regarded as one failure* and the actual MFS. For example, rule 2, i.e., $If\ c_m = c_{origin}\ and\ c_{new} \prec c_{origin}, then\ \exists c'_m, s.t., c_{new} \prec c'_m$, indicates that the schema c_{new} that is gotten by *regarded as one failure* strategy will be the subschema of some actual MFS, if it is the subschema of the schema c_{origin} that is gotten by strategy *knowing masking effects*, and c_{origin} is identical to some actual MFS. Some rules may offer more than one possible relationship between c_{new} and the actual MFS. For example, rule 4 can make c_{new} either be the subschema of the actual MFS or be irrelevant to all the actual MFS. In this case, we divide the rule into several sub-rules. As for rule 4 we divide this rule into two sub-rules: 4a and 4b, each of which represents one possibility relationship between the c_{new} and the actual MFS c'_m.

Among these rules, only two can make $\exists c'_m, s.t.\ c_{new} = c'_m$ or $c'_m \prec c_{new}$, which are rules 1 and 3 respectively. This can be easily understood, as to make $c_{new} = c'_m$ or $c'_m \prec c_{new}$, according to Proposition 3.13 and 3.14, it must have $\mathcal{T}(c_{new}) \subset (B_1 \bigcup B_2)$. Assume $c_{new} \prec c_{origin}$, then we must have $\exists t \in B_3, s.t., t \in \mathcal{T}(c_{new})$, otherwise, there should be no schema c_{new} that is the sub-schema of c_{origin}. Consequently, $\mathcal{T}(c_{new}) \not\subset (B_1 \bigcup B_2)$ if $c_{new} \prec c_{origin}$. So to make $c_{new} = c'_m$ or $c'_m \prec c_{new}$, the schema c_{new} must be identical to c_{origin}, i.e., $c_{new} = c_{origin}$ and there must be $c_{origin} = c_m$ or $c_m \prec c_{origin}$, correspondingly. Apart from these two rules, the remaining rules indicate that either $c_{new} \prec c'_m$ or c_{new} *is irrelevant to all the actual* MFS, which implies the schemas that are gotten by strategy *regarded as one failure* tends to be more subschemas or irrelevant schemas of the actual MFS when compared to that of the *knowing masking effects* strategy.

Table XIV. Example of the influence of the regarded as one failure for FCI approach

A	$B_1 \bigcup B_2 \bigcup C$	B_3	D
(0,0,0,1,0,0)	(1,1,1,0,0,0)	(1,0,1,0,0,0)	(1,1,0,0,0,0)
(0,0,0,1,1,0)	(1,1,1,0,1,0)	(1,0,1,0,1,0)	(1,1,0,0,1,0)
(0,0,1,0,0,0)	(1,1,1,1,0,0)	(0,0,1,0,1,0)	(1,1,0,1,0,0)
(0,0,1,1,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,0)	(1,1,0,1,1,0)
	(1,0,1,1,0,0)	(0,1,0,0,0,0)	(0,0,1,1,0,1)
	(1,0,1,1,1,0)	(0,1,0,1,0,0)	(0,1,1,1,0,1)
	(0,0,0,0,1,0)	(0,1,1,0,0,0)	
	(0,0,0,0,0,0)	(0,1,1,1,0,0)	
	(0,0,1,1,1,1)	(1,0,0,0,0,0)	
	(0,1,1,1,1,1)	(1,0,0,0,1,0)	
		(1,1,1,1,1,1)	
		(1,0,1,1,1,1)	

actual MFS	*knowing masking effects*	*one failure*
$\mathcal{C}(B_1 \bigcup B_2 \bigcup C \bigcup D)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$
(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)
(1,-,1,1,-,0)	(1,-,1,1,-,0)	(0,0,-,-,-,0)
(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,-,-,-,0,0)
(0,-,1,1,-,1)	(0,0,-,-,0,0)	(-,0,1,-,-,0)
	(-,0,1,1,0,0)	(-,-,1,-,0,0)
	(0,-,1,1,1,1)	(-,0,-,0,-,0)
		(-,-,1,1,1,1)
		(1,-,1,1,1,-)
		(-,0,1,1,1,-)

rules	c_m	c_{origin}	c_{new}	c'_m
2	(1,-,1,1,-,0)	(1,-,1,1,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
4a	(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
4b	(0,-,1,1,-,1)	(0,-,1,1,1,1)	(-,-,1,1,1,1)	*
6	(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
8a	*	(0,0,-,-,0,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
8b	*	(-,0,1,1,0,0)	(-,0,1,-,-,0)	*
9a	*	*	(-,0,-,0,-,0)	(0,0,0,0,-,0)
9b	*	*	(1,-,1,1,1,-)	*

Next we will give examples to depict these rules except these that have the condition '$c_{new} = c_{origin}$' (rule 1, 3, 5, 7). This is because these rules will simply result in that the relationship between c_{new} and actual MFS will be the same as the relationship between c_{origin} and the actual MFS. Table XIV presents the examples of all the remaining rules. This table consists of three parts, with the upper part giving the test cases for each area in the abstract FCI model. Note that we only list the union of areas B_1, B_2 and C. This is because the union is the common element for computing the MFS of three approaches – *actual MFS, knowing masking effects, regarded as one failure*. The middle part of this table shows the minimal schemas using this particular method. And last, the lower part depicts the sample of each possible rule in Table XIII. In this part, the left column indicates the specific rule id, i.e., 2,4a,4b,6,8a,8b,9a,9b. The column 'c_m', 'c_{origin}', 'c_{new}', 'c'_m', respectively, indicates the schema which satisfies the rule in the corresponding row. The mark $*$ means the rule is irrelevant to this schema. For example, for rule 8b, i.e., if c_{origin} $irrelevant\ all$ c_m and $c_{new} \prec c_{origin}$ then c_{new} $irrelevant\ to\ all$ c'_m, we marked '*' in the column 'c_m' and 'c'_m'.

4.2.2. using distinguish strategy. For the second strategy, *distinguishing failures*, the influence can be described as shown in Table XV.

Similar to Table XIII, this table also lists the possible relationships among the schemas obtained by strategy *distinguishing failures*, schemas obtained by *knowing masking effects* strategy, and the actual MFS. As with the *distinguishing failures* strategy, the minimal schemas identified are actually $\mathcal{C}(A \bigcup B_1 \bigcup C)$. Obviously $A \bigcup B_1 \bigcup C \subset A \bigcup B_1 \bigcup B_2 \bigcup C$. So with this strategy, $c_{new} \in \mathcal{C}(A \bigcup B_1 \bigcup C)$ should be

Table XV. Masking effects influence on FCI with distinguishing failures strategy

1	If $c_m = c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} = c'_m$
2	If $c_m = c_{origin}$ and $c_{origin} \prec c_{new}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
3	If $c_m \prec c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
4	If $c_m \prec c_{origin}$ and $c_{origin} \prec c_{new}$	Then, $c'_m, s.t., c'_m \prec c_{new}$
5	If $c_{origin} \prec c_m$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
6a	If $c_{origin} \prec c_m$ and $c_{origin} \prec c_{new}$	Then either $\exists c'_m, s.t., c_{new} = c'_m$
6b		Or $\exists c'_m, s.t., c'_m \prec c_{new}$
6c		Or $\exists c'_m, s.t., c_{new} \prec c'_m$
6d		Or c_{new} *irrelevant all* c'_m
7	If c_{origin} *irrelevant all* c_m and $c_{new} = c_{origin}$	Then, c_{new} *irrelevant to all* c'_m
8a	If c_{origin} *irrelevant all* c_m and $c_{origin} \prec c_{new}$	Then either,$\exists c'_m, s.t., c'_m \prec c_{new}$
8b		Or,c_{new} *irrelevant to all* c'_m
9	It may have c_{origin} *irrelevant to all* c_{new}	

either the parent-schema or identical to $c_{origin} \in \mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$. The main difference between the rules in Table XV and rules in Table XIII is that most rules with strategy *distinguishing failures* result in $c'_m \prec c_{new}$. This is because with strategy *distinguishing failures*, the test cases that are used for identifying the MFS is less than that of *knowing masking effects*. So it is more likely to have $\mathcal{T}(c_{new}) \subset \mathcal{T}(c'_m)$, and hence $c'_m \prec c_{new}$.

We take an example to illustrate these rules of strategy *distinguishing failures*, as depicted in Table XVI. Similar to our previous strategy, we omit the samples that have the condition '$c_{origin} = c_{new}$'.

One rule to note is rule 9, which can make some c_{origin} *removed* from the newly minimal schemas, i.e., $\nexists c_{new}, s.t., c_{new} = c_{origin}$ *or* $c_{origin} \prec c_{new}$. For the Table XVI example, in the last row for rule 9, we can find the schema $c_{origin} - -$(1,1,0,0,1,-), which is identical to the one in actual MFS, there exists no c_{new} which is identical to or the parent-schema of this schema. Consequently, in this condition, this strategy may ignore some actual MFS compared with *knowing masking effects*.

In fact, besides rule 9, there is another condition which can result in the FCI approach ignoring some actual MFS. It is when a c_{new} is the only schema that is *related* to a c_{origin}, (*related* means not irrelevant, and in this case it is either identical to or the parent-schema), and the corresponding c_{origin} is the only schema which is related to one actual MFS c_m. Then, if the c_{new} is irrelevant to all the actual MFS, we will ignore the actual MFS c_m. Note that this *ignoring* event is based on the condition that c_{new} is irrelevant to all the actual MFS, and this condition is common for both the two strategies. But rule 9, which can also lead to MFS ignored, only can be realized when applying *distinguishing failures* strategy on the FCI approaches. This indicates that this strategy has a larger chance to ignore some actual MFS than *regarded as one failure* strategy.

4.3. Summary of the masking effects on the FCI approach

From the analysis of the formal model, we can learn that masking effect does influence the FCI approaches, and even worse, both the *regarded as one failure* and *distinguishing failures* strategies have their own problems in handling this effect. Specifically when compared with the *knowing masking effects* condition, the strategy *regarded as one failure* has a larger possibility of getting more sub-schemas of the actual MFS and getting more schemas which are irrelevant to the MFS, while strategy *distinguishing failures* may get more parent schemas of the MFS and can also get more irrelevant MFS. Further, both strategies may ignore the actual MFS with the *distinguishing failures* strategy more likely to ignore the MFS than the *regarded as one failure* strategy.

Table XVI. Example the influence of distinguishing failures for FCI approach

A	$B_1 \bigcup C$	B_2	D
(0,0,0,1,1,0)	(0,0,0,0,0,0)	(0,0,1,1,1,0)	(0,1,1,0,0,0)
(0,0,1,1,0,0)	(0,0,0,0,1,0)	(1,1,0,1,0,0)	(0,1,1,0,1,0)
(0,0,1,0,1,0)	(0,0,0,1,0,0)	(1,0,1,1,0,1)	(0,1,1,1,0,0)
(0,0,1,0,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,1)	(0,1,1,1,1,0)
(1,1,0,0,0,0)	(1,1,0,0,1,0)	(1,1,0,0,1,1)	(1,1,1,1,1,1)
(0,0,1,1,0,1)	(1,1,0,1,1,0)	(0,1,0,0,1,1)	(0,1,1,1,1,1)
	(1,1,1,0,0,0)	(1,0,1,0,1,1)	(1,1,1,1,0,1)
	(1,1,1,1,0,0)		(1,0,0,1,1,1)
	(1,1,1,0,1,0)		
	(1,0,1,1,1,1)		
	(0,0,0,0,1,1)		
	(1,0,0,0,1,1)		

$actual\ MFS$	$knowing\ masking\ effects$	$distinguishing failures$
$\mathcal{C}(B_1 \bigcup B_2 \bigcup C \bigcup D)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$	$\mathcal{C}(A \bigcup B_1 \bigcup C)$
(0,-,1,1,1,-)	(1,1,-,-,-,0)	(0,0,0,-,-,0)
(1,1,-,1,-,0)	(0,0,-,-,-,0)	(0,0,-,-,0,0)
(-,-,0,0,1,1)	(-,0,1,1,-,1)	(0,0,-,0,-,0)
(0,0,0,-,0,0)	(0,0,1,1,-,-)	(1,1,1,-,-,0)
(0,0,0,0,-,0)	(0,0,0,0,1,-)	(1,1,-,-,1,0)
(1,0,-,-,1,1)	(1,1,0,0,1,-)	(1,1,-,0,-,0)
(1,1,0,0,1,-)	(1,0,1,-,1,1)	(0,0,1,1,0,-)
(-,1,1,-,-,0)	(1,0,-,0,1,1)	(1,0,1,1,1,1)
(-,-,1,1,1,1)	(-,-,0,0,1,1)	(-,0,0,0,1,1)
(1,1,-,-,1,0)		(0,0,0,0,1,-)
(-,1,1,1,1,-)		
(1,1,1,1,-,-)		
(1,-,1,1,-,1)		
(0,0,0,0,1,-)		

rules	c_m	c_{origin}	c_{new}	c'_m
2	(-,-,0,0,1,1)	(-,-,0,0,1,1)	(-,0,0,0,1,1)	(-,-,0,0,1,1)
4	(1,0,-,-,1,1)	(1,0,1,-,1,1)	((1,0,1,1,1,1))	(1,0,-,-,1,1)
6d	(1,1,-,1,-,0)	(1,1,-,-,-,0)	(1,1,-,0,-,0)	*
6c	(0,0,0,0,-,0)	(0,0,-,-,-,0)	(0,0,0,-,-,0)	(0,0,0,0,-,0)
6b	(1,1,-,1,-,0)	(1,1,-,-,-,0)	(1,1,1,-,-,0)	(-,1,1,-,-,0)
6a	(1,1,-,-,1,0)	(1,1,-,-,-,0)	(1,1,-,-,1,0)	(1,1,-,-,1,0)
8a	*	(-,0,1,1,-,1)	(1,0,1,1,1,1)	(1,-,1,1,-,1)
8b	*	(0,0,1,1,-,-)	(0,0,1,1,0,-)	*
9	(1,1,0,0,1,-)	(1,1,0,0,1,-)	*	*

5. TEST CASE REPLACING STRATEGY

The main reason that the FCI approach fails to work properly is that it cannot determine B_2 and B_3, i.e., if the test case trigger other failures which are different from the current one, it cannot figure out whether this test case will trigger the current expected failure as the masking effects may prevent that. So to limit the impact of this effect on the FCI approach, it is important to reduce the number of test cases that trigger other different failures, in order to can reduce the possibility that expected failure may be masked by other failures.

In the exhaustive testing, as all the test cases will be used to identify the MFS, there is no room left to improve the performance unless we fix other failures and re-execute all the test cases. However, when using part of all the test cases to identify the MFS (which is how the traditional FCI approach works), we can adjust the test cases by selecting the proper ones. By doing this, we can limit the size of $\mathcal{T}(mask_{F_m})$ to be as small as possible.

5.1. Replacing test cases that trigger unexpected failures

The basic idea is to pick the test cases that trigger other failures and generate new test cases to replace them. These regenerated test cases should either pass in the execution or trigger F_m. The replacement must satisfy the condition that the newly generated ones will not negatively influence the original identifying process.

Normally, when we replace the test case that triggers an unexpected failure with a new test case, we should keep some part in the original test case. We call this part the *fixed part*, and mutate the other part with different values from the original one. For example, if a test case (1,1,1,1) triggered an unexpected failure, and the fixed part is (-,-,1,1). Then, we can replace it with a test case (0,0,1,1) which may either pass or trigger the expected failure.

The *fixed part* can vary for different FCI approaches, e.g, for the OFOT [Nie and Leung 2011a] algorithm, the parameter values are the fixed part except for the one that needs to be validated, while for the FIC_BS [Zhang and Zhang 2011] approach, the parameter values that should not be mutated are fixed for the test case in the next iteration of the FIC_BS process.

We note that this replacement may need to be executed multiple times for one fixed part as we could not always find a test case that coincidentally satisfied our requirement. One replacement method is randomly choosing test cases until the satisfied test case is found. While this method may be simple and straightforward, however, it also may require trying too many times to get the satisfied one. So to handle this problem and reduce the cost, we proposed a replacement approach by computing the *strength* of the test case with the other failures, and then we selected the test case from a group of candidate test cases that has the least *strength* related to the other failures.

To explain the *strength* notion, we first introduce the *strength* that a parameter values is related to a particular failure. We use $all(o)$ to represent the number of executed test cases that contain this parameter value, and $m(o)$ to indicate the number of test cases that trigger the failure F_m and contain this parameter value. Then, the *strength* that a parameter value is related to a particular failure, i.e., $S(o, F_m)$, is $\frac{m(o)}{all(o)+1}$. This heuristic formula is based on the idea that if a parameter value frequently appears in the test cases that trigger the particular failure, then it is more likely to be the inducing factor that triggers this type of failure. We add 1 in the denominator for two reasons: (1) avoid division by zero when the parameter value has never appeared before, (2) reduce the bias when a parameter value rarely appears in the test set but coincidentally appears in a failing test case with a particular failure.

With this *strength* definition for a parameter value, we then define that the *strength* of a test case f is related to a particular failure F_m as:

$$S(f, F_m) = \frac{1}{k} \sum_{o \in f} S(o, F_m)$$

where k is the number of parameters in the test case f, and o is the specific parameter value in f. Then this formula defines the *strength* that a test case is related to a failure as the average *strength* of the relevance between each parameter value in the test case and this failure. For a selected test case, we want its ability to trigger another failure to be as small as possible. In practice, the relevance *strength* varies between test case with different failures. As a result, we cannot always find a test case that, for any failure, the *strength* that this test case is related to that failure is the least. With this in mind, we have to settle for a test case, such that the maximal possible failure (except for the one that is currently analysed) it can trigger should be the least likely to be triggered when compared with that of other test cases. In another word,

we need to find a test case, so that the maximal *strength* that it is related to a failure is minimal. Formally, we should choose a test case f, s.t.,

$$\min_{f \in R} \max_{m \leq L \& m \neq n} S(f, F_m) \tag{EQ1}$$

where L is the number of failures, and n is the current analysed failure. R is the set of all possible test cases that contain the $fixed$ part except those that have been tested. To choose the test case from the set R is because the FCI approach needs to keep the $fixed$ part when generating additional test cases and surely we need to select a test case that has not been executed yet. Obviously $|R| = \prod_{i \notin fixed}(v_i) - |\{t \mid t\ contains\ the\ fixed\ part\ \&\ t\ is\ executed\}|$.

We can further resolve this problem. Consider the test case we get – f satisfied the EQ1. Without loss of generality, we assume that the failure $F_k, k \neq n$ is the failure with which the test case f has the maximal related *strength* compared to the other failures. Then, a natural property for f is that any other test case f' which satisfies that failure F_k is the maximal related failure for this test case and must have $S(f, F_k) <= S(f', F_k)$. Formally, to get such a test case is to solve the following formula:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & f \in R \\ & S(f, F_k) > S(f, F_i), \quad 1 \leq i \leq L \ \&\ i \neq k, n \end{aligned} \tag{EQ2}$$

With this formula, to solve EQ1, we just need to find a particular failure F_k, and the corresponding test case f_k that satisfies EQ2, such that the related *strength* between f_k and F_k is smaller than the related *strength* between any other failures and their corresponding test cases that satisfy EQ2. Formally, we need to find:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & 1 \leq k \leq L \ \&\ k \neq n \\ & f, F_k\ satisfies\ EQ2 \end{aligned} \tag{EQ3}$$

According to EQ3, the problem to get such a test case lies in solving EQ2 because if EQ2 is solved we just need to rank the one that has the minimal value from the solutions to EQ2. As to EQ2, it can be formulated as an 0-1 integer linear programming (ILP) problem. Assume the SUT has K parameters in which the ith parameter has V_i values. And the SUT has L failures. We then define the variable x_{ij} as:

$$x_{ij} = \begin{cases} 1 & the\ ith\ parameter\ of\ the\ test\ case\ take\ the\ jth\ value\ for\ that\ parameter \\ 0 & otherwise \end{cases}$$

We then take $o_{m_{ij}}$ to be the related *strength* between the *j*th value of the *i*th parameter of the SUT and the failure F_m. And we use a set R of parameter values to define the fixed part in the test case we should not change, i.e., $R = \{(i,j) | i\ is\ the\ fixed\ parameter\ in\ the\ test\ case, j\ is\ the\ corresponding\ value\}$. As we can generate redundant test cases, so we keep a set of test cases $T_{executed}$ to guide the generation of different test cases. Then EQ2 can be detailed as following ILP formula:

ALGORITHM 1: Replacing test cases triggering unexpected failures

Input: failure type F_m,fixed part s_{fixed}, set of values that each option can take $Param$, the related strength matrix $o_1...o_L$
Output: t_{new} the regenerate test case,The frequency number

```
while not MeetEndCriteria() do
    optimal ← MAX ; t_new ← null ;
    forall the F_k ∈ F_1, ...F_m, F_{m+1}...F_L do
        solver ← setup(s_fixed, Param, F_m, o_1...o_L);
        (optimal', t'_new) ← solver.getOptimalTest() ;
        if optimal' < optimal then
            t_new ← t'_new;
        end
    end
    result ← execute(t_new);
    updatetRelatedStrengthMatrix(t_new) ;
    if result == PASS or result == F_m then
        return t_new;
    else
        continue;
    end
end
return null
```

$$\min \quad \frac{1}{|K|}\sum_{i=0}^{K}\sum_{j=0}^{V_i} o_{m_{ij}} \times x_{ij} \tag{EQ4}$$

$$\text{s.t.} \quad 0 \le x_{ij} \le 1 \qquad i = 0..K-1, j = 0,..V_i - 1 \tag{1}$$

$$x_{ij} \in \mathbb{Z} \qquad i = 0..K-1, j = 0,..V_i - 1 \tag{2}$$

$$\sum_{j=0}^{V_j} x_{ij} = 1 \qquad i = 0...K-1 \tag{3}$$

$$x_{ij} = 1 \qquad (i,j) \in R \tag{4}$$

$$\sum_{i=0}^{K}\sum_{j=0}^{V_i}(o_{m_{ij}} - o_{m'_{ij}}) \times x_{ij} \ge 0 \qquad 1 \le m' \ne m \le L \tag{5}$$

$$\sum_{(i,j)\in t} x_{ij} < K \qquad t \in T_{existed} \tag{6}$$

In EQ4, constraints (1) and (2) indicate that the variable x_{ij} is a 0-1 integer. Constraint (3) indicates that a parameter in one test case can only take on one value. Constraint (4) indicates that the test case should not change values of the fixed part. Constraint (5) indicates that the related strength between the test case and Failure F_m is higher than that of the other failures. Constraint (6) indicates the test cases generated should not be the same as the test cases in $T_{existed}$.

As we have formulated the problem into a 0-1 integer programming problem, we just need to utilize an ILP solver to solve this formula. In this paper, we use the solver introduced in [Berkelaar et al. 2004], which is a mixed Integer Linear Programming (MILP) solver that can handle satisfaction and optimization problems.

The complete process of replacing a test case with a new one while keeping some fixed part is depicted in Algorithm 1. The inputs for this algorithm consist of the failure type we focus on – F_m, the fixed part of which we want to keep from the original test case – s_{fixed} , the set of values that each parameter can take from respectively – $Param$ and the set of matrix $o_1, ...o_L$, for any element in which, say o_m, is recorded the related strength between each specific parameter with each value and the failure F_m, i.e., $o_m = \{o_{m_{ij}} | 0 \leq i \leq K-1, 0 \leq j \leq V_i\}$. The output of this algorithm is a test case t_{new} which either triggers the expected F_m or passes.

This algorithm has an outer loop (lines 1 - 19) containing two parts:

The first part (lines 2 - 9) generates a new test case which is supposed to be least likely to trigger failures different from F_m. The basic idea for this part is to search each failure different from F_m (line 3) and find the best test case that has the least related strength with other failures. In detail, for each failure we set up an ILP solver (line 4) and use it to get an optimal test case for that failure according to EQ4 (line 5). We compare the optimal value for each failure, and choose the one with less strength related to other failures (lines 6 - 9).

The second part is to check whether the newly generated test case is as expected (lines 10 - 16). We first execute the SUT under the newly generated test case (line 10) and update the related strength matrix ($o_1...o_L$) for each parameter value that is involved in this newly generated test case (line 11). We then check the execution result. If either the test case passes or triggers the same failure – F_m, we will get an satisfied test case (line 12), and we will directly return this test case (line 13). Otherwise, we will repeat the process, i.e., generate a new test case and check again (lines 14 - 15).

Note that this algorithm has another exit, besides finding an expected test case (line 12), which is when the function *MeetEndCriteria()* returns *true* (line 1). We didn't explicitly show what the function *MeetEndCriteria()* is like, because this is dependent on the computing resource and how accurate we want the identifying result to be. In detail, if we want to get a high quality result and have enough computing resource, it is desirable to try many times to get the expected test case; otherwise, a relatively small number of attempts is recommended.

In this paper, we just set 3 as the greatest number of repeats for this function. When it ends with *MeetEndCriteria()* returning true, we will return null(line 18), which means we cannot find an expected test case.

5.2. A case study using the replacement strategy

Suppose we have to test a system with eight parameters, each of which has three options. And when we execute the test case T_0 – (0, 0, 0, 0, 0, 0, 0, 0), a failure–$e1$ is triggered. Next, we will use the FCI approach – FIC_BS [Zhang and Zhang 2011] with replacement strategy to identify the MFS for the $e1$. Furthermore, there are two more potential failures, $e2$ and $e3$, that may be triggered during the testing; and they will mask the desired failure $e1$. The process is shown in Figure 5. In this figure, there are two main columns. The left main column indicates the executed test cases during testing as well as the executed results, and each executed test case corresponds to a specific label, T_1 – T_8, at the left. The right main column lists the related strength matrix when a test case triggers $e2$ or $e3$. In detail, the matrix records the related strength between each parameter (columns $P1$ – $P8$) for each value it can take (column v) with the unexpected failure (column F). The executed test case, which is in bold, indicates the one that triggers the other failure and should be replaced in the next iteration.

The completed MFS identifying process listed in Figure 5 works as follows: firstly the original FCI approach determines which $fixed$ part that is needed to be test in each iteration. Then the extra test case will be generated to fill in the remaining part. After

Table XVII. The number of test cases each FCI approach needed to identify MFS

Method	number of test cases to identify MFS
Charles ELA	depends on the covering array
Martinez with safe value [Martínez et al. 2008; 2009]	$O(\hat{d}logk + \hat{d}^2)$
Martinez without safe values [Martínez et al. 2008; 2009]	$O(d^2 + dlogk + log^c k)$
Martinez' ELA [Martínez et al. 2008; 2009]	$O(ds^v logk)$
Shi SOFOT [Shi et al. 2005]	$O(k)$
Nie OFOT [Nie and Leung 2011a]	$O(k \times d)$
Ylimaz classification tree	depends on the covering array
FIC [Zhang and Zhang 2011]	$O(k)$
FIC_BS [Zhang and Zhang 2011]	$O(t(logk + 1) + 1)$
Ghandehari's suspicious based [Ghandehari et al. 2012]	depends on the number and size of MFS
TRT [Niu et al. 2013]	$O(d \times t \times logk + t^d)$

executing the extra test case, if the result of the execution is normal, i.e., didn't trigger unexpected failure(e_2, e_3), then the original FCI process will continue until the MFS is identified. Otherwise, the replacement strategy will be initiated if one unexpected failure is triggered. The replacement process will mutate the parameter values that is not in the $fixed$ part according to Algorithm 1. After the replacement process, the control for the MFS identifying process will be passed to the original FCI approach. Next we will specifically explain how the replacement works with an example in this figure.

From Figure 5, for the test case that triggered $e2$ – (2, 1, 1, 1, 0, 0, 0, 0) (in this case, the fixed part of the test case is (-, -, -, -, 0, 0, 0, 0), in which the last four parameter values are the same as the original test case T_0), we generate the related matrix at left. Each element in this matrix is computed as $\frac{m(o)}{all(o)+1}$; for example, for the $P7$ parameter with value 0, we can find two test cases that contain this element, i.e., T_0 and T_1, so the all(o) is 2. And only one test case triggers the failure $e2$, which means m(o) = 1. So the final related strength between this parameter value with $e2$ is $\frac{1}{2+1} = 0.33$. All the related strength with $e3$ is labeled with a short slash as there is no test case triggering this failure in this iteration. After this matrix has been determined, we can obtain an optimal test case with the ILP solver, which is T_1'–(1, 2, 2, 2, 0, 0, 0, 0), with its related strength 0.167, which is smaller than that of the others.

This replacement process trigged each time a new test case that triggered another failure until we finally get the MFS. Sometimes we could not find a satisfied replacing test case in just one trial like T_1 to T_1'. When this happened, we needed to repeat searching the proper test case we desired. For example, for T_4 which triggered $e3$, we tried three times– T_4', T_4'', T_4''' to finally get a satisfied one T_4''' which passes the testing. Note that the matrix continues to change with the test case generated and executed so that we can adaptively find an optimal one in the current process.

5.3. Complexity analysis

This complexity relies on two values: the number of test cases that triggered other failures which need to be replaced, and the number of test cases that need to be tried to generate a non-masking-effects test case. The complexity is the product of these two values.

The first value is dependent on the extra test cases that are needed to identify the MFS, and this number varies in different FCI approaches. Table XVII lists the number of test cases that each algorithm needed to get the MFS. In this table, d indicates the number of MFS in the SUT. k means the number of the parameters of the SUT. t is the degree of MFS in the SUT. c is an upper bound, and satisfies $d \leq \frac{c}{2} loglogk$. v is the number of values that a parameter can take.

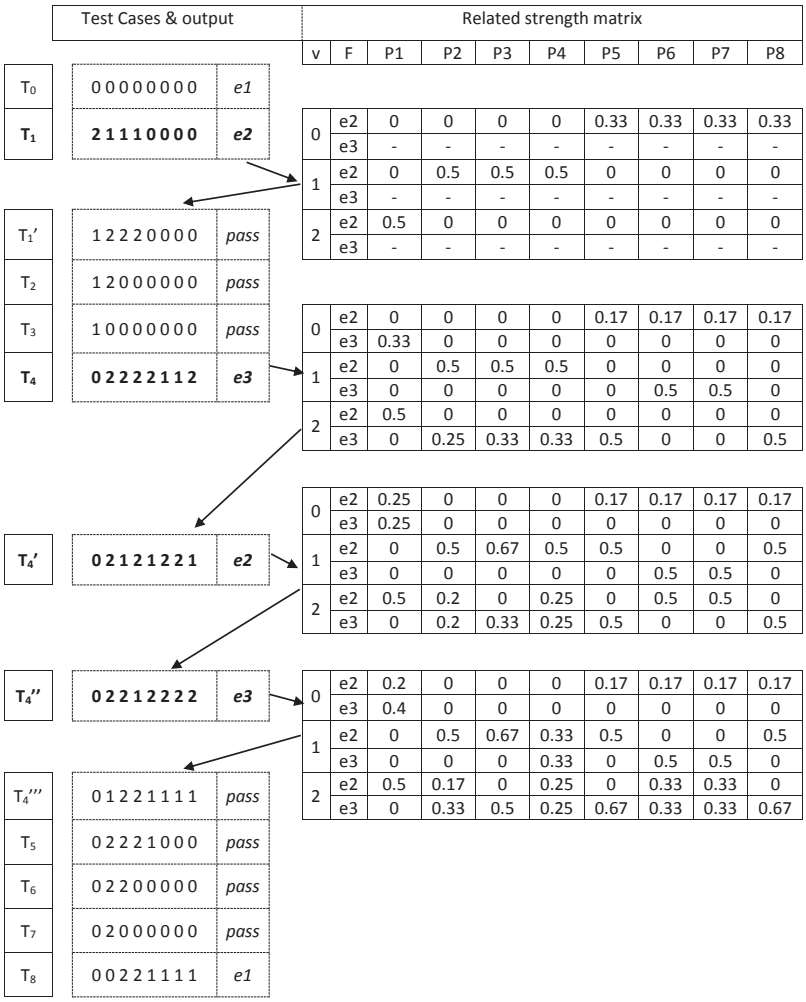


Fig. 5. A case study using our approach

It must be noted that each algorithm may have some restrictions to , details of which are shown in [Zhang and Zhang 2011].

To get the magnitude of the second value, we need to figure out the possibility of a test case that could trigger other failure. The first thing we need to consider is the *fixed* part, as the additional generated test case should somehow contain this part. As we have mentioned before, we can generate $(v-1)^{k-p}$ (*p* is the number of parameter values in the *fixed* part) possible test cases that contain the *fixed* part. Apart from the one that needs to be replaced, there remain $(v-1)^{k-p}-1$ candidate test cases, which indicates the complexity is $O((v-1)^{k-p}-1)$. However, to avoid the exponential computational complexity, in this algorithm we use the $MeetEndCriteria()$ (line 1) function to end the algorithm when the trying times is over a prior given constant, say N, so the final complexity for the second part is $O(min(N,(v-1)^{k-p}-1))$.

We note that exponent $k-p$ has an significant effect on the complexity of the second value. The greater p is, the less test cases that can be generated. For a different

Table XVIII. The complexity of the second part

Method	fixed part
Charles ELA	$O(min(N,(v-1)^{k-t}-1))$
Martinez with safe value [Martínez et al. 2008; 2009]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
Martinez without safe values [Martínez et al. 2008; 2009]	–
Martinez' ELA [Martínez et al. 2008; 2009]	$O(min(N,(v-1)^{k-t}-1))$
Shi SOFOT [Shi et al. 2005]	$O(min(N,(v-1)-1)))$
Nie OFOT [Nie and Leung 2011a]	$O(min(N,(v-1)-1)))$
Ylimaz classification tree	$O(min(N,(v-1)^{k-t}-1)$
FIC [Zhang and Zhang 2011]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
FIC_BS [Zhang and Zhang 2011]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
Ghandehari's suspicious based [Ghandehari et al. 2012]	$O(min(N,(v-1)^{k-t}-1))$
TRT [Niu et al. 2013]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$

approach, p is different. For example, for the OFOT approach, p is a fixed number, which is $k-1$. And for the FIC_BS approach, the p varies in the test cases it generates, ranging from $k-1$ to 1. While for the non-adaptive approaches, as the $fixed$ part is commonly the schemas that are needed to be covered, so the p for these approaches is at least equal to t. We have listed all of them in Table XVIII. It is noted that the approach – $Martinez\ without\ safevalues$ has no such complexity, because this approach works when $v=2$, and this results in not having other test cases to be replaced if we test a fixed part when triggering other failures.

Above all, the cost of replacement strategy varies in different FCI approaches. Note that this cost is computed for the worst case, and in practice much less test cases are needed to identify the MFS. This is because first, not every test case generated by the FCI approach needs to be replaced; and second, a satisfied test case can usually be found before the whole searching space is explored. From complexity analysis, in fact, we cannot determine which approach is better than others, as the cost of each approach is dependent on different factors. For example in Table XVII, the extra test cases needed for OFOT[Nie and Leung 2011a] and FIC_BS[Zhang and Zhang 2011] are $O(k \times d)$ and $O(t(logk+1)+1)$, respectively. Values t, k and d determine which one is better in practice. So for different SUT with different MFS, the priority of the cost of these FCI approaches may change. Besides, the less cost in practice is not always good for identifying MFS. A potential problem is, with less candidate test cases, the replacement strategy may not find a satisfied test case, which will result in a low quality of the identified schemas.

6. EMPIRICAL STUDIES

To investigate the impact of masking effects for FCI approaches in real software testing scenarios and to evaluate the performance of our approach in handling this effect, we conducted several empirical studies. Each of the studies focuses on addressing one particular issue, as follows:

Q1: Do masking effects exist in real software that contains multiple failures?

Q2: How well does our approach perform compared to traditional approaches?

Q3: Is the ILP-based test case searching technique efficient compared to random selection?

Q4: Compared to another masking effects handling approach – the FDA-CIT [Yilmaz et al. 2013], does our approach have any advantages ?

6.1. The existence and characteristics of masking effects

In the first study, we surveyed two kinds of open-source software systems to gain an insight into the existence of multiple failures and their effects. The software under study were: HSQLDB and JFlex. The first is database management software written

Table XIX. Software under survey

software	versions	LOC	classes	bug pairs
HSQLDB	2.0rc8	139425	495	#981 & #1005
	2.2.5	156066	508	#1173 & #1179
	2.2.9	162784	525	#1286 & #1280
JFlex	1.4.1	10040	58	#87 & #80
	1.4.2	10745	61	#98 & #93

in pure Java and the second is a lexical analyser generator. The reason why we chose these two systems is because they both contain different versions and are all highly configurable so that the options and their combinations can affect their behaviour. Additionally, they all have a developer community so that we can easily obtain the real bugs reported in the bug tracker forum. Table XIX lists the program, the versions we surveyed, number of lines of uncommented code, number of classes in the project, and the bug's id [3]for each of the software we studied.

6.1.1. Study setup. We first looked through the bug tracker forum of each software and focused on the bugs which are caused by the options combination. For each such bug, we will derive its MFS by analysing the bug description report and the associated test file which can reproduce the bug. For example, through analysing the source code of the test file of bug#981 for HSQLDB, we found the failure-inducing combination for this bug is: (*preparestatement*, *placeHolder*, *Long string*). These three parameter values together form the condition that triggers the bug. The analysed results will be later regarded as the "prior MFS".

We further built the testing scenario for each version of the software listed in Table XIX. The testing scenario is properly constructed so that we can reproduce different failures by controlling the inputs to the test file. For each version, the source code of the testing file as well as other detailed experiment information is available at– https://code.google.com/p/merging-bug-file.

Next, we built the input model which consists of the options related to the failure-inducing combinations and additional noise options. The detailed model information is shown in Tables XX and XXI for HSQLDB and JFLex, respectively. Each table is organised into three groups: (1)*common options*, which lists the options as well as their values under which every version of this software can be tested; (2)*specific options*, under which only the specific version of that software can be tested; and (3)*configure space*, which depicts the input model for each version of the software, presented in the abbreviated form $\#values^{\#number\ of\ parameters} \times ...$, e.g., $2^9 \times 3^2 \times 4^1$ indicates the software has 9 parameters that can take 2 values, 2 parameters can take 3 values, and only one parameter that can take 4 values.

We then generated the exhaustive test suite consisting of all possible combinations of these options, and for each of them, we executed the prepared testing file. We recorded the output of each test case to observe whether there were test cases containing prior MFS that did not produce the corresponding bug. Later we will refer to those test cases that contain the MFS but did not trigger the expected failure as the *masked* test cases.

6.1.2. Results and discussion. Table XXII lists the results of our survey. Column "all tests" give the total number of test cases executed. Column "failure" indicate the number of test cases that failed during testing, and column "masking" indicates the num-

[3]http://sourceforge.net/p/hsqldb/bugs
http://sourceforge.net/p/jflex/bugs

Table XX. Input model of HSQLDB

common options		**values**
Server Type		server, webserver, inprocess
existed form		mem, file
resultSetTypes		forwad, insensitive, sensitive
resultSetConcurrencys		read_only, updatable
resultSetHoldabilitys		hold, close
StatementType		statement, prepared
sql.enforce_strict_size		true, false
sql.enforce_names		true, false
sql.enforce_refs		true, false
versions	**specific options**	**values**
2.0rc8	more	true, false
	placeHolder	true, false
	cursorAction	next,previous,first,last
2.2.5	multiple	one, multi, defailure
	placeHolder	true, false
2.2.9	duplicate	dup, single, defailure
	defailure_commit	true, false
versions	**Config space**	
2.0rc8	$2^9 \times 3^2 \times 4^1$	
2.2.5	$2^8 \times 3^3$	
2.2.9	$2^8 \times 3^3$	

Table XXI. Input model of JFlex

common options		**values**
generation		switch, table, pack
charset		default, 7bit, 8bit, 16bit
public		true, false
apiprivate		true, false
cup		true, false
caseless		true, false
char		true, false
line		true, false
column		true, false
notunix		true, false
yyeof		true, false
versions	**specific options**	**values**
1.4.1	hasReturn	has, non, default
	normal	true, false
1.4.2	lookAhead	one, multi, default
	type	true, false
	standalone	true, false
versions	**Config space**	
1.4.1	$2^{10} \times 3^2 \times 4^1$	
1.4.2	$2^{11} \times 3^2 \times 4^1$	

Table XXII. Number of failures and their masking effects

software	versions	all tests	failure	masking
HSQLDB	2cr8	18432	4608	768 (16.7%)
-	2.2.5	6912	3456	576 (16.7%)
-	2.2.9	6912	3456	1728 (50%)
JFlex	1.4.1	36864	24576	6144 (25%)
-	1.4.2	73728	36864	6144 (16.7%)

ber of masked test cases. The percentage in the parentheses indicates the proportion of masked test cases and the failing test cases.

We observed that for each version of the software under analysis listed in the Table XXII, test cases with masking effects do exist, i.e., test cases containing MFS did not trigger the corresponding bug. For example, there are about 768 out of 4608 test cases (16.7%) in hsqldb with 2rc8 version. This rate is about 16.7%, 50%, 25%, and 16.7%, respectively, for the remaining software versions, which is not trivial.

So the answer to **Q1** is that in practice, when SUT have multiple failures, masking effects do exist widely in the test cases.

It is notable that in Yilmaz's [Yilmaz et al. 2013] paper, a similar work about the the existence of the masking effects has been conducted. The main difference between that work with ours is that, Yilmaz's work quantify impact of the masking effects as the number of t-degree schemas that only appear in the test cases that triggered other failures. Here, the t-degree schemas can be either MFS or not. Our work, however, quantify the masking effects as the number of test cases that are masked by unexpected failures. And these test cases should contain some MFS, i.e., should have triggered the expected failure if it did not trigger the unexpected failure. The reason why we quantify the masking effects in such way is because that our work seeks to handle the masking effects in the MFS identifying process, and the masking for these test cases which contain the MFS will significantly affect the MFS identifying result, so that this metric can better reflect the impact of the masking effects on the FCI approach.

6.2. Comparing our approach with traditional algorithms

In the second study, our aim was to compare the performance of our approach with traditional approaches in identifying MFS under the impact of masking effects. To conduct this study, we needed to apply our approach and traditional algorithms to identify MFS in a group of software and evaluate their results. The five prepared versions of software in Table XIX used as test objects are far from the requirement for a general evaluation. However, to construct such real testing objects is time-consuming as we must carefully study the tutorial of that software as well as the bug tracker report. So to give a desirable result based on more testing objects, we then synthesize 10 more such testing objects. The testing objects we synthesized are ten toy programs which can directly return outputs when executed with given inputs. To make the synthetic objects as similar as possible to the real software, we firstly analysed the characterizations, such as the number of parameters, the number of failures, and the possible masking effects, of the real software. As a result, we found that the number of parameters of the SUT ranged from 8 to 30, the number of different failures in the SUT ranged from 2 to 4, and the number of MFS for a failure ranged from 1 to 2, in which the degree of the MFS ranged from 1 to 6. Then for each characterization, we randomly select one value in the corresponding range and assign it to the input model by adjusting the relationships between the inputs and outputs of these toy programs. After that, the testing objects that share the similar characterizations of the real software are prepared.

Table XXIII lists the testing model for both the real and synthesizing testing objects. In this table, column 'Object' indicates the SUT under test. For the real SUT listed in Table XIX, we label the five software as $H2cr8$, $H2.2.5$, $H2.2.9$, $J1.4.1$, $J1.4.2$, respectively. While for the synthesizing ones, we label them in the form '*syn+ id*'. Column 'Model' presents the input space for that testing object. Column 'Failures' shows the different failures in the software and their masking relationships. In this column, '$>$' means the left failure will mask the right failure, i.e., if the left failure triggered, then the right failure will not have chance to be triggered. Furthermore, '$>$' is transitive so that the left failure can mask all the failures that are in the right. For example, for the $H2cr8$ object, we can find three failures : e_1, e_2, and e_3. By using the formula $e_1 > e_2 > e_3$, we indicate that the failure e_2 will mask e_3 and e_1 will mask both e_2, e_3.

Table XXIII. The testing models used in the case study

Object	Model	Failures	MFS for each failure
H2cr8	$2^9 \times 3^2 \times 4^1$	$e_1 > e_2 > e_3$	$(5_1, 6_0, 7_0)_{e_1}, (5_1, 8_2, 9_2)_{e_2}, (5_1, 8_2, 9_1)_{e_2}, (5_1, 8_3, 9_2)_{e_3}, (5_1, 8_3, 9_1)_{e_3}$
H2.2.5	$2^8 \times 3^3$	$e_1 > e_2$	$(6_1, 7_0)_{e_1}, (5_2)_{e_2}$
H2.2.9	$2^8 \times 3^3$	$e_1 > e_2 > e_3$	$(6_0)_{e_1}, (0_1, 5_1, 7_0)_{e_2}, (0_0, 5_1, 7_0)_{e_2}, (5_1, 7_0)_{e_3}$
J1.4.1	$2^{10} \times 3^2 \times 4^1$	$e_1 > e_2$	$(0_0)_{e_1}, (1_0)_{e_2}$
J1.4.2	$2^{11} \times 3^2 \times 4^1$	$e_1 > e_2$	$(1_0, 2_1)_{e_1}, (0_1)_{e_2}$
syn1	$2^5 \times 3^3 \times 4^1$	$e_1 > e_2$	$(2_1, 3_0)_{e_1}, (1_1, 2_1)_{e_2}, (1_0, 3_0)_{e_2}$
syn2	$2^6 \times 3^2 \times 4^1$	$e_1 > e_2 > e_3$	$(4_1, 6_0, 7_1, 8_0)_{e_1}, (1_1, 3_1, 5_1)_{e_2}, (2_0, 3_1, 6_0)_{e_3}$
syn3	$2^5 \times 3^3$	$e_1 > e_2 > e_3$	$(2_1, 3_0)_{e_1}, (1_0)_{e_2}, (4_1)_{e_2}, (6_0, 7_0)_{e_3}$
syn4	$2^7 \times 3^2 \times 4^1$	$e_1 > e_2 > e_3$	$(0_1, 2_1, 5_0, 6_1)_{e_1}, (2_1, 4_0)_{e_2}, (6_1, 7_0)_{e_2}, (3_0, 4_0, 5_0)_{e_3}$
syn5	$2^4 \times 3^3 \times 4^2$	$e_1 > e_2$	$(0_0, 1_1, 3_0, 6_1, 8_0)_{e_1}, (2_0, 3_0, 4_1)_{e_2}$
syn6	$2^9 \times 3^2$	$e_1 > e_2 > e_3 > e_4$	$(2_0, 7_1, 8_1)_{e_1}, (3_1, 5_1)_{e_2}, (4_0)_{e_2}, (3_1, 6_0, 7_1)_{e_3}, (3_1, 7_1, 8_0)_{e_4}$
syn7	$2^{10} \times 3^1 \times 4^1$	$e_1 > e_2 > e_3$	$(3_1, 4_0, 5_0)_{e_1}, (2_0, 4_0, 7_1, 9_0)_{e_2}, (6_1, 10_0, 11_1)_{e_3}$
syn8	$2^{11} \times 3^1 \times 4^1$	$e_1 > e_2$	$(1_0, 3_1, 4_0, 7_1, 9_0, 12_1)_{e_1}, (0_0, 2_1, 3_1, 7_1, 10_0, 11_1)_{e_2}$
syn9	$2^4 \times 4^3$	$e_1 > e_2$	$(3_1, 5_0)_{e_1}, (5_0, 6_1)_{e_2}$
syn10	$2^7 \times 3^3 \times 4^1$	$e_1 > e_2$	$(0_1, 3_0, 4_1, 7_0)_{e_1}, (2_0, 3_0, 5_1)_{e_2}, (2_0, 3_0, 5_0)_{e_2}$

Here for the simplicity of the experiment, we didn't build more complex testing scenarios such as the masking effects can happened in the form $e_1 > e_2,\ e_2 > e_3,\ e_3 > e_1$ or even $e_1 > e_2,\ e_2 > e_1$. Later we will discuss for these complex masking effects scenarios. The last column shows the MFS for each failure. The MFS is presented in an abbreviated form $\{\#index_{\#value}\}_{failure}$, e.g., for the object $H2cr8$, $(5_1, 6_0, 7_0)_{e_1}$ actually means (-, -, -, -, -, 1, 0, 0, -, -, -, -) is the MFS for the failure e_1.

6.2.1. Study setup. After preparing the objects under testing, we then apply our approach (augment the FIC_BS with replacing strategy) to identify the MFS of each SUT listed in Table XXIII. Specifically, for each SUT we select each test case that failed during testing and feed these into our FCI approach as the input. Then, after the identifying process is over, we record the identified MFS and the extra test cases it needed. For the traditional FIC_BS approach, we designed the same experiment as what are used for our approach. But as the objects being tested have multiple failures for which the traditional FIC_BS can not be applied directly, we adopted two traditional strategies on the FIC_BS algorithm, i.e., *regarded as one failure* and *distinguishing failures* described in Section 3.2. The purpose of recording the generated additional test cases is to later quantify the additive cost of our approach.

We next compared the identified MFS of each approach with the prior MFS to quantify the degree that each suffers from masking effects, such that we can figure out how much our approach performs better than traditional ones when the SUT contains potential masking effects. There are five metrics used in this study, which are as follows:

(1) The number of the identified MFS which are actual prior MFS. We denote this metric as *accurate number* later.
(2) The number of the identified MFS that are the parent schemas of some prior MFS. We refer to this metric as the *parent number*.
(3) The number of the identified MFS that are the sub schemas of some prior MFS, which is referred to as the *sub number*.
(4) The number of ignored prior MFS. This metric counts the schemas in prior MFS, which are irrelevant to the identified MFS. We label the metric as *ignored number*.

(5) The number of irrelevant identified MFS. This metric counts the schemas in the identified MFS that are irrelevant to the prior MFS. It is referred to as the *irrelevant number*.

Among these five metrics, the *accurate number* directly indicates how effectively the FCI approaches performed, and to identify as much actual MFS as possible is the target for every FCI approach. Metrics *ignored number* and *irrelevant number* indicate the extent of deviation for the FCI approaches, specifically, the former indicates how much information about the MFS will miss, while the latter indicates how serious the distraction would be due to the useless schemas identified by the FCI approach. *Parent number* and *sub number* are the metrics in between, i.e., to identify some schemas that is *parent* or *sub* schemas of the actual MFS is better than identifying *irrelevant* ones or ignoring some MFS, but it is worse than identifying the schema that is identical to some actual MFS. This is a intuitive point, as for the *parent* / *sub* schemas, we just need to *remove* / *add* some elements of the original schemas to get the actual MFS. While for the *irrelevant* or *ignore* ones, however, more efforts will be needed (e.g., both *adding* and *removing* operations will be needed to revise the irrelevant schemas to the actual MFS).

Besides these specific metrics, we also give a composite criteria to measure the overall performance of each approach. The composite criteria is defined as follows:

$$\frac{accurate + related(parent) + related(sub)}{accurate + parent + sub + irrelevant + ignored}$$

In this formula, *accurate, parent, sub, irrelevant,* and *ignored* represent the value of each specific metric. To refine the evaluation of different *parent* / *sub* schemas, we design a *related* function which gives the similarity between the schemas (either parent or sub) and the real MFS, such that we can quantify the specific efforts for changing a *parent* / *sub* schema to the real MFS. The similarity between two schemas S_A and S_B is computed as:

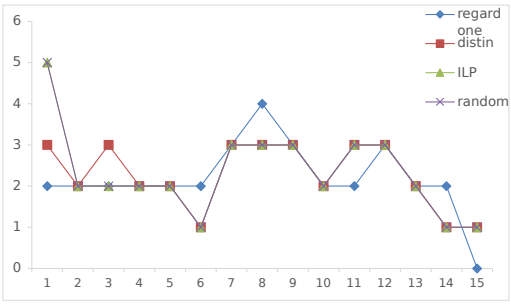
$$Similarity(S_A, S_B) = \frac{the\ same\ elements\ in\ S_A\ and\ S_B}{\max(Degree(S_A), Degree(S_B))}$$

.

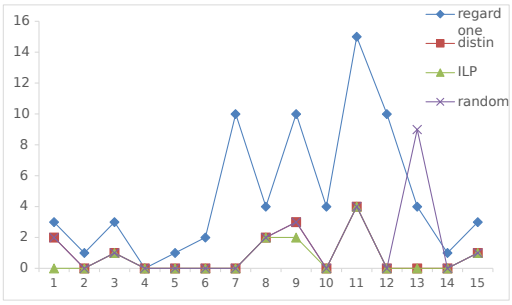
For example, the similarity of (- 1 2 - 3) and (- 2 2 - 3) is $\frac{2}{3}$. This is because the same elements of these two schemas are the third and last elements, and both schemas are three-degree.

So the *related* function is the summation of similarity of all the parent or sub schemas with their corresponding MFS.

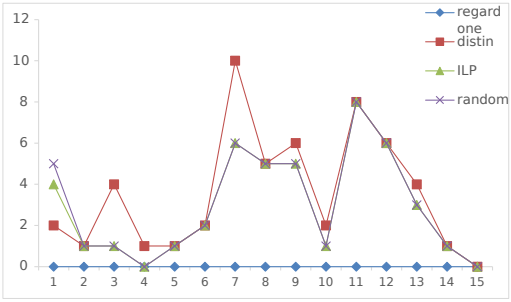
6.2.2. Results and discussion. Figure 6 depicts the results of the second case study. There are seven sub-figures in this figure, i.e., Figure 6(a) to Figure 6(g). They indicate the results of : the number of accurate MFS each approach identified, the number of identified schemas which are the sub-schema / parent-schema of some prior MFS, the number of ignored prior MFS, the number of identified schemas which are irrelevant to all the prior MFS, the metric which gives the overall evaluation of each approach, and the extra test cases each algorithm needed, respectively. For each sub-figure, there are four polygonal lines, each of which shows the results for one of the four strategies: *regarded as one failure*, *distinguishing failures*, *replacement strategy based on ILP searching*, *replacement strategy based on random searching* (The last one will be discussed in the next case study). Specifically, each point in the polygonal line indicates the specific result of a particular strategy for the corresponding testing object. For example in Figure 6(a), the point marked with '♦' at (1,2) indicates that the approach



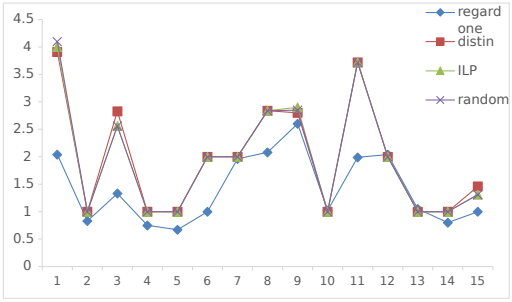
(a) Result of the number of accurately identified MFS



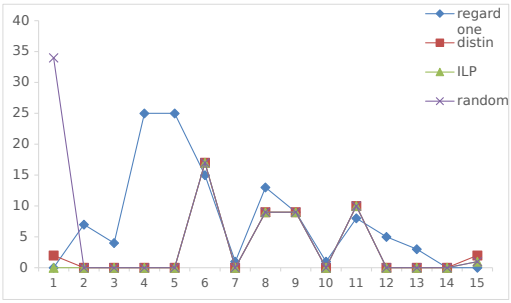
(b) Result of the number of identified sub-schemas of the MFS



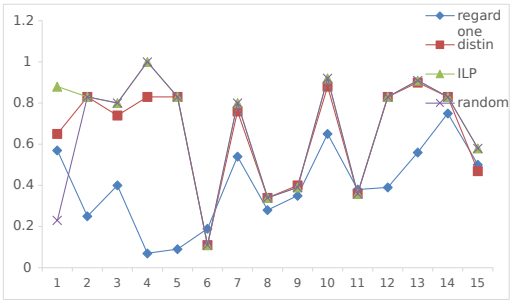
(c) Result of the number of identified parent-schemas of the MFS



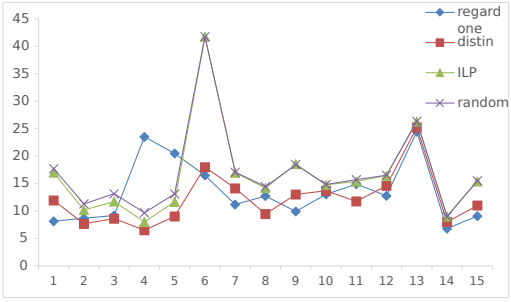
(d) Result of the number of ignored MFS



(e) Result of the number of identified irrelevant schemas of the MFS



(f) The aggregative result for the five metrics



(g) Needed test cases

Fig. 6. Result of the evaluation of each approach

using *regarded as one failure* strategy identified 2 accurate MFS in the testing object–HSQLDB 2cr8. The raw data for this experiment can be found in Table XXIV in the Appendix. Note that all the data except for the metric *ignored number* are based on all failing test cases for each testing object, i.e., we got the data by comparing the union of the schemas identified in each the failing test cases to the prior actual MFS. As for metric *ignored number*, however, we found that if we merged all the schemas identified in each failing test case, there is no MFS ignored, so instead we use the average score of metric *ignored number* for each failing test case, which can be seen in the parentheses in Column *ignore* of Table XXIV. Next we will discuss the results for each metric.

Accurate number: Figure 6(a) shows the number of accurate schemas that each approach achieved. We observe that there is no outstanding strategy among others, i.e., not existing a strategy that can always perform better or worse than others in all these testing objects. For example, for the testing object 1, *ILP* performed the best in obtaining the accurate MFS, while for the testing object 2, *distinguishing failures* identified the most accurate MFS and for testing object 3, *regarded as one failure* did the best. However, upon closer inspection, we can find that strategy *distinguishing failures* performed a little better than strategy *regarded as one failure*. This can be reflected in that there are four testing objects (1, 3, 11, and 15) on which the strategy *distinguishing failures* performed better, while for strategy *regarded as one failure* there are only three superior testing objects (6, 8, and 14). This subtle difference can be explained by our formal analysis in Section 4. Specifically, for the strategy *regarded as one failure*, only rule 1 in Table XIII can result in the FCI approaches identifying the schemas that are identical to some actual MFS, while for *distinguishing failures*, there are two rules (rule 1 and 6a in Table XV). As a result, *distinguishing failures* strategy has a slighter larger chance than *regard as one failure* to identify the schemas that are identical to the actual MFS.

We can further find that strategies *replacement strategy based on ILP searching* (short for *ILP* later) and *distinguishing failures*, have similar results about the number of identified accurate MFS. This can be easily understood, as strategy *ILP* is actually a refinement version of the strategy *distinguishing failures*, which also make the failures distinguished with each other. The main difference between *ILP* and *distinguishing failures* is that the former has to replace the test cases that are not satisfied for identifying the MFS while the latter will not change the generated test cases. As a result, the comparison of other metrics (sub, parent, ignore, irrelevant numbers) also showed the similarity between strategy *ILP* and *distinguishing failures*.

Sub number & parent number: Figure 6(b) and 6(c) depicts the results for metrics *sub number* and *parent number*, respectively. These two figures firstly showed a clear trend for strategies *regarded as one failure* and *distinguishing failures*, i.e., the former identified more sub schemas of actual MFS than the latter, while the latter identified more parent schemas of actual MFS than the former. This is consistent with our formal analysis. Specifically, there are 6 rules (rules 2, 4a, 5, 6, 8a, 9a listed in Table XIII) for strategy *regarded as one fault* that can lead to the identified schemas being sub schemas of actual MFS, while *distinguishing failures* strategy has 2 such rules (rules 5, 6c in Table XV). But for the rules that can result in the schemas being parent schemas of actual MFS, strategy *regarded as one failure* only has one (rule 3 in Table XIII), while *distinguishing failure* has 5 such rules (rules 2, 3, 4, 6b, 8a in Table XV).

Although it offers similar result as the *distinguishing failures* strategy, our strategy *ILP* tend to identify fewer sub schemas and parent schemas of actual MFS than strategy *distinguishing failures* (testing objects 1, 9 in Figure 6(b) and testing objects 3, 4,7,9,10 13 in Figure 6(c)). We believe this is an improvement, as too many sub

schemas and parent schemas will make it harder to get the actual MFS. One issue is the redundancy problem, as many sub or parent schemas in fact point to the same actual MFS.

Ignore number & irrelevant number: The results of the two negative performance metrics are given in Figure 6(d) and 6(e), respectively.

There are two main observations for these two metrics: first, for strategies *regarded as one failure* and *distinguishing failures*, we can find that the former identified more irrelevant schemas of the actual MFS, while the latter ignored more actual MFS. This observation is as expected, because in our formal analysis the strategy *regarded as one failure* has more rules than *distinguish failures* that can lead to the schemas being irrelevant to the actual MFS, in detail, the former has 4 such rules (rules 4b, 7, 8b, 9b in Table XIII) while the latter has three (rules 6d, 7,8b in Table XV). And for strategy *distinguish failures*, rule 9 in Table XV increases the chance to ignore the actual MFS than the strategy *regarded as one failure*.

The second observation is that our *ILP* approach did a good job at reducing the scores for these two negative metrics. Specifically, for the metric *ignored number*, our approach performed better than strategy *distinguishing failures* at testing object 3, 15 in Figure 6(d), but is not as good as strategy *regarded as one failure*. In fact, strategy *regarded as one failure* has a significant advantage at reducing the number of ignored MFS as it tends to associate the failures with all the failing test cases. However, when we consider the metric *irrelevant number*, we can find our approach is the best among the three strategies (better than *distinguishing failures* at testing object 1 in Figure 6(e), and better than strategy *regarded as one failure* at the most testing objects) . We believe this improvement is caused by our test cases replacing strategy, as it can increase the test cases that are useful for identifying the MFS and decrease those useless test cases.

Aggregative for the five metrics: The composite results for these five metrics are given in Figure 6(f). This metric is an overall evaluation of quality of the identified schemas. From this figure, we can find our *ILP* approach performed the best, next the *distinguishing failures*, the last is the *regarded as one failure* (See the testing object 1, 3, 4 in Figure 6(f)).

It is as expected that *ILP* performed better than *distinguishing failures* as former strategy is actually the refinement version of latter. It is a bit of surprise, however, that strategy *distinguishing failures* performed better than *regarded as one failure* at almost all the testing objects. This result cannot be derived from the formal analysis. A possible explanation is that in these testing objects we constructed, the possibility of triggering a masking effect is relatively small in a test case. Consequently if we take the strategy *regarded as one failure*, we are more likely to misjudge a test case which triggered other failures to be the failing test case for the failure we currently focus on.

Test cases: The number of test cases generated for identifying the MFS indicates the cost of FCI approach. The result for this metric is listed in Figure 6(g). We can obviously find that the strategy *ILP* generated more test cases than the other strategies. In specific, the gap between the *ILP* and other two strategies is ranged from about 2 to 5 (except for the 6th testing object, which exceeds 20), this is acceptable when comparing to the all the test cases that each approach needed. The increase in test cases for our approach is necessary, as additional test cases must be generated when some test cases are not satisfied for identifying the MFS of the currently analysed failure. As for strategies *distinguishing failures* and *regarded as one failure*, there is no significant difference between the number of test cases generated.

Above all, we draw three conclusions, which help to answer **Q2**:

1)The results of strategy *distinguishing failures* and *regard one failure* are consistent with the previous formal analysis.

2)Considering the quality of the MFS each approach identified, we can find that our *ILP* approach get the best performance, followed by the strategy *distinguishing failures*.

3)Although our approach need more test cases than the other two strategies, it is an acceptable number.

6.3. Evaluating the ILP-based test case searching method

The third empirical study aims to evaluate the efficiency of the ILP-based test case searching component in our approach. To conduct this study, we implemented an FCI approach which is also augmented by the *replacing test cases* strategy, but the test case is randomly replaced.

6.3.1. Study setup. The setup of this case study is based on the second case study, and uses the same SUT model as shown in Table XXIII. We apply the new random searching based FCI approach to identify the MFS in these prepared SUTs. To avoid the bias coming from the randomness, we repeat the new approach 30 times to identify the MFS in each failing test case. We then compute the average additional test cases as well as other metrics listed in the section 6.2.1 for the random-based approach.

6.3.2. Results and discussion. The evaluation of this random-based approach is also shown in Figure 6, in which the polygonal line marked with '$\times$' points in each subfigure indicates the results. The raw data can be also found in the column 'R' of Table XXIV in the appendix.

Compared to our ILP-based approach, we can firstly observe that there is little distinction between them in terms of the metrics: accurate schemas, parent-schemas, sub-schemas, ignored schemas, irrelevant schemas (for some particular cases the ILP-based approach performs slightly better, e.g., in Figure 6(b) for the first testing object, the ILP-based approach identified less sub schemas than that of the Random-based approach and in Figure 6(c) still for the first object the ILP -based approach identified less parent schemas than that of the random-based approach). The similarity quality of the identified MFS between these two approaches is conceivable as they both use the *test case replacement* strategy, so when examining a schema, both approaches may obtain the same result, although the test cases generated will be different.

Secondly, when considering the cost of each approach, we find the ILP-based approach performs better, which can reduce on average 1 or 2 test cases less compared to the random-based procedure. It is an evidence that our integer programming based searching technique can find a satisfied test case more rapidly than the random approach.

In summary, the answer for **Q3** is that: searching for a satisfied test case does affect the performance of our approach, especially regarding the number of extra test cases, and the ILP-based test cases can handle the masking effects at a relatively smaller cost than the random-based approach.

6.4. Comparison with Feedback driven combinatorial testing

The *FDA-CIT* [Yilmaz et al. 2013] approach can handle masking effects so that the generated covering array can cover all the t-degree schemas without being masked by the MFS. There is an integrated FCI approach in the FDA-CIT, of which this FCI approach has two versions, i.e., *ternary-class* and *multiple-class*. In this paper, we use the multiple-class version for our comparative approach, as Yilmaz claims that it performs better than the former [Yilmaz et al. 2013].

The FDA-CIT process starts with generating a t-way covering array(In [Yilmaz et al. 2013], this is a test case-aware covering array [Yilmaz 2013]). After executing the test cases in this covering array, it records the outcome of each test case and then

applies the classification tree method (Wekas implementation of C4.5 algorithm(J48) [Hall et al. 2009]) on the test cases to characterize the MFS for each failure. It then labels these MFS as the schemas that can trigger masking effects. And later if the interaction coverage is not satisfied (here the interaction coverage criteria is different from the traditional covering array, details see [Yilmaz et al. 2013]), it will re-generate a covering array that aims to cover these schemas that were masked by these MFS labeled as masking effects and then repeat the previous steps.

The main target of FDA-CIT is to make the generated test cases to cover all the t-degree schemas. In order to achieve this goal, FDA-CIT needs to repeatedly identify the schemas that can trigger the masking effects. So to make the two approaches (FDA-CIT and ILP) comparable, we need to collect all the MFS that FDA-CIT characterized in each iteration and then compare them with the MFS identified by our approach.

6.4.1. Study setup. As the FCI approach of FDA-CIT use a post-analysis(classification tree) technique on covering arrays, we first generated 2 to 4 ways covering arrays. The covering array generating method is based on augmented simulated annealing [Cohen et al. 2003], as it can be easily extended with constraint dealing and seed injecting [Cohen et al. 2007b], which is needed in the FDA-CIT process. As different test cases will influence the results of the characterization process, we generated 30 different 2 to 4 way covering arrays and fed them to the FDA-CIT. Then after running the FDA-CIT, we recorded the MFS identified, and by comparing them with prior actual MFS, we can evaluate the quality of the identified schemas according to the metrics mentioned in the previous case study.

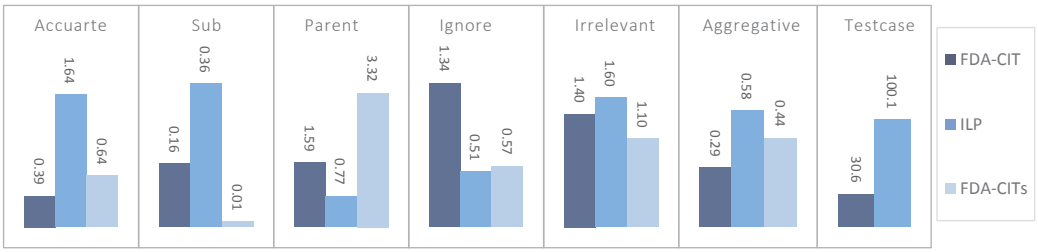
Besides the FDA-CIT, we also applied our ILP-based approach to the generated covering array. Specifically, for each failing test case in the covering array, we separately applied our approach to identify the MFS in that case. In fact, we can reduce the number of extra test cases if we utilize the other test cases in the covering array [Li et al. 2012]), but we didn't utilize the information to simplify the experiment. The same as FDA-CIT, we then recorded the MFS that are identified by our approach, and evaluate them according to the corresponding metrics. In addition, we recorded the overall test cases (including the initially generated covering array) that this approach needed and compare the magnitude of these test cases with that of the approach FDA-CIT.

As mentioned before, the FCI approach in FDA-CIT, i.e., classification tree algorithm, is a post-analysis technique. Given different set of test cases, the results identified by the classification tree algorithm are also different. Then a nature question is, what the schemas identified by FDA-CIT will be if the classification tree method is applied on the the test cases generated by our approach ILP? To figure this question out is of importance as first, we can learn that whether the test cases generated by ILP can help FDA-CIT approach to improve its quality of the identified schemas; second, the comparison between ILP and FDA-CIT will be more fair as they share the same test cases. For this, a new approach that based on FDA-CIT is introduced, which is augmented by replacing the original test cases in FDA-CIT with those generated by ILP approach. Then the schemas identified by the classification tree algorithm in FDA-CIT are recorded and evaluated. This new approach is referred to *FDA-CITs* later.

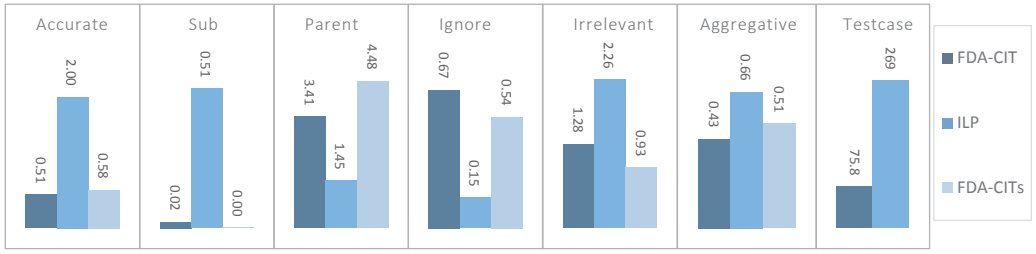
6.4.2. Result and discussion. The result is listed in Figure 7. We conducted three groups of experiments. The first one generated 30 different 2-way cover arrays for each testing object, and then for each covering array we applied the three approaches to identify the MFS. The average evaluation results for the experiments based on 30 covering arrays is recorded and listed in the Sub-figure 7(a). The other two groups of experiments starts with 3-way covering arrays and 4-way covering arrays, of which their results are depicted in Sub-figure 7(b) and 7(c) respectively.

In each sub-figure, there are 7 columns, indicates the outcomes for the previous mentioned 6 metrics and one more metric (Column *Testcase*), which indicates the overall test cases that each approach needed. Each column has three bars (Except for the Column *Testcase*, as the overall test cases for approach ILP and FDA-CITs are the same), which indicates the results for approach FDA-CIT, ILP and FDA-CITs, respectively.

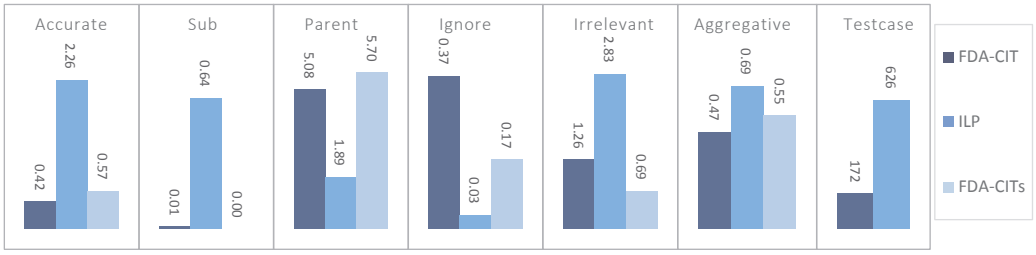
It is noted that in Figure 7, the results for each metric is the average evaluation for all the results of the experiments on the 15 testing objects in Table XXIII. The raw data for each testing object are listed in Table XXV in the appendix . The raw data is organised the same as Table XXIV, except that we added a column t which indicates the strength of the covering array generated for this experiment.



(a) Result for the 2-way covering array



(b) Result for the 3-way covering array



(c) Result of the 4-way covering array

Fig. 7. Three approaches augmented with the replacing strategy

With respect to the relationships between the results and the degree t of the covering arrays, we make the following observations:

First, for every metric in our study, the order of the performance of each approach is stable against the change of degree t. Take for example the metric *accurate number*. No matter what t is (2 , 3 or 4), *ILP* always obtained the most schemas that are identical to the actual MFS, and then is *FDA-CITs*, and the last is *FDA-CIT*. This observation

indicates that the difference between the performance of these approaches is not dependent on the characteristics of the covering array, but instead on the approaches themselves.

Second, with increasing t, the overall performance of each approach is improved. For example, the score of the aggregative metric of *ILP* is 0.55, 0.66 and 0.69, respectively, for t is 2, 3 and 4. The improvement is mainly because with increasing t, the number of test cases also increased, based on which, the approach will observed more failing test cases (See area B in Figure 3), so that we can get the schemas more close to the actual MFS.

Third, for different approaches in our study, the effect of the change of t on the scores of other metrics varies. Specifically, for *ILP*, with increasing t, metrics *accurate number, sub number, parent number, irrelevant number* also increase, while metric *ignore number* decreases. This is mainly because that *ILP* is based on the FCI approach – *FIC_BS* [Zhang and Zhang 2011], which works on single failing test case. As a result, when t increases, the number of test cases increase. Then when applying our approach, more different schemas may be identified from these increased failing test cases, so the number of accurate MFS, sub-schemas of the actual MFS, parent-schemas of the actual MFS, schemas that irrelevant to the actual MFS will increase, and also some actual MFS that had been ignored before may be obtained. For *FDA-CIT* and *FDA-CITs*, however, we find that *sub number, irrelevant number* decrease with increasing t. We believe this result is due to that their FCI approach is the classification tree method. A typical classification tree works by partitioning the test cases according to some aspects. Here, the aspect is the parameter value of the SUT. And one path (conjunction of nodes from the root to one leaf in the tree) in this tree is deemed as an MFS. So when test cases increase, the classification may need more nodes to classify the test cases. This is the so-called 'over fitting' problem. As a result, the schemas that identified by *FDA-CIT* and *FDA-CITs* tend to be the parent schemas of the actual MFS, leading to a decrease of metrics *sub number,* and *irrelevant number*.

Besides the degree t of covering array, the comparison between the three approachers at other metrics are also needed to note:

First, when compared with the original approach *FDA-CIT*, *FDA-CITs* has obvious advantages at almost all the metrics except for metric *parent number*. In detail, *FDA-CITs* obtained more schemas that are identical to the actual MFS (*accurate number*), identified less schemas that are the sub schemas of actual MFS (*sub number*), and has lower scores for the two negative metrics (*ignored number* and *irrelevant number*). At last, the schemas identified by *FDA-CITs* showed an overall higher quality than that of the original *FDA-CIT* (*aggregative metric*). We have discussed previously that *FDA-CIT* tends to identify parent-schemas of actual MFS if the test cases increase. So for metric *parent number*, it is no surprise that *FDA-CITs* identified more parent schemas of actual MFS than *FDA-CIT*, because it used the test cases generated by *ILP*, which were more than that of the original FDA-CIT. The difference between the overall performance of *FDA-CIT* and *FDA-CITs* is also expected; in fact, this result is consistent with our previous observation that when t increases, the overall performance for each approach also increases.

Second, in terms of the quality of the MFS that each approach identified, we can clearly find that our approach performed better than the other approaches. This is mainly manifested in that our approach obtained more accurate schemas and identified less irrelevant ones. We believe this gap is mainly caused by the FCI approach. Because for *ILP* and *FDA-CITs*, the test cases used to identify the MFS are the same. The only difference is how they utilize them to identify MFS. However, this result doesn't mean that the FCI approach – FIC_BS is better than the classification tree method at all the conditions. The classification tree method used in the FCI approach has its

own advantage, i.e., it doesn't need to generate additional test cases, and as a result, *FDA-CIT* generated less test cases than that of *ILP*.

In fact, another reason why our approach generated more test cases is that the FCI approach, i.e., FIC_BS , works on single test case, so when there are many failing test cases in the covering array, we need to repeatedly use our approach to identify the MFS for each failing test case. This process may produce many redundant test cases, because many failing test cases contain the same MFS, and when we have already identified the MFS in one test case, there is no need to identify it again in other failing test cases. Jieli [Li et al. 2012] introduced a method that utilizes the previous generated test cases to reduce such redundance. Here we didn't use this technique to simplify this experiment. We believe if we utilize the MFS that are already identified in previous iteration, the overall number of test cases will decrease.

Above all, we can conclude three points in this experiment, which provide answer for **Q4**:

1)The degree t of the covering array doesn't determine the order of the performance of different approaches, but for one specific approach, the bigger the t is, the better that the approach will perform.

2)When taking the test cases generated by our approach *ILP*, *FDA-CITs* performed better than the original *FDA-CIT* approach.

3)Considering the quality of the MFS each approach identified, *ILP* performed better than the other two approaches, but our approach needed more test cases.

Based on these observations, a recommendation for selecting masking handling techniques in practice is that to get a more precise identification of the MFS in the SUT, *ILP* is preferred, and for a small size of test cases due to the cost, *FDA-CIT* is a better choice.

6.5. Threats to validity

6.5.1. internal threats. There are two threats to internal validity. First, the characteristics of the actual MFS in the SUT can affect the FCI results. This is because the magnitude and location of the MFS can make the FCI approaches generate different test cases. And as a result, it can make the observed failing test cases and predicted failing test cases different (See Figure 3). In the worst case, there is a chance that the FCI approach happens to identify the exact actual MFS, and in that condition our test case replacing strategy is of no use. In this paper, we took 15 testing objects, in which 5 are real software systems with real faults and 10 synthetic ones with injected faults. To reduce the influence caused by different characteristics of the MFS, we need to build more testing objects and injected more different type of faults for a more comprehensive study of our approach.

The second threat is that we just applied our test case replacing strategy on one FCI approach – FIC_BS [Zhang and Zhang 2011]. Although we believe the test case replacing strategy can also improve the quality of the identified MFS for other FCI approaches when the testing object is suffering from masking effects, the extent to which their results can be refined may vary for different FCI approaches. For example, for the FCI approach FIC_BS [Zhang and Zhang 2011] we used in this paper, there are about $(v-1)$ to $(v-1)^{k-1}$ candidate test cases that can be replaced (See Table XVIII) when one test case triggered other failures, while for the approach OFOT [Nie and Leung 2011a], we only have $(v-1)$ candidates. As a result, FIC_BS can have a higher chance than OFOT to find a satisfied test case. So to learn the difference between the improvement of different FCI approaches when applying our test case replacing strategy, we need to try more FCI approaches in the future.

6.5.2. external threats. One threat to external validity raised from the real software we used. In this paper we have only surveyed two types of open-source software with five different versions, of which the program scale is medium-sized. This may impact the generality of our results.

Another important threat is that our approach is based on the assumption that different errors in the software can be easily distinguished by information such as exception traces, state conditions, or the like. If we cannot directly distinguish them, our approach does not work. In such case, one potential solution is to take the clustering techniques to classify the failures according to available information [Zheng et al. 2006; Jones et al. 2007; Podgurski et al. 2003]. By the way, if we cannot classify them for we do not have enough programming executing information (e.g., the black box testing) or it is too costly, we believe the only approach is to take the *regarded as one failure* strategy. With this strategy, we must take care that the MFS we identified are likely to be sub-schemas or irrelevant schemas of the actual MFS.

The third threat comes from the possible masking relationships between multiple failures in the real software. In this paper, we just focus on the condition that the masking effects are transitive, i.e., if failure A masks B, failure B masks C, then failure A must mask the failure C. In practice, the relationships between multiple failures may be more complicated. One possible scenario is that two failures are in a loop, for which the two failures can even mask each other in a particular condition. Such a case will make our formal analysis invalidate and will significantly complicate the relationships between schemas and their corresponding test cases. A new formal model should be proposed to handle that type of masking effects.

7. RELATED WORKS

Shi and Nie presented an approach for failure revealing and failure diagnosis in CT [Shi et al. 2005], which first tests the SUT with a covering array, then reduces the value schemas contained in the failing test case by eliminating those appearing in the passing test cases. If the failure-causing schema is found in the reduced schema set, failure diagnosis is completed with the identification of the specific input values which caused the failure; otherwise, a further test suite based on SOFOT is developed for each failing test cases,testing is repeated, and the schema set is then further reduced, until no more failures are found or the failure is located. Based on this work, Wang proposed an AIFL approach which extended the SOFOT process by adaptively mutating factors in the original failing test cases in each iteration to characterize failure-inducing combinations [Wang et al. 2010].

Nie et al. introduced the notion of Minimal Failure-causing Schema(MFS) and proposed the OFOT approach which is an extension of SOFOT that can isolate the MFS in SUT [Nie and Leung 2011a]. The approach mutates one value with different values for that parameter, hence generating a group of additional test cases each time to be executed. Compared with SOFOT, this approach strengthen the validation of the factor under analysis and also can detect the newly imported faulty combinations.

Delta debugging proposed by Zeller [Zeller and Hildebrandt 2002] is an adaptive divide-and-conquer approach to locate interaction failure. It is very efficient and has been applied in real software environment. Zhang et al. also proposed a similar approach that can efficiently identify the failure-inducing combinations that has no overlapped part [Zhang and Zhang 2011]. Later, Li improved the delta-debugging based failure-inducing combination by exploiting useful information in the executed covering array [Li et al. 2012].

Colbourn and McClary proposed a non-adaptive method [Colbourn and McClary 2008]. Their approach extends a covering array to the locating array to detect and locate interaction failures. C. Martinez proposed two adaptive algorithms. The first

one requires safe value as the assumption and the second one removes the assumption when the number of values of each parameter is equal to 2 [Martínez et al. 2008; 2009]. Their algorithms focus on identifying faulty tuples that have no more than 2 parameters.

Ghandehari et al. defined the suspiciousness of tuple and suspiciousness of the environment of a tuple [Ghandehari et al. 2012]. Based on this, they rank the possible tuples and generate the test configurations. They further utilized the test cases generated from the inducing combination to locate the failures inside the source code [Ghandehari et al. 2013].

Yilmaz proposed a machine learning method to identify inducing combinations from a combinatorial testing set [Yilmaz et al. 2006]. They constructed a classification tree to analyze the covering arrays and detect potential faulty combinations. Beside this, Fouché [Fouché et al. 2009] and Shakya [Shakya et al. 2012] made some improvements in identifying failure-inducing combinations based on Yilmaz's work.

Our previous work [Niu et al. 2013] proposed an approach that utilizes the tuple relationship tree to isolate the failure-inducing combinations in a failing test case. One novelty of this approach is that it can identify the overlapped faulty combinations. This work also alleviates the problem of introducing new failure-inducing combinations in additional test cases.

In addition to the studies that aim at identifying the failure-inducing combinations in test cases, there are others that focus on working around the masking effects.

Constraints handling become more and more popular in CT these years. A constraint is a invalid combination that should not appear in the test case. It can be deemed as the masking effect which are known in prior [Yilmaz et al. 2013]. Cohen [Cohen et al. 2007a; 2007b; 2008] studied the impact of the constraints that render some generated test cases invalid in CT. They also proposed an approach that integrates the incremental SAT solver with the covering arrays's generation algorithm to avoid those invalidate combinations. Further study was conducted [Petke et al. 2013]to show that with considering constrains, higher-strength covering arrays with early failure detection are practical.

Besides, additional works that aim to study the impacts of constraints for CT were as follows: [Garvin et al. 2011; Bryce and Colbourn 2006; Calvagna and Gargantini 2008; Grindal et al. 2006; Yilmaz 2013]. Among them, [Bryce and Colbourn 2006] distinguished the constraints into two types: *hard* and *soft*, which the former cannot be included in the test case, while the latter can be permitted, but not desirable. [Grindal et al. 2006] comprehensively compared the performance of four strategies at handling the constraints in the covering array. [Calvagna and Gargantini 2008] proposed an heuristic strategy to handle the constraints. It can support an ad-hoc inclusion or exclusion of combinations such that the user can customize output of the covering array. [Garvin et al. 2011] refined the simulated annealing algorithm to efficiently construct the covering array with considering the constraints. [Yilmaz 2013] introduced the test case-specific constraints, differing from the system-wide constraints, this constraint can only be triggered in some specific test cases.

Chen et al. addressed the issues of shielding parameters in combinatorial testing and proposed the Mixed Covering Array with Shielding Parameters (MCAS) to solve the problem caused by shielding parameters [Chen et al. 2010]. The shielding parameters can disable some parameter values to expose additional interaction errors, which can be regarded as a special case of masking effects.

Dumlu and Yilmaz proposed a feedback-driven approach to work around the masking effects [Dumlu et al. 2011]. Specifically, they first used classification tree approach to classify the possible failure-inducing combinations and then eliminate them and generate new test cases to detect possible masked interaction in the next iteration.

They further extended their work [Yilmaz et al. 2013] by proposing a multiple-class CTA approach to distinguishing failures in SUT. In addition, they empirically studied the impacts on both ternary-class and multiple-class CTA approaches.

These works can be categorized into 3 groups according to their relationships with our work. First, the works that aim to identifying the MFS in the SUT. It is known that our work also focus on identifying the MFS, but instead of single failure, our work considered the impacts of multiple failures on the FCI approaches, and based on this, a test case replacement strategy is proposed that can assist these FCI approaches in reducing the negative effects. Second, the works that aims to handling the constraints. As discussed before the constraints can be deemed as a special masking effects. Our work differs from them in that the masking effects we handled in this paper is those can be dynamically triggered, that is, we didn't know them in prior. Another difference between our work with these constraints handling works is that their target is to avoid the constraints when generating covering array, however, our work aims to removing the masking effects of the FCI approaches. Last, the work that is most similar to our work [Yilmaz et al. 2013], which also considered the masking effects that are dynamically appeared in test cases. But different from our work, it mainly focus on reducing the masking effects in the covering array, so that the covering array can support a comprehensive validation of all the t-degree schemas. The approach it used to reduce this negative effect is to take the FCI approach to identify the schemas that can trigger this effect in each iteration. Our approach, however, aims to handling the masking effects that happened in these FCI approaches themselves, and the approach we used to alleviate the masking effects is t o augment the FCI approaches with test case replacement strategy.

8. CONCLUSIONS

Masking effects of multiple failures in SUT can bias the results of traditional failure-inducing combinations identifying approaches. In this paper, we formally analysed the impact of masking effects on FCI approaches and showed that the two traditional strategies, i.e., *regarded as one fault* and *distinguishing failures*, are both inefficient in handling such impact. We further presented a test case replacement strategy for FCI approaches to alleviate such impact.

In our empirical studies, we extended FCI approach – FIC_BS [Zhang and Zhang 2011] with our strategy. The comparison between our approach and traditional approaches was performed on several kinds of open-source software. The results indicated that our strategy assists the traditional FCI approach in achieving better performance when facing masking effects in SUT. We also empirically evaluated the efficiency of the test case searching component by comparing it with the random searching based FCI approach. The results showed that the ILP-based test case searching method can perform more efficiently. Last, we compared our approach with existing technique for handling masking effects – FDA-CIT [Yilmaz et al. 2013], and observed that our approach achieved a more precise result which can better support debugging, though our approach generated more test cases than FDA-CIT.

As for the future work, we need to do more empirical studies to make our conclusions more general. Our current experiments focus on middle-sized software. We would like to extend our approach to more complicated, large-scaled testing scenarios. Another promising work in the future is to combine the white-box testing technique to facilitate obtaining more accurate results from the FCI approaches when handling masking effects. We believe that figuring out the failure levels of different bugs through the white-box testing technique is helpful to reduce misjudgement in the identification of failure-inducing combinations. And last, because the extent to which the FCI suffers from masking effects varies with different algorithms, a combination of different F-

CI approaches would be desired in the future to further improve identifying MFS for multiple failures.

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Online Appendix to: Identifying minimal failure-causing schemas for multiple failures

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A. THE DETAIL OF THE EXPERIMENTS

Table XXIV. Result of the evaluation of each appraoch

Subject	**accurate**				**sub**				**parent**				**ignore**				**irrelevant**				**overall**				**test cases**			
	O [1]	D [2]	I [3]	R [4]	O	D	I	R	O	D	I	R	O	D	I	R	O	D	I	R	O	D	I	R	O	D	I	R
HSQLDB 2cr8	2	3	5	5	3	2	0	2	0	2	4	5	0(2.04)	0(3.91)	0(4.0)	0(4.1)	0	2	0	34	0.57	0.65	0.88	0.23	8.125	11.92	17	17.72
HSQLDB 2.2.5	2	2	2	2	1	0	0	0	0	1	1	1	0(0.83)	0(1.0)	0(1.0)	0(1.0)	7	0	0	0	0.25	0.83	0.83	0.83	8.67	7.67	10.17	11.3
HSQLDB 2.2.9	2	3	2	2	3	1	1	1	0	4	1	1	0(1.33)	0(2.83)	0(2.56)	0(2.56)	4	0	0	0	0.4	0.74	0.8	0.8	9.167	8.61	11.72	13.14
JFlex 1.4.1	2	2	2	2	0	0	0	0	0	1	0	0	0(0.75)	0(1.0)	0(1.0)	0(1.0)	25	0	0	0	0.07	0.83	1	1	23.5	6.5	8	9.68
JFlex 1.4.2	2	2	2	2	1	0	0	0	0	1	1	1	0(0.67)	0(1.0)	0(1.0)	0(1.0)	25	0	0	0	0.09	0.83	0.83	0.83	20.5	9	11.67	13.12
synthez 1	2	1	1	1	2	0	0	0	0	2	2	2	0(1.0)	0(2.0)	0(2.0)	0(2.0)	15	17	17	17	0.19	0.11	0.11	0.11	16.5	18	41.75	41.75
synthez 2	3	3	3	3	10	0	0	0	0	10	6	6	0(1.96)	0(2.0)	0(2.0)	0(2.0)	1	0	0	0	0.54	0.76	0.8	0.8	11.19	14.12	16.96	17.08
synthez 3	4	3	3	3	4	2	2	2	0	5	5	5	0(2.08)	0(2.84)	0(2.84)	0(2.84)	13	9	9	9	0.28	0.34	0.34	0.34	12.73	9.46	14.18	14.44
synthez 4	3	3	3	3	10	3	2	3	0	6	5	5	0(2.6)	0(2.8)	0(2.9)	0(2.85)	9	9	9	9	0.35	0.4	0.39	0.39	9.91	13.02	18.55	18.45
synthez 5	2	2	2	2	4	0	0	0	0	2	1	1	0(1.02)	0(1.0)	0(1.0)	0(1.0)	1	0	0	0	0.65	0.88	0.92	0.92	13.04	13.7	14.77	14.84
synthez 6	2	3	3	3	15	4	4	4	0	8	8	8	0(1.99)	0(3.72)	0(3.72)	0(3.72)	8	10	10	10	0.38	0.36	0.36	0.36	14.91	11.75	15.37	15.71
synthez 7	3	3	3	3	10	0	0	0	0	6	6	6	0(2.04)	0(2.0)	0(2.0)	0(2.0)	5	0	0	0	0.39	0.83	0.83	0.83	12.77	14.59	16.44	16.53
synthez 8	2	2	2	2	4	0	0	9	0	4	3	3	0(1.05)	0(1.0)	0(1.0)	0(1.0)	3	0	0	0	0.56	0.9	0.91	0.91	24.45	25.25	26.27	26.37
synthez 9	2	1	1	1	1	0	0	0	0	1	1	1	0(0.8)	0(1.0)	0(1.0)	0(1.0)	0	0	0	0	0.75	0.83	0.83	0.83	6.8	8	9	9
synthez 10	0	1	1	1	3	1	1	1	0	0	0	0	0(1.0)	0(1.46)	0(1.31)	0(1.31)	0	2	1	1	0.5	0.47	0.58	0.58	9.08	11	15.38	15.53

[1] O is for the strategy *regarded as one failure*.
[2] D is for the strategy *distinguishing failures*.
[3] I is for the replacement strategy based on ILP searching.
[4] R is for the replacement strategy based on randomly searching.

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Table XXV. Comparison with FDA-CIT

Subject		**accurate**			**sub**			**parent**			**ignore**			**irrelevant**			**overall**			**test cases**		
	t	F [1]	I [2]	Fs [3]	F	I	Fs	F	I	F	F	I	Fs	F	I	Fs	F	I	Fs	F	I	Fs
HSQL2cr8	2	0.17	2.27	1.57	0.57	0	0	0.17	0.4	2.17	3.87	2.3	2	2.53	0	1.97	0.12	0.51	0.39	23.6	70.1	70.1
	3	1.47	3.67	1	0	0	0	4.67	2	6.07	0.63	0.3	0.17	3	0	1.47	0.51	0.87	0.6	76.6	241.8	241.8
	4	0.83	4.8	1	0	0	0	9.03	3.37	8	0	0	0	0.97	0	0	0.65	0.9	0.71	183.5	606.6	606.6
HSQL2.2.5	2	1	1.97	0.37	0	0	0	2.4	0.73	3.8	0.4	0	0	1.4	0	0.1	0.38	0.87	0.56	26.7	68.8	68.8
	3	0	2	0.4	0	0	0	5	1	3.8	0	0	0	0	0	0	0.52	0.83	0.56	67	202.4	202.4
	4	0	2	0.33	0	0	0	5	1	4	0	0	0	0	0	0	0.53	0.83	0.56	130.1	503.3	503.3
HSQL2.2.9	2	0.9	1.77	0.9	0	0.77	0	1.47	0.47	6.8	1.93	0.53	0	2.37	0	0.2	0.28	0.72	0.58	29.2	78.3	78.3
	3	1	2	0.83	0	1	0	5.13	0.93	7.1	0.2	0	0	0.1	0	0	0.61	0.8	0.61	72.8	221.7	221.7
	4	1	2	1	0	1	0	5.87	1	6.7	0	0	0	0	0	0	0.64	0.8	0.62	129.8	560.3	560.3
JFlex 1.4.1	2	0	2	0	0	0	0	4.03	0	4	0	0	0	0	0	0	0.49	1	0.5	30.5	87.3	87.3
	3	0	2	0	0	0	0	4	0	0	0	0	4	0	0	0	0.5	1	0.5	73.4	269.2	269.2
	4	0	2	0	0	0	0	4	0	0	0	0	0	0	0	0	0.5	1	0.5	190.6	724.7	724.7
JFlex 1.4.2	2	0.3	1.97	0.93	0	0	0	3.6	1	2.16	0.03	0	0	0.63	0	0	0.5	0.83	0.62	34.3	106.9	106.9
	3	0	2	0.97	0	0	0	5	1	2.1	0	0	0	0.03	0	0	0.52	0.83	0.61	72.3	305.7	305.7
	4	0	2	1	0	0	0	5	1	2	0	0	0	0	0	0	0.53	0.83	0.61	186.8	836.9	836.9
synthez 1	2	0.97	1	1	0	0	0	1.7	1.93	2	0	0.07	0	0.33	14.3	0	0.66	0.13	0.78	40.3	342.87	342.87
	3	1	1	1	0	0	0	2	2	2	0	0	0	0	16.73	0	0.78	0.12	0.78	93.4	809.1	809.1
	4	1	1	1	0	0	0	2	2	2	0	0	0	0	17	0	0.78	0.12	0.78	218.8	1532.8	1532.8
synthez 2	2	0.17	1.3	0.73	0.37	0	0	0	0.4	2.37	2.27	1.2	1.03	1.37	0	1.2	0.11	0.52	0.4	19.77	54.4	54.4
	3	0.73	2.23	0.5	0	0	0	1.9	1.3	7.1	1.2	0.43	0.53	2.2	0	1.33	0.36	0.82	0.52	59.5	171.5	171.5
	4	0.63	2.97	0.1	0	0	0	5.3	2.33	16.1	0.53	0	0	2.6	0	1	0.44	0.89	0.54	152.7	415.1	415.1
synthez 3	2	0.43	2.97	0.73	0	0.93	0	4.3	1.73	5.3	0.47	0.17	0.5	1.03	3.77	1.13	0.37	0.46	0.37	48.6	138.7	138.7
	3	0.2	3	0.87	0	1.57	0	7.2	3.67	6.57	0.07	0	0	0.83	6.77	0.07	0.38	0.38	0.44	106.3	315.3	315.3
	4	0.03	3	1	0	1.97	0	10.4	3	6	0	0	0	0.43	8.56	0	0.38	0.34	0.45	147.9	565.7	565.7
synthez 4	2	0.3	2.3	0.33	0.07	0.63	0	2.63	1.97	7.7	1.93	0.63	0.4	3.4	1.4	1.97	0.24	0.6	0.44	42.7	142.2	142.2
	3	0.37	2.97	0.07	0	1.26	0	6.5	3.53	10.97	0.83	0.07	0	2.5	3.43	1.03	0.39	0.54	0.51	86.5	373.2	373.2
	4	0.07	3	0	0	1.77	0	11.7	4.67	11.4	0	0	0	1.33	6.73	0.03	0.48	0.44	0.55	202.2	899.7	899.7
synthez 5	2	0.2	1.2	0.8	0.3	0	0	0.1	0.03	0.83	1.4	0.77	0.97	0.7	0	1	0.2	0.59	0.4	21.9	46.9	46.9
	3	0.87	1.4	0.53	0	0	0	0.5	0.23	3.03	1	0.6	0.77	0.37	0	1.63	0.46	0.71	0.43	76.9	150.3	150.3
	4	0.7	1.9	0.37	0	0	0	1.77	0.33	6.5	0.9	0.1	0.03	1.87	0	2.03	0.34	0.92	0.54	232.9	433.2	433.2
synthez 6	2	0.23	2.63	0.17	0.2	2	0	2.93	1.63	9.63	2.6	0.5	0.4	3.03	3.7	2	0.19	0.42	0.37	45.7	132.6	132.6
	3	0.1	3	0.1	0	2.83	0	7.4	3.83	12.5	1.2	0.17	0.03	2.3	6.5	0.67	0.31	0.38	0.43	99.5	338.9	338.9
	4	0	3	0	0	3.8	0	10.2	6.03	14.5	0.47	0	0	1.8	9.1	0.03	0.37	0.36	0.44	152.6	781.9	781.9
synthez 7	2	0.13	1.43	0.83	0.23	0	0	0.1	0.63	1.4	2.53	1.03	0.93	1.93	0	1.97	0.09	0.61	0.38	20.3	58.8	58.8
	3	0.87	2.17	0.93	0	0	0	0.43	1.23	2.97	1.77	0.17	0.13	3.2	0	2.87	0.2	0.88	0.44	52.6	164.7	164.7
	4	1	2.87	1	0	0	0	3.23	2.53	4.6	0.27	0	0	4.5	0	2.27	0.35	0.9	0.51	145.3	413.1	413.1
synthez 8	2	0	0.2	0.17	0.03	0	0	0	0	0.03	0.3	0.13	0.13	0.17	0	0.3	0.01	0.1	0.05	16.1	45.2	45.2
	3	0	0.6	0.5	0.1	0	0	0	0	0.03	0.97	0.47	0.53	0.63	0	0.87	0.02	0.3	0.17	43.1	64.3	64.3
	4	0	1.33	0.8	0.1	0	0	0	0.07	0.67	1.53	0.4	0.5	1.4	0	0.93	0.04	0.67	0.41	109.3	145.6	145.6
synthez 9	2	1	1	1	0	0	0	0.46	0.6	0.77	0.53	0	0.23	0.63	0.6	0.67	0.54	0.7	0.6	36.2	43.4	43.4
	3	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0.83	0.83	0.83	84.3	145	145
	4	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0.83	0.83	0.83	188	291.6	291.6
synthez10	2	0	0.63	0	0.6	1	0.2	0	0	0.83	1.8	0.37	1.9	1.5	0.3	3.97	0.23	0.61	0.17	23.4	84.9	84.9
	3	0.07	0.97	0	0.23	1	0.03	0.36	0	1.9	2.23	0.03	1.97	4.03	0.53	3.97	0.13	0.66	0.2	73.4	263.2	263.2
	4	0	1	0	0.07	1	0	1.7	0	2	1.87	0	2	4.03	1	4	0.21	0.58	0.2	202.2	685.9	685.9

[1] F is for the FDA-CIT approach.
[2] I is for the our approach with replacement strategy based on ILP searching.
[3] Fs is for the FDA-CITs approach.