

Identifying minimal failure-causing schemas for multiple failures

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Combinatorial testing(CT) has been proven to be effective to reveal the potential failures caused by the interaction of the inputs or options of the System Under Test (SUT). To extend and fully use CT, the theory of Minimal Failure-Causing Schema (MFS) is proposed. The use of MFS helps to isolate the root cause of a failure after detected by CT. Most MFS-based algorithms focus on handling a single failure in the SUT, however, we argue that multiple failures are the more common testing scenario, and under which masking effects may be triggered so that some expected failures will not be observed. Traditional MFS theory, as well as the related identifying algorithms, lack a mechanism to handle such effects; hence, they may incorrectly isolate the MFS in the SUT. To address this problem, we propose a new MFS model that takes into account multiple failures. We first formally analyse the impact of the multiple failures on extant MFS isolating algorithms, especially in situations where masking effects are triggered by these multiple failures. Based on this, we then develop an approach that can assist traditional algorithms to better handle multiple failure testing scenarios. Empirical studies were conducted using several kinds of open-source software, which showed that multiple failures with masking effects do negatively affect traditional MFS identifying approaches and that our approach can help to alleviate these effects.

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1. INTRODUCTION

With the increasing complexity and size of modern software, many factors, such as input parameters and configuration options, can affect the behaviour of the SUT. The unexpected failures caused by the interaction of these factors can make software testing challenging if the interaction space is too large. In the worst case, we need to examine every possible combination of these factors as each combination can contain unique failure [Song et al. 2012]. While conducting exhaustive testing is ideal and necessary in theory, it is impractical and uneconomical. One remedy for this problem is

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Table I. MS word example

id	Highlight	Status bar	Bookmarks	Smart tags	Outcome
1	On	On	On	Off	PASS
2	On	Off	Off	On	PASS
3	Off	On	Off	Off	Fail
4	Off	Off	On	Off	PASS
5	Off	On	On	On	PASS

combinatorial testing, which systematically samples the interaction space and selects a relatively small set of test cases that cover all valid iterations, with the number of factors involved in the interaction no more than a prior fixed integer, i.e., the *strength* of the interaction. Many works in CT aim to construct the smallest set of efficient testing objects [Cohen et al. 1997; Bryce et al. 2005; Cohen et al. 2003], which also called *covering array*.

Once failures are detected by the covering array, the failure-inducing combinations in these failed test cases must be isolated. This task is important in CT as it can facilitate debugging efforts by reducing the code scope that needed for inspection [Ghandehari et al. 2012]. However, information from the covering array sometimes does not clearly identify the location and magnitude of the failure-inducing combinations [Colbourn and McClary 2008]. Thus, deeper analysis is needed. Consider the following example [Bach and Schroeder 2004], Fig I presents a two-way covering array for testing an MS-Word application in which we want to examine various combinations of options for the MS-Word ‘Highlight’, ‘Status Bar’, ‘Bookmarks’ and ‘Smart tags’. Assume the third test case failed. We then can get five two-way suspicious combinations that may be responsible for this failure. They are respectively (Highlight: Off, Bookmarks: Off), (Highlight: Off, Smart tags: Off), (Status Bar: On, Bookmarks: Off), (Status Bar: On, Smart tags: Off), and (Bookmarks: Off, Smart tags: Off). (Note that (Highlight: Off, Status Bar: On) is excluded in this set as it appeared in the fifth passing test case). Without any more information, we cannot figure out which one or more of the combinations in this suspicious set caused the failure. In fact, taking into account the higher strength combination, e.g., (Highlight: Off, Status Bar: On, Smart tags: Off), the problem becomes more complicated.

To address this problem, prior work [Nie and Leung 2011a] specifically studied the properties of the minimal failure-causing schemas in SUT, based on which a further diagnosis by generating additional test cases was applied that can identify the MFS in the test case. Other works have proposed ways to identify the MFS in SUT, which include approaches such as building a tree model [Yilmaz et al. 2006], generating additional test cases by mutating one factor of the original failed test case at a time [Nie and Leung 2011a], ranking suspicious combinations based on some rules [Ghandehari et al. 2012], using graphic-based deduction [Martínez et al. 2008], among others. These approaches can be partitioned into two categories [Colbourn and McClary 2008] according to how the additional test cases are generated: *adaptive*—additional test cases are chosen based on the outcomes of the executed tests [Nie and Leung 2011a; Ghandehari et al. 2012; Niu et al. 2013; Zhang and Zhang 2011; Shakya et al. 2012; Wang et al. 2010; Li et al. 2012] or *nonadaptive*—additional test cases are chosen independently and can be executed parallel [Yilmaz et al. 2006; Colbourn and McClary 2008; Martínez et al. 2008; 2009; Fouché et al. 2009].

The MFS methodology as well as other MFS-identifying approaches mainly focus on the ideal scenario in which SUT only contains one failure, i.e., the test case either fails or passes the testing. However, in this paper, we argue that SUT with multiple distinguished failures is the more common testing scenario in practice, and moreover, this impacts the Failure-inducing Combinations Identifying (FCI) approaches. One main

impact of multiple failures on FCI approaches is the masking effect. A masking effect [Dumlu et al. 2011; Yilmaz et al. 2013] is an effect in which some failures prevent test cases from normally checking combinations that are supposed to be tested. Take the Linux command *Grep* for example. We noticed that there are two different failures reported in the bug tracker system. The first ¹ claims that *Grep* incorrectly matches unicode patterns with '`\<\>`', while the second ² claims an incompatibility between option '`-c`' and '`-o`'. When we put these two scenarios into one test case, only one failure will be observed, which means another failure is masked by the observed failure. This effect will prevent test cases from executing normally, resulting incorrect judgments of the correlation between the combinations checked in the test case and the failure that been masked and therefore not observed.

As we know that masking effects negatively affect the performance of FCI approaches, a natural question is how this effect biases the results of these approaches. In this paper, we formalize the process of identifying the MFS under the circumstances in which masking effects exist in the SUT and try to answer this question. One insight from the formal analysis is that we cannot completely get away from the impact of masking effects even if we do exhaustive testing. Even worse, either ignore the masking effects or regard multiple failures as one failure is detrimental to the FCI process.

Based on this concern, we propose a strategy to alleviate this impact. This strategy adopts the divide and conquer framework, i.e., separately handles each failure in the SUT. For a particular failure under analysis, when applying traditional FCI approaches to identify the failure-inducing combinations, we pick the test cases generated by FCI approaches that trigger unexpected failures and replace them with newly regenerated test cases. These new test cases should either pass or trigger the same failure under analysis.

The key to our approach is to search for a test case that does not trigger unexpected failures which may introduce the masking effect. To guide the search process, i.e., to reduce the possibility that the extra generated test case will trigger an unexpected failure, a natural idea is to take some characteristics from the existing test cases and make the characteristics of the newly searched test case as much as different from the existing test cases which triggered the unexpected failure. To reach this target, we define the *related strength* between the factor and the failures. The higher the *related strength* between a factor and a particular failure, the greater the likelihood that the factor will trigger this failure. We then use the integer linear programming (ILP) technique to find a test case which has the least *related strength* with the unexpected failure.

To evaluate the performance of our approach, we applied our strategy on the FCI approach FIC_BS [Zhang and Zhang 2011]. The subjects we used were several open-source software found in the developers' forum in the Source-Forge community. Through studying their bug reports in the bug tracker system as well as their user's manuals, we built a testing model which can reproduce the reported bugs with specific test cases. We then compared the FCI approach augmented with our strategy to the traditional FCI approach with these subjects. We further empirically studied the performance of the important component of our strategy – searching satisfied test cases. To conduct this study, we compare our approach with the augmented FCI approach by randomly searching satisfied test cases. We finally compare our approach with the existing masking handling technique – FDA-CIT[Yilmaz et al. 2013]. All of these empirical studies showed that our replacing strategy as well as the searching test case component achieved a better performance than these traditional approaches when the

¹<http://savannah.gnu.org/bugs/?29537>

²<http://savannah.gnu.org/bugs/?33080>

```

public float foo(int a, int b, int c, int d){
    //step 1 will cause an exception when b == c
    float x = (float)a / (b - c);

    //step 2 will cause an exception when c < d
    float y = Math.sqrt(c - d);

    return x+y;
}

```

Fig. 1. A toy program with four input parameters

subject suffered multiple failures, especially when these failures can import masking effects.

The main contributions of this paper are:

- We studied the impact of the masking effects among multiple failures on the isolation of the failure-inducing combinations in SUT.
- We proposed a divide and conquer strategy of selecting test cases to alleviate the impact of these effects.
- We designed an efficient test case searching method which can rapidly find a test case that does not trigger an unexpected failure.
- We conducted several empirical studies and showed that our strategy can assist FCI approaches to achieve better performance in identifying failure-inducing combinations in SUT with masking effects.

2. MOTIVATING EXAMPLE

For convenience, we constructed a small program example to illustrate the motivation of our approach. Assume we have a method *foo* which has four input parameters: *a*, *b*, *c*, and *d*. The four parameter types are all integers and the values that they can take are: $v_a = \{7, 11\}$, $v_b = \{2, 4, 5\}$, $v_c = \{4, 6\}$, $v_d = \{3, 5\}$. The code detail the method is shown in Fig 1.

Considering the simple code in Fig 1, we can find two potential failures: first, in the step 1 we can get an *Arithmetic Exception* when *b* is equal to *c*, i.e., $b = 4$ and $c = 4$, that makes division by zero. Second, another *Arithmetic Exception* will be triggered in step 2 when $c < d$, i.e., $c = 4$ and $d = 5$, which makes square roots of negative numbers. So the expected failure-inducing combinations in this example should be $(-, 4, 4, -)$ and $(-, -, 4, 5)$.

Traditional FCI algorithms do not consider the code detail; instead, they apply black-box testing to test this program, i.e., feed inputs to those programs and execute them to observe the result. The basic justification behind those approaches is that the failure-inducing combinations for a particular failure can only appear in those inputs that trigger this failure. Traditional FCI approaches aim at using as few inputs as possible to get the same (or approximate) result as exhaustive testing, so the results derived from an exhaustive testing set are the best that these FCI approaches can achieve. Next, we will show how exhaustive testing works to identify the failure-inducing combinations in the program.

We first generate every possible input listed in the column “test inputs” of Table II, and the execution results are listed in the result column of Table II. In this column, *PASS* means that the program runs without any exception under the input in the same row. *Ex 1* indicates that the program triggered an exception corresponding to step 1 and *Ex 2* indicates the program triggered an exception corresponding to step

Table II. test inputs and their corresponding result

id	test inputs	results	id	test inputs	result
1	(7, 2, 4, 3)	PASS	13	(11, 2, 4, 3)	PASS
2	(7, 2, 4, 5)	Ex 2	14	(11, 2, 4, 5)	Ex 2
3	(7, 2, 6, 3)	PASS	15	(11, 2, 6, 3)	PASS
4	(7, 2, 6, 5)	PASS	16	(11, 2, 6, 5)	PASS
5	(7, 4, 4, 3)	Ex 1	17	(11, 4, 4, 3)	Ex 1
6	(7, 4, 4, 5)	Ex 1	18	(11, 4, 4, 5)	Ex 1
7	(7, 4, 6, 3)	PASS	19	(11, 4, 6, 3)	PASS
8	(7, 4, 6, 5)	PASS	20	(11, 4, 6, 5)	PASS
9	(7, 5, 4, 3)	PASS	21	(11, 5, 4, 3)	PASS
10	(7, 5, 4, 5)	Ex 2	22	(11, 5, 4, 5)	Ex 2
11	(7, 5, 6, 3)	PASS	23	(11, 5, 6, 3)	PASS
12	(7, 5, 6, 5)	PASS	24	(11, 5, 6, 5)	PASS

Table III. Identified failure-inducing combinations and their corresponding Exception

Failure-inducing combinations	Exception
(-, 4, 4, -)	Ex 1
(-, 2, 4, 5)	Ex 2
(-, 5, 4, 5)	Ex 2

2. From the data listed in Table II, we can determine that $(-, 4, 4, -)$ must be the failure-inducing combination of Ex 1 as all the inputs that triggered Ex 1 contain this combination. Similarly, the combination $(-, 2, 4, 5)$ and $(-, 5, 4, 5)$ must be the failure-inducing combinations of Ex 2. We list these three combinations and the corresponding exceptions in Table III.

Note that in this case we did not get the expected result with traditional FCI approaches. The failure-inducing combinations we got for Ex 2 are $(-, 2, 4, 5)$ and $(-, 5, 4, 5)$, respectively, instead of the expected combination $(-, -, 4, 5)$. So why did we fail to get the $(-, -, 4, 5)$? The reason lies in *input 6* (7,4,4,5) and *input 18* (11,4,4,5). These two inputs contain the combination $(-, -, 4, 5)$, but they didn't trigger Ex 2; instead, Ex 1 was triggered.

Now let us get back to the source code of *foo*. We can find that if Ex 1 is triggered, it will stop executing the remaining code and report the exception information. In another word, Ex 1 has a higher failure level than Ex 2, so Ex 1 may mask Ex 2. Let us re-examine the combination $(-, -, 4, 5)$. If we suppose that *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1, then we can conclude that $(-, -, 4, 5)$ should be the failure-inducing combination of the Ex 2, which is identical to the expected one.

Unless we fix the code that triggers Ex 1 and re-execute all the test cases, however, we cannot validate the supposition that *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1. So in practice, when we do not have enough resources to execute all the test cases repeatedly or can only do black-box testing, a more economical and efficient approach to alleviate the masking effect on FCI approaches is desired.

3. FORMAL MODEL

This section presents some definitions and propositions for a formal model to solve the FCI problem.

3.1. Failure-causing Schemas in CT

Assume that the SUT is influenced by k parameters, and each parameter p_i has a_i discrete values from the finite set V_i , i.e., $a_i = |V_i|$ ($i = 1, 2, \dots, k$). Some of the definitions below were originally defined in [Nie and Leung 2011b].

Definition 3.1. A *test case* of the SUT is an array of k values, one for each parameter of the SUT, which is denoted as a k -tuple (v_1, v_2, \dots, v_k) , where $v_1 \in V_1, v_2 \in V_2 \dots v_k \in V_k$.

In practice, these parameters in the test case can represent many factors, such as input variables, run-time options, building options, or various combinations of them. We need to execute the SUT with these test cases to ensure the correctness of the software behaviour.

We consider the abnormal execution of a test case to be a *failure*. It can be a thrown exception, compilation error, assertion failure or constraint violation. When failures are triggered by some test cases, we need to determine the cause of these failures; hence, some subsets of the test case must be analysed.

Definition 3.2. For the SUT, the t -tuple $(-, v_{k_1}, \dots, v_{k_t}, \dots)$ is called a t -value *schema* ($0 < t \leq k$) when some t parameters have fixed values and the others can take on their respective allowable values, represented as “-”.

In effect a test case itself is a t -value *schema*, when $t = k$. Furthermore, if a test case contains a *schema*, i.e., every fixed value in the combination is in this test case, we say this test case *contains the schema*.

Definition 3.3. Let c_l be a l -value schema, c_m be an m -value schema in SUT, and $l < m$. If all the fixed parameter values in c_l are also in c_m , then c_m *subsumes* c_l . In this case, we can also say that c_l is a *sub-schema* of c_m , and c_m is a *parent-schema* of c_l , which can be denoted as $c_l \prec c_m$.

For example, in the motivation example section, the two-value schema $(-, 4, 4, -)$ is a sub-schema of the three-value schema $(-, 4, 4, 5)$, that is, $(-, 4, 4, -) \prec (-, 4, 4, 5)$.

Definition 3.4. If all test cases contain a schema, say c , and trigger a particular failure, say F , then we call this schema c the *failure-causing schema* for F . Additionally, if none of the sub-schema of c is the *failure-causing schema* for F , we then call the schema c the *Minimal Failure-causing Schema*, i.e., the MFS for F .

In fact, MFS is identical to the failure-inducing combinations we discussed previously. Figuring this out can eliminate all details that are irrelevant to the cause of the failure and, hence, facilitate the debugging efforts.

Some notions used later are listed below for convenient reference:

- k : the number of parameters that influence the SUT.
- V_i : the set of discrete values that the i th factor of the SUT can take.
- T^* : The exhaustive set of test cases for the SUT. For a SUT with k factors, and each factor can take $|V_i|$ values, the number of this set of test cases T^* is $\prod_{i=1}^{i \leq k} |V_i|$.
- L : the number of failures contained in the SUT.
- F_m : the m th failure in the SUT; for different failures, we can differentiate them from their exception traces or other buggy information.
- T_{F_m} : All the test cases that can trigger the failure F_m .
- $\mathcal{T}(c)$: All the test cases that can contain the schema c . Based on the definition of MFS, we know that if schema c is MFS for F_m , then $\mathcal{T}(c)$ must be subsumed in T_{F_m} .
- $\mathcal{I}(t)$: All the schemas that are contained in the test case t , e.g., $\mathcal{I}((111)) = \{(1-), (-1-), (- -1), (11-), (1-1), (-11), (111)\}$.
- $\mathcal{I}(T)$: All the schemas that are contained in a set of test cases T , i.e., $\mathcal{I}(T) = \bigcup_{t \in T} \mathcal{I}(t)$.
- $S(T)$: All the schemas that are only contained in the set of test cases. It is important to note that this set is different from $\mathcal{I}(T)$, as the schemas contained by the test cases in T can also be contained by other test cases that do not belong to this set. In fact, $S(T)$ is computed by $\{c | c \in \mathcal{I}(T) \text{ and } c \notin \mathcal{I}(T^* \setminus T)\}$.

Table IV. Example of proposition 3.5

c	
$(0, 0, -, -)$	$\mathcal{T}(c)$
$(0, 0, 0, 0)$	$(0, 0, 0, 0)$
$(0, 0, 0, 1)$	$(0, 0, 0, 1)$
$(0, 0, 1, 0)$	$(0, 0, 1, 0)$
$(0, 0, 1, 1)$	$(0, 0, 1, 1)$
$(0, 1, 0, 0)$	$(0, 1, 0, 0)$
$(0, 1, 0, 1)$	$(0, 1, 0, 1)$
$(0, 1, 1, 0)$	$(0, 1, 1, 0)$
$(0, 1, 1, 1)$	$(0, 1, 1, 1)$

— $\mathcal{C}(T)$: the minimal schemas that are only contained by the set of test cases T . This set is the sub-set of $\mathcal{S}(T)$, which is defined as $\{c | c \in \mathcal{S}(T) \text{ and } \nexists c' \prec c, c' \in \mathcal{S}(T)\}$.

first important proposition : every is the subset!

PROPOSITION 3.5. *For l -value schema c_l and m -value schema c_m , if $c_l \prec c_m$, then we can have all the test cases that contain c_m must also contain c_l , i.e., $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$.*

PROOF. For $\forall t \in \mathcal{T}(c_m)$, we then have that t contains c_m . Then as $c_l \prec c_m$, we must also have that t contains c_l . This is because all the elements in c_l are also in c_m , which are contained in the test case t . Therefore, we get $t \in \mathcal{T}(c_l)$. Thus $t \in \mathcal{T}(c_m)$ implies $t \in \mathcal{T}(c_l)$, so it follows that $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$. \square

Table IV illustrates an example of the SUT with four binary parameters (unless otherwise specified, the following examples also assume a SUT with binary parameters). The left column lists the schema $(0,0,-,-)$ as well as all the test cases that contain this schema, while the right column lists the test cases for schema $(0,-,-,-)$. We can observe that $(0,-,-,-) \prec (0,0,-,-)$, and the set of test cases which contain $(0,-,-,-)$ includes the set of test cases that contain $(0,0,-,-)$.

first important proposition : every is the subset!

PROPOSITION 3.6.

For any set T of test cases of a SUT, we can always get a set of minimal schemas $\mathcal{C}(T) = \{c | \nexists c' \in \mathcal{C}(T), s.t. c' \prec c\}$, such that,

$$T = \bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c)$$

PROOF. We prove this by producing this set of schemas.

We have denoted the exhaustive test cases for SUT as T^* and let $T^* \setminus T$ be the test cases that are in T^* but not in T . Obviously for $\forall t \in T$, we can always find at least one schema which is contained in t , i.e., we can find $c \in \mathcal{I}(t)$, such that $c \notin \mathcal{I}(T^* \setminus T)$. Specifically, at least the test case t itself as schema holds.

Then we collect all the satisfied schemas which are only contained by the test cases in the test cases of T , which can be denoted as: $\mathcal{S}(T) = \{c | c \in \mathcal{I}(T) \text{ and } c \notin \mathcal{I}(T^* \setminus T)\}$.

For the schemas in $\mathcal{S}(T)$, we can have $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) = T$. This is because first, for $\forall t \in \mathcal{T}(c), c \in \mathcal{S}(T)$, it must have $t \in T$. This is because if not so, then $t \in T^* \setminus T$, which contradicts with the definition of $\mathcal{S}(T)$. So $t \in T$. Hence, $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) \subset T$.

Then second, for any test case t in T , as we have learned at least find one c' in $\mathcal{I}(t)$, such that c' in $\mathcal{S}(T)$ (The t itself as a schema holds). In another word, the test case t contains the schema c' , which implies $t \in \mathcal{T}(c'), c' \in \mathcal{S}(T)$. And obviously $\mathcal{T}(c') \subset \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, so $t \in \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, therefore, $T \subset \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$.

Table V. Example of the minimal schemas

T	$S(T)$	$\mathcal{C}(T)$
(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, -)
(0, 0, 0, 1)	(0, 0, 0, 1)	(0, 0, -, 0)
(0, 0, 1, 0)	(0, 0, 1, 0)	
	(0, 0, 0, -)	
	(0, 0, -, 0)	

Since $\bigcup_{c \in S(T)} \mathcal{T}(c) \subset T$ and $T \subset \bigcup_{c \in S(T)} \mathcal{T}(c)$, so it follows $\bigcup_{c \in S(T)} \mathcal{T}(c) = T$.

Then we denote the minimal schemas of $S(T)$ as $M(S(T)) = \{c | c \in S(T) \text{ and } \nexists c' \prec c, s.t., c' \in S(T)\}$. For this set, we can still have $\bigcup_{c \in M(S(T))} \mathcal{T}(c) = T$. We also prove this by two steps, first and obviously, $\bigcup_{c \in M(S(T))} \mathcal{T}(c) \subset \bigcup_{c \in S(T)} \mathcal{T}(c)$. Then we just need to prove that $\bigcup_{c \in S(T)} \mathcal{T}(c) \subset \bigcup_{c \in M(S(T))} \mathcal{T}(c)$.

In fact by definition of $M(S(T))$, for $\forall c' \in S(T) \setminus M(S(T))$, we can have some $c \in M(S(T))$, such that $c \prec c'$. According to the Proposition 3.5, $\mathcal{T}(c') \subset \mathcal{T}(c)$. So for any test case $t \in \bigcup_{c \in S(T)} \mathcal{T}(c)$, as we have either $\exists c' \in S(T) \setminus M(S(T))$, s.t., $t \in \mathcal{T}(c')$ or $\exists c \in M(S(T))$, s.t., $t \in \mathcal{T}(c)$. Both cases can deduce $t \in \bigcup_{c \in M(S(T))} \mathcal{T}(c)$. So, $\bigcup_{c \in S(T)} \mathcal{T}(c) \subset \bigcup_{c \in M(S(T))} \mathcal{T}(c)$.

Hence, $\bigcup_{c \in S(T)} \mathcal{T}(c) = \bigcup_{c \in M(S(T))} \mathcal{T}(c)$, and $M(S(T))$ is the set of schemas that holds this proposition. \square

For example, Table V lists the $S(T)$ and minimal schemas $\mathcal{C}(T)$ for the set of test cases T . We can see that for any other schema not in $\mathcal{C}(T)$, either we can find a test case not in T contains the schema, e.g., (0,0,-,-) with the test case (0,0,1,1) not in T , or that is the parent schema of one of the two minimal schemas, e.g., (0,0,0,0) the parent schema of both (0,0,0,-) and (0,0,-,0).

Let T_{F_m} denotes the set of all the test cases triggering failure F_m , then $\mathcal{C}(T_{F_m})$ actually is the set of MFS of F_m by definition of MFS.

From the construction process of $\mathcal{C}(T)$, one observation is that the minimal schema set $\mathcal{C}(T)$ is the subset of the schema set $S(T)$, i.e., $\mathcal{C}(T) \subset S(T)$, and for any schema in $S(T)$, it either belongs to $\mathcal{C}(T)$, or is the parent schema of one element of $\mathcal{C}(T)$. Then, we can have the following proposition.

PROPOSITION 3.7. *For any test case set T and schema c , if every test case contains c is in the set T , i.e., $\mathcal{T}(c) \subset T$, then it must be that $c \in S(T)$.*

PROOF. We first have $c \in \mathcal{C}(\mathcal{T}(c))$, this is obviously and in fact the minimal schemas for the test cases set $\mathcal{T}(c)$ only contain one schema, which is exactly c itself. As discussed previously we have $\mathcal{C}(\mathcal{T}(c)) \subset S(\mathcal{T}(c))$, so it must be $c \in S(\mathcal{T}(c))$.

Then as $\mathcal{T}(c) \subset T$, it follows $S(\mathcal{T}(c)) \subset S(T)$ by definition. In detail, $S(\mathcal{T}(c)) = \{c | c \in \mathcal{I}(\mathcal{T}(c)) \text{ and } c \notin \mathcal{I}(T^* \setminus \mathcal{T}(c))\}$, so $S(\mathcal{T}(c)) \subset \{c | c \in \mathcal{I}(T) \text{ and } c \notin \mathcal{I}(T^* \setminus T)\}$, which is exactly $S(T)$.

So as $c \in S(\mathcal{T}(c))$ and hence $c \in S(T)$. \square

For two different sets of test cases, there exist some relationships between the minimal schemas of these two sets that, varies in relevancy with respect to the two different sets of test cases. In fact, there are three possible associations between two different sets of test cases: *inclusion*, *disjointed*, and *intersection*, as listed in Fig 2. We did not list the condition for two sets that are identical, because on that condition the minimal schemas must also be identical. To discuss the properties of the relationship of the minimal schemas between two different sets of test cases is important as we will learn later the masking effects between multiple failures will make the MFS identifying process work incorrectly, i.e., these FCI approaches may isolate the minimal schemas for

Table VI. Example of the scenarios

T_l	T_k	T_l	T_k
(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)	(0, 1, 0)	(0, 1, 0)
	(0, 1, 1)		(1, 1, 0)
			(1, 1, 1)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$	$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, 0, -)	(0, -, -)	(0, 0, -)	(0, 0, -)
(0, -, 0)		(0, -, 0)	(0, -, 0)
			(1, 1, -)

the set of test cases which are different from the expected failing set of test cases. And these properties can help us to figure out the impact of masking effects on the FCI approaches. Next, we will separately discuss the relationship between minimal schemas under the three conditions.

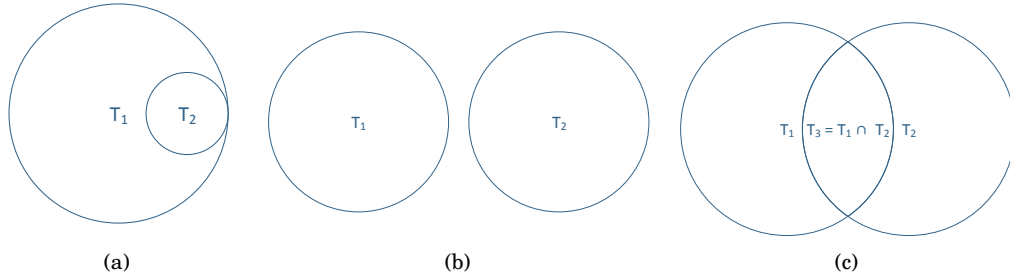


Fig. 2. Test suite relationships

3.2. Inclusion

It is the first relationship corresponding to Fig. 2(a). We can have the following proposition with two sets of test cases which have an inclusion relationship.

PROPOSITION 3.8. *For two sets of test cases T_l and T_k , assume that $T_l \subset T_k$. Then, we have*

$$\forall c_l \in \mathcal{C}(T_l) \text{ either we have } c_l \in \mathcal{C}(T_k) \text{ or have } \exists c_k \in \mathcal{C}(T_k), s.t., c_k \prec c_l.$$

PROOF. Obviously for $\forall c_l \in \mathcal{C}(T_l)$, we can get $\mathcal{T}(c_l) \subset T_l \subset T_k$. According to proposition 3.7, we can have $c_l \in \mathcal{S}(T_k)$. So this proposition holds as the schema in $\mathcal{S}(T_k)$ either is also in $\mathcal{C}(T_k)$, or must be the parent of some schemas in $\mathcal{C}(T_k)$. \square

Based on this proposition, in fact, the schema $c_k \in \mathcal{C}(T_k)$ remains with the following three possible relationships with $\mathcal{C}(T_l)$: (1) $c_k \in \mathcal{C}(T_l)$, or (2) $\exists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$, or (3) $\nexists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$ or $c_k = c_l$, or $c_l \prec c_k$. For the third case, c_k is called to be *irrelevant* to $\mathcal{C}(T_l)$.

We illustrate these scenarios in Table VI. There are two parts in this table, with each part showing two sets of test cases: T_l and T_k , which have $T_l \subset T_k$. For the left part, we can see that in the schema in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0), both are the parent of the schema of the one in $\mathcal{C}(T_k)$: (0, -, -). While for the right part, the schemas in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0) are both also in $\mathcal{C}(T_k)$. Furthermore, one schema in $\mathcal{C}(T_k)$: (1, 1, -) is irrelevant to $\mathcal{C}(T_l)$.

Table VII. Disjoint Example

T_l	T_k
(0, 0, 0)	(1, 0, 0)
(0, 1, 0)	(1, 0, 1)
	(1, 1, 0)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, -, 0)	(1, 0, -)
	(1, -, 0)

3.3. Disjoint

This relationship corresponds to Fig.2(b). For two different sets of test cases, one obvious property is listed as follows:

PROPOSITION 3.9. *For two test cases set T_1, T_2 , if $T_1 \cap T_2 = \emptyset$, we have, $\mathcal{S}(T_1) \cap \mathcal{S}(T_2) = \emptyset$.*

PROOF. If $\mathcal{S}(T_1) \cap \mathcal{S}(T_2) \neq \emptyset$. Without loss of generality, we let $c \in \mathcal{S}(T_1) \cap \mathcal{S}(T_2)$ we can learn that $\mathcal{T}(c)$ must both in T_1 and T_2 , which is contradiction. \square

This property tells that the minimal schemas of two disjointed test cases should be irrelevant to each other. Table VII shows an example of this scenario. We can learn from this table that for two different test case sets T_l, T_k , their minimal schemas, i.e., (0, -, 0) and (1, 0, -), (1, -, 0), respectively, are irrelevant to each other.

3.4. Intersect

This relationship corresponds to Fig.2(c). This scenario is the most common scenario for two sets of test cases, but is also the most complicated scenario for analysis. To conveniently illustrate the properties of the minimal schemas of this scenario, we assume that $T_1 \cap T_2 = T_3$ as depicted in Fig.2(c). Then, we can have the following properties:

PROPOSITION 3.10. *For two intersecting sets of test cases T_1 and T_2 (these two sets are neither identical nor do the members subsume each other), it must have $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, s.t. c_1 and c_2 are irrelevant.*

PROOF. First, we can learn that $\mathcal{C}(T_1 \setminus T_3)$ are irrelevant to $\mathcal{C}(T_2 \setminus T_3)$, as $(T_1 \setminus T_3) \cap (T_2 \setminus T_3) = \emptyset$.

As the schemas in $\mathcal{C}(T_1 \setminus T_3)$ are either identical to some schemas in $\mathcal{C}(T_1)$ or parent schemas of them. Now, if some of them are identical, i.e., $\exists c', s.t., c' \in \mathcal{C}(T_1 \setminus T_3)$ and $c' \in \mathcal{C}(T_1)$, then schemas c' must be irrelevant to these schemas in $\mathcal{C}(T_2)$ as $(T_1 \setminus T_3) \cap T_2 = \emptyset$. This also holds if $\mathcal{C}(T_2 \setminus T_3)$ is identical to some schemas in $\mathcal{C}(T_2)$.

Next, if both $\mathcal{C}(T_1 \setminus T_3)$ and $\mathcal{C}(T_2 \setminus T_3)$ are parent schemas of some of $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, respectively. Without loss of generality, we let $c_1 \prec c_{1-3}$, ($c_{1-3} \in \mathcal{C}(T_1 \setminus T_3)$ and $c_1 \in \mathcal{C}(T_1)$) and $c_2 \prec c_{2-3}$, ($c_{2-3} \in \mathcal{C}(T_2 \setminus T_3)$ and $c_2 \in \mathcal{C}(T_2)$). Then, these corresponding sub-schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, i.e., c_1 and c_2 respectively, must also be irrelevant to each other. This is because $\mathcal{T}(c_1) \supset \mathcal{T}(c_{1-3})$ and $\mathcal{T}(c_2) \supset \mathcal{T}(c_{2-3})$. And as $\mathcal{T}(c_{1-3}) \cap \mathcal{T}(c_{2-3}) = \emptyset$, so $\mathcal{T}(c_1)$ and $\mathcal{T}(c_2)$ are neither identical nor subsuming each other. This also implies that c_1 and c_2 are irrelevant to each other. \square

For example, Table VIII shows two test cases that interact with each other at test case (1,0,0), but their minimal schemas, (1,0,-) and (1,-,0), respectively, are irrelevant to each other.

PROPOSITION 3.11. *For two intersecting sets of test cases T_1 and T_2 , and let $T_3 = T_1 \cap T_2$, if we can find $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, s.t., c_1 is identical to c_2 , then it must have $c_1 = c_2 \in \mathcal{C}(T_3)$*

Table VIII. Example of Intersection by irrelevant examples

T_1	T_2
(1, 0, 0)	(1, 0, 0)
(1, 0, 1)	(1, 1, 0)
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$
(1, 0, -)	(1, -, 0)

Table IX. Example of Intersection by identical examples

T_1	T_2	$T_3 = T_1 \cap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 1, 0)
(1, 1, 0)	(0, 0, 1)	(1, 1, 1)
(1, 1, 1)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(0, 0, -)	(1, 1, -)
(1, 1, -)	(1, 1, -)	

Table X. Example of Intersect by subsuming examples

T_1	T_2	$T_3 = T_1 \cap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 1)
(1, 0, 1)	(1, 0, 1)	(1, 1, 0)
(1, 1, 0)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(-, 0, 0)	(1, 0, -)
(1, 0, -)	(1, -, -)	(1, -, 0)
(1, -, 0)		

PROOF. As we see that identical schema must share identical test cases, then the only identical test case between T_1 and T_2 is $T_1 \cap T_2 = T_3$. So the only possible identical schema between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$ is in $\mathcal{C}(T_3)$. \square

We must know that this proposition holds when some schemas in $\mathcal{C}(T_1 \cap T_2)$ are identical to some schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$.

For example, Table IX shows two test cases that interact with each other at test cases (1,1,0) and (1,1,1), and they have identical minimal schema, i.e., (1,1,-), which is also the minimal schema in $\mathcal{C}(T_3)$.

PROPOSITION 3.12. *For two intersecting sets of test cases T_1 and T_2 , let $T_3 = T_1 \cap T_2$, if we can find $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, s.t., c_1 is the parent-schema of c_2 , then it must have $c_1 \in \mathcal{C}(T_3)$. (and vice versa).*

PROOF. We have proved previously if two schemas have a subsuming relationship, then their test cases must also have an inclusion relationship. And as the only inclusion relationship between T_1 and T_2 is that $T_3 \subset T_1$ and $T_3 \subset T_2$. So the parent schemas must be in $\mathcal{C}(T_3)$. \square

It is noted that this proposition holds when some schemas in $\mathcal{C}(T_3)$ are also in $\mathcal{C}(T_1)$ (or $\mathcal{C}(T_2)$), and simultaneously the same schemas in $\mathcal{C}(T_3)$ must be the parent-schema of the minimal schemas of another set of test cases, i.e., $\mathcal{C}(T_2)$ (or $\mathcal{C}(T_1)$).

Table X illustrates this scenario, in which, the minimal schemas of T_1 : (1,0,-),(1,-,0), which are also the schemas in $\mathcal{C}(T_3)$, is the parent schema of the minimal schema of T_2 : (1,-,-).

It is noted that these three conditions can simultaneously appears when two sets of test cases intersect with each other.

3.5. Identify the MFS

According to this analysis, we can determine that $\mathcal{C}(T_{F_m})$ actually is the set of failure-causing schemas of F_m . Then in theory, if we want to accurately figure out the MFS in the SUT, we need to exhaustively execute each possible test case, and collect the failed test cases T_{F_m} . This is impossible in practice, especially when the testing space is very large.

So for traditional FCI approaches, they need to select a subset of the exhaustive test cases, and then either use some assumptions to predict the remaining test cases or just give a suspicious ranking. As giving a suspicious ranking can also be regard as a special case of making a prediction (with computing the possibility), so we next only formally describe the mechanism of FCI approaches belonging to the first type. We refer to the observed failed test case as $T_{fail_{observed}}$, and refer to the remaining failed test cases based on prediction as $T_{fail_{predicted}}$. We also denote the actual entire failed test cases as T_{fail} . Then the MFS identified by FCI approaches can be depicted as:

$$MFS = \mathcal{C}(T_{fail_{observed}} \cup T_{fail_{predicted}}).$$

For each FCI approach, the way it predicts the $T_{fail_{predicted}}$ according to observed failed test cases varies; furthermore, as the test cases it generates are different, the failed test cases observed by different test cases, i.e., $T_{fail_{observed}}$ also varies. We offer an example using the OFOT approach to illustrate this formula.

Suppose that the SUT has 3 parameters, each of which can take 2 values. And assume the test case (1, 1, 1) failed. Then, we can describe the FCI process as shown in Table XI. In this table, test case t failed, and OFOT mutated one factor of the test case t at a time to generate new test cases: $t_1; t_2; t_3$. It found the t_1 passed, which indicates that this test case breaks the MFS in the original test case t . So, the (1,-,-) should be one failure-causing factor, and as the other mutating processes all failed, this means no other failure-inducing factors were broken; therefore, the MFS in t is (1,-,-).

Now let us explain this process with our formal model. Obviously the $T_{fail_{observed}}$ is $\{(1,1,1), (1,0,1), (1,1,0)\}$. And as having found (0,-,-) broke the MFS, hence by theory[Nie and Leung 2011a], all the test cases that contain (0,-,-) should pass the testing (This conclusion is built on the assumption that the SUT just contain one failure-causing schema). As a result, (0,1,1), (0,0,1), (0,1,0), (0,0,0) should pass the testing. Further, as obviously the test case either passes or fails the testing (we label skipping the testing as a special case of failing), so the remaining test case (1,0,0), will be predicted to fail, i.e., $T_{fail_{predicted}}$ is $\{(1,0,0)\}$. Taken together, the MFS using the OFOT strategy can be described as: $\mathcal{C}(T_{fail_{observed}} \cup T_{fail_{predicted}}) = \mathcal{C}(\{(1, 1, 1), (1, 0, 1), (1, 1, 0), (1, 0, 0)\}) = (1, -, -)$, which is identical to the one it got previously.

Table XI. OFOT with our strategy

original test case				Outcome
t	1	1	1	Fail
observed				
t_1	0	1	1	Pass
t_2	1	0	1	Fail
t_3	1	1	0	Fail
predicted				
t_4	0	0	1	Pass
t_5	0	1	0	Pass
t_6	1	0	0	Fail
t_7	0	0	0	Pass

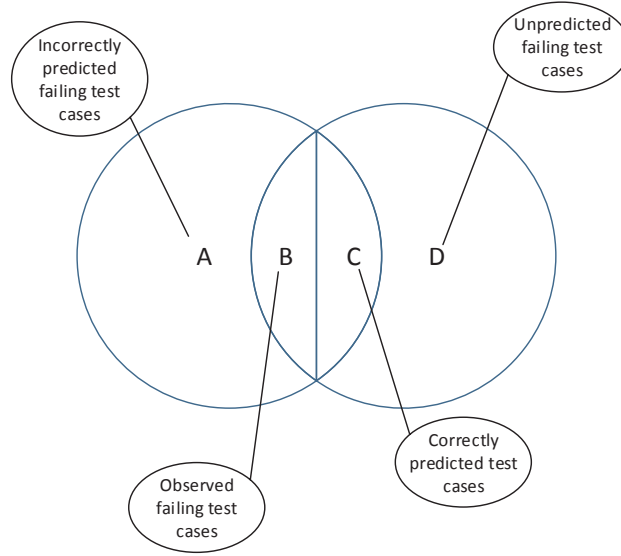


Fig. 3. Generally model of FCI

Similarly, other FCI approaches can also be modeled into this formal description. We will not discuss in detail how to model each FCI approach as this is not the point of this paper. It is noted that the test cases FCI predicts to be failing are not always identical to the actually failed test cases. In fact, we can generally depict the process of FCI approaches as shown in Fig. 3.

We can see in Fig. 3 that area A denotes the test cases that should have passed testing but were predicted to be fail, area B depicts the test cases that the approach observed to be failed test cases, area C refers to the failed test cases that were not observed to be but were predicted to be failed test cases, and area D shows the failed test cases that are neither observed nor predicted. This figure is actually one sample of the condition in which two sets of test cases intersect with each other; in specific, areas $A \cup B \cup C = T_1$, $D \cup B \cup C = T_2$ and $B \cup C = T_1 \cap T_2 = T_3$.

We learned previously that this scenario makes the schemas identified in T_1 biased from the expected MFS in T_2 ; specifically they must be irrelevant schemas between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, which means that the FCI approach will identify some minimal schemas that are irrelevant to the actual MFS, and must ignore some actual MFS. Moreover, under the appropriate conditions listed in propositions 3.11 and 3.12, FCI may identify the identical schemas or parent-schema or sub-schema of the actual MFS. So to identify the schemas as accurately as possible, the FCI approach needs to make T_1 as similar as possible to T_2 ; specifically, it must make area B and area C as large as possible, and make area A as small as possible.

However, even though each FCI approach tries its best to identify the MFS as accurately as possible, masking effects raised from the test cases will result in their efforts being in vain. We next will discuss the masking problem and how it affects the FCI approaches.

Table XII. masking effects for exhaustive testing

T_1	$T_{mask(1)}$	T_*
(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 1, 0, 0)
(1, 1, 1, 0)	(0, 1, 1, 1)	(0, 0, 0, 0)
(1, 1, 0, 1)		(1, 0, 0, 0)
		(1, 0, 1, 1)
		(0, 0, 1, 1)
actual MFS for 1	regarded as one failure	distinguishing failures
$\mathcal{C}(T_1 \cup T_{mask(1)})$	$\mathcal{C}(T_1 \cup T_{mask(1)} \cup T_*)$	$\mathcal{C}(T_1)$
(1, 1, -, -)	(-, -, 0, 0)	(1, 1, -, 1)
(-, 1, 1, 1)	(1, 1, -, -)	(1, 1, 1, -)
	(-, -, 1, 1)	

4. MASKING EFFECT

Definition 4.1. A *masking effect* is an effect that results when a test case t contains an MFS for a particular failure, but the t does not trigger the expected failure because another failure was triggered ahead of it that prevents t from being normally checked.

Taking the masking effects into account, when identifying the MFS for a specific failure, say, F_m , we should not ignore these test cases which should have triggered F_m if they didn't trigger other failures. We call these test cases $T_{mask(F_m)}$. Hence, the MFS for failure F_m should be $\mathcal{C}(T_{F_m} \cup T_{mask(F_m)})$.

As an example, in the motivation example in section 2, the $F_{mask(F_{Ex2})}$ is $\{(7,4,4,5), (11,4,4,5)\}$. So the MFS for $Ex2$ is $\mathcal{C}(T_{F_{Ex2}} \cup T_{mask(F_{Ex2})})$, which is $(-, -, 4, 5)$.

In practice with masking effects, however, it is not possible to correctly identifying the MFS, unless we fix some bugs in the SUT and re-execute the test cases to figure out $T_{mask(F_m)}$.

In effect for traditional FCI approaches, without the knowledge of $T_{mask(F_m)}$, only two strategies can be adopted when facing the multiple failures problem. We will separately analyse the two strategies under exhaustive testing condition and normal FCI testing condition.

4.1. Masking effects for exhaustive testing

4.1.1. Regarded as one failure. The first is the most common strategy, as it does not distinguish the failures, i.e., it treats all of the types of failures as one failure-*failure*, and others as *pass*.

With this strategy, the minimal schemas we identify are the set $\mathcal{C}(\bigcup_{i=1}^L T_{F_i})$, L is the number of all the failures in the SUT. Obviously, $T_{F_m} \cup T_{mask(F_m)} \subset \bigcup_{i=1}^L T_{F_i}$. So in this case, by Proposition 3.8, some schemas we get may be the sub-schemas of some of the actual MFS, or be irrelevant to the actual MFS.

As an example, consider the test cases in Table XII. In this example, assume we need to characterize the MFS for error 1. All the test cases that triggered error 1 are listed in column T_1 ; similarly, we list the test cases that triggered other failures in column $T_{mask(1)}$ and T_* , respectively, in which the former masked the error 1, while the latter did not. Actually the MFS for error 1 should be $(1,1,-,-)$ and $(-,1,1,1)$ as we listed them in the column *actual MFS for 1*. However, when we use the *regarded as one failure* strategy, the minimal schemas we get will be $(-, -, 0, 0)$, $(1,1,-,-)$, $(-, -, 1, 1)$, in which the $(-, -, 0, 0)$ is irrelevant to the actual MFS for error 1, and the $(-, -, 1, 1)$ is the sub-schema of the actual MFS $(-, 1, 1, 1)$.

4.1.2. Distinguishing failures. Distinguishing the failures by the exception traces or error code can help make the MFS related to particular failure. Yilmaz [Yilmaz et al. 2013] proposed the *multiple-class* failure characterizing method instead of the *ternary-class*

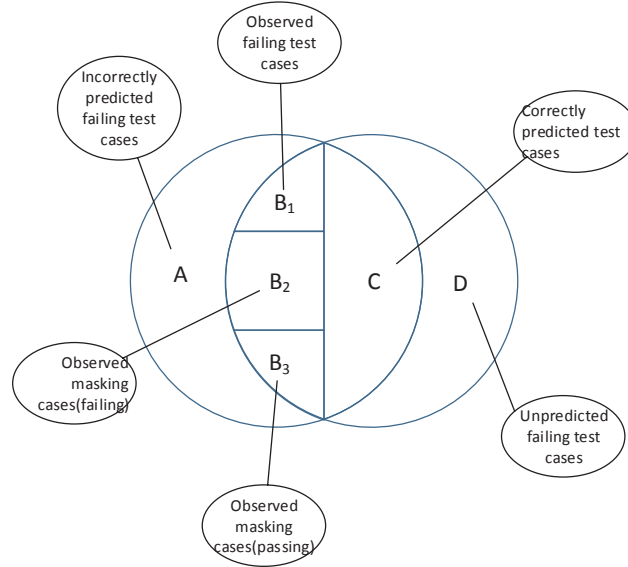


Fig. 4. FCI with masking effects

approach to make the characterizing process more accurate. Besides, other approaches can also be easily extended with this strategy for testing SUT with multiple failures.

This strategy focuses on identifying the set of $\mathcal{C}(T_{F_m})$, and as $T_{F_m} \cup T_{mask(F_m)} \supset T_{F_m}$, consequently, some schemas that get through this strategy may be the parent-schema of some actual MFS. Moreover, some MFS may be irrelevant to the schemas we get with this strategy, which means this strategy will ignore these actual MFS.

For the simple example in Table XII, when we use this strategy, we will get the minimal schemas (1, 1, -, 1) and (1, 1, 1, -), which are both the parent schemas of the actual MFS (1,1,-,-), and we will observe that no schemas gotten by this strategy have any relationship with the actual MFS (-,1,1,1), which means it was ignored.

It is noted that the motivation example in section 2 actually adopted this strategy, so we see that the schemas identified for Ex 2: (-,2,4,5), (-,3,4,5) are the parent-schemas of the correct MFS(-,-,4,5).

4.2. Masking effects for FCI approaches

With masking effects, the scenario of traditional FCI approaches is a bit more complicated than the previous two exhaustive testing scenarios, and is depicted in the Fig 4. In this figure, areas A, C and D are the same as in Fig 3, and area B is divided into three sub-areas in which B_1 still represents the observed failed test cases for the current analysed failure, area B_2 represents the test cases that triggered other failures which masked the current failure, and area B_3 represents the test cases that triggered other failures which did not mask the current failure. It can be found that the actual MFS set for the SUT is $\mathcal{C}(B_1 \cup B_2 \cup C \cup D)$.

With this model, if we know which test cases mask the expected failure, i.e., if we have figured out the B_2 and B_3 areas, then the schemas that the FCI approach will identify can be described as $\mathcal{C}(A \cup B_1 \cup B_2 \cup C)$. We next denote this result as *knowing masking effects*. However, as we discussed before, to get this result is not possible without human involvement. Correspondingly, when using the *regarded as one failure* strategy, the set of MFS traditional FCI identify is $\mathcal{C}(A \cup B_1 \cup B_2 \cup B_3 \cup C)$. And for

the *distinguishing failures* strategy, the MFS is $\mathcal{C}(A \cup B_1 \cup C)$. Next, we will discuss the influence of masking effects on the two strategies.

4.2.1. Using the regarded as one failure strategy. For the first strategy: *regarded as one failure*, the impact of masking effects on FCI approaches can be described as shown in Table XIII. To understand the content of this table, let us go back to the relationship between the minimal schemas of two different sets of test cases. For the *knowing masking effects* condition, the test cases that are used for identifying the MFS, i.e., $A \cup B_1 \cup B_2 \cup C$, intersect the test cases that are used to compute the actual MFS ($B_1 \cup B_2 \cup C \cup D$). It means that the minimal schemas get in this condition can be identical, parent-schema, sub-schema, and irrelevant to the actual MFS. And if we apply the *regarded as one failure*, the minimal schemas we get are $\mathcal{C}(A \cup B_1 \cup B_2 \cup B_3 \cup C)$. Obviously, we have $A \cup B_1 \cup B_2 \cup B_3 \cup C \supset A \cup B_1 \cup B_2 \cup C$. So the minimal schema gotten by this strategy is either the sub-schema or identical to some schemas from the ones gotten by *known masking effects*, or alternatively, existing some schemas that are irrelevant to all of them. Taking these two properties together, we will get what is shown in Table XIII.

Table XIII. Masking effects influence on FCI with regarded as one failure strategy

1	If $c_m = c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} = c'_m$
2	If $c_m = c_{origin}$ and $c_{new} \prec c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
3	If $c_m \prec c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
4a	If $c_m \prec c_{origin}$ and $c_{new} \prec c_{origin}$	Then either, $c'_m, s.t., c_{new} \prec c'_m$
4b		Or, c_{new} irrelevant to all c'_m
5	If $c_{origin} \prec c_m$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
6	If $c_{origin} \prec c_m$ and $c_{new} \prec c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
7	If c_{origin} irrelevant all c_m and $c_{new} = c_{origin}$	Then, c_{new} irrelevant to all c'_m
8a	If c_{origin} irrelevant all c_m and $c_{new} \prec c_{origin}$	Then either, $\exists c'_m, s.t., c_{new} \prec c'_m$
8b		Or, c_{new} irrelevant to all c'_m
9a	If c_{new} irrelevant to all c_{origin}	Then either, $\exists c'_m, s.t., c_{new} \prec c'_m$
9b		Or, c_{new} irrelevant to all c'_m

There are totally 9 rules for this strategy, which are labeled from 1 to 9 respectively. In these rules, c_m and c'_m are the actual MFSs, i.e., $c_m, c'_m \in \mathcal{C}(B_1 \cup B_2 \cup C \cup D)$. c_{origin} is the minima schema that are gotten by *knowing masking effects* strategy, i.e., $c_{origin} \in \mathcal{C}(A \cup B_1 \cup B_2 \cup C)$. c_{new} is the schema that are gotten by *regarded as one failure* strategy, i.e., $c_{new} \in \mathcal{C}(A \cup B_1 \cup B_2 \cup B_3 \cup C)$. c_{new} satisfies that $c_{new} = c_{origin}$ or $c_{new} \prec c_{origin}$. Each rule in Table XIII describes one possible relationship between the schemas gotten by strategy *regarded as one failure* and the actual MFS. For example, rule 2, i.e., If $c_m = c_{origin}$ and $c_{new} \prec c_{origin}$, then $\exists c'_m, s.t., c_{new} \prec c'_m$, indicates that the schema c_{new} that is gotten by *regarded as one failure* strategy will be the subschema of some actual MFS, if it is the subschema of the schema c_{origin} that is gotten by strategy *knowing masking effects*, and c_{origin} is identical to some actual MFS. Some rules may result in that there are more than one possible relationship between the c_{new} and the actual MFS. For example, rule 4 can make c_{new} either be the subschema of the actual MFS or be irrelevant to all the actual MFS. In this case, we divide the rule into several sub-rules. As for rule 4 we divide this rule into two sub-rules: 4a and 4b, each of which represents one possibility relationship between the c_{new} and the actual MFS c'_m .

In these rules, only two can make $\exists c'_m, s.t., c_{new} = c'_m$ or $c'_m \prec c_{new}$, which are rules 1 and 3 respectively. This can be easily understood, as to make $c_{new} = c'_m$ or $c'_m \prec c_{new}$, according to proposition 3.11 and 3.12, it must have $\mathcal{T}(c_{new}) \subset (B_1 \cup B_2)$. Assume we get $c_{new} \prec c_{origin}$, then we must have $\exists t \in B_3, s.t., t \in \mathcal{T}(c_{new})$, otherwise, there should be no schema c_{new} that is the sub-schema of c_{origin} . Consequently, $\mathcal{T}(c_{new}) \not\subset (B_1 \cup B_2)$ if we have $c_{new} \prec c_{origin}$. So to make $c_{new} = c'_m$ or $c'_m \prec c_{new}$, the schema c_{new}

Table XIV. Example of the influence of the regarded as one failure for FCI approach

A	$B_1 \cup B_2 \cup C$	B_3	D	
(0,0,0,1,0,0)	(1,1,1,0,0,0)	(1,0,1,0,0,0)	(1,1,0,0,0,0)	
(0,0,0,1,1,0)	(1,1,1,0,1,0)	(1,0,1,0,1,0)	(1,1,0,0,1,0)	
(0,0,1,0,0,0)	(1,1,1,1,0,0)	(0,0,1,0,1,0)	(1,1,0,1,0,0)	
(0,0,1,1,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,0)	(1,1,0,1,1,0)	
	(1,0,1,1,0,0)	(0,1,0,0,0,0)	(0,0,1,1,0,1)	
	(1,0,1,1,1,0)	(0,1,0,1,0,0)	(0,1,1,1,0,1)	
	(0,0,0,0,1,0)	(0,1,1,0,0,0)		
	(0,0,0,0,0,0)	(0,1,1,1,0,0)		
	(0,0,1,1,1,1)	(1,0,0,0,0,0)		
	(0,1,1,1,1,1)	(1,0,0,0,1,0)		
		(1,1,1,1,1,1)		
		(1,0,1,1,1,1)		
<i>actual MFS</i> $\mathcal{C}(B_1 \cup B_2 \cup C \cup D)$	<i>knowing masking effects</i> $\mathcal{C}(A \cup B_1 \cup B_2 \cup C)$	<i>one failure</i> $\mathcal{C}(A \cup B_1 \cup B_2 \cup B_3 \cup C)$		
(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)		
(1,-,1,1,-,0)	(1,-,1,1,-,0)	(0,0,-,-,-,0)		
(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,-,-,-,0,0)		
(0,-,1,1,-,1)	(0,0,-,-,0,0)	(-,0,1,-,-,0)		
	(-,0,1,1,0,0)	(-,1,-,0,0,0)		
	(0,-,1,1,1,1)	(-,0,-,0,-,0)		
		(-,1,1,1,1,1)		
		(1,-,1,1,1,-)		
		(-,0,1,1,1,-)		
rules	c_m	c_{origin}	c_{new}	c'_m
2	(1,-,1,1,-,0)	(1,-,1,1,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
4a	(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
4b	(0,-,1,1,-,1)	(0,-,1,1,1,1)	(-,1,1,1,1,1)	*
6	(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
8a	*	(0,0,-,-,0,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
8b	*	(-,0,1,1,0,0)	(-,0,1,-,-,0)	*
9a	*	*	(-,0,-,0,-,0)	(0,0,0,0,-,0)
9b	*	*	(1,-,1,1,1,-)	*

must be identical to c_{origin} , i.e., $c_{new} = c_{origin}$ and there must be $c_{origin} = c_m$ or $c_m \prec c_{origin}$, correspondingly. Apart from these two rules, the remaining rules indicate that either $c_{new} \prec c'_m$ or c_{new} is irrelevant to all the actual MFS, which implies the schemas that are gotten by strategy *regarded as one failure* tends to be more subschemas or irrelevant schemas of the actual MFS when compared to that of the *knowing masking effects* strategy.

Next we will give examples to depict these rules except these that have the condition ' $c_{new} = c_{origin}$ ' (rule 1, 3, 5, 7). This is because these rules will simply result in that the relationship between c_{new} and actual MFS will be the same as the relationship between c_{origin} and the actual MFS. Table XIV presents the examples of all the remaining rules. This table consists of three parts, with the upper part giving the test cases for each area in the abstract FCI model. Note that we only list the union of areas B_1 , B_2 and C . This is because the union is the common element for computing the MFS of three approaches – *actual MFS*, *knowing masking effects*, *regarded as one failure*. The middle part of this table shows the minimal schemas using this particular method. And last, the lower part depicts the sample of each possible rule in Table XIII. In this part, In this part, the left column indicates the specific rule id, i.e., 2,4a,4b,6,8a,8b,9a,9b. The column ' c_m ', ' c_{origin} ', ' c_{new} ', ' c'_m ', respectively, indicates the schema which satisfies the rule in the corresponding row. The mark * in thee column means the rule is irrelevant to this schema. For example, for rule 8b, i.e., if c_{origin} irrelevant all c_m and $c_{new} \prec c_{origin}$ then c_{new} irrelevant to all c'_m , we marked "*" in the column ' c_m ' and ' c'_m '.

4.2.2. *using distinguish strategy.* And for the second strategy, *distinguishing failures*, the influence can be described as in Table XV.

Table XV. Masking effects influence on FCI with distinguishing failures strategy

1	If $c_m = c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} = c'_m$
2	If $c_m = c_{origin}$ and $c_{origin} \prec c_{new}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
3	If $c_m \prec c_{origin}$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c'_m \prec c_{new}$
4	If $c_m \prec c_{origin}$ and $c_{origin} \prec c_{new}$	Then, $c'_m, s.t., c'_m \prec c_{new}$
5	If $c_{origin} \prec c_m$ and $c_{new} = c_{origin}$	Then, $\exists c'_m, s.t., c_{new} \prec c'_m$
6a	If $c_{origin} \prec c_m$ and $c_{origin} \prec c_{new}$	Then either $\exists c'_m, s.t., c_{new} = c'_m$
6b		Or $\exists c'_m, s.t., c'_m \prec c_{new}$
6c		Or $\exists c'_m, s.t., c_{new} \prec c'_m$
6d		Or c_{new} irrelevant all c'_m
7	If c_{origin} irrelevant all c_m and $c_{new} = c_{origin}$	Then, c_{new} irrelevant to all c'_m
8a	If c_{origin} irrelevant all c_m and $c_{origin} \prec c_{new}$	Then either, $\exists c'_m, s.t., c'_m \prec c_{origin}$
8b		Or, c_{new} irrelevant to all c'_m
9	It may have c_{origin} irrelevant to all c_{new}	

Similar to Table XIII, this table also lists the possible relationships among the schemas that are gotten by strategy *distinguishing failures*, schemas that are gotten by *knowing masking effects* strategy, and the actual MFS. As with the *distinguishing failures* strategy, the minimal schemas identified are actually $\mathcal{C}(A \cup B_1 \cup C)$. Obviously $A \cup B_1 \cup C \subset A \cup B_1 \cup B_2 \cup C$. So with this strategy, $c_{new} \in \mathcal{C}(A \cup B_1 \cup C)$ should be either the parent-schema or identical to $c_{origin} \in \mathcal{C}(A \cup B_1 \cup B_2 \cup C)$. The main difference between the rules in Table XV and rules in Table XIII is that most rules with strategy *distinguishing failures* result in that $c'_m \prec c_{new}$. This is because with strategy *distinguishing failures*, the test cases that are used for identifying the MFS is less than that of *knowing masking effects*. So more likely it may have $\mathcal{T}(c_{new}) \subset \mathcal{T}(c'_m)$, and hence $c'_m \prec c_{new}$.

We take an example to illustrate these rules of strategy *distinguishing failures*, which is depicted in Table XVI. Similar to our previous strategy, we omit the samples that have the condition ' $c_{origin} = c_{new}$ '.

One rule that is needed to be noted is rule 10, which can make some c_{origin} removed from the newly minimal schemas, i.e., $\nexists c_{new}, s.t., c_{new} = c_{origin}$ or $c_{origin} \prec c_{new}$. For the Table XVI example, in the last row for the rule 10, we can find the schema $c_{origin} - (1,1,0,0,1,-)$, which is identical to the one in actual MFS, there exists no c_{new} which is identical to or is the parent-schema of this schema. Consequently, in this condition, this strategy may ignore some actual MFS compared with *knowing masking effects*.

In fact, besides this special case that may result in the FCI approach ignoring some actual MFS, there exist some other cases that can also achieve the same effect. For example, when the c_{new} is the only schema that is related to c_{origin} , (related means not irrelevant, and in this case it is either identical to or the parent-schema). And the corresponding parent-schema c_{origin} is the only schema which is related to one actual MFS c_m . Then, if the c_{new} is irrelevant to all the actual MFS, we will ignore the actual MFS c_m . This *ignored* event is caused by the c_{new} growing into the irrelevant schemas, which can also appear in the strategy *regarded as one failure*. However, the aforementioned cause of *ignored*—the c_{origin} is removed from the newly minimal schemas can only happen in the strategy *distinguishing failures*.

4.3. Summary of the masking effects on the FCI approach

From the analysis of the formal model, we can learn that masking effects do influence the FCI approaches, and even worse, both the *regarded as one failure* and *distinguishing failures* strategy are harmful. Specifically when compared with the *knowing masking effects* strategy, the former has a large possibility of getting more sub-schemas of

Table XVI. Example the influence of distinguishing failures for FCI approach

A	$B_1 \cup C$	B_2	D	
(0,0,0,1,1,0)	(0,0,0,0,0,0)	(0,0,1,1,1,0)	(0,1,1,0,0,0)	
(0,0,1,1,0,0)	(0,0,0,0,1,0)	(1,1,0,1,0,0)	(0,1,1,0,1,0)	
(0,0,1,0,1,0)	(0,0,0,1,0,0)	(1,0,1,1,0,1)	(0,1,1,1,0,0)	
(0,0,1,0,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,1)	(0,1,1,1,1,0)	
(1,1,0,0,0,0)	(1,1,0,0,1,0)	(1,1,0,0,1,1)	(1,1,1,1,1,1)	
(0,0,1,1,0,1)	(1,1,0,1,1,0)	(0,1,0,0,1,1)	(0,1,1,1,1,1)	
	(1,1,1,0,0,0)	(1,0,1,0,1,1)	(1,1,1,1,0,1)	
	(1,1,1,1,0,0)		(1,0,0,1,1,1)	
	(1,1,1,0,1,0)			
	(1,0,1,1,1,1)			
	(0,0,0,0,1,1)			
	(1,0,0,0,1,1)			
$actual\ MFS$ $\mathcal{C}(B_1 \cup B_2 \cup C \cup D)$	$knowing\ masking\ effects$ $\mathcal{C}(A \cup B_1 \cup B_2 \cup C)$	$distinguishing\ failures$ $\mathcal{C}(A \cup B_1 \cup C)$		
(0,-,1,1,1,-)	(1,1,-,-,0)	(0,0,0,-,-,0)		
(1,1,-,1,-,0)	(0,0,-,-,0)	(0,0,-,-,0,0)		
(-,0,0,1,1)	(-,0,1,1,-,1)	(0,0,-,0,-,0)		
(0,0,0,-,0,0)	(0,0,1,1,-,-)	(1,1,1,-,-,0)		
(0,0,0,0,-,0)	(0,0,0,0,1,-)	(1,1,-,-,1,0)		
(1,0,-,-,1,1)	(1,1,0,0,1,-)	(1,1,-,0,-,0)		
(1,1,0,0,1,-)	(1,0,1,-,1,1)	(0,0,1,1,0,-)		
(-,1,1,-,-,0)	(1,0,-,0,1,1)	(1,0,1,1,1,1)		
(-,1,1,1,1)	(-,0,0,0,1,1)	(-,0,0,0,1,1)		
(1,1,-,-,1,0)		(0,0,0,0,1,-)		
(-,1,1,1,1,-)				
(1,1,1,1,-,-)				
(1,-,1,1,-,1)				
(0,0,0,0,1,-)				
rules	c_m	c_{origin}	c_{new}	c'_m
2	(-,0,0,1,1)	(-,0,0,1,1)	(-,0,0,0,1,1)	(-,0,0,1,1)
4	(1,0,-,-,1,1)	(1,0,1,-,1,1)	((1,0,1,1,1,1))	((1,0,-,-,1,1))
6d	(1,1,-,1,-,0)	(1,1,-,-,0)	(1,1,-,0,-,0)	*
6c	(0,0,0,0,-,0)	(0,0,-,-,0)	(0,0,0,-,-,0)	(0,0,0,0,-,0)
6b	(1,1,-,1,-,0)	(1,1,-,-,0)	(1,1,1,-,-,0)	(-,1,1,-,-,0)
6a	(1,1,-,-,1,0)	(1,1,-,-,0)	(1,1,-,-,1,0)	(1,1,-,-,1,0)
8a	*	(-,0,1,1,-,1)	(1,0,1,1,1,1)	(1,-,1,1,-,1)
8b	*	(0,0,1,1,-,-)	(0,0,1,1,0,-)	*
9	(1,1,0,0,1,-)	(1,1,0,0,1,-)	*	*

the actual MFS and getting more schemas which are irrelevant to the MFS, while the latter may get more parent schemas of the MFS and can also get more irrelevant MFS. Further, both strategies can ignore the actual MFS and the *distinguishing failures* strategy is more likely to ignore the MFS than the *regarded as one failure* strategy.

Note that our discussion is based on a SUT using deterministic software, i.e., the random failing information of a test case will be ignored. The non-deterministic problem will result in a more complex test scenario, which will not be discussed in this paper.

5. TEST CASE REPLACING STRATEGY

The main reason why the FCI approach fails to properly work is that we cannot determine the areas B_2 and B_3 , i.e., if the test case trigger other failures which is different from the current one, we cannot figure out whether this test case will trigger the current expected failure as the masking effects may prevent that. So to limit the impact of this effect on the FCI approach, we need to reduce the number of test cases that trigger other failures as much as possible.

In the exhaustive testing, as all the test cases will be used to identify the MFS, there is no room left to improve the performance without fixing the other failures and re-executing all the test cases. However, when you just need to select part of all the test cases to identify the MFS (which is how the traditional FCI approach works), we can adjust the test cases we need to use by selecting the proper ones. By doing this, we can limit the size of $\mathcal{T}(mask_{F_m})$ to be as small as possible.

5.1. Replacing test cases that trigger unexpected failures

The basic idea is to pick the test cases that trigger other failures and generate new test cases to replace them. These regenerated test cases should either pass in the execution or trigger F_m . The replacement must satisfy the condition that the newly generated ones will not negatively influence the original identifying process.

Commonly, when we replace the test case that triggers an unexpected failure with a new test case, we should keep some part in the original test case. We call this part the *fixed part*, and mutate the other part with different values from the original one. For example, if a test case (1,1,1,1) triggered an unexpected failure, and the fixed part is (-, -,1,1). Then, we can replace it with a test case (0,0,1,1) which may either pass or trigger the expected failure.

The *fixed part* can vary for different FCI approaches, e.g, for the OFOT [Nie and Leung 2011a] algorithm, the factors are the fixed part except for the one that needs to be validated, while for the FIC_BS [Zhang and Zhang 2011] approach, we will fix the factors that should not be mutated for the test case in the next iteration of the FIC_BS process.

We note that this replacement may need to be executed multiple times for one fixed part as we could not always find a test case that coincidentally satisfied our requirement. One replacement method is randomly choosing test cases until the satisfied test case is found. While this method may be simple and really works, however, it also may require trying too many times to get the satisfied one. So to handle this problem and reduce the cost, we proposed a replacement approach by computing the *strength* of the test case with the other failures, and then we selected the test case from a group of candidate test cases that has the least *strength* related to the other failures.

To explain the *strength* notion, we need first to introduce the *strength* that a factor is related to a particular failure. We use $all(o)$ to represent the number of executed test cases that contain this factor, and $m(o)$ to indicate the number of test cases that trigger the failure F_m and contain this factor. Then, the *strength* that a factor is related to a particular failure, i.e., $S(o, F_m)$, is $\frac{m(o)}{all(o)+1}$. This heuristic formula is based on the idea that if a factor frequently appears in the test cases that trigger the particular failure, then it is more likely to be the inducing factor that triggers this type of failure. We add 1 in the denominator for two facts: (1) avoid division by zero when the factor has never appeared before, (2) reduce the bias when a factor rarely appears in the test set but by coincidence appears in a failed test case with a particular failure.

With this factor *strength*, we then define that the *strength* of a test case f is related to a particular failure F_m as:

$$S(f, F_m) = \frac{1}{k} \sum_{o \in f} S(o, F_m)$$

In this formula, k is the number of factors in the test case f , o is the specific factor in f . This formula computes the average *strength* of all the factors in the test case as the *strength* for this test case that is related to a particular failure. For a test case that is selected to be tested, we want that the ability of that test case to trigger another failure

to be as small as possible. In practice, one test case can have different related *strength* to different failures, so we cannot always find a test case that has the least relating *strength* to all the failures when compared other test cases. With this in mind, our target changes to find a test case for which the maximal *strength* of the related failure among other failures is the least compared other test cases. Formally, we should choose a test case f , s.t.,

$$\min_{f \in R} \max_{m \leq L \& m \neq n} S(f, F_m) \quad (\text{EQ1})$$

In this formula, L is the number of all the failures, and n is the current analysed failure. R is the set of all the possible test cases that contain the *fixed* part except those ones that have been tested. Obviously $|R| = \prod_{i \notin \text{fixed}(v_i)} |t|$ where t contains the *fixed* part & t is tested.

We can further resolve this problem. Consider the test case we get – f satisfied the EQ1. Without loss of generality, we assume that the failure $F_k, k \neq n$ is the failure with which the test case f has the maximal related *strength* compared to the other failures. Then, a natural property for f is that any other test case f' which satisfies that failure F_k is the maximal related failure for this test case and must have $S(f, F_k) \leq S(f', F_k)$. Formally, to get such a test case is to solve the following formula:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & f \in R \\ & S(f, F_k) > S(f, F_i), \quad 1 \leq i \leq L \& i \neq k, n \end{aligned} \quad (\text{EQ2})$$

With this formula, to solve EQ1, we just need to find the particular failure F_k , such that the related *strength* between the test case f that satisfies EQ2 and this failure is the smallest than that of the other failures. Formally, we need to find:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & 1 \leq k \leq L \& k \neq n \\ & f, F_k \text{ satisfies EQ2} \end{aligned} \quad (\text{EQ3})$$

According to EQ3, the problem to get such a test case lies in solving EQ2 because if EQ2 is solved we just need to rank the one that has the minimal value from the solutions to EQ2. As to EQ2, it can be formulated as an 0-1 integer linear programming (ILP) problem. Assume the SUT we test has K factors in which the i th factor has V_i values it can take from. And the SUT has L failures. We then define the variable x_{ij} as:

$$x_{ij} = \begin{cases} 1 & \text{the } i\text{th factor of the test case take the } j\text{th value for that factor} \\ 0 & \text{otherwise} \end{cases}$$

We then take $o_{m_{ij}}$ to be the related *strength* between the j th value of the i th factor of the SUT and the failure F_m . And we use a set R of factors with its values to define the fixed part in the test case we should not change, i.e., $R = \{(i, j) | i \text{ is the fixed factor in the test case, } j \text{ is the corresponding value}\}$. As we can generate redundant test cases, so we keep a set of test cases $T_{executed}$ to guide to generate different test cases. Then EQ2 can be detailed as the in following ILP formula:

$$\min \quad \frac{1}{|K|} \sum_{i=0}^K \sum_{j=0}^{V_i} o_{m_{ij}} \times x_{ij} \quad (\text{EQ4})$$

$$\text{s.t.} \quad 0 \leq x_{ij} \leq 1 \quad i = 0..K-1, j = 0, ..V_i - 1 \quad (1)$$

$$x_{ij} \in \mathbb{Z} \quad i = 0..K-1, j = 0, ..V_i - 1 \quad (2)$$

$$\sum_{j=0}^{V_j} x_{ij} = 1 \quad i = 0..K-1 \quad (3)$$

$$x_{ij} = 1 \quad (i, j) \in R \quad (4)$$

$$\sum_{i=0}^K \sum_{j=0}^{V_i} (o_{m_{ij}} - o_{m'_{ij}}) \times x_{ij} \geq 0 \quad 1 \leq m' \neq m \leq L \quad (5)$$

$$\sum_{(i,j) \in t} x_{ij} < K \quad t \in T_{existed} \quad (6)$$

In this formula, constraints (1) and (2) indicate that the variable x_{ij} is a 0-1 integer. Constraint (3) indicates that a factor in one test case can only take one value. Constraint (4) indicates the test case should not change values of the fixed part. Constraint (5) indicates that the related strength between Failure F_m and the test case is maximal than the others. Constraint (6) indicates the test cases generated should not be the same as the test cases in $T_{existed}$.

As we have formulated the problem into a 0-1 integer programming problem, we just need to utilize an ILP solver to solve this formula. In this paper, we use the solver introduced in [Berkelaar et al. 2004], which is a mixed Integer Linear Programming (MILP) solver that can handle satisfaction and optimization problems.

The complete process of replacing a test case with a new one while keeping some fixed part is depicted in Algorithm 1:

The inputs for this algorithm consist of the failure type we focus on – F_m , the fixed part of which we want to keep from the original test case – s_{fixed} , the set of values that each factor can take from respectively – $Param$ and the set of matrix o_1, \dots, o_L , for any element in which, say o_m , is recorded the related strength between each specific factor with each value and the failure F_m , i.e., $o_m = \{o_{m_{ij}} | 0 \leq i \leq K-1, 0 \leq j \leq V_i\}$. The output of this algorithm is a test case t_{new} which either triggers the expected F_m or passes.

This algorithm is an outer loop (lines 1 - 19) containing two parts:

The first part (lines 2 - 9) generates a new test case which is supposed to be least likely to trigger failures different from F_m . The basic idea for this part is to search each failure different from F_m (line 3) and find the best test case that has the least related strength with other failures. In detail, for each failure we set up an ILP solver (line 4) and use it to get an optimal test case for that failure according to EQ4 (line 5). We compare the optimal value for each failure, and choose the one has less strength related to other failures (lines 6 - 9).

The second part is to check whether the newly generated test case is as expected (lines 10 - 16). We first execute the SUT under the newly generated test case (line 10) and update the related strength matrix ($o_1 \dots o_L$) for each factor that is involved in this newly generated test case (line 11). We then check the execute result. If either the test case passes or triggers the same failure – F_m , we will get an satisfied test case (line 12), and we will directly return this test case (line 13). Otherwise, we will repeat the process, i.e., generate a new test case and check again (lines 14 - 15).

ALGORITHM 1: Replacing test cases triggering unexpected failures

Input: failure type F_m , fixed part s_{fixed} , values set that each option can take $Param$, the related strength matrix $o_1 \dots o_L$

Output: t_{new} the regenerate test case, The frequency number

```

1 while not MeetEndCriteria() do
2    $optimal \leftarrow MAX$ ;  $t_{new} \leftarrow null$ ;
3   forall the  $F_k \in F_1, \dots, F_m, F_{m+1} \dots F_L$  do
4      $solver \leftarrow setup(s_{fixed}, Param, F_m, o_1 \dots o_L)$ ;
5      $(optimal', t'_{new}) \leftarrow solver.getOptimalTest()$ ;
6     if  $optimal' < optimal$  then
7        $t_{new} \leftarrow t'_{new}$ ;
8     end
9   end
10   $result \leftarrow execute(t_{new})$ ;
11   $updateRelatedStrengthMatrix(t_{new})$ ;
12  if  $result == PASS$  or  $result == F_m$  then
13    return  $t_{new}$ ;
14  else
15    continue;
16  end
17 end
18 return null

```

Note that this algorithm has another exit, besides we find an expected test case (line 12), which is when the function *MeetEndCriteria()* returns *true* (line 1). We didn't explicitly show what the function *MeetEndCriteria()* is like, because this is dependent on the computing resource and how accurate you want the identifying result to be. In detail, if you want to get a high quality result and you have enough computing resource, you can try many times to get the expected test case; otherwise, a relatively small number of attempts is recommended.

In this paper, we just set 3 as the greatest number of repeats for this function. When it ends with *MeetEndCriteria()* is true, we will return null (line 18), which means we cannot find an expected test case.

5.2. A case study using the replacement strategy

Suppose we have to test a system with eight parameters, each of which has three options. And when we execute the test case $T_0 = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$, a failure— e_1 is triggered. Next, we will use the FCI approach – FIC-BS [Zhang and Zhang 2011] with replacement strategy to identify the MFS for the e_1 . Furthermore, there are two more potential failures, e_2 and e_3 , that may be triggered during the testing; and they will mask the desired failure e_1 . The process is shown in Fig 5. In this figure, there are two main column. The left main column indicates the executed test cases during testing as well as the executed results, and each executed test case corresponds to a specific label, $T_1 - T_8$, at the left. The right main column lists the related strength matrix when a test case triggers e_2 or e_3 . In detail, the matrix records the related strength between each factor (columns $O1 - O8$) for each value it can take (column v) with the unexpected failure (column F). The executed test case, which is in bold, indicates the one that triggers the other failure and should be replaced in the next iteration.

From Fig 5, for the test case that triggered $e_2 = (2 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0)$ (in this case, the fixed part of the test case is $(- - - 0 \ 0 \ 0 \ 0)$, in which the last four factors are the same as the original test case T_0), we generate the related matrix at left. Each element in this matrix is computed as the $\frac{m(o)}{all(o)+1}$; for example, for the $O7$ factor with value 0, we can

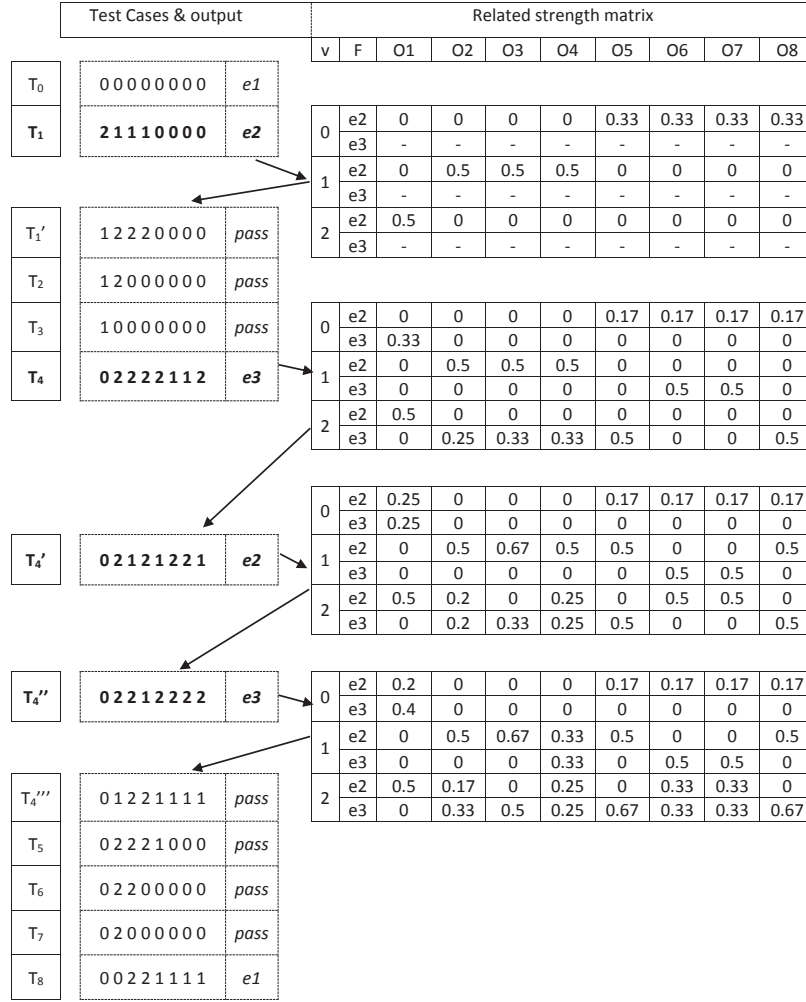


Fig. 5. A case study using our approach

find two test cases that contain this element, i.e., T_0 and T_1 , so the $\text{all}(o)$ is 2. And only one test case triggers the failure $e2$, which means $m(o) = 1$. So the final related strength between this factor with $e2$ is $\frac{1}{2+1} = 0.33$. All the related strength with $e3$ is labeled with a short slash as there is no test case triggering this failure in this iteration. After this matrix has been determined, we can obtain a optimal test case with the ILP solver, which is $T_1' - (1\ 2\ 2\ 2\ 0\ 0\ 0\ 0)$, with its related strength 0.167, which is smaller than that of the others.

This replacement process triggered each time a new test case that triggered another failure until we finally get the MFS. Sometimes we could not find a satisfied replacing test case in just one trial like T_1 to T_1' . When this happened, we needed to repeat searching the proper test case we desired. For example, for T_4 which triggered $e3$, we tried three times— T_4' , T_4'' , T_4''' to finally get a satisfied one T_4''' which passes the testing. Note that the matrix continues to change with the test case generated and executed so that we can adaptively find an optimal one in the current process.

Table XVII. The number of test cases each FCI approach needed to identify MFS

Method	number of test cases to identify MFS
Charles ELA	depends on the covering array
Martinez with safe value [Martínez et al. 2008; 2009]	$O(d \log k + d^2)$
Martinez without safe values [Martínez et al. 2008; 2009]	$O(d^2 + d \log k + \log^c k)$
Martinez' ELA [Martínez et al. 2008; 2009]	$O(ds^v \log k)$
Shi SOFOT [Shi et al. 2005]	$O(k)$
Nie OFOT [Nie and Leung 2011a]	$O(k \times d)$
Yilmaz classification tree	depends on the covering array
FIC [Zhang and Zhang 2011]	$O(k)$
FIC.BS [Zhang and Zhang 2011]	$O(t(\log k + 1) + 1)$
Ghandehari's suspicious based [Ghandehari et al. 2012]	depends on the number and size of MFS
TRT [Niu et al. 2013]	$O(d \times t \times \log k + t^d)$

5.3. Complexity analysis

This complexity relies on two facts: the number of test cases that triggered other failures which need to be replaced, and the number of test cases that need to be tried to generate a non-masking-effects test case. The complexity is the product of these two facts.

The first fact is comparable to the extra test cases that are needed to identify the MFS, and this number varies in different FCI approaches. Table XVII lists the number of test cases that each algorithm needed to get the MFS. In this table, d indicates the number of MFS in the SUT. k means the number of the parameters of the SUT. t is the number of MFS factors in the SUT. c is an upper bond, and satisfies $d \leq \frac{c}{2} \log \log k$. v is the number of values one parameter can take.

It must be noted that each algorithm may be limited to some restrictions to identify the result, details of which are shown in [Zhang and Zhang 2011].

To get the magnitude of the second fact, we need to figure out the possibility of a test case that could trigger other failure. The first thing we need to consider is the *fixed* part, as the additional generated test case should somehow contain this part. As we have mentioned before, we can generate $(v - 1)^{k-p}$ (p is the number of factors in the *fixed* part) possible test cases that contain the *fixed* part. Apart from the one that needs to be replaced, there remain $(v - 1)^{k-p} - 1$ candidate test cases, which indicates the complexity is $O((v - 1)^{k-p} - 1)$. However, to avoid the exponential computational complexity, in this algorithm we use the method *MeetEndCriteria()* (line 1) function to end the algorithm when the trying times is over a prior given constant, say N , so the final complexity for the second part is $O(\min(N, (v - 1)^{k-p} - 1))$.

We note that exponential factor $k - p$ directly affects the complexity of the second factor. The greater p is, the less test cases that can be generated. For a different approach, p is different. For example, for the OFOT approach, p is a fixed number, which is $k - 1$. And for the FIC.BS approach, the p varies in the test cases it generates, ranging from $k - 1$ to 1. While for the non-adaptive approaches, as the *fixed* part is commonly the schemas that are needed to be covered, so the p for these approaches is at least equal to t . We have listed all of them in Table XVIII. It is noted that the approach – *Martinez without safe values* has no such complexity, because this approach works when $v = 2$, and this results in not having other test cases to be replaced if we test a fixed part when triggering other failures.

6. EMPIRICAL STUDIES

To investigate the impact of masking effects for FCI approaches in real software testing scenarios and to evaluate the performance that how well our approach handles this effect, we conducted several empirical studies which we discuss in this section. Each of the studies focuses on addressing one particular issue, as follows:

Table XVIII. The complexity of the second part

Method	fixed part
Charles ELA	$O(\min(N, (v-1)^{k-t} - 1))$
Martinez with safe value [Martínez et al. 2008; 2009]	$O(\min(N, (v-1) - 1)) \sim O(\min(N, (v-1)^{k-1} - 1))$
Martinez without safe values [Martínez et al. 2008; 2009]	—
Martinez' ELA [Martínez et al. 2008; 2009]	$O(\min(N, (v-1)^{k-t} - 1))$
Shi SOFOT [Shi et al. 2005]	$O(\min(N, (v-1) - 1))$
Nie OFOT [Nie and Leung 2011a]	$O(\min(N, (v-1) - 1))$
Yilmaz classification tree	$O(\min(N, (v-1)^{k-t} - 1))$
FIC [Zhang and Zhang 2011]	$O(\min(N, (v-1) - 1)) \sim O(\min(N, (v-1)^{k-1} - 1))$
FIC_BS [Zhang and Zhang 2011]	$O(\min(N, (v-1) - 1)) \sim O(\min(N, (v-1)^{k-1} - 1))$
Ghandehari's suspicious based [Ghandehari et al. 2012]	$O(\min(N, (v-1)^{k-t} - 1))$
TRT [Niu et al. 2013]	$O(\min(N, (v-1) - 1)) \sim O(\min(N, (v-1)^{k-1} - 1))$

Table XIX. Software under survey

software	versions	LOC	classes	bug pairs
HSQLDB	2.0rc8	139425	495	#981 & #1005
	2.2.5	156066	508	#1173 & #1179
	2.2.9	162784	525	#1286 & #1280
JFlex	1.4.1	10040	58	#87 & #80
	1.4.2	10745	61	#98 & #93

Q1: Do masking effects exist in real software that contains multiple failures?

Q2: How well does our approach perform compared to traditional approaches?

Q3: Is the ILP-based test case searching technique efficient compared to random selection?

Q4: Compared to another masking effects handling approach – the FDA-CIT [Yilmaz et al. 2013], does our new approach have any advantages ?

6.1. The existence and characteristics of masking effects

In the first study, we surveyed two kinds of open-source software to gain an insight into the existence of multiple failures and their effects. The software under study were: HSQLDB and JFlex. The first is database management software written in pure Java and the second is a lexical analyser generator. Each contains different versions and are all highly configurable so that the options and their combinations can affect their behaviour. Additionally, they all have a developers' community so that we can easily obtain the real bugs reported in the bug tracker forum. Table XIX lists the program, the number of versions we surveyed, number of lines of uncommented code, number of classes in the project, and the bug's id³ for each of the software we studied.

6.1.1. Study setup. We first looked through the bug tracker forum of each software and focused on the bugs which are caused by the options combination. For each such bug, we will derive its MFS by analysing the bug description report and the attached test file which can reproduce the bug. For example, through analysing the source code of the test file of bug#981 for HSQLDB, we found the failure-inducing combination for this bug is: (*preparestatement*, *placeholder*, *Long string*). These three factors together form the condition that triggers the bug. The analysed results will be later regarded as the “prior MFS”.

We further built the testing scenario for each version of the software listed in Table XIX. The testing scenario is properly constructed so that we can reproduce different

³<http://sourceforge.net/p/hsqldb/bugs>
<http://sourceforge.net/p/jflex/bugs>

failures by controlling the inputs to the test file. For each version of the software, the source code of the testing file as well as other detailed experiment information is available at– <https://code.google.com/p/merging-bug-file>.

Next, we built the input model which consists of the options related to the failure-inducing combinations and additional noise options. The detailed model information is in Tables XX and XXI for HSQLDB and JFlex, respectively. Each table is organised into four groups: (1)*common options*, which lists the options as well as their values under which every version of this software can be tested; (2)*common Boolean options*, which lists additional common options whose data type is Boolean; (3)*specific options*, under which only the specific version of that software can be tested; and (4)*configure space*, which depicts the input model for each version of the software, the input model is presented in the abbreviated form $\#values^{\#number\ of\ parameters} \times \dots$, e.g., $2^9 \times 3^2 \times 4^1$ indicates the software has 9 parameters that can take 2 values, 2 parameters can take 3 values, and only one factor that can take 4 values.

Table XX. Input model of HSQLDB

common options		values
Server Type	existed form	server, webserver, inprocess mem, file
resultSetTypes		forwad, insensitive, sensitive
resultSetConcurrencys		read_only, updatable
resultSetHoldabilitys		hold, close
StatementType		statement, prepared
common Boolean options		
sql.enforce_strict_size, sql.enforce_names, sql.enforce_refs		
versions	specific options	values
2.0rc8	more	true, false
	placeholder	true, false
	cursorAction	next,previous,first,last
2.2.5	multiple	one, multi, defailure
	placeholder	true, false
2.2.9	duplicate	dup, single, defailure
	defailure_commit	true, false
versions	Config space	
2.0rc8	$2^9 \times 3^2 \times 4^1$	
2.2.5	$2^8 \times 3^3$	
2.2.9	$2^8 \times 3^3$	

Table XXI. Input model of JFlex

common options		values
generation		switch, table, pack
charset		defailure, 7bit, 8bit, 16bit
common boolean options		
public, apiprivate,cup,caseless,char,line,column,notunix, yyeof		
versions	specific options	values
1.4.1	hasReturn	has, non, defailure
	normal	true, false
1.4.2	lookAhead	one, multi, defailure
	type	true, false
	standalone	true, false
versions	Config space	
1.4.1	$2^{10} \times 3^2 \times 4^1$	
1.4.2	$2^{11} \times 3^2 \times 4^1$	

We then generated the exhaustive test suite consisting of all possible combinations of these options, and under each of them, we executed the prepared testing file. We

Table XXII. Number of failures and their masking effects

software	versions	all tests	failure	masking
HSQldb	2cr8	18432	4608	768
-	2.2.5	6912	3456	576
-	2.2.9	6912	3456	1728
JFlex	1.4.1	36864	24576	6144
-	1.4.2	73728	36864	6144

recorded the output of each test case to observe whether there were test cases containing prior MFS that did not produce the corresponding bug.

6.1.2. Results and discussion. Table XXII lists the results of our survey. Column “all tests” give the total number of test cases we executed. Column “failure” indicate the number of test cases that failed during testing, and column “masking” indicates the number of test cases which triggered the masking effect.

We observed that for each version of the software under analysis that we listed in the Table XXII, test cases with masking effects do exist, i.e., test cases containing MFS did not trigger the corresponding bug. In effect, there are about 768 out of 4608 test cases (16.7%) in hsqldb with 2rc8 version. This rate is about 16.7%, 50%, 25%, and 16.7%, respectively, for the remaining software versions, which is not trivial.

So the answer to **Q1** is that in practice, when SUT have multiple failures, masking effects do exist widely in the test cases.

6.2. Comparing our approach to traditional algorithms

In the second study, our aim was to compare the performance of our approach to traditional approaches in identifying MFS under the impact of masking effects. To conduct this study, we needed to apply our approach and traditional algorithms to identify MFS in a group of software and evaluate their identifying results. The five prepared versions of software in Table XIX used as test objects are far from a general evaluation of such objects. However, to construct such real testing scenarios is time-consuming as we must carefully study the tutorial of that software as well as the bug tracker report. So to give a desirable result based on more testing objects, we then synthesize a number of such testing scenarios of which the characterizations, such as the number of factors, the number of failures, and the possible masking effects, are similar to that of the real software. In detail, we set the number of parameters k of the SUT to a range from 8 to 30. We limited the scale of the SUT to a relatively small size because we needed to exhaustively execute each possible test case of the SUT to select the failed test cases which we then fed into the FCI approach. We then randomly choose 10 such SUTs, and for each SUT we injected 2 to 5 different MFS that can mask each other. The degree of the MFS we injected ranged from 1 to 6.

Above all, Table XXIII lists the testing model for both the real and synthesizing testing scenario. In this table, the column ‘software’ indicates the SUT under test. For the real SUT, we label it with the form ‘*name + version*’, while for the synthesizing ones, we label them as ‘*synthes+ id*’. The column ‘Model’ presents the model of the input space for that software. The last column shows the MFS as well as the masking sequence for each testing object. The MFS is presented in an abbreviated form $\{\#index\#value\}$, e.g., $(5_1, 6_0, 7_0)$ actually means $(- - - - 1, 0, 0, -, -, -)$ for HSQldb of version ‘2cr8’. It is noted that we use ‘ \rightarrow ’ and ‘ $=$ ’ to describe the masking sequence of each MFS, in which ‘ \rightarrow ’ means the left MFS in this operator can mask the right MFS of this operator, e.g., $(5_1, 6_0, 7_0) \rightarrow (5_1, 8_2, 9_2)$ means if $(5_1, 6_0, 7_0)$ appears in the test case, then $(5_1, 8_2, 9_2)$ will not be triggered. Operator ‘ $=$ ’ means that these two MFS will not mask each other.

Table XXIII. The testing models used in the case study

software	Model	MFS& masking sequence
HSQldb 2cr8	$2^9 \times 3^2 \times 4^1$	$(5_1, 6_0, 7_0) \rightarrow (5_1, 8_2, 9_2) = (5_1, 8_2, 9_1) \rightarrow (5_1, 8_3, 9_2) = (5_1, 8_3, 9_1)$
HSQldb 2.2.5	$2^8 \times 3^3$	$(6_1, 7_0) \rightarrow (5_2)$
HSQldb 2.2.9	$2^8 \times 3^3$	$(6_0) \rightarrow (0_1, 5_1, 7_0) = (0_0, 5_1, 7_0) \rightarrow (5_1, 7_0)$
JFlex 1.4.1	$2^{10} \times 3^2 \times 4^1$	$(0_0) \rightarrow (1_0)$
JFlex 1.4.2	$2^{11} \times 3^2 \times 4^1$	$(1_0, 2_1) \rightarrow (0_1)$
synthex 1	$2^5 \times 3^3 \times 4^1$	$(2_1, 3_0) \rightarrow (1_1, 2_1) = (1_0, 3_0)$
synthex 2	$2^6 \times 3^2 \times 4^1$	$(4_1, 6_0, 7_1, 8_0) \rightarrow (1_1, 3_1, 5_1) \rightarrow (2_0, 3_1, 6_0)$
synthex 3	$2^5 \times 3^3$	$(2_1, 3_0) \rightarrow (1_0) = (4_1) \rightarrow (6_0, 7_0)$
synthex 4	$2^7 \times 3^2 \times 4^1$	$(0_1, 2_1, 5_0, 6_1) \rightarrow (2_1, 4_0) = (6_1, 7_0) \rightarrow (3_0, 4_0, 5_0)$
synthex 5	$2^4 \times 3^3 \times 4^2$	$(0_0, 1_1, 3_0, 6_1, 8_0) \rightarrow (2_0, 3_0, 4_1)$
synthex 6	$2^9 \times 3^2$	$(2_0, 7_1, 8_1) \rightarrow (3_1, 5_1) = (4_0) \rightarrow (3_1, 6_0, 7_1) \rightarrow (3_1, 7_1, 8_0)$
synthex 7	$2^{10} \times 3^1 \times 4^1$	$(3_1, 4_0, 5_0) \rightarrow (2_0, 4_0, 7_1, 9_0) \rightarrow (6_1, 10_0, 11_1)$
synthex 8	$2^{11} \times 3^1 \times 4^1$	$(1_0, 3_1, 4_0, 7_1, 9_0, 12_1) \rightarrow (0_0, 2_1, 3_1, 7_1, 10_0, 11_1)$
synthex 9	$2^4 \times 4^3$	$(3_1, 5_0) \rightarrow (5_0, 6_1)$
synthex 10	$2^7 \times 3^3 \times 4^1$	$(0_1, 3_0, 4_1, 7_0) \rightarrow (2_0, 3_0, 5_1) = (2_0, 3_0, 5_0)$

6.2.1. Study setup. After preparing the subjects under testing, we then apply our approach (augment the FIC_BS with replacing strategy) to identify the MFS of each SUT listed in Table XXIII. Specifically, for each SUT we select each test case that failed during testing and feed these into our FCI approach as the input. Then, after the identifying process is over, we record the MFS each got (referred to as *identified MFS*, for convenience) and the extra test cases it needed. For the traditional FIC_BS approach, we designed the same experiment as what are used for our approach. But as the objects being tested have multiple failures for which the traditional FIC_BS can not be applied directly, we adopted two traditional strategies on the FIC_BS algorithm, i.e., *regarded as one failure* and *distinguishing failures* described in Section 3.2. The purpose of recording the generated additional test cases is to later quantify the additive cost of our approach.

We next compared the identified MFS of each approach with the prior MFS to quantify the degree that each suffers from masking effects, such that we can figure out how much our approach performs better than traditional ones when the SUT contains potential masking effects. There are five metrics we need to calculate in this study, which are as follows:

- (1) The number of the common combinations that appear in both identified MFS and prior MFS. We denote this metric as *accurate number* later.
- (2) The number of the identified combinations which are the parent combinations of some prior failure-inducing combinations. We refer to this metric as the *parent number*.
- (3) The number of the identified combinations that are the sub combinations of some prior failure-inducing combinations, which are referred to as the *sub number*.
- (4) The number of ignored failure-inducing combinations. This metric counts these combinations in prior failure-inducing combinations, which are irrelevant to the identified combinations. We label the metric as *ignored number*.
- (5) The number of irrelevant combinations. This metric counts the combinations in these identified combinations that are irrelevant to the prior failure-inducing combinations. It is referred to as the *irrelevant number*.

Among these five metrics, the high *accurate number* value indicates FCI approaches that perform effectively, while the *ignored number* and *irrelevant number* indicate the degree of deviation for the FCI approaches. The *parent number* and *sub number* indicate the FCI approaches that can determine part parameter values for the failure-inducing combinations, although with additional noisy information.

Besides these specific metrics, we also give a composite criteria to measure the overall performance of each approach. The computing formula for the composite criteria is as follows:

$$\frac{\text{accurate} + \text{related}(\text{parent}) + \text{related}(\text{sub})}{\text{accurate} + \text{parent} + \text{sub} + \text{irrelevant} + \text{ignored}}$$

In this formula, *accurate*, *parent*, *sub*, *irrelevant*, and *ignored* represent the value of each specific metric. The *related* function gives the similarity between the schemas (either parent or sub) and the real MFS. The similarity between two schemas S_A and S_B is computed as:

$$\text{Similarity}(S_A, S_B) = \frac{\text{the same elements in } S_A \text{ and } S_B}{\max(\text{Degree}(S_A), \text{Degree}(S_B))}$$

For example, the similarity of (- 1 2 - 3) and (- 2 2 - 3) is $\frac{2}{3}$. This is because the same elements of this two schemas are the third and last elements, and both of these two schemas are three-degree.

So the *related* function is the summation of similarity of all the parent or sub schemas with their corresponding MFS.

6.2.2. Results and discussion. Fig 6 depicts the results of the second case study. There are seven sub-figures in this figure, i.e., Fig. 6(a) to Fig. 6(g). They indicate the results of : the number of accurate MFS each approach identified, the number of identified schemas which are the sub-schema parent-schema of some prior MFS, the number of ignored prior MFS, the number of identified schemas which are irrelevant to all the prior MFS, the metric which gives the overall evaluation of each approach, and the extra test cases each algorithm needed, respectively. For each sub-figure, there are four polygonal lines in it, each of which shows the results for one of the four strategies: *regarded as one failure*, *distinguishing failures*, replacement strategy based on ILP searching, replacement strategy based on randomly searching (The last one will be discussed in the next case study). Specifically, each point in the polygonal line indicate the specific result of a particular strategy for the corresponding testing object. For example at Fig 6(a), the point marked with ‘♦’ at (1,2) indicates that the approach using *regarded failures* identified 2 accurate MFS in the testing object-HSQLDB 2cr8. The raw data for this experiment can be found in Table XXIV in the Appendix. Next we will discuss these results of these sub-figures.

accurate number: Fig .6(a) shows that the number of accurate schemas that each approach achieved. We can firstly find there is no special ones, i.e., not existing an approach that can always perform better/worse others in all these subjects. This is because the accurate schemas is depend on the failure test cases it can figure out(either observed or predicted) as we discussed in the formal analysis. Based on this, the two strategies *regarded as one failure* and *distinguishing failures* will both, in specific, *regarded as one failure* will and *distinguishing failures* will.

We can further find that for the two approaches, ILP and distin, have similar results about the number of identified accurate MFS. This can be easily understood, as our approach ILP derived from the distin, which also make the failure distinguished with each other. And with some more refinement strategy. The

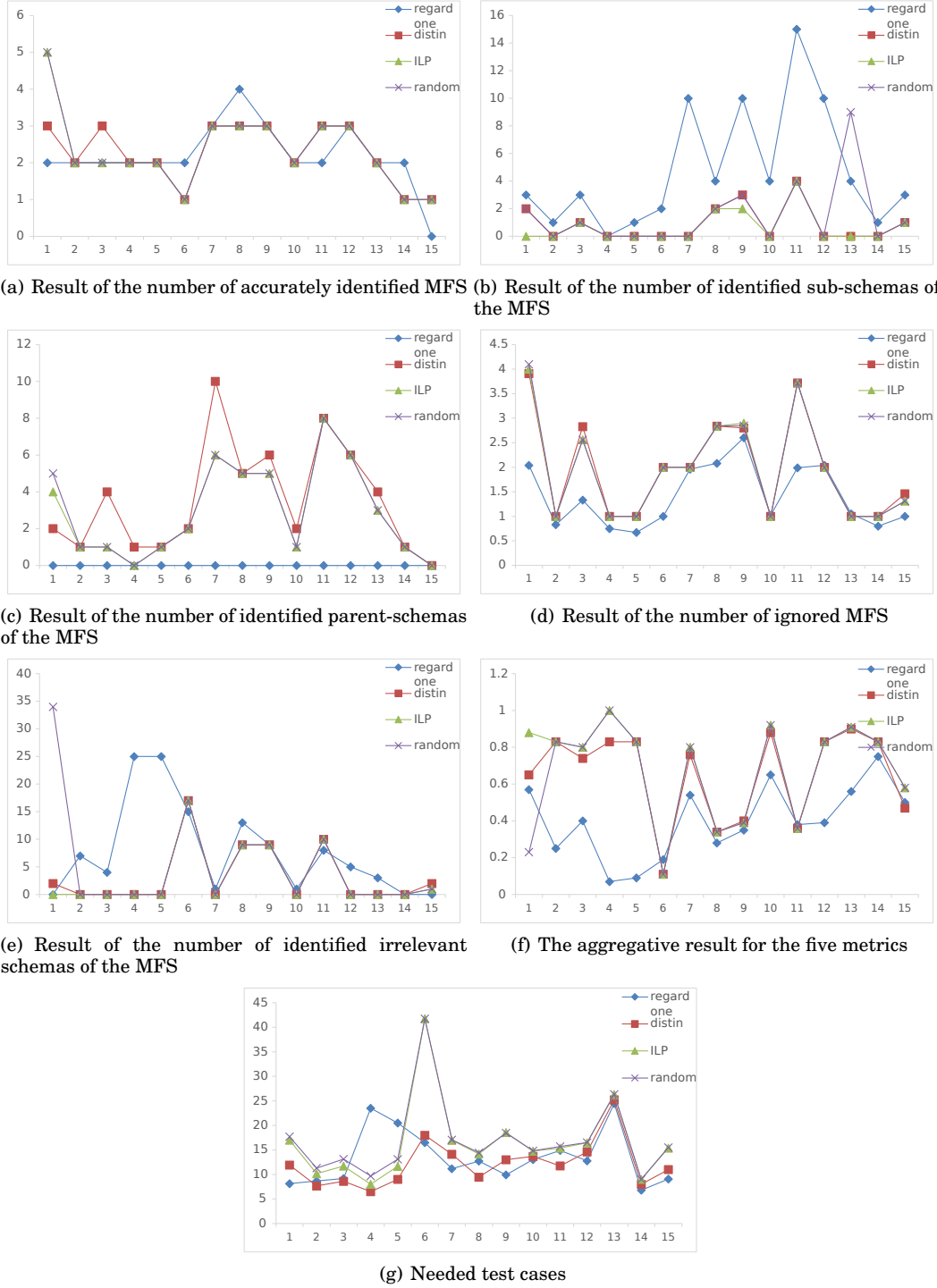


Fig. 6. Result of the evaluation of each approach

sub number:

Fig. 6(b) depicts the results belong to this category. It shows a more clear trend than *accurate number*. We can easily find that the strategy *regarded as one fault* performs the worst when compared to the ILP and *distin* approach. In fact, except for the fourth subject (JFlex 1.4.1), *regarded as one failure* achieved more sub schemas than the other two strategies on all the remaining subjects. With respect to results for strategy *regarded as one failure* and *distinguishing failures*, we find that the result, i.e., *regarded as one fault* will produce more sub schemas than *distinguishing failures*, is as expected as discussed in section 4.

When compared our approach ILP and the *distinguishing failures*, we can learn that first the results are really similar to each other, and a step further we can find our approach perform a little better than that of *distin* (at subject 1 and 9). We believe the little improvement is based on the fact our approach take the better test cases when encountering the not current fault condition, and the better test cases will get a good chance that the FCI will make better prediction of the failure test cases. So at last we can find that our approach can get less sub schemas.

parent number: The result in Fig. 6(c) shows that the *regarded as one fault* performed better than the other two approaches. This is consistent with the formal analysis as this strategy the approach does not discard these test cases that may have masking effects, which make it less chances than *distin* that will not make the MFS in the intersection area not be parent. Although we have predicted it can perform better than the other two approaches, we didn't expect it does not have even a parent schema than. In fact all these subjects is 0. It is interesting which drive us to deep insight what happened. And finally we get that.

Another observation about this metric is that similar to the sub number, when comparing ILP with *distin*, we can find that our approach can also perform a little better than *distin*. It is a evidence that the replacement process really improving the generated test cases quality and make the prediction better.

ignore number:

As respect to this metric, Fig 6(d) gives two viewpoints we can conclude: first, *regarded as one failure* performs better than the other two approaches, this is also consistent with the formal analysis in section, because for the *distin* strategy, it will discard some important test cases which can directly make the MFS ignored; second, ILP performs better than the *distin*.

irrelevant number: Fig 6(e) shows the results for this metric. We can firstly observed that except the point 1, 6, 9, 11, 14, 15 that *regarded as one failure* have a similar result as ILP and *distin*, the remaining have a significance gap comparing to *distin* and ILP (*regarded as one failure* normally identified have 10 to 20 more schemas that are irrelevant to the real MFS). This tell us *regarded as one failure* have a poorer performance than the other two approaches at this metric. This result is also as expected as we in the formal analysis section got that *regarded as one failure* has more rules that can lead the schemas to be irrelevant ones.

As for the ILP and *distin*, we can still find ILP a little better than the *distin*. This can not be easily observed at this figure, but when we look at the raw data at Table XXIV, we can find that ILP has on average identified 0.XXXX less irrelevant test cases.

aggregative for the five metrics: When we composite these five metrics using the formula aforementioned and to comprehensive investigate the schemas that each approach get, we finally get Fig 6(f). This figure gives a clear trend for the optimal and worse order of the approaches, i.e., ILP > *distin* > *regard*. In specific, for every subject except for the very similar ones such as 6, 8, 9, 11, the data shows this trend.

We can expect that ILP can perform better than the *distin* because the ILP is actually based on the *distin*, and do some refinement by getting higher quality test cases. As

for distin better than regarded as one failure. We have not get it in the formal analysis, but however, we have a strong belief in that this is because this divide and conquer at least make the FCI approaches be more sensitive for the failures than the regarded as one failure.

test cases: The above analysis, in a word, gives the comparable of the quality that each approach can result. This metric gives us the comparable ways about the cost, i.e., the extra test cases that each approach will generated. When we look at the Fig. 6(g), we can obviously find that the ILP will cost more test cases than distin and regard one. In specific, the gap between the ILP and distin and regard is ranged from about 2 to 5(except for the 6 subject, which exceeds 20), this is acceptable when comparing to the all the test cases that each test cases needed. This gap is expected as our ILP approaches needed to generate additional test cases when some test cases are not satisfied the requirement. When we looked back at the distin and regard one, there is no clear trend of which is better than another.

Above all, we can conclude about three points in this experiment, and which can be an answer for the **Q2**:

- 1)the results of distin and regard one are consistent with the formal analysis.
- 2)Considering the quality of the MFS each approach identified, we can find our approach can get the best, and then the distin.
- 3)Our approach need more test cases than ILP and distin, but which is acceptable.

6.3. Evaluating the ILP-based test case searching method

The third empirical study aims to evaluate the efficiency of the ILP-based test case searching component in our approach. To conduct this study, we implemented an FCI approach which is also augmented by the *replacing test cases* strategy, but the test case replacing process is by random.

6.3.1. Study setup. The setup of this case study is based on the second case study, and uses the same SUT model as in Table XXIII. Then, we apply the new random searching based FCI approach to identify the MFS in these prepared SUTs. To avoid the bias that comes from the randomness, we repeat the new approach 30 times to identify the MFS in each failed test case. We will compute the average additional test cases as well as other metrics listed in the previous section of the random-based approach.

6.3.2. Results and discussion. The evaluation of this random-based approach is also shown in Fig. 6, in which the polygonal line marked with 'x' points in each sub-figure indicates this type of results. The raw data can be also found in the column 'R' of Table XXIV in the appendix section.

Compared to our ILP-based approach, we can firstly observe that there is little distinction between them in terms of the metrics: accurate schemas, parent-schemas, sub-schemas, ignored schemas, irrelevant schemas (for some particular cases the ILP-based approach performs slightly better, e.g., in Fig. 6(b) for the first subject, the ILP-based approach identified less sub schemas than that of the Random-based approach and in Fig. 6(c) still for the first object the ILP -based approach identified less parent schemas than that of the random-based approach). The similarity quality of the identified MFS between this two approaches is conceivable as the two approaches both use the *test case replacement* strategy, so when examining a schema, both of these two approaches may obtain the same result, although the test cases generated will be different.

Secondly, when considering the cost of each approach, we find the ILP-based approach performs better, which can reduce in the average to 1 or 2 test cases less compared to the random-based procedure. It is an evidence of our integer programming

based searching technique can find a satisfied test case more rapidly than the random based one.

In summary, the answer for **Q3** is that: searching for a satisfied test case does have affect the performance of our approach, especially regarding the number of extra test cases, and the ILP-based test cases can handle the masking effects at a relatively smaller cost than the random-based approach.

6.4. Comparison with Feedback driven combinatorial testing

The FDA-CIT [Yilmaz et al. 2013] approach can handle masking effects so that the covering array it generates can cover all the t-way schemas without being masked by the MFS. There is an integrated FCI approach in the FDA-CIT, of which this FCI approach has two versions, i.e., ternary-class and multiple-class. In this paper, we use the multiple-class version for our comparative approach, as Yilmaz claims that the multiple-class version performs better than the former [Yilmaz et al. 2013].

6.4.1. Study setup. As the FCI approach of FDA-CIT use a post-analysis(classified tree) technique on given test cases, in this paper we fed the FDA-CIT the covering array as the input just as was done in the Yilmaz study [Yilmaz et al. 2013]. The covering arrays we generated ranged from 2 to 4 ways. The covering array generating method we used is that contained in [Cohen et al. 2003], as it can be easily extended with constraint dealing and seed injecting [Cohen et al. 2007b], which is needed in the FDA-CIT process. As different test cases will influence the results of the characterization process, we generating 30 different 2 to 4 way covering arrays and fed them into the FDA-CIT. Then, we recorded the results of this approach, which consists of the metrics mentioned in the second case study.

Besides the FDA-CIT, we also applied our ILP-based approach to the generated covering array. Specifically, for each failed test case in the covering array, we separately applied our approach to identify the MFS in that case. In fact, we can reduce the number of extra test cases if we utilize the other test cases in the covering array [Li et al. 2012]), but we didn't utilize the information to simplify the experiment. We then merged all the test cases that our approach needed for each failed test case in the covering array, and we also merged other metrics listed in the second case study for each failed test case.

As our approach generated different test cases from the FDA-CIT, we also used the multiple-class FCI approach of FDA-CIT to characterize the MFS using the test cases generated by our approach, so that we could obtain a fair result with which to evaluate the FCI approach.

6.4.2. Result and discussion. For each strength t (ranged from 2 to 4), we conducted the experiment repeated 30 times for different covering arrays, we then recorded the average data(included accurate, sub, parent, irrelevant, ignored, overall and needed test cases) for the FDA-CIT, ILP-based approach, and FDA-CIT using our test cases (FDAs), respectively, for each subject. To conveniently and clearly visualize the result, we merged all the results of all the subjects for each approach and list them in the figure 7. The raw data which can show the result for each subject can be found in Table XXV in the appendix section. The result is organised the same as in Table XXIV, except that we added a column t which indicates the strength of the covering array we generated for this experiment.

As a first view of this figure, we can immediately get a conclusion that the trend that different approach performs is never changed with the change of the covering array strength. For example, focusing the metric *ignored schemas*, we can find that for any of the t , there is always $FDA > FDA-s > ILP$. Next we will discuss these metrics separately.

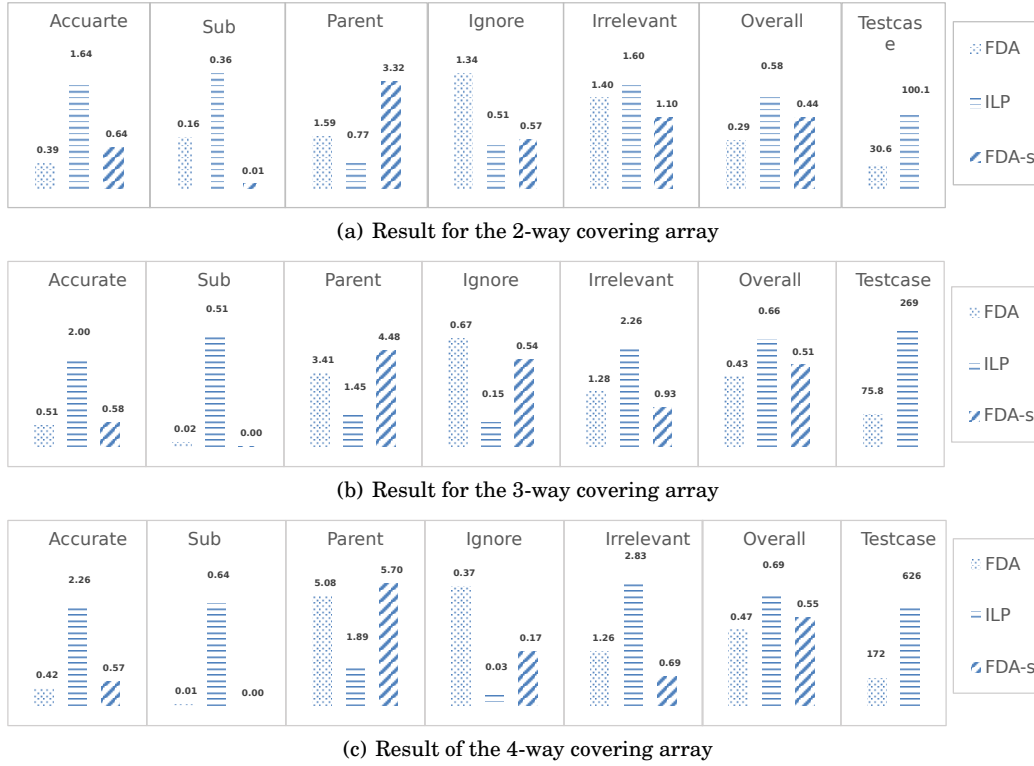


Fig. 7. Three approaches augmented with the replacing strategy

accurate number:

sub number:

parent number:

ignore number:

irrelevant number:

aggregative for the five metrics:

test cases:

Above all, we can conclude about three points in this experiment, and which can be an answer for the **Q4**:

1)the general trend of the performance that each approach is never changed with the changing of the t.

2)Considering the quality of the MFS each approach identified, we can find our approach can get the best, and then the distin.

3)Our approach need more test cases than ILP and distin, but which is accepta

From this result, we can first observe that in all the cases our ILP-based approach can more accurately identify the MFS and ignored less MFS than the FDA-CIT approach. For the metric ‘parent-schema’, ‘sub-schema’, and ‘irrelevant schemas’, there are ups and downs on both sides. With respect to the ‘overall metric’, we find our approach has a significant advantage over the FDA-CIT, but it also requires many more test cases than FDA-CIT.

We note that, when applying the multiple-class FCI to characterize the MFS with the test cases that were generated using our approach, their ‘overall’ metric is still not

as good as our ILP-based approach, but may show some improvement over the original FDA-CIT.

Another interesting observation is that overall performance in most cases is increasing with the t , which can be easily understood. More test cases will contain more information about the MFS, so that we can utilize them to identify the MFS more precisely.

So the answer for **Q4** is that: our approach can achieved a more precise result for the MFS, and the FDA-CIT can perform the identifying process using a small amount extra test cases. Both of these two approaches have their ups and downs, choosing which approach in practice will depend on the specific scenario you test.

6.5. Threats to validity

There are several threats to the validity for these empirical studies. First, we have only surveyed two types of open-source software with five different versions, of which the program scale is medium-sized. This may impact the generality of our observations. Although we believe it is quite possibly a common phenomenon in most software that contains multiple failures which can mask each other, we need to investigate more software to support our belief.

The second threat comes from the input model we built. As we focused on the options related to the perfect combinations and only augmented it with some noise options, there is a chance we will get different results if we choose other noise options. More options needed to be tested to see whether our result is common or just appears in some particular input models.

The third threat is that we just evaluated one FCI approach – FIC-BS [Zhang and Zhang 2011], so further works needed to examine more algorithms in this field to obtain a more general result.

7. RELATED WORKS

Shi and Nie presented an approach for failure revealing and failure diagnosis in CT [Shi et al. 2005], which first tests the SUT with a covering array, then reduces the value schemas contained in the failed test case by eliminating those appearing in the passed test cases. If the failure-causing schema is found in the reduced schema set, failure diagnosis is completed with the identification of the specific input values which caused the failure; otherwise, a further test suite based on SOFOT is developed for each failed test cases, testing is repeated, and the schema set is then further reduced, until no more failures are found or the failure is located. Based on this work, Wang proposed an AIFL approach which extended the SOFOT process by adaptively mutating factors in the original failing test cases in each iteration to characterize failure-inducing combinations [Wang et al. 2010].

Nie et al. introduced the notion of Minimal Failure-causing Schema (MFS) and proposed the OFOT approach which is an extension of SOFOT that can isolate the MFS in SUT [Nie and Leung 2011a]. The approach mutates one value with different values for that parameter, hence generating a group of additional test cases each time to be executed. Compared with SOFOT, this approach strengthen the validation of the factor under analysis and also can detect the newly imported faulty combinations.

Delta debugging proposed by Zeller [Zeller and Hildebrandt 2002] is an adaptive divide-and-conquer approach to locate interaction failure. It is very efficient and has been applied in real software environment. Zhang et al. also proposed a similar approach that can efficiently identify the failure-inducing combinations that has no overlapped part [Zhang and Zhang 2011]. Later, Li improved the delta-debugging based failure-inducing combination by exploiting useful information in the executed covering array [Li et al. 2012].

Colbourn and McClary proposed a non-adaptive method [Colbourn and McClary 2008]. Their approach extends a covering array to the locating array to detect and locate interaction failures. C. Martinez proposed two adaptive algorithms. The first one requires safe value as the assumption and the second one removes the assumption when the number of values of each parameter is equal to 2 [Martínez et al. 2008; 2009]. Their algorithms focus on identifying faulty tuples that have no more than 2 parameters.

Ghandehari et al. defined the suspiciousness of tuple and suspiciousness of the environment of a tuple [Ghandehari et al. 2012]. Based on this, they rank the possible tuples and generate the test configurations. They further utilized the test cases generated from the inducing combination to locate the failures inside the source code [Ghandehari et al. 2013].

Yilmaz proposed a machine learning method to identify inducing combinations from a combinatorial testing set [Yilmaz et al. 2006]. They constructed a classification tree to analyze the covering arrays and detect potential faulty combinations. Beside this, Fouché [Fouché et al. 2009] and Shakya [Shakya et al. 2012] made some improvements in identifying failure-inducing combinations based on Yilmaz's work.

Our previous work [Niu et al. 2013] proposed an approach that utilizes the tuple relationship tree to isolate the failure-inducing combinations in a failed test case. One novelty of this approach is that it can identify the overlapped faulty combinations. This work also alleviates the problem of introducing new failure-inducing combinations in additional test cases.

In addition to the studies that aim at identifying the failure-inducing combinations in test cases, there are others that focus on working around the masking effects.

With having known masking effects in prior, Cohen [Cohen et al. 2007a; 2007b; 2008] studied the impact of the masking effects that render some generated test cases invalid in CT. They also proposed an approach that integrates the incremental SAT solver with the covering arrays's generation algorithm to avoid masking effects. Further study was conducted [Petke et al. 2013] to show that with considering constraints, higher-strength covering arrays with early failure detection are practical. Besides, additional works that aim to study the impacts of constraints for CT were as follows: [Garvin et al. 2011; Bryce and Colbourn 2006; Calvagna and Gargantini 2008; Grindal et al. 2006; Yilmaz 2013].

Chen et al. addressed the issues of shielding parameters in combinatorial testing and proposed the Mixed Covering Array with Shielding Parameters (MCAS) to solve the problem caused by shielding parameters [Chen et al. 2010]. The shielding parameters can disable some parameter values to expose additional interaction errors, which can be regarded as a special case of masking effects.

Dumlu and Yilmaz proposed a feedback-driven approach to work around the masking effects [Dumlu et al. 2011]. Specifically, they first used classification tree approach to classify the possible failure-inducing combinations and then eliminate them and generate new test cases to detect possible masked interaction in the next iteration. They further extended their work [Yilmaz et al. 2013] by proposing a multiple-class CTA approach to distinguishing failures in SUT. In addition, they empirically studied the impacts on both ternary-class and multiple-class CTA approaches.

Our work differs from these mainly in that we formally studied the masking effects on FCI approaches and further proposed a divide-and-conquer strategy to alleviate this impact.

8. CONCLUSIONS

Masking effects of multiple failures in SUT can bias the results of traditional failure-inducing combinations identifying approaches. In this paper, we formally analysed the

impact of masking effects on FCI approaches and showed that the two traditional strategies, i.e., *regarded as one fault* and *distinguishing failures*, are both inefficient in handling such impact. We further presented a divide-and-conquer strategy for FCI approaches to alleviate such impact.

In our empirical studies, we extended FCI approach – FIC_BS[Zhang and Zhang 2011] with our strategy. The comparison between our approach and traditional approaches was performed on several kinds of open-source software. The results indicated that our strategy assists the traditional FCI approach in achieving better performance when facing masking effects in SUT. We also empirically evaluated the efficiency of the test case searching component by comparing it with the random searching based FCI approach. The results showed that the ILP-based test case searching method can perform much more efficiently. Last, we compared our approach with existing technique for handling masking effects – FDA-CIT[Yilmaz et al. 2013], and observed that our approach achieved a more precise result which can support better debugging aids, though our approach generated more test cases than FDA-CIT.

As for the future work, we need to do more empirical studies to make our conclusions more general. Our current experiments focus on middle-sized software. We would like to extend our approach to more complicated, large-scaled testing scenarios. Another promising work in the future is to combine the white-box testing technique to facilitate obtaining more accurate results from the FCI approaches when handling masking effects. We believe that figuring out the failure levels of different bugs through the white-box testing technique is helpful to reduce misjudgement in the failure-inducing combinations identifying process. And last, because the extent to which the FCI suffers from masking effects varies with different algorithms, a combination of different FCI approaches would be desired in the future to further improve identifying MFS for multiple failures.

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Online Appendix to: Identifying minimal failure-causing schemas for multiple failures

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A. THE DETAIL OF THE EXPERIMENTS

Table XXIV. Result of the evaluation of each approach

Subject	accurate	sub	parent	ignore	irrelevant	overall	test cases
	O ¹ D ² I ³ R ⁴	O D I R	O D I R	O D I R	O D I R	O D I R	O D I R
HSQLDB 2cr8	2 3 5 5	3 2 0 2	0 2 4 5	0(2.04) 0(3.91) 0(4.0) 0(4.1)	0 2 0 34	0.57 0.65 0.88 0.23	8.125 11.92 17 17.72
HSQLDB 2.2.5	2 2 2 2	1 0 0 0	0 1 1 1	0(0.83) 0(1.0) 0(1.0) 0(1.0)	7 0 0 0	0.25 0.83 0.83 0.83	8.67 7.67 10.17 11.3
HSQLDB 2.2.9	2 3 2 2	3 1 1 1	0 4 1 1	0(1.33) 0(2.83) 0(2.56) 0(2.56)	4 0 0 0	0.4 0.74 0.8 0.8	9.167 8.61 11.72 13.14
JFlex 1.4.1	2 2 2 2	0 0 0 0	0 1 0 0	0(0.75) 0(1.0) 0(1.0) 0(1.0)	25 0 0 0	0.07 0.83 1 1	23.5 6.5 8 9.68
JFlex 1.4.2	2 2 2 2	1 0 0 0	0 1 1 1	0(0.67) 0(1.0) 0(1.0) 0(1.0)	25 0 0 0	0.09 0.83 0.83 0.83	20.5 9 11.67 13.12
synthex 1	2 1 1 1	2 0 0 0	0 2 2 2	0(1.0) 0(2.0) 0(2.0) 0(2.0)	15 17 17 17	0.19 0.11 0.11 0.11	16.5 18 41.75 41.75
synthex 2	3 3 3 3	10 0 0 0	0 10 6 6	0(1.96) 0(2.0) 0(2.0) 0(2.0)	1 0 0 0	0.54 0.76 0.8 0.8	11.19 14.12 16.96 17.08
synthex 3	4 3 3 3	4 2 2 2	0 5 5 5	0(2.08) 0(2.84) 0(2.84) 0(2.84)	13 9 9 9	0.28 0.34 0.34 0.34	12.73 9.46 14.18 14.44
synthex 4	3 3 3 3	10 3 2 3	0 6 5 5	0(2.6) 0(2.8) 0(2.9) 0(2.85)	9 9 9 9	0.35 0.4 0.39 0.39	9.91 13.02 18.55 18.45
synthex 5	2 2 2 2	4 0 0 0	0 2 1 1	0(1.02) 0(1.0) 0(1.0) 0(1.0)	1 0 0 0	0.65 0.88 0.92 0.92	13.04 13.7 14.77 14.84
synthex 6	2 3 3 3	15 4 4 4	0 8 8 8	0(1.99) 0(3.72) 0(3.72) 0(3.72)	8 10 10 10	0.38 0.36 0.36 0.36	14.91 11.75 15.37 15.71
synthex 7	3 3 3 3	10 0 0 0	0 6 6 6	0(2.04) 0(2.0) 0(2.0) 0(2.0)	5 0 0 0	0.39 0.83 0.83 0.83	12.77 14.59 16.44 16.53
synthex 8	2 2 2 2	4 0 0 9	0 4 3 3	0(1.05) 0(1.0) 0(1.0) 0(1.0)	3 0 0 0	0.56 0.9 0.91 0.91	24.45 25.25 26.27 26.37
synthex 9	2 1 1 1	1 0 0 0	0 1 1 1	0(0.8) 0(1.0) 0(1.0) 0(1.0)	0 0 0 0	0.75 0.83 0.83 0.83	6.8 8 9 9
synthex 10	0 1 1 1	3 1 1 1	0 0 0 0	0(1.0) 0(1.46) 0(1.31) 0(1.31)	0 2 1 1	0.5 0.47 0.58 0.58	9.08 11 15.38 15.53

¹ O is for the strategy *regarded as one failure*.

² D is for the strategy *distinguishing failures*.

³ I is for the replacement strategy based on ILP searching.

⁴ R is for the replacement strategy based on randomly searching.

Table XXV. Comparison with FDA-CIT

Subject		accurate			sub			parent			ignore			irrelevant			overall			test cases		
	t	F ¹	I ²	Fs ³	F	I	Fs	F	I	F	F	I	Fs	F	I	Fs	F	I	Fs	F	I	Fs
HSQl2cr8	2	0.17	2.27	1.57	0.57	0	0	0.17	0.4	2.17	3.87	2.3	2	2.53	0	1.97	0.12	0.51	0.39	23.6	70.1	70.1
	3	1.47	3.67	1	0	0	0	4.67	2	6.07	0.63	0.3	0.17	3	0	1.47	0.51	0.87	0.6	76.6	241.8	241.8
	4	0.83	4.8	1	0	0	0	9.03	3.37	8	0	0	0	0.97	0	0	0.65	0.9	0.71	183.5	606.6	606.6
HSQl2.2.5	2	1	1.97	0.37	0	0	0	2.4	0.73	3.8	0.4	0	0	1.4	0	0.1	0.38	0.87	0.56	26.7	68.8	68.8
	3	0	2	0.4	0	0	0	5	1	3.8	0	0	0	0	0	0	0.52	0.83	0.56	67	202.4	202.4
	4	0	2	0.33	0	0	0	5	1	4	0	0	0	0	0	0	0.53	0.83	0.56	130.1	503.3	503.3
HSQl2.2.9	2	0.9	1.77	0.9	0	0.77	0	1.47	0.47	6.8	1.93	0.53	0	2.37	0	0.2	0.28	0.72	0.58	29.2	78.3	78.3
	3	1	2	0.83	0	1	0	5.13	0.93	7.1	0.2	0	0	0.1	0	0	0.61	0.8	0.61	72.8	221.7	221.7
	4	1	2	1	0	1	0	5.87	1	6.7	0	0	0	0	0	0	0.64	0.8	0.62	129.8	560.3	560.3
JFlex 1.4.1	2	0	2	0	0	0	0	4.03	0	4	0	0	0	0	0	0	0.49	1	0.5	30.5	87.3	87.3
	3	0	2	0	0	0	0	4	0	0	0	0	4	0	0	0	0.5	1	0.5	73.4	269.2	269.2
	4	0	2	0	0	0	0	4	0	0	0	0	0	0	0	0	0.5	1	0.5	190.6	724.7	724.7
JFlex 1.4.2	2	0.3	1.97	0.93	0	0	0	3.6	1	2.16	0.03	0	0	0.63	0	0	0.5	0.83	0.62	34.3	106.9	106.9
	3	0	2	0.97	0	0	0	5	1	2.1	0	0	0	0.03	0	0	0.52	0.83	0.61	72.3	305.7	305.7
	4	0	2	1	0	0	0	5	1	2	0	0	0	0	0	0	0.53	0.83	0.61	186.8	836.9	836.9
synthez 1	2	0.97	1	1	0	0	0	1.7	1.93	2	0	0.07	0	0.33	14.3	0	0.66	0.13	0.78	40.3	342.87	342.87
	3	1	1	1	0	0	0	2	2	2	0	0	0	0	16.73	0	0.78	0.12	0.78	93.4	809.1	809.1
	4	1	1	1	0	0	0	2	2	2	0	0	0	0	17	0	0.78	0.12	0.78	218.8	1532.8	1532.8
synthez 2	2	0.17	1.3	0.73	0.37	0	0	0	0.4	2.37	2.27	1.2	1.03	1.37	0	1.2	0.11	0.52	0.4	19.77	54.4	54.4
	3	0.73	2.23	0.5	0	0	0	1.9	1.3	7.1	1.2	0.43	0.53	2.2	0	1.33	0.36	0.82	0.52	59.5	171.5	171.5
	4	0.63	2.97	0.1	0	0	0	5.3	2.33	16.1	0.53	0	0	2.6	0	1	0.44	0.89	0.54	152.7	415.1	415.1
synthez 3	2	0.43	2.97	0.73	0	0.93	0	4.3	1.73	5.3	0.47	0.17	0.5	1.03	3.77	1.13	0.37	0.46	0.37	48.6	138.7	138.7
	3	0.2	3	0.87	0	1.57	0	7.2	3.67	6.57	0.07	0	0	0.83	6.77	0.07	0.38	0.38	0.44	106.3	315.3	315.3
	4	0.03	3	1	0	1.97	0	10.4	3	6	0	0	0	0.43	8.56	0	0.38	0.34	0.45	147.9	565.7	565.7
synthez 4	2	0.3	2.3	0.33	0.07	0.63	0	2.63	1.97	7.7	1.93	0.63	0.4	3.4	1.4	1.97	0.24	0.6	0.44	42.7	142.2	142.2
	3	0.37	2.97	0.07	0	1.26	0	6.5	3.53	10.97	0.83	0.07	0	2.5	3.43	1.03	0.39	0.54	0.51	86.5	373.2	373.2
	4	0.07	3	0	0	1.77	0	11.7	4.67	11.4	0	0	0	1.33	6.73	0.03	0.48	0.44	0.55	202.2	899.7	899.7
synthez 5	2	0.2	1.2	0.8	0.3	0	0	0.1	0.03	0.83	1.4	0.77	0.97	0.7	0	1	0.2	0.59	0.4	21.9	46.9	46.9
	3	0.87	1.4	0.53	0	0	0	0.5	0.23	3.03	1	0.6	0.77	0.37	0	1.63	0.46	0.71	0.43	76.9	150.3	150.3
	4	0.7	1.9	0.37	0	0	0	1.77	0.33	6.5	0.9	0.1	0.03	1.87	0	2.03	0.34	0.92	0.54	232.9	433.2	433.2
synthez 6	2	0.23	2.63	0.17	0.2	2	0	2.93	1.63	9.63	2.6	0.5	0.4	3.03	3.7	2	0.19	0.42	0.37	45.7	132.6	132.6
	3	0.1	3	0.1	0	2.83	0	7.4	3.83	12.5	1.2	0.17	0.03	2.3	6.5	0.67	0.31	0.38	0.43	99.5	338.9	338.9
	4	0	3	0	0	3.8	0	10.2	6.03	14.5	0.47	0	0	1.8	9.1	0.03	0.37	0.36	0.44	152.6	781.9	781.9
synthez 7	2	0.13	1.43	0.83	0.23	0	0	0.1	0.63	1.4	2.53	1.03	0.93	1.93	0	1.97	0.09	0.61	0.38	20.3	58.8	58.8
	3	0.87	2.17	0.93	0	0	0	0.43	1.23	2.97	1.77	0.17	0.13	3.2	0	2.87	0.2	0.88	0.44	52.6	164.7	164.7
	4	1	2.87	1	0	0	0	3.23	2.53	4.6	0.27	0	0	4.5	0	2.27	0.35	0.9	0.51	145.3	413.1	413.1
synthez 8	2	0	0.2	0.17	0.03	0	0	0	0	0.03	0.3	0.13	0.13	0.17	0	0.3	0.01	0.1	0.05	16.1	45.2	45.2
	3	0	0.6	0.5	0.1	0	0	0	0	0.03	0.97	0.47	0.53	0.63	0	0.87	0.02	0.3	0.17	43.1	64.3	64.3
	4	0	1.33	0.8	0.1	0	0	0	0.07	0.67	1.53	0.4	0.5	1.4	0	0.93	0.04	0.67	0.41	109.3	145.6	145.6
synthez 9	2	1	1	1	0	0	0	0.46	0.6	0.77	0.53	0	0.23	0.63	0.6	0.67	0.54	0.7	0.6	36.2	43.4	43.4
	3	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0.83	0.83	0.83	84.3	145	145
	4	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0.83	0.83	0.83	188	291.6	291.6
synthez10	2	0	0.63	0	0.6	1	0.2	0	0	0.83	1.8	0.37	1.9	1.5	0.3	3.97	0.23	0.61	0.17	23.4	84.9	84.9
	3	0.07	0.97	0	0.23	1	0.03	0.36	0	1.9	2.23	0.03	1.97	4.03	0.53	3.97	0.13	0.66	0.2	73.4	263.2	263.2
	4	0	1	0	0.07	1	0	1.7	0	2	1.87	0	2	4.03	1	4	0.21	0.58	0.2	202.2	685.9	685.9

¹ F is for the strategy *regarded as one failure*.² I is for the replacement strategy based on ILP searching.³ Fs is for the strategy *distinguishing failures*.