Identifying minimal failure-causing schemas in the presence of multiple faults

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Combinatorial testing (CT) has been proven effective in revealing the failures caused by the interaction of factors that affect the behavior of a system. The theory of Minimal Failure-Causing Schema (MFS) has been proposed to isolate the root cause of a failure after CT. Most algorithms that aim to identify MFS focus on handling a single fault in the System Under Test (SUT). However, we argue that multiple faults are more common in practice, under which masking effects may be triggered so that some failures cannot be observed. The traditional MFS theory lacks a mechanism to handle such effects; hence, they may incorrectly isolate the MFS. To address this problem, we propose a new MFS model that takes into account multiple faults. We first formally analyse the impact of the multiple faults on existing MFS identifying algorithms, especially in situations where masking effects are triggered by multiple faults. We then develop an approach that can assist traditional algorithms to better handle multiple faults. Empirical studies were conducted using several kinds of open-source software, which showed that multiple faults with masking effects do negatively affect traditional MFS identifying approaches and that our approach can help to alleviate these effects.

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1. INTRODUCTION

With the increasing complexity and size of modern software, many factors, such as input parameters and configuration options, can affect the behaviour of the SUT. The failures caused by the interaction of these factors can make software testing challenging, especially when the interaction space is large. In the worst case, we need to examine every possible interaction of these factors as each interaction may cause unique

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Table I. MS word example

id	*Highlight*	*Status bar*	*Bookmarks*	*Smart tags*	**Outcome**
1	On	On	On	On	PASS
2	Off	Off	On	On	PASS
3	Off	On	Off	Off	Fail
4	On	Off	Off	On	PASS
5	On	Off	On	Off	PASS

failure [Song et al. 2012]. While exhaustive testing achieves maximal test coverage, it is impractical and uneconomical. One remedy for this problem is Combinatorial Testing (CT)[1], which systematically samples the interaction space and selects a relatively small set of test cases that cover all valid interactions, with the number of factors involved in each interaction no more than a prior fixed integer, i.e., the *strength* of the interaction. Many works in CT aim to construct the smallest set of test cases [Cohen et al. 1997; Bryce et al. 2005; Cohen et al. 2003; Lei et al. 2008], which is also called *covering array*.

Once failures are detected by a covering array, the failure-inducing interactions in the failing test cases should be isolated. This task is important as it can facilitate debugging efforts by reducing the scope of code that is needed for inspection. [Ghandehari et al. 2012]. However, information from a covering array sometimes is not sufficient to identify the location and number of the failure-inducing interactions [Colbourn and McClary 2008]. Thus, additional information is often needed. Consider the following example [Bach and Schroeder 2004], Table I presents a two-way covering array for testing an MS-Word application in which we want to examine various interactions of options for 'Highlight', 'Status Bar', 'Bookmarks' and 'Smart tags'. Assume the third test case failed. We can get six two-way suspicious interactions that may be responsible for this failure. They are (Highlight: Off, Status Bar: On), (Highlight: Off, Bookmarks: Off), (Highlight: Off, Smart tags: Off), (Status Bar: On, Bookmarks: Off), (Status Bar: On, Smart tags: Off), and (Bookmarks: Off, Smart tags: Off). Without additional information, it is difficult to figure out the specific interactions in this suspicious set that caused the failure. In fact, considering that the higher strength interactions could also be failure-inducing interactions, e.g., (Highlight: Off, Status Bar: On, Smart tags: Off), the problem becomes more complicated.

To address this problem, prior work [Nie and Leung 2011a] specifically studied the properties of the failure-inducing interactions in the SUT, based on which additional test cases were generated to identify them. Other approaches to identify the failure-inducing interactions in the SUT include building a tree model [Yilmaz et al. 2006], adaptively generating additional test cases according to the outcome of the last test case [Zhang and Zhang 2011], ranking suspicious interactions based on some rules [Ghandehari et al. 2012], and using graphic-based deduction [Martínez et al. 2008], among others.

Most existing approaches mainly focus on the ideal scenario in which SUT only contains one fault, under which the outcomes of test cases can be simply categorized into failure or pass. However, in this paper, we argue that SUT with multiple faults is the more common testing scenario in practice, which will result in many distinguished failures as outcomes of test cases, and moreover, this affects the effectiveness of Failure-inducing Interactions Identifying (FII) approaches. The major challenge imposed by multiple faults is dealing with the masking effect. A masking effect [Dumlu et al. 2011; Yilmaz et al. 2014] occurs when some failures prevent test cases from checking interactions that are supposed to be tested. Take the Linux command *Grep* for exam-

[1]Another term for CT is Combinatorial Interaction Testing, which is abbreviated as CIT. In this paper, they are uniformly cited as Combinatorial testing (CT).

ple. We notice that there are two different failures reported in the bug tracker system. The first [2] claims that Grep incorrectly matches unicode patterns with '\<\>', while the second [3] claims an incompatibility between option '-c' and '-o'. When we put these two scenarios into one test case, only one failure will be observed, which means the other failure is masked by the observed failure. This effect prevents test cases from being executed normally, leading to incorrect judgment of the correlation between the interactions checked in the test case and the failure that has been masked. This effect was firstly noted by Dumlu and Yilmaz in [Dumlu et al. 2011], in which they found that the masking effect in CT can prevent traditional covering array in testing some interactions.

As masking effect can negatively affect the performance of FII approaches, a natural question is how this effect biases the results of these approaches. In this paper, we formalize the process of identifying the failure-inducing interactions under the circumstances in which masking effects exist in the SUT and try to answer this question. One insight from the formal analysis is that we cannot completely avoid the impact of masking effects even if we do exhaustive testing. Even worse, either ignoring the masking effects or treating different failures as the same failure is detrimental to the FII process.

To address this concern, we propose a strategy to alleviate this impact by adopting a divide and conquer framework. With this framework, FII approaches are scheduled to separately handle each failure in the SUT. Specifically, for a particular failure, FII approaches only focus on the test cases that either pass or trigger the same failure under analysis. Test cases that triggered other different failures will be replaced with some newly generated test cases. In this way, FII approaches can properly work with little interference from the negative masking effects.

The key to our strategy is to search for a test case that does not trigger *unexpected* failures, i.e., failures different from the one under analysis. To guide the search process, a natural idea is to take some characteristics from the existing test cases, and make the characteristics of the newly searched test case as disparate from the test cases which triggered unexpected failures as possible. To achieve this target, we define the *suspiciousness* between a factor and the failure. The higher the *suspiciousness* a factor is related to a particular failure, the greater the likelihood that the factor will trigger this fault. We then use the integer linear programming (ILP) technique to find a test case which has the least *suspiciousness* with unexpected failures.

To evaluate the effectiveness of our approach, we applied our strategy on the FII approach FIC_BS [Zhang and Zhang 2011]. The subjects used were two open-source software systems found in the developers' forum in the Source-Forge community. Through studying their bug reports in the bug tracker system as well as their user manuals, we built a testing model which can reproduce the reported bugs with given test cases. We then compared the FII approach augmented with our strategy to the original FII approach. We further empirically studied the performance of the important component of our strategy – searching for desired test cases. To conduct this study, we compared our approach with the augmented FII approach by randomly searching for desired test cases. We finally compared our approach with the only existing masking handling technique – FDA-CIT [Yilmaz et al. 2014]. Our studies show that our replacing strategy as well as the test case searching component achieved a better performance than the traditional approaches when the subject encountered multiple faults, especially when these faults can induce masking effects.

The main contributions of this paper are:

[2]http://savannah.gnu.org/bugs/?29537
[3]http://savannah.gnu.org/bugs/?33080

```
public float foo(int a, int b, int c, int d){
  //step 1 will cause an exception when b == c
  float x = (float)a / (b - c);

  //step 2 will cause an exception when c < d
  float y = Math.sqrt(c - d);

  return x+y;
  }
```

Fig. 1. A simple program *foo* with four input parameters

— We formally analysed the relationships between failure-inducing interactions and test sets. (Section 3)
— We provide an unified framework for explaining how traditional FII approaches work. (Section 3.3)
— We studied the impact of the masking effects caused by multiple faults on FII approaches. (Section 4)
— We proposed an efficient test case replacement strategy to alleviate the impact of these effects (Section 5).
— We conducted several empirical studies and showed that our strategy can assist FII approaches to achieve better performance in identifying failure-inducing interactions in the SUT with masking effects. (Section 8)

2. MOTIVATING EXAMPLE

We use a small example to motivate our approach. Assume a method *foo* has four input parameters: *a, b, c,* and *d*. The four parameter types are all integers and their values are: $v_a = \{7, 11\}, v_b = \{2, 4, 5\}, v_c = \{4, 6\}, v_d = \{3, 5\}$. The code of *foo* is shown in Figure 1.

There are two potential failures of *foo*: first, in step 1 we can get an *Arithmetic Exception* when b is equal to c, i.e., $b = 4$ and $c = 4$, that causes a division by zero. Second, another *Arithmetic Exception* will be triggered in step 2 when $c < d$, i.e., $c = 4$ and $d = 5$, taking square root of a negative number. So the expected failure-inducing interactions in this example should be (-, 4, 4, -) and (-, -, 4, 5).

FII approaches do not consider the code detail; instead, they apply black-box testing, i.e., feed inputs to the programs and execute them to observe the result. The basic proposition of FII approaches is that the failure-inducing interactions for a particular failure can only appear in those test cases that trigger this fault. FII approaches often aim at using as few test cases as possible to get the same (or approximate) result as exhaustive testing, so the results derived from an exhaustive testing set are the best that FII approaches can achieve. Here, we will show how exhaustive testing works to identify the failure-inducing interactions for the program.

We first generate every possible test case listed in the column "test case" of Table II. The execution results are listed in the result column of Table II. In this column, *PASS* means that the program runs without any exception. *Ex 1* indicates that the program triggered an exception corresponding to step 1 and *Ex 2* indicates the program triggered an exception corresponding to step 2. From the data listed in Table II, we can determine that (-, 4, 4, -) must be the failure-inducing interaction of Ex 1 as all the test cases that triggered Ex 1 contain this interaction. Similarly, interactions (-, 2, 4, 5) and (-, 5, 4, 5) must be the failure-inducing interactions of Ex 2. We list these interactions and the corresponding exceptions in Table III.

Table II. Test cases and their corresponding result

id	test case	result	id	test case	result
1	(7, 2, 4, 3)	PASS	13	(11, 2, 4, 3)	PASS
2	(7, 2, 4, 5)	Ex 2	14	(11, 2, 4, 5)	Ex 2
3	(7, 2, 6, 3)	PASS	15	(11, 2, 6, 3)	PASS
4	(7, 2, 6, 5)	PASS	16	(11, 2, 6, 5)	PASS
5	(7, 4, 4, 3)	Ex 1	17	(11, 4, 4, 3)	Ex 1
6	**(7, 4, 4, 5)**	**Ex 1**	**18**	**(11, 4, 4, 5)**	**Ex 1**
7	(7, 4, 6, 3)	PASS	19	(11, 4, 6, 3)	PASS
8	(7, 4, 6, 5)	PASS	20	(11, 4, 6, 5)	PASS
9	(7, 5, 4, 3)	PASS	21	(11, 5, 4, 3)	PASS
10	(7, 5, 4, 5)	Ex 2	22	(11, 5, 4, 5)	Ex 2
11	(7, 5, 6, 3)	PASS	23	(11, 5, 6, 3)	PASS
12	(7, 5, 6, 5)	PASS	24	(11, 5, 6, 5)	PASS

Table III. Identified failure-inducing interactions and their corresponding Exception

Failure-inducing interaction	Exception
(-, 4, 4, -)	Ex 1
(-, 2, 4, 5)	Ex 2
(-, 5, 4, 5)	Ex 2

Note that in this example we did not get the expected result with traditional FII approaches. The failure-inducing interactions for Ex 2 are (-,2,4,5) and (-,5,4,5), respectively, instead of the expected interaction (-,-,4,5). So why did we fail to get the (-,-,4,5)? The reason lies in *test case 6* (7,4,4,5) and *test case 18* (11,4,4,5). These two test cases contain the interaction (-,-,4,5), but they did not trigger Ex 2; instead, Ex 1 was triggered.

Now consider the source code of *foo*. If Ex 1 is triggered, it will stop executing the remaining code and report the exception result. In other words, Ex 1 may mask Ex 2. Let us re-examine the interaction (-,-,4,5). If we suppose that *test case 6* and *test case 18* should trigger Ex 2 if they did not trigger Ex 1, then we can conclude that (-,-,4,5) should be the failure-inducing interaction of Ex 2, which is identical to the expected one. However, we cannot get this result, unless we fix the code that triggers Ex 1 and re-execute all the test cases.

So in practice, *when we lack resources to execute all the test cases repeatedly or can only do black-box testing, a more economical and efficient approach to alleviate the masking effect on FII approaches is desired*.

3. FORMAL MODEL OF MINIMAL FAILURE-CAUSING SCHEMA

This section presents some definitions and propositions for a formal description of failure-inducing interactions and test sets.

3.1. Minimal Failure-causing Schemas

Assume that the behaviour of a SUT is influenced by k parameters, and each parameter p_i has a_i discrete values from the finite set V_i, i.e., $a_i = |V_i|$ (i = 1,2,...,k). In practice, these parameters can represent many factors, such as input variables, run-time options, building options, etc. Next we will give some formal definitions, Definitions 3.1, 3.3, 3.4 were originally defined in [Nie and Leung 2011b].

Definition 3.1. A *test case* of a SUT is a set of k values, one for each parameter of the SUT, which is denoted as a k-tuple (v_1, v_2,...,v_k), where $v_1 \in V_1$, $v_2 \in V_2$ ... $v_k \in V_k$.

For the example in Section 2, ($a = 7$, $b = 2$, $c = 4$, $d = 3$) is a test case, which is actually a group of values being assigned to each input parameter.

Definition 3.2. A *failure* is an abnormal execution of a test case.

In CT, such a *failure* can be a thrown exception, compilation error, assertion failure or constraint violation. In this paper, failures are classified according to the the specific *fault* information. For example, if failures have the same exception traces information, they are treated as the same failure. The main point in this paper is to study the impact of multiple different *failures* on failure-inducing interactions identification.

To facilitate our discussion, we introduce the following assumptions that will be used throughout this paper:

ASSUMPTION 1. *The execution result of a test case is deterministic.*

This assumption is a common assumption of CT fault diagnosis[Wang et al. 2010; Zhang and Zhang 2011; Ghandehari et al. 2012; Li et al. 2012; Niu et al. 2013]. It indicates that the outcome of executing a test case is reproducible and will not be affected by some random events.

ASSUMPTION 2. *Different failures in the SUT can be distinguished by various information such as exception traces, state conditions, or the like.*

This assumption indicates that the testers can detect different failures during testing. As different failures will complicate fault diagnosis tasks, distinguishing them is the first step to resolve them.

ASSUMPTION 3. *All the tests are valid in the SUT, i.e., there is no* inter-option *constraints in the SUT.*

Here *option* indicates the parameter of a SUT. This assumption shows that we can always determine the outcome after executing a test case, i.e., either pass or fail with some exceptions.

In practice, these assumptions may not be satisfied. In Section 7, we will discuss their impact on the theories and approach proposed in this paper, as well as the measures to alleviate them. Particularly for the third assumption, we will show that one type of inter-option constraint is essentially equal to a specific failure. Thus we can directly use the approach in this paper to solve it.

Now let us consider the condition that some failures are triggered by some test cases. It is then desirable to determine the cause of these failures and hence some parameter values of the failing test cases must be analysed.

Definition 3.3. For the SUT, the τ-tuple (-,v_{b_1},...,v_{b_τ},...) is called a τ-degree *schema* ($0 < \tau \leq k$) when some τ parameters have fixed values and the others can take on their respective allowable values, represented as "-".

When $\tau = k$, τ-degree *schema* is actually a test case. Furthermore, if every fixed value in a schema is in a test case, we say this test case *contains* the schema.

For example, (-, 4, 4, -) in Table III is a 2-degree schema. And the test case (7, 4, 4, 3) contains this schema.

Definition 3.4. Let c_1 be a *m*-degree schema, c_2 be an *n*-degree schema in the SUT, and $m < n$. If all the fixed parameter values in c_1 are also in c_2, then c_2 *subsumes* c_1. In this case, we can also say that c_1 is a *sub-schema* of c_2, and c_2 is a *super-schema* of c_1, denoted as $c_1 \prec c_2$.

For example, in the motivating example, the 2-degree schema (-, 4, 4, -) is a sub-schema of the 3-degree schema (-, 4, 4, 5), that is, (-,4,4,-) $\prec$ (-,4,4,5).

According to definition 3.4, it is obvious that the subsuming relationship '$\prec$' is transitive, i.e., if $c_1 \prec c_2, c_2 \prec c_3$, then $c_1 \prec c_3$.

Definition 3.5. If for any test case, as long as it contains the schema c, it will trigger a particular fault F. Then we call c the *failure-causing schema* of F. Additionally, if none of the sub-schema of c is the *failure-causing schema* of F, we then call c the *Minimal Failure-causing Schema* (MFS) of F.

In fact, MFS is identical to the failure-inducing interactions discussed previously. Identifying MFS helps to focus on the root cause of a failure and thus facilitates the debugging efforts. Note that all the failures discussed in this paper are option-related [Yilmaz et al. 2006]. That is, all the failures are caused by the interactions of the parameter values in the SUT. Another noteworthy point is that, in practice, the MFS definition (Section 3.5) may not be valid. This is because in some specific cases, e.g., the masking problems which we will discussed later, we may not determine whether some test cases will trigger the particular failure of F, or not.

Some notations used later are listed below for ease of reference:

— k : The number of parameters that influence the SUT.
— V_i : The set of discrete values that the ith parameter of the SUT can take.
— T^* : The exhaustive set of test cases for the SUT. For a SUT with k parameters, and each parameter can take $|V_i|$ values, the number of test cases in T^* is $\prod_{i=1}^{k} |V_i|$. Note that some test cases may be invalid if there exists constraints among the parameters.
— T : A set of test cases. (Similarly for $T_i,\ T_j, \ldots.$)
— $\bar{T}$: The complementary test set of T, i.e., $\bar{T} \bigcup T = T^*,\ \bar{T} \bigcap T = \varnothing$.
— $A \backslash B$: The set of elements that belong to set A but not to B. For example $T_i \backslash T_j$ indicates the set of test cases that belong to set T_i, but not to T_j.
— L : The number of faults contained in the SUT.
— F_m : The mth fault in the SUT ($1 \leq m \leq L$); A failure which is classified to *fault* F_m is called *failure* of F_m.
— T_{F_m} : All the test cases that can trigger the failure of F_m in the SUT.
— $\mathcal{T}(c)$: All the test cases that contain the schema c in the SUT. Based on the definition of MFS, we know that if schema c is an MFS of F_m, then $\mathcal{T}(c) \subseteq T_{F_m}$. Note that, there may be multiple MFS for F_m
— $\mathcal{I}(t)$: All the schemas that are contained in the test case t, e.g., $\mathcal{I}(\ (111)\ ) = \{(1--)(-1-)(--1)(11-)(1-1)(-11)(111)\}$.
— $\mathcal{I}(T)$: All the schemas that are contained in test set T, i.e., $\mathcal{I}(T) = \bigcup_{t \in T} \mathcal{I}(t)$.
— $\mathcal{S}(T)$: All the schemas that are only contained in test set T (Referred to as *Special schemas*); $\mathcal{S}(T) = \mathcal{I}(T) \backslash \mathcal{I}(\bar{T})$.
— $\mathcal{C}(T)$: A set of the minimal schemas that are only contained in test set T (Referred to as *Minimal schemas*); $\mathcal{C}(T) = \{c | c \in \mathcal{S}(T)\ and\ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$.

3.2. Relations between schemas and test sets

PROPOSITION 3.6 (SMALLER SCHEMA c HAS A LARGER $\mathcal{T}(c)$). *For schemas c_1, c_2, if $c_1 \prec c_2$, then all the test cases that contain c_2 must also contain c_1, i.e., $\mathcal{T}(c_2) \subseteq \mathcal{T}(c_1)$.*

We omit the proof of this proposition as it is quite obvious. Suppose a SUT with four binary parameters, which can be denoted as SUT(2^4). Table IV illustrates an example of the Proposition 3.6. The left column lists the schema c_2 = (0,0,-,-) as well as all the test cases in $\mathcal{T}(c_2)$, while the right column lists the schema c_1 = (0,-,-,-) and $\mathcal{T}(c_1)$. We can see that when $c_1 \prec c_2$, $\mathcal{T}(c_2) \subseteq \mathcal{T}(c_1)$.

PROPOSITION 3.7 (PROPERTY OF S(T): SPECIAL SCHEMA SET OF TEST SET T). *For any test set T of the SUT, $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) = T$.*

Table IV. Example of Proposition 3.6

c_2
(0, 0, -, -)
$\mathcal{T}(c_2)$
(**0**, **0**, 0, 0)
(**0**, **0**, 0, 1)
(**0**, **0**, 1, 0)
(**0**, **0**, 1, 1)

c_1
(0, -, -, -)
$\mathcal{T}(c_1)$
(**0**, 0, 0, 0)
(**0**, 0, 0, 1)
(**0**, 0, 1, 0)
(**0**, 0, 1, 1)
(**0**, 1, 0, 0)
(**0**, 1, 0, 1)
(**0**, 1, 1, 0)
(**0**, 1, 1, 1)

PROOF. As $\mathcal{S}(T) = \mathcal{I}(T) \backslash \mathcal{I}(\bar{T})$, $\forall c \in \mathcal{S}(T),\ c \in \mathcal{I}(T)\ and\ c \notin \mathcal{I}(\bar{T})$. Then $\forall t \in \mathcal{T}(c), t\ contains\ c$, indicating that $t \in T$. Hence, $\mathcal{T}(c) \subseteq T$. Then $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) \subseteq T$.

On the other hand, $\forall t \in T$, $\exists c' \in \mathcal{I}(t)$, such that $c' \notin \mathcal{I}(\bar{T})$ (at least it holds when $c' = t$). Hence, $c' \in \mathcal{S}(T)$. Obviously $t \in \mathcal{T}(c') \subseteq \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$. Therefore, $T \subseteq \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$. □

PROPOSITION 3.8 (PROPERTY OF C(T): MINIMAL SCHEMA SET OF TEST SET T). *For any test set T of the SUT, $\bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c) = T$.*

PROOF. As $\mathcal{C}(T) = \{c | c \in \mathcal{S}(T)\ and\ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$, indicating that $\mathcal{C}(T) \subseteq \mathcal{S}(T)$. It is then obviously $\bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c) \subseteq \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$. Hence, we just need to prove that $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) \subseteq \bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c)$.

$\forall t \in \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, $\exists c \in \mathcal{S}(T),\ s.t., t \in \mathcal{T}(c)$. According to the definition of $\mathcal{C}(T)$, $\exists c' \in \mathcal{C}(T), s.t.,\ c' = c\ or\ c' \prec c$. Correspondingly $\mathcal{T}(c') = \mathcal{T}(c),\ or\ \mathcal{T}(c) \subseteq \mathcal{T}(c')$ by Proposition 3.6. Hence, $t \in \mathcal{T}(c') \subseteq \bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c)$.

Therefore, $\bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c) = \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) = T$. □

Table V gives an example of T^*, T, $\bar{T}$, $\mathcal{S}(T)$ and $\mathcal{C}(T)$ in SUT(2^3). We can find that all the schemas in $\mathcal{S}(T)$ and $\mathcal{C}(T)$ are only contained in test set T, and for any $t \in T$, it contains at least one schema in $\mathcal{S}(T)$ and $\mathcal{C}(T)$. Additionally, $\mathcal{C}(T)$ is a minimal schema set which filters those super schemas in $\mathcal{S}(T)$.

Table V. Example of the special and minimal schemas

T^*	T	$\bar{T}$	$\mathcal{S}(T)$	$\mathcal{C}(T)$
(0, 0, 0)	(0, 0, 0)		(0, 0, -)	(0, 0, -)
(0, 0, 1)	(0, 0, 1)		(0, -, 0)	(0, -, 0)
(0, 1, 0)	(0, 1, 0)		(0, 0, 0)	
(0, 1, 1)		(0, 1, 1)	(0, 0, 1)	
(1, 0, 0)		(1, 0, 0)	(0, 1, 0)	
(1, 0, 1)		(1, 0, 1)		
(1, 1, 0)		(1, 1, 0)		
(1, 1, 1)		(1, 1, 1)		

Let T_{F_m} denote the set of all the test cases triggering failure of F_m, then $\mathcal{C}(T_{F_m})$ actually is the MFS set of F_m.

According to the definition of $\mathcal{C}(T)$, one observation is $\mathcal{C}(T) \subseteq \mathcal{S}(T)$, and for any schema in $\mathcal{S}(T)$, it either belongs to $\mathcal{C}(T)$, or is the super schema of one element of $\mathcal{C}(T)$, i.e., $\forall c \in \mathcal{S}(T), \exists c' \in \mathcal{C}(T), s.t., c' = c,\ or\ c' \prec c$.

PROPOSITION 3.9 (MINIMAL SCHEMA OF THE SUBSET OF TEST SET T). *For any test set T and schema c of the SUT, if $\mathcal{T}(c) \subseteq T$, then $c \in \mathcal{S}(T)$.*

PROOF. Assume $c \notin \mathcal{S}(T)$, i.e., $c \notin \mathcal{I}(T) \backslash \mathcal{I}(\bar{T})$, then $c \in \mathcal{I}(\bar{T})$. It indicates that $\exists t \in \bar{T}, t \in \mathcal{T}(c)$, which contradicts that $\mathcal{T}(c) \subseteq T$. Therefore, $c \in \mathcal{S}(T)$. $\square$

Table VI shows an example of this proposition for SUT(2^3). In this table, the test set $\mathcal{T}(c)$ of schema c is the subset of test set T. As a result, the special schema set $\mathcal{S}(T)$ of T contains this schema c = (0, 0, -).

Table VI. Example of a minimal schema of the subset of a test set

c	$\mathcal{T}(c)$	T	$\mathcal{S}(T)$
(0, 0, -)	(0, 0, 0)	(0, 0, 0)	(0, -, -)
	(0, 0, 1)	(0, 0, 1)	(0, -, 0)
		(0, 1, 0)	(0, -, 1)
		(0, 1, 1)	(0, 0, -)
			(0, 1, -)
			(0, 0, 0)
			(0, 0, 1)
			(0, 1, 0)
			(0, 1, 1)

Based on Proposition 3.9, as long as $\mathcal{T}(c) \subseteq T$ for any schema c and any test set T in the SUT, c either belongs to $C(T)$ or is the super-schema of some schema in $C(T)$. Considering a more general scenario, i.e., two test sets T_1, T_2 with $T_2 \subseteq T_1$, we then can easily obtain the relationship between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$ according to Proposition 3.9.

PROPOSITION 3.10 (MINIMAL SCHEMAS IN THE SMALLER TEST SET). *For T_1 and T_2 of the SUT with $T_2 \subseteq T_1$, $\forall c_2 \in \mathcal{C}(T_2)$, $\exists c_1 \in \mathcal{C}(T_1), s.t.,\ either\ c_1 = c_2\ or\ c_1 \prec c_2$.*

PROOF. $\forall c_2 \in \mathcal{C}(T_2)$, $\mathcal{T}(c_2) \subseteq T_2 \subseteq T_1$. According to Proposition 3.9, $c_2 \in \mathcal{S}(T_1)$. By definitions of $\mathcal{S}(T)$ and $\mathcal{C}(T)$, $\exists c_1 \in \mathcal{C}(T_1), s.t., c_1 = c_2,\ or\ c_1 \prec c_2$. $\square$

PROPOSITION 3.11 (MINIMAL SCHEMAS IN THE LARGER TEST SET). *For T_1 and T_2 of the SUT, $T_2 \subseteq T_1$. Then $\forall c_1 \in \mathcal{C}(T_1)$, $\exists c_2 \in \mathcal{C}(T_2), s.t.,\ (1)\ c_1 = c_2$, or $(2)\ c_1 \prec c_2$, or $(3)\ \nexists c_2 \in \mathcal{C}(T_2), s.t., c_2 \prec c_1\ or\ c_2 = c_1,\ or\ c_1 \prec c_2$.*

PROOF. Assume that $\exists c_1 \in \mathcal{C}(T_1)$, s.t., $\exists c_2 \in \mathcal{C}(T_2), c_2 \prec c_1$. According to Proposition 3.10, as $T_2 \subseteq T_1$, so $\forall c_2 \in \mathcal{C}(T_2)$, $\exists c_1' \in \mathcal{C}(T_1), s.t.,\ either\ c_1' = c_2\ or\ c_1' \prec c_2$. Combining them, we can get that $c_1' \prec c_1$. This is a obvious contradiction, as c_1 and c_1' are both minimal schemas in $\mathcal{C}(T_1)$.

Hence, apart from the impossible relationship $c_2 \prec c_1$, then $\forall c_1 \in \mathcal{C}(T_1)$, $\exists c_2 \in \mathcal{C}(T_2), s.t.,\ (1)\ c_1 = c_2$, or $(2)\ c_1 \prec c_2$, or $(3)\ \nexists c_2 \in \mathcal{C}(T_2), s.t., c_2 \prec c_1\ or\ c_2 = c_1,\ or\ c_1 \prec c_2$. $\square$

We need to note the third case, i.e., $\nexists c_2 \in \mathcal{C}(T_2), s.t., c_1 \prec c_2\ or\ c_1 = c_2,\ or\ c_2 \prec c_1$. We refer to this case as c_1 is *irrelevant* to $\mathcal{C}(T_2)$. Furthermore, we can also say a schema is *irrelevant* to another schema if these two schemas are neither identical nor subsuming each other.

We illustrate these scenarios with examples in Table VII for SUT(2^3). There are two parts in this table, with each part showing two test sets: T_1 and T_2, which have $T_2 \subseteq T_1$. In the left part, the schemas in $\mathcal{C}(T_2)$: (0, 0, -) and (0, -, 0), both are the super-schemas of the one in $\mathcal{C}(T_1)$:(0,-,-). While in the right part, the schemas in $\mathcal{C}(T_2)$: (0, 0, -) and (0, -, 0) are both in $\mathcal{C}(T_1)$. Furthermore, one schema in $\mathcal{C}(T_1)$: (1,1,-) is irrelevant to $\mathcal{C}(T_2)$.

In summary, these propositions provide the foundation of MFS identification (Propositions 3.7 and 3.8), and more importantly, they clarify the relationships between the minimal schemas of two different test sets (Propositions 3.11 and 3.10), which are the keys to explain the impact of masking effects later.

Table VII. Minimal schemas of two subsuming test sets

T_2	T_1
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(0, 1, 1)
$\mathcal{C}(T_2)$	$\mathcal{C}(T_1)$
(0, 0, -)	(0, -, -)
(0, -, 0)	

T_2	T_1
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(1, 1, 0)
	(1, 1, 1)
$\mathcal{C}(T_2)$	$\mathcal{C}(T_1)$
(0, 0, -)	(0, 0, -)
(0, -, 0)	(0, -, 0)
	(1, 1, -)

3.3. MFS identification

In this section, we will study how traditional MFS identification approaches, i.e., FII approaches, identify the MFS. Let T_F indicate all the failing test cases and T_P all the passing test cases (Note that $T_F \bigcap T_P = \varnothing$ and $T_F \bigcup T_P = T^*$). Then the MFS for the SUT is $\mathcal{C}(T_F)$. Theoretically, to figure out all the MFS in the SUT, we need to exhaustively execute each possible test case, and collect the failing test cases T_F. This is impossible in practice, especially when the testing space is very large.

As it is impractical to get all the MFS, an appropriate strategy is to determine some MFS of them. With the following proposition, we can learn that with the existing subset of test cases, we can also identify some of them.

PROPOSITION 3.12 (EQUAL MINIMAL SCHEMAS). *For T_1 and T_2, $T_1 \subset T_2$. If $\exists c \in \mathcal{C}(T_1)$, such that $c \in \mathcal{C}(T_2)$. Then, for any test set T_3, such that $T_1 \subset T_3 \subset T_2$, we have $c \in \mathcal{C}(T_3)$.*

PROOF. According to Proposition 3.11, $\forall c \in \mathcal{C}(T_1)$, there exists $c' \in \mathcal{C}(T_3)$, such that either $c' \prec c$ or $c' = c$. Let us assume that $c' \prec c$. As there exists $c'' \in \mathcal{C}(T_2)$, such that $c = c''$, then $c' \prec c''$. This is contradiction as $T_3 \subset T_2$. Hence, the assumption $c' \prec c$ does not hold, indicating that $c' = c$. □

For example, Table VIII shows the minimal schemas for T_1, T_2, T_3 and T_3', where $T_1 \subset T_3 \subset T_2$ and $T_1 \subset T_3' \subset T_2$. We can find that $\mathcal{C}(T_1)$, i.e., (0, 0, -) is identical to that one in $\mathcal{C}(T_2)$, which also in $\mathcal{C}(T_3)$ and $\mathcal{C}(T_3')$.

Table VIII. Identical minimal schemas of subsuming test sets

T_1			T_2			T_3			T_3'		
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	1	0	0	1
			1	1	1	1	1	1	1	1	0
			1	1	0						
$\mathcal{C}(T_1)$			$\mathcal{C}(T_2)$			$\mathcal{C}(T_3)$			$\mathcal{C}(T_3)$'		
0	0	-	0	0	-	0	0	-	0	0	-
			1	1	-	1	1	1	1	1	0

Based on Proposition 3.12, let $T_{F-known}$ be the given test cases which are known to be failing, and $T_{P-known}$ be the known passing test cases. $T_{unknown} = T^* \setminus (T_{F-known} \bigcup T_{P-known})$ be the test cases that are not known to be failing or not, we have the following lemma:

LEMMA 3.13 (DETERMINE SOME MFS). *$\forall c \in \mathcal{C}(T_{F-known})$, if $\exists c' \in \mathcal{C}(T_{F-known} \bigcup T_{unknown})$, such that $c = c'$, then c must be MFS.*

The proof of this lemma is obvious. This is because whatever the outcomes of the test cases $T_{unknown}$ will be, the actual failing test cases (denoted as $T_{F-acutal}$), must have $T_{F-known} \subset T_{F-acutal} \subset (T_{F-known} \bigcup T_{unknown})$. Based on Proposition 3.12, this lemma holds. With this lemma, we can determine some MFS in the SUT without executing all the test cases. However, to identify an MFS c, there still needs many test cases to be executed; at least, we should execute all the test cases in $\mathcal{T}(c)$. For example, for SUT(2^8), to identify the MFS (1, 1, -, -, -, -, -, -), we should make sure all the test cases contain this schema must fail, of which the number is $|T_{F-known}| = 2^{8-2} = 64$. This number grows exponentially with the number of parameters in the SUT. Hence, traditional FII approaches will make some assumptions to further reduce the number of needed test cases to identify the MFS.

Generally, with proper assumptions, apart from the test cases which have been executed, we can infer some of the remaining test cases to be passing or failing. Formally, let $T_{F-infer}$ refer to the test cases that can be inferred to be failing and $T_{P-infer}$ the inferred passing test cases. With the test cases whose outcomes can be inferred, the test cases which are known to failing is increased to be $T_{F-known} \bigcup T_{F-infer}$, and the unknown test cases decreased to be $T_{unknown} \setminus (T_{F-infer} \bigcup T_{P-infer})$. Based on Lemma 3.13, $\forall c \in \mathcal{C}(T_{F-known} \bigcup T_{F-infer})$, if it is identical to some schema in $\mathcal{C}((T_{F-known} \bigcup T_{F-infer}) \bigcup (T_{unknown} \setminus (T_{F-infer} \bigcup T_{P-infer})))$, then c must be MFS.

The assumptions and corresponding inferring process vary with different FII approaches. To specifically describe the inferring processes of different FII approaches is beyond this paper. Next, we will only focus on describing the FIC_BS appraoch[Zhang and Zhang 2011]–an efficient adaptive MFS identification approach.

The algorithm of FIC_BS is listed in the appendix, as well as the corresponding interpretation. We will give a simple example to illustrate how FIC_BS works. For SUT(2^4), assume the test case (1, 1, 1, 1) failed, then the FIC_BS approach can be illustrated in Table IX. In this table, to identify the MFS in t_0, FIC_BS generated 4 more test cases, i.e., t_1, t_2, t_3 and t_4. Each of these test cases consists of two parts: the same part as in the original failing test case, called the *fixed* part against t_0; and the part with values different from the original failing one, called the *mutated* part against t_0. For example, for test case t_1, the *fixed* part (against t_0) is (-, -, 1, 1) and the *mutated* part is (0, 0, -, -). Note that these mutated parts do not necessarily have to be value (0, 0, -, -). We just need make them different from what are in t_0 (This is important to the approach we later propose in the paper). The MFS obtained by FIC_BS is (-, 1, -, 1). This conclusion is based on the following assumption.

Table IX. FIC_BS running example

ID	Test Case				outcome
t_0	1	1	1	1	fail
t_1	0	0	1	1	pass
t_2	0	1	1	1	fail
t_3	0	1	0	0	pass
t_4	0	1	0	1	fail

ASSUMPTION 4. *None of the parameter values of the mutated part of each additional test cases is the part of any MFS.*

We later refer to the parameter values which is not the part of MFS as *safe values*. Hence, this assumption indicates that the mutated part of each additional test cases are all safe values. Based on this assumption, we have the following two lemmas:

LEMMA 3.14. *$\forall t \in T_{unknown}$, if $\exists t' \in T_{P-known}$, such that t is obtained by mutating some parameter values of t', and the mutated part are all safe values, then t must be a passing test case.*

PROOF. Assume that t is a failing test case. As $\{ t \} \subset T_F$, $\exists c \in \mathcal{C}(T_F)$, such that $c \prec t$ or $c = t$ (Based on Proposition 3.9). In other words, $\forall e \in c$, it has $e \in t$. As c has no parameter value that is safe value (c is an MFS), so $\forall e \in c$, it has $e \in (t \setminus mutate)$, where $mutate$ indicates the mutated part of t against t'. However, as t' also contain $(t \setminus mutate)$, so t' should be a failing test case, which is contradiction. Hence, t must be a passing test case. □

For example, if (0, 0, 0, 0) is known to be a passing test case, and (-, -, -, 1) is safe value, then test case (0, 0, 0, 1) must be a passing test case.

LEMMA 3.15. *$\forall t \in T_{unknown}$, if $\exists t' \in T_{F_known}$, such that t is obtained by mutating some parameter values of t', and the mutated part are all safe values, then t must be a failing test case.*

This lemma can be proved according to Lemma 3.14. As an example, assume (0, 0, 0, 0) is known to be a failing test case, and (-, -, -, 0) is safe value, then test case (0, 0, 0, 1) must be a failing test case.

Based on these two lemmas, we can easily infer that some unknown test cases to be failing or not. In table IX, FIC_BS assumes that (0, -, -, -),(-, 0, -, -) ,(-, -, 0, -) and (-, -, -, 0) are *safe values*. Then those inferred test cases are listed in Table X with column 'State' of *infer*. For example, the mutated part of t_2 against the known passing test case t_4 is the (-, -, 0, -), which is safe value, so t_2 should be inferred to be passing; the mutated part of the known failing test case t_{16} against the unknown test case t_{14} is still the safe value (-, -, 0, -), so t_{14} should be inferred to be failing.

Table X. Test outcomes for MFS(- 1 - 1)

ID	Test Case				Outcome	State
t_1	**0**	**0**	**0**	**0**	**pass**	**infer**
t_2	**0**	**0**	**0**	**1**	**pass**	**infer**
t_3	**0**	**0**	**1**	**0**	**pass**	**infer**
t_4	0	0	1	1	pass	known
t_5	0	1	0	0	pass	known
t_6	0	1	0	1	fail	known
t_7	0	1	1	0	-	unkonwn
t_8	0	1	1	1	fail	known
t_9	1	0	0	0	-	unkonwn
t_{10}	1	0	0	1	-	unkonwn
t_{11}	1	0	1	0	-	unkonwn
t_{12}	1	0	1	1	-	unkonwn
t_{13}	1	1	0	0	-	unkonwn
t_{14}	**1**	**1**	**0**	**1**	**fail**	**infer**
t_{15}	1	1	1	0	-	unkonwn
t_{16}	1	1	1	1	fail	known

After inferring these test cases, we can find that $\mathcal{C}(T_{F-known} \bigcup T_{F-infer})$ = $\mathcal{C}(\{t_6, t_8, t_{14}, t_{16}\})$ = (-, 1, -, 1), and $\mathcal{C}((T_{F-known} \bigcup T_{F-infer}) \bigcup (T_{unknown} \setminus (T_{F-infer} \bigcup T_{P-infer})))$ = $\mathcal{C}(\{t_6, t_8, t_{14}, t_{16}, t_7, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{15}\})$ = (1, -, -, -), (-, 1, 1, -), (-, 1, -, 1). Hence, we can derive that (-, 1, -, 1) must be MFS.

This example illustrates that the FII approach does not execute all the test cases, but with proper assumptions, the schemas identified by FII can be approximate to or even the same as the exactly correct MFS in the SUT. Other FII approaches can also be built into our formal model for MFS identification, but this is beyond the scope of

this paper. Even though each FII approach tries to identify the MFS as accurately as possible, masking effects will negatively affect the observed outcomes and the inferred results. We next discuss the masking problem.

4. MASKING EFFECT

As discussed before, $\mathcal{C}(T_{F_m})$ is considered to be the MFS set of fault F_m in theory. When accounting for the masking effects between multiple faults, however, this consideration is not correct.

Definition 4.1. A *masking effect* occurs when a test case t contains an MFS of a particular fault, but it does not trigger the expected fault because other unexpected event, such as the other fault or unaccounted control dependency, was triggered earlier that prevents t from being normally checked.

Taking the masking effects into account, when identifying the MFS of a specific fault F_m, we must not ignore those test cases which did not trigger F_m but should have triggered it. We call these test cases $T_{mask(F_m)}$. Hence, the MFS of fault F_m should be revised as $\mathcal{C}(T_{F_m} \bigcup T_{mask(F_m)})$.

Going back to the motivating example in Section 2, as *test case 6* and *test case 18* should trigger Ex 2 in case they did not trigger Ex 1, $T_{mask(F_2)}$ is { (7,4,4,5),(11,4,4,5)}. Hence, the MFS of $Ex2$ is $\mathcal{C}(T_{F_2} \bigcup T_{mask(F_2)})$, which is (-,-,4,5) instead of the incorrect schema set $\{(-,2,4,5),(-,5,4,5)\}$.

In practice with masking effects, however, it is not possible to correctly identifying the MFS, unless we fix some bugs in the SUT and re-execute some test cases to figure out $T_{mask(F_m)}$.

For traditional FII approaches, without the knowledge of $T_{mask(F_m)}$, two common strategies can be adopted to deal with the multiple faults problem, i.e., *regarded as same failure* and *distinguishing failures*. The former strategy treats all types of faults as one fault, and hence all the failures are considered the same, while the latter distinguishes the failures but with no special consideration of the masking effects, i.e., if a test case fails with a particular type of fault, this strategy presumes it does not contain other types of faults.

4.1. Regarded as same failure strategy

This is the most common strategy. With this strategy, the minimal schemas are the set $\mathcal{C}(\bigcup_{i=1}^{L} T_{F_i})$, where L is the number of all the faults in the SUT. Obviously, $T_{F_m} \bigcup T_{mask(F_m)} \subseteq \bigcup_{i=1}^{L} T_{F_i}$. By Proposition 3.11, some schemas obtained by this strategy may be the sub-schemas of some of the actual MFS, or be irrelevant to the actual MFS.

As an example, consider the test cases in Table XI. Assume we need to characterize the MFS of *fault 1*. All the test cases that triggered *fault 1* are listed in column T_{F_1}; similarly, we list the test cases that triggered failures of other faults in column $T_{mask(F_1)}$ and $T_{non\_mask(F_1)}$, respectively, in which the former masked *fault 1*, while the latter did not. Actually the MFS of *fault 1* should be (1,1,-,-) and (-,1,1,1) as listed in the column 'actual MFS of *fault 1*'. However, when we use the *regarded as same failure* strategy, the minimal schemas obtained will be (-,-,0,0), (1,1,-,-), (-,-,1,1), in which (-,-,0,0) is irrelevant to the actual MFS of *fault 1*, and (-,-,1,1) is a sub-schema of the actual MFS (-,1,1,1).

4.2. Distinguishing failures strategy

Distinguishing the failures by the exception traces or error code can help identify the MFS related to a particular failure. Yilmaz [Yilmaz et al. 2014] proposed the *multiple-*

Table XI. Masking effects for exhaustive testing

T_{F_1}	$T_{mask(F_1)}$	$T_{non\_mask(F_1)}$
(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 1, 0, 0)
(1, 1, 1, 0)	(0, 1, 1, 1)	(0, 0, 0, 0)
(1, 1, 0, 1)		(1, 0, 0, 0)
		(1, 0, 1, 1)
		(0, 0, 1, 1)
actual MFS of *Fault 1*	regarded as same failure	distinguishing failures
$\mathcal{C}(T_{F_1} \bigcup T_{mask(F_1)})$	$\mathcal{C}(T_{F_1} \bigcup T_{mask(F_1)} \bigcup T_{non\_mask(F_1)})$	$\mathcal{C}(T_{F_1})$
(1, 1, -, -)	(-, -, 0, 0)	(1, 1, -, 1)
(-, 1, 1, 1)	(1, 1, -, -)	(1, 1, 1, -)
	(-, -, 1, 1)	

class failure characterizing method instead of the *ternary-class* approach to make the characterizing process more accurate. Besides, other approaches can also be easily extended using this strategy for testing the SUT with multiple faults.

This strategy focuses on identifying the set of $\mathcal{C}(T_{F_m})$. As $T_{F_m} \bigcup T_{mask(F_m)} \supseteq T_{F_m}$, by Proposition 3.10, some schemas obtained by this strategy may be the super-schema of some actual MFS. Moreover, some actual MFS may be irrelevant to the schemas obtained by this strategy according to Proposition 3.11, which means that this strategy will *ignore* these actual MFS.

For the simple example shown in Table XI, when using this strategy, we will get the minimal schemas (1, 1, -, 1) and (1, 1, 1, -), which are both super schemas of the actual MFS (1,1,-,-). Furthermore, no schemas obtained by this strategy have any relationship with the actual MFS (-,1,1,1), which means it was ignored.

It is noted that the motivating example in section 2 actually adopted this strategy. As a result, the schemas identified for Ex 2: (-,2,4,5), (-,3,4,5) are the super-schemas of the correct MFS(-,-,4,5).

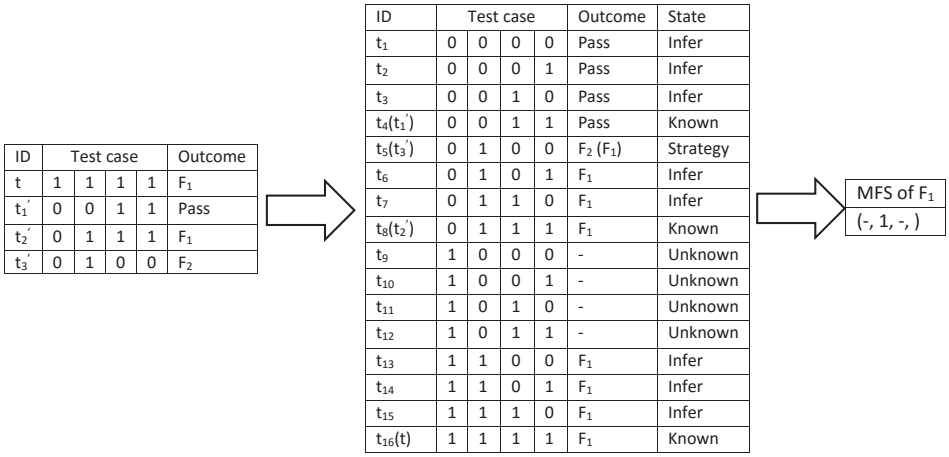
4.3. Masking effects for FII approaches

Based on previous analysis, even though exhaustive testing is conducted to obtain T_{F_m}, we cannot determine the MFS set because of the masking effects. For traditional MFS identification approaches, i.e., FII approaches, masking effects can make problems worse. This is because these approaches, with masking effects, are not only unable to determine $T_{mask(F_m)}$ as discussed before, but also unable to correctly infer the outcomes of some unexecuted test cases, i.e., wrong $T_{F-infer}$ and $T_{P-infer}$ as mentioned in Section 3.3.

Consider a FII example that still use the FIC_BS method [Zhang and Zhang 2011]. Similar to the example listed in Table IX, assume the test case (1, 1, 1, 1) failed with F_1 (The MFS is still assumed to be (-, 1, -, 1) as Table IX). Then assume that there exists an MFS (-, 1, 0, -) that triggers F_2, where fault F_2 can mask F_1. Next we will illustrate how FIC_BS works using strategies the *regarded as same failure* strategy and *distinguishing failures* strategy, respectively.

First, for the *regarded as same failure* strategy, we list how it works in Fig. 2, as well as the masking effects it suffers. With this strategy, it just needs three additional test cases to determine the MFS in the original failing test case t, i.e., t'_1, t'_2, and t'_3 (listed in the left part of Fig. 2). The right part of Fig. 2 shows the MFS it obtained is (-, 1, -, -), which is obviously wrong (See the correct result in Table X, which should be (-, 1, -, 1), but this strategy treats its sub-schema (-, 1, -, -) to be MFS). The middle part of Fig. 2 explains why this strategy obtained this result. Specifically, it treats all failures as the same failure, i.e., failure of F_1. Hence, test case t_5 in the middle of Fig. 2 (Marked *Strategy* in the column 'State ') is determined to trigger F_1. Here, the test case t_5 corresponds to the test case t'_3 in the left part of Fig.2. According to the Assumption

3.14 and 3.15, test cases t_1, t_2, t_3, t_6, t_7, t_{13}, t_{14}, and t_{15} are determined to be pass or fail with F_1, respectively. Afterward, we can determine that (-, 1, -, -) should be the MFS. This is because, $\mathcal{C}(T_{F_1})$ = $\mathcal{C}(\{t_5, t_6, t_7, t_8, t_{13}, t_{14}, t_{15}, t_{16}\})$ = (-, 1, -, -), which has been included in $\mathcal{C}(T_{F_1} \bigcup T_{unknown}$ = $\mathcal{C}(\{t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{16}\})$ = (1, -, -, -), (-, 1, -, -).



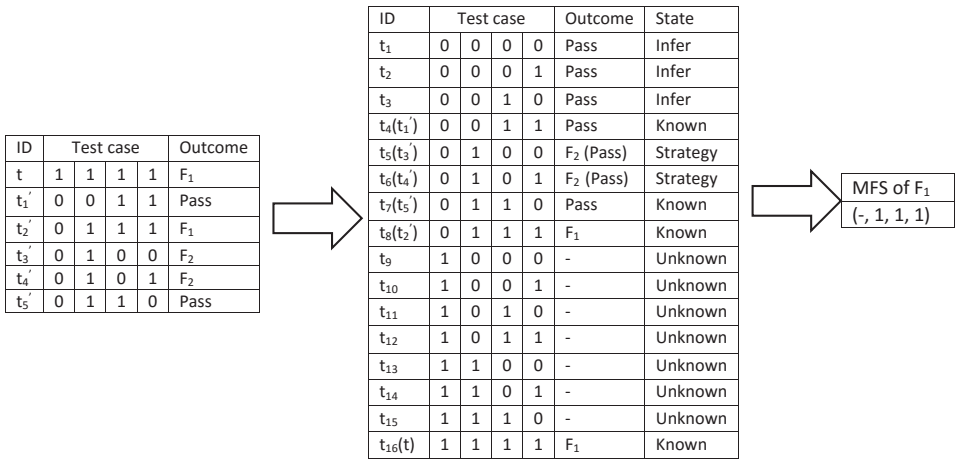
ID	Test case				Outcome
t	1	1	1	1	F_1
t_1'	0	0	1	1	Pass
t_2'	0	1	1	1	F_1
t_3'	0	1	0	0	F_2

ID	Test case				Outcome	State
t_1	0	0	0	0	Pass	Infer
t_2	0	0	0	1	Pass	Infer
t_3	0	0	1	0	Pass	Infer
$t_4(t_1')$	0	0	1	1	Pass	Known
$t_5(t_3')$	0	1	0	0	F_2 (F_1)	Strategy
t_6	0	1	0	1	F_1	Infer
t_7	0	1	1	0	F_1	Infer
$t_8(t_2')$	0	1	1	1	F_1	Known
t_9	1	0	0	0	-	Unknown
t_{10}	1	0	0	1	-	Unknown
t_{11}	1	0	1	0	-	Unknown
t_{12}	1	0	1	1	-	Unknown
t_{13}	1	1	0	0	F_1	Infer
t_{14}	1	1	0	1	F_1	Infer
t_{15}	1	1	1	0	F_1	Infer
t_{16}(t)	1	1	1	1	F_1	Known

MFS of F_1
(-, 1, -,)

Fig. 2. Masking effects of strategy Regarded as same failure

For the second strategy, i.e., *distinguishing failures*, we list the result in Fig. 3. Different from the first strategy, it treats the different failures other than F_1 as pass. As a result, test cases t_5 and t_6 in the middle part of Fig. 3 are determined to be pass. Then combining the inferred test cases, it finally gets the MFS (-, 1, 1, 1), which is also not correct (This schema is the super-schema of the original MFS of F_1 (-, 1, -, 1)).



ID	Test case				Outcome
t	1	1	1	1	F_1
t_1'	0	0	1	1	Pass
t_2'	0	1	1	1	F_1
t_3'	0	1	0	0	F_2
t_4'	0	1	0	1	F_2
t_5'	0	1	1	0	Pass

ID	Test case				Outcome	State
t_1	0	0	0	0	Pass	Infer
t_2	0	0	0	1	Pass	Infer
t_3	0	0	1	0	Pass	Infer
$t_4(t_1')$	0	0	1	1	Pass	Known
$t_5(t_3')$	0	1	0	0	F_2 (Pass)	Strategy
$t_6(t_4')$	0	1	0	1	F_2 (Pass)	Strategy
$t_7(t_5')$	0	1	1	0	Pass	Known
$t_8(t_2')$	0	1	1	1	F_1	Known
t_9	1	0	0	0	-	Unknown
t_{10}	1	0	0	1	-	Unknown
t_{11}	1	0	1	0	-	Unknown
t_{12}	1	0	1	1	-	Unknown
t_{13}	1	1	0	0	-	Unknown
t_{14}	1	1	0	1	-	Unknown
t_{15}	1	1	1	0	-	Unknown
t_{16}(t)	1	1	1	1	F_1	Known

MFS of F_1
(-, 1, 1, 1)

Fig. 3. Masking effects of strategy Distinguishing failures

Such deviations from these two strategies are caused by the reduction of the test cases which are known to be failing with specific fault. Here, test cases (0, 1, 0, 0)

and (0, 1 0, 1) both triggered other faults, among which the second test case (0, 1, 0, 1) should trigger F_1 if fault F_2 was not triggered first, while the first test case not. Hence, neither strategy *Regarded as same failure* nor *Distinguishing failures* can make a perfect determination of the result of these two test cases. Besides this, the incorrect determination of these test cases negatively affect the the inference of FII approach–FIC_BS. For example, with strategy *Regarded as same failure*, it wrongly inferred the test case t_{15} (1, 1, 1 ,0) to be fail with F_1 (See the middle part of Fig. 2), which should be marked as 'Unknown' in Table X. With strategy *Distinguishing failures*, however, it marked test case t_{14} (1, 1, 0, 1) as 'Unknown' (See the middle part of Fig. 3), which should be inferred to be fail with F_1 in Table X.

As masking effect has significant impact on the FII approaches, alleviating this negative effect is desired to improve the quality of the identified MFS.

5. TEST CASE REPLACING STRATEGY

The main reason that the FII approach fails to work properly is that it cannot determine $T_{mask(F_m)}$. In other words, if the test case triggers other unexpected failures which are different from the currently analysed F_m, it cannot figure out whether this test case will trigger F_m because of the masking effects. So to limit the impact of this effect on the FII approach, it is important to reduce the number of test cases that trigger other different failures, as this can reduce the probability that the expected failure may be masked by other different failures.

For the exhaustive testing model, i.e., $\mathcal{C}(T_{F_m})$, as all the test cases will be used to identify the MFS, there is no room left to improve its performance unless we fix other different failures and re-execute all the test cases. However, if only a subset of all test cases is used to identify the MFS (which is how the traditional FII approach works), it is important to make the right selection to limit the size of $T_{mask(F_m)}$ to be as small as possible.

5.1. Replacing test cases that trigger unexpected failures

The basic idea is to pick the test cases that trigger other faults and generate new test cases to replace them. The regenerated test cases should either pass in the execution or trigger F_m.

Normally, when we replace the test case that triggers an unexpected failure with a new test case, we should keep some part of the original test case. We call this part the *fixed part*, and mutate the other part with different values from the original one. For example, assume that a test case (1,1,1,1) triggered an unexpected failure, and the fixed part is (-,-,1,1). Then, we can replace it with a test case (0,0,1,1) which may either pass or trigger the same failure as currently analysed.

The *fixed part* can vary for different FII approaches. For example, for the OFOT [Nie and Leung 2011a] algorithm, all parameter values are the fixed part except for the one that needs to be validated, while for the FIC_BS [Zhang and Zhang 2011] approach, the fixed parts can be dynamically changed, depending on the outcome of the execution of last generated test case.

This replacement process may need to be executed multiple times for one fixed part as it may not always be possible to find a test case that satisfies our requirement. One replacement method is randomly choosing test cases until the satisfied test case is found. While this method may be simple and straightforward, however, it may require many tries. To address this problem and reduce the cost, we propose a replacement approach by computing the *suspiciousness* of the test case with the other faults, and then selecting the test case from a group of candidate test cases that has the least *suspiciousness* with other faults.

To explain the *suspiciousness* notion, we first introduce the *suspiciousness* between a parameter value o and a particular fault $F_i, 1 \leq i \leq L$. We use $all(o)$ to represent the number of executed test cases that contain this parameter value, and $f_i(o)$ to indicate the number of test cases that trigger the failure of F_i, and contain this parameter value. Then, the *suspiciousness* of a parameter value o with respect to the given fault F_i, i.e., $\mathcal{SP}(o, F_i)$, is $\frac{f_i(o)}{all(o)+1}$. This heuristic formula is based on the idea that if a parameter value frequently appears in the test cases that trigger a particular failure, then it is more likely to be an inducing factor that triggers this fault. We add 1 in the denominator for two reasons: (1)to avoid division by zero when the parameter value has never appeared before, (2)to reduce the bias when a parameter value rarely appears in the test set but coincidentally appears in a failing test case with a particular fault.

Table XII gives an example to compute the suspiciousness between parameter values and particular faults. The left part of this table gives 4 executed test cases with their outcomes, in which faults F_1 and F_2 are triggered. The right part shows the suspiciousness between each parameter value and these two faults. Specifically, consider the parameter P_1 taking value 0. There are three test cases t_1, t_2, and t_4 that contain this parameter value, and only t_2 triggered F_1. Hence, $\mathcal{SP}(P_1 = 0, F_1)$ = $\frac{f_1(o)}{all(o)+1}$ = $\frac{1}{3+1}$ = 1/4.

Table XII. Suspiciousness example

ID	Test cases executed	Outcomes
t_1	(0, 0, 0)	$Pass$
t_2	(0, 1, 1)	F_1
t_3	(1, 1, 0)	F_2
t_4	(0, 1, 1)	F_2

Suspiciousness for $F1$

	P_1	P_2	P_3
0	1/4	0	0
1	0	1/4	1/3

Suspiciousness for $F2$

	P_1	P_2	P_3
0	1/4	0	1/3
1	1/2	1/2	1/3

With the *suspiciousness* associated with a parameter value, we then define the *suspiciousness* between a test case t and a particular fault F_i as:

$$\mathcal{SP}(t, F_i) = \frac{1}{k} \sum_{o \in t} \mathcal{SP}(o, F_i) \quad \textbf{(EQ1)}$$

where k is the number of parameters in t, and o is the specific parameter value in t. The *suspiciousness* between a test case and a fault is actually the average *suspiciousness* between each parameter value in the test case and this fault. For example, considering an unexecuted test case (1, 0, 0). With the suspiciousness of all the parameter values listed in Table XII, $\mathcal{SP}((1,0,0), F_1)$ = $1/3 \times (0+0+0) = 0$ and $\mathcal{SP}((1,0,0), F_2)$ = $1/3 \times (1/2 + 0 + 1/3) = 5/18$.

For a selected test case, we want its ability to trigger failures of other faults to be as small as possible, such that the masking effects can be alleviated. In practice, the *suspiciousness* varies between test cases and different faults. As a result, we cannot always find a test case that, for any fault, the *suspiciousness* between this test case and that fault is the least.

Table XIII illustrates such a scenario for SUT(2^4). Suppose the FII approach is analysing MFS for fault F_1, and needs to replace test case (0,0,0,0) with fixed part (0, -, -, -) that triggers failures of other faults, i.e., F_2 or F_3. This table lists five candidate test cases, with their suspiciousness with F_2 and F_3 given in the corresponding columns. It is obvious that t_1 has the least suspiciousness with fault F_3 and t_2 has the least suspiciousness with F_2. These two test cases, however, should not be selected

because their suspiciousness with another fault is too high. Instead, t_3 is a good choice as both its suspiciousness with F_2 and F_3 is not high (The higher one is just 0.4).

Table XIII. Select minimal maximal suspiciousness

ID	Candidate test cases	$\mathcal{SP}(t, F_2)$	$\mathcal{SP}(t, F_3)$	$Max\ \mathcal{SP}(t, F_m), m = 2, 3$
t_1	(0, 0, 0, 1)	0.7	0.2	$\mathcal{SP}(t_1, F_2) : 0.7$
t_2	(0, 0, 1, 0)	0.2	0.6	$\mathcal{SP}(t_2, F_3) : 0.6$
$\mathbf{t_3}$	**(0, 0, 1, 1)**	**0.4**	**0.3**	$\mathcal{SP}(\mathbf{t_3}, \mathbf{F_2}) : \mathbf{0.4}$
t_4	(0, 1, 0, 0)	0.3	0.5	$\mathcal{SP}(t_4, F_3) : 0.5$
t_5	(0, 1, 0, 1)	0.5	0.3	$\mathcal{SP}(t_5, F_2) : 0.5$

With this in mind, we have to settle for a test case, such that the most likely fault (except for the one that is currently analysed) it can trigger should be the least likely to be triggered when compared with that of other test cases. In other words, we need to find a test case, so that the maximal *suspiciousness* of this test case w.r.t a given fault is minimized. Formally, we should choose a test case t, such that,

$$\min_{t \in R} \max_{1 \leq i \leq L \& i \neq m} \mathcal{SP}(t, F_i) \tag{EQ2}$$

where L is the number of faults, and m is the current analysed fault. R is the set of all possible test cases that contain the $fixed$ part except those that have been tested. As the $fixed$ part is a set of parameter values which can be deemed as a schema, then obviously $R = \mathcal{T}(fixed) \backslash T_{executed}$, where $\mathcal{T}(fixed)$ represents all the test cases that contain this fixed part and $T_{executed}$ represents those executed test cases. Additional test cases need to be selected in set R, so that FII approaches can work properly.

The complete process of replacing a test case with a new one while keeping some fixed part is depicted in Algorithm 1.

ALGORITHM 1: Replacing test cases triggering unexpected failures

Input: fault F_m, all the candidate test cases R, the suspiciousness matrix $\mathcal{SP}$
Output: t_{new} the regenerate test case

```
1  while not R is empty do
2      t_new ← min_{t∈R} max_{1≤i≤L&i≠m} SP(t, F_i);
3      result ← execute(t_new);
4      R ← R\t_new;
5      update_SP(t_new) ;
6      if result == PASS or result == F_m then
7          return t_new;
8      else
9          continue;
10     end
11 end
12 return null
```

The inputs to this algorithm consist of the fault F_m under analysis, the candidate test cases $R = \mathcal{T}(fixed) \backslash T_{executed}$ and the suspiciousness matrix $\mathcal{SP}$, which records the suspiciousness between each factor o and each fault F_i, i.e., $\mathcal{SP}(o, F_i)$ $(1 \leq i \leq L)$. The output of this algorithm is a test case t_{new} which either triggers the expected F_m or passes.

The outer loop of this algorithm (lines 1 - 11) contains three parts:

The first part (lines 2 - 3) generates and executes a new test case which is supposed to be least likely to trigger faults different from F_m. The new test case is generated

according to EQ2. In our implementation, we use the solver introduced in [Berkelaar et al. 2004], which is a mixed Integer Linear Programming (MILP) solver suitable for satisfaction and optimization problems. (Note that a simpler linear search algorithm can also be applied in our approach to find a proper test case. However, as the problem to search a proper test case is related to Integers (a test case consists of discrete parameters with discrete values), we use Integer Linear Programming (ILP) technique instead.) When a test case is generated, we remove it from the candidate test case set R to avoid redundancy (line 4).

The second part (line 5) updates the suspiciousness matrix ($\mathcal{SP}$) for each parameter value that is involved in this newly generated test case (line 5). Specifically, for a particular parameter value o, the number of executed test cases that contain o, i.e., $all(o)$, increases by 1. Additionally, if this test case triggers failure of F_i ($1 \leq i \leq L$) , then the number of test cases that contain o and trigger failure of F_i, i.e., $f_i(o)$, increases by 1. At last, the suspiciousness value will be re-computed according to formula $\mathcal{SP}(o, F_i) = \frac{f_i(o)}{all(o)+1}$ ($1 \leq i \leq L$).

The last part (lines 6 - 10) checks whether the newly generated test case is as expected. Specifically, if the test case passes or triggers the same failure of fault – F_m, a satisfied test case is obtained (line 6) and returned (line 7). Otherwise, we will repeat the process, i.e., generate a new test case and check again (lines 8 - 9).

6. ILLUSTRATION OF THE APPROACH

This section will illustrate the complete MFS identification approach that combines the traditional MFS identification procedure with test cases replacement strategy.

6.1. MFS identification with replacement strategy

Algorithm 2 shows the procedure of our approach. The inputs to this algorithm are the fault F_m that is currently focused on and an original failing test case t. The output of this algorithm is to give the MFS for fault F_m in the test case t. Variable $\mathcal{SP}$ is to record the suspicious value between each parameter value with each fault. $T_{Unknown}$ is the set of test cases that is not yet determined to be fail with fault F_m or not, and $T_{F-Known}$ is the set of test cases that are deemed to trigger the fault F_m. This algorithm loops until some MFS are determined (line 25 - 27). This is based on Lemma 3.13 in Section 3.3. Specifically, by computing the minimal schemas of $T_{F-Known}$, i.e., $\mathcal{C}(T_{F-Known})$, and minimal schemas of $T_{F-Known} \bigcup T_{Unknown}$, i.e., $\mathcal{C}(T_{F-Known} \bigcup T_{Unknown})$, respectively, we can determine $\mathcal{C}(T_{F-Known})$ are MFS if they are contained in $\mathcal{C}(T_{F-Known} \bigcup T_{Unknown})$.

In each iteration, this algorithm will generate one additional test case (line 7) to execute. This test case is randomly selected from a candidate set R which has the same fixed part (line 6). The fixed part is different in each iteration (See Algorithm 3 in the Appendix for detail). Then the approach will execute this test case (line 8) and update the suspiciousness matrix (line 9) according to the execution result. If the result is pass or fail with the same fault F_m, then the generated test case is a proper test case for MFS identification; otherwise, we should run the replacement strategy (line 10 - 15) based on Algorithm 1. Note that if we cannot find a proper test case by Algorithm 1, we need to randomly select one test case in R (line 13), and regard it as fail with F_m (line 14). Next we will infer the result of some unknown test cases according to the result of the generated test case t_{next} (line 17 - 24). This inferring procedure is exactly based on Lemma 3.14 and 3.15. After that, we update the $T_{F-known}$ and $T_{Unknown}$ (line 19 - 20, line 23), and re-check if the ending condition is satisfied (line 27).

ALGORITHM 2: MFS identification with Replacing test cases strategy

Input: fault F_m, original failing test case t
Output: MFS returning the determined MFS

```
1  SP ← init_Suspicious_Matrix(t) ;
2  T_Unknown ← T_ALL \ t ;
3  T_F−Known ← {t} ;
4  MFS ← empty ;
5  while true do
6      R ← T(current_fixed_part) ;
7      t_next ← random_pick(R);
8      result ← execute(t_next);
9      update_SP(t_next) ;
10     if result ≠ PASS and result ≠ F_m then
11         t_next ← Replacing(F_m, SP, R);
12         if t_next == null then
13             t_next ← random_pick(R);
14             result ← F_m
15         end
16     end
17     if result == F_m then
18         T_infer−F ← InferringFailing(t_next, SV);
19         T_F−Known ← T_F−Known ⋃ t_next ⋃ T_infer−F;
20         T_Unknown ← T_Unknown \ (t_next ⋃ T_infer−F) ;
21     else
22         T_infer−P ← InferringPassing(t_next, SV);
23         T_Unknown ← T_Unknown \ (t_next ⋃ T_infer−P) ;
24     end
25     MFS ← C(T_F−Known) ;
26     M_Possible ← C(T_F−Known ⋃ T_Unknown) ;
27     if MFS ⊆ M_Possible then
28         break;
29     end
30 end
31 return M_Candidate
```

6.2. A case study using the replacement strategy

In this section, we will give a case study to illustrate our approach. Suppose we want to test a system with eight parameters, each of which has three options, i.e., SUT(3^8). When we execute the test case t_0 – (0, 0, 0, 0, 0, 0, 0, 0), a failure of fault–$e1$ is triggered. Furthermore, there are two more potential faults, $e2$ and $e3$, that may be triggered during the testing and they will mask the desired fault $e1$. Next, we will use FIC_BS [Zhang and Zhang 2011] with replacement strategy to identify the MFS of $e1$. The process is shown in Figure 4. In this figure, there are two main columns. The left main column indicates the executed test cases during testing as well as the executed results, with each executed test case corresponding to a specific label, t_1 – t_8, at the left. The underline part for each test case is the $fixed$ part according to FIC_BS [Zhang and Zhang 2011]. The right main column lists the suspiciousness matrix when a test case triggers $e2$ or $e3$. The executed test case, shown in bold, indicates the one that triggers the failure of other faults and should be replaced in the next iteration.

The completed MFS identifying process listed in Figure 4 works as follows. Firstly the original FII approach determines which $fixed$ part needed to be tested in each iteration. Then an extra test case will be generated to fill in the remaining part. After executing the extra test case, if the execution did not trigger any unexpected faults

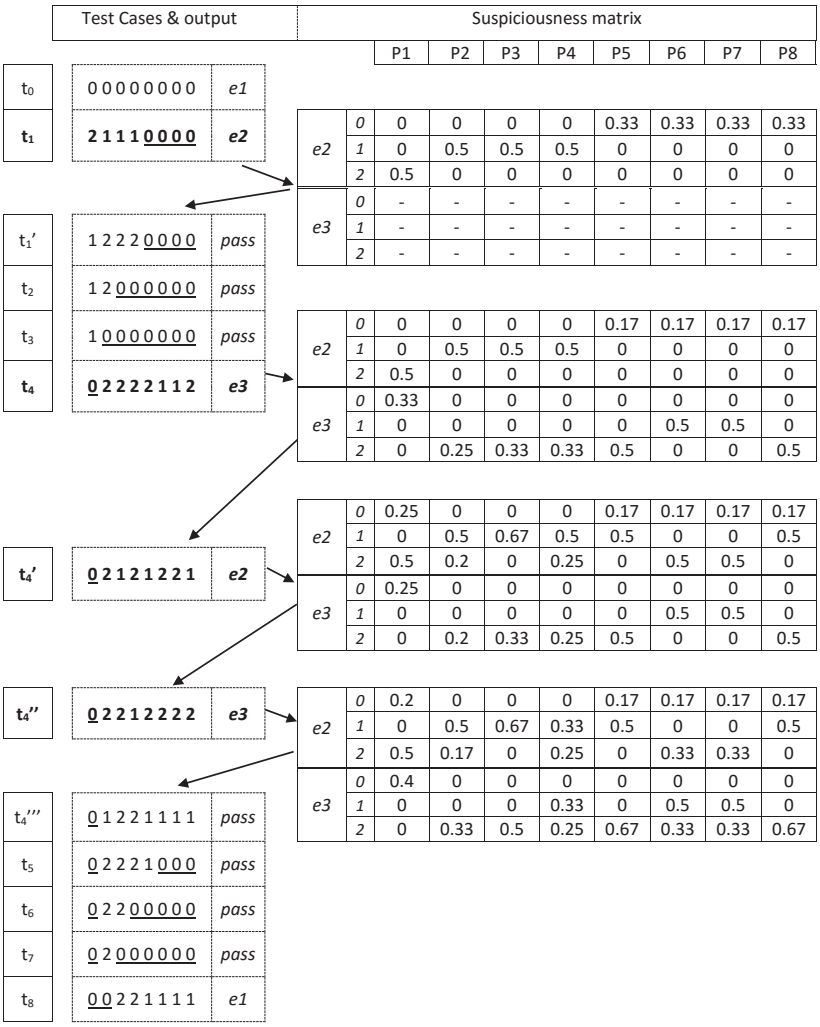


Fig. 4. A case study using our approach

(e_2, e_3), the original FII process will continue until the MFS is identified. Otherwise, the replacement strategy starts when an unexpected failure of other faults is triggered. The replacement process will mutate the parameter values that are not in the $fixed$ part according to EQ2. After the replacement process, the control for the MFS identifying process will be passed back to the original FII approach. Next we will specifically explain how the replacement works with an example in Figure 4.

From Figure 4, for the test case that triggered $e2$ – (2, 1, 1, 1, 0, 0, 0, 0) (in this case, the fixed part of the test case is (-, -, -, -, 0, 0, 0, 0), in which the last four parameter values are the same as the original test case t_0), we generate the related matrix at left. Each element in this matrix is computed according to EQ1. All the suspiciousness with $e3$ is labeled with a short slash as there is no test case triggering this fault in this iteration. After this matrix is determined, we can obtain the optimal test case with the ILP solver, which is t_1'–(1, 2, 2, 2, 0, 0, 0, 0), with its suspiciousness 0.167, which is smaller than all other test cases.

This replacement process is started each time a new test case triggered another fault until we finally get a proper test case. Sometimes we could not find a satisfied replacement test case in just one trial like t_1 to t_1'. When this happens, we need to repeat searching for the proper test case. For example, for t_4 which triggered $e3$, we tried three times– t_4', t_4'', t_4''' to finally get a satisfied t_4''' which passes the testing. Note that the *suspiciousness* matrix continues to change as the test case is generated and executed so that we can adaptively find an optimal one.

With the replacement test cases, the FII approach can work properly. At last, the MFS identified for fault $e1$ is (0,0, -, -, -, -, -) (Test cases passed when we mutated the first two parameter values of the original failing cases). Note that, we did not show the complete process to obtain the MFS (0, 0, -, -, -, -, -, -). This is because we need to list all the possible test cases ($3^8 = 6561$ test cases in total) to illustrate this process, which is not possible in this paper.

Another notable point is that, in this example, test cases that trigger $e2$ and $e3$ do not contribute to the MFS identification. In a real-world scenario, however, it is appealing to iteratively start MFS identification for $e2$ and $e3$ based on these test cases.

7. DISCUSSION ABOUT THE INFLUENCE OF THE ASSUMPTIONS

In section 3.1, we introduced three assumptions that can simplify our formal modeling and proposed approach, and in section 3.3, we introduced the safe value assumption which can reduce the number of test cases to identify MFS. In this section, we will discuss their influence on our propositions and approach, as well as some measures to alleviate their impact.

7.1. Deal with non-deterministic problem

The first assumption is that the outcomes of all the tests are deterministic. In practice, re-executing the same test case may result in different outcomes. For example, if the program using some random variables, different runs of the test case will assign different values to these random variables. As a result, the control flow of the program may be changed and hence the outcome will be different. We called this type of failure the non-deterministic failure [Yilmaz et al. 2006; Fouché et al. 2009]. Non-deterministic failure will complicate the MFS identification, and even worse, it may lead to an unreliable result of the MFS identification. Consider the the following example in Table XIV, the only difference between the left and the right test set is the outcome of test case (0, 1, 1). This subtle difference leads to different results of the MFS identification. Hence, if there is one or more test cases which have non-deterministic executing results, the MFS identification is not reliable.

Table XIV. MFS of two similar test sets

T_{fail}	T_{pass}	MFS
(0, 0, 0)		(0, 0, -)
(0, 0, 1)		(0, -, 0)
(0, 1, 0)		
	(0, 1, 1)	
	(1, 0, 0)	
	(1, 0, 1)	
	(1, 1, 0)	
	(1, 1, 1)	

T_{fail}	T_{pass}	MFS
(0, 0, 0)		(0, -, -)
(0, 0, 1)		
(0, 1, 0)		
(0, 1, 1)		
	(1, 0, 0)	
	(1, 0, 1)	
	(1, 1, 0)	
	(1, 1, 1)	

Inspired by the idea that using multiple same-way covering arrays to identify the MFS [Fouché et al. 2009], one potential solution to alleviate this non-deterministic failure is by adding redundancy, i.e., through re-executing the same test case to obtain the relatively stable outcome. However, this measure will increase the overall cost of

the MFS identification, so the tradeoff between cost and the quality of MFS identification should be further studied.

7.2. Deal with failures distinguishing problem

The second assumption is that different errors in the software can be easily distinguished by information such as exception traces, state conditions, or the like. If we cannot directly distinguish them, our approach does not work. This is because we cannot determine which test case should be replaced and with what. In such case, one potential solution is to use the clustering techniques to classify the failures according to available information [Zheng et al. 2006; Jones et al. 2007; Podgurski et al. 2003]. If we cannot classify them because we do not have enough information (e.g., the black box testing) or it is too costly, we believe the only approach is to take the *regarded as same failure* strategy. With this strategy, we must aware that the MFS identified are likely to be sub-schemas or irrelevant schemas of the actual MFS.

7.3. Deal with inter-option constraints

The third assumption is that all the test cases are valid, i.e., SUT does not have any inter-option constraint. This is a very strong assumption. In this section, we will try to remove this assumption. In fact, with constraints known in prior, many studies in CT have proposed approaches to avoid them when generating test cases [Cohen et al. 2007a; 2007b; 2008; Calvagna and Gargantini 2008; Yu et al. 2013; Petke et al. 2013; Yu et al. 2015]. Generally, with constraints known in prior, not all test cases in the SUT are valid. We set those valid test cases, i.e., do not contain any of the constraints as T_{valid}. Hence, the test cases we generated for MFS identification or replacement are limited to those in T_{valid}. Note that this can also make MFS identification invalid. For example, in Table XV, assume test case(0, 1, 1) is invalid. Then without any more information, we cannot determine the MFS (See two different results in Table XIV).

Table XV. MFS identification with constraints

T_{fail}	T_{pass}	$T_{invalid}$
(0, 0, 0)		
(0, 0, 1)		
(0, 1, 0)		
		(0, 1, 1)
	(1, 0, 0)	
	(1, 0, 1)	
	(1, 1, 0)	
	(1, 1, 1)	

Another type of constraint includes those that can not be found at first, but triggered a failure later in testing[Yilmaz et al. 2014]. The same as the approach proposed in [Yilmaz et al. 2014], we can just treat them as one type of failure, and then use the MFS identification approach to identify them when encountering any invalid test cases.

7.4. Deal with safe values

The last assumption is the safe value, i.e., each parameter has one value that is not the part of any MFS. With this assumption, based on Lemma 3.15 and 3.14, we can infer the result of many test cases without executing them. On the contrary, if this assumption does not hold, those test cases may be inferred incorrectly. As a result, the MFS that identified by FII approaches are not accurate.

In theory, to solve this problem, we should exhaustively execute all the test cases of which the results could be inferred. This is a costly process, and hence, some approaches have been proposed to handle such problem. Martínez [Martínez et al. 2008; 2009] proposed two approaches without the safe value assumption, but both of them needs to know the number of MFS or the degrees of them in prior. Then either through generating higher-way covering array or adaptively generating additional test case, they can determine the MFS. We [Niu et al. 2013] have also proposed an approach to alleviate this problem. In that work, we repeatedly check the same schema to reduce the impact of non-safe values.

8. EMPIRICAL STUDIES

To investigate the impact of masking effects on FII approaches in real software testing scenarios and to evaluate the performance of our approach in handling this effect, we conducted several empirical studies. Each of the studies focuses on addressing one particular issue, as follows:

Q1: Do masking effects exist in real software that contains multiple faults?

Q2: How well does our approach perform compared to traditional approaches?

Q3: Is the ILP-based test case searching technique efficient compared to the random selection?

Q4: Compared to the masking effects handling approach FDA-CIT [Yilmaz et al. 2014], does our approach have any advantages ?

8.1. The existence and characteristics of masking effects

In the first study, we surveyed three kinds of open-source software systems to gain an insight into the existence of multiple faults and their effects. The software under study were HSQLDB, JFlex and Grep. The first is a database management software written in pure Java, the second is a lexical analyser generator, and the last one is a command-line utility for searching plain-text data sets for lines matching a regular expression. The reason that we chose these systems is because they contain different versions and are all highly configurable so that the options and their interactions can affect their behaviour. Additionally, they all have a developer community so that we can easily obtain the real bugs reported in the bug tracker forum. Table XVI lists the program, the versions surveyed, number of lines of code, number of classes in the project, and the bug's ids [4] for each of the software.

Table XVI. Software under survey

software	versions	LOC	classes	bugs' id
HSQLDB	2.0rc8	139425	495	#981 & #1005
	2.2.5	156066	508	#1173 & #1179
	2.2.9	162784	525	#1286 & #1280
JFlex	1.4.1	10040	58	#87 & #80
	1.4.2	10745	61	#98 & #93
Grep	2.6.3	27046	156	#7600 & #29537
	2.22	48101	297	#33080 & #28588

[4] http://sourceforge.net/p/hsqldb/bugs
http://sourceforge.net/p/jflex/bugs
http://savannah.gnu.org/bugs/

8.1.1. Study setup. We first looked through the bug tracker forum and focused on the bugs which are caused by the options interactions. For each of them, we derived its MFS by analysing the bug description report and the associated test file which can reproduce the bug. For example, through analysing the source code of the test file of bug#981 for HSQLDB, we found the failure-inducing interaction for this bug is (*preparestatement*, *placeHolder*, *Long string*). These three parameter values together form the condition that triggers the bug. The analysed result was referred to as the "prior MFS" later.

We further built the testing scenario for each version of the software listed in Table XVI. The testing scenario is constructed so that we can reproduce different failures by controlling the inputs to the test file. For each version, the source code of the testing file as well as other detailed information is available at http://gist.nju.edu.cn/doc/multi/.

Next, we built the input model which consists of the options related to the failure-inducing interactions and additional options that are commonly used. The detailed model information is shown in Table XVII for HSQLDB, JFLex and Grep, respectively. Each table is organised into three groups: (1) *common options*, which lists the options as well as their values under which every version of this software can be tested; (2) *specific options*, under which only the specific version can be tested; and (3) *configure space*, which depicts the input model for each version of the software, presented in the abbreviated form $\#values^{\#number\ of\ parameters} \times ...$, e.g., $2^9 \times 3^2 \times 4^1$ indicates the software has 9 parameters that can take 2 values, 2 parameters 3 values, and only one parameter 4 values.

Table XVII. Input models of HSQLDB, JFlex and Grep

HSQLDB

common options		values
Server Type		3
existed form		2
resultSetTypes		3
resultSetConcurrencys		2
resultSetHoldabilitys		2
StatementType		2
sql.enforce_strict_size		2
sql.enforce_names		2
sql.enforce_refs		2
versions	**specific options**	**values**
2.0rc8	more	2
	placeHolder	2
	cursorAction	4
2.2.5	multiple	3
	placeHolder	2
2.2.9	duplicate	3
	defailure_commit	2
versions	**Config space**	
2.0rc8	$2^9 \times 3^2 \times 4^1$	
2.2.5	$2^8 \times 3^3$	
2.2.9	$2^8 \times 3^3$	

JFlex

common options		values
generation		3
charset		4
public		2
apiprivate		2
cup		2
caseless		2
char		2
line		2
column		2
notunix		2
yyeof		2
versions	**specific options**	**values**
1.4.1	hasReturn	3
	normal	2
1.4.2	lookAhead	3
	type	2
	standalone	2
versions	**Config space**	
1.4.1	$2^{10} \times 3^2 \times 4^1$	
1.4.2	$2^{11} \times 3^2 \times 4^1$	

Grep

common options		values
-E		2
-i		2
-V or –color		3
versions	**specific options**	**values**
2.6.3	ascii	2
	command	2
	word	2
	charset	2
2.22	-A or -B or -C	3
	value	4
	only-matching	2
	–count	2
versions	**Config space**	
2.6.3	$2^5 \times 3^1 \times 4^1$	
2.22	$2^4 \times 3^2 \times 4^1$	

We then generated the exhaustive test set consisting of all possible interactions of these options. For each of them, we executed the prepared testing file. We recorded the output of each test case to observe whether there were test cases containing prior MFS that did not produce the corresponding bug. Later we refer to those test cases that contain the MFS but did not trigger the expected failure as the *masked* test cases.

Table XVIII. Number of failures and their masking effects

software	versions	all tests	failure			masking	total
HSQLDB	2rc8	18432	#1(2304)	#2(1152)	#3(1152)	#1>#2#3(768)	768 (16.7%)
-	2.2.5	6912	#1(1728)	#2(1728)	-	#1>#2(576)	576 (16.7%)
-	2.2.9	6912	#1(2304)	#2(768)	#3(384)	#1>#2#3(960) #2>#3(768)	1728 (50%)
JFlex	1.4.1	36864	#1(12288)	#2(12288)	-	#1>#2(6144)	6144 (25%)
-	1.4.2	73728	#1(18432)	#2(18432)	-	#1>#2(6144)	6144 (16.7%)
Grep	2.6.3	384	#1(128)	#2(64)	#3(72)	#1>#2#3(80) #2>#3(16)	96 (36.4%)
-	2.22	576	#1(192)	#2(64)	#3(80)	#1>#2#3(80) #2>#3(16)	6144 (28.6%)

8.1.2. Results and discussion. Table XVIII lists the results of our survey. Column "all tests" gives the total number of test cases executed. Column "failure" indicates the number of test cases that failed during testing. Specifically, we give the specific number of failing test cases of each fault (labeled in the form #n). Note that the faults we listed include some uncontrolled dependencies, so there are more faults than Table XVI. Column "masking" indicates the specific number of test cases that are masked by fault. In this column, we use the form (#m $>$ #n#n'...) to indicate that fault #m masks faults (#n#n'...). The last column "total" shows the number of masked test cases in total (for all the faults). The percentage in the parentheses in this column indicates the proportion of masked test cases and the failing test cases.

We observed that for each version of the software under analysis listed in Table XVIII, test cases with masking effects do exist, i.e., test cases containing MFS did not trigger the corresponding bug. In fact, there are 768 out of 4608 test cases (about 16.7%) in hsqldb with 2rc8 version. This rate is about 16.7%, 50%, 25%, 16.7%, 36.4%, and 28.6% respectively, for the remaining software versions.

So the answer to **Q1** is that in practice, when SUTs have multiple faults, masking effects do exist widely.

It is notable that in Yilmaz's [Yilmaz et al. 2014] paper, a similar study about the existence of the masking effects has been conducted. The main difference between that work and ours is that their work quantifies the impact of the masking effects as the number of τ-degree schemas that only appear in the test cases that triggered failures of other faults. Here, the τ-degree schemas may not be MFS. Our work, however, quantifies the masking effects as the number of test cases that are masked by different failures. These test cases should contain some MFS, i.e., they should have triggered the expected failure if they did not trigger any other different failure. The reason that we quantify the masking effects in such way is because our work seeks to overcome the masking effects in the MFS identifying process. As these test cases which contain the MFS but fail to produce the corresponding failure will significantly affect the MFS identifying results, their number can better reflect the impact of the masking effects on the FII approach.

8.2. Comparing our approach with traditional approaches

The second study aims to compare the performance of our approach with traditional approaches in identifying MFS under the impact of masking effects. To conduct this study, we need to apply our approach and traditional algorithms to identify MFS in a variety of software and evaluate their results. The seven versions of software in Table XVI used as test objects are far from the requirement for a general evaluation. However, to construct real testing objects for evaluations is time-consuming. This is because we must carefully study the detail of software systems as well as their bug tracker reports. To compromise, we synthesized 5 more testing objects. These synthesized objects are five small programs which can directly return outputs when executed with given

Table XIX. The testing models used in the case study

Object	Model	Faults	MFS of each fault
H2cr8	$2^9 \times 3^2 \times 4^1$	$e_1 > e_2 > e_3$	$(5_1, 6_0, 7_0)_{e_1}, (5_1, 8_2, 9_2)_{e_2}, (5_1, 8_2, 9_1)_{e_2}, (5_1, 8_3, 9_2)_{e_3}, (5_1, 8_3, 9_1)_{e_3}$
H2.2.5	$2^8 \times 3^3$	$e_1 > e_2$	$(6_1, 7_0)_{e_1}, (5_2)_{e_2}$
H2.2.9	$2^8 \times 3^3$	$e_1 > e_2 > e_3$	$(6_0)_{e_1}, (0_1, 5_1, 7_0)_{e_2}, (0_0, 5_1, 7_0)_{e_2}, (5_1, 7_0)_{e_3}$
J1.4.1	$2^{10} \times 3^2 \times 4^1$	$e_1 > e_2$	$(0_0)_{e_1}, (1_0)_{e_2}$
J1.4.2	$2^{11} \times 3^2 \times 4^1$	$e_1 > e_2$	$(1_0, 2_1)_{e_1}, (0_1)_{e_2}$
G2.6.3	$2^5 \times 3^1 \times 4^1$	$e_1 > e_2 > e_3$	$(0_0)_{e_1}, (1_1, 2_1)_{e_2}, (3_0, 4_1)_{e_3}, (3_1, 4_1)_{e_3}, (3_2, 4_1)_{e_3}$
G2.22	$2^4 \times 3^2 \times 4^1$	$e_1 > e_2 > e_3$	$(0_0)_{e_1}, (1_0, 2_3)_{e_2}, (1_1, 2_3)_{e_2}, (3_0, 4_0)_{e_3}$
syn1	$2^7 \times 3^2 \times 4^1$	$e_4 > e_3 > e_2 > e_1$	$(2_0, 7_0)_{e_1}, (3_1, 5_1)_{e_2}, (4_0)_{e_2}, (6_0, 7_2)_{e_3}, (6_1, 8_2)_{e_4}$
syn2	$2^4 \times 3^2 \times 4^1$	$e_3 > e_2 > e_1$	$(2_0, 3_0)_{e_1}, (2_0, 5_1)_{e_1}, (4_0, 6_1)_{e_2}, (3_1, 6_0)_{e_2}, (2_2, 4_3)_{e_3}$
syn3	$2^4 \times 3^3 \times 4^1$	$e_4 > e_3 > e_2 > e_1$	$(0_0, 1_0)_{e_1}, (1_1, 2_1)_{e_2}, (2_1, 3_1)_{e_2}, (4_1, 7_1)_{e_3}, (5_2, 6_2)_{e_4}$
syn4	$2^7 \times 3^2 \times 4^1$	$e_5 > e_4 > e_3 > e_2 > e_1$	$(0_0)_{e_1}, (1_1, 3_0)_{e_2}, (2_1)_{e_3}, (4_0, 5_0)_{e_4}, (6_0)_{e_5}$
syn5	3^7	$e_4 > e_3 > e_2 > e_1$	$(0_0)_{e_1}, (2_0, 3_0)_{e_2}, (2_1, 4_1)_{e_2}, (1_2, 2_2)_{e_3}, (0_2, 6_0)_{e_4}$

inputs (details of them are also available at http://gist.nju.edu.cn/doc/multi/). These synthesized objects are created such that their testing models are similar to those real systems (We increase the number of faults in these synthesized objects in order to better show the performance of these approaches).

Table XIX lists the testing model for both the real and synthetic testing objects. In this table, column 'Object' indicates the SUT under test. For the real SUT listed in Table XVI, we label the seven software as $H2cr8$, $H2.2.5$, $H2.2.9$, $J1.4.1$, $J1.4.2$, $G2.6.3$ and $G2.22$ respectively. While for the synthesized ones, we label them in the form of '*syn+ id*'. Column 'Model' presents the input space for each testing object. Column 'Faults' shows the different faults in the software and their masking relationships. In this column, '$>$' means the left fault can mask the right fault, i.e., if a failure of the left fault is triggered, then the failure of the right fault will not be triggered. Furthermore, '$>$' is transitive so that the left fault can mask all the faults in the right. For example, for the $H2cr8$ object, we can find three faults : e_1, e_2, and e_3. The notation of $e_1 > e_2 > e_3$ indicates that fault e_2 will mask e_3; and e_1 will mask both e_2 and e_3. Here for the simplicity of the experiment, we did not build more complex testing scenarios such as masking effects in the form $e_1 > e_2,\ e_2 > e_3,\ e_3 > e_1$ or even $e_1 > e_2,\ e_2 > e_1$. The last column shows the MFS of each fault. The MFS is presented in an abbreviated form $\{\#index_{\#value}\}_{fault}$, e.g., for the object $H2cr8$, $(5_1, 6_0, 7_0)_{e_1}$ actually means (-, -, -, -, -, 1, 0, 0, -, -, -, -) is the MFS of fault e_1.

8.2.1. Study setup. After preparing the objects under testing, we then applied our approach (FIC_BS with replacement strategy) to identify the MFS. Specifically, for each SUT we selected each test case that failed during testing and fed it into our FII approach as the input. Then, after the identifying process was completed, we recorded the identified MFS and the extra test cases needed. For the traditional FIC_BS approach, we designed the same experiment. But as the objects being tested have multiple faults for which the traditional FIC_BS can not be applied directly, we adopted two traditional strategies on the FIC_BS algorithm, i.e., *regarded as same failure* and *distinguishing failures* as described in Section 4.3. The purpose of recording the generated additional test cases is to quantify the additive cost of our approach.

We next compared the identified MFS of each approach with the prior MFS to quantify the degree that each approach suffers from masking effects. There are five metrics used in this study, listed as follows:

(1) *Accurate number* : the number of identified MFS which are actual prior MFS.

(2) *Super number*: the number of identified MFS that are the super schemas of some prior MFS.
(3) *Sub number* : the number of identified MFS that are the sub schemas of some prior MFS.
(4) *Ignored number* : the number of schemas that are in the prior MFS, but irrelevant to the identified MFS.
(5) *Irrelevant number* : the number of schemas in the identified MFS that are irrelevant to the prior MFS.

Among these five metrics, the *accurate number* directly indicates the effectiveness of the FII approaches, since the target for every FII approach is to identify as many actual MFS as possible. Metrics *ignored number* and *irrelevant number* indicate the extent of deviation for the FII approaches. Specifically, the former indicates how much information about the MFS will miss, while the latter indicates how serious the distraction would be due to the "useless" schemas identified by the FII approach. *Super number* and *sub number* are the metrics in between, i.e., to identify some schemas that is *super* or *sub* schemas of the actual MFS is better than identifying *irrelevant* ones or ignoring some MFS, but it is worse than identifying the schema that is identical to the actual MFS. This is intuitive, as given the *super* / *sub* schemas, we just need to *remove* / *add* some elements of the original schemas to get the actual MFS. While for the *irrelevant* or *ignore* schemas, however, more efforts will be needed (e.g., both *adding* and *removing* operations will be needed to revise the irrelevant schemas to the actual MFS).

Besides these specific metrics, we also define a composite metric to measure the overall performance of each approach. The composite metric *aggregate* is defined as follows:

$$Aggregate = \frac{accurate + related(super) + related(sub)}{accurate + super + sub + irrelevant + ignored}$$

In this formula, *accurate, super, sub, irrelevant,* and *ignored* represent the value of specific metric. To refine the evaluation of different *super* / *sub* schemas, we design a *related* function which gives the similarity between the schemas (either super or sub) and the real MFS, so that we can quantify the specific effort for changing a *super* / *sub* schema to the real MFS. The similarity between two schemas c_1 and c_2 is computed as:

$$Similarity(c_1, c_2) = \frac{number\ of\ same\ elements\ in\ c_1\ and\ c_2}{\max(Degree(c_1), Degree(c_2))}$$

For example, the similarity of (- 1 2 - 3) and (- 2 2 - 3) is $\frac{2}{3}$. This is because (- 1 2 - 3) and (- 2 2 - 3) have the same third and last elements, and both of them are 3-degree.

The *related* function gives the summation of similarity of all the super or sub schemas with their corresponding MFS.

8.2.2. Results and discussion. Figure 5 depicts the results of the second case study. There are seven sub-figures in this figure, i.e., Figure 5(a) to Figure 5(g). They indicate the results of the number of accurate MFS each approach identified, the number of identified schemas which are the sub-schema / super-schema of some prior MFS, the number of ignored prior MFS, the number of identified schemas which are irrelevant to all the prior MFS, the aggregate value, and the extra test cases each algorithm needed, respectively.

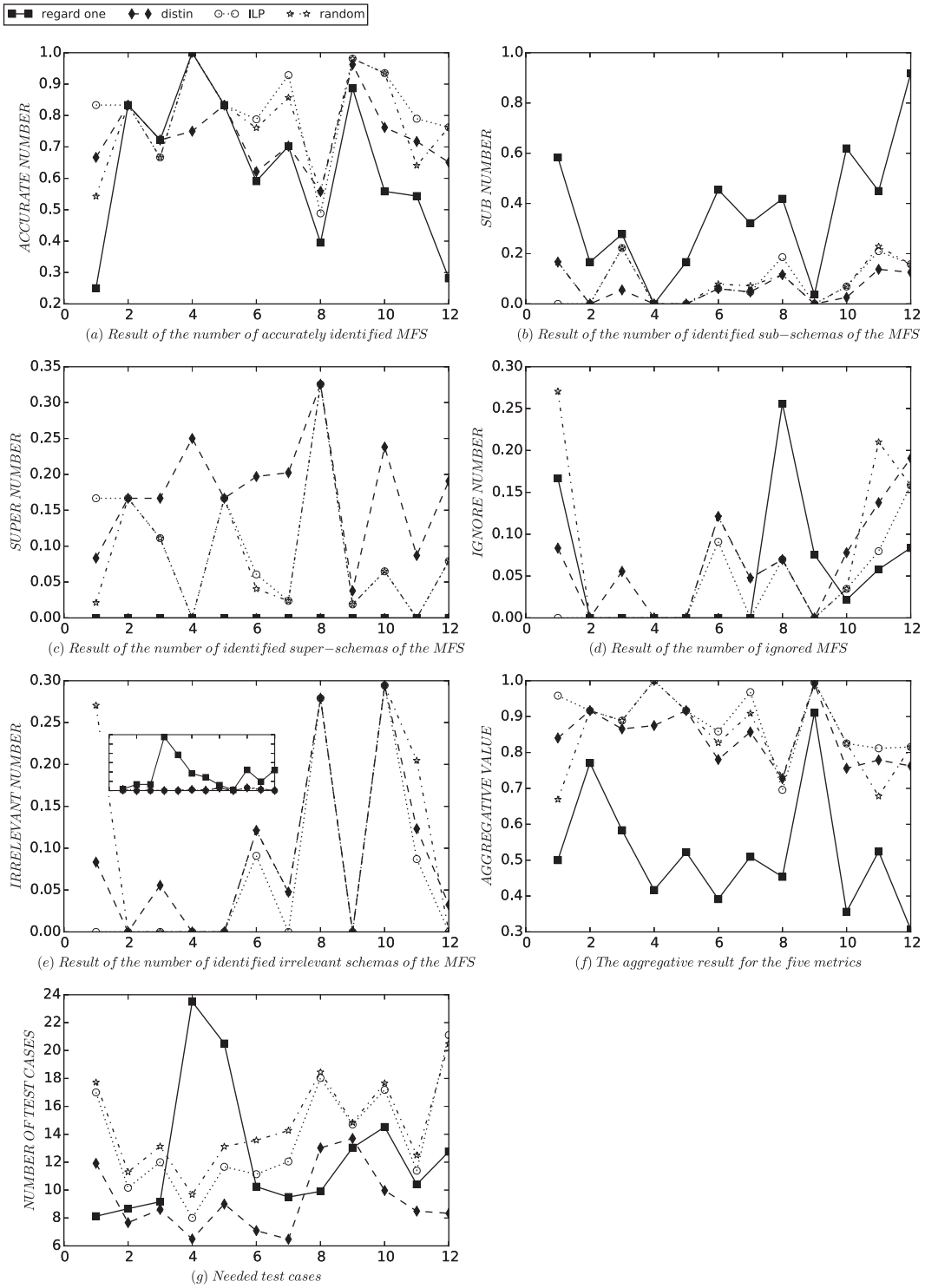


Fig. 5. Result of the evaluation for the second case study

In each sub-figure, there are four polygonal lines, each of which shows the results for one of the four strategies: *regarded as same failure*, *distinguishing failures*, *replacement strategy based on ILP searching*, *replacement strategy based on random searching* (The last one will be discussed in the next case study). Specifically, each point in the polygonal line indicates the specific result of a particular strategy for the corresponding testing object. For example in Figure 5(a), the point marked with '■' at (1,0.25) indicates that the approach using *regarded as same failure* strategy identified 0.25 accurate MFS on average for each failing test case of the testing object–HSQLDB 2cr8. The raw data for this experiment can be found in Table XXI of the Appendix. Note that all these data are average values, i.e., the average performance of these approaches when identifying the MFS for each failing test case.

Accurate number: Figure 5(a) shows the average number of accurate schemas that each approach achieved. It appears that *ILP* performed the best among the three approaches. In fact, for most testing objects (testing objects 1, 4, 5, 6, 7, 9, 10, 11 and 12), *ILP* either obtained the most number of accurate MFS, or tie with the most number of accurate MFS. The second best approach is *distinguishing failures*, which obtained better results than *regarded as same failure* for testing objects 1, 6, 8, 9, 10, 11, and 12.

Sub number & super number: Figure 5(b) and 5(c) depict the results for *sub number* and *super number*, respectively. These two figures firstly showed a clear trend for strategies *regarded as same failure* and *distinguishing failures*, i.e., the former identified more sub schemas of actual MFS than the latter, while the latter identified more super schemas of actual MFS than the former. This is consistent with our formal analysis in Section 4.1 and Section 4.2.

The performance of our strategy *ILP* for these two metrics are in between. Specifically, ILP identify more sub schemas than strategy *distinguishing failures* but fewer than *regarded as same failure*; and ILP identify more super schemas than *regarded as same failure*, but fewer than *distinguishing failures*.

Ignore number & irrelevant number: The results of the two negative performance metrics are given in Figure 5(d) and 5(e), respectively. One observation is that, comparing with strategy *regarded as same failure*, *distinguishing failures* obtained fewer irrelevant schemas, but ignored more actual MFS. This is also consistent with the formal analysis in Section 4. Note that for metric *irrelevant number*, strategy *regarded as same failure* obtained significantly much more irrelevant schemas than the remaining strategies, which make it hard to distinguish each other if we post them together on one figure. Hence, we draw this figure with only three polygonal lines (for *distinguishing failures*, *ILP* , and *random* respectively). Besides this, we offer an additional smaller figure in Figure 5(e), which includes the strategy *regarded as same failure*, to depicts the comparison of all these approaches.

The second observation is that *ILP* did a good job at reducing the scores for these two negative metrics. Specifically, for *ignored number*, our approach performed better than strategy *distinguishing failures* for testing objects (1, 3, 7, 10, 11, 12) in Figure 5(d), but is not as good as strategy *regarded as same failure*. In fact, strategy *regarded as same failure* has a significant advantage at reducing the number of ignored MFS as it tends to associate the failures with all the failing test cases. However, when we consider the *irrelevant number*, our approach is the best among all three strategies (better than *distinguishing failures* at testing objects 1, 3, 6, 7, 11, 12 in Figure 5(e), and better than strategy *regarded as same failure* for almost all the testing objects) . We believe this improvement is caused by our test cases replacing strategy, as it can increase the test cases that are useful for identifying the MFS and decrease those useless test cases.

Aggregative for the five metrics: The composite results are given in Figure 5(f). This metric gives an overall evaluation of the quality of the identified schemas. From this figure, we can find that *ILP* performed the best, next the *distinguishing failures*, the last is the *regarded as same failure* (See the testing objects 1, 3, 4, 6, 7, 10, 11, and 12 in Figure 5(f)).

It is as expected that *ILP* performed better than *distinguishing failures* as it is actually the refinement version of latter. In fact, *ILP* also make the failures distinguished from each other. The main difference between *ILP* and *distinguishing failures* strategy is that the former has to replace the test cases that triggered any failure other than the currently analysed one while the latter will not change the generated test cases.

It is a bit of surprise to find, however, that strategy *distinguishing failures* performed better than *regarded as same failure* at almost all the testing objects. This result cannot be derived from the formal analysis. We believe the reason is that the masking effects are *monotonic* in the testing objects we constructed for evaluation. That is, our study includes only the case that bug A always mask bug B, and does not consider cases that bug A can mask bug B and bug B can also mask bug A. This condition is favorable for the *distinguishing failures* strategy. For example, assume bug A masks bug B; then when we identify the MFS of bug A, the *distinguishing failures* is the correct strategy, as if there is a test case trigger the bug B, then it must not trigger the bug A (otherwise, bug B will not be triggered). Hence, the probability that *distinguishing failures strategy* makes the correct operation is at least 50% .

Test cases: The number of test cases generated for identifying the MFS indicates the cost of FII approach. The result is shown in Figure 5(g). We can find that strategy *ILP* generated more test cases than the other strategies. Specifically, the gap between *ILP* and the other two strategies ranged from about 2 to 5. This is acceptable when comparing to all the test cases that each approach needed. The increase in test cases for our approach is necessary, as additional test cases must be generated when some test cases cannot help to identify the MFS of the currently analysed failure. As for strategies *distinguishing failures* and *regarded as same failure*, there is no significant difference between them.

Above all, we draw three conclusions, which help to answer **Q2**:

1) *Distinguishing failures* strategy obtained more *super number* and *ignored number* than *regarded as same failure* strategy, while the latter identified more *sub number* and *irrelevant number* than the former. This result is consistent with the previous formal analysis in Section 4.

2) Considering the quality of the MFS each approach identified, we can find that our *ILP* approach achieves the best performance, followed by the strategy *distinguishing failures*.

3) Although our approach need more test cases than the other two strategies, it is acceptable.

8.3. Evaluating the ILP-based test case searching method

The third empirical study aims to evaluate the efficiency of the ILP-based test case searching component of our approach. To conduct this study, we implemented an additional FII approach that is augmented by the *replacing test cases* strategy with test case randomly replaced.

8.3.1. Study setup. The setup of this case study is based on the second case study, and uses the same SUT model as shown in Table XIX. We applied the new random searching based FII approach to identify the MFS in the prepared SUTs. To avoid the bias coming from the randomness, we repeated the new approach 30 times to identify the MFS in each failing test case. We then computed the average additional test cases

as well as other metrics listed in section 8.2.1 for the random-based approach. As the approach involves the random replacement, we conducted t-test on the original 30 groups of data to test the difference between the random-based approach and the ILP-based approach.

8.3.2. Results and discussion. The evaluation of this random-based approach is also shown in Figure 5, in which the polygonal line marked with '⋆' indicates the results. The raw data can also be found in the Row 'R' of Table XXI in the appendix. It is noted that for the random approach we just give the average result of 30 tries. We offer the p-value in the Row 'P(I,R)' of Table XXI. Note that A p-value smaller than 0.05 indicates that the performance of these two approaches are statistically different with each other at 95% confidence.

Compared to the ILP-based approach, we can firstly observe that there is little distinction between them in terms of the metrics super-schemas and sub-schemas. For metric accurate number, the ILP-based approach performs slightly better (see testing objects 1, 6, 7, 11, where ILP obtained more accurate MFS), and for metrics ignore and irrelevant number, the ILP-based approach also has a slightly better performance, i.e., ILP obtained smaller number of these schemas which are of no-use for testing. Combining these results, we conclude that the ILP-based approach has a similar performance as random-based approach at the quality of identified MFS (The former may perform slightly better at some testing objects). This is consistent to Figure 5(f).

The similar quality of MFS identification of these two approaches can also be embodied with the p-values we obtained. From Table XXI in the Appendix, we can learn that, for all these metrics except for 'testNum'(which will be discussed later), there exists some p-values that are *NaN* (Not a number). This is because for these testing objects, the results are the same for all the 30 tries. The reason that the results are the same is because the only random part of the approach is the test case replacement. This random process normally does not affect the determination of whether a schema is failure-inducing or not, as no matter which test case we choose, the replacement has the same ending condition, i.e., to find some test case that triggers the same failure or pass (The only exception is that different test cases for replacement may introduce some newly MFS of the same failure, which will interfere with the MFS identification process [Zhang and Zhang 2011]). As a result, the MFS identified will be the same for the 30 tries for most cases. Then as the random part normally does not affect the quality of the MFS identification, it is conceivable that these two approaches have a similar performance as they both use the *test case replacement* strategy.

Secondly, when considering the cost, we find that the ILP-based approach performs better, as it can reduce on average 1 to 2 test cases compared to the random-based procedure (See testing objects of Figure 5(g)). This conclusion is statistically significant, as we can learn that the p-value of these two approaches for this metric is extremely small, which ranged from 1.74E-26 to 1.26E-71 (See Row 'testNum' of Table XXI in the Appendix).

To be precise, we next compared the test cases only used for replacement of the two approaches. By this we can eliminate the interference of other test cases for identifying MFS, and focus on the performance of the key part of these two approaches — the replacement strategy. Figure 6 shows the result. We can learn that on average, for each call of the replacement algorithm, ILP can save about 1 test case. This may be trivial. But considering the masking status in Table XVIII, this cost reduction is significant for MFS identifying in practice. We listed the original data and p-values in Table XX. It can be easily learned that the difference between these two approaches is also statistically significant, as the Row '$\mathcal{P}$' shows all p-values are very small.

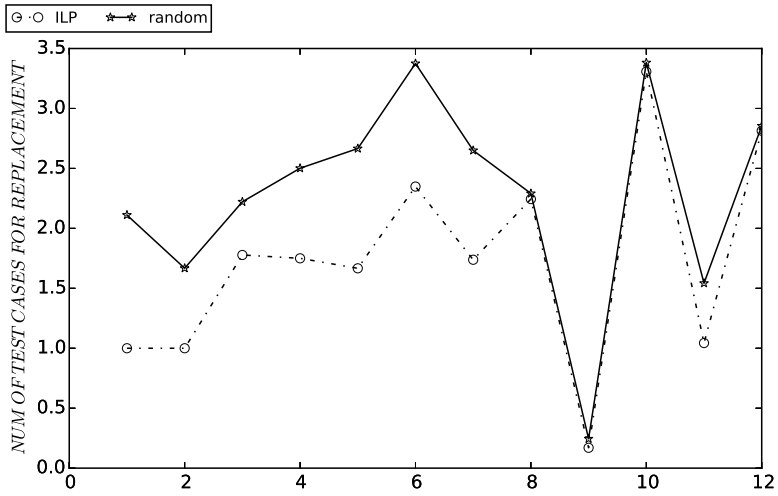


Fig. 6. The comparison of the number of replacement test cases

Table XX. The average test cases needed for one replacement

	H2cr8	H2.2.5	H2.2.9	J1.4.1	J1.4.2	G2.6.3	G2.22	syn1	syn2	syn3	syn4	syn5
ILP	1	1	1.77	1.75	1.66	2.34	1.73	2.24	0.16	3.3	1.04	2.81
Rand	2.11	1.66	2.22	2.5	2.66	3.37	2.64	2.29	0.24	3.38	1.54	2.85
$\mathcal{P}$	1.6E-60	1.1E-51	1.3E-51	2.5E-67	3.9E-70	8.0E-37	1.5E-42	2.5E-28	2.1E-28	1.1E-28	4.3E-45	3.2E-16

In summary, the answer for **Q3** is that searching for a satisfied test case affects the performance of our approach, especially regarding the number of extra test cases, and the ILP-based approach can handle the masking effects at a relatively smaller cost than the random-based approach.

8.4. Comparison with Feedback driven combinatorial testing

The *FDA-CIT* [Yilmaz et al. 2014] approach handle masking effects by generating covering array that can cover all the τ-degree schemas without being masked by the MFS. There is an integrated FII approach in the FDA-CIT, which has two versions, i.e., *ternary-class* and *multiple-class*. In this paper, we use the multiple-class version for comparison, as it performs better than the former [Yilmaz et al. 2014].

The FDA-CIT process starts with generating a covering array (In [Yilmaz et al. 2014], this is a test case-aware covering array [Yilmaz 2013]). After executing the test cases in this covering array, it records the outcome of each test case and then applies the classification tree method on the test cases to characterize the MFS of each fault. It then labels these MFS as the schemas that can trigger masking effects. Later, if the interaction coverage is not satisfied (here the interaction coverage criteria is different from the traditional covering array [Yilmaz et al. 2014]), it will re-generate a covering array that aims to cover these schemas that were masked by the MFS and then repeat the previous steps.

The main target of FDA-CIT is to guarantee that the generated test cases should cover all the τ-degree schemas. To achieve this goal, FDA-CIT needs to repeatedly identify the schemas that can trigger the masking effects. So to make the two approaches (FDA-CIT and ILP) comparable, we need to collect all the MFS that FDA-CIT characterized in each iteration and then compare them with the MFS identified by our approach.

8.4.1. Study setup. As FDA-CIT used a post-analysis (classification tree) technique on covering arrays, we first generated 2 to 4 ways covering arrays. The covering array generating method is based on augmented simulated annealing [Cohen et al. 2003], as it can be easily extended with constraint dealing and seed injection [Cohen et al. 2007b], which is needed by the FDA-CIT process. As different test cases will influence the results of the characterization process, we generated 30 different 2 to 4 way initial covering arrays. Then before we feed them to the FDA-CIT approach, we extend the corresponding τ-way covering array to τ+1-way covering array. This is because FDA-CIT needs higher-way covering arrays to identify MFS [Yilmaz et al. 2014]. The classification tree method which is integrated into FDA-CIT is implemented by Weka J48 [Hall et al. 2009], of which the configuration options are set as follows: the n-fold of cross validation is set to 10, the accuracy cutoff is set to be 1 and the default confidence factor is 0.25 [Yilmaz et al. 2006]. After running FDA-CIT, we recorded the MFS identified, and by comparing them with prior actual MFS, we can evaluate the quality of the identified schemas according to the metrics mentioned in the previous case study.

Besides the FDA-CIT, we also applied our ILP-based approach to the initial τ-way covering array. Specifically, for each failing test case (except those test cases which contain existed identified MFS) in the covering array, we separately applied our approach to identify the MFS of that case. In fact, we can reduce the number of extra test cases if we utilize the other test cases in the covering array [Li et al. 2012]), but we did not utilize the information to simplify the experiment. Similarly, we then recorded the MFS identified by our approach, and evaluated them according to the corresponding metrics. In addition, we recorded the overall test cases (including the initially generated covering array) that this approach needed and compared the magnitude of these test cases with that of FDA-CIT.

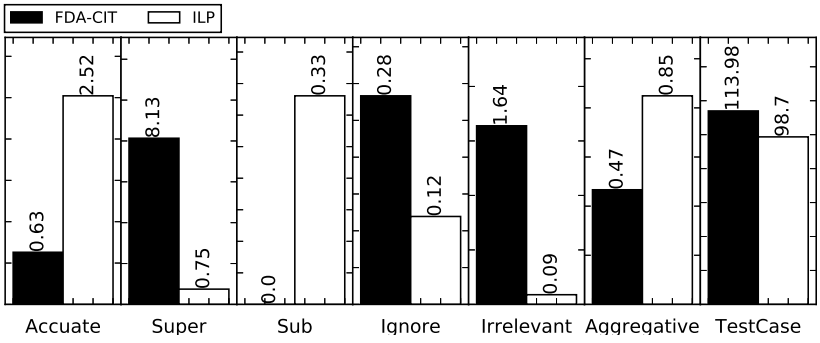
8.4.2. Result and discussion. The result is shown in Figure 7. There are three subfigures of Figure 7, which describe the results of the experiments based on 2-way, 3-way and 4-way covering arrays, respectively. In each sub-figure, there are 7 columns, showing the outcomes for the previous mentioned 6 metrics and one more metric (Column *Testcase*), which indicates the overall test cases that each approach needed. Each column has two bars, which indicate the results for approach FDA-CIT, and ILP respectively.

Note that in Figure 7, the results for each metric is the average evaluation for all the results of the experiments on the 12 testing objects in Table XIX. The raw results (as well as the p-values) for each testing object are listed in Table XXII in the appendix. The raw data is organised the same way as Table XXI.

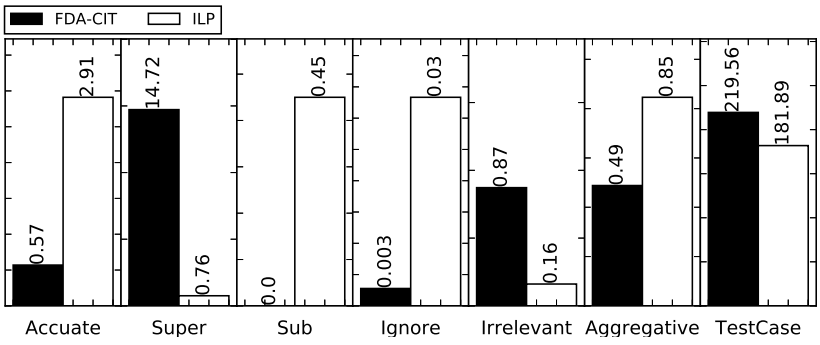
From Figure 7, we have the following observations:

First, for most metrics in our study, the performance of each approach is relatively stable against the change of degree τ. The only exceptions are metrics *ignore number* and *irrelevant number*. With increasing τ, the value of ignore number and irrelevant number of approach *FDA-CIT* decrease rapidly, while the performance of *ILP* is relatively steady when compared to *FDA-CIT*. Apart from these two metrics, the relationships between the two approaches for the other metrics are stable. Take for example the metric *accurate number*. No matter what τ is (2, 3 or 4), *ILP* always obtained more accurate schemas than *FDA-CIT*. This observation indicates that the difference between the performance of these approaches is not dependent on the characteristics of the covering array, but instead on the approaches themselves.

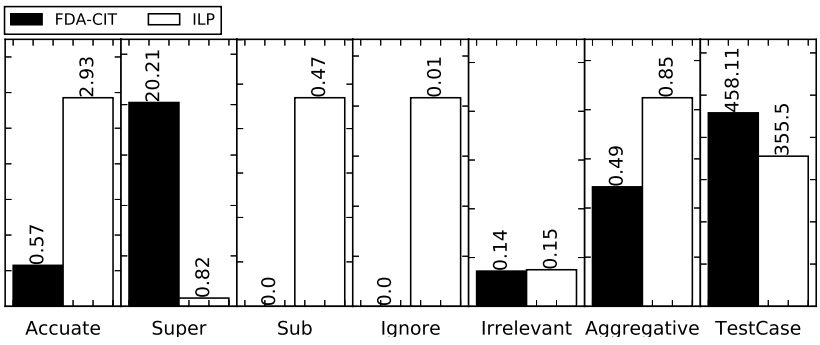
From the perspective of p-values, we can also get the same observation. For all the metrics except *ignore number* and *irrelevant number*, the p-values are relatively small. Note that there also exist some test objects that the p-values are *NaN*. This is because that the MFS of these testing objects are relatively simple (between 2 to 3 MFS). As a



(a) Result for the 2-way covering array



(b) Result for the 3-way covering array



(c) Result of the 4-way covering array

Fig. 7. Three approaches augmented with the replacing strategy

result, both approaches obtained the same schemas for the 30 tries. Besides the *NaN* values, other values of these metrics indicates that the difference between these two approaches are statistically significant. As for metrics *ignore number* and *irrelevant number*, the p-values are relatively large. For example, for the 2-way experiment, some p-values of metric *ignore* are around 0.3 (Column '*syn1*') and 0.1 (Column '*syn3*'), which show that there are no significant different between the two approaches for this metric.

Second, comparing to *ILP*, the schemas identified by *FDA-CIT* tend to be super schemas rather than sub schemas of the original MFS. We believe this result is due to the use of the classification tree analysis. Note that the way that CTA constructing decision trees usually work top-down, by choosing a parameter value at each step

that best splits the set of test cases [Rokach and Maimon 2005]. After that, one path (conjunction of nodes from the root to a leaf in the tree) in this tree is deemed as an MFS. Consequently, the MFS identified by FDA-CIT tends to share some common nodes (here, a node represents a parameter value), e.g. the root node, which result in the identified schemas containing some unnecessary parameter value. This induces the so-called 'over fitting' problem. As a result, the schemas identified by *FDA-CIT* tend to be the super schemas of the actual MFS.

Third, in terms of the quality of the MFS identified, we can clearly find that our approach performed better than FDA-CIT. This is manifested in that our approach obtained more accurate schemas (the differences are statistically significant for all 3 degrees). We believe this gap is mainly caused by the FII approach. However, this result does not mean that FIC_BS is better than the classification tree method under all conditions. In fact, the classification tree method has its own advantage, i.e., it is flexible and does not need to generate additional test cases. As a result, FDA-CIT can be applied to some complex testing scenarios, e.g., occasional failures, and incompatibilities between test cases and configurations [Yilmaz et al. 2014].

At last, when considering the cost of these two approaches, our approach needs fewer test cases. However, it should be noted that FDA-CIT is conducted on τ+1 way covering arrays. Hence, FDA-CIT can get better performance when using τ-way covering array.

Above all, we can conclude three points in this experiment, which provide answer to **Q4**:

1)The degree τ of the covering array does not affect the overall performance of approaches, but for FDA-CIT, it may get better performance at reducing the *ignore number* and *irrelevant number* when using higher-τ-way covering array.

2)When taking account of the quality of MFS identification, *ILP* approach preforms better.

3)Without additional measures, FDA-CIT is a better choice when handling some complex testing scenarios.

It is noted that FDA-CIT's primary concern is to avoid masking effects and to give every τ-degree a fair chance to be tested, not to perform fault characterization. FDA-CIT is also an established technique which can work with non-deterministic failures and in the presence of inter-option constraints. So even though our approach *ILP* has shown a better performance at MFS identification, it is appealing and effective to use FDA-CIT to guide the test generation when taking into account the masking effects. In fact, considering FDA-CIT can work with other fault characterization approaches, it may also be appealing to combine these two approaches for MFS identification.

8.5. Threats to validity

8.5.1. internal threats. There are two threats to internal validity. First, the characteristics of the actual MFS in the SUT can affect the FII results. This is because the magnitude and location of the MFS can make the FII approaches generate different test cases. As a result, it can lead to different observed failing test cases and inferred failing test cases. In the worst case, the FII approach happens to identify the exact actual MFS, then our test case replacing strategy is of no use. In this paper, we used 12 testing objects, in which 5 are real software systems with real faults and 10 synthetic ones with injected faults. To reduce the influence caused by different characteristics of the MFS, we need to build more testing objects and inject additional types of faults for a more comprehensive study of our approach.

The second threat is that we just applied our test case replacing strategy on one FII approach – FIC_BS [Zhang and Zhang 2011]. Although we believe the test case replacing strategy can also improve the quality of the identified MFS for other FII approaches when the testing object is suffering from masking effects, the extent to

which their results can be refined may vary for different FII approaches. For example, for FIC_BS [Zhang and Zhang 2011] used in this paper, there are about $(v-1)$ to $(v-1)^{k-1}$ (k is the number of parameters in a test case, v is the number of values each parameter can take) candidate test cases that can be replaced when one test case triggered failure of other fault, while for OFOT [Nie and Leung 2011a], there are $(v-1)$ candidates. As a result, FIC_BS can have a higher chance than OFOT to find a satisfied test case. To learn the difference between the improvement of various FII approaches when applying our test case replacing strategy, we need to try more FII approaches in the future.

8.5.2. external threats. One threat to external validity comes from the real software we used. In this paper we have only surveyed two types of open-source software with five different versions, of which the program scale is medium-sized. This may impact the generality of our results.

The second threat comes from the possible masking relationships between multiple faults in the real software. In this paper, we just focus on the condition that the masking effects are transitive, i.e., if fault A masks B, and fault B masks C, then fault A must mask fault C. In practice, the relationships between multiple faults may be more complicated. One possible scenario is that two faults are in a loop, for which they can even mask each other in a particular condition. Such a case will make our formal analysis invalid and will significantly complicate the relationships between schemas and their corresponding test cases. A new formal model should be proposed to handle that type of masking effects.

The third threat is that all the failures in the experiments are option-related. In practice, some failures may not be related to the parameters modeled in the SUT. For example, consider the Internet Explorer option-compliant testing problem [Nie et al. 2013]. Initially we may not properly model the options we tested, as a result, it can happen that the explorer will always crash no matter we change which option in the initial model. This will cause the MFS identification invalid, as all the test cases fail during testing. To solve this problem, one potential solution is to re-model the options we should test, as the error may be related to other options in the SUT (For example, those options which are set to default value). Or alternately, we should try other testing techniques to assist original CT. For example, if the error is related to the internal code instead of those configuration options, we may try program slicing technique [Weiser 1981] or spectrum-based approaches [Naish et al. 2011]. In such case, the parameters that we model for the SUT should not only be limited to configuration options or simple inputs, but also those variables, predicates, or other logical structures and data in the software under test.

9. RELATED WORKS

Shi and Nie [Shi et al. 2005] presented an approach for failure revealing and failure diagnosis in CT , which first tests the SUT with a covering array, then reduces the value schemas contained in the failing test case by eliminating those appearing in the passing test cases. If the failure-causing schema is found in the reduced schema set, failure diagnosis is completed with the identification of the specific input values which caused the failure; otherwise, a further test suite based on SOFOT is developed for each failing test case, and the schema set is then further reduced, until no more faults are found or the fault is located. Based on this work, Wang [Wang et al. 2010] proposed an AIFL approach which extended the SOFOT process by adaptively mutating factors in the original failing test cases in each iteration to characterize failure-inducing interactions.

Nie et al. [Nie and Leung 2011a] introduced the notion of Minimal Failure-causing Schema(MFS) and proposed the OFOT approach which is an extension of SOFOT that can isolate the MFS in the SUT. This approach mutates one value of that parameter at a time, hence generating a group of additional test cases each time to be executed. Compared with SOFOT, this approach strengthens the validation of the factor under analysis and can also detect the newly imported faulty interactions.

Delta debugging [Zeller and Hildebrandt 2002] is an adaptive divide-and-conquer approach to locate interaction failure. It is very efficient and has been applied to real software environment. Zhang et al. [Zhang and Zhang 2011] also proposed a similar approach that can efficiently identify the failure-inducing interactions that have no overlapped part. Later, Li [Li et al. 2012] improved the delta-debugging based approach by exploiting useful information in the executed covering array.

Colbourn and McClary [Colbourn and McClary 2008] proposed a non-adaptive method. Their approach extends a covering array to the locating array to detect and locate interaction failures. Martínez [Martínez et al. 2008; 2009] proposed two adaptive algorithms. The first one requires safe value as the assumption and the second one removes this assumption when the number of values of each parameter is equal to 2. Their algorithms focus on identifying faulty tuples that have no more than 2 parameters.

Ghandehari et al. [Ghandehari et al. 2012] defined the suspiciousness of tuple and suspiciousness of the environment of a tuple. Based on this, they ranked the possible tuples and generated the test configurations. They [Ghandehari et al. 2013] further utilized the test cases generated from the inducing interaction to locate the fault.

Yilmaz [Yilmaz et al. 2006] proposed a machine learning method to identify inducing interactions from a combinatorial testing set. They constructed a classification tree to analyze the covering arrays and detect potential faulty interactions. Beside this, Fouché [Fouché et al. 2009] and Shakya [Shakya et al. 2012] made some improvements in identifying failure-inducing interactions based on Yilmaz's work.

Our previous work [Niu et al. 2013] proposed an approach that utilizes the tuple relationship tree to isolate the failure-inducing interactions in a failing test case. One novelty of this approach is that it can identify the overlapped faulty interaction. This work also alleviates the problem of introducing new failure-inducing interactions in additional test cases.

In addition to the studies that aim at identifying the failure-inducing interactions in test cases, there are others that focus on working around the masking effects.

Constraints handling becomes more and more popular in CT these years. A constraint is an invalid interaction that should not appear in the test case. It can be deemed as the masking effect which are known in prior [Yilmaz et al. 2014]. Cohen [Cohen et al. 2007a; 2007b; 2008] studied the impact of the constraints that render some generated test cases invalid in CT. They also proposed an approach that integrates the incremental SAT solver with the covering arrays generating algorithm to avoid those invalid interactions. Further study was conducted [Petke et al. 2013] to show that with consideration of constraints, higher-strength covering arrays with early failure detection are practical.

Besides, there are additional works that aim to study the impact of constraints for CT [Garvin et al. 2011; Bryce and Colbourn 2006; Calvagna and Gargantini 2008; Grindal et al. 2006; Yilmaz 2013]. Among them, [Bryce and Colbourn 2006] distinguished the constraints into two types: *hard* and *soft*, which the former cannot be included in the test case, while the latter can be permitted, but not desirable. [Grindal et al. 2006] comprehensively compared the performance of four strategies at handling the constraints in the covering array. [Calvagna and Gargantini 2008] proposed an heuristic strategy to handle the constraints. It can support an ad-hoc inclusion or ex-

clusion of interactions such that the user can customize output of the covering array. [Garvin et al. 2011] refined the simulated annealing algorithm to efficiently construct the covering array while considering the constraints. [Yilmaz 2013] introduced the test case-specific constraints; differing from the system-wide constraints, these constraints can only be triggered in some specific test cases.

Chen et al. [Chen et al. 2010] addressed the issues of shielding parameters in combinatorial testing and proposed the Mixed Covering Array with Shielding Parameters (MCAS) to solve the problem caused by shielding parameters. The shielding parameters can disable some parameter values to expose additional interaction errors, which can be regarded as a special case of masking effects.

Dumlu and Yilmaz [Dumlu et al. 2011]. proposed a feedback-driven approach to work around the masking effects Specifically, they first used classification tree to classify the possible failure-inducing interactions and eliminate them. Then they generate new test cases to detect possible masked interaction in the next iteration. They [Yilmaz et al. 2014] further extended their work by proposing a multiple-class CTA approach to distinguish failures in the SUT. In addition, they empirically studied the impact of masking effects on both ternary-class and multiple-class CTA approaches.

All works above can be categorized into 3 groups according to their relationships with our work. First, we discuss the works that aim to identifying the MFS in the SUT. Our work also focuses on identifying the MFS, but instead of single fault, our work considers the impact of multiple faults on the FII approaches, and based on this, a test case replacement strategy is proposed that can assist these FII approaches in reducing the negative effects. Second, the works that aim to deal with the constraints. As discussed before the constraints can be deemed as a special masking effect. Our work differs from them in that the masking effects handled in this paper are those that can be dynamically triggered; that is, we did not know them in prior. Another difference between our work with these constraints handling works is that their target is to avoid the constraints when generating covering array. However, our work aims to remove the masking effects of the FII approaches. Last, the work[Yilmaz et al. 2014] that is most similar to our work, which also considered the masking effects that are dynamically appeared in test cases. But different from our work, it mainly focused on reducing the masking effects in the covering array, so that the covering array can support a comprehensive validation of all the τ-degree schemas. The approach used to reduce this negative effect is to use the FII approach to identify the schemas that can trigger this effect in each iteration. Our approach, however, addresses the masking effects that happened in these FII approaches themselves, and our approach alleviates the masking effects by augmenting the FII approaches with a test case replacement strategy.

10. CONCLUSIONS

Masking effects of multiple faults in the SUT can bias the results of traditional failure-inducing interactions identification approaches. In this paper, we formally analysed the impact of masking effects on FII approaches and showed that the two traditional strategies, i.e., *regarded as same failure* and *distinguishing failures*, are both inefficient in handling such impact. We further presented a test case replacement strategy for FII approaches to alleviate such impact.

In our empirical studies, we extended FIC_BS [Zhang and Zhang 2011] with our strategy. A comparison between our approach and traditional approaches was performed on several open-source software. The results indicate that our strategy assists the traditional FII approach in achieving better performance when facing masking effects in the SUT. We also empirically evaluated the efficiency of the test case searching component by comparing it with the random searching based FII approach. The result-

s showed that the ILP-based test case searching method can perform more efficiently. Last, we compared our approach with existing technique for handling masking effects – FDA-CIT [Yilmaz et al. 2014], and observed that our approach achieved a more precise result which can better support debugging, though our approach required more test cases than FDA-CIT.

As for the future work, we plan to do more empirical studies to make our conclusions more general. Our current experiments focus on medium-sized software. We would like to extend our approach to more complicated, large-scaled testing scenarios. Another promising work in the future is to integrate the white-box testing technique into the FII approaches. We believe gaining insight into source code can help figure out the relationships between multiple faults, and hence facilitate the FII approaches obtaining more accurate results. And last, because the extent to which the FII suffers from masking effects varies with different algorithms, combining these different FII approaches would be desired in the future to further improve identifying MFS of multiple faults.

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Online Appendix to: Identifying minimal failure-causing schemas in the presence of multiple faults

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A. THE DETAIL ALGORITHMS OF FIC_BS

ALGORITHM 3: The algorithm of FIC_BS

Input: Failing test case $t_{original}$, Safe values of parameter SV
Output: The MFS in the $t_{original}$

```
1  C_free ← NULL ;
2  MFS ← NULL ;
3  while true do
4  |  C_cand ← t_original \ C_free \ MFS ;
5  |  while |C_cand| > 1 && ContainMFS_factor( C_cand ) do
6  |  |  (C_low, C_high) ← BinaryPartition(C_cand) ;
7  |  |  testCase ← Mutate(t_original, SV, C_low ⋃ C_free);
8  |  |  if Run(testCase) == PASS then
9  |  |  |  C_cand ← C_low;
10 |  |  else
11 |  |  |  C_cand ← C_high;
12 |  |  |  C_free ← C_low ⋃ C_free;
13 |  |  end
14 |  end
15 |  if C_cand == NULL then
16 |  |  break ;
17 |  end
18 |  MFS ← MFS ⋃ C_cand ;
19 end
20 return MFS ;
```

Here, C_{free} indicates the parameter values in $t_{original}$ that is not related to the failure. C_{cand} represents the set of parameter values in $t_{original}$ that is needed to check in one iteration (Note that C_{cand} must contain at least one factor of MFS (line 5)). This algorithm consists of two loops. The outer loop (line 3 - 19) repeatedly searches failure-inducing parameter values in $t_{original}$ and appends them in MFS (line 18), until none of failure-inducing parameter value is found (line 15). The inter loop (line 5 - 14) focuses on finding one failure-inducing parameter value in C_{cand} (Those parameters which have been determined to be part of MFS or in C_{free} is omitted (line 4)). To reach this target, the inter loop repeatedly uses binary search technique to reduce the scope of parameter values (C_{cand}) that need to be checked (line 5 - 14). In each iteration, C_{cand} is split into two equally size parts, i.e., C_{low} and C_{high} (line 6). Then

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a new test case is generated by mutating the C_{free} part and C_{low} part of the original failing test case $t_{original}$ (line 7). For example, assume the original failing test case is (1, 1, 1, 1). The C_{free} is the second parameter value, i.e., (-, 1, -, -) and C_{low} part is the third parameter value (-, -, 1, -). Then a new test case may be generated like (1, 0, 0, 1). Note that the mutated values should be one of safe values set (SV), and the parameter values that are not mutated in this iteration is called *fixed* part. If the newly generated test case passed (This indicates that MFS was broken. As C_{free} is not related to the failure, hence, at least one element in C_{low} is failure-inducing), then the scope of failure-inducing elements will be reduced to C_{low} (line 8 - 9). Otherwise, it will be reduced to C_{high} part (line 11) (This is because MFS was not broken, indicating that MFS is in C_{high}). In this case, C_{low} will be appended to the C_{free} part (line 12), as it does not relate to the MFS. The inter loop ends when C_{cand} has just one element (one failure-inducing parameter value is found, and should appended to MFS (line 18)), or is empty (no more failure-inducing parameter value can be found, and the algorithm ends (line 15- 17)).

B. THE DETAIL OF THE EXPERIMENTS

Table XXI. Result of the evaluation

		HSQL1	HSQL2	HSQL3	JFlex1	Jflex2	Grep1	Grep2	syn1	syn2	syn3	syn4	syn5
accurate	O [1]	0.25	0.83	0.72	1	0.83	0.59	0.7	0.39	0.88	0.55	0.54	0.28
	D [2]	0.66	0.83	0.72	0.75	0.83	0.62	0.7	0.55	0.96	0.76	0.71	0.65
	I [3]	0.83	0.83	0.66	1	0.83	0.78	0.92	0.48	0.98	0.93	0.78	0.76
	R [4]	0.54	0.83	0.66	1	0.83	0.76	0.85	0.55	0.98	0.93	0.64	0.76
	p(I,R)[5]	2.34E-57	NaN	NaN	NaN	NaN	3.29E-15	2.61E-27	1.87E-38	NaN	NaN	8.04E-42	NaN
super	O	0	0	0	0	0	0	0	0	0	0	0	0
	D	0.08	0.16	0.16	0.25	0.16	0.19	0.2	0.32	0.03	0.23	0.08	0.19
	I	0.16	0.16	0.11	0	0.16	0.06	0.02	0.32	0.01	0.06	0	0.07
	R	0.02	0.16	0.11	0	0.16	0.04	0.02	0.32	0.01	0.06	0	0.07
	p(I,R)	2.32E-56	NaN	NaN	NaN	NaN	2.92E-10	NaN	NaN	NaN	NaN	NaN	NaN
sub	O	0.58	0.16	0.27	0	0.16	0.45	0.32	0.41	0.03	0.61	0.44	0.91
	D	0.16	0	0.05	0	0	0.06	0.04	0.11	0	0.02	0.13	0.12
	I	0	0	0.22	0	0	0.06	0.04	0.18	0	0.06	0.21	0.15
	R	0.16	0	0.22	0	0	0.07	0.07	0.11	0	0.06	0.22	0.15
	p(I,R)	7.84E-47	NaN	NaN	NaN	NaN	1.25E-06	5.15E-12	1.87E-38	NaN	NaN	1.15E-19	NaN
ignore	O	0.16	0	0	0	0	0	0	0.25	0.07	0.02	0.05	0.08
	D	0.08	0	0.05	0	0	0.12	0.04	0.06	0	0.07	0.13	0.19
	I	0	0	0	0	0	0.09	0	0.06	0	0.03	0.07	0.15
	R	0.27	0	0	0	0	0.12	0.04	0.06	0	0.03	0.2	0.15
	p(I,R)	7.23E-56	NaN	NaN	NaN	NaN	2.15E-12	1.25E-22	NaN	NaN	NaN	5.90E-40	NaN
irrelevant	O	0.16	0.66	0.66	5.75	3.83	1.84	1.42	0.58	0.07	2.22	0.95	2.2
	D	0.08	0	0.05	0	0	0.12	0.04	0.27	0	0.29	0.12	0.03
	I	0	0	0	0	0	0.09	0	0.27	0	0.29	0.08	0
	R	0.27	0	0	0	0	0.12	0.04	0.27	0	0.29	0.2	0
	p(I,R)	7.23E-56	NaN	NaN	NaN	NaN	2.15E-12	1.25E-22	NaN	NaN	NaN	1.16E-36	NaN
aggregate	O	0.49	0.77	0.58	0.41	0.52	0.39	0.5	0.45	0.91	0.35	0.52	0.3
	D	0.84	0.91	0.86	0.87	0.91	0.78	0.85	0.72	0.99	0.75	0.77	0.76
	I	0.95	0.91	0.88	1	0.91	0.85	0.96	0.69	0.99	0.82	0.81	0.81
	R	0.66	0.91	0.88	1	0.91	0.82	0.9	0.73	0.99	0.82	0.67	0.81
	p(I,R)	2.00E-60	NaN	NaN	NaN	NaN	1.65E-17	1.59E-28	1.87E-38	NaN	NaN	2.64E-40	NaN
testNum	O	8.12	8.66	9.16	23.5	20.5	10.24	9.5	9.9	13.03	14.51	10.4	12.75
	D	11.91	7.66	8.61	6.5	9	7.09	6.47	13.02	13.69	9.96	8.49	8.33
	I	17	10.16	12	8	11.66	11.13	12.04	18.03	14.69	17.18	11.39	21.1
	R	17.72	11.29	13.13	9.67	13.11	13.56	14.26	18.45	14.83	17.66	12.49	20.47
	p(I,R)	7.19E-47	5.07E-50	8.32E-54	1.26E-71	8.87E-73	6.97E-40	2.75E-37	5.28E-39	1.74E-26	6.31E-42	5.79E-38	3.86E-44

[1] O denotes the strategy *regarded as same failure*.
[2] D denotes the strategy *distinguishing failures*.
[3] I denotes the replacement strategy based on ILP searching.
[4] R denotes the replacement strategy based on random searching.
[4] $p(I, R)$ denotes the p-value of a test of significance for the probability that the results of ILP and Random are equal

Table XXII. Comparison with FDA-CIT

			HSQL1	HSQL2	HSQL3	JFlex1	Jflex2	Grep1	Grep2	syn1	syn2	syn3	syn4	syn5
2-way	accurate	Fda-cit	1.63	0	1	0	0	0.7	1	0.3	0.83	0.13	0.96	1
		ILP	3.66	2	2	2	2	2.2	3.93	1.23	1.56	4.03	1	4.63
		p-value	4.41E-12	NaN	NaN	NaN	NaN	1.12E-17	4.18E-28	5.95E-05	4.38E-08	1.77E-26	3.26E-01	4.14E-27
	super	Fda-cit	3.23	5.16	4.83	4	5.03	9.26	8.6	6.5	0.53	19.16	12.06	19.2
		ILP	1.33	0.36	0.26	0	0.4	0.8	0.03	1.6	0	0.53	3.63	0
		p-value	2.95E-05	1.83E-41	3.47E-20	NaN	1.25E-34	1.04E-16	2.93E-18	4.53E-12	3.65E-02	9.08E-18	4.86E-13	1.7E-26
	sub	Fda-cit	0	0	0	0	0	0	0	0	0	0	0	0
		ILP	0	0	1	0	0	0.66	0.03	0.9	0	0.63	0	0.73
		p-value	NaN	NaN	NaN	NaN	NaN	2.14E-08	3.26E-01	1.31E-07	NaN	8.72E-08	NaN	8.05E-10
	ignore	Fda-cit	1.2	0	0.2	0	0	0.09	0.06	0.93	0.63	0.06	0.2	0
		ILP	0.3	0	0	0	0	0.46	0	0.23	0	0.03	0.4	0
		p-value	1.78E-04	NaN	8.31E-02	NaN	NaN	2.02E-02	3.26E-01	7.9E-06	8.72E-08	5.62E-01	1.20E-01	NaN
	irrelevant	Fda-cit	2.53	0	0.1	0	0	2	1.93	3.1	0.7	4.63	1.56	3.1
		ILP	0	0	0	0	0	0.46	0	0.56	0	0	0	0.03
		p-value	1.48E-13	NaN	8.31E-02	NaN	NaN	2.05E-07	2.22E-07	1.59E-09	4.89E-07	2.32E-10	1.02E-09	1.61E-07
	aggregate	Fda-cit	0.46	0.51	0.61	0.5	0.52	0.47	0.48	0.36	0.58	0.36	0.31	0.41
		ILP	0.88	0.93	0.82	1	0.93	0.63	0.98	0.59	1	0.86	0.6	0.92
		p-value	1.29E-16	6.32E-26	1.14E-13	NaN	8.79E-23	2.42E-09	4.88E-35	1.04E-07	1.04E-07	2.14E-30	1.13E-25	3.06E-43
	testNum	Fda-cit	72.33	62.13	61.76	73	72	106.36	162.53	83.76	77.76	204.53	120.16	271.4
		ILP	132.86	57.13	63.7	61.93	73.53	74.06	92.43	103.56	92.06	161.53	153.8	117.73
		p-value	1.65E-16	2.68E-01	6.66E-01	1.95E-26	4.13E-01	1.05E-03	1.24E-04	5.75E-03	6.61E-09	2.95E-02	1.99E-02	4.47E-06
3-way	accurate	Fda-cit	0.86	0	1	0	0	0.96	1	0	1	0	1	1
		ILP	4.63	2	2	2	2	2.3	3.73	3.13	3.43	4.03	1	4.66
		p-value	4.14E-35	NaN	NaN	NaN	NaN	5.14E-15	1.89E-19	2.93E-21	7.95E-14	3.04E-22	NaN	1.7E-27
	super	Fda-cit	8.8	5	5.96	4	5	12.53	13.66	19.66	23.6	38.9	21.43	18.1
		ILP	1.33	0.4	0.43	0	0.4	0.83	0.1	1.2	0	0.43	3.96	0
		p-value	2.01E-14	7.79E-30	3.94E-37	NaN	7.79E-30	2.5E-21	3.5E-22	5.22E-25	9.59E-25	1.74E-30	2.69E-25	3.38E-53
	sub	Fda-cit	0	0	0	0	0	0	0	0	0	0	0	0
		ILP	0	0	1	0	0	0.9	0.13	0.9	1.2	0.66	0	0.6
		p-value	NaN	NaN	NaN	NaN	NaN	4.88E-16	4.34E-02	1.31E-07	8.42E-11	2.14E-08	NaN	3.15E-07
	ignore	Fda-cit	0	0	0	0	0	0	0	0	0	0	0.03	0
		ILP	0	0	0	0	0	0.1	0	0.2	0	0	0.09	0
		p-value	NaN	NaN	NaN	NaN	NaN	8.31E-02	NaN	3.14E-02	NaN	NaN	3.10E-01	NaN
	irrelevant	Fda-cit	0.8	0	0	0	0	1.2	0.96	1.23	2.16	2.53	1.13	0.43
		ILP	0	0	0	0	0	0.46	0	1.06	0.36	0	0	0.03
		p-value	7.43E-06	NaN	NaN	NaN	NaN	1.05E-04	8.56E-07	5.51E-01	2.42E-11	5.4E-10	1.78E-06	0.066167
	aggregate	Fda-cit	0.66	0.52	0.63	0.5	0.52	0.48	0.48	0.38	0.49	0.38	0.27	0.48
		ILP	0.94	0.93	0.81	1	0.93	0.66	0.96	0.67	0.77	0.86	0.64	0.93
		p-value	5.71E-30	5.47E-22	1.68E-35	NaN	5.47E-22	3.63E-17	5.37E-29	2.67E-10	6.57E-16	2.42E-24	2.53E-35	9.19E-38
	testNum	Fda-cit	182.36	130	129.13	190.03	186.96	185.73	228.8	243.8	359.8	310.16	171.8	316.13
		ILP	219.3	114.2	122.53	143.13	156.13	113.66	142.93	217.73	267.83	235.5	205.43	244.23
		p-value	2.69E-08	1.32E-11	8.66E-04	7.77E-54	2.31E-21	4.26E-04	2.10E-03	1.59E-04	9.18E-35	1.21E-02	2.55E-03	6.07E-10
4-way	accurate	Fda-cit	0.9	0	1	0	0	1	1	0	1	0	1	1
		ILP	5	2	2	2	2	2.16	3.76	3.2	3.5	3.86	1	4.63
		p-value	1.59E-34	NaN	NaN	NaN	NaN	1.59E-16	8.79E-21	7.47E-23	1.56E-12	5.52E-21	NaN	4.14E-27
	super	Fda-cit	8.9	5	6	4	5	22	19.26	25.7	29.23	66.96	32.46	18
		ILP	1.5	0.36	0.36	0	0.33	0.76	0.16	1.5	0	0.5	4.3	0
		p-value	5.78E-16	3.95E-30	1.43E-32	NaN	1.71E-30	1.71E-43	6.47E-37	8.4E-28	9.14E-25	4.1E-36	3.6E-28	NaN
	sub	Fda-cit	0	0	0	0	0	0	0	0	0	0	0	0
		ILP	0	0	1	0	0	1	0.13	0.93	1.13	0.7	0	0.73
		p-value	NaN	NaN	NaN	NaN	NaN	NaN	4.34E-02	1.35E-07	8.04E-09	4.54E-09	NaN	1.67E-08
	ignore	Fda-cit	0	0	0	0	0	0	0	0	0	0	0	0
		ILP	0	0	0	0	0	0	0	0.09	0	0	0	0
		p-value	NaN	NaN	NaN	NaN	NaN	NaN	NaN	1.84E-01	NaN	NaN	NaN	NaN
	irrelevant	Fda-cit	0	0	0	0	0	0	0	0.5	0	1.03	0.2	0
		ILP	0	0	0	0	0	0.56	0	1.1	0.13	0	0	0
		p-value	NaN	NaN	NaN	NaN	NaN	1.03E-06	NaN	1.42E-02	4.34E-02	2.28E-06	3.14E-02	NaN
	aggregate	Fda-cit	0.7	0.52	0.63	0.5	0.52	0.44	0.47	0.37	0.5	0.36	0.26	0.49
		ILP	0.94	0.93	0.82	1	0.94	0.64	0.95	0.67	0.81	0.85	0.64	0.93
		p-value	4.17E-40	2.36E-22	2.29E-32	NaN	8.75E-23	1.59E-18	4.52E-25	3.47E-11	1.54E-15	3.86E-24	2.02E-39	3.52E-30
	testNum	Fda-cit	421.8	295.13	295.73	451.66	456.4	241.16	358.56	490.2	975.93	562.76	298.7	649.3
		ILP	403.93	237.6	244.16	331.86	352.66	174.46	243.86	378.33	662.9	401.56	304.3	530.36
		p-value	4.32E-03	2.33E-36	4.39E-27	2.29E-64	1.62E-41	8.32E-04	2.41E-03	2.02E-19	2.46E-04	4.99E-06	7.41E-01	1.48E-19