Identifying minimal failure-causing schemas for multiple faults

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Combinatorial testing(CT) has been proven to be effective to reveal the potential failures caused by the interaction of the inputs or options of the System Under Test (SUT). To extend and fully use CT, the theory of Minimal Failure-Causing Schema (MFS) is proposed. The use of MFS helps to isolate the root cause of a failure after CT detection. Most algorithms that have been based on MFS theory focus on handling a single fault in the SUT, however, we argue that multiple faults are the more common testing scenario, and under which masking effects may be triggered so that some expected faults will not be observed. Traditional MFS theory, as well as the related identifying algorithms, lack a mechanism to handle such effects; hence, they may incorrectly isolate the MFS in the SUT. To address this problem, we propose a new MFS model that takes into account multiple faults. We first formally analyse the impact of the multiple faults on extant MFS isolating algorithms, especially in situations where masking effects are triggered by these multiple faults. Based on this, we then develop an approach that can assist traditional algorithms to better handle multiple fault testing scenarios. Empirical studies were conducted using several kinds of open-source software, which showed that multiple faults with masking effects do negatively affect traditional MFS identifying approaches and that our approach can help to alleviate these effects.

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1. INTRODUCTION

With the increasing complexity and size of modern software, many factors, such as input parameters and configuration options, can affect the behaviour of the SUT. The unexpected faults caused by the interaction of these factors can make software testing challenging if the interaction space is too large. In the worst case, we need to examine every possible combination of these factors as each combination can contain unique faults [Song et al. 2012]. While conducting exhaustive testing is ideal and necessary in theory, it is impractical and uneconomical. One remedy for this problem is com-

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Table I. MS word example

id	*Highlight*	*Status bar*	*Bookmarks*	*Smart tags*	**Outcome**
1	On	On	On	Off	PASS
2	On	Off	Off	On	PASS
3	Off	On	Off	Off	Fail
4	Off	Off	On	Off	PASS
5	Off	On	On	On	PASS

binatorial testing, which systematically samples the interaction space and selects a relatively small set of test cases that cover all valid iterations, with the number of factors involved in the interaction no more than a prior fixed integer, i.e., the *strength* of the interaction. Many works in CT aim to construct the smallest set of efficient testing objects [Cohen et al. 1997; Bryce et al. 2005; Cohen et al. 2003], which also called *covering array*.

Once failures are detected by the covering array, the failure-inducing combinations in these failed test cases must be isolated. This task is important in CT as it can facilitate debugging efforts by reducing the code scope that needed for inspection [Ghandehari et al. 2012]. However, information from the covering array sometimes does not clearly identify the location and magnitude of the failure-inducing combinations [Colbourn and McClary 2008]. Thus, deeper analysis is needed. Consider the following example [Bach and Schroeder 2004], Fig I presents a two-way covering array for testing an MS-Word application in which we want to examine various combinations of options for the MS-Word 'Highlight', 'Status Bar', 'Bookmarks' and 'Smart tags'. Assume the third test case failed. We then can get five two-way suspicious combinations that may be responsible for this failure: (Highlight: Off, Bookmarks: Off), (Highlight: Off, Smart tags: Off), (Status Bar: On, Bookmarks: Off), (Status Bar: On, Smart tags: Off), and (Bookmarks: Off, Smart tags: Off). (Note that (Highlight: Off, Status Bar: On) is excluded in this set as it appeared in the fifth passing test case). Without any more information, we cannot figure out which one or more of the combinations in this suspicious set caused the failure. In fact, taking into account the higher strength combination, e.g., (Highlight: Off, Status Bar: On, Smart tags: Off), the problem becomes more complicated.

To address this problem, prior work [Nie and Leung 2011a] specifically studied the properties of the minimal failure-causing schemas in SUT, based on which a further diagnosis by generating additional test cases was applied that can identify the MFS in the test case. Other works have proposed ways to identify the MFS in SUT, which include approaches such as building a tree model [Yilmaz et al. 2006], exploiting the methodology of minimal failure-causing schema [Nie and Leung 2011a], ranking suspicious combinations based on some rules [Ghandehari et al. 2012], using graphic-based deduction [Martínez et al. 2008], among others. These approaches can be partitioned into two categories [Colbourn and McClary 2008]: *adaptive*–additional test cases are chosen based on the outcomes of the executed tests [Nie and Leung 2011a; Ghandehari et al. 2012; Niu et al. 2013; Zhang and Zhang 2011; Shakya et al. 2012; Wang et al. 2010; Li et al. 2012]or *nonadaptive*–additional test cases are chosen independently and can be executed parallel [Yilmaz et al. 2006; Colbourn and McClary 2008; Martínez et al. 2008; 2009; Fouché et al. 2009].

The MFS methodology as well as other MFS-identifying approaches mainly focus on the ideal scenario in which SUT only contains one fault, i.e., the test case either fails or passes the testing. However, in this paper, we argue that SUT with multiple distinguished faults is the more common testing scenario in practice, and moreover, this impacts the Failure-inducing Combinations Identifying(FCI) approaches. One main impact of multiple faults on FCI approaches is the masking effect. A masking effect [Dumlu et al. 2011; Yilmaz et al. 2013] is an effect in which some failures prevent test cases from normally checking combinations that are supposed to be tested. Take the Linux command *Grep* for example. We noticed that there are two different faults reported in the bug tracker system. The first [1] claims that Grep incorrectly matches unicode patterns with '\<\>', while the second [2] claims an incompatibility between option '-c' and '-o'. When we put these two scenarios into one test case, only one fault will be observed, which means another fault is masked by the observed fault. This effect will prevent test cases from executing normally, resulting incorrect judgments of the correlation between the combinations checked in the test case and the fault that been masked and therefore not observed.

As we know that masking effects negatively affect the performance of FCI approaches, a natural question is how this effect biases the results of these approaches. In this paper, we formalize the process of identifying the MFS under the circumstances in which masking effects exist in the SUT and try to answer this question. One insight from the formal analysis is that we cannot completely get away from the impact of masking effects even if we do exhaustive testing. But, both ignoring the masking effects and regarding multiple faults as one fault are detrimental to the FCI process.

Based on this concern, we propose a strategy to alleviate this impact. This strategy adopts the divide and conquer framework, i.e., separately handles each fault in the SUT. For a particular fault under analysis, when applying traditional FCI approaches to identify the failure-inducing combinations, we pick the test cases generated by FCI approaches that trigger unexpected faults and replace them with newly regenerated test cases. These new test cases should either pass or trigger the same fault under analysis.

The key to our approach is to search for a test case that does not trigger unexpected faults which may import the masking effect. To guide the search process, i.e., to reduce the possibility that the extra generated test case will trigger an unexpected fault, a natural idea is to take some characteristics from the existing test cases and make the characteristics of the newly searched test case as much as different from the existing test cases which triggered the unexpected fault. To reach this target, we define the *related strength* between the factor and the faults. The more the *related strength* between a factor and a particular fault, the greater the likelihood that the factor will trigger this fault. We then use the integer linear programming (ILP) technique to find a test case which has the least *related strength* with the unexpected fault.

To evaluate the performance of our approach, we applied our strategy on the FCI approach FIC_BS [Zhang and Zhang 2011]. The subjects we used were several open-source software found in the developers' forum in the Source-Forge community. Through studying their bug reports in the bug tracker system as well as their user's manuals, we built a testing model which can reproduce the reported bugs with specific test cases. We then compare the FCI approach augmented with our strategy to the traditional FCI approach with these subjects. We further empirically studied the performance of the important component of our strategy – searching satisfied test cases. To conduct this study, we compare our approach with the augmented FCI approach by randomly searching satisfied test cases. We finally compare our approach with the existing masking handling technique – FDA-CIT[Yilmaz et al. 2013]. All of these empirical studies showed that our replacing strategy as well as the searching test case component achieved a better performance than these traditional approaches when the subject suffered multiple faults, especially when these faults can import masking effects.

[1]http://savannah.gnu.org/bugs/?29537
[2]http://savannah.gnu.org/bugs/?33080

```
public float foo(int a, int b, int c, int d){
  //step 1 will cause an exception when b == c
  float x = (float)a / (b - c);

  //step 2 will cause an exception when c < d
  float y = Math.sqrt(c - d);

  return x+y;
  }
```

Fig. 1. A toy program with four input parameters

The main contributions of this paper are:

— We studied the impact of the masking effects among multiple faults on the isolation of the failure-inducing combinations in SUT.
— We proposed a divide and conquer strategy of selecting test cases to alleviate the impact of these effects.
— We designed an efficient test case searching method which can rapidly find a test case that does not trigger an unexpected fault.
— We conducted several empirical studies and showed that our strategy can assist FCI approaches to achieve better performance in identifying failure-inducing combinations in SUT with masking effects.

2. MOTIVATING EXAMPLE

For convenience, we constructed a small program example to illustrate the motivation of our approach. Assume we have a method *foo* which has four input parameters: *a, b, c,* and *d*. The four parameter types are all integers and the values that they can take are: $v_a = \{7, 11\}, v_b = \{2, 4, 5\}, v_c = \{4, 6\}, v_d = \{3, 5\}$. The code detail the method is shown in Figure 1.

Considering the simple code in Figure 1, we can find two potential faults: first, in the step 1 we can get an *Arithmetic Exception* when b is equal to c, i.e., $b = 4$ and $c = 4$, that makes division by zero. Second, another *Arithmetic Exception* will be triggered in step 2 when $c < d$, i.e., $c = 4$ and $d = 5$, which makes square roots of negative numbers. So the expected failure-inducing combinations in this example should be (-, 4, 4, -) and (-, -, 4, 5).

Traditional FCI algorithms do not consider the code detail; instead, they apply black-box testing to test this program, i.e., feed inputs to those programs and execute them to observe the result. The basic justification behind those approaches is that the failure-inducing combinations for a particular fault can only appear in those inputs that trigger this fault. Traditional FCI approaches aim at using as few inputs as possible to get the same (or approximate) result as exhaustive testing, so the results derived from an exhaustive testing set are the best that these FCI approaches can achieve. Next, we will show how exhaustive testing works to identify the failure-inducing combinations in the program.

We first generate every possible input listed in the column "test inputs" of Table II, and the execution results are listed in the result column of Table II. In this column, *PASS* means that the program runs without any exception under the input in the same row. *Ex 1* indicates that the program triggered an exception corresponding to step 1 and *Ex 2* indicates the program triggered an exception corresponding to step 2. From the data listed in Table II, we can determine that (-, 4 , 4, -) must be the failure-inducing combination of Ex 1 as all the inputs that triggered Ex 1 contain this

Table II. test inputs and their corresponding result

id	test inputs	results	id	test inputs	result
1	(7, 2, 4, 3)	PASS	13	(11, 2, 4, 3)	PASS
2	(7, 2, 4, 5)	Ex 2	14	(11, 2, 4, 5)	Ex 2
3	(7, 2, 6, 3)	PASS	15	(11, 2, 6, 3)	PASS
4	(7, 2, 6, 5)	PASS	16	(11, 2, 6, 5)	PASS
5	(7, 4, 4, 3)	Ex 1	17	(11, 4, 4, 3)	Ex 1
6	(7, 4, 4, 5)	Ex 1	18	(11, 4, 4, 5)	Ex 1
7	(7, 4, 6, 3)	PASS	19	(11, 4, 6, 3)	PASS
8	(7, 4, 6, 5)	PASS	20	(11, 4, 6, 5)	PASS
9	(7, 5, 4, 3)	PASS	21	(11, 5, 4, 3)	PASS
10	(7, 5, 4, 5)	Ex 2	22	(11, 5, 4, 5)	Ex 2
11	(7, 5, 6, 3)	PASS	23	(11, 5, 6, 3)	PASS
12	(7, 5, 6, 5)	PASS	24	(11, 5, 6, 5)	PASS

Table III. Identified failure-inducing combinations and their corresponding Exception

Failure-inducing combinations	Exception
(-, 4, 4, -)	Ex 1
(-, 2, 4, 5)	Ex 2
(-, 3, 4, 5)	Ex 2

combination. Similarly, the combination (-, 2, 4, 5) and (-, 3, 4, 5) must be the failure-inducing combinations of Ex 2. We list these three combinations and the corresponding exceptions in Table III.

Note that in this case we did not get the expected result with traditional FCI approaches. The failure-inducing combinations we got for Ex 2 are (-,2,4,5) and (-,3,4,5), respectively, instead of the expected combination (-,-,4,5). So why did we fail to get the (-,-,4,5)? The reason lies in *input 6* (7,4,4,5) and *input 18* (11,4,4,5). These two inputs contain the combination (-,-,4,5), but they didn't trigger Ex 2; instead, Ex 1 was triggered.

Now let us get back to the source code of *foo*. We can find that if Ex 1 is triggered, it will stop executing the remaining code and report the exception information. In another word, Ex 1 has a higher fault level than Ex 2, so Ex 1 may mask Ex 2. Let us re-examine the combination (-,-,4,5). If we suppose that *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1, then we can conclude that (-,-,4,5) should be the failure-inducing combination of the Ex 2, which is identical to the expected one.

However, we cannot validate the supposition, i.e., *input 6* and *input 18* should trigger Ex 2 if they didn't trigger Ex 1, unless we fix the code that triggers Ex 1 and then re-execute all the test cases. So in practice, when we do not have enough resources to re-execute all the test cases again and again or can only do black-box testing, a more economical and efficient approach to alleviate the masking effect on FCI approaches is desired.

3. FORMAL MODEL

This section presents some definitions and propositions for a formal model to solve the FCI problem.

3.1. Failure-causing Schemas in CT

Assume that the SUT is influenced by *k* parameters, and each parameter p_i has a_i discrete values from the finite set V_i, i.e., a_i = $|V_i|$ (i = 1,2,..k). Some of the definitions below were originally defined in [Nie and Leung 2011b].

Definition 3.1. A *test case* of the SUT is an array of *k* values, one for each parameter of the SUT, which is denoted as a *k*-tuple (v_1, v_2,...,v_k), where $v_1 \in V_1$, $v_2 \in V_2$ ... $v_k \in V_k$.

In practice, these parameters in the test case can represent many factors, such as input variables, run-time options, building options, or various combinations of them. We need to execute the SUT with these test cases to ensure the correctness of the software behaviour.

We consider the abnormally execution of a test case to be a *fault*. It can be a thrown exception, compilation error, assertion failure or constraint violation. When faults are triggered by some test cases, we need to determine the cause of these faults; hence, some subsets of the test case must be analysed.

Definition 3.2. For the SUT, the *t*-tuple (-,v_{k_1},...,v_{k_t},...) is called a *t*-value *schema* ($0 < t \leq k$) when some t parameters have fixed values and the others can take on their respective allowable values, represented as "-".

In effect a test case itself is a t-value *schema*, when t = k. Furthermore, if a test case contains a *schema*, i.e., every fixed value in the combination is in this test case, we say this test case *hits* the *schema*.

Definition 3.3. Let c_l be a *l*-value schema, c_m be an *m*-value schema in SUT, and $l < m$. If all the fixed parameter values in c_l are also in c_m, then c_m *subsumes* c_l. In this case, we can also say that c_l is a *sub-schema* of c_m, and c_m is a *parent-schema* of c_l, which can be denoted as $c_l \prec c_m$.

For example, in the motivation example section, the two-value schema (-, 4, 4, -) is a sub-schema of the three-value schema (-, 4, 4, 5), that is, (-,4,4,-) $\prec$ (-,4,4,5).

Definition 3.4. If all test cases contain a schema, say c, and trigger a particular fault, say F, then we call this schema c the *failure-causing schema* for F. Additionally, if none of the sub-schema of c is the *failure-causing schema* for F, we then call the schema c the *Minimal Failure-causing Schema*, i.e., the MFS for F.

In fact, MFS is identical to the failure-inducing combinations we discussed previously. Figuring this out can eliminate all details that are irrelevant to the cause of the failure and, hence, facilitate the debugging efforts.

Some notions used later are listed below for convenient reference:

— k : the number of parameters that influence the SUT.
— V_i : the set of discrete values that the ith factor of the SUT can take.
— T^* : The exhaustive set of test cases for the SUT. For a SUT with k factors, and each factor can take $|V_i|$ values, the number of this set of test cases T^* is $\prod_{i=1}^{i<=k} |V_i|$.
— L : the number of faults contained in the SUT.
— F_m : the mth fault in the SUT; for different faults, we can differentiate them from their exception traces or other buggy information.
— T_{F_m} : All the test cases that can trigger the fault F_m.
— $\mathcal{T}(c)$: All the test cases that can hit (contain) the schema c. Based on the definition of MFS, we know that if schema c is MFS for F_m, then $\mathcal{T}(c)$ must be subsumed in T_{F_m}.
— $\mathcal{I}(t)$: All the schemas that are hit in the test case t, e.g., $\mathcal{I}(\ (111)\ ) = \{(1--)(-1-)(--1)(11-)(1-1)(-11)(111)\}$.
— $\mathcal{I}(T)$: All the schemas that are hit in a set of test cases T, i.e., $\mathcal{I}(T) = \bigcup_{t \in T} \mathcal{I}(t)$.
— $\mathcal{S}(T)$: All the schemas that are only hit in the set of test cases. It is important to note that this set is different from $\mathcal{I}(T)$, as the schemas hit by the test cases in T can also be hit by other test cases that do not belong to this set. In fact, $\mathcal{S}(T)$ is computed by $\{c | c \in \mathcal{I}(T)\ and\ c\ \notin \mathcal{I}(T^* \backslash T\}$.
— $\mathcal{C}(T)$: the minimal schemas only hit by the set of test cases T, this set is the sub-set of $\mathcal{S}(T)$, which is defined as $\{c | c \in \mathcal{S}(T)\ and\ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$.

Table IV. Example of proposition 3.5

c
(0, 0, - , -)
$\mathcal{T}(c)$
(0, 0, 0 ,0)
(0, 0, 0 ,1)
(0, 0, 1 ,0)
(0, 0, 1 ,1)

c
(0, -, - , -)
$\mathcal{T}(c)$
(0, 0, 0 ,0)
(0, 0, 0 ,1)
(0, 0, 1 ,0)
(0, 0, 1 ,1)
(0, 1, 0 ,0)
(0, 1, 0 ,1)
(0, 1, 1 ,0)
(0, 1, 1 ,1)

PROPOSITION 3.5. *For l-value schema c_l and m-value schema c_m, if $c_l \prec c_m$, then we can have all the test cases that hit c_m must also hit c_l, i.e., $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$.*

PROOF. Suppose $\forall t \in \mathcal{T}(c_m)$, we have that t hits c_m. Then as $c_l \prec c_m$, it must have t also hit c_l. This is because all the elements in c_l are also in c_m, which are contained in the test case t. Therefore, we get $t \in \mathcal{T}(c_l)$. Thus $t \in \mathcal{T}(c_m)$ implies $t \in \mathcal{T}(c_l)$, so it follows that $\mathcal{T}(c_m) \subset \mathcal{T}(c_l)$. □

Table IV illustrates an example of the SUT with four binary parameters (unless otherwise specified, the following examples also assume a SUT with binary parameters). The left column lists the schema (0,0,-,-) as well as all the test cases that hit this schema, while the right column lists the test cases for schema (0,-,-,-). We can observe that (0,-,-,-) $\prec$ (0,0,-,-), and the set of test cases which hit (0,-,-,-) contains the set that hits (0,0,-,-).

PROPOSITION 3.6.
For any set T of test cases of a SUT, we can always get a set of minimal schemas $\mathcal{C}(T)$ = $\{c|\ \nexists c' \in \mathcal{C}(T), s.t. c' \prec c\}$, such that,

$$T = \bigcup_{c \in \mathcal{C}(T)} \mathcal{T}(c)$$

PROOF. We prove this by producing this set of schemas.

We have denoted the exhaustive test cases for SUT as T^* and let $T^* \backslash T$ be the test cases that are in T^* but not in T. It is obviously $\forall t \in T$, we can always find at least one schema which hit by t, i.e., $c \in \mathcal{I}(t)$, such that $c \notin \mathcal{I}(T^* \backslash T)$. Specifically, at least the test case t itself as schema holds.

Then we collect all the satisfied schemas which only hit by the test cases in the test cases of T, which can be denoted as: $\mathcal{S}(T) = \{c | c \in \mathcal{I}(T)\ and\ c \notin \mathcal{I}(T^* \backslash T)\}$.

For the schemas in $\mathcal{S}(T)$, we can have $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) = T$. This is because first, for $\forall t \in \mathcal{T}(c), c \in \mathcal{S}(T)$, it must have $t \in T$. This is because if not so, then $t \in T^* \backslash T$, which contradict with the definition of $\mathcal{S}(T)$. So $t \in T$. Hence, $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) \subset T$.

Then second, for any test case t in T, as we have learned at least find one c' in $\mathcal{I}(t)$, such that c' in $\mathcal{S}(T)$ (The t itself as a schema holds). In another word, the test case t hit the schema c', which implies $t \in \mathcal{T}(c'), c' \in \mathcal{S}(T)$. And obviously $\mathcal{T}(c') \subset \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, so $t \in \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, therefore, $T \subset \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$.

Since $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) \subset T$ and $T \subset \bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c)$, so it follows $\bigcup_{c \in \mathcal{S}(T)} \mathcal{T}(c) = T$.

Then we denote the minimal schemas of $\mathcal{S}(T)$ as $M(\mathcal{S}(T)) = \{c | c \in \mathcal{S}(T)\ and\ \nexists c' \prec c, s.t., c' \in \mathcal{S}(T)\}$. For this set, we can still have $\bigcup_{c \in M(\mathcal{S}(T))} \mathcal{T}(c) = T$. We also prove this

Table V. Example of the minimal schemas

T	$\mathcal{S}(T)$	$\mathcal{C}(T)$
(0, 0, 0 ,0)	(0, 0, 0, 0)	(0, 0, 0, -)
(0, 0, 0 ,1)	(0, 0, 0, 1)	(0, 0, -, 0)
(0, 0, 1 ,0)	(0, 0, 1, 0)	
	(0, 0, 0, -)	
	(0, 0, -, 0)	

by two steps, first and obviously, $\bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c) \subset \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$. Then we just need to prove that $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$.

In fact by definition of $M(\mathcal{S}(T))$, for $\forall c' \in \mathcal{S}(T)\backslash M(\mathcal{S}(T))$, we can have some $c \in M(\mathcal{S}(T))$, such that $c \prec c'$. According to the Proposition 3.5, $\mathcal{T}(c') \subset \mathcal{T}(c)$. So for any test case $t \in \bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c)$, as we have either $\exists c' \in \mathcal{S}(T)\backslash M(\mathcal{S}(T)), s.t., t \in \mathcal{T}(c')$ or $\exists c \in M(\mathcal{S}(T)), s.t., t \in \mathcal{T}(c)$. Both cases can deduce $t \in \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$. So, $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) \subset \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$.

Hence, $\bigcup_{c\in\mathcal{S}(T)} \mathcal{T}(c) = \bigcup_{c\in M(\mathcal{S}(T))} \mathcal{T}(c)$, and $M(\mathcal{S}(T))$ is the set of schemas that holds this proposition. □

For example, Table V lists the $\mathcal{S}(T)$ and minimal schemas $\mathcal{C}(T)$ for the set of test cases T. We can see that for any other schema not in $\mathcal{C}(T)$, either we can find a test case not in T hit the schema, e.g.,(0,0,-,-) with the test case (0,0,1,1) not in T, or that is the parent schema of one of the two minimal schemas, e.g.,(0,0,0,0) the parent schema of both (0,0,0,-) and (0,0,-,0).

Let T_{F_m} denotes the set of all the test cases triggering fault F_m , then $\mathcal{C}(T_{F_m})$ actually is the set of MFS of F_m by definition of MFS.

From the construction process of $\mathcal{C}(T)$, one observation is that the minimal schema set $\mathcal{C}(T)$ is the subset of the schema set $\mathcal{S}(T)$, i.e., $\mathcal{C}(T) \subset \mathcal{S}(T)$, and for any schema in $\mathcal{S}(T)$, it either belongs to $\mathcal{C}(T)$, or is the parent schema of one element of $\mathcal{C}(T)$. Then, we can have the following proposition.

PROPOSITION 3.7. *For any test case set T and schema c, if any test case hit c is in the set T, i.e., $\mathcal{T}(c) \subset T$, then it must be that $c \in \mathcal{S}(T)$.*

PROOF. We first have $c \in \mathcal{C}(\mathcal{T}(c))$, this is obviously and in fact the minimal schemas for the test cases set $\mathcal{T}(c)$ only contain one schema, which is exactly c itself. As discussed previously we have $\mathcal{C}(\mathcal{T}(c)) \subset \mathcal{S}(\mathcal{T}(c))$, so it must be $c \in \mathcal{S}(\mathcal{T}(c))$.

Then as $\mathcal{T}(c) \subset T$, it follows $\mathcal{S}(\mathcal{T}(c)) \subset \mathcal{S}(T)$ by definition. In detail, $\mathcal{S}(\mathcal{T}(c)) = \{c|c \in \mathcal{I}(\mathcal{T}(c))\ and\ c \notin \mathcal{I}(T^*\backslash\mathcal{T}(c))\}$, so $\mathcal{S}(\mathcal{T}(c)) \subset \{c|c \in \mathcal{I}(T)\ and\ c \notin \mathcal{I}(T^*\backslash T)\}$, which is exactly $\mathcal{S}(T)$.

So as $c \in \mathcal{S}(\mathcal{T}(c))$ and hence $c \in \mathcal{S}(T)$. □

For two different sets of test cases, there exist some relationships between the minimal schemas of these two sets that, varies in relevancy with respect to the two different sets of test cases. In fact, there are three possible associations between two different sets of test cases: *inclusion*, *disjointed*, and *intersection*, as listed in Figure 2. We did not list the condition for two sets that are identical, because on that condition the minimal schemas must also be identical. To discuss the properties of the relationship of the minimal schemas between two different sets of test cases is important as we will learn later the masking effects between multiple faults will make the MFS identifying process work incorrectly, i.e., these FCI approaches may isolate the minimal schemas for the set of test cases which are biased from the expected failing set of test cases. And these properties can help us to figure out the impact of masking effects on the

Table VI. Example of the scenarios

T_l	T_k
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(0, 1, 1)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, 0, -)	(0, -, -)
(0, -, 0)	

T_l	T_k
(0, 0, 0)	(0, 0, 0)
(0, 0, 1)	(0, 0, 1)
(0, 1, 0)	(0, 1, 0)
	(1, 1, 0)
	(1, 1, 1)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, 0, -)	(0, 0, -)
(0, -, 0)	(0, -, 0)
	(1, 1, -)

FCI approaches. Next, we will separately discuss the relationship between minimal schemas under the three conditions.

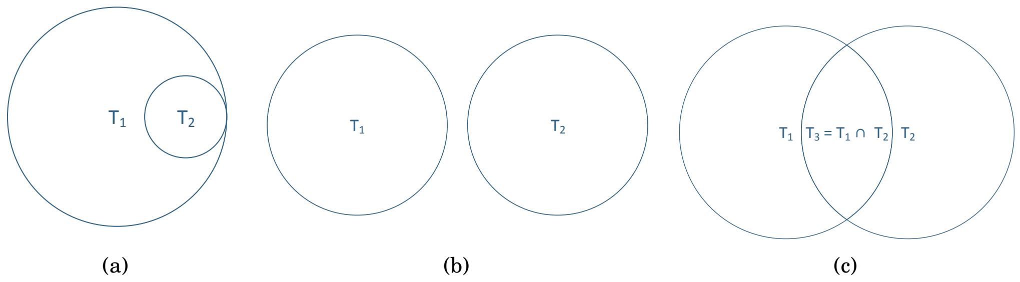


Fig. 2. Test suite relationships

3.2. Inclusion

It is the first relationship corresponding to Fig. 2(a). We can have the following proposition with two sets of test cases which have the an inclusion relationship.

PROPOSITION 3.8. *For two sets of test cases T_l and T_k, assume that $T_l \subset T_k$. Then, we have*

$$\forall c_l \in \mathcal{C}(T_l) \text{ either we have } c_l \in \mathcal{C}(T_k) \text{ or have } \exists c_k \in \mathcal{C}(T_k), s.t., c_k \prec c_l.$$

PROOF. Obviously for $\forall c_l \in \mathcal{C}(T_l)$, we can get $\mathcal{T}(c_l) \subset T_l \subset T_k$. According to proposition 3.7, we can have $c_l \in \mathcal{S}(T_k)$. So this proposition holds as the schema in $\mathcal{S}(T_k)$ either is also in $\mathcal{C}(T_k)$, or must be the parent of some schemas in $\mathcal{C}(T_k)$. □

Based on this proposition, in fact, the schema $c_k \in \mathcal{C}(T_k)$ remains with the following three possible relationships with $\mathcal{C}(T_l)$: (1) $c_k \in \mathcal{C}(T_l)$, or (2) $\exists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l$, or (3) $\nexists c_l \in \mathcal{C}(T_l), s.t., c_k \prec c_l \text{ or } c_k = c_l, \text{ or } c_l \prec c_k$. For the third case, we call c_k is *irrelevant* to $\mathcal{C}(T_l)$

We illustrate these scenarios in Table VI. There are two parts in this table, with each part showing two sets of test cases: T_l and T_k, which have $T_l \subset T_k$. For the left part, we can see that in the schema in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0), both are the parent of the schema of the one in $\mathcal{C}(T_k)$:(0,-,-). While for the right part, the schemas in $\mathcal{C}(T_l)$: (0, 0, -) and (0, -, 0) are both also in $\mathcal{C}(T_k)$. Furthermore, one schema in $\mathcal{C}(T_k)$: (1,1,-) is irrelevant to $\mathcal{C}(T_l)$.

Table VII. Disjoint Example

T_l	T_k
(0, 0, 0)	(1, 0, 0)
(0, 1, 0)	(1, 0, 1)
	(1, 1, 0)
$\mathcal{C}(T_l)$	$\mathcal{C}(T_k)$
(0, -, 0)	(1, 0, -)
	(1, -, 0)

3.3. Disjoint

This relationship corresponds to Figure.2(b). For two different sets of test cases, one obvious property is listed as follows:

PROPOSITION 3.9. *For two test cases set* T_1,T_2, *if* $T_1 \bigcap T_2 = \emptyset$, *we have,* $\mathcal{S}(T_1) \bigcap \mathcal{S}(T_2) = \emptyset$.

PROOF. If $\mathcal{S}(T_1) \bigcap \mathcal{S}(T_2) \neq \emptyset$. Without loss of generality, we let $c \in \mathcal{S}(T_1) \bigcap \mathcal{S}(T_2)$ we can learn that $\mathcal{T}(c)$ must both in T_1 and T_2, which is contradiction. □

This property tells that the minimal schemas of two disjointed test cases should be irrelevant to each other. Table VII shows an example of this scenario. We can learn from this table that for two different test case sets T_l,T_k, their minimal schemas, i.e., (0, - , 0) and (1, 0, -),(1, -, 0), respectively, are irrelevant to each other.

3.4. Intersect

This relationship corresponds to Figure.2(c). This scenario is the most common scenario for two sets of test cases, but is also the most complicated scenario for analysis. For convenience to illustrate the properties of the minimal schemas of this scenario, we assume that $T_1 \bigcap T_2 = T_3$ as depicted in Fig.2(c). Then, we can have the following properties:

PROPOSITION 3.10. *For two intersecting sets of test cases* T_1 *and* T_2 *(this two sets are neither identical nor do the members subsume each other), it must have* $\exists c_1 \in \mathcal{C}(T_1)$ *and* $c_2 \in \mathcal{C}(T_2)$. *s.t.* c_1 *and* c_2 *are irrelevant.*

PROOF. First, we can learn that $\mathcal{C}(T_1 \backslash T_3)$ are irrelevant to $\mathcal{C}(T_2 \backslash T_3)$, as $(T_1 \backslash T_3) \bigcap (T_2 \backslash T_3) = \emptyset$.

As $\mathcal{C}(T_1 \backslash T_3)$ is either identical to some schemas in $\mathcal{C}(T_1)$ or be parent schemas of them, then if some of them are identical, i.e., $\exists c', s.t., c' \in \mathcal{C}(T_1 \backslash T_3)\ and\ c' \in \mathcal{C}(T_1)$, then these schemas c' must be irrelevant to $\mathcal{C}(T_2)$ as $(T_1 \backslash T_3) \bigcap T_2 = \emptyset$. This also holds if $\mathcal{C}(T_2 \backslash T_3)$ is identical to some schemas in $\mathcal{C}(T_2)$.

Next, if both $\mathcal{C}(T_1 \backslash T_3)$ and $\mathcal{C}(T_2 \backslash T_3)$ are parent schemas of some of $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, respectively, without loss of generality, we let $c_1 \prec c_{1-3}$, $(c_{1-3} \in \mathcal{C}(T_1 \backslash T_3)\ and\ c_1 \in \mathcal{C}(T_1))$ and $c_2 \prec c_{2-3}$, $(c_{2-3} \in \mathcal{C}(T_2 \backslash T_3)\ and\ c_2 \in \mathcal{C}(T_2))$. Then, these corresponding sub-schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, i.e., c_1 and c_2 respectively, must also be irrelevant to each other. This is because $\mathcal{T}(c_1) \supset \mathcal{T}(c_{1-3})$ and $\mathcal{T}(c_2) \supset \mathcal{T}(c_{1-3})$. And as $\mathcal{T}(c_{1-3}) \bigcap \mathcal{T}(c_{2-3}) = \emptyset$, so $\mathcal{T}(c_1)$ and $\mathcal{T}(c_2)$ are neither identical nor subsume each other, this also implies that c_1 and c_2 are irrelevant to each other. □

For example, Table VIII shows two test cases that interact with each other at test case (1,0,0), but their minimal schemas, (1,0,-) and (1,-,0), respectively, are irrelevant to each other.

Table VIII. Example of Intersection by irrelevant examples

T_1	T_2
(1, 0, 0)	(1, 0, 0)
(1, 0, 1)	(1, 1, 0)
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$
(1, 0, -)	(1, -, 0)

Table IX. Example of Intersection by identical examples

T_1	T_2	$T_3 = T_1 \bigcap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 1, 0)
(1, 1, 0)	(0, 0, 1)	(1, 1, 1)
(1, 1, 1)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(0, 0, -)	(1, 1, -)
(1, 1, -)	(1, 1, -)	

PROPOSITION 3.11. *For two intersecting sets of test cases T_1 and T_2, and let $T_3 = T_1 \bigcap T_2$, if we can find $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, s.t., c_1 is identical to c_2, then it must have $c_1 = c_2 \in \mathcal{C}(T_3)$*

PROOF. As we see that identical schema must share identical test cases, then the only identical test case between T_1 and T_2 is $T_1 \bigcap T_2 = T_3$. So the only possible identical schema between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$ is in $\mathcal{C}(T_3)$. $\square$

We must know that this proposition holds when some schemas in $\mathcal{C}(T_1 \bigcap T_2)$ are identical to some schemas in $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$.

For example, Table IX shows two test cases that interact with each other at test cases (1,1,0) and (1,1,1), and they have identical minimal schema, i.e., (1,1,-), which is also the minimal schema in $\mathcal{C}(T_3)$.

PROPOSITION 3.12. *For two intersecting sets of test cases T_1 and T_2, let $T_3 = T_1 \bigcap T_2$, if we can find $\exists c_1 \in \mathcal{C}(T_1)$ and $c_2 \in \mathcal{C}(T_2)$, s.t., c_1 is the parent-schema of c_2, then it must have $c_1 \in \mathcal{C}(T_3)$. (and vice versa).*

PROOF. We have proved previously if two schemas have have a subsuming relationship, then their test cases must also have an inclusion relationship. And as the only inclusion relationship between T_1 and T_2 is that $T_3 \subset T_1$ and $T_3 \subset T_2$. So the parent schemas must be in $\mathcal{C}(T_3)$. $\square$

It is noted that this proposition holds when some schemas in $\mathcal{C}(T_3)$ are also in $\mathcal{C}(T_1)$ (or $\mathcal{C}(T_2)$), and simultaneously the same schemas in $\mathcal{C}(T_3)$ must be the parent-schema of the minimal schemas of another set of test cases, i.e., $\mathcal{C}(T_2)$ (or $\mathcal{C}(T_1)$).

Table X illustrates this scenario, in which, the minimal schemas of T_1: (1,0,-),(1,-,0),which are also the schemas in $\mathcal{C}(T_3)$, is the parent schema of the minimal schema of T_2: (1,-,-).

It is noted that these three conditions can simultaneously appears when two sets of test cases intersect with each other.

3.5. Identify the MFS

According to this analysis, we can determine that $\mathcal{C}(T_{F_m})$ actually is the set of failure-causing schemas of F_m. Then in theory, if we want to accurately figure out the MFS in the SUT, we need to exhaustively execute each possible test case, and collect the failed

Table X. Example of Intersect by subsuming examples

T_1	T_2	$T_3 = T_1 \bigcap T_2$
(0, 1, 0)	(0, 0, 0)	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 1)
(1, 0, 1)	(1, 0, 1)	(1, 1, 0)
(1, 1, 0)	(1, 1, 0)	
	(1, 1, 1)	
$\mathcal{C}(T_1)$	$\mathcal{C}(T_2)$	$\mathcal{C}(T_3)$
(-, 1, 0)	(-, 0, 0)	(1, 0, -)
(1, 0, -)	(1, -, -)	(1, -, 0)
(1, -, 0)		

test cases T_{F_m}. This is impossible in practice, especially when the testing space is very large.

So for traditional FCI approaches, they need to select a subset of the exhaustive test cases, and then either use some assumptions to predict the remaining test cases or just give a suspicious ranking. As giving a suspicious ranking can also be regard as a special case of making a prediction (with computing the possibility), so we next only formally describe the mechanism of FCI approaches belonging to the first type. We refer to the observed failed test case as $T_{fail_{observed}}$, and refer to the remaining failed test cases based on prediction as $T_{fail_{predicted}}$. We also denote the actual entire failed test cases as T_{fail}. Then the MFS identified by FCI approaches can be depicted as:

MFS = $\mathcal{C}(T_{fail_{observed}} \bigcup T_{fail_{predicted}})$.

For each FCI approach, the way it predicts the $T_{fail_{predicted}}$ according to observed failed test cases varies; further more, as the test cases it generates are different, the failed test cases observed by different test cases, i.e., $T_{fail_{observed}}$ also varies. We offer an example using the OFOT approach to illustrate this formulae.

Suppose that the SUT has 3 parameters, each of which can take 2 values. And assume the test case (1, 1, 1) failed. Then, we can describe the FCI process as shown in Table XI. In this table, test case *t* failed, and OFOT mutated one factor of the *t* one time to generate new test cases: $t_1; t_2; t_3$, It found the t_1 passed, which indicates that this test case breaks the MFS in the original test case *t*. So, the (1,-,-) should be one failure-causing factor, and as the other mutating processes all failed, this means no other failure-inducing factors were broken; therefore, the MFS in *t* is (1,-,-).

Now let us explain this process with our formal model. Obviously the $T_{fail_{observed}}$ is {(1,1,1),(1,0,1),(1,1,0)}. And as having found (0,-,-) broke the MFS, hence by theory[Nie and Leung 2011a], all the test cases that contain (0,-,-) should pass the testing (This conclusion is built on the assumption that the SUT just contain one failure-causing schema). As a result, (0,1,1),(0,0,1),(0,1,0),(0,0,0) should pass the testing. Further, as obviously the test case either passes or fails the testing (we label skipping the testing as a special case of failing), so the remaining test case(1,0,0), will be predicted to fail, i.e., $T_{fail_{predicted}}$ is {(1,0,0)}. Taken together, the MFS using the OFOT strategy can be described as: $\mathcal{C}(T_{fail_{observed}} \bigcup T_{fail_{predicted}})$ = $\mathcal{C}(\{(1,1,1),(1,0,1),(1,1,0),(1,0,0)\})$ = (1,-,-), which is identical to the one it got previously.

Similarly, other FCI approaches can also be modeled into this formal description. We will not discuss in detail how to model each FCI approach as this is not the point of this paper. It is noted that the test cases FCI predicts to be failing are not always identical to the actually failed test cases. In fact, we can generally depict the process of FCI approaches as shown in Figure. 3.

We can see in Figure. 3 that area A denotes the test cases that should have passed testing but were predicted to be fail, area B depicts the test cases that the approach observed to be failed test cases, area C refers to the failed test cases that were not

Table XI. OFOT with our strategy

original test case				**Outcome**
t	1	1	1	Fail
observed				
t_1	0	1	1	Pass
t_2	1	0	1	Fail
t_3	1	1	0	Fail
predicted				
t_4	0	0	1	Pass
t_5	0	1	0	Pass
t_6	1	0	0	Fail
t_7	0	0	0	Pass

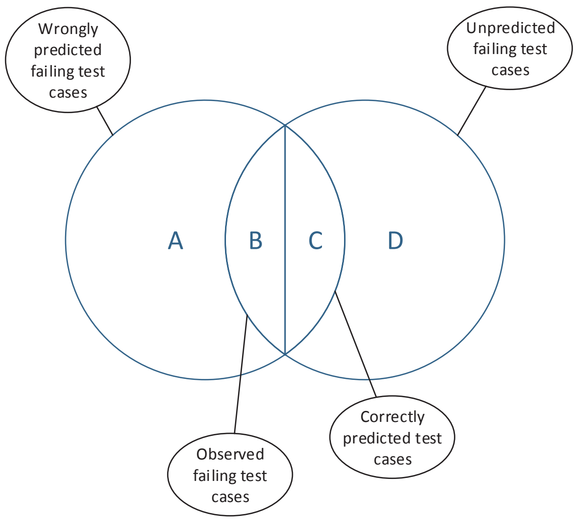

Wrongly predicted failing test cases
Unpredicted failing test cases
A
B
C
D
Correctly predicted test cases
Observed failing test cases

Fig. 3. generally model of FCI

observed to be but were predicted to be failed test cases, and area D shows the failed test cases that are neither observed nor predicted. This figure is actually one sample of the condition in which two sets of test cases intersect with each other; in specific, areas $A \bigcup B \bigcup C = T_1$, $D \bigcup B \bigcup C = T_2$ and $B \bigcup C = T_1 \bigcap T_2 = T_3$.

We learned previously that this scenario makes the schemas identified in T_1 biased from the expected MFS in T_2; specifically they must be irrelevant schemas between $\mathcal{C}(T_1)$ and $\mathcal{C}(T_2)$, which means that the FCI approach will identify some minimal schemas that are irrelevant to the actual MFS, and must ignore some actual MFS. Moreover, under the appropriate conditions listed in propositions 3.11 and 3.12, FCI may identify the identical schemas or parent-schema or sub-schema of the actual MFS. So to identify the schemas as accurately as possible, the FCI approach needs to make T_1 as similar as possible to T_2; specifically, it must make area B and area C as large as possible, and make area A as small as possible.

However, even though each FCI approach tries its best to identify the MFS as accurately as possible, masking effects raised from the test cases will result in their efforts being in vain. We next will discuss the masking problem and how it affects the FCI approaches.

Table XII. masking effects for exhaustive testing

T_1	$T_{mask(1)}$	T_*
(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 1, 0, 0)
(1, 1, 1, 0)	(0, 1, 1, 1)	(0, 0, 0, 0)
(1, 1, 0, 1)		(1, 0, 0, 0)
		(1, 0, 1, 1)
		(0, 0, 1, 1)
actual MFS for 1	regarded as one fault	distinguishing faults
$\mathcal{C}(T_1 \bigcup T_{mask(1)})$	$\mathcal{C}(T_1 \bigcup T_{mask(1)} \bigcup T_*)$	$\mathcal{C}(T_1)$
(1, 1, -, -)	(-, -, 0, 0)	(1, 1, -, 1)
(-, 1, 1, 1)	(1, 1, -, -)	(1, 1, 1, -)
	(-, -, 1, 1)	

4. MASKING EFFECT

Definition 4.1. A *masking effect* is an effect that results when a test case t hits an MFS for a particular fault, but the t does not trigger the expected fault because another fault was triggered ahead of it that prevents t from being normally checked.

Taking the masking effects into account, when identifying the MFS for a specific fault, say, F_m, we should not ignore these test cases which should have triggered F_m if they didn't trigger other faults. We call these test cases $T_{mask(F_m)}$. Hence, the MFS for fault F_m should be $\mathcal{C}(T_{F_m} \bigcup T_{mask(F_m)})$

As an example, in the motivation example in section 2, the $F_{mask(F_{Ex\ 2})}$ is $\{$ (7,4,4,5),(11,4,4,5)$\}$, So the MFS for $Ex2$ is $\mathcal{C}(T_{F_{Ex\ 2}} \bigcup T_{mask(F_{Ex\ 2})})$, which is (-,-,4,5).

In practice with masking effects, however, it is not possible to correctly identifying the MFS, unless we fix some bugs in the SUT and re-execute the test cases to figure out $T_{mask(F_m)}$.

In effect for traditional FCI approaches, without the knowledge of $T_{mask(F_m)}$, only two strategies can be adopted when facing the multiple faults problem. We will separately analyse the two strategies under exhaustive testing condition and normal FCI testing condition.

4.1. Masking effects for exhaustive testing

4.1.1. Regarded as one fault. The first is the most common strategy, as it does not distinguish the faults, i.e., it treats all of the types of faults as one fault–*failure*, and others as *pass*.

With this strategy, the minimal schemas we identify are the set $\mathcal{C}(\bigcup_{i=1}^{L} T_{F_i})$, L is the number of all the faults in the SUT. Obviously, $T_{F_m} \bigcup T_{mask(F_m)} \subset \bigcup_{i=1}^{L} T_{F_i}$. So in this case, by Proposition 3.8, some schemas we get may be the sub-schemas of some of the actual MFS, or be irrelevant to the actual MFS.

As an example, consider the test cases in Table XII. In this example, assume we need to characterize the MFS for error *1*. All the test cases that triggered error 1 are listed in column T_1; similarly, we list the test cases that triggered other faults in column $T_{mask(1)}$ and T_*, respectively, in which the former masked the error 1, while the latter did not. Actually the MFS for error 1 should be (1,1,-,-) and (-,1,1,1) as we listed them in the column *actual MFS for 1*. However, when we use the *regarded as one fault* strategy, the minimal schemas we get will be (-,-,0,0), (1,1,-,-), (-,-,1,1), in which the (-,-,0,0) is irrelevant to the actual MFS for error 1, and the (-,-,1,1) is the sub-schema of the actual MFS (-,1,1,1).

4.1.2. Distinguishing faults. Distinguishing the faults by the exception traces or error code can help make the MFS related to particular fault. Yilmaz [Yilmaz et al. 2013] proposed the *multiple-class* failure characterizing method instead of the *ternary-class*

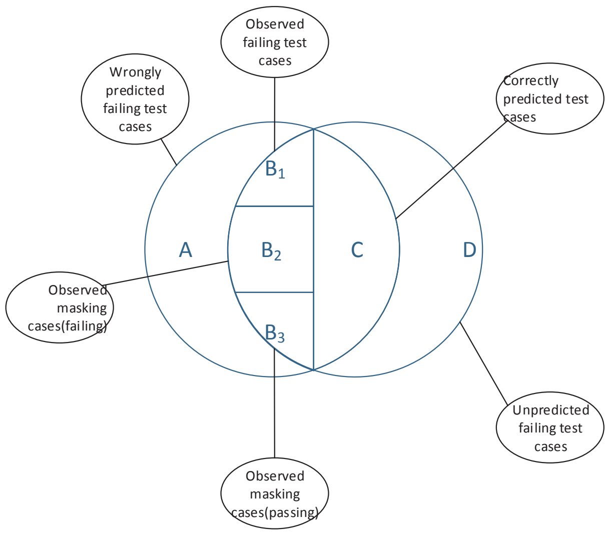

Observed failing test cases
Wrongly predicted failing test cases
Correctly predicted test cases
B1
A
B2
C
D
Observed masking cases(failing)
B3
Unpredicted failing test cases
Observed masking cases(passing)

Fig. 4. FCI with masking effects

approach to make the characterizing process more accurate. Besides, other approaches can also be easily extended with this strategy for testing SUT with multiple faults.

This strategy focuses on identifying the set of $\mathcal{C}(T_{F_m})$, and as $T_{F_m} \bigcup T_{mask(F_m)} \supset T_{F_m}$, consequently, some schemas that get through this strategy may be the parent-schema of some actual MFS. Moreover, some MFS may be irrelevant to the schemas we get with this strategy, which means this strategy will ignore these actual MFS.

For the simple example in Table XII, when we use this strategy, we will get the minimal schemas (1, 1, -, 1) and (1, 1, 1, -), which are both the parent schemas of the actual MFS (1,1,-,-), and we will observe that no schemas gotten by this strategy have any relationship with the actual MFS (-,1,1,1), which means it was ignored.

It is noted that the motivation example in section 2 actually adopted this strategy, so we see that the schemas identified for Ex 2: (-,2,4,5), (-,3,4,5) are the parent-schemas of the correct MFS(-,-,4,5).

4.2. Masking effects for FCI approaches

With masking effects, the scenario of traditional FCI approaches is a bit more complicated than the previous two exhaustive testing scenarios, and is depicted in the Figure 4. In this figure, areas A, C and D are the same as in Figure 3, and area B is divided into three sub-areas in which B_1 still represents the observed failed test cases for the current analysed fault, area B_2 represents the test cases that triggered other faults which masked the current fault, and area B_3 represents the test cases that triggered other faults which did not mask the current fault.

With this model, if we know which test cases mask the expected fault, i.e., if we have figured out the B2 and B3 areas, then the schemas that the FCI approach will identify can be described as $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$. We next denote this result as *knowing masking effects*. However, as we discussed before, to get this result is not possible without human involvement. Correspondingly, when using the *regarded as one fault* strategy, the set of MFS traditional FCI identify is $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$. And for the *distinguishing faults* strategy, the MFS is $\mathcal{C}(A \bigcup B_1 \bigcup C)$. Next, we will discuss the influence of masking effects on the two strategies.

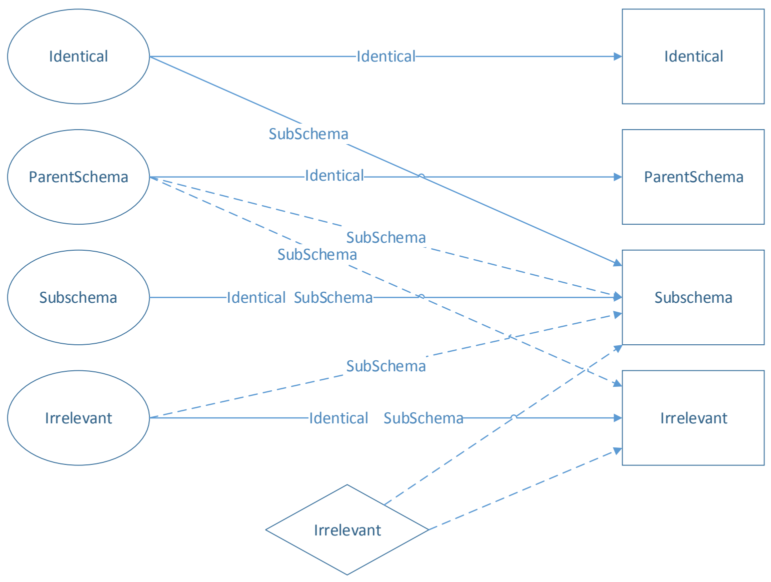


Fig. 5. Masking effects influence on FCI with regarded as one fault

4.2.1. Using the regarded as one fault strategy. For the first strategy: *regarded as one fault*, the impact of masking effects on FCI approaches can be described as shown in Figure 5. To understand the content of this figure, let us go back to the relationship between the minimal schemas of two different sets of test cases. For the *knowing masking effects* condition, the result identified by the FCI approach, i.e., $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$, is one case of the *intersect* scenario. It means that the minimal schemas get in this condition can be identical, parent-schema, sub-schema, and irrelevant to the actual MFS. And if we apply the *regarded as one fault*, the minimal schemas we get are $\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$. Obviously, we have $A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C \supset A \bigcup B_1 \bigcup B_2 \bigcup C$. So the minimal schema gotten by this strategy is either the sub-schema or identical to some schemas from the ones gotten by *known masking effects*, or exist some schemas irrelevant to all of them. Taking these two properties together, we will get what is shown in Figure 5.

The ellipses in the left column of this figure illustrate the relationship between the schemas identified under the *knowing masking effects* condition with the actual MFS, which includes the four possibilities: identical, sub-schema, parent-schema and irrelevant. For each relationship in this part, without of loss of generality, we consider c_{origin} and c_m. c_{origin} is the minima schema gotten by *knowing masking effects*, and c_m is the actual MFS. Then, when we apply the *regarded as one fault* strategy, we may get some schemas which are identical or sub-schema of c_{origin}, say c_{new} s.t., $c_{new} = c_{origin}$ or $c_{new} \prec c_{origin}$. In this figure, we use the directed line to represent these two transformations with different labels in each line. As for the schemas c_{new} that are irrelevant to all the c_{origin}, we in the bottom of this figure use a rhombus labeled ”Irrelevant” to represent them. Then, we must have some relationship between the c_{new} and actual MFS, which also has four possibilities represented as rectangles in the right column.

Note that there exist two types of directed line – solid line and dashed line, in which the former indicates that this transformation is deterministic, e.g., if $c_{origin} \prec c_m$ and $c_{new} \prec c_{origin}$, then we must have $c_{new} \prec c_m$. The latter type means the transformation is just one of all the possibilities in such condition, e.g., if c_{origin} *irrelevant* c_m

and $c_{new} \prec c_{origin}$, then we can have either $\exists c_{m'}\ is\ one\ actual\ MFS, s.t. c_{new} \prec c_m$ or $c_{new}\ irrelevant\ to\ all\ actual\ MFS$. (We will later give illustrative examples.)

In this figure, only two transformations can make $\exists c_{m'}\ is\ one\ actual\ MFS, s.t.\ c_{new} = c_{m'}$ or $c_{m'} \prec c_{new}$, which are when $c_{origin} = c_m$, $c_{new} = c_{origin}$ or $c_m \prec c_{origin}$, $c_{new} = c_{origin}$ respectively. This can be easily understood, as to make $c_{new} = c_{m'}$ or $c_{m'} \prec c_{new}$, according to proposition 3.11 and 3.12, it must have $\mathcal{T}(c_{new}) \subset (B_1 \bigcup B_2)$. If after transformation, we get $c_{new} \prec c_{origin}$, then we must have $\exists t \in B_3, s.t., t \in \mathcal{T}(c_{new})$, otherwise, there should be no changing to the c_{origin}, and it must have no new schema c_{new} that is the sub-schema of c_{origin}. Consequently, $\mathcal{T}(c_{new}) \not\subset (B_1 \bigcup B_2)$ if we have $c_{new} \prec c_{origin}$, to make e $c_{new} = c_{m'}$ or $c_{m'} \prec c_{new}$, the transformation can only be identical, i.e., $c_{new} = c_{origin}$ and must have $c_{origin} = c_m$ or $c_m \prec c_{origin}$, correspondingly.

As the identical transformation is simple, so we will ignore it and just take examples to illustrate the other transformations. Table XIII presents all these possibilities except the identical transformation. This table consists of three parts, with the upper part giving the test cases for each area in the abstract FCI model. Note that we merged the B_1, B_2 and C areas into one column because the way of computing the minimal schemas of the three approaches – *actual MFS, knowing masking effects, regarded as one fault* are all dependent on the merged test cases of these three areas, not on their specific distribution. The middle part of this table shows the minimal schemas using this particular method. And last, the lower part depicts the sample of each possible result of the transformation in Figure 5. In this part, $c_m = c_{origin} \rightarrow c_{new} \prec c_{m'}$ indicates the scenario that the schema c_{origin} gotten by *knowing masking effects* is identical to one actual MFS c_m. Then, taking *regarded as one fault*, we get $c_{new} \prec c_{origin}$, so that we can have $\exists c_{m'}\ is\ one\ actual\ MFS, s.t., c_{new} \prec c_m$. Other formulae in this column can be explained in this way, in which $c_{new}\ irrele$ and $c_{origin}\ irrele$ mean that the schema c_{new} and c_{origin} is irrelevant to all the actual MFS respectively. The mark $*$ in c_m and $c_{m'}$,respectively, represent these two conditions. The formulae in the last two rows, $c_{new_{irrele}} \prec c_{m'}$ and $c_{new_{irrele}} irrele$, indicates the schemas $c_{new_{irrele}}$ obtained by *regarded as one fault* are irrelevant to all the c_{origin}, in which the former is the subschema of some actual MFS $c_{m'}$ while the latter is irrelevant to all of the actual MFS. The $*$ in the c_{origin} column means that the c_{new} is irrelevant to all the c_{origin}.

4.2.2. using distinguish strategy. And for the second strategy, *distinguishing faults*, the influence can be described as in Figure 6.

This figure is organised the same way as Figure 5. As with the distinguishing faults strategy, the minimal schemas identified are actually $\mathcal{C}(A \bigcup B_1 \bigcup C)$. Obviously $A \bigcup B_1 \bigcup C \subset A \bigcup B_1 \bigcup B_2 \bigcup C$. So under this transformation, the c_{new} should be either the parent-schema or identical to the c_{origin}.

We take an example to illustrate this type of transformation, which is depicted in Table XIV. Similar to our previous strategy, we omit the samples that belong to the identical transformation.

We noted that in addition to the transformations corresponding each directed line that is depicted in Figure 6, there is one more transformation can appear in this strategy, which can make some c_{origin} *removed* from the newly minimal schemas, i.e., $\nexists c_{new}, s.t., c_{new} = c_{origin}\ or\ c_{origin} \prec c_{new}$. For the Table XIV example, in the last row we used a formulae $c_{origin}\ ignored$ to represent this condition, with the mark $*$ in the c_{new} indicating that the c_{origin} is irrelevant to all the c_{new}. In this row, we can find the schema $c_{origin} - -$(1,1,0,0,1,-), which is identical to the one in actual MFS, there exists no c_{new} which is identical to or is the parent-schema of this schema. Consequently, in this condition, this strategy may ignore some actual MFS compared with *knowing masking effects*.

Table XIII. Example of the influence of the regarded as one fault for FCI approach

A	$B_1 \bigcup B_2 \bigcup C$	B_3	D
(0,0,0,1,0,0)	(1,1,1,0,0,0)	(1,0,1,0,0,0)	(1,1,0,0,0,0)
(0,0,0,1,1,0)	(1,1,1,0,1,0)	(1,0,1,0,1,0)	(1,1,0,0,1,0)
(0,0,1,0,0,0)	(1,1,1,1,0,0)	(0,0,1,0,1,0)	(1,1,0,1,0,0)
(0,0,1,1,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,0)	(1,1,0,1,1,0)
	(1,0,1,1,0,0)	(0,1,0,0,0,0)	(0,0,1,1,0,1)
	(1,0,1,1,1,0)	(0,1,0,1,0,0)	(0,1,1,1,0,1)
	(0,0,0,0,1,0)	(0,1,1,0,0,0)	
	(0,0,0,0,0,0)	(0,1,1,1,0,0)	
	(0,0,1,1,1,1)	(1,0,0,0,0,0)	
	(0,1,1,1,1,1)	(1,0,0,0,1,0)	
		(1,1,1,1,1,1)	
		(1,0,1,1,1,1)	

$actual\ MFS$	$knowing\ masking\ effects$	$one\ fault$
$\mathcal{C}(B_1 \bigcup B_2 \bigcup C \bigcup D)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup B_3 \bigcup C)$
(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)
(1,-,1,1,-,0)	(1,-,1,1,-,0)	(0,0,-,-,-,0)
(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,-,-,-,0,0)
(0,-,1,1,-,1)	(0,0,-,-,0,0)	(-,0,1,-,-,0)
	(-,0,1,1,0,0)	(-,-,1,-,0,0)
	(0,-,1,1,1,1)	(-,0,-,0,-,0)
		(-,-,1,1,1,1)
		(1,-,1,1,1,-)
		(-,0,1,1,1,-)

transformation	c_m	c_{origin}	c_{new}	$c_{m'}$
$c_m = c_{origin} \rightarrow c_{new} \prec c_{m'}$	(1,-,1,1,-,0)	(1,-,1,1,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
$c_m \prec c_{origin} \rightarrow c_{new} \prec c_{m'}$	(1,1,-,-,-,0)	(1,1,1,-,-,0)	(1,-,1,-,-,0)	(1,-,1,1,-,0)
$c_m \prec c_{origin} \rightarrow c_{new}\ irrele$	(0,-,1,1,-,1)	(0,-,1,1,1,1)	(-,-,1,1,1,1)	*
$c_{origin} \prec c_m \rightarrow c_{new} \prec c_{m'}$	(0,0,0,0,-,0)	(0,0,0,-,-,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
$c_{origin}\ irrele \rightarrow c_{new} \prec c_{m'}$	*	(0,0,-,-,0,0)	(0,0,-,-,-,0)	(0,0,0,0,-,0)
$c_{origin}\ irrele \rightarrow c_{new}\ irrele$	*	(-,0,1,1,0,0)	(-,0,1,-,-,0)	*
$c_{new_{irrele}} \prec c_{m'}$	*	*	(-,0,-,0,-,0)	(0,0,0,0,-,0)
$c_{new_{irrele}}\ irrele$	*	*	(1,-,1,1,1,-)	*

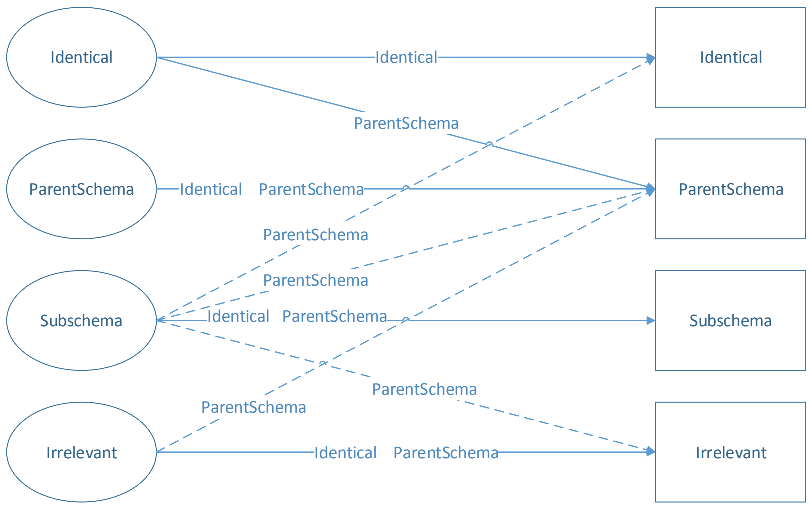


Fig. 6. masking effects influence on FCI with distinguishing faults

Table XIV. Example the influence of distinguishing faults for FCI approach

A	$B_1 \bigcup C$	B_2	D
(0,0,0,1,1,0)	(0,0,0,0,0,0)	(0,0,1,1,1,0)	(0,1,1,0,0,0)
(0,0,1,1,0,0)	(0,0,0,0,1,0)	(1,1,0,1,0,0)	(0,1,1,0,1,0)
(0,0,1,0,1,0)	(0,0,0,1,0,0)	(1,0,1,1,0,1)	(0,1,1,1,0,0)
(0,0,1,0,0,0)	(1,1,1,1,1,0)	(0,0,1,1,1,1)	(0,1,1,1,1,0)
(1,1,0,0,0,0)	(1,1,0,0,1,0)	(1,1,0,0,1,1)	(1,1,1,1,1,1)
(0,0,1,1,0,1)	(1,1,0,1,1,0)	(0,1,0,0,1,1)	(0,1,1,1,1,1)
	(1,1,1,0,0,0)	(1,0,1,0,1,1)	(1,1,1,1,0,1)
	(1,1,1,1,0,0)		(1,0,0,1,1,1)
	(1,1,1,0,1,0)		
	(1,0,1,1,1,1)		
	(0,0,0,0,1,1)		
	(1,0,0,0,1,1)		

actual MFS	*knowing masking effects*	*distinguishingfaults*
$\mathcal{C}(B_1 \bigcup B_2 \bigcup C \bigcup D)$	$\mathcal{C}(A \bigcup B_1 \bigcup B_2 \bigcup C)$	$\mathcal{C}(A \bigcup B_1 \bigcup C)$
(0,-,1,1,1,-)	(1,1,-,-,-,0)	(0,0,0,-,-,0)
(1,1,-,1,-,0)	(0,0,-,-,-,0)	(0,0,-,-,0,0)
(-,-,0,0,1,1)	(-,0,1,1,-,1)	(0,0,-,0,-,0)
(0,0,0,-,0,0)	(0,0,1,1,-,-)	(1,1,1,-,-,0)
(0,0,0,0,-,0)	(0,0,0,0,1,-)	(1,1,-,-,1,0)
(1,0,-,-,1,1)	(1,1,0,0,1,-)	(1,1,-,0,-,0)
(1,1,0,0,1,-)	(1,0,1,-,1,1)	(0,0,1,1,0,-)
(-,1,1,-,-,0)	(1,0,-,0,1,1)	(1,0,1,1,1,1)
(-,-,1,1,1,1)	(-,-,0,0,1,1)	(-,0,0,0,1,1)
(1,1,-,-,1,0)		(0,0,0,0,1,-)
(-,1,1,1,1,-)		
(1,1,1,1,-,-)		
(1,-,1,1,-,1)		
(0,0,0,0,1,-)		

transformation	c_m	c_{origin}	c_{new}	$c_{m'}$
$c_m = c_{origin} \rightarrow c_{m'} \prec c_{new}$	(-,-,0,0,1,1)	(-,-,0,0,1,1)	(-,0,0,0,1,1)	(-,-,0,0,1,1)
$c_m \prec c_{origin} \rightarrow c_{m'} \prec c_{new}$	(1,0,-,-,1,1)	(1,0,1,-,1,1)	((1,0,1,1,1,1))	((1,0,-,-,1,1))
$c_{origin} \prec c_m \rightarrow c_{new}$ *irrele*	(1,1,-,1,-,0)	(1,1,-,-,-,0)	(1,1,-,0,-,0)	*
$c_{origin} \prec c_m \rightarrow c_{new} \prec c_{m'}$	(0,0,0,0,-,0)	(0,0,-,-,-,0)	(0,0,0,-,-,0)	(0,0,0,0,-,0)
$c_{origin} \prec c_m \rightarrow c_{m'} \prec c_{new}$	(1,1,-,1,-,0)	(1,1,-,-,-,0)	(1,1,1,-,-,0)	(-,1,1,-,-,0)
$c_{origin} \prec c_m \rightarrow c_{m'} = c_{new}$	(1,1,-,-,1,0)	(1,1,-,-,-,0)	(1,1,-,-,1,0)	(1,1,-,-,1,0)
c_{origin} *irrele* $\rightarrow c_{m'} \prec c_{new}$	*	(-,0,1,1,-,1)	(1,0,1,1,1,1)	(1,-,1,1,-,1)
c_{origin} *irrele* $\rightarrow c_{new}$ *irrele*	*	(0,0,1,1,-,-)	(0,0,1,1,0,-)	*
c_{origin} *ignored*	(1,1,0,0,1,-)	(1,1,0,0,1,-)	*	*

In fact, besides this special case that may result in the FCI approach ignoring some actual MFS, there exist some other cases that can also achieve the same effect. For example, when the c_{new} is the only schema that is *related* to c_{origin}, (*related* means not irrelevant, and in this case it is either identical to or the parent-schema). And the corresponding parent-schema c_{origin} is the only schema which is related to one actual MFS c_m. Then, if the c_{new} is irrelevant to all the actual MFS, we will ignore the actual MFS c_m. This *ignored* event is caused by the c_{new} growing into the irrelevant schemas, which can also appear in the strategy *regarded as one fault*. However, the aforementioned cause of *ignored*–the c_{origin} is *removed* from the newly minimal schemas can only happen in the strategy *distinguishing faults*.

4.3. Summary of the masking effects on the FCI approach

From the analysis of the formal model, we can learn that masking effects do influence the FCI approaches, and even worse, both the *regarded as one fault* and *distinguishing faults* strategy are harmful. Specifically when compared with the *knowing masking effects* strategy, the former has a large possibility of getting more sub-schemas of the actual MFS and getting more schemas which are irrelevant to the MFS, while the latter may get more parent schemas of the MFS and can also get more irrelevant MFS. Further, both strategies can ignore the actual MFS and the *distinguishing faults* strategy is more likely to ignore the MFS than the *regarded as one fault* strategy.

Note that our discussion is based on a SUT using deterministic software, i.e., the random failing information of a test case will be ignored. The non-deterministic problem will result in a more complex test scenario, which will not be discussed in this paper.

5. TEST CASE REPLACING STRATEGY

The main reason why the FCI approach fails to properly work is that we cannot determine the areas B_2 and B_3, i.e., if the test case under test trigger other faults which is different from the current one, we cannot figure out whether this test case will trigger the current expected fault as the masking effects may prevent that. So to limit the impact of this effect on the FCI approach, we need to reduce the number of test cases that trigger other faults as much as possible.

In the exhaustive testing, as all the test cases will be used to identify the MFS, there is no room left to improve the performance without fixing the other faults and re-executing all the test cases. However, when you just need to select part of all the test cases to identify the MFS, which is how the traditional FCI approach works, we can adjust the test cases we need to use by selecting the proper ones, thereby limiting the size of $\mathcal{T}(mask_{F_m})$ to be as small as possible.

5.1. Replacing test cases that trigger unexpected faults

The basic idea is to pick the test cases that trigger other faults and generate new test cases to replace them. These regenerated test cases should either pass in the execution or trigger F_m. The replacement must satisfy the condition that the newly generated ones will not negatively influence the original identifying process.

Commonly, when we replace the test case that triggers an unexpected fault with a new test case, we should keep some part in the original test case. We call this part the *fixed part*, and mutate the other part with different values from the original one. For example, if a test case (1,1,1,1) triggered an unexpected fault, and the fixed part is (-,-,1,1). Then, we can replace it with a test case (0,0,1,1) which may either pass or trigger the expected fault.

The *fixed part* can vary for different FCI approaches, e.g, for the OFOT [Nie and Leung 2011a] algorithm, the factors are the fixed part except for the one that needs to be validated, while for the FIC_BS [Zhang and Zhang 2011] approach, we will fix the factors that should not be mutated for the test case in the next iteration of the FIC_BS process.

We note that this replacement may need to be executed multiple times for one fixed part as we could not always find a test case that by coincidence satisfied our requirement. One replacement method is randomly choosing test cases until the satisfied test case is found. While this method may be simple and really works, however, it also may require trying too many times to get the satisfied one. So to handle this problem and reduce the cost, we proposed a replacement approach by computing the *strength* of the test case with the other faults, and then we selected the test case from a group of candidate test cases that has the least *strength* related to the other faults.

To explain the *strength* notation, we need first to introduce the *strength* that a factor is related to a particular fault. We use $all(o)$ to represent the number of executed test cases that contain this factor, and $m(o)$ to indicate the number of test cases that trigger the fault F_m and contain this factor. Then, a the *strength* that a factor is related to a particular fault, i.e., $S(o, F_m)$, is $\frac{m(o)}{all(o)+1}$. This heuristic formulae is based on the idea that if a factor frequently appears in the test cases that trigger the particular fault, then it is more likely to be the inducing factor that triggers this type of fault. We add 1 in the denominator for two facts: (1) avoid division by zero when the factor has never appeared before, (2) reduce the bias when a factor rarely appears in the test set but by coincidence appears in a failed test case with a particular fault.

With this factor *strength*, we then define that the *strength* of a test case f is related to a particular fault F_m as:

$$S(f, F_m) = \frac{1}{k} \sum_{o \in f} S(o, F_m)$$

In this formulae, k is the number of factors in the test case f, o is the specific factor in f. This formulae computes the average *strength* of all the factors in the test case as the *strength* for this test case that is related to a particular fault. For a test case that is selected to be tested, we want that the ability of that test case to trigger another fault to be as small as possible. In practice, one test case can have different related *strength* to different faults, so we cannot always find a test case that has the least relating *strength* to all the faults when compared other test cases. With this in mind, our target changes to find a test case for which the maximal *strength* of the related fault among other faults is the least compared other test cases. Formally, we should choose a test case f, s.t.,

$$\min_{f \in R} \max_{m \leq L \& m \neq n} S(f, F_m) \tag{EQ1}$$

In this formulae, L is the number of all the faults, and n is the current analysed fault. R is the set of all the possible test cases that contain the $fixed$ part except those ones that have been tested. Obviously $|R| = \prod_{i \notin fixed}(v_i) - |t \mid t\ contains\ the\ fixed\ part\ \&\ t\ is\ tested|$.

We can further resolve this problem. Consider the test case we get – f satisfied the EQ1. Without loss of generality, we assume that the fault $F_k, k \neq n$ is the fault with which the test case f has the maximal related *strength* compared to the other faults. Then, a natural property for f is that any other test case f' which satisfies that fault F_k is the maximal related fault for this test case and must have $S(f, F_k) <= S(f', F_k)$. Formally, to get such a test case is to solve the following formulae:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & f \in R \\ & S(f, F_k) > S(f, F_i), \quad 1 \leq i \leq L \ \&\ i \neq k, n \end{aligned} \tag{EQ2}$$

With this formulae, to solve EQ1, we just need to find the particular fault F_k, such that the related *strength* between the test case f that satisfies EQ2 and this fault is the smallest than that of the other faults. Formally, we need to find:

$$\begin{aligned} \min \quad & S(f, F_k) \\ \text{s.t.} \quad & 1 \leq k \leq L \ \&\ k \neq n \\ & f, F_k\ satisfies\ EQ2 \end{aligned} \tag{EQ3}$$

According to EQ3, the problem to get such a test case lies in solving EQ2 because if EQ2 is solved we just need to rank the one that has the minimal value from the solutions to EQ2. As to EQ2, it can be formulated as an 0-1 integer linear programming (ILP) problem. Assume the SUT we test has K factors in which the ith factor has V_i

values it can take from. And the SUT has L faults. We then define the variable x_{ij} as:

$$x_{ij} = \begin{cases} 1 & the\ ith\ factor\ of\ the\ test\ case\ take\ the\ jth\ value\ for\ that\ factor \\ 0 & otherwise \end{cases}$$

We then take $o_{m_{ij}}$ to be the related *strength* between the *j*th value of the *i*th factor of the SUT and the fault F_m. And we use a set R of factors with its values to define the fixed part in the test case we should not change, i.e., $R = \{(i,j)|i\ is\ the\ fixed\ factor\ in\ the\ test\ case, j\ is\ the\ corresponding\ value\}$. As we can generate redundant test cases, so we keep a set of test cases $T_{executed}$ to guide to generate different test cases. Then EQ2 can be detailed as the in following ILP formulae:

$$\min \quad \frac{1}{|K|}\sum_{i=0}^{K}\sum_{j=0}^{V_i} o_{m_{ij}} \times x_{ij} \qquad \textbf{(EQ4)}$$

$$\begin{aligned}
\text{s.t.}\quad & 0 \le x_{ij} \le 1 && i = 0..K-1, j = 0,..V_i - 1 && (1)\\
& x_{ij} \in \mathbb{Z} && i = 0..K-1, j = 0,..V_i - 1 && (2)\\
& \sum_{j=0}^{V_j} x_{ij} = 1 && i = 0...K-1 && (3)\\
& x_{ij} = 1 && (i,j) \in R && (4)\\
& \sum_{i=0}^{K}\sum_{j=0}^{V_i}(o_{m_{ij}} - o_{m'_{ij}}) \times x_{ij} \ge 0 && 1 \le m' \ne m \le L && (5)\\
& \sum_{(i,j)\in t} x_{ij} < K && t \in T_{existed} && (6)
\end{aligned}$$

In this formulae, constraints (1) and (2) indicate that the variable x_{ij} is a 0-1 integer. Constraint (3) indicates that a factor in one test case can only take one value. Constraint (4) indicates the test case should not change values of the fixed part. Constraint (5) indicates that the related strength between Fault F_m and the test case is maximal than the others. Constraint (6) indicates the test cases generated should not be the same as the test cases in $T_{existed}$

As we have formulated the problem into a 0-1 integer programming problem, we just need to utilize an ILP solver to solve this formulae. In this paper, we use the solver introduced in [Berkelaar et al. 2004], which is a mixed Integer Linear Programming (MILP) solver that can handle satisfaction and optimization problems.

The complete process of replacing a test case with a new one while keeping some fixed part is depicted in Algorithm 1:

The inputs for this algorithm consist of the fault type we focus on – F_m, the fixed part of which we want to keep from the original test case – s_{fixed} , the values sets that each factor can take from respectively – $Param$ and the set of matrix $o_1, ...o_L$, for any element in which, say o_m, is recorded the related strength between each specific factor with each value and the fault F_m, i.e., $o_m = \{o_{m_{ij}}|0 \le i \le K-1, 0 \le j \le V_i\}$. The output of this algorithm is a test case t_{new} which either triggers the expected F_m or passes.

This algorithm is an outer loop (lines 1 - 19) containing two parts:

The first part (lines 2 - 9) generates a new test case which is supposed to be least likely to trigger faults different from F_m. The basic idea for this part is to search each fault different from F_m (line 3) and find the best test case that has the least related strength with other faults. In detail, for each fault we set up an ILP solver (line 4) and

ALGORITHM 1: Replacing test cases triggering unexpected faults

```
Input: fault type $F_m$,fixed part $s_{fixed}$, values set that each option can take $Param$, the related
       strength matrix $o_1...o_L$
Output: $t_{new}$ the regenerate test case,The frequency number
1  while not MeetEndCriteria() do
2      $optimal \leftarrow MAX$ ; $t_{new} \leftarrow null$ ;
3      forall the $F_k \in F_1, ...F_m, F_{m+1}...F_L$ do
4          $solver \leftarrow setup(s_{fixed}, Param, F_m, o_1...o_L)$;
5          $(optimal', t'_{new}) \leftarrow solver.getOptimalTest()$ ;
6          if $optimal' < optimal$ then
7              $t_{new} \leftarrow t'_{new}$;
8          end
9      end
10     $result \leftarrow execute(t_{new})$;
11     $updatetRelatedStrengthMatrix(t_{new})$ ;
12     if $result == PASS$ or $result == F_m$ then
13         return $t_{new}$;
14     else
15         continue;
16     end
17 end
18 return null
```

use it to get an optimal test case for that fault according to EQ4 (line 5). We compare the optimal value for each fault, and choose the one has less strength related to other faults (lines 6 - 9).

The second part is to check whether the newly generated test case is as expected (lines 10 - 16). We first execute the SUT under the newly generated test case (line 10) and update the related strength matrix ($o_1...o_L$) for each factor that is involved in this newly generated test case (line 11). We then check the execute result. If either the test case passes or triggers the same fault – F_m, we will get an satisfied test case (line 12), and we will directly return this test case (line 13). Otherwise, we will repeat the process, i.e., generate a new test case and check again (lines 14 - 15).

Note that this algorithm has another exit, besides we find an expected test case (line 12), which is when the function *MeetEndCriteria()* returns *true* (line 1). We didn't explicitly show what the function *MeetEndCriteria()* is like, because this is dependent on the computing resource and how accurate you want the identifying result to be. In detail, if you want to get a high quality result and you have enough computing resource, you can try many times to get the expected test case; otherwise, a relatively small number of attempts is recommended.

In this paper, we just set 3 as the greatest number of repeats for this function. When it ends with *MeetEndCriteria()* is true, we will return null(line 18), which means we cannot find an expected test case.

5.2. A case study using the replacement strategy

Suppose we have to test a system with eight parameters, each of which has three options. And when we execute the test case T_0 – (0 0 0 0 0 0 0 0), a failure–$e1$ is triggered. Next, we will use the FCI approach – FIC_BS [Zhang and Zhang 2011] with replacement strategy to identify the MFS for the $e1$. Furthermore, there are two more potential faults, $e2$ and $e3$, that may be triggered during the testing; and they will mask the desired fault $e1$. The process is shown in Figure 7. In this figure, there are two main column. The left main column indicates the executed test cases during testing as well

Table XV. The number of test cases each FCI approach needed to identify MFS

Method	**number of test cases to identify MFS**
Charles ELA	depends on the covering array
Martinez with safe value [Martínez et al. 2008; 2009]	$O(\hat{d}logk + \hat{d}^2)$
Martinez without safe values [Martínez et al. 2008; 2009]	$O(d^2 + dlogk + log^c k)$
Martinez' ELA [Martínez et al. 2008; 2009]	$O(ds^v logk)$
Shi SOFOT [Shi et al. 2005]	$O(k)$
Nie OFOT [Nie and Leung 2011a]	$O(k \times d)$
Ylimaz classification tree	depends on the covering array
FIC [Zhang and Zhang 2011]	$O(k)$
FIC_BS [Zhang and Zhang 2011]	$O(t(logk + 1) + 1)$
Ghandehari's suspicious based [Ghandehari et al. 2012]	depends on the number and size of MFS
TRT [Niu et al. 2013]	$O(d \times t \times logk + t^d)$

as the executed results, and each executed test case corresponds to a specific label, T_1 – T_8, at the left. The right main column lists the related strength matrix when a test case triggers $e2$ or $e3$. In detail, the matrix records the related strength between each factor (columns $O1$ – $O8$) for each value it can take (column v) with the unexpected fault (column F). The executed test case, which is in bold, indicates the one that triggers the other fault and should be replaced in the next iteration.

From Figure 7, for the test case that triggered $e2$ – (2 1 1 1 0 0 0 0) (in this case, the fixed part of the test case is (- - - - 0 0 0 0), in which the last four factors are the same as the original test case T_0), we generate the related matrix at left. Each element in this matrix is computed as the $\frac{m(o)}{all(o)+1}$; for example, for the $O7$ factor with value 0, we can find two test cases that contain this element, i.e., T_0 and T_1, so the all(o) is 2. And only one test case triggers the fault $e2$, which means m(o) = 1. So the final related strength between this factor with $e2$ is $\frac{1}{2+1} = 0.33$. All the related strength with $e3$ is labeled with a short slash as there is no test case triggering this fault in this iteration. After this matrix has been determined, we can obtain a optimal test case with the ILP solver, which is T_1'–(1 2 2 2 0 0 0 0), with its related strength 0.167, which is smaller than that of the others.

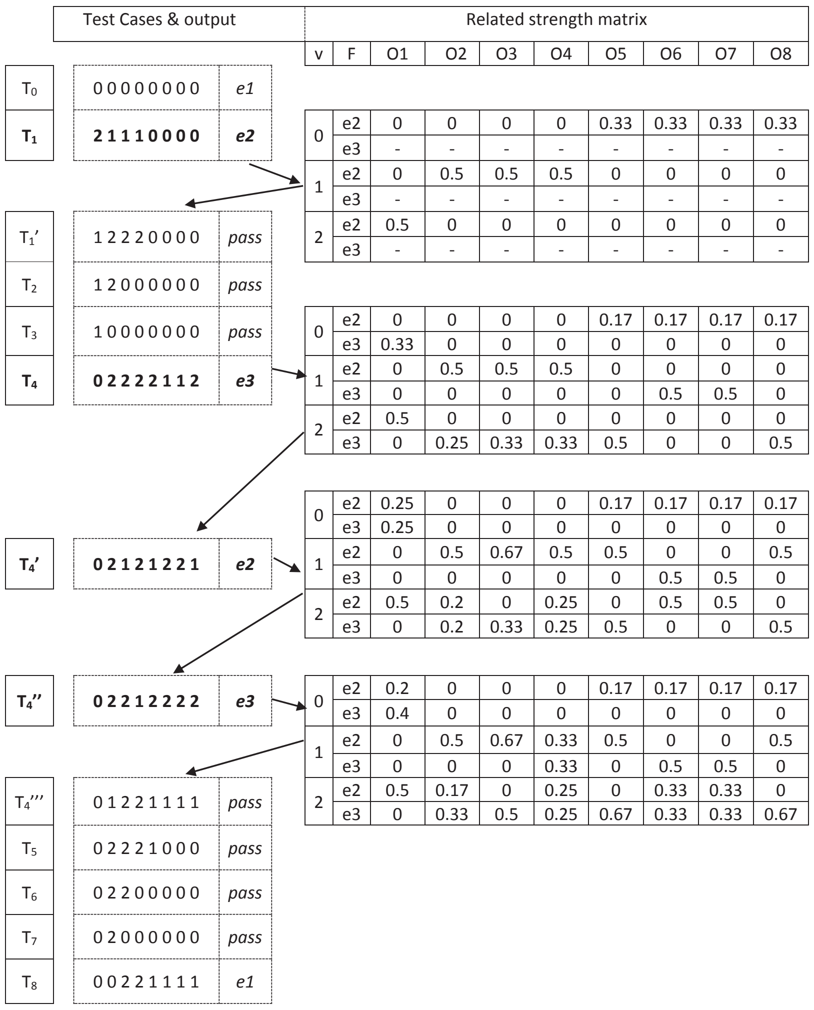
This replacement process trigged each time a new test case that triggered another fault until we finally get the MFS. Sometimes we could not find a satisfied replacing test case in just one trial like T_1 to T_1'. When this happened, we needed to repeat searching the proper test case we desired. For example, for T_4 which triggered $e3$, we tried three times– T_4', T_4'', T_4''' to finally get a satisfied one T_4''' which passes the testing. Note that the matrix continues to change with the test case generated and executed so that we can adaptively find an optimal one in the current process.

5.3. Complexity analysis

This complexity relies on two facts: the number of test cases that triggered other fault-s which need to be replaced, and the number of test cases that need to be tried to generate a non-masking-effects test case. The complexity is the product of these two facts.

The first fact is comparable to the extra test cases that are needed to identify the MFS, and this number varies in different FCI approaches. Table XV lists the number of test cases that each algorithm needed to get the MFS. In this table, *d* indicates the number of MFS in the SUT. *k* means the number of the parameters of the SUT. *t* is the number of MFS factors in the SUT. *c* is an upper bond, and satisfies $d \leq \frac{c}{2}loglogk$. *v* is the number of values one parameter can take.

It must be noted that each algorithm may be limited to some restrictions to identify the result, details of which are shown in [Zhang and Zhang 2011].



	Test Cases & output	
T_0	00000000	*e1*
T_1	**21110000**	***e2***
T_1'	12220000	*pass*
T_2	12000000	*pass*
T_3	10000000	*pass*
T_4	**02222112**	***e3***
T_4'	**02121221**	***e2***
T_4''	**02212222**	***e3***
T_4'''	01221111	*pass*
T_5	02221000	*pass*
T_6	02200000	*pass*
T_7	02000000	*pass*
T_8	00221111	*e1*

Related strength matrix

v	F	O1	O2	O3	O4	O5	O6	O7	O8
0	e2	0	0	0	0	0.33	0.33	0.33	0.33
	e3	-	-	-	-	-	-	-	-
1	e2	0	0.5	0.5	0.5	0	0	0	0
	e3	-	-	-	-	-	-	-	-
2	e2	0.5	0	0	0	0	0	0	0
	e3	-	-	-	-	-	-	-	-

v	F	O1	O2	O3	O4	O5	O6	O7	O8
0	e2	0	0	0	0	0.17	0.17	0.17	0.17
	e3	0.33	0	0	0	0	0	0	0
1	e2	0	0.5	0.5	0.5	0	0	0	0
	e3	0	0	0	0	0	0.5	0.5	0
2	e2	0.5	0	0	0	0	0	0	0
	e3	0	0.25	0.33	0.33	0.5	0	0	0.5

v	F	O1	O2	O3	O4	O5	O6	O7	O8
0	e2	0.25	0	0	0	0.17	0.17	0.17	0.17
	e3	0.25	0	0	0	0	0	0	0
1	e2	0	0.5	0.67	0.5	0.5	0	0	0.5
	e3	0	0	0	0	0	0.5	0.5	0
2	e2	0.5	0.2	0	0.25	0	0.5	0.5	0
	e3	0	0.2	0.33	0.25	0.5	0	0	0.5

v	F	O1	O2	O3	O4	O5	O6	O7	O8
0	e2	0.2	0	0	0	0.17	0.17	0.17	0.17
	e3	0.4	0	0	0	0	0	0	0
1	e2	0	0.5	0.67	0.33	0.5	0	0	0.5
	e3	0	0	0	0.33	0	0.5	0.5	0
2	e2	0.5	0.17	0	0.25	0	0.33	0.33	0
	e3	0	0.33	0.5	0.25	0.67	0.33	0.33	0.67

Fig. 7. A case study using our approach

To get the magnitude of the second fact, we need to figure out the possibility of a test case that could trigger other fault. The first thing we need to consider is the *fixed* part, as the additional generated test case should somehow contain this part. As we have mentioned before, we can generate $(v-1)^{k-p}$ (p is the number of factors in the *fixed* part) possible test cases that contain the *fixed* part. Apart from the one that needs to be replaced, there remain $(v-1)^{k-p}-1$ candidate test cases, which indicates the complexity is $O((v-1)^{k-p}-1)$. However, to avoid the exponential computational complexity, in this algorithm we use the method $MeetEndCriteria()$ (line 1) function to end the algorithm when the trying times is over a prior given constant, say N, so the final complexity for the second part is $O(min(N,(v-1)^{k-p}-1))$.

We note that exponential factor $k-p$ directly affects the complexity of the second factor. The greater p is, the less test cases that can be generated. For a different approach, p is different. For example, for the OFOT approach, p is a fixed number, which is $k-1$. And for the FIC_BS approach, the p varies in the test cases it generates, ranging from

Table XVI. The complexity of second part

Method	fixed part
Charles ELA	$O(min(N,(v-t)^{k-p}-1))$
Martinez with safe value [Martínez et al. 2008; 2009]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
Martinez without safe values [Martínez et al. 2008; 2009]	–
Martinez' ELA [Martínez et al. 2008; 2009]	$O(min(N,(v-t)^{k-p}-1))$
Shi SOFOT [Shi et al. 2005]	$O(min(N,(v-1)-1)))$
Nie OFOT [Nie and Leung 2011a]	$O(min(N,(v-1)-1)))$
Ylimaz classification tree	$O(min(N,(v-t)^{k-p}-1)$
FIC [Zhang and Zhang 2011]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
FIC_BS [Zhang and Zhang 2011]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$
Ghandehari's suspicious based [Ghandehari et al. 2012]	$O(min(N,(v-t)^{k-p}-1))$
TRT [Niu et al. 2013]	$O(min(N,(v-1)-1)) \sim O(min(N,(v-1)^{k-1}-1))$

$k-1$ to 1. While for the non-adaptive approaches, as the $fixed$ part is the coverage degree, we list them as t. We have listed all of them in Table XVI. It is noted that the $Martinezwithoutsafevalues$ has no such complexity, because this approach works when $v = 2$, and this results in not having other test cases to be replaced if we test a fixed part when triggering other faults.

6. EMPIRICAL STUDIES

To investigate the impact of masking effects for FCI approaches in real software testing scenarios and to evaluate the performance that how well our approach handles this effect, we conducted several empirical studies which we discuss in this section. Each of the studies focuses on addressing one particular issue, as follows:

Q1: Do masking effects exist in real software that contains multiple faults?

Q2: How well does our approach perform compared to traditional approaches?

Q3: Is the ILP-based test case searching technique efficient compared to random selection?

Q4: Compared to another masking effects handling approach – the FDA-CIT [Yilmaz et al. 2013], does our new approach have any advantages ?

6.1. The existence and characteristics of masking effects

In the first study, we surveyed two kinds of open-source software to gain an insight into the existence of multiple faults and their effects. The software under study were: HSQLDB and JFlex. The first is database management software written in pure Java and the second is a lexical analyser generator. Each contains different versions and are all highly configurable so that the options and their combinations can affect their behaviour. Additionally, they all have a developers' community so that we can easily obtain the real bugs reported in the bug tracker forum. Table XVII lists the program, the number of versions we surveyed, number of lines of uncommented code, number of classes in the project, and the bug's id [3]for each of the software we studied.

6.1.1. Study setup. We first looked through the bug tracker forum of each software and focused on the bugs which are caused by the options combination. For each such bug, we will derive its MFS by analysing the bug description report and the attached test file which can reproduce the bug. For example, through analysing the source code of the test file of bug#981 for HSQLDB, we found the failure-inducing combination for this bug is: (*preparestatement*, *placeHolder*, *Long string*). These three factors together form the condition that triggers the bug. The analysed results will be later regarded as the "prior MFS".

[3]http://sourceforge.net/p/hsqldb/bugs
http://sourceforge.net/p/jflex/bugs

Table XVII. Software under survey

software	versions	LOC	classes	bug pairs
HSQLDB	2.0rc8	139425	495	#981 & #1005
	2.2.5	156066	508	#1173 & #1179
	2.2.9	162784	525	#1286 & #1280
JFlex	1.4.1	10040	58	#87 & #80
	1.4.2	10745	61	#98 & #93

We further built the testing scenario for each version of the software listed in Table XVII. The testing scenario is properly constructed so that we can reproduce different faults by controlling the inputs to the test file. For each version of the software, the source code of the testing file as well as other detailed experiment information is available at– https://code.google.com/p/merging-bug-file.

Next, we built the input model which consists of the options related to the failure-inducing combinations and additional noise options. The detailed model information is in Tables XVIII and XIX for HSQLDB and JFLex, respectively. Each table is organised into four groups: (1)"common options", which lists the options as well as their values under which every version of this software can be tested; (2)"common Boolean options", which lists additional common options whose data type is Boolean; (3)"specific options", under which only the specific version of that software can be tested; and (4)"configure space", which depicts the input model for each version of the software, the input model is presented in the abbreviated form $\#values^{\#number\ of\ parameters} \times ...$, e.g., $2^9 \times 3^2 \times 4^1$ indicates the software has 9 parameters that can take 2 values, 2 parameters can take 3 values, and only one factor that can take 4 values.

Table XVIII. Input model of HSQLDB

common options		**values**
Server Type		server, webserver, inprocess
existed form		mem, file
resultSetTypes		forwad, insensitive, sensitive
resultSetConcurrencys		read_only, updatable
resultSetHoldabilitys		hold, close
StatementType		statement, prepared
common Boolean options		
sql.enforce_strict_size, sql.enforce_names, sql.enforce_refs		
versions	**specific options**	**values**
2.0rc8	more	true, false
	placeHolder	true, false
	cursorAction	next,previous,first,last
2.2.5	multiple	one, multi, default
	placeHolder	true, false
2.2.9	duplicate	dup, single, default
	default_commit	true, false
versions	**Config space**	
2.0rc8	$2^9 \times 3^2 \times 4^1$	
2.2.5	$2^8 \times 3^3$	
2.2.9	$2^8 \times 3^3$	

We then generated the exhaustive test suite consisting of all possible combinations of these options, and under each of them, we executed the prepared testing file. We recorded the output of each test case to observe whether there were test cases containing prior MFS that did not produce the corresponding bug.

Table XIX. Input model of JFlex

common options		**values**
generation		switch, table, pack
charset		default, 7bit, 8bit, 16bit
common boolean options		
public, apiprivate,cup,caseless,char,line,column,notunix, yyeof		
versions	**specific options**	**values**
1.4.1	hasReturn	has, non, default
	normal	true, false
1.4.2	lookAhead	one, multi, default
	type	true, false
	standalone	true, false
versions	**Config space**	
1.4.1	$2^{10} \times 3^2 \times 4^1$	
1.4.2	$2^{11} \times 3^2 \times 4^1$	

Table XX. Number of faults and their masking effects

software	versions	all tests	failure	masking
HSQLDB	2cr8	18432	4608	768
-	2.2.5	6912	3456	576
-	2.2.9	6912	3456	1728
JFlex	1.4.1	36864	24576	6144
-	1.4.2	73728	36864	6144

6.1.2. Results and discussion. Table XX lists the results of our survey. Column "all tests" give the total number of test cases we executed. Column "failure" indicate the number of test cases that failed during testing, and column "masking" indicates the number of test cases which triggered the masking effect.

We observed that for each version of the software under analysis that we listed in the Table XX, test cases with masking effects do exist, i.e., test cases containing MFS did not trigger the corresponding bug. In effect, there are about 768 out of 4608 test cases (16.7%) in hsqldb with 2rc8 version. This rate is about 16.7%, 50%, 25%, and 16.7%, respectively, for the remaining software versions, which is not trivial.

So the answer to **Q1** is that in practice, when SUT have multiple faults, masking effects do exist widely in the test cases.

6.2. Comparing our approach to traditional algorithms

In the second study, our aim was to compare the performance of our approach to traditional approaches in identifying MFS under the impact of masking effects. To conduct this study, we needed to apply the our approach and traditional algorithms to identify MFS in a group of software and evaluate their identifying results. The five prepared versions of software in Table XVII used as test objects are far from a general evaluation of such objects. However, to construct such real testing scenarios is time-consuming as we must carefully study the tutorial of that software as well as the bug tracker report. So to give a desirable result based on more testing objects, we then synthesize a number of such testing scenarios of which the characterizations, such as the number of factors, the number of faults, and the possible masking effects, are similar to that of the real software. In detail, we set the number of parameters k of the SUT to a range from 8 30. We limited the scale of the SUT to a relatively small size because we needed to exhaustively execute each possible test case of the SUT to select the failed test cases which we then fed into the FCI approach. We then randomly choose 10 such SUTs, and for each SUT we injected 2 to 5 different MFS that can mask each other. The degree of the MFS we injected ranged from 1 to 6.

Table XXI. The testing models used in the case study

software	Model	MFS& masking sequence
HSQLDB 2cr8	$2^9 \times 3^2 \times 4^1$	$(5_1, 6_0, 7_0) \rightarrow (5_1, 8_2, 9_2) = (5_1, 8_2, 9_1) \rightarrow (5_1, 8_3, 9_2) = (5_1, 8_3, 9_1)$
HSQLDB 2.2.5	$2^8 \times 3^3$	$(6_1, 7_0) \rightarrow (5_2)$
HSQLDB 2.2.9	$2^8 \times 3^3$	$(6_0) \rightarrow (0_1, 5_1, 7_0) = (0_0, 5_1, 7_0) \rightarrow (5_1, 7_0)$
JFlex 1.4.1	$2^{10} \times 3^2 \times 4^1$	$(0_0) \rightarrow (1_0)$
JFlex 1.4.2	$2^{11} \times 3^2 \times 4^1$	$(1_0, 2_1) \rightarrow (0_1)$
synthez 1	$2^5 \times 3^3 \times 4^1$	$(2_1, 3_0) \rightarrow (1_1, 2_1) = (1_0, 3_0)$
synthez 2	$2^6 \times 3^2 \times 4^1$	$(4_1, 6_0, 7_1, 8_0) \rightarrow (1_1, 3_1, 5_1) \rightarrow (2_0, 3_1, 6_0)$
synthez 3	$2^5 \times 3^3$	$(2_1, 3_0) \rightarrow (1_0) = (4_1) \rightarrow (6_0, 7_0)$
synthez 4	$2^7 \times 3^2 \times 4^1$	$(0_1, 2_1, 5_0, 6_1) \rightarrow (2_1, 4_0) = (6_1, 7_0) \rightarrow (3_0, 4_0, 5_0)$
synthez 5	$2^4 \times 3^3 \times 4^2$	$(0_0, 1_1, 3_0, 6_1, 8_0) \rightarrow (2_0, 3_0, 4_1)$
synthez 6	$2^9 \times 3^2$	$(2_0, 7_1, 8_1) \rightarrow (3_1, 5_1) = (4_0) \rightarrow (3_1, 6_0, 7_1) \rightarrow (3_1, 7_1, 8_0)$
synthez 7	$2^{10} \times 3^1 \times 4^1$	$(3_1, 4_0, 5_0) \rightarrow (2_0, 4_0, 7_1, 9_0) \rightarrow (6_1, 10_0, 11_1)$
synthez 8	$2^{11} \times 3^1 \times 4^1$	$(1_0, 3_1, 4_0, 7_1, 9_0, 12_1) \rightarrow (0_0, 2_1, 3_1, 7_1, 10_0, 11_1)$
synthez 9	$2^4 \times 4^3$	$(3_1, 5_0) \rightarrow (5_0, 6_1)$
synthez 10	$2^7 \times 3^3 \times 4^1$	$(0_1, 3_0, 4_1, 7_0) \rightarrow (2_0, 3_0, 5_1) = (2_0, 3_0, 5_0)$

Above all, Table XXI lists the testing model for both the real and synthesizing testing scenario. In this table, the column ‘software’ indicates the SUT under test. For the real SUT, we label it with the form ‘ *name + version*’, while for the synthesizing ones, we label them as ‘*synthez+ id*’. The column ‘Model’ presents the model of the input space for that software. The last column shows the MFS as well as the masking sequence for each testing object. The MFS is presented in an abbreviated form $\{\#index_{\#value}\}$, e.g., $(5_1, 6_0, 7_0)$ actually means (- - - - - 1, 0, 0, -, -, -, -) for HSQLDB of version ’2cr8’. It is noted that we use ’$\rightarrow$’ and ’=’ to describe the masking sequence of each MFS, in which ’$\rightarrow$’ means the left MFS in this operator can mask the right MFS of this operator, e.g., $(5_1, 6_0, 7_0) \rightarrow (5_1, 8_2, 9_2)$ means if $(5_1, 6_0, 7_0)$ appears in the test case, then $(5_1, 8_2, 9_2)$ will not be triggered. Operator ’=’ means that these two MFS will not mask each other.

6.2.1. Study setup. After preparing the subjects under testing, we then apply our approach (augment the FIC_BS with replacing strategy) to identify the MFS of each SUT listed in Table XXI. Specificcally, for each SUT we select each test case that failed during testing and feed these into our FCI approach as the input. Then, after the identifying process is over, we record the MFS each got (referred to as *identified MFS*, for convenience) and the extra test cases it needed. For the traditional FIC_BS approach, we designed the same experiment as hat used for our approach, but as the objects being tested have multiple faults for which the traditional FIC_BS can not be applied directly, we adopted two traditional strategies on the FIC_BS algorithm, i.e., *regarded as one fault* and *distinguishing faults* described in Section 3.2. The purpose of recording the generated additional test cases is to later quantify the additive cost of our approach.

We next compared the identified MFS of each approach with the prior MFS to quantify the degree that each suffers from masking effects, such that we can figure out how much our approach performs better than traditional ones when the SUT contains potential masking effects. There are five metrics we need to calculate in this study, which are as follows:

(1) The number of the common combinations that appear in both identified MFS and prior MFS. We denote this metric as *accurate number* later.

(2) The number of the identified combinations which are the parent combinations of some prior failure-inducing combinations. We refer this metric as the *parent number*.
(3) The number of the identified combinations that are the sub combinations of some prior failure-inducing combinations, which are referred to as the *sub number*.
(4) The number of ignored failure-inducing combinations. This metric counts these combinations in prior failure-inducing combinations, which are irrelevant to the identified combinations. We label the metric as *ignored number*.
(5) The number of irrelevant combinations. This metric counts the combinations in these identified combinations that are irrelevant to the prior failure-inducing combinations. It is referred to as the *irrelevant number*.

Among these five metrics, the high *accurate number* value indicates FCI approaches that perform effectively, while the *ignored number* and *irrelevant number* indicate the degree of deviation for the FCI approaches. The *parent number* and *sub number* indicate the FCI approaches that can determine part parameter values for the failure-inducing combinations, although with additional noisy information.

Besides these specific metrics, we also give a composite criteria to measure the overall performance of each approach. The computing formulae for the composite criteria is as follows:

$$\frac{accurate + related(parent) + related(sub)}{accurate + parent + sub + irrelevant + ignored}$$

In this formulae, *accurate, parent, sub, irrelevant,* and *ignored* represent the value of each specific metric. The *related* function gives the similarity between the schemas (either parent or sub) and the real MFS. The similarity between two schemas S_A and S_B is computed as:

$$Similarity(S_A, S_B) = \frac{the\ same\ elements\ in\ S_A\ and\ S_B}{\max(Degree(S_A), Degree(S_B))}$$

.

For example, the similarity of (- 1 2 - 3) and (- 2 2 - 3) is $\frac{2}{3}$. This is because the same elements of this two schemas are the third and last elements, and both of these two schemas are three-degree.

So the *related* function is the summation of similarity of all the parent or sub schemas with their corresponding MFS.

6.2.2. Results and discussion. Table XXII depicts the results of the second case study. There are eight main columns in this table, (we break it into two parts), which indicate: the object to which we apply the FCI approach, the number of accurate MFS each approach identified, the number of identified schemas which are the sub-schema parent-schema of some prior MFS, the number of ignored prior MFS, the number of identified schemas which are irrelevant to all the prior MFS, the metric which gives the overall evaluation of each approach, and the extra test cases each algorithm needed. For each main column, there are four sub-columns which depict the results for the FCI with *regarded as one fault* strategy, FCI with *distinguishing faults* strategy, FCI with replacement strategy based on ILP test case searching and FCI with replacement strategy based on randomly searching. The last one is discussed in the next case study.

We first observed that, the results of two traditional strategies, *regarded as one fault* and *distinguishing faults*, coincide with the formal analysis in section 4. Specifically, the former has more sub-schemas of MFS than the latter for all the 15 subjects, and the latter has more parent-schemas of MFS than the former. For the ‘ignored MFS’ metric, as we have executed all the failed test cases, we get 0 ignored MFS for all the

Table XXII. Result of the evaluation of each appraoch

Subject	accurate				sub				parent				ignore			
	One	Distin	ILP	Rand	One	Distin	ILP	Rand	One	Distin	ILP	Rand	One	Distin	ILP	Rand
HSQL2cr8	2	3	5	5	3	2	0	2	0	2	4	5	0(2.04)	0(3.91)	0(4.0)	0(4.1)
HSQL2.2.5	2	2	2	2	1	0	0	0	0	1	1	1	0(0.83)	0(1.0)	0(1.0)	0(1.0)
HSQL2.2.9	2	3	2	2	3	1	1	1	0	4	1	1	0(1.33)	0(2.83)	0(2.56)	0(2.56)
JFlex1.4.1	2	2	2	2	0	0	0	0	0	1	0	0	0(0.75)	0(1.0)	0(1.0)	0(1.0)
JFlex 1.4.2	2	2	2	2	1	0	0	0	0	1	1	1	0(0.67)	0(1.0)	0(1.0)	0(1.0)
synthez 1	2	1	1	1	2	0	0	0	0	2	2	2	0(1.0)	0(2.0)	0(2.0)	0(2.0)
synthez 2	3	3	3	3	10	0	0	0	0	10	6	6	0(1.96)	0(2.0)	0(2.0)	0(2.0)
synthez 3	4	3	3	3	4	2	2	2	0	5	5	5	0(2.08)	0(2.84)	0(2.84)	0(2.84)
synthez 4	3	3	3	3	10	3	2	3	0	6	5	5	0(2.6)	0(2.8)	0(2.9)	0(2.85)
synthez 5	2	2	2	2	4	0	0	0	0	2	1	1	0(1.02)	0(1.0)	0(1.0)	0(1.0)
synthez 6	2	3	3	3	15	4	4	4	0	8	8	8	0(1.99)	0(3.72)	0(3.72)	0(3.72)
synthez 7	3	3	3	3	10	0	0	0	0	6	6	6	0(2.04)	0(2.0)	0(2.0)	0(2.0)
synthez 8	2	2	2	2	4	0	0	9	0	4	3	3	0(1.05)	0(1.0)	0(1.0)	0(1.0)
synthez 9	2	1	1	1	1	0	0	0	0	1	1	1	0(0.8)	0(1.0)	0(1.0)	0(1.0)
synthez 10	0	1	1	1	3	1	1	1	0	0	0	0	0(1.0)	0(1.46)	0(1.31)	0(1.31)

Subject	irrelevant				overall				test cases			
	One	Distin	ILP	Rando	One	Distin	ILP	Rand	One	Distin	ILP	Rand
HSQL2cr8	0	2	0	34	0.57	0.65	0.88	0.23	8.125	11.92	17	17.72
HSQL2.2.5	7	0	0	0	0.25	0.83	0.83	0.83	8.67	7.67	10.17	11.3
HSQL2.2.9	4	0	0	0	0.4	0.74	0.8	0.8	9.167	8.61	11.72	13.14
JFlex1.4.1	25	0	0	0	0.07	0.83	1	1	23.5	6.5	8	9.68
JFlex 1.4.2	25	0	0	0	0.09	0.83	0.83	0.83	20.5	9	11.67	13.12
synthez 1	15	17	17	17	0.19	0.11	0.11	0.11	16.5	18	41.75	41.75
synthez 2	1	0	0	0	0.54	0.76	0.8	0.8	11.19	14.12	16.96	17.08
synthez 3	13	9	9	9	0.28	0.34	0.34	0.34	12.73	9.46	14.18	14.44
synthez 4	9	9	9	9	0.35	0.4	0.39	0.39	9.91	13.02	18.55	18.45
synthez 5	1	0	0	0	0.65	0.88	0.92	0.92	13.04	13.7	14.77	14.84
synthez 6	8	10	10	10	0.38	0.36	0.36	0.36	14.91	11.75	15.37	15.71
synthez 7	5	0	0	0	0.39	0.83	0.83	0.83	12.77	14.59	16.44	16.53
synthez 8	3	0	0	0	0.56	0.9	0.91	0.91	24.45	25.25	26.27	26.37
synthez 9	0	0	0	0	0.75	0.83	0.83	0.83	6.8	8	9	9
synthez 10	0	2	1	1	0.5	0.47	0.58	0.58	9.08	11	15.38	15.53

approaches. So to evaluate this metric for the approaches, we record it for each FCI one test case at a time and take the average value as the result, which is listed in parentheses. We also found in most cases (except synthez 5 and synthez 6) that the approach with *distinguish* strategy get more ignored MFS than the other, which also coincides with the formal analysis. And for the 'irrelevant MFS', we found *regarded as one fault* strategy in most cases got more 'irrelevant MFS' than the other, which was also as expected.

We then observed that our approach can perform better than the two traditional strategies. The advantages are shown in the more accurate MFS and less irrelevant schemas. Our approach also identified the least number of sub-schemas. As for the 'parent-schema' metric, our approach performed better than 'distinguishing faults' but not as well as 'regarded as one fault'. This is because our strategy is based on the 'distinguishing faults' strategy (the main difference is that our strategy filters the unsatisfied test cases). However, our approach is poor at the 'ignored MFS' metric. The possible reason for this may be that the unsatisfied test cases we discard may contain some useful information about the MFS. Above all, our approach achieves the best performance compared with the other two strategies, which can be shown in the 'overall' metric. The overall metric indicates that our approach performs better than 'distinguishing faults' strategy, which is better than the 'regarded as one fault' strategy.

We further observed that, the extra cost (of generating test cases) for our approach is acceptable. In fact, when compared to the *'distinguishing faults'* strategy, our approach needed an average of just 3 or 4 more test cases, and when compared to the *'regarded as one fault'* strategy, our approach needed less test cases in some instances (JFlex 1.4.1, JFlex 1.4.2, synthez 3 and synthez 6). This is because the extra test cases the FCI needed lies in what the MFS is, the difference at the identified MFS for FCI approaches will make their needed test cases differs greatly, so that the cost for replacing test cases in our approach may have little influence on the number of final test cases needed.

Therefore, the answer we got for **Q2** is that: our approach achieves better performance than two traditional strategies when handling masking effects at an acceptable extra cost.

6.3. Evaluating the ILP-based test case searching method

The third empirical study aims to evaluate the efficiency of the ILP-based test case searching component in our approach. To conduct this study, we implemented an FCI approach which is also augmented by the *replacing test cases* strategy, but the test case replacing process is by random.

6.3.1. Study setup. The setup of this case study is based on the second case study, and uses the same SUT model as in TableXXI. Then, we apply the new random searching based FCI approach to identify the MFS in these prepared SUTs. To avoid the bias that comes from the randomness, we repeat the new approach 30 times to identify the MFS in each failed test case. We will compute the average additional test cases as well as other metrics listed in the precise section of the random-based approach.

6.3.2. Results and discussion. The evaluation of this random-based approach is also shown in Table XXII. Compared to our ILP-based approach, we observed that there is little distinction between them in terms of the metrics: accurate schemas, parent-schemas, sub-schemas, ignored schemas, irrelevant schemas (the ILP-based approach performs slightly better, e.g., for subject HSQL2cr8, the ILP-based approach identified less sub, parent and irrelevant schemas than the random-based procedure). This is because the two approaches both use the *test case replacement* strategy, so when examining a schema, both of this two approaches may obtain the same result, although the test cases generated will be different. However, when considering the cost of each approach, we find the ILP-based approach performs better, which can reduce in the average to 1 or 2 test cases less than random-based procedure.

In summary, the answer for **Q3** is that: searching for a satisfied test case does have affect the performance of our approach, especially regarding the number of extra test cases, and the ILP-based test cases can handle the masking effects at a relatively smaller cost than the random-based approach.

6.4. Comparison with Feedback driven combinatorial testing

The FDA-CIT [Yilmaz et al. 2013] approach can handle masking effects so that the covering array it generates can cover all the t-way schemas without being masked by the MFS. There is an integrated FCI approach in the FDA-CIT, of which this FCI approach has two versions, i.e., ternary-class and multiple-class. In this paper, we use the multiple-class version for our comparative approach, as Yilmaz claims that the multiple-class version performs better than the former [Yilmaz et al. 2013].

6.4.1. Study setup. As the FCI approach of FDA-CIT use a post-analysis(classified tree) technique on given test cases, in this paper we fed the FDA-CIT the covering array as the input just as was done in the Yilmaz study [Yilmaz et al. 2013]. The covering arrays we generated ranged from 2 to 4 ways. The covering array generating method we used is that contained in [Cohen et al. 2003], as it can be easily extended with constraint dealing and seed injecting [Cohen et al. 2007b], which is needed in the FDA-CIT process. As different test cases will influence the results of the characterization process, we generating 30 different 2-4 way covering arrays and fed them into the FDA-CIT. Then, we recorded the results of this approach, which consists of the metrics mentioned in the second case study.

Besides the FDA-CIT, we also applied our ILP-based approach to the generated covering array. Specifically, for each failed test case in the covering array, we separately applied our approach to identify the MFS in that case. In fact, we can reduce the number of extra test cases if we utilize the other test cases in the covering array [Li et al. 2012]), but we didn't utilize the information to simplify the experiment. We then merged all the test cases that our approach needed for each failed test case in the covering array, and we also merged other metrics listed in the second case study for each failed test case.

As our approach generated different test cases from the FDA-CIT, we also used the multiple-class FCI approach of FDA-CIT to characterize the MFS using the test cases generated by our approach, so that we could obtain a fair result with which to evaluate the FCI approach.

6.4.2. Result and discussion. We list the average result of the 30 times experiment for the FDA-CIT, ILP-based approach, and FDA-CIT using our test cases (FDAs), respectively, in Table XXIII. The result is organised as in Table XXII, except that we added a column *t* which indicates the strength of the covering array we generated for this experiment.

Table XXIII: Comparison with FDA-CIT

Subject		accurate			sub			parent			ignore		
	t	FDA	ILP	FDA-s	FDA	ILP	FDA-s	FDA	ILP	FDA-s	FDA	ILP	FDA-s
HSQL2cr8	2	0.17	2.27	1.57	0.57	0	0	0.17	0.4	2.17	3.87	2.3	2
	3	1.47	3.67	1	0	0	0	4.67	2	6.07	0.63	0.3	0.17
	4	0.83	4.8	1	0	0	0	9.03	3.37	8	0	0	0
HSQL2.2.5	2	1	1.97	0.37	0	0	0	2.4	0.73	3.8	0.4	0	0
	3	0	2	0.4	0	0	0	5	1	3.8	0	0	0
	4	0	2	0.33	0	0	0	5	1	4	0	0	0
HSQL2.2.9	2	0.9	1.77	0.9	0	0.77	9	1.47	0.47	6.8	1.93	0.53	0
	3	1	2	0.83	0	1	0	5.13	0.93	7.1	0.2	0	0
	4	1	2	1	0	1	0	5.87	1	6.7	0	0	0
JFlex 1.4.1	2	0	2	0	0	0	0	4.03	0	4	0	0	0
	3	0	2	0	0	0	0	4	0	0	0	0	4
	4	0	2	0	0	0	0	4	0	0	0	0	0
JFlex 1.4.2	2	0.3	1.97	0.93	0	0	0	3.6	1	2.16	0.03	0	0
	3	0	2	0.97	0	0	0	5	1	2.1	0	0	0

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	4	0	2	1	0	0	0	5	1	2	0	0	0
synthez 1	2	0.97	1	1	0	0	0	1.7	1.93	2	0	0.07	0
	3	1	1	1	0	0	0	2	2	2	0	0	0
	4	1	1	1	0	0	0	2	2	2	0	0	0
synthez 2	2	0.17	1.3	0.73	0.37	0	0	0	0.4	2.37	2.27	1.2	1.03
	3	0.73	2.23	0.5	0	0	0	1.9	1.3	7.1	1.2	0.43	0.53
	4	0.63	2.97	0.1	0	0	0	5.3	2.33	16.1	0.53	0	0
synthez 3	2	0.43	2.97	0.73	0	0.93	0	4.3	1.73	5.3	0.47	0.17	0.5
	3	0.2	3	0.87	0	1.57	0	7.2	3.67	6.57	0.07	0	0
	4	0.03	3	1	0	1.97	0	10.4	3	6	0	0	0
synthez 4	2	0.3	2.3	0.33	0.07	0.63	0	2.63	1.97	7.7	1.93	0.63	0.4
	3	0.37	2.97	0.07	0	1.26	0	6.5	3.53	10.97	0.83	0.07	0
	4	0.07	3	0	0	1.77	0	11.7	4.67	11.4	0	0	0
synthez 5	2	0.2	1.2	0.8	0.3	0	0	0.1	0.03	0.83	1.4	0.77	0.97
	3	0.87	1.4	0.53	0	0	0	0.5	0.23	3.03	1	0.6	0.77
	4	0.7	1.9	0.37	0	0	0	1.77	0.33	6.5	0.9	0.1	0.03
synthez 6	2	0.23	2.63	0.17	0.2	2	0	2.93	1.63	9.63	2.6	0.5	0.4
	3	0.1	3	0.1	0	2.83	0	7.4	3.83	12.5	1.2	0.17	0.03
	4	0	3	0	0	3.8	0	10.2	6.03	14.5	0.47	0	0
synthez 7	2	0.13	1.43	0.83	0.23	0	0	0.1	0.63	1.4	2.53	1.03	0.93
	3	0.87	2.17	0.93	0	0	0	0.43	1.23	2.97	1.77	0.17	0.13
	4	1	2.87	1	0	0	0	3.23	2.53	4.6	0.27	0	0
synthez 8	2	0	0.2	0.17	0.03	0	0	0	0	0.03	0.3	0.13	0.13
	3	0	0.6	0.5	0.1	0	0	0	0	0.03	0.97	0.47	0.53
	4	0	1.33	0.8	0.1	0	0	0	0.07	0.67	1.53	0.4	0.5
synthez 9	2	1	1	1	0	0	0	0.46	0.6	0.77	0.53	0	0.23
	3	1	1	1	0	0	0	1	1	1	0	0	0
	4	1	1	1	0	0	0	1	1	1	0	0	0
synthez10	2	0	0.63	0	0.6	1	0.2	0	0	0.83	1.8	0.37	1.9
	3	0.07	0.97	0	0.23	1	0.03	0.36	0	1.9	2.23	0.03	1.97
	4	0	1	0	0.07	1	0	1.7	0	2	1.87	0	2

	irrelevant			overall			test cases		
t	FDA	ILP	FDA-s	FDA	ILP	FDA-s	FDA	ILP	FDA-s
2	2.53	0	1.97	0.12	0.51	0.39	23.6	70.1	70.1
3	3	0	1.47	0.51	0.87	0.6	76.6	241.8	241.8
4	0.97	0	0	0.65	0.9	0.71	183.5	606.6	606.6
2	1.4	0	0.1	0.38	0.87	0.56	26.7	68.8	68.8
3	0	0	0	0.52	0.83	0.56	67	202.4	202.4
4	0	0	0	0.53	0.83	0.56	130.1	503.3	503.3
2	2.37	0	0.2	0.28	0.72	0.58	29.2	78.3	78.3
3	0.1	0	0	0.61	0.8	0.61	72.8	221.7	221.7
4	0	0	0	0.64	0.8	0.62	129.8	560.3	560.3
2	0	0	0	0.49	1	0.5	30.5	87.3	87.3
3	0	0	0	0.5	1	0.5	73.4	269.2	269.2
4	0	0	0	0.5	1	0.5	190.6	724.7	724.7
2	0.63	0	0	0.5	0.83	0.62	34.3	106.9	106.9

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3	0.03	0	0	0.52	0.83	0.61	72.3	305.7	305.7
4	0	0	0	0.53	0.83	0.61	186.8	836.9	836.9
2	0.33	14.3	0	0.66	0.13	0.78	40.3	342.87	342.87
3	0	16.73	0	0.78	0.12	0.78	93.4	809.1	809.1
4	0	17	0	0.78	0.12	0.78	218.8	1532.8	1532.8
2	1.37	0	1.2	0.11	0.52	0.4	19.77	54.4	54.4
3	2.2	0	1.33	0.36	0.82	0.52	59.5	171.5	171.5
4	2.6	0	1	0.44	0.89	0.54	152.7	415.1	415.1
2	1.03	3.77	1.13	0.37	0.46	0.37	48.6	138.7	138.7
3	0.83	6.77	0.07	0.38	0.38	0.44	106.3	315.3	315.3
4	0.43	8.56	0	0.38	0.34	0.45	147.9	565.7	565.7
2	3.4	1.4	1.97	0.24	0.6	0.44	42.7	142.2	142.2
3	2.5	3.43	1.03	0.39	0.54	0.51	86.5	373.2	373.2
4	1.33	6.73	0.03	0.48	0.44	0.55	202.2	899.7	899.7
2	0.7	0	1	0.2	0.59	0.4	21.9	46.9	46.9
3	0.37	0	1.63	0.46	0.71	0.43	76.9	150.3	150.3
4	1.87	0	2.03	0.34	0.92	0.54	232.9	433.2	433.2
2	3.03	3.7	2	0.19	0.42	0.37	45.7	132.6	132.6
3	2.3	6.5	0.67	0.31	0.38	0.43	99.5	338.9	338.9
4	1.8	9.1	0.03	0.37	0.36	0.44	152.6	781.9	781.9
2	1.93	0	1.97	0.09	0.61	0.38	20.3	58.8	58.8
3	3.2	0	2.87	0.2	0.88	0.44	52.6	164.7	164.7
4	4.5	0	2.27	0.35	0.9	0.51	145.3	413.1	413.1
2	0.17	0	0.3	0.01	0.1	0.05	16.1	45.2	45.2
3	0.63	0	0.87	0.02	0.3	0.17	43.1	64.3	64.3
4	1.4	0	0.93	0.04	0.67	0.41	109.3	145.6	145.6
2	0.63	0.6	0.67	0.54	0.7	0.6	36.2	43.4	43.4
3	0	0	0	0.83	0.83	0.83	84.3	145	145
4	0	0	0	0.83	0.83	0.83	188	291.6	291.6
2	1.5	0.3	3.97	0.23	0.61	0.17	23.4	84.9	84.9
3	4.03	0.53	3.97	0.13	0.66	0.2	73.4	263.2	263.2
4	4.03	1	4	0.21	0.58	0.2	202.2	685.9	685.9

From this result, we can first observe that in all the cases our ILP-based approach can more accurately identify the MFS and ignored less MFS than the FDA-CIT approach. For the metric 'parent-schema', 'sub-schema', and 'irrelevant schemas', there are ups and downs on both sides. With respect to the 'overall metric', we find our approach has a significant advantage over the FDA-CIT, but it also requires many more test cases than FDA-CIT.

We note that, when applying the multiple-class FCI to characterize the MFS with the test cases that were generated using our approach, their 'overall' metric is still not as good as our ILP-based approach, but may show some improvement over the original FDA-CIT.

Another interesting observation is that overall performance in most cases is increasing with the t, which can be easily understood. More test cases will contain more information about the MFS, so that we can utilize them to identify the MFS more precisely.

So the answer for **Q4** is that: our approach can achieved a more precise result for the MFS, and the FDA-CIT can perform the identifying process using a small amount extra test cases. Both of these two approaches have their ups and downs, choosing which approach in practice will depend on the specific scenario you test.

6.5. Threats to validity

There are several threats to the validity for these empirical studies. First, we have only surveyed two types of open-source software with five different versions, of which the program scale is medium-sized. This may impact the generality of our observations. Although we believe it is quite possibly a common phenomenon in most software that contains multiple faults which can mask each other, we need to investigate more software to support our belief.

The second threat comes from the input model we built. As we focused on the options related to the perfect combinations and only augmented it with some noise options, there is a chance we will get different results if we choose other noise options. More options needed to be tested to see whether our result is common or just appears in some particular input models.

The third threat is that we just evaluated one FCI approach – FIC_BS [Zhang and Zhang 2011], so further works needed to examine more algorithms in this field to obtain a more general result.

7. RELATED WORKS

Shi and Nie presented a further testing strategy for fault revealing and failure diagnosis[Shi et al. 2005], which first tests the SUT with a covering array, then reduces the value schemas contained in the failed test case by eliminating those appearing in the passed test cases. If the failure-causing schema is found in the reduced schema set, failure diagnosis is completed with the identification of the specific input values which caused the failure; otherwise, a further test suite based on SOFOT is developed for each failed test cases,testing is repeated, and the schema set is then further reduced, until no more failures are found or the fault is located. Based on this work, Wang proposed an AIFL approach which extended the SOFOT process by mutating the changed strength in each iteration that characterized failure-inducing combinations[Wang et al. 2010].

Nie et al. introduced the notion of Minimal Failure-causing Schema(MFS) and proposed the OFOT approach which is an extension of SOFOT that can isolate the MFS in SUT[Nie and Leung 2011a]. The approach mutates one value with different values for that parameter, hence generating a group of additional test cases each time to be executed. Compared with SOFOT, this approach strengthen the validation of the factor under analysis and also can detect the newly imported faulty combinations.

Delta debugging proposed by Zeller [Zeller and Hildebrandt 2002] is an adaptive divide-and-conquer approach to locate interaction fault. It is very efficient and has been applied in real software environment. Zhang et al. also proposed a similar approach that can efficiently identify the failure-inducing combinations that has no overlapped part [Zhang and Zhang 2011]. Later, Li improved the delta-debugging based failure-inducing combination by exploiting useful information in the executed covering array[Li et al. 2012].

Colbourn and McClary proposed a non-adaptive method [Colbourn and McClary 2008]. Their approach extends the covering array to the locating array to detect and locate interaction faults. C. Martinez proposed two adaptive algorithms. The first one requires safe value as the assumption and the second one remove the assumption when the number of values of each parameter is equal to 2[Martínez et al. 2008; 2009]. Their algorithms focus on identifying faulty tuples that have no more than 2 parameters.

Ghandehari et al. defined the suspiciousness of tuple and suspiciousness of the environment of a tuple[Ghandehari et al. 2012]. Based on this, they rank the possible tuples and generate the test configurations. They further utilized the test cases generated from the inducing combination to locate the faults inside the source code[Ghandehari et al. 2013].

Yilmaz proposed a machine learning method to identify inducing combinations from a combinatorial testing set [Yilmaz et al. 2006]. They constructed a classified tree to analyze the covering arrays and detect potential faulty combinations. Beside this, Fouché [Fouché et al. 2009] and Shakya [Shakya et al. 2012] made some improvements in identifying failure-inducing combinations based on Yilmaz's work.

Our previous work [Niu et al. 2013] proposed an approach that utilizes the tuple relationship tree to isolate the failure-inducing combinations in a failed test case. One novelty of this approach is that it can identify the overlapped faulty combinations. This work also alleviates the problem of introducing new failure-inducing combinations in additional test cases.

In addition to the studies that aim at identifying the failure-inducing combinations in test cases, there are others that focus on working around the masking effects.

With having known masking effects in prior, Cohen [Cohen et al. 2007a; 2007b; 2008] studied the impact of the masking effects that render some generated test cases invalid in CT. They also proposed an approach that integrates the incremental SAT solver with the covering array generation algorithm to avoid masking effects. Further study was conducted [Petke et al. 2013]to show that with considering constrains, higher-strength covering arrays with early fault detection are practical. Besides, additional constraints impacting CT were studied in the following works: [Garvin et al. 2011; Bryce and Colbourn 2006; Calvagna and Gargantini 2008; Grindal et al. 2006; Yilmaz 2013].

Chen et al. addressed the issues of shielding parameters in combinatorial testing and proposed the Mixed Covering Array with Shielding Parameters (MCAS) to solve the problem caused by shielding parameters[Chen et al. 2010]. The shielding parameters can disable some parameter values to expose additional interaction errors, which can be regarded as a special case of masking effects.

Dumlu and Yilmaz proposed a feedback-driven approach to work around the masking effects[Dumlu et al. 2011]. Specifically, they first used CTA to classify the possible failure-inducing combinations and then eliminate them and generate new test cases to detect possible masked interaction in the next iteration. They further extended their work [Yilmaz et al. 2013] by proposing a multiple-class CTA approach to distinguishing faults in SUT. In addition, they empirically studied the impacts on both ternary-class and multiple-class CTA approaches.

Our work differs from these mainly in that we formally studied the masking effects on FCI approaches and further proposed a divide-and-conquer strategy to alleviate this impact.

8. CONCLUSIONS

Masking effects of multiple faults in SUT can bias the results of traditional failure-inducing combinations identifying approaches. In this paper, we formally analysed the impact of masking effects on FCI approaches and showed that the two traditional strategies are both inefficient in handling such impact. We further presented a divide-and-conquer strategy for FCI approaches to alleviate such impact.

In our empirical studies, we extended FCI approach – FIC_BS[Zhang and Zhang 2011] with our strategy. The comparison between our approach and traditional approaches was performed on several kinds of open-source software. The results indicated that our strategy assists the traditional FCI approach in achieving better performance when facing masking effects in SUT, we also empirically evaluated the efficiency of the test case searching component by comparing it with the random searching based FCI approach. The results showed that the ILP-based test case searching method can perform much more efficiently. Last, we compared our approach with existing technique for handling masking effects – FDA-CIT[Yilmaz et al. 2013], and observed that our approach achieved a more precise result which can support better debugging aids, though our approach generated more test cases than FDA-CIT.

As for the future work, we need to do more empirical studies to make our conclusions more general. Our current experiments focus on middle-sized software. We would like to extend our approach to more complicated, large-scaled testing scenarios. Another promising work in the future is to combine the white-box testing technique to facilitate obtaining more accurate results from the FCI approaches when handling masking effects. We believe that figuring out the fault levels of different bugs through the white-box testing technique is helpful to reduce misjudgement in the failure-inducing combinations identifying process. And last, because the extent to which the FCI suffers from masking effects varies with different algorithms, a combination of different FCI approaches would be desired in the future to further improve identifying MFS for multiple faults.

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