1. I captured 10 images of vehicles queued at the ramp lights of SH1 Greenlane Interchange between 4:20 pm and 4:33 pm. The ramp road has two lanes, one that lets vehicles from the Greenlane East roundabout enter the system and the other from the Greenlane East Road. **At peak times the ramp road often gets congested leading to increased traffic in the Greenlane East Road and the roundabout** as the rate at which the vehicles arrive at the ramp is greater than the rate at which they are served.

(a) Queuing theory provides a mathematical framework for analysing and predicting the behaviour of queues, such as waiting lines. In this scenario, we can apply queuing theory to model the number of cars waiting at the ramp lights, analyze factors affecting the queue length, and optimize traffic flow by adjusting the parameters.

To model this type of data using queuing theory, we utilize parameters such as :

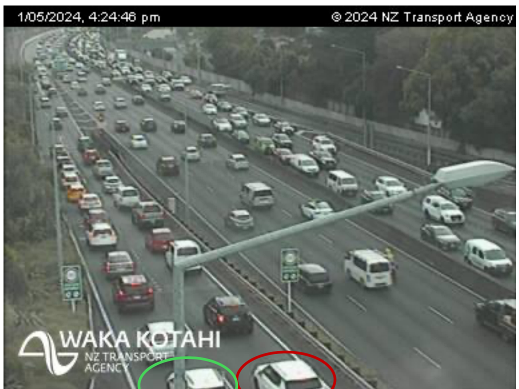
- Arrival Rate (λ): The rate at which vehicles arrive at the ramp lights.
- Service Rate (μ): The rate at which vehicles are served and proceed through the ramp lights.
- Queue Length: The number of vehicles waiting at the ramp lights at any given time.
- Service Time: The time it takes for a vehicle to proceed through the ramp lights once it begins moving.
- Utilization Factor (ρ): The ratio of the arrival rate to the service rate ($\rho = \lambda/\mu$).

To collect data for modelling the queuing system at the ramp lights, I refreshed the website periodically to obtain snapshots with relatively consistent intervals between them. Since the website updates the snapshots randomly, the time gaps between the images vary. However, we can observe some vehicles repeating in 2 consecutive pictures with shorter time intervals between them. Based on their position in the queue and the time gap between the these snaps, we can calculate the arrival and service rates. The queue length during each interval is also calculated by counting the number of vehicles. I have calculated the parameters separately for both lanes. This is because I have taken **two independent queues having their respective serving points**. As the vehicles enter their respective queues from different locations they do not have a choice in the selection of queues. This approach allows us to understand the vehicle flow at the ramp lights and reduce congestion during peak times more precisely.

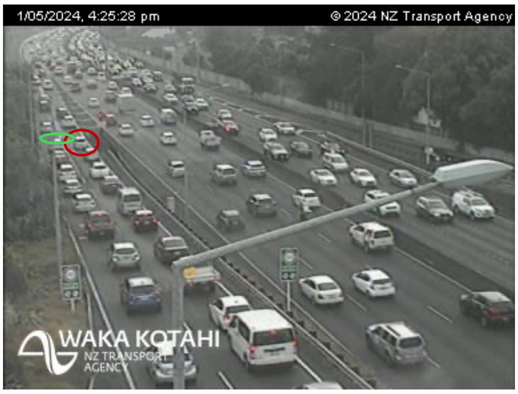
(b) Snapshots of the vehicles at the ramp in queue along with the number of vehicles in each lane. The left lane is taken as lane 1 and the right one as lane 2. The circles in green show the vehicles spotted in consecutive images in Lane 1 and the one in red show the vehicles spotted in consecutive images in Lane 2.

TIME	TIME INTERVAL	Number of vehicles in Lane 1	Number of vehicles in Lane 1
4:23:19 PM		12	10
4:24:46 PM	00:01:27	12	10
4:25:28 PM	00:00:42	9	11
4:26:53 PM	00:01:25	11	11
4:27:35 PM	00:00:42	11	11
4:28:18 PM	00:00:43	9	9
4:29:43 PM	00:01:25	10	11
4:30:30 PM	00:00:47	10	11
4:31:17 PM	00:00:47	9	10
4:32:41 PM	00:01:24	9	10

LANE 1: 12
LANE 2: 10

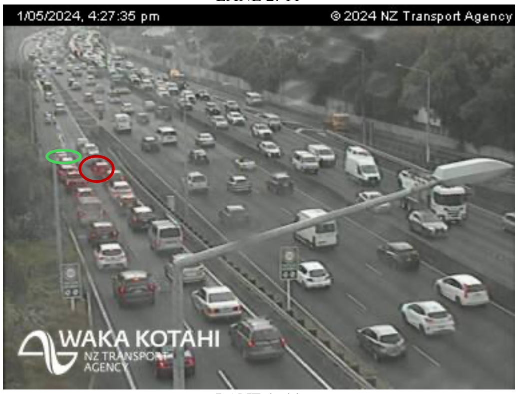


LANE 1: 12
LANE 2: 10



LANE 1:9
LANE 2: 11

LANE 1: 11
LANE 2: 11



LANE 1: 11
LANE 2: 11

LANE 1: 9
LANE 2: 9

LANE 1: 10
LANE 2: 11

LANE 1: 10
LANE 2: 11

LANE 1: 9
LANE 2: 10

LANE 1: 9
LANE 2: 10

Assumptions:

- The duration of red and green light remains constant between the taken time period: 4:20 pm and 4:33 pm .
- The arrival and service processes are Poisson.
- Available infinite calling population
- Service time is exponentially distributed

Calculation of parameters

- From the 4th and 5th snaps, in LANE 1 we can see that the vehicle in the 11th position of the vehicle of the 4th snap is in the 2nd position of the 4th snap. Thus, we can say that the 9 vehicles ahead of it were served. Number of new vehicles behind it counts to 10, which is the newly arrived vehicles.
  Therefore, we can say that in **42 seconds 9 vehicles were served and 10 vehicles arrived.**
- Similarlly in LANE 2, we can see that the vehicle in the 11th position of the vehicle of the 4th snap is in the 3rd position of the 4th snap. Thus, we can say that the 9 vehicles ahead of it were served. Number of new vehicles behind it counts to 8, which is the newly arrived vehicles.
  Therefore, we can say that in **42 seconds 9 vehicles were served and 8 vehicles arrived.**
- Similarly from 2nd and 3rd snap we can say for lane 1, in **42 seconds 9 vehicles were served and 9 vehicles arrived** and for lane 2, in **42 seconds 9 vehicles were served and 10 vehicles arrived.** Likewise from snap 8 and 9 we can say that, for lane 1 in **47 seconds 10 vehicles were served and 11 vehicles arrived** and for lane 2 in **47 seconds 10 vehicles were served and 9 vehicles arrived .**
- Here the inter-arrival times and the service times are distributed exponentially. According to the property of exponential distribution which says that the minimum of independent exponentially distributed random variables is exponentially distributed we can say that Arrivals with exponentially distributed interarrival times from n different sources, each with arrival rates $\lambda_{1,}\, \lambda_{2,}\, \lambda_{3,} \dots \dots \lambda_n$, can be considered as a single homogeneous process with exponentially distributed interarrival times and an overall arrival rate of $\boldsymbol{\lambda_= \lambda_1 + \lambda_2 + \cdots .. \lambda_n}$, .Similarly, a service facility with n servers operating in parallel, each with exponentially distributed service times and rates $\mu_{1,}\, \mu_{2,}\, \mu_{3,} \dots \dots \mu_n$,, can be modeled as a single server with an overall service rate of $\boldsymbol{\mu_= \mu_1 + \mu_2 + \cdots .. \mu_n}$.

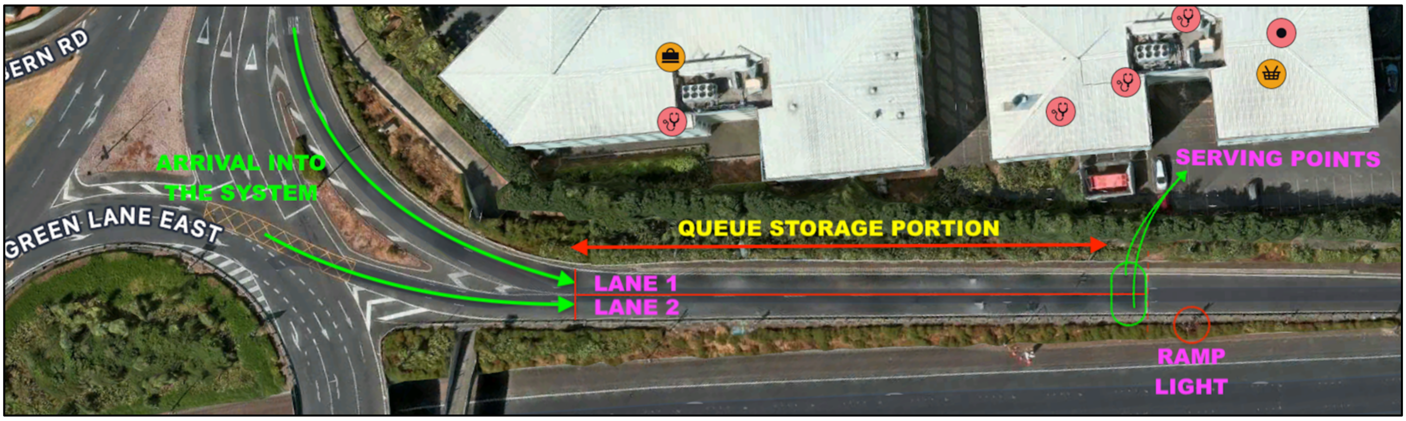
The parameters required to model the system can be calculated by the following equations with the values obtained from the data.

- Arrival Rate (λ): Mean($\frac{Number\ of\ vehicles\ arrived}{time\ Interval}$)
  - $\lambda_1 = \frac{\left(\frac{10}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3}$ = 0.22
  - $\lambda_2 = \frac{\left(\frac{8}{42}+\frac{10}{42}+\frac{11}{47}\right)}{3}$ = 0.22
  - $\boldsymbol{\lambda = 0.44}$ vehicles per second
- Inter- arrival time= 1/0.22= 4.5 seconds per vehicle
- Service Rate (μ): Mean($\frac{Number\ of\ vehicles\ served}{time\ Interval}$)
  - $\mu_1 = \frac{\left(\frac{9}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3}$ = 0.21
  - $\mu_2 = \frac{\left(\frac{9}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3}$ = 0.21
  - $\boldsymbol{\mu = 0.42}$ vehicles per second
- Average Service Time: $\frac{1}{\mu}$ = 1/0.21= 4.76 seconds per vehicle
- Utilization Factor (ρ): $\frac{\lambda}{\mu} = \frac{0.44}{0.42}$ = **1.0476** vehicles per second

2.

(a) **QUEUING PROCESS**

- The **calling population** : The calling population refers to the vehicles entering the queuing system. In this scenario, the calling population is considered **infinite**, as there is an endless supply of vehicles arriving from the Greenlane roundabout and Greenlane East Road at the peak time.
- The **arrival process**: The vehicles from the roundabout arrive into Lane 2 and the other to Lane 1. The vehicles arrive into lanes with an average inter-arrival time of approximately 4.5. The number of arrivals in any given time period follows a Poisson distribution. Given that vehicles enter their respective lanes from different locations and do not have the option to change lanes once queued, this setup simplifies the analysis as each lane operates independently with its own arrival rate. The Poisson process is suitable here because the arrivals are random and memoryless.
- There are **2 queues** with respective serving points, **infinite** length and the **queue discipline is FIFO** (First-In-First-Out), where vehicles are served in the order they arrive. This principle ensures fairness and predictability in the queue management.
- The job of this queuing model is to stop the vehicle at red light and pass on green. **1 vehicle from each lane is set to depart from the ramp on every green light.**
- The **service mechanism**: Here the **server is defined as the position where the first vehicle in each lane waits to be served**. The queue consists of the second and subsequent positions behind the server. The ramp light regulates the flow of vehicles departing from the ramp.
- The below diagram shows the queuing process. (Ref: Image obtained from iMaps)



(b) **QUEUE CONFIGURATION:**

- The ramp road has two queues corresponding to its two lanes. One lane serves vehicles coming from the Greenlane East roundabout, and the other serves vehicles coming from Greenlane East Road.
- The server is the position where the first vehicle in each lane waits to be served or to enter the main road. The queue starts at the second position in each lane, directly behind the server. The queue includes all subsequent vehicles waiting in line to be served.
- The queues begin from the second position in each lane, directly behind the server. They extend backwards along their respective lanes towards the Greenlane East roundabout and Greenlane East Road.
- Vehicles do not have the option to choose between the two queues as their entry into a specific lane is determined by their point of origin (either the Greenlane East roundabout or Greenlane East Road). Once vehicles enter the queue, they cannot exit or leave the queue before being served, due to the road layout. Changing lanes within the queue is highly discouraged and practically impossible during peak times due to the high density of vehicles. Lane changes could exacerbate congestion and are generally avoided.
- The maximum size of the queue is considered infinite. This assumption helps in understanding the scenario where vehicle arrival exceeds the service rate without any upper limit on the number of waiting vehicles.

  In summary, the queue configuration at the Greenlane East ramp road is defined by two separate queues aligned with the two lanes serving vehicles from different points of origin. The configuration is such that vehicles cannot switch lanes once in the queue, and there is no option for balking or reneging.

(c) **QUEUING MODEL**

To analyse the traffic problem at the Greenlane Interchange I have used an **M/M/1 Queuing Model.**

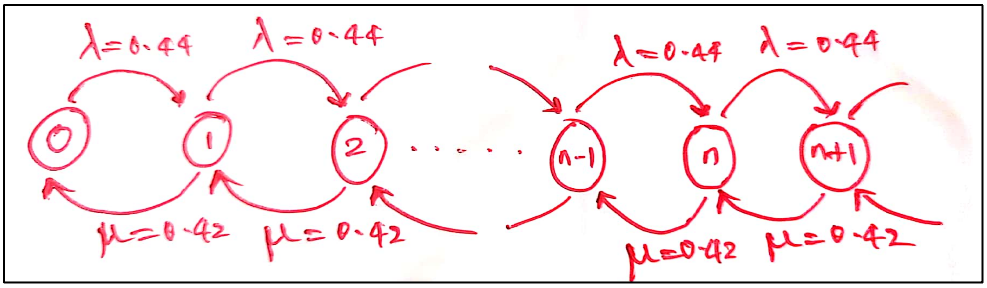
- **Markovian Arrival Process (M):**
  Poisson Arrival Process: The vehicles arrive randomly at the ramp, and the number of arrivals in a fixed time interval follows a Poisson distribution. This implies the time between consecutive arrivals is exponentially distributed. The arrival of vehicles from both the Greenlane roundabout and Greenlane East Road can be independently modelled as Poisson processes, with vehicles arriving at random intervals.
- **Markovian Service Process (M):**
  Exponential Service Time: The time taken for a vehicle to be served and proceed is exponentially distributed. The time vehicles wait at the ramp light is random but follows an average rate, which fits the exponential distribution.
- **Single Server (1):**
  Single Service Point: Although there are two lanes and 2 serving points, according to the properties of Poisson Distribution a service facility with n servers operating in parallel, each with exponentially distributed service times can be modelled as a single server. So,

$$\boldsymbol{\lambda}_{=} \boldsymbol{\lambda_1} + \boldsymbol{\lambda_2} + \cdots .. \boldsymbol{\lambda_n}$$
$$\boldsymbol{\mu}_{=} \boldsymbol{\mu_1} + \boldsymbol{\mu_2} + \cdots .. \boldsymbol{\mu_n}$$

As calculated earlier,

- Arrival Rate (λ): Mean($\frac{Number\ of\ vehicles\ arrived}{time\ Interval}$): The average rate at which vehicles enter the system.
  - $\lambda_1 = \frac{\left(\frac{10}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3} = 0.22$
  - $\lambda_2 = \frac{\left(\frac{8}{42}+\frac{10}{42}+\frac{11}{47}\right)}{3} = 0.22$
  - $\boldsymbol{\lambda = 0.44}$ vehicles per second
- Inter- arrival time= 1/0.22= 4.5 seconds per vehicle for each lane

- Service Rate (μ): Mean($\frac{Number\ of\ vehicles\ served}{time\ Interval}$): The average rate at which vehicles are served.
  - $\mu_1 = \frac{\left(\frac{9}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3} = 0.21$
  - $\mu_2 = \frac{\left(\frac{9}{42}+\frac{9}{42}+\frac{10}{47}\right)}{3} = 0.21$
  - $\boldsymbol{\mu = 0.42}$ vehicles per second
- Average Service Time: $\frac{1}{\mu}$ = 1/0.21= 4.76 seconds per vehicle for each lane



Rate diagram for the system

- Utilization Factor (ρ): $\frac{\lambda}{\mu} = \frac{0.44}{0.42}$ = **1.0476** vehicles per second. Indicates the fraction of time the server is busy.
- A utilization factor (ρ) greater than 1 indicates that the system is overloaded, as the arrival rate exceeds the service rate. This results in increasing queue lengths and waiting times, leading to congestion at the roundabout and Greenlane East Road.
- Average queue length at time t = 9.2 and 9.4 for lane 1 and 2 respectively.
- Average system length at time t = 10.2 and 10.4 in lane 1 and 2 respectively.
- Average time a vehicle spends in the system =4.76* 10.2= 48.552 seconds for lane 1 and 49.504 seconds for lane 2.

Assumptions:

- The calling population is considered infinite.
- Both service and arrival times are exponentially distributed, with arrivals following a Poisson Process.
- The system operates with a single First-In-First-Out queue configuration of infinite length.
- There are no instances of balking, reneging, or retrial within the system.
- Although there are two lanes and 2 serving points, as both are exponentially distributed we are modelling the system as one with single server and single queue.

3.

(a) The queuing model employed for analysis is the M/M/1 model, which assumes a Markovian arrival process, Markovian service process, and a single server system. This model is suitable for analyzing systems such as traffic flow at the Greenlane Interchange.

- The arrival of vehicles from both the Greenlane roundabout and Greenlane East Road follows a Poisson distribution, with vehicles entering the system at random intervals. The average inter-arrival time for vehicles is approximately 4.5 seconds per vehicle.
- The service time for vehicles at the interchange follows an exponential distribution. The average service time is approximately 4.76 seconds per vehicle.
- Although there are two lanes and serving points, the system is modeled as having a single server as per the properties of the Poisson distribution.
- The utilization factor is calculated to be approximately 1.0476, indicating that the system is overloaded, as the arrival rate exceeds the service rate.
- The average queue length at the interchange is measured to be approximately 9.2 and 9.4 vehicles for lanes 1 and 2, respectively.
- The average system length, including vehicles in the queue and being served, is approximately 10.2 and 10.4 vehicles for lanes 1 and 2, respectively.
- The average time a vehicle spends in the system is calculated by multiplying the average service time by the average system length. For lane 1, the average waiting time is approximately 48.552 seconds, and for lane 2, it is approximately 49.504 seconds.

Steady-State Condition:

- The steady-state condition of the queuing system is reached when the arrival rate equals the service rate, resulting in a stable equilibrium. For M/M/1 queuing model,

Steady State condition: ρ = (λ/μ) < 1

- In this analysis, the utilization factor (**ρ=1.0476**) which is greater than 1, indicating that the system is overloaded.
- This increases the traffic in the origin roads causing greater waiting for the vehicles as well as distrupting the traffic on those roads.
- This can be achieved by increasing the service rates of the system.
- After achieving a stead state we can calculate the following parameters:
    - **L** = Expected number of customers in the system (in steady state)
    - **Lq** = Expected number of customers in the queue (in steady state)
    - **W** = Expected time a job spends in the system
    - **Wq**= Expected time a job spends in the queue

(b) There are various practical solutions that can address the situation.
Our aim is to increase the service rate, i.e., to increase the number of vehicles that can pass through the ramp signal at a particular time.

SOLUTION 1: ADJUSTING RAMP LIGHTS DURATION

Currently, the average service time is 4.76 seconds per vehicle for each lane. That is in every 4.76 seconds 1 vehicle from each lane passes the signal. Thus we can conclude that the duration of 1 red and green cycle is 4.76 seconds. The duration of a red light can vary, ranging from as brief as 2 seconds to as lengthy as 14 seconds, contingent upon traffic volumes. Meanwhile, adhering to international standards, the green phase lasts for 1.3 seconds, enabling the departure of one vehicle from the ramp. Thus, in our situation assuming the duration of the green light is set at 1.3 seconds, we can say that the duration of red light is 4.76-1.3= 3.46 seconds.

CASE 1

- To increase the service rate we can reduce the duration of the red light, say 3 seconds
- Then the average service time will be equal to 4.3 seconds per vehicle
- Thus making the service rate = 2*(1/4.3)= 0.465 vehicles per second
- Now utilization factor (ρ= **0.44/0.465 = 0.946**) which is less than 1 making our system in steady state.
- LIMITATION: the 3.46 seconds of red light is the time that two vehicles will take to merge in the transition distance before they enter into the motorway. When reducing the duration of red light we should make sure that there is enough time for the lanes to merge. This can be done by increasing the transitional distance.

$$\lambda=0.44$$
$$\mu = 0.46$$
$$\rho= 0.946$$
$$L = \rho/(1-\rho)=17.52 \text{ vehicles}$$
$$L_q = \mathrm{L} - \rho= 16.574 \text{ vehicles}$$
$$W = L/\lambda= 39.81 \text{ seconds}$$
$$w_q = L_q/\lambda = 37.66 \text{ seconds}$$

CASE 2

- Now 2 vehicles pass the ramp light at every 4.76 seconds and we can say that it takes 9.52 seconds for 4 vehicles to pass the lights.
- If we increase both the red and green lights by 1 second each, making it possible for 2 vehicles per lane to pass the signal for every green light, then the service time for 2 vehicles per lane will be ((1+1.3)+(3.46+1))=6.76
- Thus making the service rate = 4*(1/6.76)= 0.59 vehicles per second
- Now utilization factor (ρ= **0.44/0.59 = 0.746**) which is less than 1 making our system in steady state.
- LIMITATION: the 3.46 seconds of red light is the time that two vehicles will take to merge in the transition distance before they enter into the motorway. When increasing the number of vehicles that should merge we should make sure that there is enough space for this to happen. This can be done by increasing the transitional distance.

$$\lambda=0.44$$
$$\mu = 0.59$$
$$\rho= 0.746$$
$$L = \rho/(1-\rho)=2.937 \text{ vehicles}$$
$$L_q = \mathrm{L} - \rho= 2.191 \text{ vehicles}$$
$$W = L/\lambda=6.675 \text{ seconds}$$
$$w_q = L_q/\lambda = 4.979 \text{ seconds}$$

SOLUTION 2: INCREASING THE NUMBER OF LANES AT THE RAMP SIGNAL

CASE 1

- Number of lanes= 3, this can be done by increasing the number of lanes entering the system from any one arrival roads.
- Since the arrivial rate doesn't change we can say that $\boldsymbol{\lambda = 0.44}$ **vehicles per second** despite the number of lanes.

- But for 1 lane the service rate doubles, i.e., say the number of vehicles in lane 1 is distributed between lane 1.1 and 1.2, then $\mu_{1.1} = 0.21$, $\mu_{1.2} = 0.21$ and $\mu_2 = 0.21$. Thus the effective service rate is $\boldsymbol{\mu = 0.63.}$
- Now utilization factor (ρ= **0.44/0.63 = 0.698**) which is less than 1 making our system in steady state.
- LIMITATION: Increasing the number of lanes is a long term solution, which is expensive as well as time consuming.

$$\lambda=0.44$$
$$\mu = 0.63$$
$$\rho= 0.698$$
$$L = \rho/(1-\rho)=2.311 \text{ vehicles}$$
$$L_q = L - \rho= 1.61 \text{ vehicles}$$
$$W = L/\lambda=5.25 \text{ seconds}$$
$$w_q = L_q/\lambda = 3.659 \text{ seconds}$$

CASE 2

- Number of lanes= 3, this can be done by increasing the number of lanes entering the system from both the roundabout and the Greenlane East.
- Since the arrivial rate doesn't change we can say that $\boldsymbol{\lambda = 0.44}$ **vehicles per second** despite the number of lanes.
- But the service rate doubles for each lanes, i.e., $\mu_1 = 0.42$ and $\mu_2 = 0.42$. Thus the effective service rate is $\boldsymbol{\mu = 0.84}$ **.**
- Now utilization factor (ρ= **0.44/0.84 = 0.524**) which is less than 1 making our system in steady state.
- LIMITATION: Increasing the number of lanes is a long term solution, which is expensive as well as time consuming.

$$\lambda=0.44$$
$$\mu = 0.84$$
$$\rho= 0.524$$
$$L = \rho/(1-\rho)=1.1008 \text{ vehicles}$$
$$L_q = L - \rho= 0.5768 \text{ vehicles}$$
$$W = L/\lambda=2.502 \text{ seconds}$$
$$w_q = L_q/\lambda = 1.312 \text{ seconds}$$

DECREASING THE TRAFFIC

- At the end decreasing the waiting time would lead to increased traffic in the motorway. Thus measures to decrease the overall traffic should be taken.
- One of the ways would be to promote carpooling. For this transit lane could be implemented in the motorways and ramp roads.
- Also by leaving the left-most lane of the motorway vacant or decreasing the number of vehicles in this lane by priority or time gap will aid in easy merging of vehicles from the acceleration distance of the ramp road.

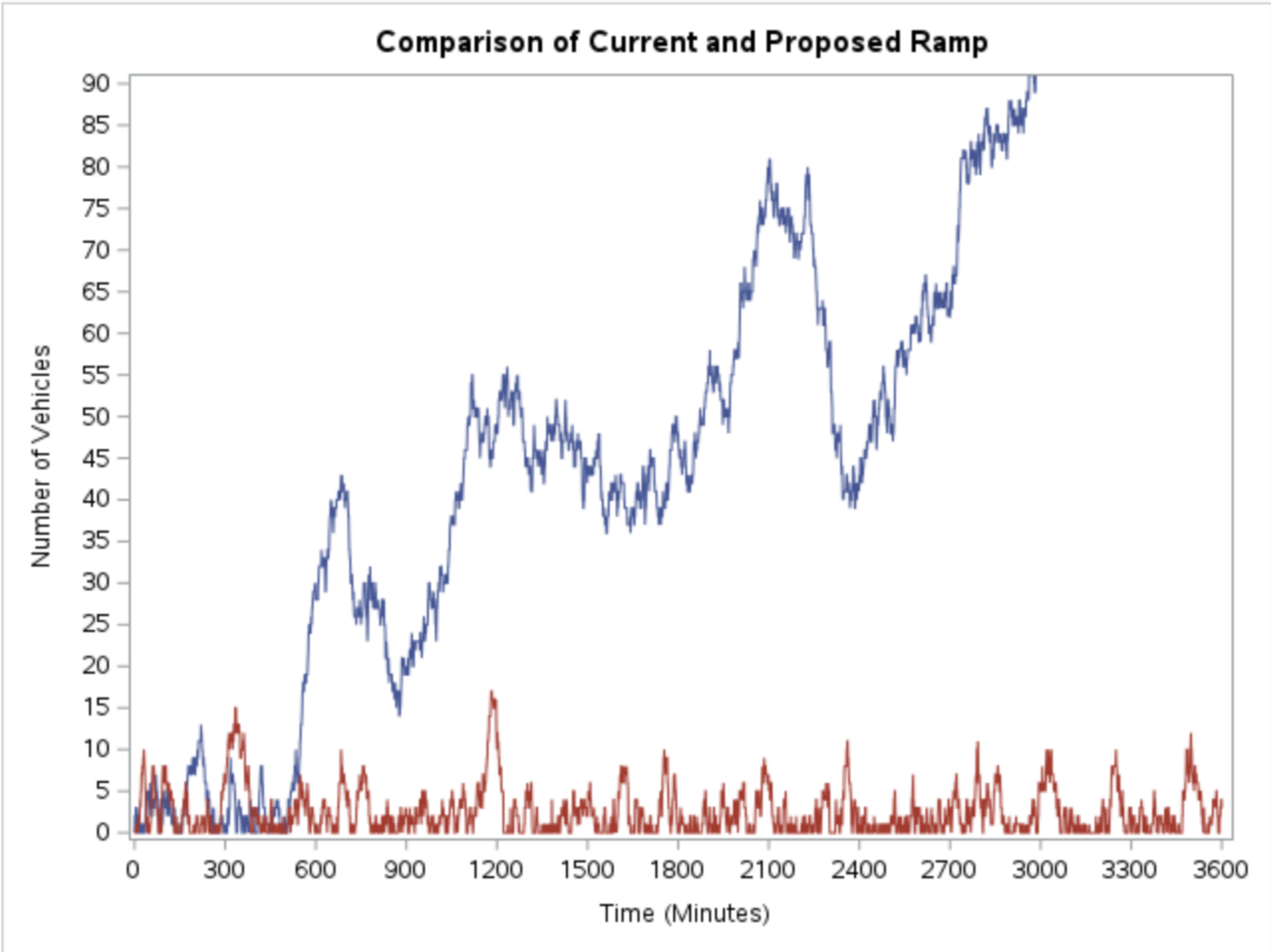
I am taking case 2 of solution 1 as the practical solution. Here the utilization factor is enough less than 1 to maintain the steady state unlike the case 1 of solution 1. Also solution 2 is a long term solution which is expensive.

(c) SIMULATION OF THE SOLUTION

```
proc iml;
start main;
   /* Set seed */
   call streaminit(123456789);
   lambda = 0.44;

/*CURRENT SITUATION*/
   mu = 0.42;
   T = 60*60;
```

```
	current = j(T+1,4,0);

	do i = 1 to T+1;
		arrivals = rand("Poisson",lambda);
		departures = rand("Poisson",(mu));

		/* Time*/
		current[i,1]=i-1;
		/* Arrivals and Departures */
		current[i,2]=arrivals;
		current[i,3]=departures;

		/* Queue Size*/
		if i = 1 then	current[1,4]=max(current[i,2]-current[i,3],0);
		else		current[i,4]=max((current[i-1,4]+(current[i,2]-
   current[i,3])),0);

		if current[i,4]=0 then	current[i,3]=0;
	end;
/*SOLN*/
	mu1 = 0.59;
	T = 60*60;
	CASE1 = j(T+1,4,0);

	do i = 1 to T+1;
		arrivals = rand("Poisson",lambda);
		departures = rand("Poisson",(mu1));

		/* Time*/
		CASE1[i,1]=i-1;
		/* Arrivals and Departures */
		CASE1[i,2]=arrivals;
		CASE1[i,3]=departures;

		/* Queue Size*/
		if i = 1 then	CASE1[1,4]=max(CASE1[i,2]-CASE1[i,3],0);
		else		CASE1[i,4]=max((CASE1[i-1,4]+(CASE1[i,2]-
CASE1[i,3])),0);

		if CASE1[i,4]=0 then  CASE1[i,3]=0;
	end;


	vars  = {"Time","Arrivals","Departures","Queue_Size"};
	vars1 = {"Time","Arrivals1","Departures1","Queue_Size1"};
```

```
      create work.current from current[colname=vars];
            append from current;
      close work.current;
create work.CASE1 from CASE1[colname=vars1];
            append from CASE1;
      close work.CASE1;

finish main;
RUN;



DATA work.compare;
merge work.current work.CASE1;
RUN;

title1 "Comparison of Current and Proposed Ramp";

PROC SGPLOT data=work.compare;
      series x=Time y=Queue_Size  / name="current";
      series x=Time y=Queue_Size1 / name="CASE1";

      xaxis      label="Time (Minutes)" values=(0 to 3600 by 300);

      yaxis      label="Number of Vehicles" values=(0 to 90 by 5);

      keylegend "ramp" "ramp1" / location=inside
                              position=top;
RUN;

title1 "Histogram of Queue Size Currently";
PROC UNIVARIATE data = work.ramp;
      histogram Queue_Size /      midpoints = 0 to 90 by 5
                          normal
                          kernel;
RUN;

title1 "Histogram of Queue Size for Proposed  solution";
PROC UNIVARIATE data = work.CASE1;
      histogram Queue_Size1 /     midpoints = 0 to 12 by 1
                          normal
                          exponential;
RUN;
```



The simulation shows a notable enhancement in traffic flow on the ramp. The proposed solution effectively prevents the queue from becoming unmanageable, achieving a steady state system that is independent of the initial conditions. The queue size stabilized at 5 vehicles.