# ARDUINO AND MATLAB IMPLEMENTATION FOR ESTIMATION OF RESPIRATION RATE USING PPG SIGNALS

**A PROJECT REPORT**

***Submitted by***


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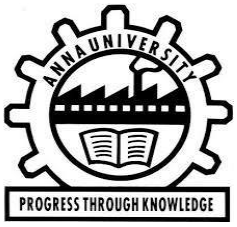
**BACHELOR OF ENGINEERING**

**IN**

**ELECTRONICS AND COMMUNICATION ENGINEERING**





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# ANNA UNIVERSITY: CHENNAI 600 025

## BONAFIDE CERTIFICATE

Certified that this project report titled "**ARDUINO AND MATLAB IMPLEMENTATION FOR ESTIMATION OF RESPIRATION RATE USING PPG SIGNALS**" is the bonafide work of "**NIVAASHINI S(710020106019), KAVYASHREE R(710020106014)**" who carried out the project work under my supervision.

**SUBMITTED TO:**

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**Dr.V.R.VIJAYKUMAR**

# ACKNOWLEDGEMENT

First and foremost, we place this project work on the feet of GOD ALMIGHTY who is the power of strength in each step of progress towards the successful completion of the project. We owe whole hearted sense of reverence and gratitude to our project guide **Dr. GANGIREDDY NARENDRA KUMAR REDDY**, Assistant Professor, IIIT Kottayam Bhubaneswar for his efficient and excellent guidance, inspiring discussion insightful and timely encouragements for the successful completion of the project. We express our thanks to our Head Of the Department of Electronics and Communication Engineering, **Dr.V.R.VIJAYKUMAR**, for giving us the opportunity and providing us with adequate infrastructure and congenial environment. We thank **Dr.M.SABARIMALAI MANIKANDAN**, IIIT Bhubaneswar for his great support with blessings. We also extended our heartfelt thanks to all faculties and staff who have rendered their valuable help in making this project successful.

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## ABSTRACT

**Objective:** This paper presents a simple automated method to estimate respiration rate (RR) from the photoplethysmography (PPG) signals. **Methods:** The method consists of preprocessing, extremely low-frequency two-pole digital resonator, fast Fourier transform, spectral magnitude computation, prominent spectral peak identification and respiration rate computation. **Validation Dataset:** The proposed RR estimation method is evaluated using standard PPG databases including MIMIC-II and CapnoBase. **Results:** The proposed method had estimation accuracy with absolute error value (median, 25th–75th percentiles) of 0.522 (0, 1.403) and 0.2746 (0, 0.475) breaths/minute for the PPG recordings from the MIMIC-II and CapnoBase databases, respectively for the 30 s segment. Results showed that the proposed method outperforms the existing methods such as the empirical mode decomposition and principal component analysis (EMD-PCA), ensemble EMD-PCA, and improved complete EEMD with adaptive noise-PCA methods. **Conclusion:** Results demonstrate that the proposed method is more accurate in estimating RR with less processing time of 0.0296 s as compared to that of the existing methods. This study further demonstrate that the two-pole digital resonator with extremely low-frequency can be simple method to estimate the RR from the PPG signals.



**Keywords:** Photoplethysmography (PPG) signal · Respiratory signal · Respiration rate (RR) · Digital resonator · Fast Fourier transform

## 1 Introduction

Respiration rate (RR) is an essential parameter for understanding the breathing patterns under normal and abnormal conditions. The RR ranges between 5 and 24 breaths/min (i.e., 0.08–0.4Hz) at resting condition for adult subjects, and that of neonates at resting ranges from 10 to 80 breaths/min (0.17–1.33 Hz). During exercise, the RR can increase to approximately 45 breaths/min. For children with age ranging between 1–5 years, the RR value above 40 breaths/min (age) is recommended as per the guidelines. The PPG signal contains cardiac rhythm with synchronous respiratory component. In the past studies, it was observed that the PPG waveform is modulated by the breathing in three ways such as the baseline wander (BW), amplitude modulation (AM), and frequency modulation (FM). The respiration induces variations in pulse rate (PR), amplitude and width of the pulsatile waveform. The PR is increased and decreased during inspiration and expiration, respectively which causes respiratory-induced frequency variation (RIFV) in the PPG signal. The PRV is modulated by respiration, which is well known as respiratory sinus arrhythmia (RSA). The respiratory synchronous blood volume variations cause the respiratory-induced intensity variation (RIIV) [21], which indicates variations in absolute amplitude of peaks of the PPG waveform [8,12]. A decrease in cardiac output with reduced ventricular filling causes change in peripheral pulse strength, which is termed as the respiratory-induced amplitude variation (RIAV) which indicates the height of pulsatile waveform [12]. The pulse amplitude is modulated due to variations in stroke volume and stiffness of the blood vessel that can also modulate the pulse width variability (PWV).

## 2 Materials and Methods

In order to evaluate the performance of the proposed method, we used three publicly-available datasets such as Multiparameter intelligent monitoring in intensive care (MIMIC) [26], and CapnoBase [27], as described below:

– **The MIMIC Dataset**: The MIMIC dataset [26] includes 72 simultaneously recorded PPG, blood pressure, respiratory and electrocardiogram (ECG) signals. The signals were digitized with a sampling rate of 125 Hz. In this study, 266 epochs of 30 s duration are manually selected from simultaneously recorded PPG and respiratory signals. For each of the epochs, we have compared RR estimated from the PPG signal with reference RR derived from the respiratory signal. In addition, we tested the algorithm's performance with 406 epochs of 20 s duration and 826 epochs of 10 s duration.

– **The CapnoBase Dataset**: The capnoBase dataset [27] contains simultaneously recorded respiratory, ECG, and PPG signals. The signals were digitized with a sampling rate of 300 Hz. In this study, 336 epochs of 30 s duration are manually selected from simultaneously recorded PPG and respiratory signals. In addition, we tested the algorithm's performance with 504 epochs of 20 s duration and 1008 epochs of 10 s duration.

### 2.1 Proposed RR Estimation Method

Figure 1 depicts a block diagram of the proposed method, which consists of the following steps:

(1) Preprocessing for mean subtraction and amplitude normalization

(2) Respiratory signal extraction using two-pole digital resonator
(3) Finding Fourier magnitude spectrum using fast Fourier transform
(4) Determining maximum spectral peak location within specified spectral range
(5) Respiration rate determination
Each of the steps of the proposed method is described in the next subsections.

**Preprocessing.** The preprocessing is performed to subtract mean from the PPG signal $y[n]$ and to normalize the zero-mean signal amplitude between –1 to 1. In this study, we consider three block duration values such as 10, 20 and 30 s to estimate the respiration rate. The mean subtraction is implemented as

$$x[n] = y[n] - \mu_y, \quad (1)$$

where $\mu y$ denotes the mean of the signal $y[n]$. Then, the zero-mean signal amplitude is normalized as

$$\hat{x}[n] = \frac{x[n]}{max_{n=0}^{N-1}\{|x[n]|\}}, \quad (2)$$

where $N$ is the number samples within a block, which is computed as $\lfloor D * Fs \rfloor$

where $D$ is the block duration, $Fs$ is the sampling rate (samples/second) and $\lfloor \bullet \rfloor$ denotes the floor.

**Respiratory Signal Extraction.** In past studies, many signal processing techniques such as digital filters, wavelet transforms, EMD, ensemble EMD, synchro squeezing transform, complementary EEMD, complete EEMD with adaptive noise (CEEMDAN), and improved CEEMDAN (ICEEMDAN), empirical wavelet transform (EWT), Fourier-Bessel series expansion-based EWT (FBSEEWT), AR models, and PCA were used for extracting respiratory signal from the PPG signal. Although the RR estimation methods based on the above signal processing technique(s) showed promising estimation results, real-time implementation of the methods was not addressed on the wearable devices which are constraint with limited resources such as processor speed, memory and battery power. Therefore, in this study, we attempt to explore a simple filtering technique to extract a respiratory component from the PPG signal. Past studies showed that the pole-zero placement is effective method for designing narrow bandwidth band-pass and band-stop filters. The placement of poles emphasizes the magnitude response and of zeros provides zero gains. A two-pole band-pass filter with the pair of complex conjugate poles near the unit circle, i.e., poles at $p1,2 = re \pm j\omega o$, where $\omega o$ denotes the resonant frequency and $r$ denotes magnitude of the poles, which controls the bandwidth of the filter.

The transfer function of the two-pole digital resonator with zeros at the origin is given by

$$H(z) = \frac{b_0}{(1 - re^{j\omega_o} z^{-1})(1 - re^{-j\omega_0} z^{-1}} \quad (3)$$

where *b*0 is the gain of the resonator. The magnitude response of the function has a large magnitude for the frequency of *fo*. By choosing suitable band-pass parameters such as *r* and *fo*, we can extract the respiratory signal having frequency range between 0.1 to 0.8 Hz (6–48 breaths/min). The ⁄*H*(*ejωo* ⁄ has its peak at or near *ω* = *ωo*. The width of the resonance centered at *ωo* can be controlled by using the parameter *r*. The Eq.(3) can be rewritten as

$$H(z) = \frac{b_0}{1 - (2r\cos\omega_o)z^{-1} + r^2 z^{-2}} \quad (4)$$

The above transfer function can be further simplified as

$$H(z) = \frac{b_0}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (5)$$

where, the desired normalization factor *b*0 can be computed as

$$b_0 = (1 - r)\sqrt{1 + r^2 - 2r\cos 2\omega_o}$$

$$a_1 = -2r\cos\omega_o \quad and \quad a_2 = r^2. \quad (6)(7)$$

In this study, the resonant frequency *ωo* and the bandwidth controlling parameter *r* are fixed as 0.3 Hz and 0.997 empirically. Then the feed backward coefficients *a*1 and *a*2 and the gain factor *b*0 are computed by using the Eqs. (6) and (7). The magnitude response and phase response of the two-pole resonator are shown in Fig. 2. From Eq. (5), the difference equation of the two-pole digital resonator can be expressed as

$$y[n] = -a_1 y[n] - a_2 y[n-2] + b_0 x[n]. \quad (8)$$

For the PPG signals taken from the MIMIC and CapnoBase datasets, the extracted respiratory signals are shown in Fig. 3. From the extracted respiratory signals, it is noted that the period of the respiratory signal is approximately similar to the period of the original respiratory signal. Further, the extracted signal contains the pulse waveform which is superimposed on the respiratory signal. Thus, it can be used for extracting both pulse rate (PR) and respiration rate(RR) from the extracted signals by using Fourier magnitude spectrum. However,
this study restricts to perform RR estimation by limiting the peak analysis of the Fourier spectrum within the range between 0.1 to 0.8 Hz.

**Fourier Magnitude Spectrum:** In this study, we use FFT for computing the magnitude spectrum of the extracted respiratory signal. Depending on the block duration and sampling rate to be analyzed, the number of points is fixed for FFT computation. For each of the datasets and their sampling rates, the Fourier magnitude spectra of the extracted respiratory signals are shown in Figs. 4 and 5. From the results, it is observed that the dominant spectral peak corresponds to the frequency of the respiratory signals. Results further show that the estimated respiratory frequency matches with the respiratory frequency estimated from the original respiratory signal.

**Spectral Peak Finding Logic:** In this study, we assume that the respiration rate varies from 6 to 48 breaths/min which equals to the frequency range between 0.1 to 0.8 Hz. Thus, we find

a location of the dominant spectral peak within the Fourier spectrum ranging from 0.1–0.8Hz. From this we find the respiratory frequency *FR* to compute the respiration rate.

**Respiration Rate Estimation:** From the estimated frequency *FR*, the respiration rate can be computed as

$$\mathrm{RR} = F_R * 60 \quad (\text{breaths/min}). \tag{9}$$

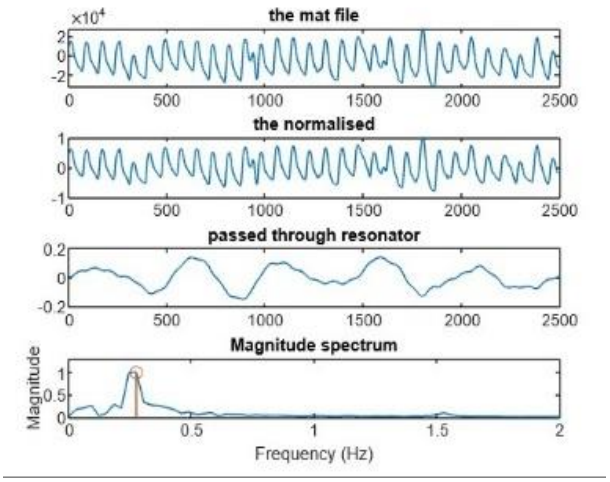
## MATLAB IMPLEMENTATION:

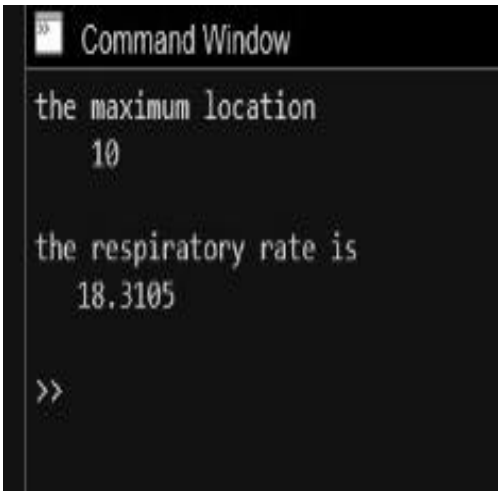
**CODE:**

```
clc;
clear all;
close all;
data=load(['bidmc01m.mat']);
x1=data.val(1,1:125*20);
x2=data.val(1,2:125*20);
figure;
subplot(4,1,1);
plot(x2);
hold on
title('the mat file');
% preprocessing
x2=x2-mean(x2);  % mean removal
d=max(abs(x2));
normsig=x2/d;  % amplitude normalization
subplot(4,1,2);
plot(normsig);
title('the normalised');
%-----------digital resonator for RR estimate--------------
 bwidth=0.3; %Hz
 Fs=125;
wo=2*pi*(bwidth)/Fs;
 r=0.997;
a0=1;a1=-2*r*cos(wo);a2=r^2;
a=[a0 a1 a2];
b0=(1-r)*sqrt(1+(r^2)-(2*r*cos(2*wo)));
%the two pole filter with zero at origin
num=[b0 0 0];
den=[a0 a1 a2];
sys=tf(num,den);
% passing the signal through the digital resonator
y=filter(num,den,normsig);
subplot(4,1,3);
plot(y);
title('passed through resonator');
% fft
L=length(y);
N=2^nextpow2(L);
```

```
y_fftmag1=abs(fft(y,N));  % two sided magnitude spectrum
y_fftmag=y_fftmag1(1:N/2); % one sided magnitude spectrum
y_fftmag=y_fftmag/max(abs(y_fftmag)); % normalized spectrum
fk=(0:length(y_fftmag)-1).*Fs/N; % convert samples into frequeny
% finding the peak value of the frequency spectrum
%fk=range from 0.1 to .8
%N =no. of samples
%fs sampling frequency
%fk=k x fs /N
k1=floor(0.1*N/Fs);
k2=floor(0.8*N/Fs);
[Kmax, Kloc1]=max(y_fftmag(k1:k2));
Kloc=Kloc1+k1-1; % kmax location
disp('the maximum location')
disp(Kloc);
%finding the breath rate
fmax=Kloc*Fs/N;
rr=fmax*60;
disp('the respiratory rate is');
disp(rr);
subplot(4,1,4);
plot( fk, y_fftmag);
hold on;
stem(fk(Kloc),y_fftmag(Kloc));
ylim([0 1.3]);
xlim([0 2]);
xlabel('Frequency (Hz)')
ylabel('Magnitude')
title('Magnitude spectrum')
```

## MATLAB OUTPUT:




**SPREADSHEET FOR ALL SAMPLES OF THE ENTIRE BIDMC DATASET**

| | | | |
|---|---|---|---|
| bidmc01m | 25.6348 | bidmc26m | 21.9727 |
| bidmc02m | 18.3105 | bidmc27m | 14.6484 |
| bidmc03m | 18.3105 | bidmc28m | 20.1416 |

| | | | |
|---|---|---|---|
| bidmc04m | 21.9727 | bidmc29m | 20.1416 |
| bidmc05m | 20.1416 | bidmc30m | 20.1416 |
| bidmc06m | 21.9727 | bidmc31m | 20.1416 |
| bidmc07m | 21.9727 | bidmc32m | 27.4658 |
| bidmc08m | 23.8037 | bidmc33m | 16.4795 |
| bidmc09m | 21.9727 | bidmc34m | 20.1416 |
| bidmc10m | 23.8037 | bidmc35m | 20.1416 |
| bidmc11m | 14.6484 | bidmc36m | 20.1416 |
| bidmc12m | 20.1416 | bidmc37m | 18.3105 |
| bidmc13m | 20.1416 | bidmc38m | 16.4795 |
| bidmc14m | 16.4795 | bidmc39m | 16.4795 |
| bidmc15m | 16.4795 | bidmc40m | 16.4795 |
| bidmc16m | 20.1416 | bidmc41m | 18.3105 |
| bidmc17m | 20.1416 | bidmc42m | 18.3105 |
| bidmc18m | 20.1416 | bidmc43m | 18.3105 |
| bidmc19m | 18.3105 | bidmc44m | 18.3105 |
| bidmc20m | 18.3015 | bidmc45m | 18.3105 |
| bidmc21m | 16.4795 | bidmc46m | 14.6484 |
| bidmc22m | 20.1416 | bidmc47m | 23.8037 |
| bidmc23m | 20.1416 | bidmc48m | 21.9727 |
| bidmc24m | 25.6348 | bidmc49m | 20.1416 |
| bidmc25m | 18.3105 | bidmc50m | 18.3105 |
| bidmc51m | 20.1416 | bidmc52m | 20.1416 |
| bidmc53m | 21.9727 | 37 | 14.6484 |

# ARDUINO IMPLEMENTATION USING ARDUINO DUE

- Then we tried without resampling in due board
- Same 125Hz signal is used
- And the implemented in Arduino due, but this time the board was much slower compared to 1500 samples

## ARDUINO CODE:

```
#include <SPI.h>
#include<SD.h>
#define PI 3.14159265358979323846264338327 95
/*https://archive.physionet.org/cgi-bin/atm/ATM */

void dft(const double inrealn[],double outreal[],double outimg[],double magi[],size_t n);//for n number of samples
float maxi(float a, float b);

double inreal[7500]={0.189,0.201,0.22,0.252,0.297,0.353,0.417,0.488,0.555,0.613,0.661,0.702,0.734,0.759,0.773,0.781,0.784,0.782,0.774,0.762,0.748,0.729,0.709,0.686,0.662,0.638,0.618,0.598,0.579,0.558,0.537,0.516,0.497,0.477,0.459,0.443,0.426,0.41,0.393,0.377,0.364,0.344,0.326,0.315,0.3,0.286,0.275,0.265,0.255,0.245,0.237,0.228,0.221,0.213,0.205,0.198,0.193,0.189,0.185,0.183,0.18,0.178,0.175,0.171,0.169,0.168,0.166,0.164,0.162,0.16,0.16,0.161,0.163,0.165,0.167,0.168,0.17,0.172,0.173,0.174,0.176,0.178,0.18,0.182,0.183,0.185,0.187,0.189,0.191,0.194,0.196,0.199,0.201,0.202,0.201,0.2,0.199,0.2,0.208,0.225,0.252,0.284,0.325,0.377,0.44,0.507,0.576,0.64,0.7,0.751,0.791,0.821,0.841,0.849,0.849,0.845,0.834,0.819,0.803,0.782,0.761,0.736,0.71,0.682,0.654,0.627,0.602,0.581,
```

0.558,0.536,0.514,0.494,0.473,0.452,0.431,0.413,0.395,0.378,0.366,0.357,0.348,0.34,0.331,0.322,0.313,0.303,0.293,0.283,0.275,0.266,0.259,0.254,0.251,0.247
,0.242,0.239,0.237,0.235,0.233,0.231,0.227,0.223,0.219,0.215,0.213,0.209,0.205,0.202,0.199,0.197,0.196,0.196,0.196,0.196,0.198,0.202,0.206,0.209,0.212,0.2
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6,0.636,0.691,0.738,0.775,0.803,0.819,0.826,0.824,0.817,0.805,0.789,0.769,0.748,0.724,0.698,0.673,0.646,0.621,0.594,0.57,0.548,0.525,0.503,0.482,0.462,0.4
44,0.426,0.409,0.389,0.368,0.347,0.329,0.314,0.296,0.281,0.268,0.256,0.241,0.228,0.218,0.207,0.198,0.189,0.18,0.173,0.167,0.161,0.155,0.15,0.145,0.142,0.1
4,0.14,0.141,0.142,0.143,0.176,0.224,0.248,0.259,0.265,0.267,0.266,0.266,0.266,0.266,0.267,0.267,0.269,0.27,0.27,0.27,0.27,0.269,0.269,0.269,0.269,0.27,0.2
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633,0.622,0.611,0.599,0.587,0.573,0.559,0.546,0.533,0.52,0.508,0.498,0.488,0.48,0.472,0.465,0.459,0.454,0.448,0.442,0.435,0.429,0.422,0.415,0.409,0.402,0.
395,0.389,0.384,0.378,0.371,0.364,0.357,0.35,0.344,0.339,0.334,0.331,0.328,0.326,0.325,0.322,0.319,0.316,0.314,0.312,0.31,0.307,0.305,0.302,0.298,0.295,0.
292,0.29,0.288,0.286,0.284,0.283,0.282,0.281,0.279,0.276,0.274,0.27,0.268,0.267,0.266,0.265,0.264,0.264,0.264,0.263,0.262,0.262,0.264,0.269,0.279,0.291,0.
309,0.333,0.366,0.403,0.444,0.486,0.527,0.567,0.602,0.631,0.655,0.671,0.679,0.685,0.688,0.689,0.686,0.68,0.673,0.664,0.652,0.639,0.626,0.612,0.599,0.587,0
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,0.357,0.351,0.346,0.341,0.336,0.332,0.328,0.324,0.32,0.317,0.314,0.312,0.31,0.308,0.306,0.304,0.302,0.3,0.298,0.295,0.293,0.291,0.289,0.286,0.283,0.281,0.
278,0.276,0.275,0.274,0.273,0.272,0.27,0.269,0.268,0.266,0.264,0.261,0.259,0.257,0.256,0.257,0.26,0.266,0.278,0.294,0.314,0.337,0.369,0.405,0.444,0.484,0.
524,0.56,0.591,0.618,0.638,0.654,0.663,0.667,0.668,0.666,0.661,0.653,0.644,0.634,0.623,0.611,0.599,0.586,0.571,0.557,0.553,0.553,0.545,0.535,0.524,0.514,0
.505,0.497,0.489,0.481,0.473,0.466,0.46,0.453,0.445,0.437,0.429,0.421,0.413,0.406,0.399,0.392,0.385,0.378,0.372,0.367,0.36,0.352,0.344,0.336,0.329,0.323,0.
318,0.313,0.31,0.307,0.305,0.303,0.301,0.298,0.295,0.292,0.289,0.285,0.283,0.279,0.276,0.273,0.271,0.27,0.268,0.266,0.264,0.261,0.259,0.257,0.255,0.254,0.
253,0.253,0.252,0.252,0.252,0.25,0.248,0.245,0.241,0.238,0.236,0.237,0.242,0.255,0.276,0.298,0.326,0.362,0.404,0.45,0.497,0.541,0.581,0.616,0.644,0.666,0.
681,0.692,0.697,0.697,0.695,0.69,0.683,0.674,0.664,0.652,0.638,0.624,0.608,0.592,0.579,0.565,0.551,0.539,0.526,0.514,0.503,0.494,0.485,0.477,0.47,0.463,0.
457,0.453,0.448,0.442,0.435,0.428,0.419,0.41,0.401,0.392,0.384,0.377,0.371,0.367,0.362,0.358,0.354,0.349,0.345,0.341,0.336,0.331,0.326,0.322,0.317,0.314,0
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66,0.265,0.262,0.26,0.259,0.261,0.266,0.277,0.294,0.32,0.352,0.385,0.419,0.461,0.505,0.55,0.592,0.63,0.663,0.688,0.708,0.72,0.727,0.729,0.727,0.722,0.715,0
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6,0.418,0.411,0.402,0.393,0.385,0.377,0.37,0.364,0.359,0.353,0.348,0.342,0.337,0.331,0.326,0.322,0.317,0.314,0.311,0.307,0.305,0.303,0.3,0.298,0.295,0.293,

```
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0.382,0.376,0.37,0.367,0.362,0.356,0.351,0.346,0.341,0.336,0.333,0.329,0.326,0.322,0.319,0.316,0.313,0.311,0.308,0.306,0.303,0.3,0.296,0.292,0.289,0.286,0.
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.532,0.522,0.512,0.504,0.497,0.49,0.483,0.476,0.468,0.46,0.452,0.445,0.437,0.428,0.418,0.41,0.4,0.391,0.382,0.374,0.368,0.361,0.353,0.346,0.34,0.336,0.332,
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265,0.264,0.263,0.262,0.26,0.258,0.256,0.253,0.252,0.251,0.25,0.252,0.258,0.27,0.288,0.315,0.348,0.389,0.433,0.48,0.527,0.566,0.599,0.632,0.662,0.685,0.70
2,0.713,0.718,0.718,0.716,0.709,0.7,0.69,0.679,0.666,0.652,0.638,0.623,0.608,0.592,0.578,0.563,0.55,0.539,0.528,0.519,0.512,0.504,0.498,0.49,0.482,0.473,0.
465,0.457,0.452,0.445,0.438,0.432,0.426,0.419,0.412,0.404,0.397,0.389,0.382,0.376,0.37,0.365,0.36,0.355,0.352,0.347,0.342,0.337,0.332,0.328,0.326,0.324,0.
322,0.32,0.317,0.315,0.313,0.311,0.308,0.304,0.3,0.297,0.294,0.292,0.291,0.29,0.289,0.288,0.288,0.288,0.287,0.287,0.287,0.286,0.286,0.285,0.283,0.282,0.28,
0.277,0.275,0.274,0.275,0.28,0.288,0.304,0.327,0.36,0.398,0.441,0.487,0.533,0.576,0.611,0.638,0.667,0.691,0.711,0.724,0.733,0.736,0.735,0.731,0.723,0.715,
0.704,0.693,0.68,0.667,0.652,0.636,0.62,0.605,0.59,0.577,0.564,0.552,0.542,0.532,0.524,0.516,0.508,0.5,0.494,0.487,0.48,0.473,0.464,0.456,0.448,0.44,0.433,
0.426,0.419,0.412,0.404,0.396,0.387,0.378,0.371,0.366,0.36,0.354,0.35,0.347,0.343,0.34,0.336,0.332,0.329,0.326,0.323,0.319,0.317,0.315,0.313,0.311,0.309,0.
306,0.301,0.296,0.292,0.289,0.288,0.287,0.287,0.286,0.285,0.283,0.283,0.283,0.283,0.283,0.283,0.283,0.282,0.281,0.279,0.277,0.275,0.275,0.279,0.285,0.299,
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82,0.375,0.37,0.365,0.36,0.354,0.349,0.345,0.341,0.336,0.332,0.327,0.324,0.319,0.316,0.313,0.311,0.309,0.307,0.306,0.304,0.302,0.299,0.296,0.293,0.291,0.2
89,0.288,0.286,0.285,0.283,0.282,0.281,0.281,0.28,0.278,0.275,0.273,0.271,0.269,0.268,0.268,0.267,0.269,0.274,0.283,0.298,0.322,0.352,0.391,0.435,0.482,0.
53,0.573,0.612,0.646,0.675,0.695,0.708,0.718,0.724,0.726,0.725,0.72,0.715,0.707,0.696,0.685,0.673,0.66,0.646,0.631,0.616,0.6,0.586,0.57,0.556,0.543,0.533,0
.523,0.514,0.506,0.499,0.492,0.484,0.475,0.466,0.457,0.45,0.441,0.433,0.424,0.415,0.405,0.396,0.388,0.379,0.371,0.366,0.36,0.354,0.349,0.344,0.338,0.332,0.
327,0.323,0.32,0.317,0.314,0.31,0.306,0.303,0.3,0.298,0.296,0.294,0.292,0.289,0.287,0.286,0.284,0.283,0.281
};

void setup() {
  // put your setup code here, to run once:
  Serial.begin(115200);
}
void loop() {
  // preprocessing
  //mean
  int n=7500;
  /*for(int i=0;i<n;i++){
    Serial.print(i);
    Serial.print(" ");
    Serial.println(inreal[i]);
  }// for plotting the original signal*/
 double sum1=0;
 for(int j=0;j<n;j++){
    sum1=sum1+inreal[j];
   }
  //Serial.println(sum1);
  //find mean
  double mean=sum1/n;
  //Serial.println(mean);          //all clear
   for(int i=0;i<n;i++)
  {
    inreal[i]=inreal[i]-mean;
  }
  /*for(int i=0;i<n;i++)
  {
     Serial.println( inreal[i]);
  }*/

  //find max value
  double maxVal = inreal[0];

  for (int i = 1; i < 7500; i++) {
      if (inreal[i] > maxVal) {
         maxVal = inreal[i];
      }
```

```
  }
  Serial.println(maxVal);  //fine

  //normalise the signal
  for(int j=0;j<n;j++){
    inreal[j]=inreal[j]/maxVal;
    //Serial.println(inreal[j]);  //fine
  }

  //this is for coefficient calculation for resonator
  double a1=-2*0.997*cos(0.3*2*PI);
  double a2=0.997*0.997;
  double b=(1-0.997)*sqrt(1+(0.997*0.997)-2*0.997*cos(2*0.3));


 //resonator
 double inrealn[7500]={};

 for(int j=2;j<n;j++){
   inrealn[j]=-a1*inrealn[j-1]-a2*inrealn[j-2]+b*inreal[j];
 }

 //dft
 //double inmg[250]={};
 double outreal[7500]={};
 double outimg[7500]={};
 double magi[7500]={};

 dft(inrealn,outreal,outimg,magi,n);//works good
 //Serial.println(magi[],6);
 /*for(int i=0;i<n;i++){
   //Serial.print(i);
   //Serial.print(",");
   Serial.println(magi[i],6);   //
 }// this is for plotting magnitude spectrum*/

 //find maximum peak in spectrum
 double fmaxi = -100000.00;

 for (int i = 0; i < n; i++) {
   fmaxi = maxi(magi[i],fmaxi);
   //Serial.println(fmaxi);

   // delay(10);
 }


 //find breathing rate
 double resp_rate=fmaxi*60;
 //Serial.println(resp_rate);
}



//for dft
```

```
void dft(const double inrealn[],double outreal[],double outimg[],double magi[],size_t n){//double inmg is removed ass it is asumed to be 0

   for (int k=0;k<n;k++){
     double sumreal=0;
     double sumimg=0;
     for (int t=0;t<n;t++){
        double angles=2*PI*t*k/n;
        sumreal+=inrealn[t]*cos(angles);
        sumimg+=-inrealn[t]*sin(angles);

     }

     outreal[k]=sumreal;
     outimg[k]=sumimg;
     magi[k]=sqrt(pow(outreal[k],2)+pow(outimg[k],2));
     //Serial.println(k);
     //Serial.println(',');
     //Serial.println(magi[k],6);  //works good
   }
}

float maxi(float a, float b){
   if(a>b){
      return a;
   }
   else{
      return b;
   }

}
```

## ARDUINO IMPLEMNTATION OUTPUT:

```
:1:11:41.656 -> 43.93
```

## ARDUINO IMPLEMENTATION USING DUE BOARD:

## CONCLUSION:

A simple method is presented for estimating the RR from a PPG signal. The proposed method is based on the two-pole digital resonator and fast Fourier transform (FFT) algorithms. The proposed method is evaluated using the benchmark metrics on the standard PPG datasets. Ardunio and matlab implementation of the same is implemented successfully.

## REFERENCES:

Dataset reference: https://archive.physionet.org/cgi-bin/atm/ATM