Practical simulation experiments using SIMULINK

Car suspension system (a worked example)

Object:

To simulate the behaviour of a car suspension system when the car is driven over an obstacle. Different situations, for example when the speed or the loading of the car changes, will be examined.

Instructions:

The car suspension for one wheel is shown in figure 1. A simplified model of a car suspension system can be described by two linear differential equations simulating the time responses of the car body and tyre displacement. Here a quarter of the car body mass(M) is taken into account but the half of the axle mass together with the mass of the wheels is neglected. The mass-spring-damper system shown in figure 2 can be developed to approximate the car suspension system.

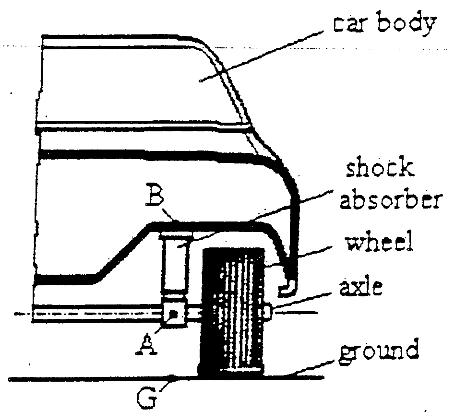


Fig. 1

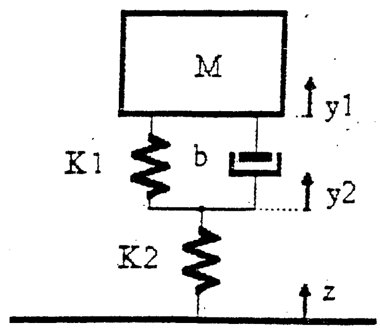


Fig. 2

The basis for the ,modelling of the system is Newton's second law of motion:

$$\sum F = Ma \quad , \qquad (1)$$

where: M = mass (kg); a = acceleration (m/s^2); F = force (N).

Two different forces operate in the system:

-the mechanical spring force F_s required to deflect a spring a distance y from its natural length:

$$F_s = ky \quad , \tag{2}$$

where k is the spring coefficient (N/m);

-the viscous damper force F_d required to move one end of the dashpot at velocity v relative to the other end:

$$F_d = bv = b\dot{y} \quad , \tag{3}$$

where b is the damping coefficient (Ns/m).

Applying these equations to the system, at the two equilibrium points A and B, the following two differential equations are obtained:

$$k_1(y_2 - y_1) + k_2(y_2 - z) + b(\dot{y}_2 - \dot{y}_1) = 0 \quad , \tag{4}$$

$$k_1(y_1 - y_2) + b(\dot{y}_1 - \dot{y}_2) = -M\ddot{y}_1 \quad , \tag{5}$$

where y_1 and y_2 are the vertical displacements of mechanical point B (car body), and A (wheel) respectively, relatively to the initial position (when the car is standing), and z is the ground reaction (given by the height of the road profile). k_1 and k_2 are the spring constants of the car and tyre and b represents the constant of the car shock absorber.

After rearrangement of (4) and (5), the following model is obtained:

$$\ddot{y}_1 = -\frac{b}{M}\dot{y}_1 - \frac{k_1}{M}y_1 + \frac{b}{M}\dot{y}_2 + \frac{k_1}{M}y_2 \quad , \tag{6}$$

$$\dot{y}_2 = -\frac{k_1 + k_2}{b}y_2 + \dot{y}_1 + \frac{k_1}{b}y_1 + \frac{k_2}{b}z \quad , \tag{7}$$

or, in terms of transfer functions:

$$Y_1(s) = G_{y1}(s)Y_2(s) \quad , \tag{8}$$

$$Y_2(s) = G_{y2}(s)Y_1(s) + G_z(s)Z(s) \quad , \tag{9}$$

where:

$$G_{y1}(s) = \frac{(b/M)s + k_1/M}{s^2 + (b/M)s + k_1/M} \quad , \tag{10}$$

$$G_{y2}(s) = \frac{s + k_1/b}{s + (k_1 + k_2)/b} \quad , \tag{11}$$

$$G_z(s) = \frac{k_2/b}{s + (k_1 + k_2)/b} \quad . \tag{12}$$

The following parameters will be used in the analysis of this system:

$M = 500$ kg;

$k_1 = 7500$ N/m;

$k_2 = 170000$ N/m;

$b = 1250$ Ns/m.

The suggested profile of the road is depicted in figure 3. Consider that the car is moving with a constant speed of v=10 m/s.

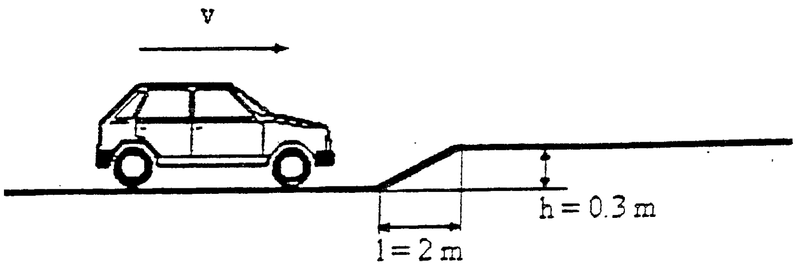


Fig. 3

Task 1:

Simulate the car body and wheel displacements over the profile shown in figure 3 by using SIMULINK.

Task 2:

Demonstrate the effect of increasing the speed (at v=20 m/s) or the loading of the car (M=700 kg);

Task 3:

Improve the behaviour of the car (maximum one oscillation with an amplitude under 15% from the obstacle's high when passing over) in the most unfavourable situation (say v=20 m/s, M=700 kg) by changing the coefficient b of the shock absorber.

Solutions (car suspension system)

Task 1:

- According to equations (8)…(12) the block diagram of the simulated car is (figure S1):

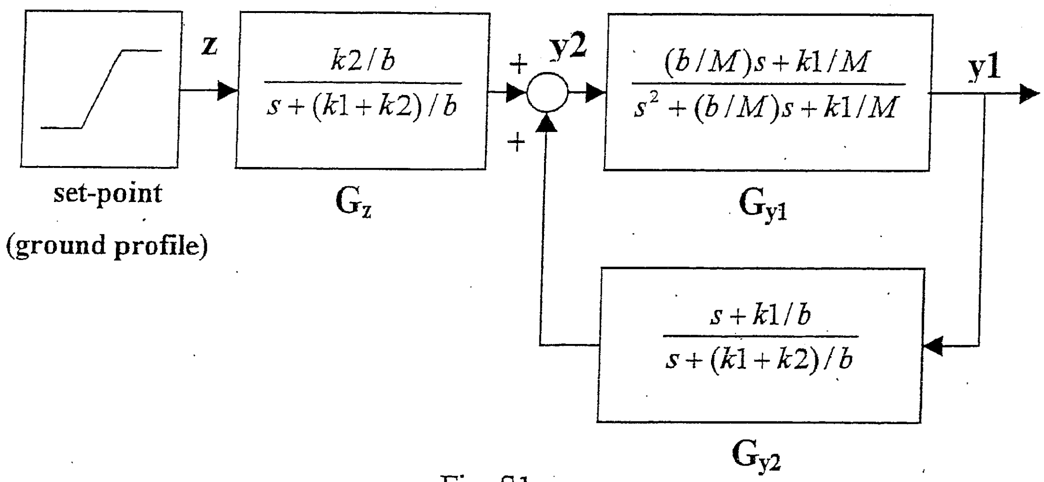


Fig. S1

- The corresponding SIMULINK model may be drawn as in figure S2.

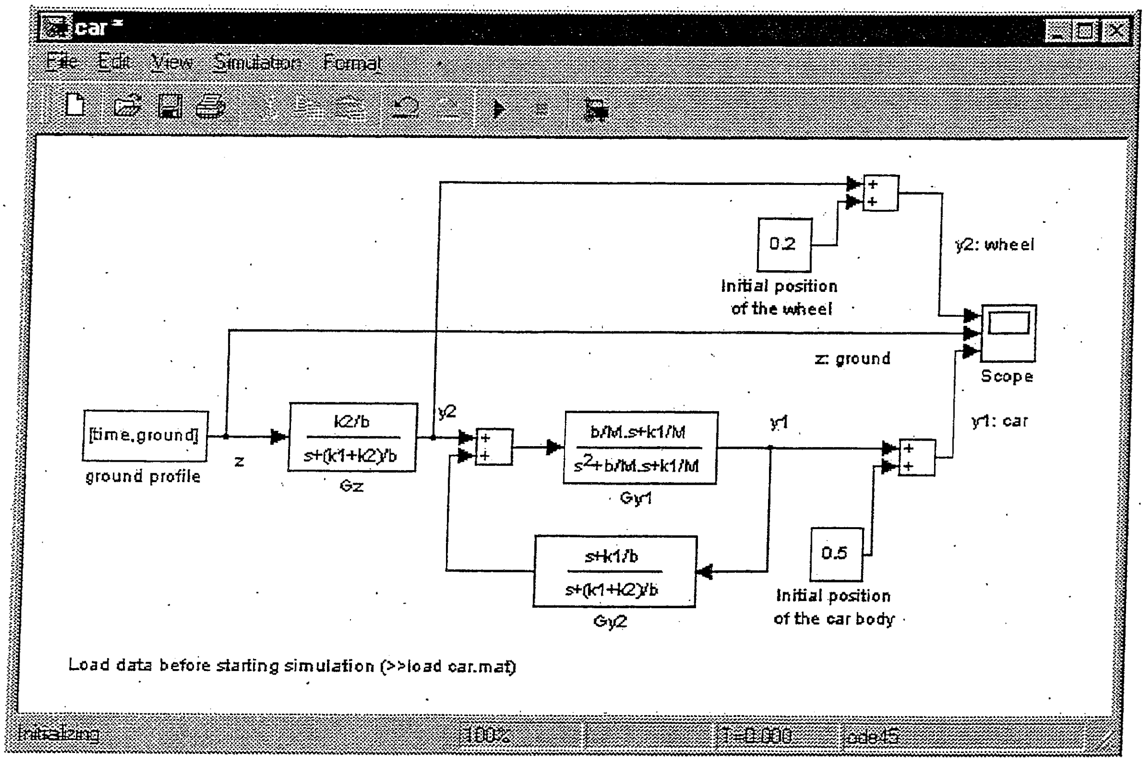


Fig. S2

NOTE: The parameters of the blocks G_{y1}, G_{y2}, and G_z could be directly given with the SIMULINK **Transfer Fcn** blocks. Since the simulation should be repeated for various system parameters, it is better to use symbolic variables or any MATLAB expression for defining the block coefficients. For example, the parameter "dialog box" associated with transfer function G_{y1} is shown in figure S3.

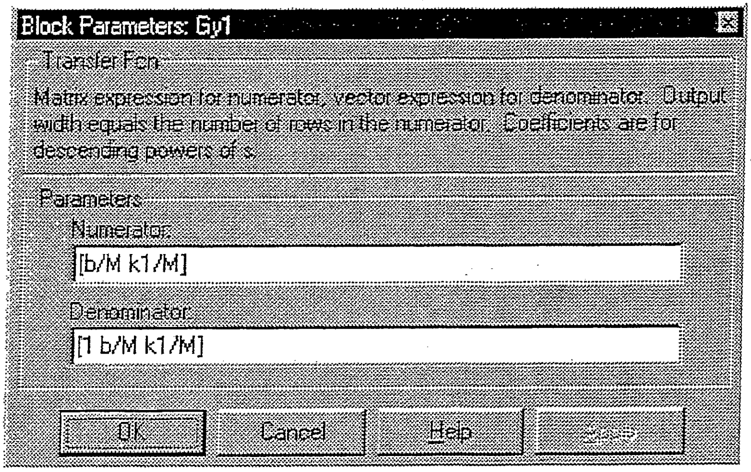


Fig. S3

Since MATLAB cannot perform symbolic algebra, before starting the simulation, the parameter values are to be defined in the MATLAB workspace:

```
M = 500;                                                (M1)
k1 = 7500;                                              (M2)
k2 = 170000;                                            (M3)
b = 1250;                                               (M4)
```

- The reference z for the simulation model reflects the road profile. Consequently, the **From Workspace** block can be used to define the time dependence of the road height.

For a road of 50 m length, assuming that the obstacle occurs after 10 m, and the car speed is v = 10 m/s, the following MATLAB statements define the corresponding reference input:

```
v = 10;                                                 (M5)
l = 2;                                                  (M6)
h = 0.3;                                                (M7)
time = [0;10/v;(10+l)/v;50/v];                          (M8)
ground = [0;0;h;h];                                     (M9)
```

- The `ode45` solver can successfully be used for this simulation. Concerning timing, only the "stop time" should be specified, corresponding to the value:

```
t_stop = 50/v;                                          (M10)
```

Note that the integration interval used in simulation is automatically set by SIMULINK.

Thus, "the dialog box" required for setting the simulation parameters is shown in figure S4.

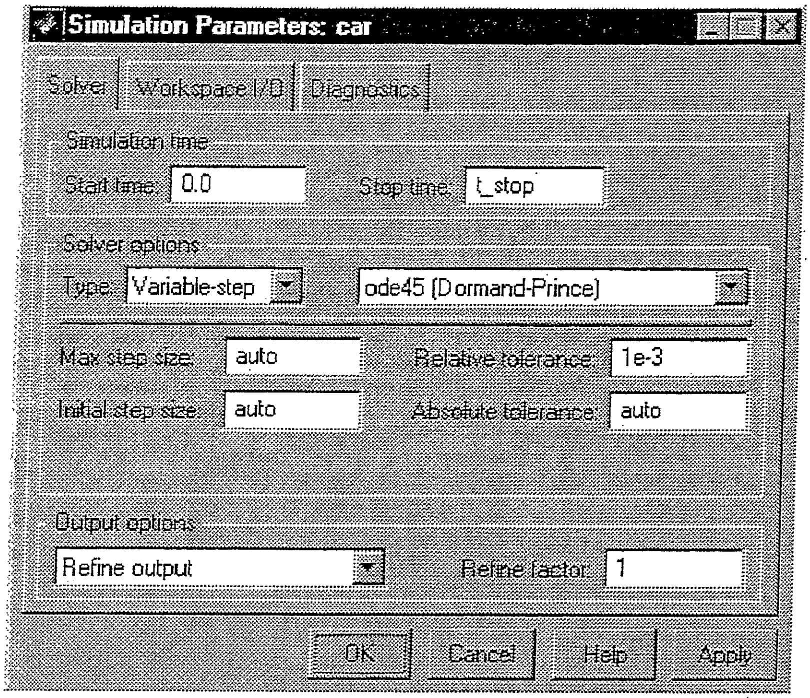
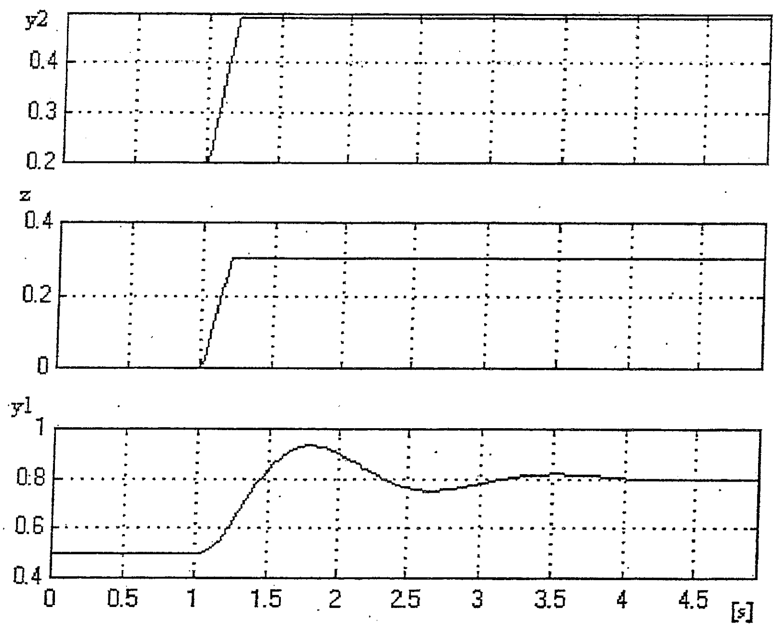


Fig. S4

- The Scope block connected to y_2 and y_1 allows one to display the behaviour of the wheel and of the car respectively.
- The simulation results are displayed in figure S5.



Task 2:

- When the speed is increased to the value v=20 m/s, the MATLAB variable v should be redefined as:

```
v = 20;                                                    (M11)
```

and the statements (M8) and (M10) should run again.

- In this case, the simulation results are shown in figure S6.

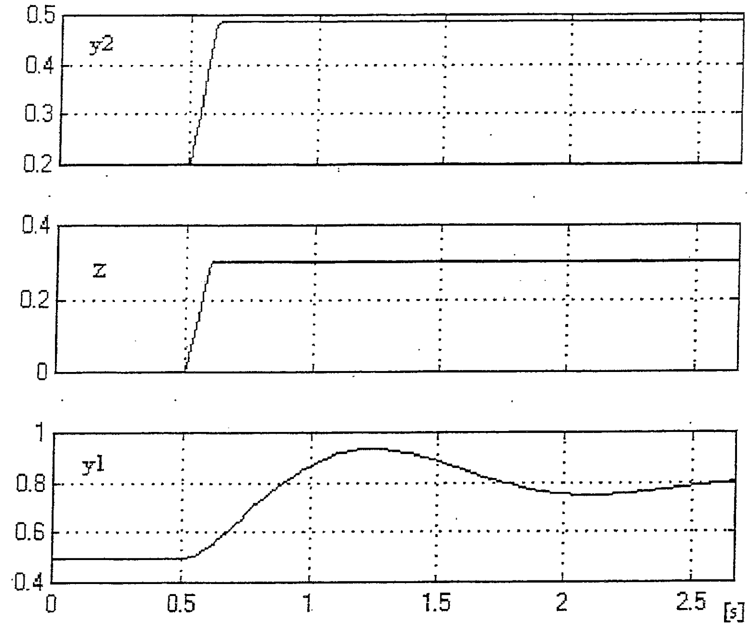


Fig. S6

- Now if the car mass M is also increased to the value 700 kg, the results are given in figure S7.

For performing the simulation, statement (M1) should run again:

```
M = 700;                                                        (M12)
```

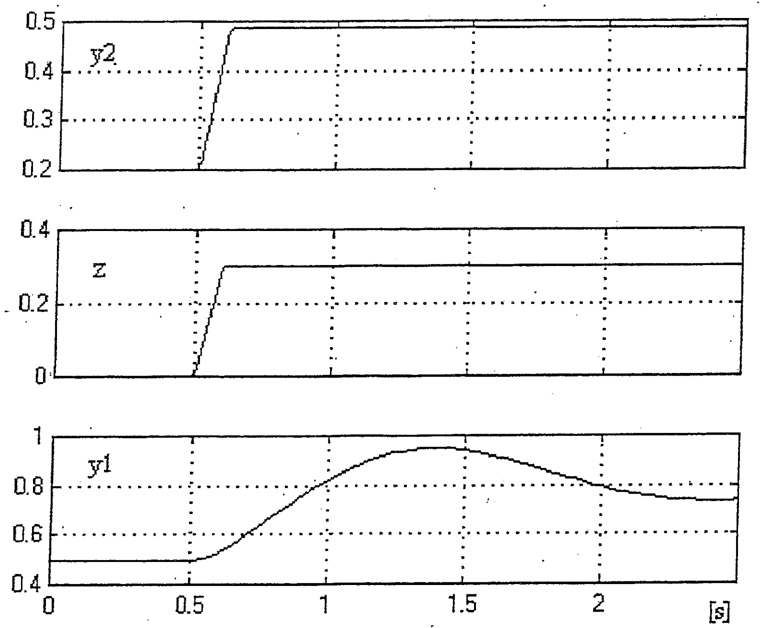


Fig. S7

Task 3:

- The improvement of the car behaviour can be achieved by changing the coefficient b of the shock absorber. The effect of this change can be studied by performing successive simulations, each of them corresponding to a given value of b.

The results obtained for three different situations:

(a) b = 1250

(b) b = 5500

(c) b = 12500

are displayed in the figure S8.

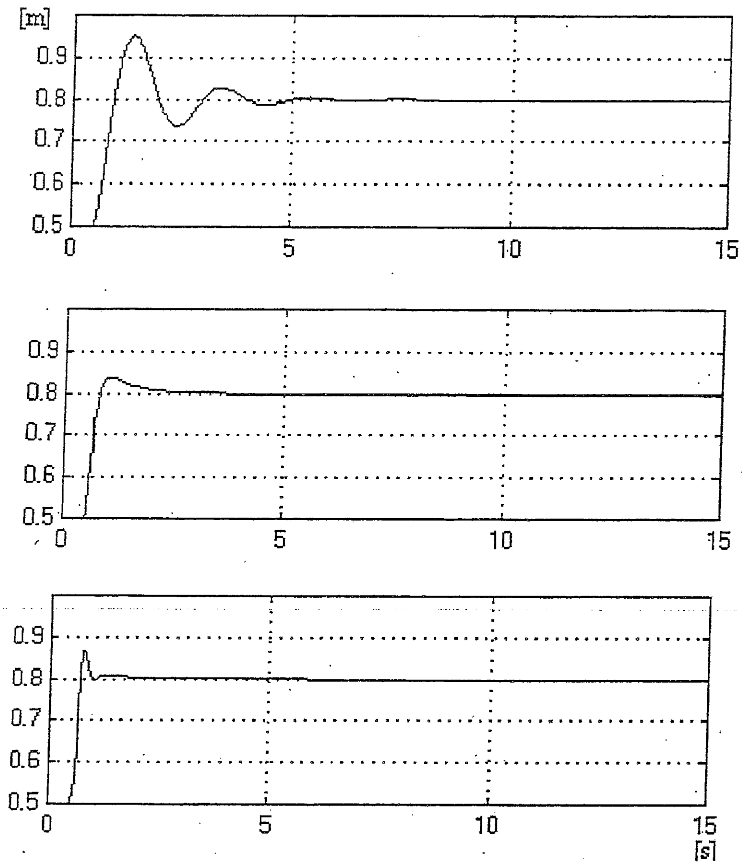


Fig. S8

Obviously, the (b) case ensures the required improvement.

Positioning system

Object:

To study the control system employed for positioning in a machine tool. Improving the behaviour of the system by parameter adjustments is also required.

Instructions:

The schematic diagram of a simplified positioning system is shown in figure 1. The angular displacement of the DC motor is translated into a linear movement by means of a mechanical transmission chain. The input voltage u applied to the armature circuit of the motor is supplied by an analogue controller based on the tracking error e between the prescribed reference z and the position y of the tool. The measured position y_m of the tool is obtained by means of a potentiometer - based transducer.

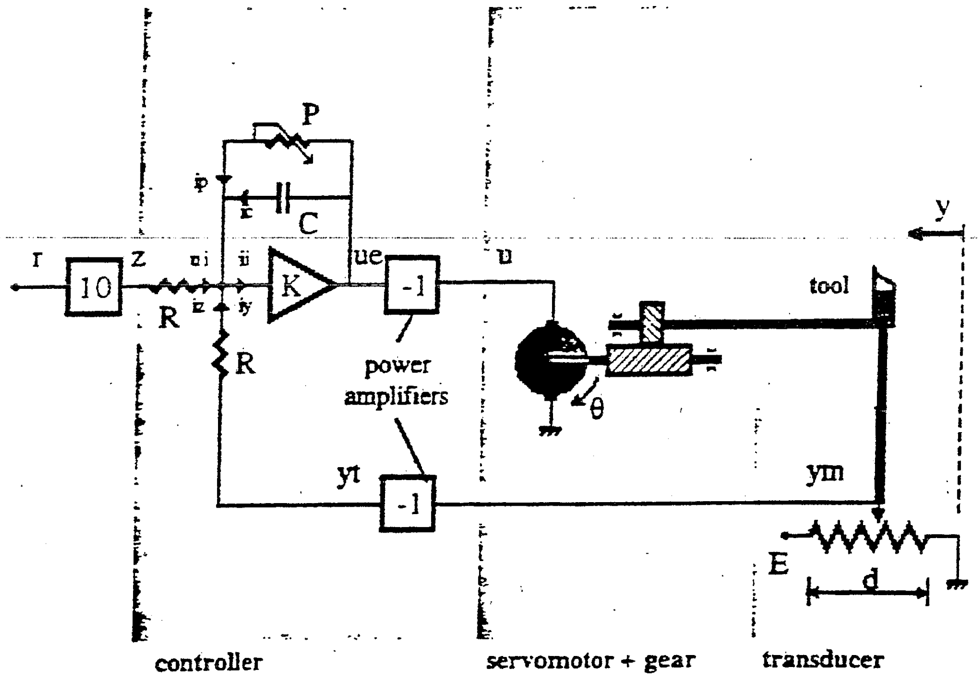


Fig. 1

Considering an ideal operational amplifier (infinite gain $K \to \infty$, infinite bandwidth, infinite input and zero output impedance, zero drift, virtual ground for the input $u_i = 0$, $i_i = 0$) Kirchhoff's law for the input summing junction describes the behaviour of the controller:

$$i_z + i_p + i_c + i_y = i_i = 0, \tag{1}$$

where,

$$i_z = \frac{z - u_i}{R} = \frac{1}{R} z \quad , \tag{2}$$

$$i_p = \frac{u_e - u_i}{P} = \frac{1}{P} u_e \quad , \tag{3}$$

$$i_c = C \frac{d}{dt} (u_e - u_i) = C \dot{u}_e \quad , \tag{4}$$

$$i_y = \frac{y_t - u_i}{R} = \frac{1}{R} y_t \quad . \tag{5}$$

Combination of equations (1)...(5) gives:

$$\dot{u}_e = -\frac{1}{PC} u_e - \frac{1}{RC} z - \frac{1}{RC} y_t \; . \tag{6}$$

Taking into account the presence of the two power amplifiers:

$$u = -u_e \quad , \tag{7}$$

$$y_t = -y_m \quad , \tag{8}$$

the complete model of the controller becomes:

$$\dot{u} = -\frac{1}{PC} u + \frac{1}{RC} z - \frac{1}{RC} y_m \quad , \tag{9}$$

which, expressed in terms of transfer function, is:

$$U(s) = G_c(s)\big(Z(s) - Y_m(s)\big) \quad , \tag{10}$$

where:

$$G_c(s) = \frac{1/(RC)}{s + 1/(PC)} \quad . \tag{11}$$

For the servo motor, an integral-type model is assumed so it follows that the motor shaft velocity is:

$$\dot{\theta} = k_s u \quad , \tag{12}$$

and therefore:

$$\theta = k_s \int_0^t u \quad . \tag{13}$$

If k_m denotes gear ratio, the transfer function of the servomechanism is:

$$Y(s) = G_m(s) U(s) \quad , \tag{14}$$

with:

$$G_m(s) = k_m k_s \frac{1}{s} \quad . \tag{15}$$

Finally, the gain of the transducer can be stated as:

$$G_t(s) = \frac{E}{d} \qquad . \tag{16}$$

where d is the maximum allowed displacement of the tool (the length of the potentiometer) and we can therefore write:

$$Y_m(s) = G_t(s)Y(s) \qquad , \tag{17}$$

The following data must be considered:

$R = 10\ k\Omega$;
$P = 100\ k\Omega$;
$C = 0.33\ \mu F$;
$k_s = 52.5$ rad/sV;
$k_m = 5\ 10^{-3}$ m/rad;
$E = 10$ V;
$d = 1$ m.

Note that the reference input z is an electric signal which takes values in the [0, 10]V domain, like the measured output y_m. On the other hand the displacement y of the tool is expressed in the [0, 1]m domain. For a proper analysis a real set point r ([0,1]m) should be considered which means that a scaling adapter must be used.

The block scheme of the system is presented in figure 2.

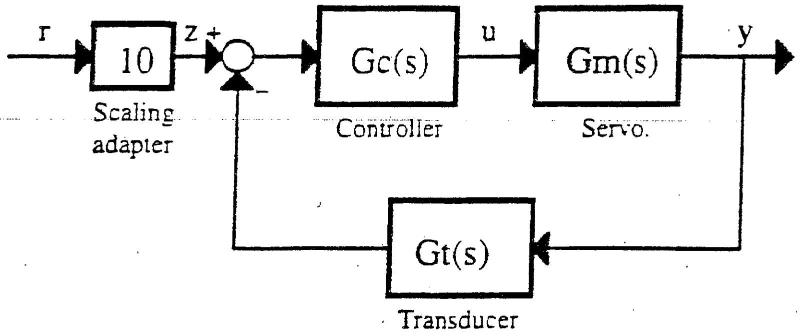


Fig. 2

Task 1:

Investigate the step response of the system (the amplitude of the step may be 0.5).

Task 2:

Improve the behaviour of the system (fastest response with no overshoot) by an appropriate adjustment of P.

Task 3:

Study the effect of saturation of the command signal: $u \in [-10, 10]$ V.

Task 4:

Observe the effect of a 0.02 backlash in the gear.

NOTE: You can find "saturation" and "backlash" blocks in the "non-linear" library.

Hydraulic tanks systems

Object:

To simulate two coupled hydraulic tanks systems and to investigate the characteristics of the resulting systems when the tanks are connected in different ways.

Instructions:

A commonly occurring control problem in the chemical process industries is the control of fluid levels in storage tanks, chemical blending and reaction vessels. A typical situation is one in which it is required to supply fluid to a chemical reactor at constant rate. To this end one or more cascaded hold-up tanks may be used to smooth out any variations in the upstream supply flow by maintaining constant levels of fluid in the reservoirs. A parallel situation may occur when the tanks are themselves reaction or blending vessels in which it is required to hold constant levels.

In this simulation exercise a cascade of two identical tanks is considered.

The fluid is pumped into first tank by a variable- speed pump driven by an electric motor. Neglecting the time constants of the pump, which are considerably smaller than the time scale of the fluid level dynamics, the pump behaviour can be expressed by the following equation:

$$q_i = k_p u \quad , \tag{1}$$

where q_i represents the output flow rate of the pump, u is the input voltage to the electric motor and k_p is a constant depending on the pump.

For a single tank the procedure of modelling is based on the mass balance principle:

accumulation = input - output.

Referring to figure 1 the above principle can be described as:

$$A\dot{h} = q_i - q_o \quad , \tag{2}$$

where q_i and q_o are the input and output flow rates, h is the level of liquid and A is the cross-section area of the tank.

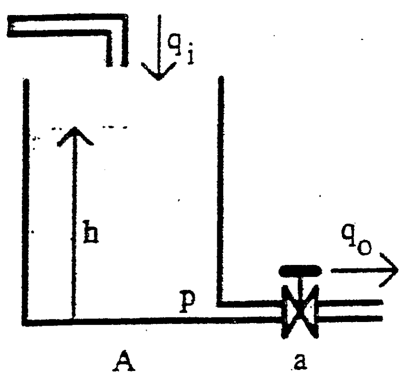


Fig. 1

q_i = flow rate of liquid into tank one;

q_1 = flow rate of liquid out of tank one;

q_o = flow rate of liquid out of tank two;

h_1 , h_2 = the level of liquid in tanks one and two;

A_1 , A_2 = cross-sectional areas of tanks one and two;

a_1 , a_2 = cross-sectional areas of tank's outlets;

C_1 , C_2 = discharge coefficients;

k_p = constant of the pump.

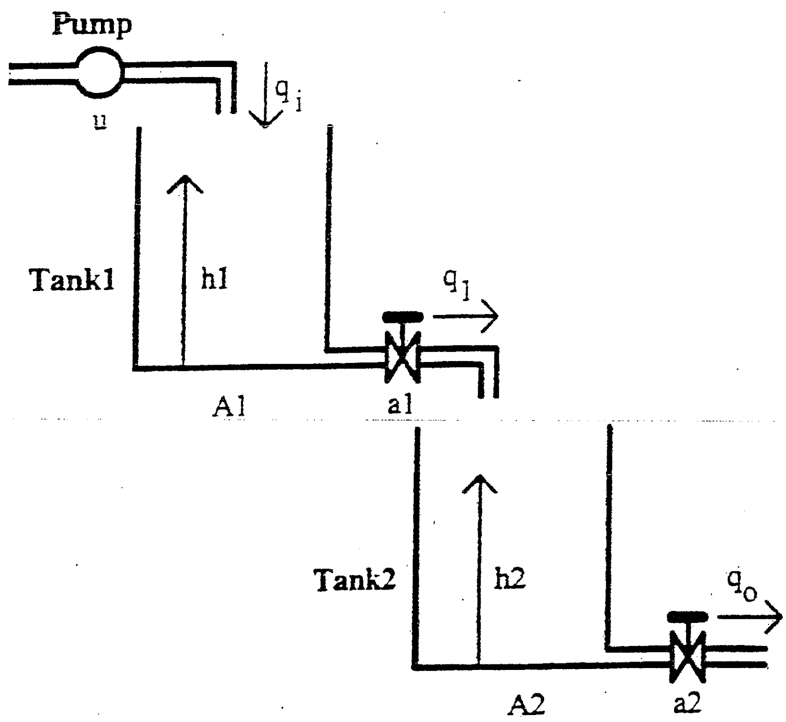


Fig. 2

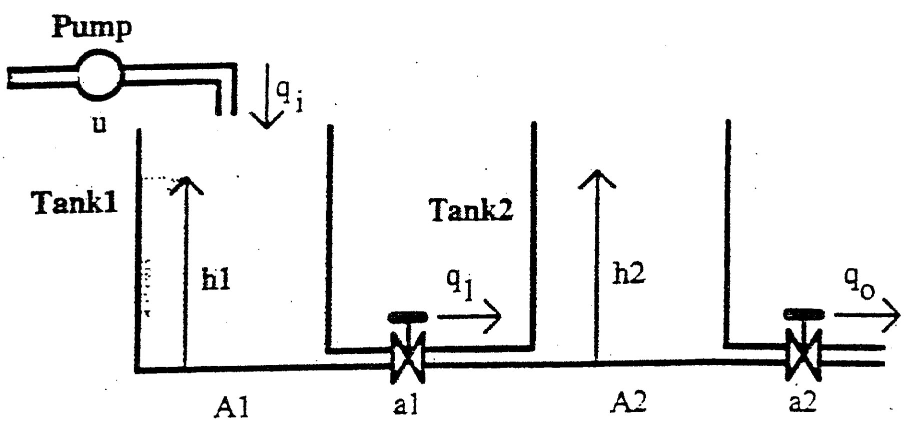


Fig. 3

The flow of liquid out of the tank can be calculated from the equation:

$$q_o = \frac{1}{R}(p - p_0) \qquad , \tag{3}$$

where p and p_0 represent the pressure at the bottom of the tank, and the atmospheric pressure, respectively and R is the hydraulic resistance of the output valve; R has a inverse dependence on the cross-sectional area 'a' of the tank's outlet:

$$R = k\frac{1}{a} \qquad . \tag{4}$$

Note that the relations (3) and (4) describe a linearized model of the valve which is acceptable under the assumptions of small variations of the pressures and laminar flow through the restriction (valve).

Since the pressure p is:

$$p = \rho g h + p_0 \qquad , \tag{5}$$

we have:

$$q_o = \frac{\rho g}{R} h = \frac{\rho g}{k} a h \qquad , \tag{6}$$

and thus:

$$q_o = Cah \qquad , \tag{7}$$

where ρ is the density of the liquid, g represents the gravitational constant and C is the discharge coefficient.

Combining equations (2) and (7) the following linearized model of the tank is obtained:

$$A\dot{h} = q_i - Cah . \tag{8}$$

The systems to be investigated are shown in figures 2 and 3 respectively. For both hydraulic plants the control variable is the input voltage u to the electric motor which drives the pump. The pump supplies the first tank with a flow rate q_i . The liquid flows out of tank one into tank two and then runs out of this tank at a rate q_o . The controlled variable is the liquid level h_2 in the second tank, while the auxiliary variable is the level h_1 in the first tank.

For the first plant under investigation (figure 2) the following equations describe the system:

$$\begin{aligned} q_i &= k_p u \\ A_1\dot{h}_1 &= q_i - q_1 = q_i - C_1 a_1 h_1 \qquad , \\ A_2\dot{h}_2 &= q_1 - q_o = C_1 a_1 h_1 - C_2 a_2 h_2 \end{aligned} \tag{9}$$

where:

u = input voltage to the motor (pump);

Performing the Laplace transformation on equations (9) we obtain:

$$Q_i(s) = k_p U(s) \quad , \tag{10}$$

$$H_1(s) = G_1(s) Q_i(s) \quad , \tag{11}$$

$$H_2(s) = G_2(s) H_1(s) \quad , \tag{12}$$

where:

$$G_1(s) = \frac{1}{A_1 s + C_1 a_1} \quad , \tag{13}$$

$$G_2(s) = \frac{C_1 a_1}{A_2 s + C_2 a_2} \quad . \tag{14}$$

The corresponding block diagram of the system is presented in figure 4:

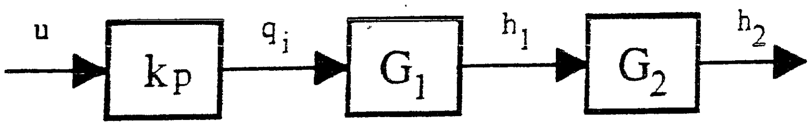


Fig . 4

The system shown in figure 3 consists of the same tanks used to form the system in figure 2, only this time the tanks are connected together by a pipe. In this new case the system is described by:

$$q_i = k_p u$$

$$A_1 \dot{h}_1 = q_i - q_1 = q_i - (q_{12} - q_{21}) = q_i - C_1 a_1 h_1 + C_1 a_1 h_2 \quad , \tag{15}$$

$$A_2 \dot{h}_2 = q_1 - q_o = (q_{12} - q_{21}) - q_0 = C_1 a_1 h_1 - C_1 a_1 h_2 - C_2 a_2 h_2$$

or, in terms of transfer functions:

$$Q_i(s) = k_p U(s) \quad , \tag{16}$$

$$H_1(s) = G_1(s) Q_i(s) + G_r(s) H_2(s) \quad , \tag{17}$$

$$H_2(s) = G_2(s) H_1(s) \quad , \tag{18}$$

where:

$$G_1(s) = \frac{1}{A_1 s + C_1 a_1} \quad , \tag{19}$$

$$G_r(s) = \frac{C_1 a_1}{A_1 s + C_1 a_1} \quad , \tag{20}$$

$$G_2(s) = \frac{C_1 a_1}{A_2 s + (C_1 a_1 + C_2 a_2)} \quad . \tag{21}$$

The corresponding block diagram of the system is presented in figure 5:

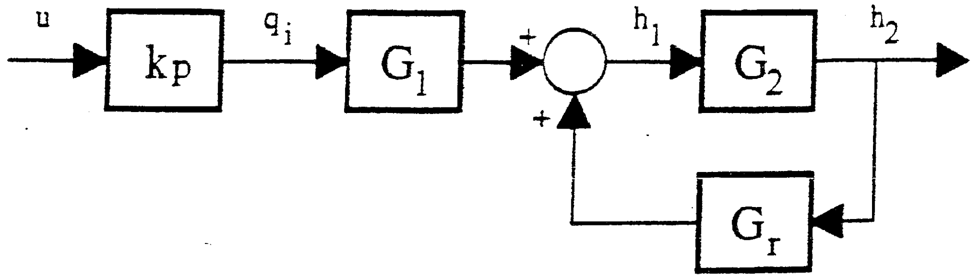


Fig. 5

The system data are:

$A_1 = A_2 = 0.01\ m^2$

$a_1 = 0.4\ 10^{-4}\ m^2$

$a_2 = 0.32\ 10^{-4}\ m^2$

$C_1 = C_2 = 12\ s^{-1}$

$k_p = 15\ 10^{-6}\ m^3/sV$

Task 1:

Simulate both systems (figures 2 and 3). Compare the performances of these two systems in terms of the response of the levels h_1 and h_2. A step input u having the amplitude 3 may be used in simulations.

Task 2:

Study the performances of the systems when the cross-sectional area a_1 decrease to a value of $0.2\ 10^{-4}\ m^2$. Demonstrate that, if system one is considered, the steady-state value of the level h_2 is not affected by this parametric change. What about the second system ?

Depth control for a torpedo

Object:

This example introduces control system methods employed to stabilise the response of the torpedo to a step change in depth setting. Adjustments of system parameters to obtain a satisfactory response are also required.

Instructions:

The torpedo can be represented in the pitch plane as shown in figure 1.

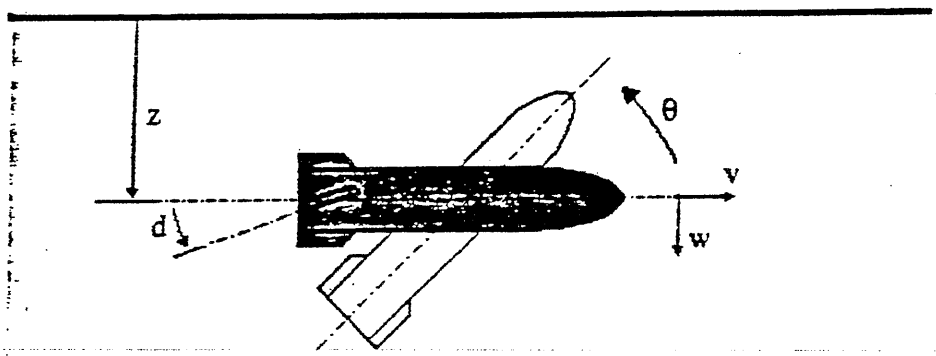


Fig. 1

The following notations are used:

- w = downward velocity;
- v = forward velocity;
- d = elevator deflection;
- θ = pitch rate;
- q = pitch rate;
- z = depth.

The sign conventions for all variables are those stated in figure 1: sign "+" for vertical displacements from up to down and sign "-" for rotations in trigonometric way.

Neglecting the gravity effects, the equations of motion in the pitch plane for a specific unstable torpedo are:

$$\dot{w} = 6.2q - 1.23w + 5.26d \quad , \tag{1}$$

$$\dot{q} = -6.18q + 1.68w + 5.9d \quad , \tag{2}$$

$$\dot{\theta} = q \quad , \tag{3}$$

$$\dot{z} = -w.\cos(\theta) + v.\sin(\theta) \quad . \tag{4}$$

The torpedo depth z is stabilised by means of a feedback controller which supplies the control variable d; the feedback law is:

$$d = z_r - (k_1\theta + k_2 z + k_3 q) \quad , \tag{5}$$

where z_r is the desired depth (set point).

For small variations of the pitch angle:

$$\theta \leq \pm 5^\circ , \tag{6}$$

equation (3) can be linearized as follows:

$$\dot{z} = -w + v\theta \quad . \tag{7}$$

The corresponding block diagram of the linearized system is shown in figure 2.

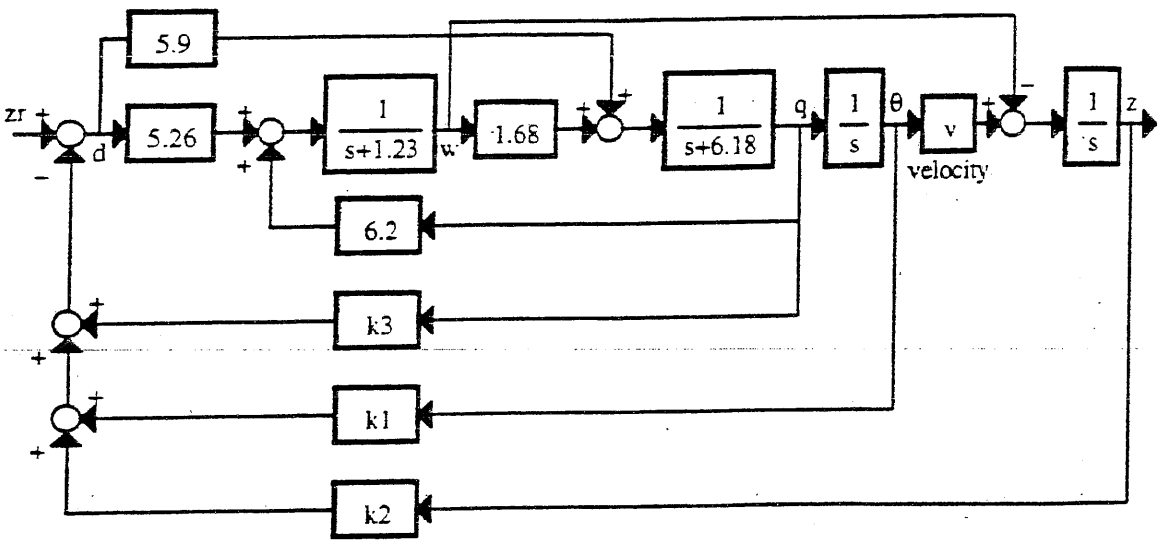


Fig. 2

Task 1:

Investigate the step response of the controlled torpedo given that v = 20 m/s, k_1 = 10, k_2 = 1, k_3 = 1. What happens if k_2 = 0.8 ?

Task 2:

Demonstrate the effect of increasing the speed of torpedo at the value v = 40 m/s. Regain a satisfactory response by adjustments of k_1 and k_3 .

Task 3:

Construct the block diagram of the system including the non-linear equation (4). Simulate the non-linear system and comment the results.

Control systems for a heating process

Object:

To simulate the behaviour of a heating process. Different methods for temperature control, such as relay feedback or PI control are investigated; optimal tuning of the parameters of a PI controller is required.

Instructions:

Temperature control had become a common problem in automatic control, widely used from domestic applications such as ironing or house warming to industrial processes such as heat treatment in heavy industry (e.g. steel, glass).

In this example warming of a room with an electrical heating device is investigated. The design of the heating device and its controller could be performed on site in the room. However, the necessary measurements are very time consuming due to the fact that time constants of such systems are in the range of hours. The exact mathematical analysis is also problematic due to the process non-linearity which creates a complicated dynamic behaviour. Here, simulation offers a very good possibility to find the optimal solution for controlling the system.

An approximate model of the heating process is:

$$\frac{d}{dt}(\theta-\theta_e)+\frac{1}{T}(\theta-\theta_e)=\frac{k}{T}p_{(t-L)} \quad , \qquad (1)$$

where θ represents the temperature in the room (°C), θ_e is the temperature of the environment (°C). p is the power transmitted by the heating device and T, k and L are the time constant, the gain and the time delay of the process, respectively.

Usually, a model which describes the conditions relative to a working point (in our case the environment temperature) is used. So the corresponding output of the process is:

$$y=\theta-\theta_e \quad , \qquad (2)$$

and the model becomes:

$$\dot{y}+\frac{1}{T}y=\frac{k}{T}p_{(t-L)} \quad , \qquad (3)$$

which can also be given in transfer function form:

$$Y(s)=G_w(s)P(s) \quad , \tag{4}$$

where:

$$G_w(s)=\frac{k}{Ts+1}e^{-Ls} \quad , \tag{5}$$

First the situation of controlling the temperature by means of a thermostat (relay feedback) is considered. The block diagram of the closed loop is shown in figure 1.

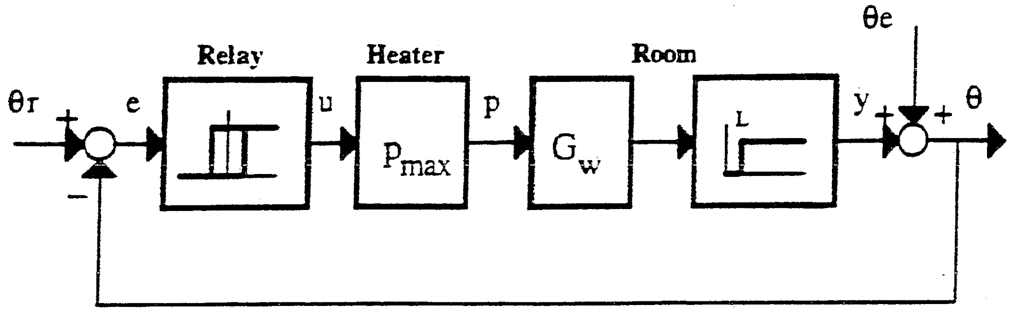


Fig. 1

The heater is switched by the thermostat output u: thus it can be described by the input-output relation:

$$p=p_{max}u \quad , \tag{6}$$

where:

$$u=\begin{cases}0 & \text{when switch off}\\ 1 & \text{when switch on}\end{cases} \quad . \tag{7}$$

The output value u of the thermostat can be set on the difference e between the desired θ_r and measured θ temperatures in the room:

$$e=\theta_r-\theta \quad . \tag{8}$$

Considering a relay with amplitudes 0 (off) and 1 (on) and hysteresis Δe (figure 2) the controller action can be described by:

$$u_{(t)}=\begin{cases}0 & e\le-\frac{\Delta e}{2}\\ 1 & e\ge\frac{\Delta e}{2}\\ u_{(t-\Delta t)} & -\frac{\Delta e}{2}\le e\le\frac{\Delta e}{2}\end{cases} \quad , \tag{9}$$

where u(t-Δt) represents the previous value of u.

Due to the control principle the regulated output will oscillate around the desired temperature. For a specific room and a given maximum power (p_{max}) of the heater the oscillations are mainly influenced by the relay hysteresis. As we desire low oscillations the hysteresis must be relatively narrow; but on the other hand, such hysteresis causes frequent switching of the heating device and this is also undesirable.

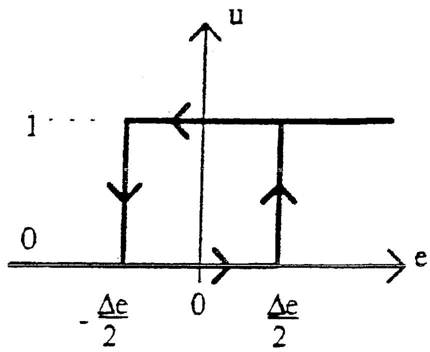


Fig. 2

Better tracking of the temperature desired profile can be obtained using PI controllers. Actually the PI control law represents one of the standard tools to solve process control problems.

Unlike the relay feedback control method discussed above, in this case the heater is supplied by a continuous input signal u taking values within the interval [0, u_{max}]. The power p transferred to the process is:

$$p_{(t)} = p_{max} u_{(t)} \quad . \tag{10}$$

The PI control law can be described by the relation:

$$u_{(t)} = k_p \left(e_{(t)} + \frac{1}{T_i} \int_0^t e_{(t)} dt \right) \quad , \tag{11}$$

where the error e has the same meaning as in (9).

Accordingly the transfer function of the PI controller is:

$$U(s) = G_{PI}(s) E(s) \quad , \tag{12}$$

where

$$G_{PI}(s) = k_p \left(1 + \frac{1}{T_i s} \right) \quad , \tag{13}$$

Figure 3 shows the structure of the PI controlled process.

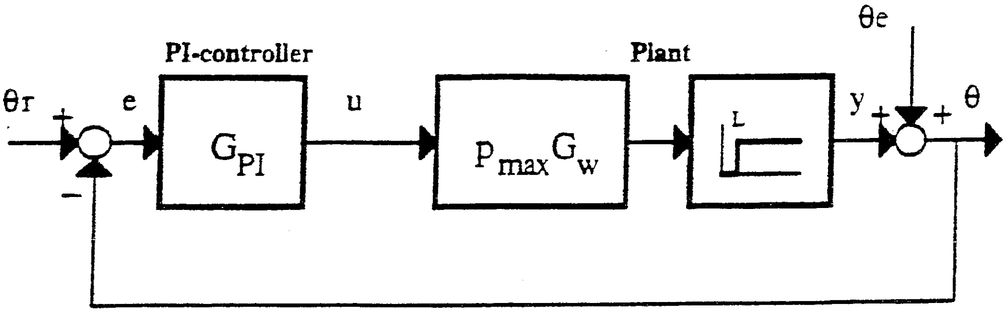


Fig. 3

Different methods for determining the coefficients k_p (proportional gain) and T_i (integration time) can be found in control literature, and are usually known as controller tuning procedures. One of the most popular is the Ziegler - Nichols tuning rule. Briefly, for PI controllers this rule relates the controller coefficients (k_p, T_i) according to:

$$k_p = 0.9 \frac{T}{L} \frac{1}{k_f} \quad , \tag{14}$$

$$T_i = 3L \quad , \tag{15}$$

where k_f denotes the gain of the plant (including the proportional gain of the heater):

$$k_f = p_{max} k \quad , \tag{16}$$

It is worth noticing that the plant parameters are usually determined by simple experiments (monitoring the step response of the plant).

Usually a plant with a PI controller tuned by Ziegler-Nichols rule will exhibit approximately 10% - 60% maximum overshoot in step response. It is possible to make fine tuning so that the closed-loop system will have better transient responses.

Starting from the tuning values determined by Ziegler-Nichols one can improve the closed-loop behaviour by heuristic adjustments of k_p and T_i based on the following remarks:

-A suitable decrease of the controller gain k_p improves the closed-loop stability and thus the overshoot will decrease.

-Due to the pure time delay of the plant, for small values of T_i, the integral action tends to approach rapidly the steady-state and therefore large oscillations of the controller output occur, yielding slightly damped oscillations of the regulated output. So the overshoot can be reduced by increasing T_i. On the other hand, it should be noticed that large values for T_i might increase the duration of the transient state.

Optimal values for k_p and T_i can be found in order to reduce both the overshoot and the time response of the closed-loop. A method which ensures optimality requires minimisation of the integral criterion:

$$IAE = \int_0^t |e_{(t)}| dt \qquad (17)$$

Since the analytical evaluation of this criterion is a difficult task, a numerical solution based on repetitive simulation experiments performed for different pairs (k_p ,T_i) may provide a map of the IAE values; in this way the graphical representation of this map allows one to find a solution in the vicinity of the analytical minimum.

The following data are to be considered:

$k = 2$ °C/kW;

$T = 1$ h;

$L = 0.5$ h;

$P_{max} = 5$ kW/V;

$\theta_e = 15$ °C;

$$\theta_r = \begin{cases} 15^\circ C & t < 0.5h \\ 20^\circ C & t \geq 0.5h \end{cases}$$

Task 1:

Simulate the behaviour of the relay-controlled heating process considering the hysteresis $\Delta e=2$. Perform the same experiment for a decreased value of hysteresis. In both simulations analyse the controller output variations.

Task 2:

Find the Ziegler-Nichols settings for k_p and T_i and simulate the PI-controlled loop.

Task 3:

Improve the performance of the PI-controlled system by using heuristic adjustments of k_p and T_i.

Population dynamics (rabbits and foxes)

Object:

To study growth and decay characteristics of different types of populations, prey and predators, in an ecosystem.

Instructions:

Assuming that in an ecological system there exists only one kind of prey (rabbits) and a single type of predator (foxes); if their population sizes are treated as continuous variables, the logarithmic growth of one population depends only on the other population. Consequently, it can be seen that the logarithmic growth of predators is directly proportional to the population of prey, while the logarithmic growth of prey is inversely proportional to the population of predators.

Considering linear relations between the population of the prey (rabbits), denoted by x_1, and the population of predators (foxes), x_2, the following mathematical model can be developed:

$$\frac{\dot{x}_1}{x_1} = (a_{11} - a_{12}x_2) \quad , \tag{1}$$

$$\frac{\dot{x}_2}{x_2} = (a_{21}x_1 - a_{22}) \quad , \tag{2}$$

or:

$$\dot{x}_1 = (a_{11} - a_{12}x_2)x_1 \quad , \tag{3}$$

$$\dot{x}_2 = (a_{21}x_1 - a_{22})x_2 \quad , \tag{4}$$

It is also assumed here that self-inhibition is absent and that in the absence of prey, the predator population declines, while in the absence of predators the prey population increases.

Here, positive constants a_{11} and a_{22} depend on corresponding growth rates, whereas the positive constants a_{12} and a_{21} represent mutual inhibition factors and are proportional to the size of the other population. These constants can be estimated with the aid of the following assumptions:

-every pair of rabbits produces an average of ten young in one year;

-every fox captures an average of twenty-five rabbits in one year;

-the average age of foxes is five years, which means that 1/5 of foxes die yearly;

-on average, the number of young foxes which survive depends on the available food (this means that the number of young foxes surviving is equal to the number of rabbits divided by 25);

-the observed ecological area is 50 km^2 where the average number of rabbits is $\overline{x_1} = 500$ and foxes $\overline{x_2} = 100$;

-the time unit is considered one year ($\Delta t = 1$).

The model coefficients become under these assumptions:

$$a_{11} = \left.\frac{\dot{x}_1}{x_1}\right|_{a_{12}=0} = \frac{\Delta x_1/\Delta t}{x_1} = \frac{10}{5} = 5 \quad , \tag{5}$$

$$a_{22} = \left.\frac{\dot{x}_2}{x_2}\right|_{a_{21}=0} = -\frac{\Delta x_2/\Delta t}{x_2} = -\frac{-1}{5} = 0.2 \quad , \tag{6}$$

$$a_{12} = \left.\frac{\dot{x}_1}{\overline{\overline{x_1 x_2}}}\right|_{a_{11}=0} = -\frac{\Delta x_1/\Delta t}{\overline{\overline{x_1 x_2}}} = -\frac{25\overline{x_2}}{\overline{\overline{x_1 x_2}}} = -\frac{25}{500} = 0.05 \,, \tag{7}$$

$$a_{21} = \left.\frac{\dot{x}_2}{\overline{\overline{x_1 x_2}}}\right|_{a_{22}=0} = \frac{\Delta x_2/\Delta t}{\overline{\overline{x_1 x_2}}} = \frac{\overline{x_1}/25}{\overline{\overline{x_1 x_2}}} = \frac{1}{25 \cdot 1000} = 0.0004 \quad , \tag{8}$$

For simulation of the ecosystem described by equation (3) and (4) the adequate model is:

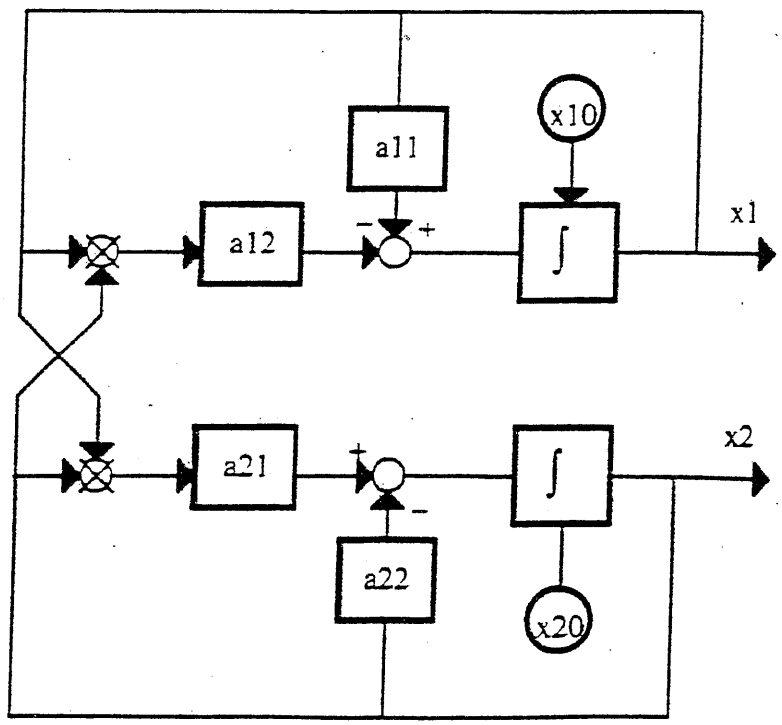


Fig. 1

where x_{10} and x_{20} are the initial conditions (the initial populations of rabbits respectively foxes).

Task 1:

Investigate the dynamics of the two populations for a 20 year period of observation; the model coefficients are those specified in equations (5)...(8) and the initial values are $x_{10} = 520$, $x_{20} = 90$.

Task 2:

Suppose that after 10 years of observation you decide to "correct" the dynamics of the ecosystem (for example you observe that the environment is not able to support without damage a population larger than 600 rabbits and thus you want to reduce the amplitude of the oscillations of the prey population). To achieve this, you must add (remove) at t=10 years a suitable number of rabbits and foxes. Attempt to do this and observe the results.