Analysis of Euler Solvers for Temperature Equation in Channel Flows

Girish Nivarti (50951094)

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I The error norms for each mesh were plotted in Fig. 1. For a mesh 400x160, and L_2 norm of 0.0014011 was found for flux calculation and 9.169×10^{-6} for source calculations. There is a great magnitude difference in these norms, and it is reflected on all mesh sizes. This is perhaps due to the fact that flux calculation requires terms involving temperature field, which adds to the overall error.

II Both explicit and implicit schemes were used to calculate the solution of the given problem with a fully developed velocity profle, and a fully developed inlflow for temperature. The norms of solution have been plotted for each mesh size, for both implicit (Fig. 4), and explicit (Fig. 3) Euler methods. These plots and with exact values in the following tables, demonstrate that the methods are 2^{nd} order accurate. Implicit method was found to converge, to the set convergence criterion of $dT_{max} = 10^{-8}$, in much fewer steps. Despite using a large time step (we will see later that much larger time steps can be taken while using Implicit methods as opposed to Explicit methods), the same values of error norms are arrived at in 30 times less steps. Something phenomenal! Twice the number of steps (Table. 2 were taken for the time step of 0.1 in the Implicit solution on a 200x80 mesh. The error profile has been plotted (Fig. 2 for the mesh, errors are similar in both meshes and mimic the fully developed velocity profile everywhere except near the entrance of the channel where the exact profile has been specified at inflow.

III The efficiency test was conducted, and the relevant features have been tabulated in Table. 3 for explicit scheme, and Table. 4 for implicit method. There was no observable maximum/limiting time step for Implicit scheme. However, as the step size increased beyond certain values, the simulation time went up, iterations up to convergence went up. This is against our intuition as with large time steps we are in effect pacing towards steady state at with bigger leaps, and must reach there sooner. This is not observed for explicit schemes, although the explicit schemes have a much lower maximum time step. This is perhaps due to the approximate factorisation, which gets large values on RHS for large time steps, possibly increasing the maximum change in solution at every iteration. Explicit schemes were simulated using the maximum possible time step which was estimated by experimenting with different possible values of dt.

IV The gradients were calculated for a slightly modified problem (with new boundary conditions on top wall, and a realistic Ec = 0.001). Evidence that the domain size of 25.0 which was chosen for this calculation, has been provided as a plot (Fig. 6) of gradient at the bottom wall, with varying channel lengths. These values lie on the same curve that has a global maximum in the region of x = 12.5. Grid convergence study was done, and has been tabulated in Table 5. A final estimate (after using Richardson extrapolation) gives us: $\frac{\partial T}{\partial y}_{max,y=0} = 1.1291 \pm 6.335 \times 10^{-5} at x = 12.5 \pm 0.006281$. The apparent order of the method was calculated as 1.80881. Plots of the gradient at bottom wall (Fig. 7), and gradient across the channel (Fig. 8) have been drawn for a mesh of size 200x80, and channel length 25.0.

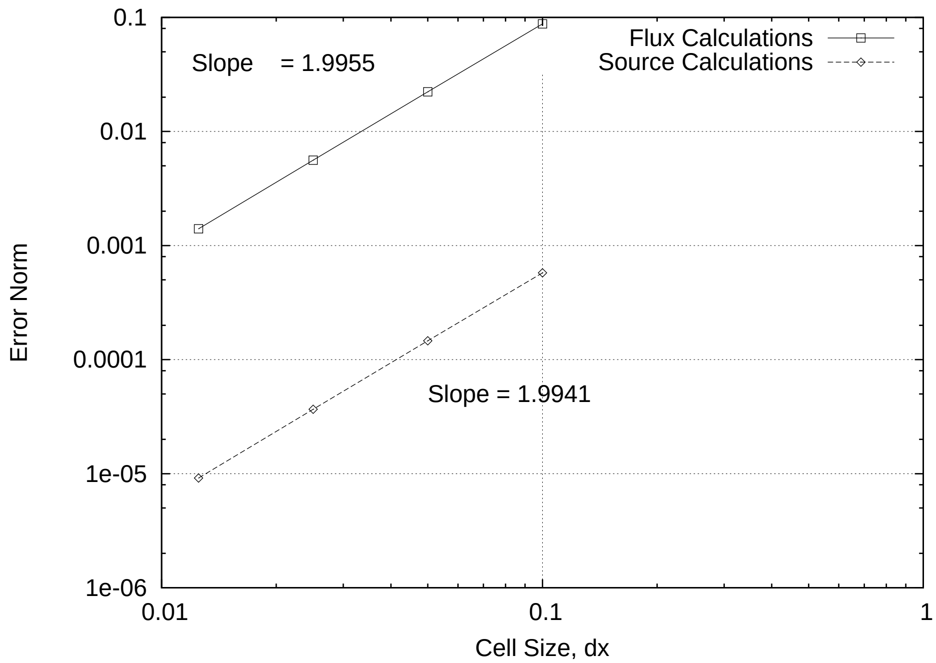

Figure 1: Norms for flux and source calculations using the finite volume method. Norms (L_2) are plotted on a log-log graph with varying cell size (in x direction) as the mesh was refined. The order of accuracy of the method is given by the slope of the line.

Mesh Size	L_2 Norm	Iterations
25x10	0.0112145	2987
50x20	0.00288149	1617
100x40	0.000725072	1467
200x80	0.000181544	1467

Table 1: Variation of error norms with mesh size for an Explicit Euler solution scheme using time step, dt = 0.002 for all meshes. The order of the method was found to be 1.9906.

Mesh Size	L_2 Norm	Iterations
25x10	0.0112145	59
50x20	0.00288149	52
100x40	0.00072508	52
200x80	0.000181553	101

Table 2: Variation of error norms with mesh size for an Implicit Euler solution scheme using time step, dt = 0.1 for all meshes. The order of the method was found to be 1.991.

Mesh Size	Maximum Time Step	Iterations	Run-Time
25x10	0.01	4377	240 ms
50x20	0.007	3206	580 ms
100x40	0.005	1168	870 ms
200x80	0.002	1467	4350 ms

Table 3: Maximum time step simulations for different mesh sizes using Explicit Euler scheme. Corresponding run time, and iterations have been reported.

Mesh Size	Maximum Time Step	Iterations	Run-Time
25x10	10.0	639	470 ms
25x10	5.0	347	250 ms
25x10	4.0	284	220 ms
50x20	5.0	775	1460 ms
100x40	5.0	1279	6980 ms
200x80	5.0	2500	43870 ms

Table 4: Maximum time step simulations for different mesh sizes using Implicit Euler scheme. Corresponding run time, and iterations have been reported. No maximum time step was noted, the time steps could be increased to any extent, yet they gave same error norms and stable solutions, with the expense of computation time.

Mesh Size	Max Temperature Gradient	Position
25x10	1.12570929088971	13.5
50x20	1.12832053502411	12.25
100x40	1.12882403056881	12.375
200x80	1.1289677379422	12.4375

Table 5: Grid convergence study for channel flow solution of temperature equation. Length of channel was extended to 25.0, after experiments confirming it to be a long enough channel.

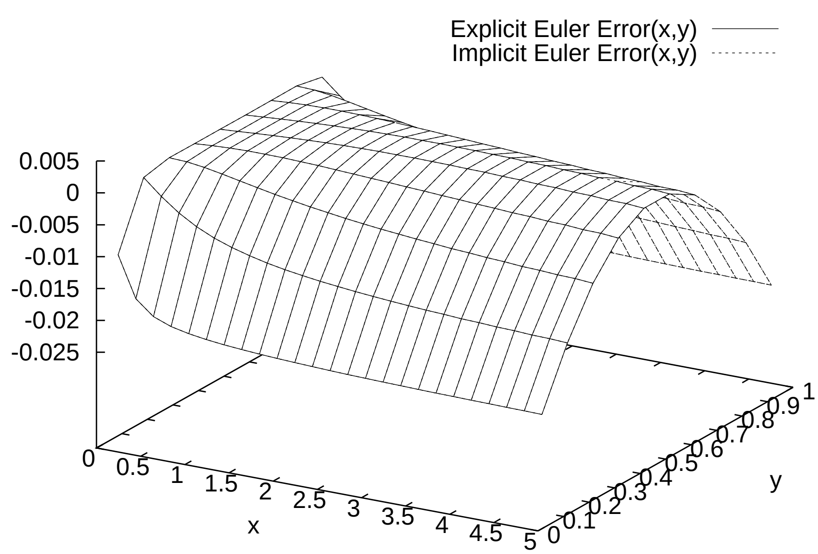

Figure 2: Explicit, and implicit methods have similar errors throughout the mesh.

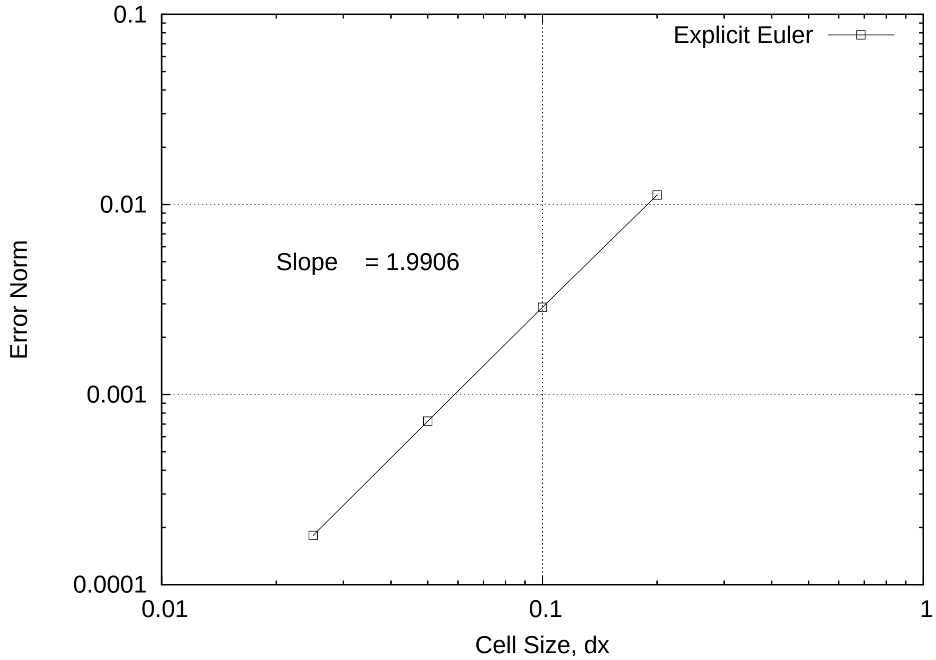

Figure 3: Error norms for implicit method tabulated in a log-log graph for different mesh sizes.

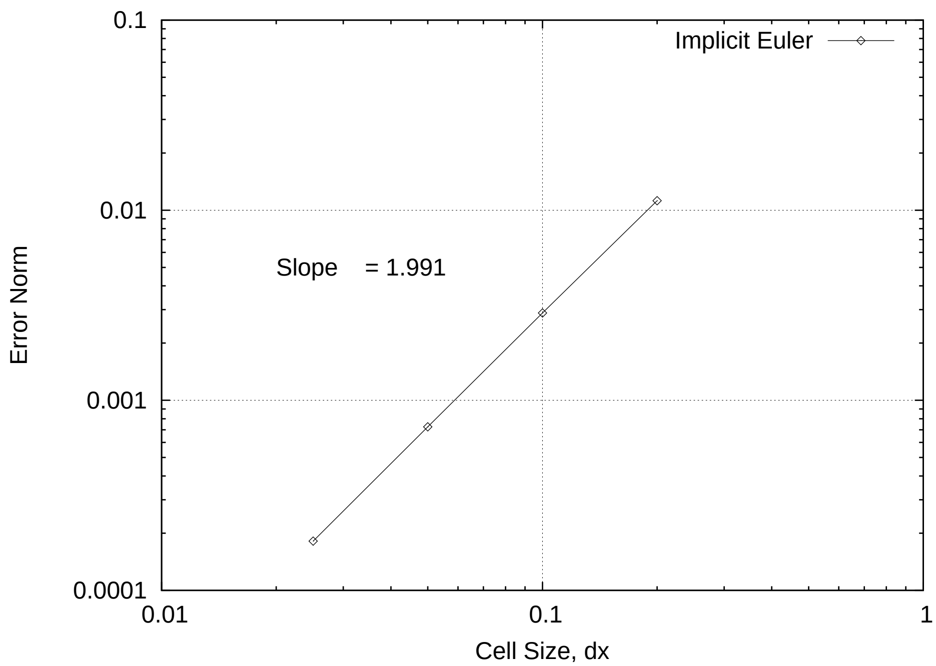

Figure 4: Error norms for implicit method tabulated in a log-log graph for different mesh sizes.

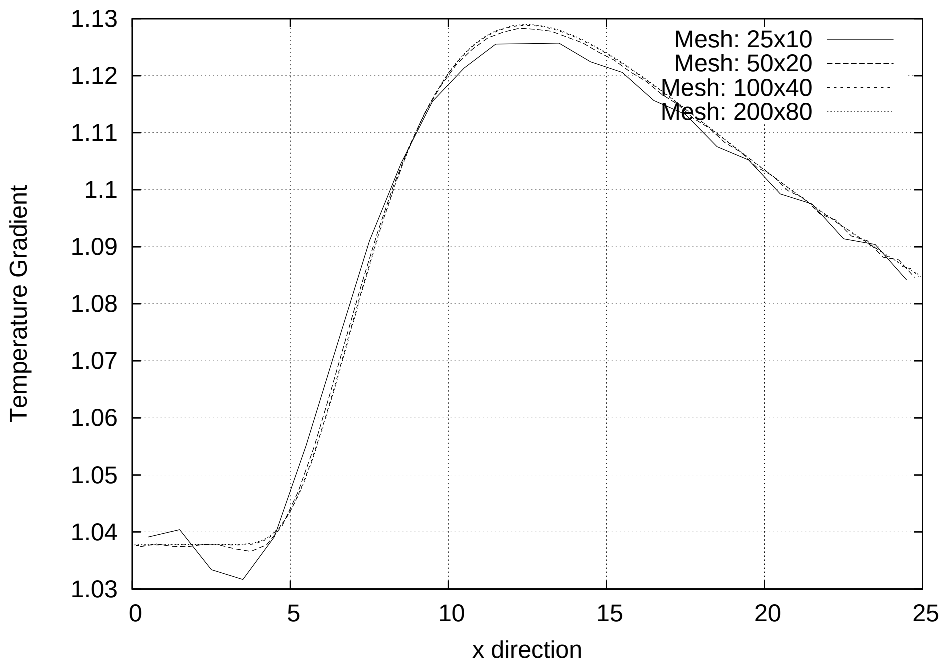

Figure 5: Grid convergence study using different mesh sizes. The temperature gradient at bottom wall is shown to converge here for a selection of meshes.

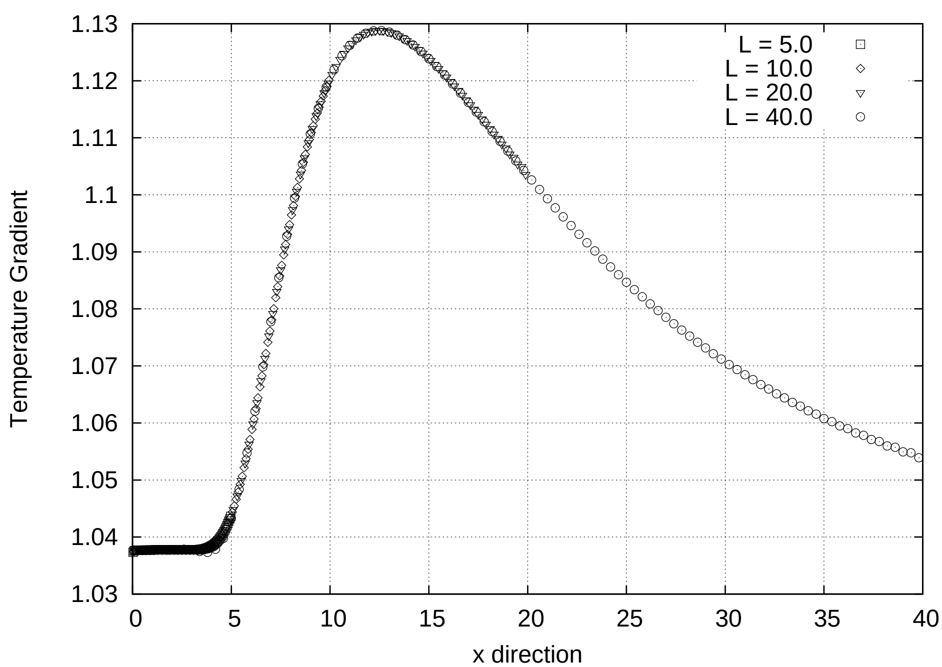

Figure 6: Experiments establishing 25.0 as an adequate channel length to make estimates of maximum bottom wall temperature gradient. Different channel lengths were used, and a global maximum was found in the neighbourhood of x = 12.5.

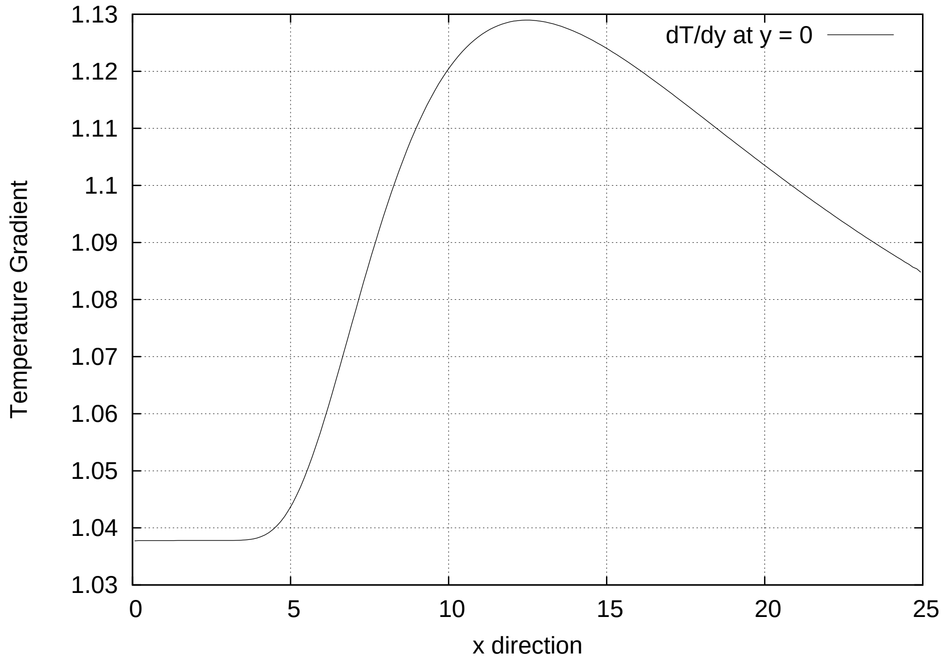

Figure 7: Temperature gradient along the bottom wall using a mesh of size 100x40, and channel width 25.0.

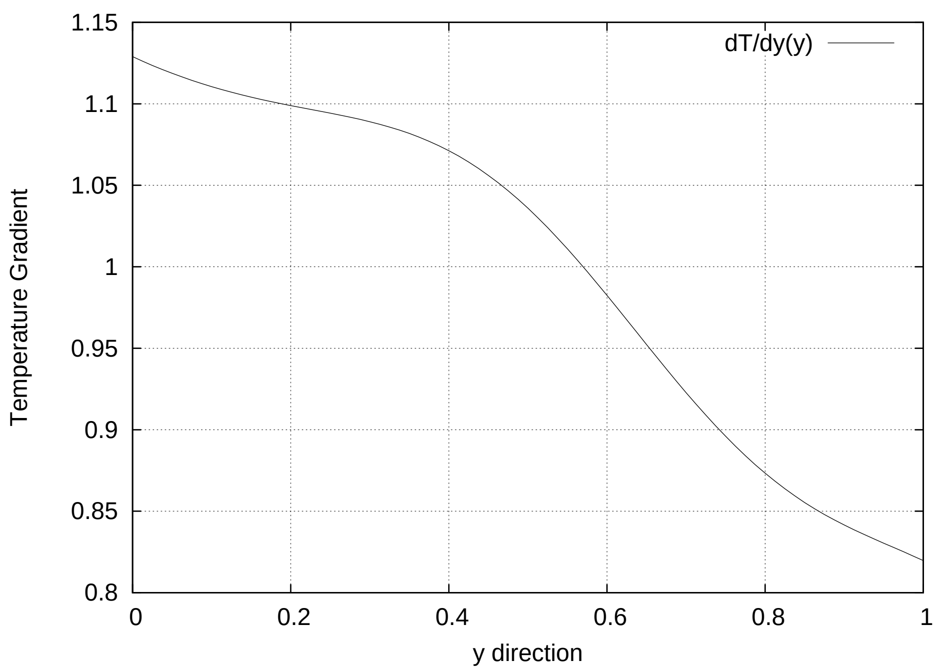

Figure 8: Gradient of temperature in y direction throughout the channel width at position of maximum bottom wall gradient.