

DP - Memo - Coin change 2

coin denominations available \leftarrow array
(infinite number of each denomination available) coins

amount \leftarrow integer

return number of ways to make up amount

(eg) coins = [1, 2, 5]

amount = 5

1+1+1+1+1

1+1+1+2

1+2+2

5

} output = 4

sol: sort coins array

$f(i, x)$ = no. of ways to make x

with coins $[i:]$ denominations

$f(i, x) = f(i, x - \text{coins}[i])$ \leftarrow use a coin $[i]$

\nearrow |coins| number of choices for i
 \nearrow amount

+ $f(i+1, x)$ \leftarrow don't use any more coin $[i]$ s...

so will need to compute |coins| |amount| number of values

$f(\text{non empty array}, < 0) = 0$

$$f([], > 0) = 0$$

$$f(\text{nonempty array}, 0) = 1$$

$$f([], 0) = 1$$

want $f(0, \text{amount})$

Dry run

$$f([1, 2, 5], 5)$$

$$= \overset{1+...}{f([1, 2, 5], 4)} + f([2, 5], 5)$$

$$= \overset{1+1+...}{f([1, 2, 5], 3)} + \overset{1+...}{f([2, 5], 4)} + \overset{2+...}{f([2, 5], 3)} + \underline{f([5], 5)}$$

$$= \overset{1+1+1+...}{f([1, 2, 5], 2)} + \overset{1+1+...}{f([2, 5], 3)} + \overset{1+2+...}{f([2, 5], 2)} + \overset{1+...}{f([5], 4)} + \overset{2+2+...}{f([2, 5], 1)} + \overset{2+...}{f([5], 3)} + \overset{5+...}{f([5], 0)} + \cancel{f([5], 5)}$$

0