Greedy strategies

(Minimizing lateness)

Problem

* 1 resource, n requests to use resource

* Request i : t(i) time to complete, d(i) deadline

* Have to schedule all requests

Request i : starts at s(i) [have to find out what s(i) works]

finishes at f(i)

$f(i) = s(i) + t(i)$

* Lateness of job i : $l(i) = \max[f(i) - d(i), 0]$

if $d(i) \geq f(i)$, 0 lateness

If $d(i) < f(i)$ Then $f(i) - d(i)$ lateness

* Penalty of a schedule $= \max_i [l(i)]$

Find a schedule with minimum penalty.

Greedy strategy

$t(1) = 1$ $d(1) = 100$

$t(2) = 10$ $d(2) = 10$

- Pick job with least t(i) ✗

- Pick job with least $d(j) - t(j)$ ✗ (least slack time)

$t(1) = 1$ $d(1) = 2$ $t(2) = 10$ $d(2) = 10$

- Pick job with earliest deadline

why does it work

Let G be sol. got by greedy sol.

- order jobs by deadlines
- schedule them in the sorted order

Let O' be an optimal sol

Goal $O' \xrightarrow[\text{argument}]{\text{exchange}} G$

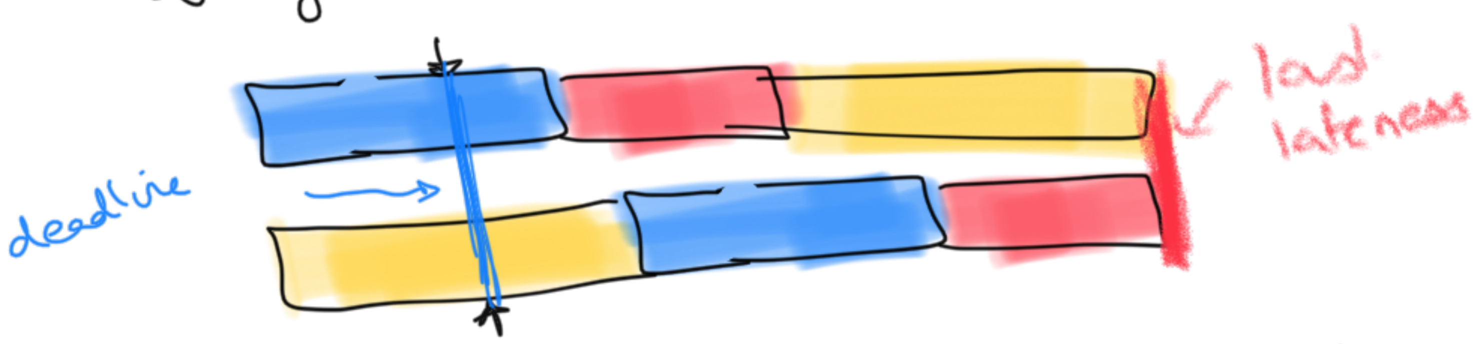
2 features of G

No idle time (jobs scheduled back to back)

No inversions (i scheduled before j $\iff d(i) < d(j)$)

* Any 2 schedules with no idle time, no inversions $\rightarrow$ same lateness penalty

Pf: only leeway is in choosing order of jobs with same deadline



same penalty =

$\exists$ optimal sol with no idle time

$O' \longrightarrow O$

jobs scheduled

no gaps

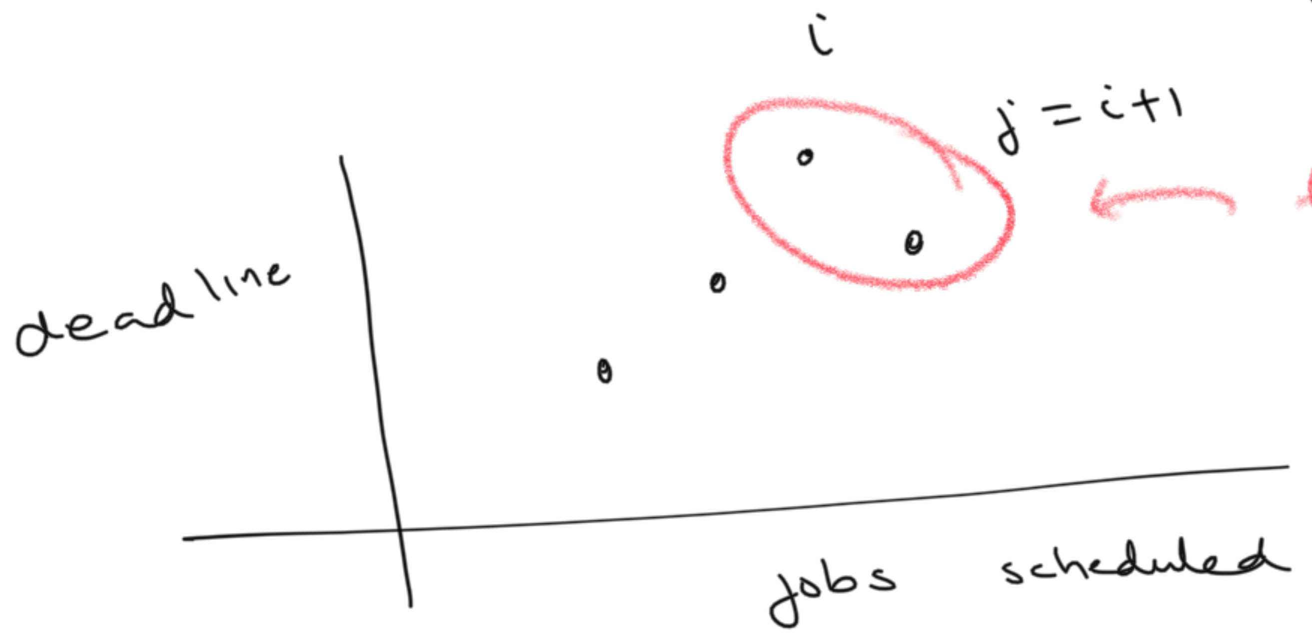
can only ↓ lateness

so assume ∃ O optimal with

no idle time

Goal : $O \xrightarrow{?} G'$ ← no idle time, no inversion

Pf



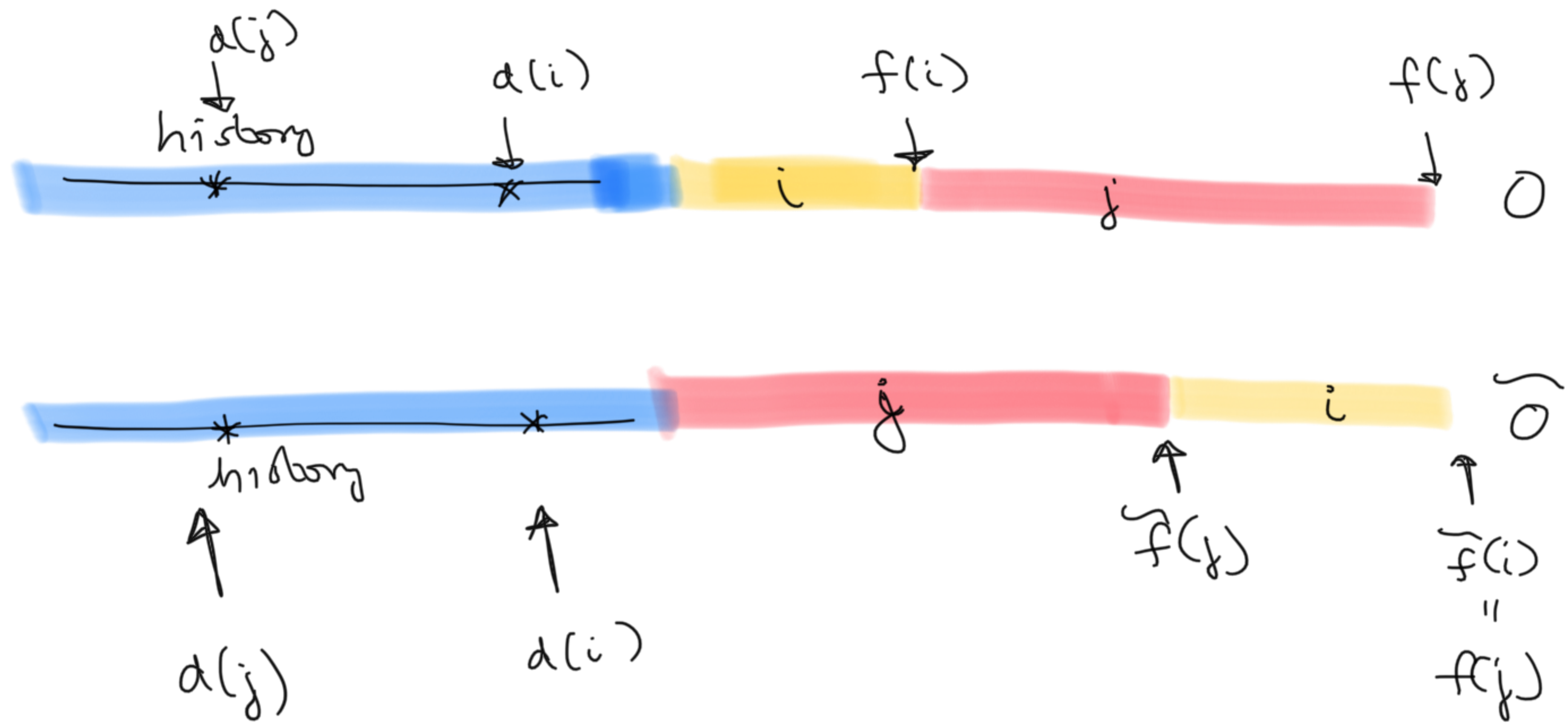
← find 2 consecutive jobs which are inversion

↓ i ↓ j

$d(i) > d(j)$

→ if you swap i, j , inversion removed ∴ new sol $\tilde{O}$

→ what happens to penalty ?



penalty in O :

$$\max \begin{pmatrix} \text{historic penalties} \\ f(i) - d(i) \\ f(j) - d(j) \\ \text{future penalties} \end{pmatrix}$$

penalty in $\tilde{O}$:

$$\max \begin{pmatrix} \text{historic penalties} \\ \tilde{f}(j) - \tilde{d}(j) \\ \tilde{f}(i) - \tilde{d}(i) \\ \text{future penalties} \end{pmatrix}$$

$$\tilde{f}(i) = f(j)$$

$$d(i) > d(j)$$

$$-d(i) < -d(j)$$

$$\begin{array}{ccc} \tilde{f}(i) - d(i) & < & f(j) - d(j) = \ell(j) \\ & & \vee \\ \tilde{f}(j) - d(j) & < & f(i) - d(i) \end{array}$$

$$\begin{array}{ccc} \tilde{O} & & O \\ \max \begin{pmatrix} \text{same} \\ \tilde{\ell}(i) < \ell(j) \\ \tilde{\ell}(j) < \ell(j) \end{pmatrix} & \leq & \max \begin{pmatrix} \text{same} \\ \ell(j) \\ \ell(i) < \ell(j) \end{pmatrix} \end{array}$$

max # of inversions

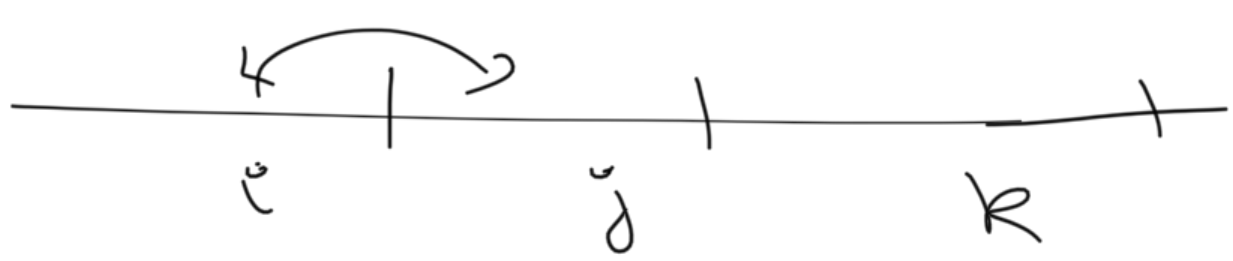
- if every pair of jobs out of order

  $\#$ of inv $\leq \dfrac{n(n-1)}{2}$

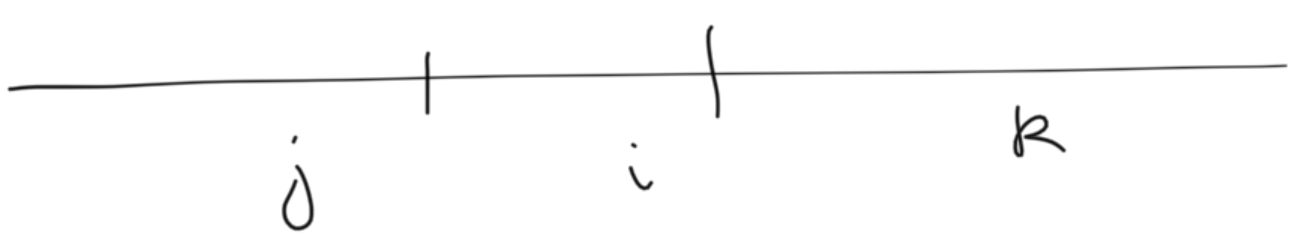
- remove each adjacent inv...

I - inversion
N - not inv.

$d(k) > d(i) > d(j)$



(i j) I
(i k) N
(j k) *



(i j) N

(i k) N

(j k) *