Bit Manipulation

```
  0110
+ 0010
------
  1000
```

```
  0011
  0101
------
    11
    0
  1100
------
  1111
```

```
  0110
+ 0110
------
0110 * 2
= 0110 0
```

```
   1
  0011
+ 0010
------
   101
```

```
  0011 ← 3
* 0011 ← 3
------
  1001 ← 9 = 8+1
```

```
  0100 = 4
* 0011
------
0011 00
```

```
  0 1 0 1
  0110
- 0011
------
    11
```

```
1101 >> 2
---------
0011
```

```
bit wise ~
1101 ^ (~1101)
--------------
1111
```

```
  0 1 1
  1000
- 0110
------
    10
```

```
1101 ^ 0101
-----------
1000
```

```
1011 &
(~0 << 2)

= 1011 &
(1100)

=
------
1000
```

Right shift

$1\cancel{001} >> 3$

$= 0001$

$\underset{1001}{\overset{9}{\shortparallel}} \;//\; 2^3 = 1$

Left shift

$1001 << 3$

$= 1001000$

$9 * 2^3$

Bitwise operations

$x \wedge 0 = x$

$x \,\&\, 0 = 0$

$x \mid 0 = x$

$x \wedge 1 = \sim x$

$x \,\&\, 1 = x$

$x \mid 1 = 1$

$x \wedge x = 0$

$x \,\&\, x = x$

$x \mid x = x$

Two's complement

positive sign bit

4 bits used say

negative sign bit

0	0000		
1	0001	-7	1001
2	0010	-6	1010
3	0011	-5	1011
4	0100	-4	1100
5	0101	-3	1101
6	0110	-2	1110
7	0111	-1	1111

n bits : 1 sign bit + n-1 bits for the number)

$$\mathrm{bin}(-k) = 1\ \underbrace{\qquad\qquad}_{n-1 \text{ bits}}$$

$k > 0$

binary rep for $2^{n-1} - k$

$$\mathrm{bin}(-3) = 1\ \underbrace{\qquad\qquad}$$

binary rep for $(2^{n-1} - 3)$

if $n = 4$, $\mathrm{bin}(-3) = 1\ \underbrace{101}$

3 bit for $2^3 - 3 = 5$

Another way to think about this

bin(−3):

bin(3) = 011

~ bin(3) = 100

+1

= 101

bin(−3) = 1 101

prepend signed bit 1.

bin(−3)

bin(3−1): 010

~ bin(3−1): 101

bin(−3) = 1 101

prepend signed bit 1

n bits : 1 sign, n−1 unsigned bits.

* $[-2^{n-1}, 2^{n-1}]$ can be rep

* bin(−k) $= 1 \underbrace{\text{bin}(2^{n-1} - k)}_{n-1 \text{ bits}}$

k>0

$= 1 \underbrace{\sim \text{bin}(k-1)}_{n-1 \text{ bits}}$

$= 1 \underbrace{\sim \text{bin}(k) + 1}_{n-1 \text{ bits}}$

i^{th} bit . . . . 1^{st} bit 0^{th} bit

Get i^{th} bit in a number (num)

$$1 << i = 0 . \_ \_ 0 \; 1 \underbrace{0 0 . . . 0}$$

$$num = \_ \_ \_ \_ * \_ \_ i \_ \quad \_$$

$$num \,\&\, (1 << i) = \boxed{0 \_ \_ 0 * 0 . . \_ . . 0}$$

If $= 0$ → i^{th} bit $= 0$

If $\neq 0$ → i^{th} bit $\neq 0$

set i^{th} bit $= 1$ in a number (num)

$$1 << i = 0 . \_ \_ 0 \; 1 \overbrace{0 0 . . . 0}^{i}$$

$$num = \_ \_ \_ \_ * \_ \_ \; \_ \quad \_$$

$$num \mid (1 << i) = \boxed{\_ \_ \_ \_ \_ \; 1 \; . \_ \_ \; . . . . \; \_}$$

↑ sets the i^{th} bit $= 1$

clear ith bit in a number (num)

$\sim(1 \ll i)$: $111111\,0\,\underbrace{111\ldots1}_{i}$

num : $\ldots\ldots * \ldots\ldots$

num & $(\sim(1 \ll i))$ $\ldots\ldots\,0\,\ldots\ldots$

update ith bit to that of a boolean bit v

$\sim(1 \ll i)$ $11111\,0\,11\ldots1$

AND

num $\ldots\ldots * \ldots\ldots$

$\ldots\ldots\,0\,\ldots\ldots$

OR

$v \ll i$ $0\ldots0\,v\,\underbrace{00\ldots0}_{i}$

$\ldots\ldots v \ldots\ldots$

$(num \,\&\, \sim(1 \ll i)) \mid (v \ll i)$

Clear $[n \rightarrow i]$ bits of a number num

$1 << i \qquad 0 \cdots 0\, 1\, \underbrace{0 \cdots 0}_{i}$

$(1 << i) - 1 \qquad 0 \cdots 0\, 0\, 0\, \underbrace{1\, 1 \cdots 1}_{i}$

$\text{num} \qquad - \cdots - \;\; * - *$

$$((1 << i) - 1) \,\&\, \text{num} = 0 \cdots 0 \;\; * \cdots *$$

Clear $[i \leftarrow 0]$ bits of a number num

$-1 \qquad 1\,1\,1\,1\,1 \cdots 1$

$-1 << i+1 \qquad 1\,1 \cdots 1\; \overbrace{0 \cdots 0}^{i+1}$

AND

$\text{num} \qquad * \cdots * \cdots\cdots$

$$(-1 << i+1) \,\&\, \text{num} = * \cdots *\; \underbrace{0 \cdots 0}_{i+1}$$