Balanced binary trees

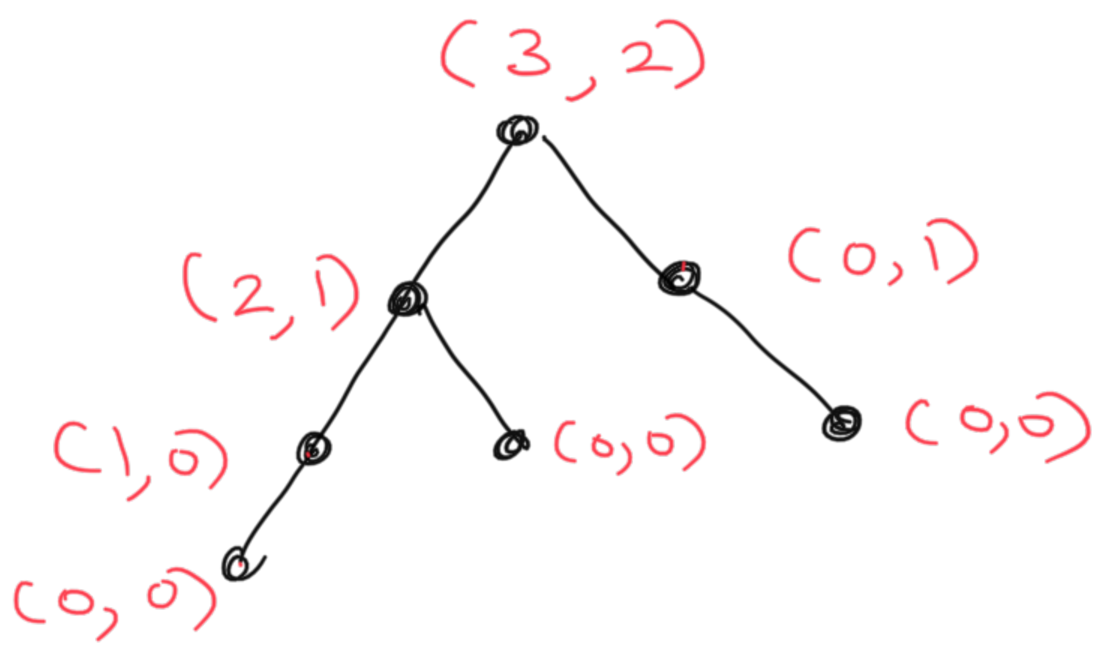
* Operations on binary search trees
  – O(height of tree)

* want to keep our binary trees (of n nodes) balanced so that $h = O(\log n)$

* Height balanced tree (AVL)

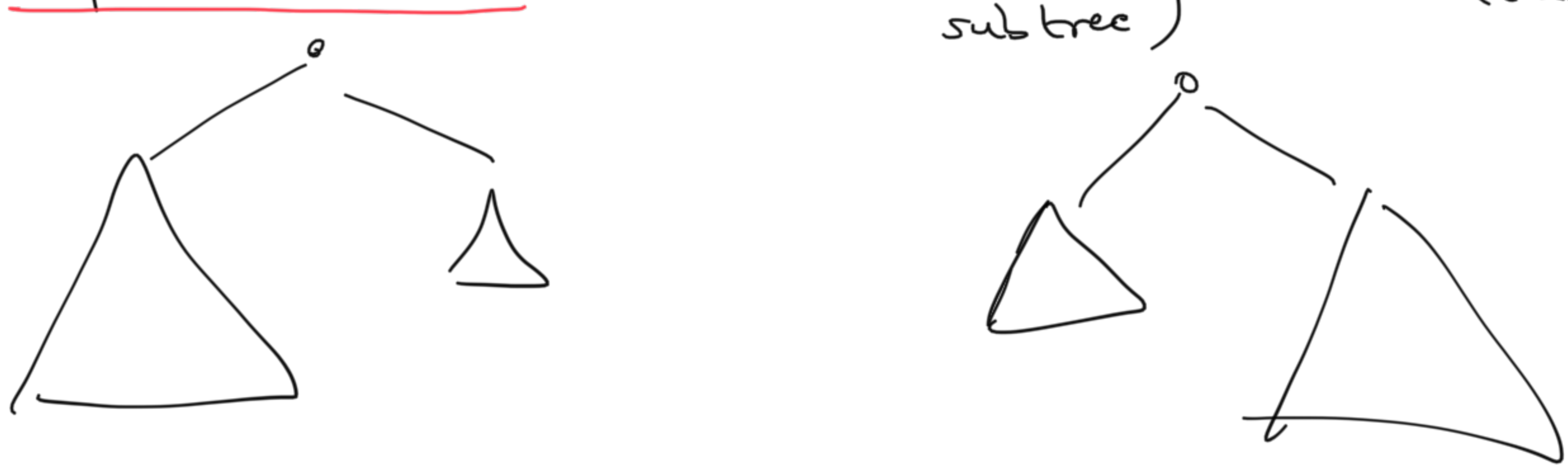
At every node, $|ht(\text{left subtree}) - ht(\text{right subtree})| \leq 1$

(eg) label (ht of left subtree, ht of right subtree) at each node



A – Adelson
V – Velsky
L – Landis

Slope of node = ht (left subtree) – ht (right subtree)



+ve slope

negative slope

v – arbit node

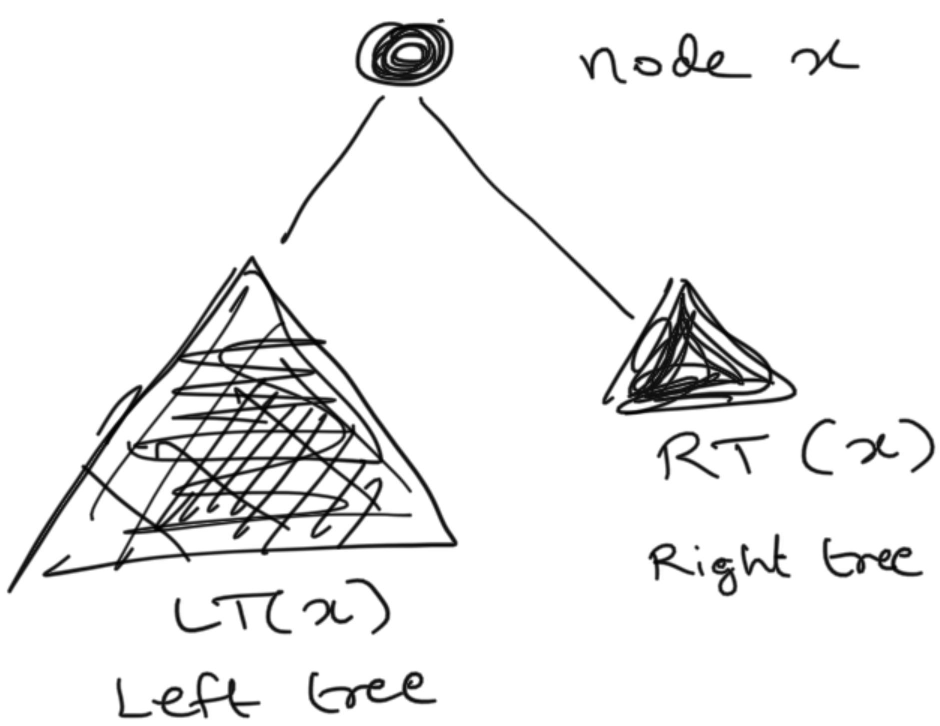
AVL tree : slope (v) $\in \{-1, 0, 1\}$

* After 1 insert / delete

$\rightsquigarrow$ slopes $\in \{-2, -1, 0, 1, 2\}$
of nodes

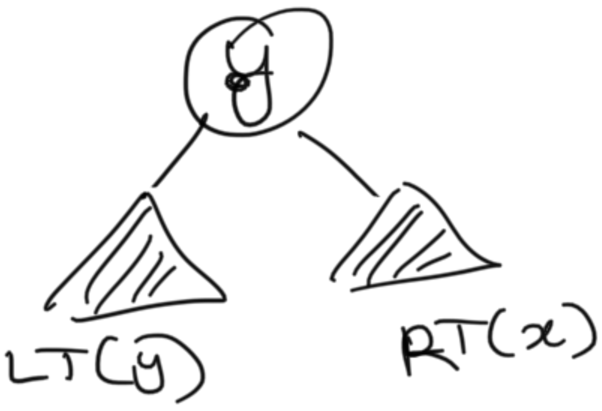
* so rebalance tree after each insert/delete operation

* Do bottom-up rebalances.



- Assume x is not balanced, slope $= 2$
- LT, RT are balanced
- so $ht(RT) = h$, $ht(LT) = h+2$

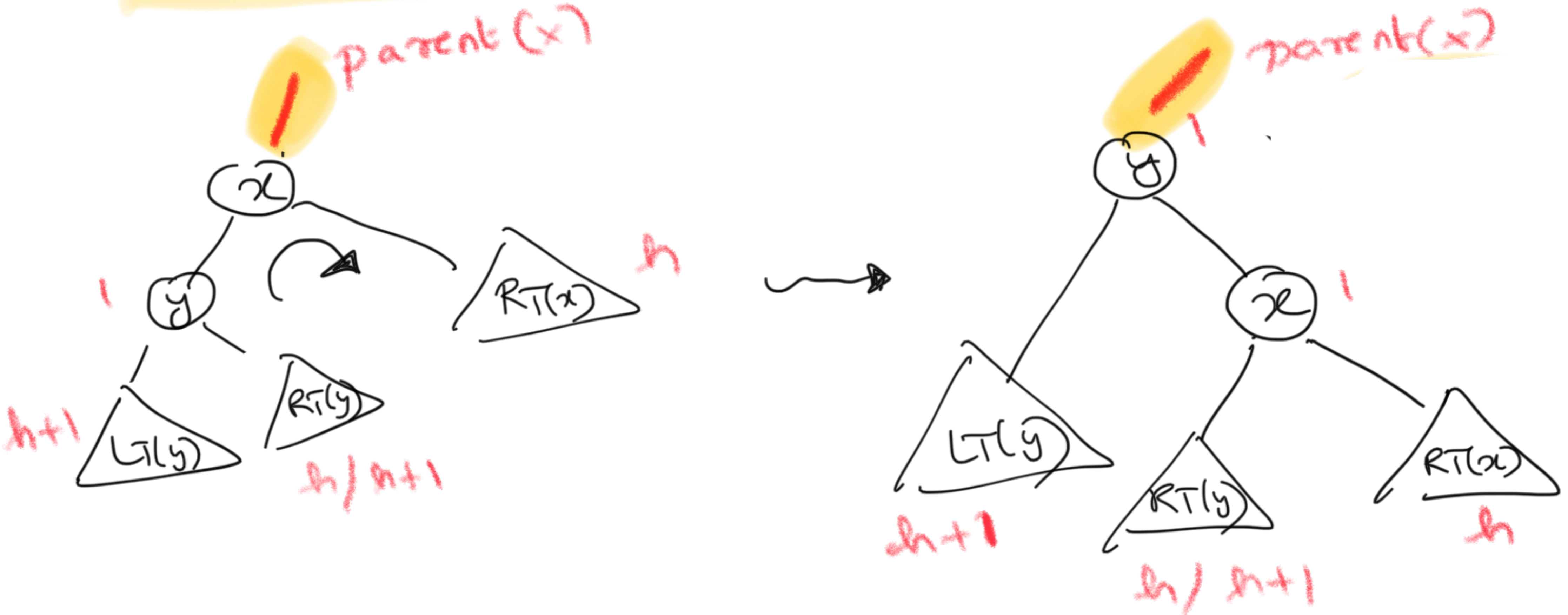
LT(x):



Case I

slope(y) = 0 or 1 : $\quad ht(RT(y)) = h+1$ or h

Rotate tree right at x



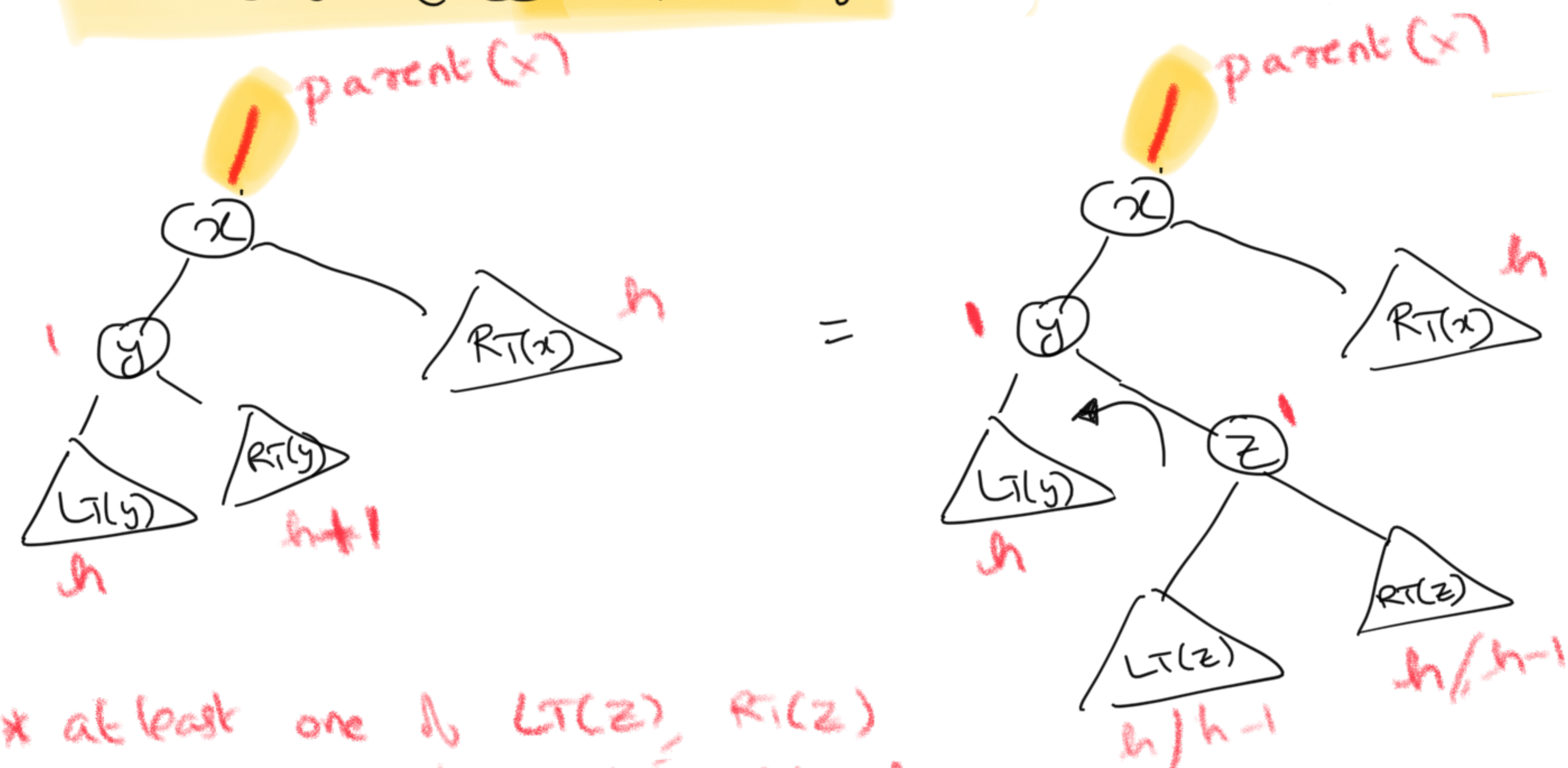
slope(y) = h+1 − [h+2 or h+1]

slope(x) = h/h+1 − h

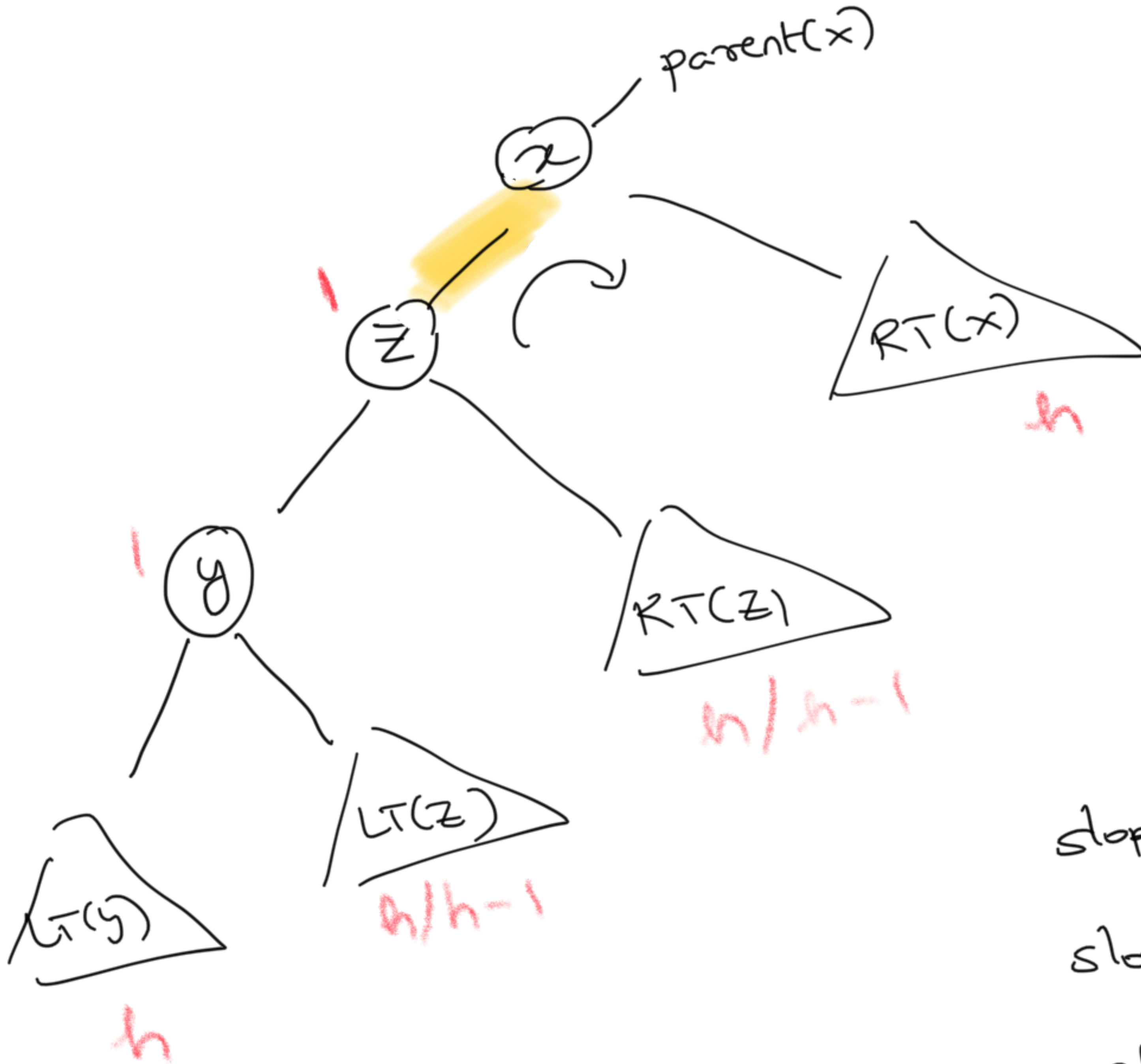
Case II

slope(y) = −1

Rotate tree left at y



* at least one of LT(z), RT(z) has to be height h

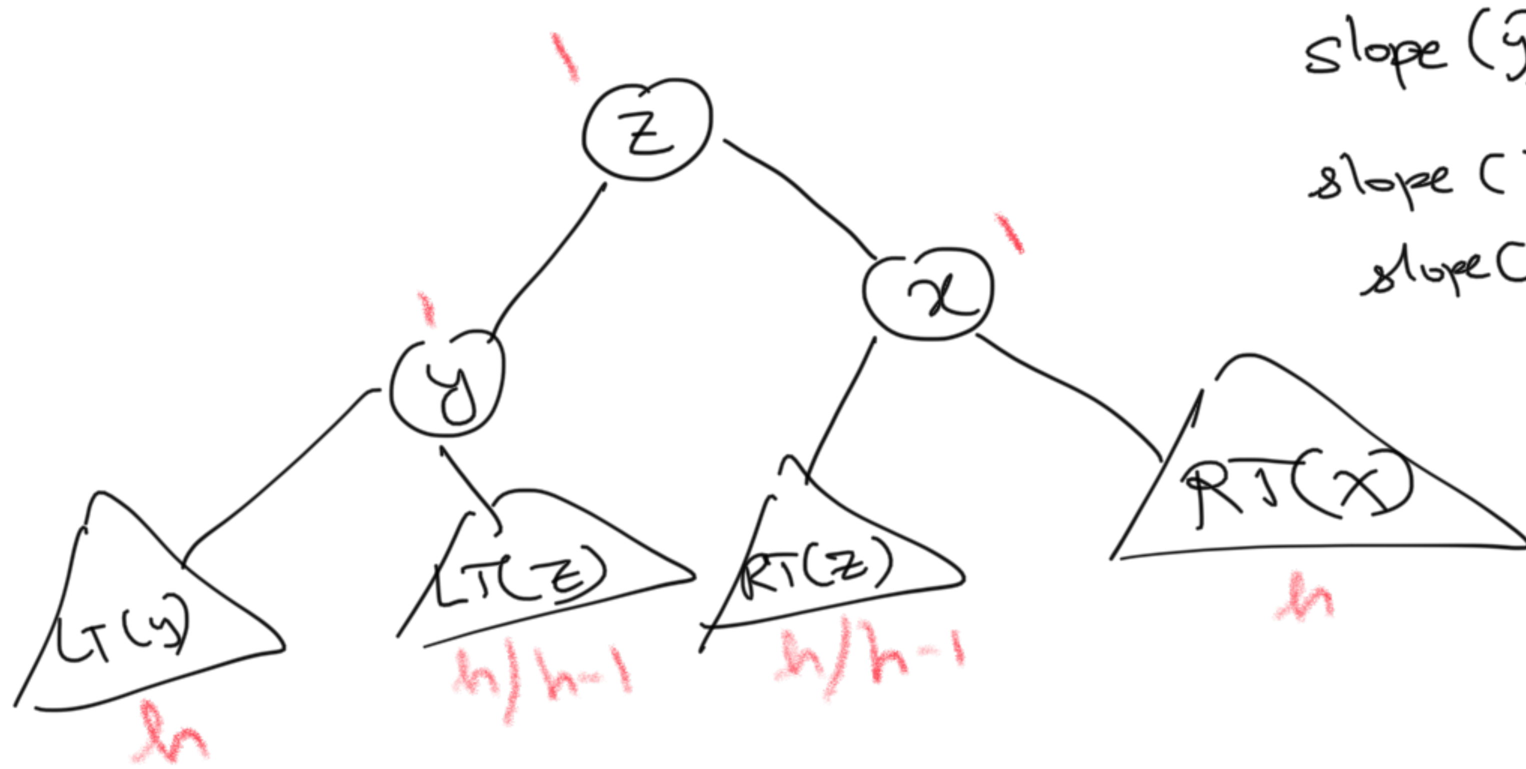


$\text{slope}(y) \in \{0/-1\}$

$\text{slope}(z) \in \{1, 2\}$

$\text{slope}(x) = 2$

Rotate right at x

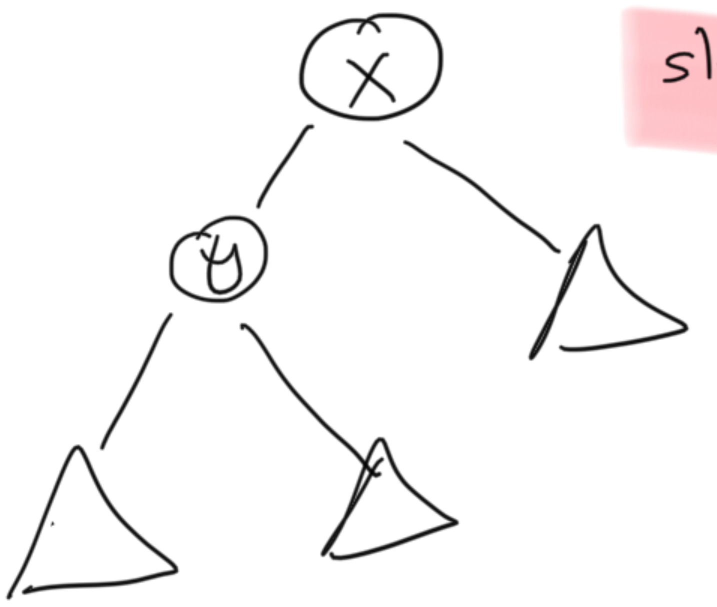


$\text{slope}(y) \in \{0, +1\}$

$\text{slope}(x) \in \{0, -1\}$

$\text{slope}(z) \in \{0\}$

Summary



slope 2

- slope $(y) \in \{0, 1\}$

  $\Rightarrow$ rotate right at x

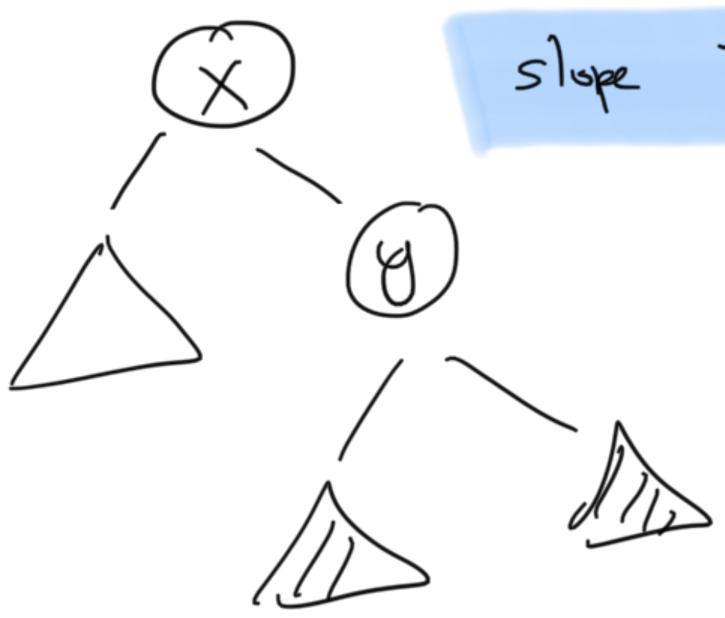
- slope $y \in \{-1\}$

  $\Rightarrow$ rotate left at y

  +

  rotate right at x

Symmetric analysis



slope -2

- slope $(y) \in \{0, -1\}$

  $\Rightarrow$ rotate left at x

- slope $(y) \in \{1\}$

  $\Rightarrow$ rotate right at y

  +

  rotate left at x

Rebalancing (at a node) $\rightarrow$ $O(1)$

In recursive insert / delete,

right after calling recursive insert / del (child)

Do rebalance (child)

* Computing ht (tree) $\rightsquigarrow$ $O(\underset{\text{tree}}{\text{size of}})$!!

* So instead store t. height at each node t

* update t. height with each insert/del $\rightsquigarrow$ $O(1)$