

Bit Manipulation

$$\begin{array}{r} + \quad 0110 \\ \quad 0010 \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 0011 \\ 0101 \\ \hline 11 \\ 1100 \\ \hline 1111 \end{array}$$

$$\begin{array}{r} + \quad 0110 \\ \quad 0110 \\ \hline 0110 * 2 \\ = 01100 \end{array}$$

$$\begin{array}{r} \quad 0011 \\ + \quad 0010 \\ \hline 101 \end{array}$$

$$\begin{array}{r} 0011 \leftarrow 3 \\ * \quad 0011 \leftarrow 3 \\ \hline 1001 \leftarrow 9 = 8+1 \end{array}$$

$$\begin{array}{r} 0100 = 4 \\ * \quad 0011 \\ \hline 001100 \end{array}$$

$$\begin{array}{r} 0110 \\ - 0011 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 1101 \gg 2 \\ \hline 0011 \end{array}$$

bit wise ~

$$\begin{array}{r} 1101 \wedge (\sim 1101) \\ \hline 1111 \end{array}$$

$$\begin{array}{r} 1000 \\ - 0110 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1101 \wedge 0101 \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 1011 \& \\ (\sim 0 \ll 2) \\ = 1011 \& \\ (1100) \\ = \\ \hline 1000 \end{array}$$

Right shift

$$\begin{array}{c} \text{1001} \\ \text{1001} \end{array} \gg 3 = 0001$$

$$9 \div 2^3 = 1$$

Left shift

$$1001 \ll 3 = 1001000$$

$$9 * 2^3$$

Bitwise operations

$$x \wedge 0 = x$$

$$x \& 0 = 0$$

$$x \mid 0 = x$$

$$x \wedge 1 = \sim x$$

$$x \& 1 = x$$

$$x \mid 1 = 1$$

$$x \wedge x = 0$$

$$x \& x = x$$

$$x \mid x = x$$

Two's complement

positive sign bit

4 bits used say

negative sign bit

0	0000		
1	0001	-7	1001
2	0010	-6	1010
3	0011	-5	1011
4	0100	-4	1100
5	0101	-3	1101
6	0110	-2	1110
7	0111	-1	1111

n bits : 1 sign bit + n-1 bits for the number)

$$\text{bin}(-k) \quad k > 0 = 1 \quad \underbrace{\hspace{2cm}}_{n-1 \text{ bits}} \quad \text{binary rep for } 2^{n-1} - k$$

$$\text{bin}(-3) = 1 \quad \underbrace{\hspace{2cm}}_{\text{binary rep for } (2^{n-1} - 3)}$$

if $n=4$, $\text{bin}(-3) = 1 \quad \underbrace{101}_{3 \text{ bit for } 2^3 - 3 = 5}$

Another way to think about this

bin(-3):

$$\text{bin}(3) = 011$$

$$\sim \text{bin}(3) = 100$$

+1

$$= 101$$

$$\text{bin}(-3) = 1101$$

↑
prepend signed bit 1.

bin(-3)

$$\text{bin}(3-1) = 010$$

$$\sim \text{bin}(3-1) = 101$$

$$\text{bin}(-3) = 1101$$

↑
prepend signed bit 1

n bits: 1 sign, n-1 unsigned bits.

* $[-2^{n-1}, 2^{n-1}]$ can be rep

$$\begin{aligned} * \quad \text{bin}(-k) &= 1 \underbrace{\text{bin}(2^{n-1} - k)}_{n-1 \text{ bits}} \\ k > 0 \end{aligned}$$

$$= 1 \underbrace{\sim \text{bin}(k-1)}_{n-1 \text{ bits}}$$

$$= 1 \underbrace{\sim \text{bin}(k) + 1}_{n-1 \text{ bits}}$$

i^{th} bit ... 1st bit 0th bit

Get i^{th} bit in a number (num)

$$1 \ll i = 0 \dots 0 \mid \underbrace{00 \dots 0}_i$$

$$\text{num} = \dots * \dots i \dots$$

$$\text{num} \& (1 \ll i) = \boxed{0 \dots 0 * 0 \dots 0}$$

if $= 0$
 i^{th} bit $= 0$

if $\neq 0$
 i^{th} bit $\neq 0$

Set i^{th} bit = 1 in a number (num)

$$1 \ll i = 0 \dots 0 \mid \underbrace{00 \dots 0}_i$$

$$\text{num} = \dots * \dots$$

$$\text{num} \mid (1 \ll i) = \boxed{\dots \mid 1 \dots}$$

↑
 sets the i^{th} bit = 1

clear i^{th} bit in a number (num)

5. $(1 \ll i)$: 

η_{hm} : — — — — *

num of $(u(1 \leq i))$ 0

update i th bit to that of a boolean bit v

$\sim (1 \ll i)$ $1 \ 1 \ 1 \ 1 \ 0 \ 1 \dots$

AND

num - - *

— — — — — 0 — — — — —

or

 $\sqrt{q} \ll i$

0... 0 9 00... 0

.....v.....

$$(\text{num } b \sim (1 \ll i) \mid (0 \ll i))$$

Clear $[n \rightarrow i]$ bits of a number num

$$1 \ll i$$



$$(1 \ll i) - 1$$



num

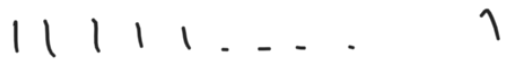


$$(1 \ll i) - 1 \& \text{num} =$$



Clear $[i \leftarrow 0]$ bits of a number num

$$-1$$



$$-1 \ll i+1$$



num



$$(-1 \ll i+1) \& \text{num} =$$



$i+1$

