Greedy strategies

(interval scheduling)

Problem

* Instructor i wants slot starting s(i) and finishes at f(i)

* Slots may overlap

* maximize # of non overlapping slots

Idea

* Pick next booking according to some greedy strategy
* Remove all conflicting slot requests to this

what greedy strategy to adopt ?

* earliest start time ✗

* shortest slot ✗

* choose slot with min # of conflicts. ✗

* earliest finish time ✓

eg,

Algorithm

B = set of slot requests which are still feasible

A = set of accepted requests

B = {every slot req} (initially)

A = ∅ (initially)

while $B \neq \emptyset$:

- pick $b \in B$ with earliest finish time
- $A = A \cup \{b\}$
- Remove all conflicts (with b) from B

<u>Why does it work ?</u>

Let O be optimal set of accepted requests

<u>Goal</u> show $|O| = |A|$

Let $A = \{ i_1, i_2, \ldots i_k \}$ (sorted)

$s(i_1) \leq f(i_1) \leq s(i_2) \leq f(i_2) \ldots s(i_k) \leq f(i_k)$

Let $O = \{ j_1, \ldots, j_m \}$ (sorted)

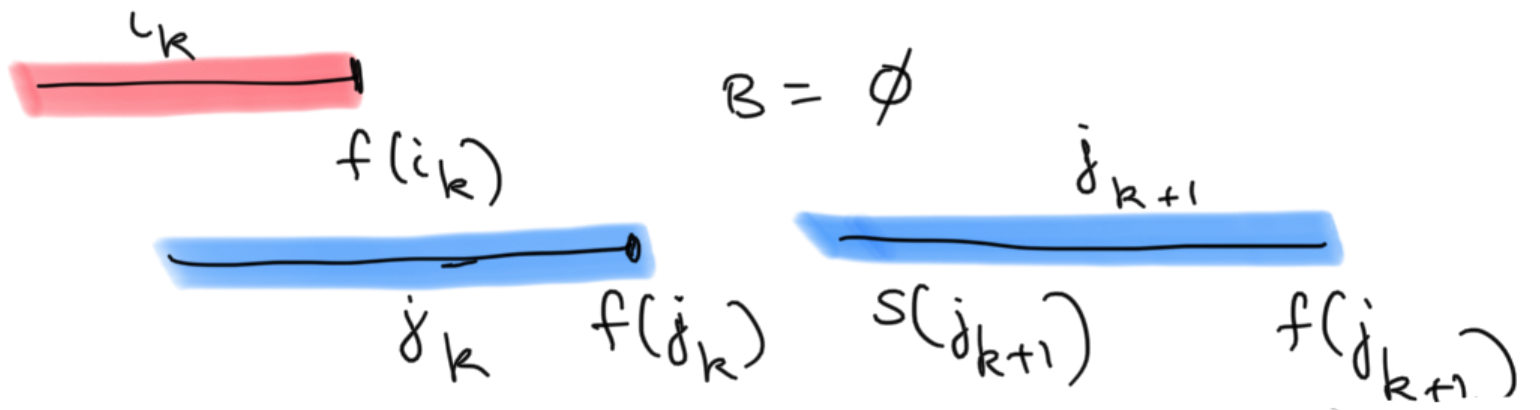
<u>Goal</u> show $k = m$

<u>Claim</u> $\forall \; r \leq k, \quad f(i_r) \leq f(j_r)$

(ie) our r^{th} job finishes earlier than optimal solution's r^{th} job

Suppose claim OK, then our greedy solution "finishes" earlier than optimal solution

Suppose $m > k$



But j_{k+1} slot req when greedy algo terminates doesn't have any conflict with any jobs in A

so $j_{k+1} \in B$ when greedy algo terminates

$\longrightarrow\leftarrow$ as it terminates only when $B = \phi$

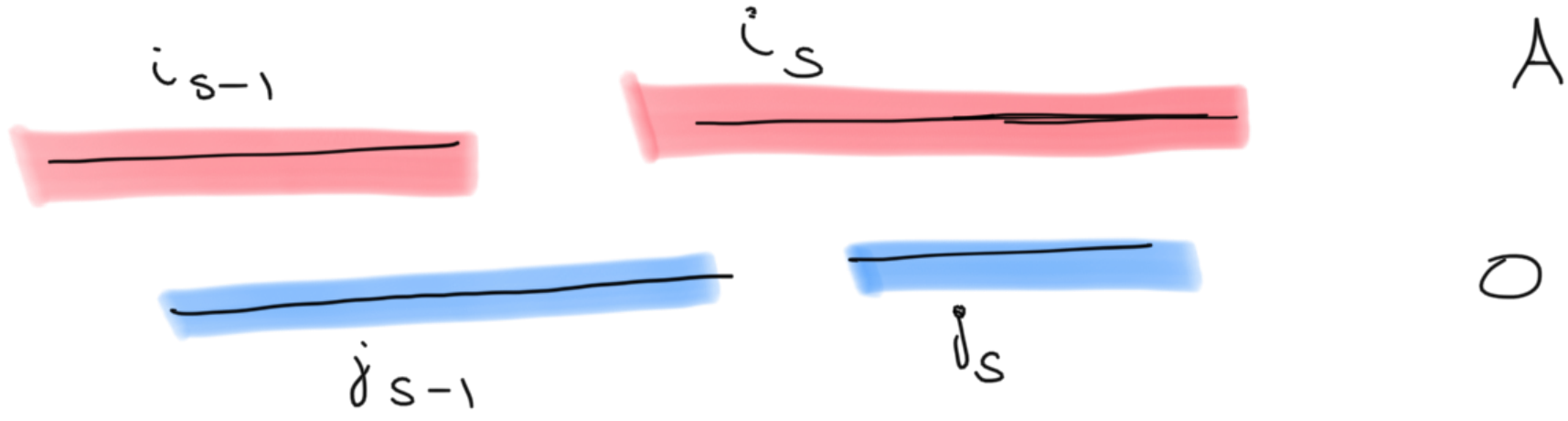
why is claim true

Induction

$r = 1$: greedy picks least finish time slot req So ok ✓

Assume ok till $r < s$.

let $r = s$



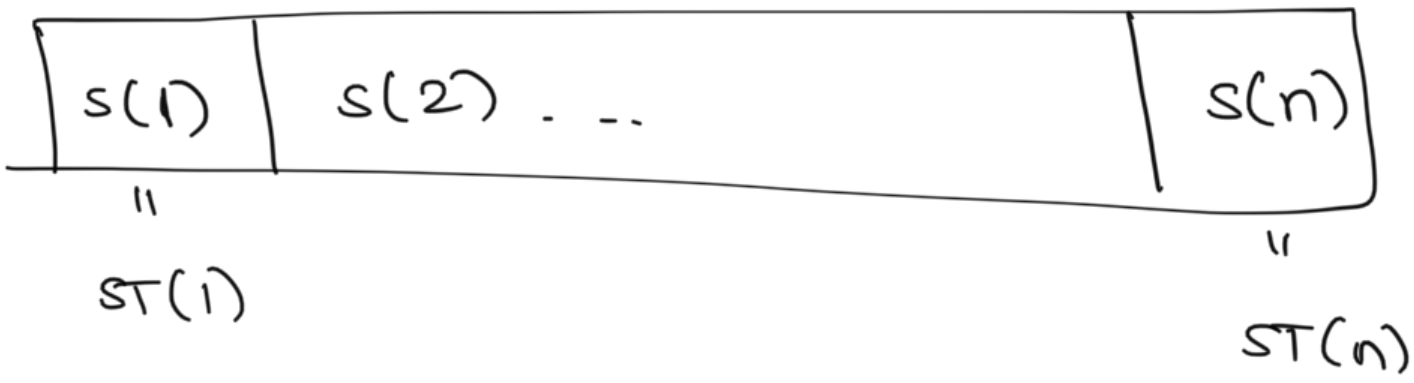
$f(i_{s-1}) \leq f(j_{s-1})$

Suppose $f(i_s) > f(j_s)$

then our algo would have picked j_s as j_s doesn't conflict with jobs in A thus far.

Implementation $|B| = n$

* sort B by finishing time $O(n \log n)$
* $B = \{1, 2, \ldots, n\}$ (renumbered) (sorted order)
* create start time array $O(n)$



* $A = \{1\}$
* select least $j_1 > 1$ with $ST(j_1) \geq f(1)$
* $A = \{1, j_1\}$
* Select least $j_2 > j_1$ with $ST(j_2) \geq f(j_1)$

$\vdots$

takes $O(n)$ time

So over all $O(n \log n)$

(ie) the slot req conflicting with 1 are $2, \ldots, j-1$ and "removed" from consideration

* if $\exists\ k > j_1$ and $ST(k) < f(1)$ even if we have not "removed" it now, note that $ST(k) < f(1) < f(\text{future accepted req})$