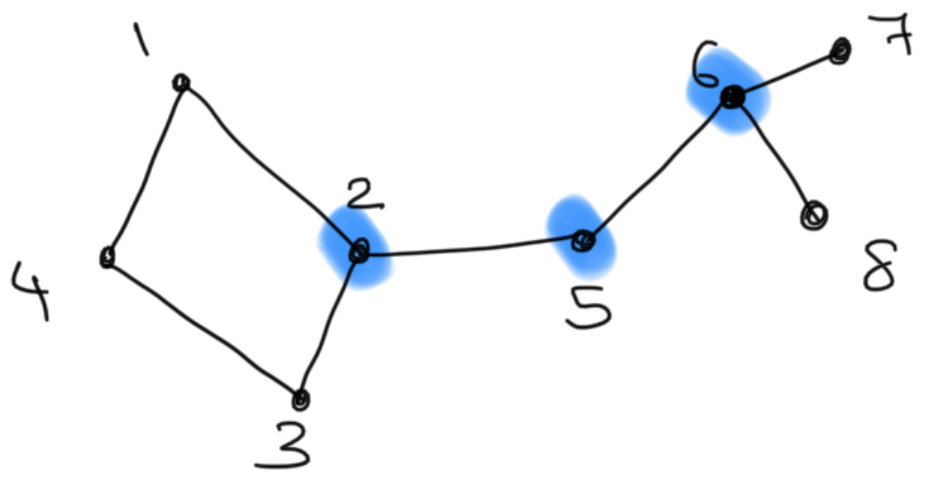
Articulation points

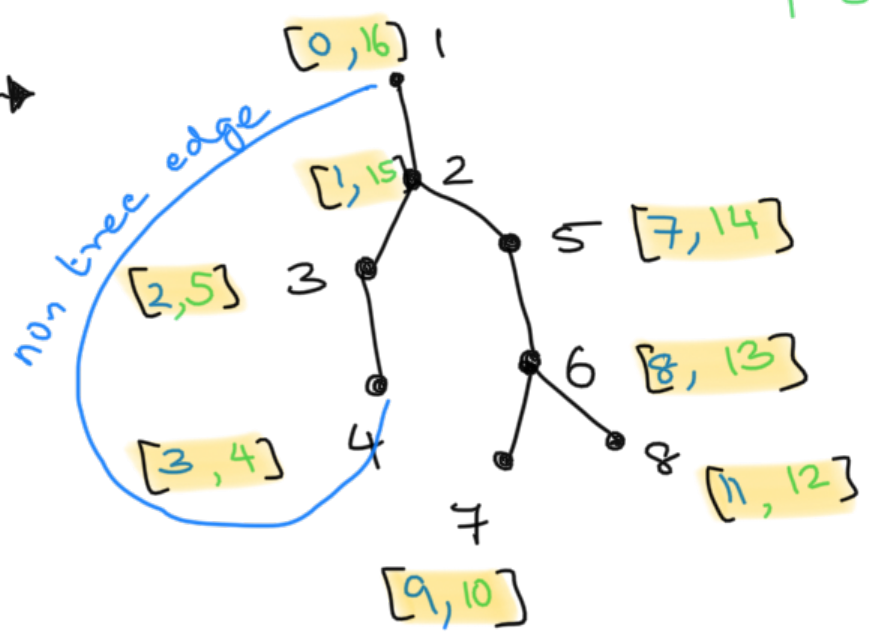


Articulation points / cut vertices : nodes in graph if removed, disconnect the graph

How to identify them ?

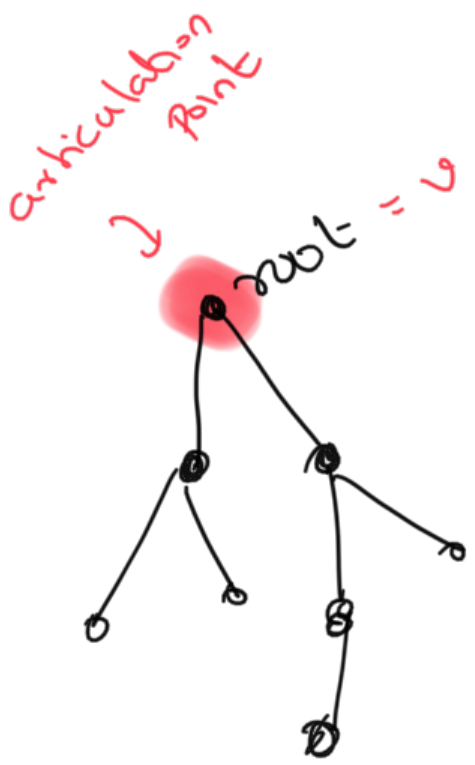
pre
post

DFS tree →

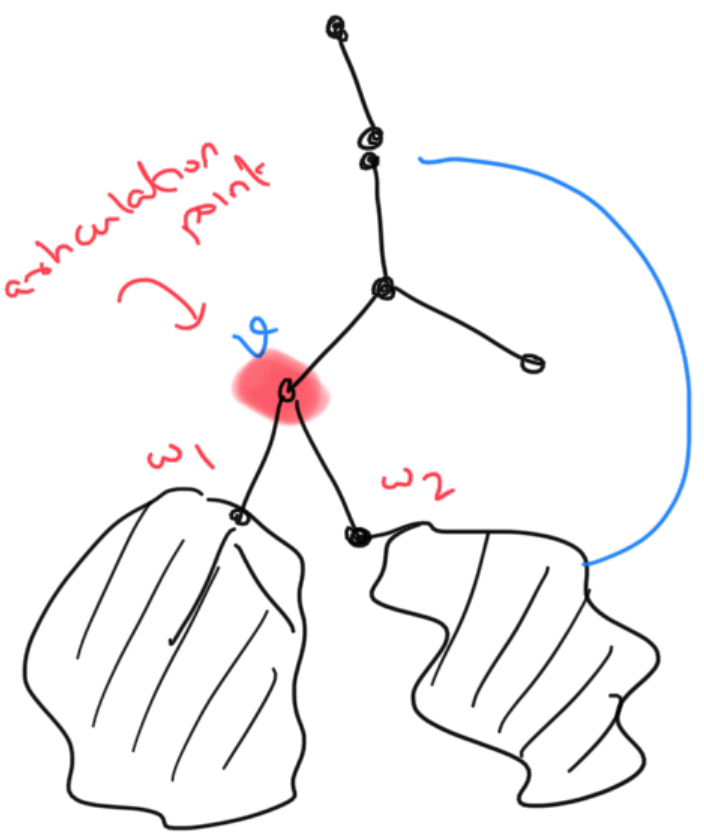


root
||
v is an articulation point $\Leftrightarrow$ it has ≥ 2 children

root
#
v is an articulation point $\Leftrightarrow$ $\exists$ some child w of v so that no vertex in subtree rooted at w has back edge that go strictly above v



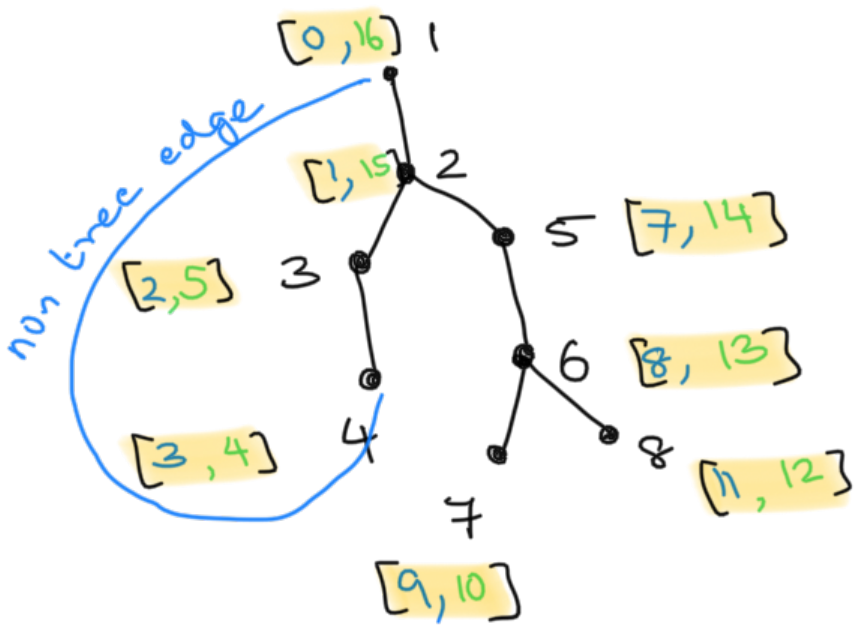
v is root, has ≥ 2 children



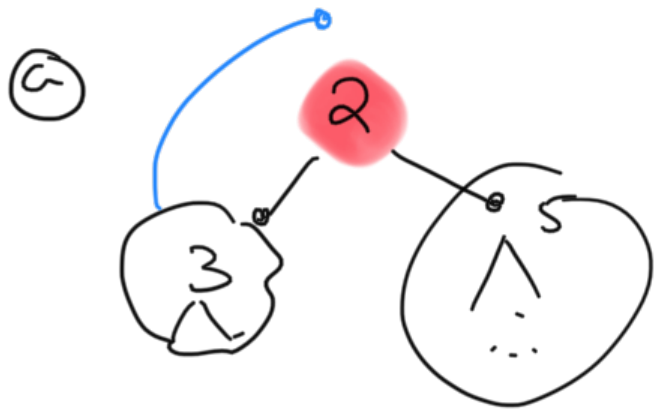
v has 2 children w1, w2.

But subtree at w1 has no back edge to > v.

So v articulation point



In our example,

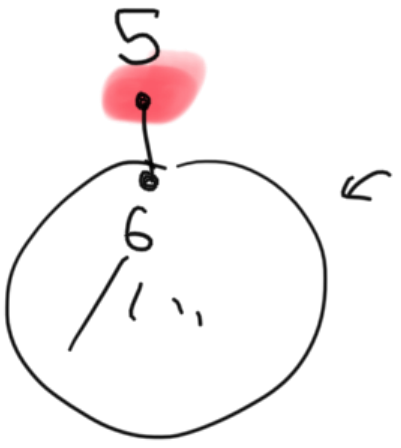


2 has child 5,

and 5's subtree has no back edge going above 2

So 2 articulation point

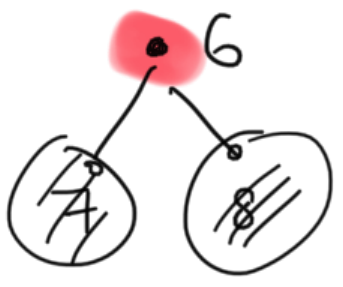
(b)



5 has child 6,
and subtree at 6 has no back edge going above 5

So 5 articulation point

(c)



6 has children 7, 8
and subtree at 7 has no back edge going above 6

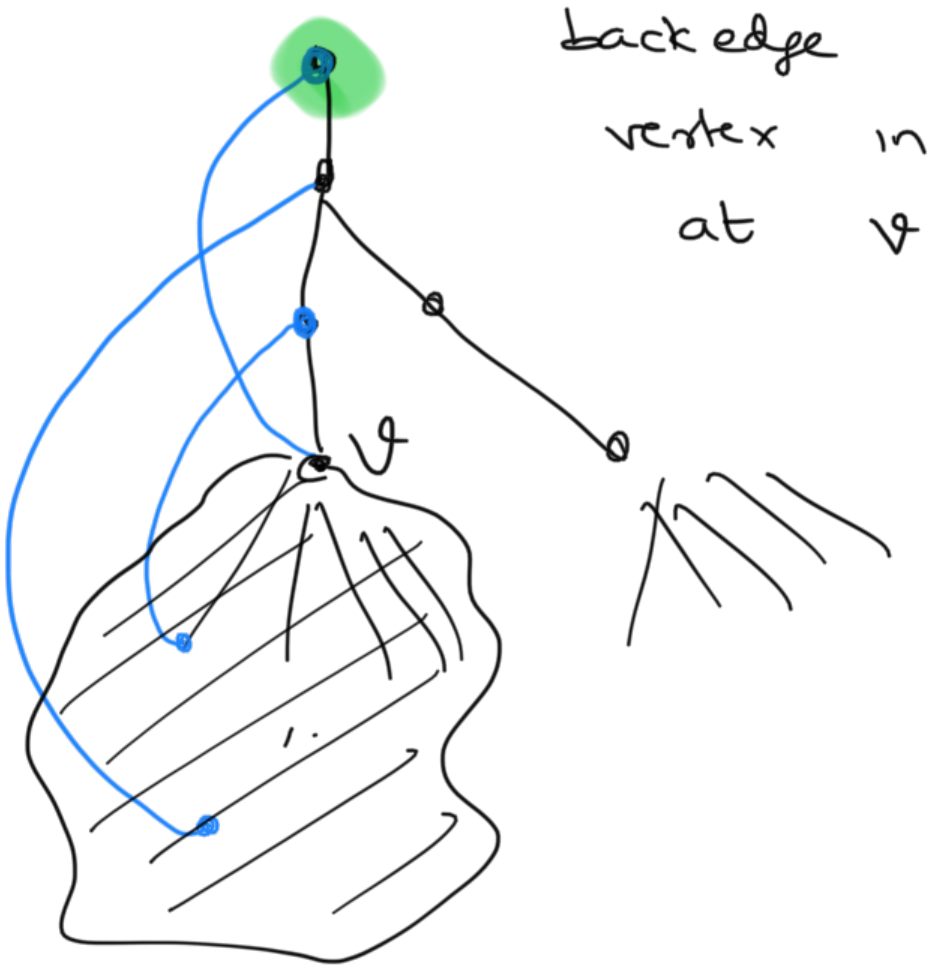
So 6 articulation point

Implementation

"back edge going above v" ⟺ non tree edge from vertex to a vertex with DFS pre number < DFS pre(v).

* just to assume low(v) ≥ pre(v)
* so if v has subtree with no back edge, low(v) = pre(v)

$$low(v) = \min \left[\begin{array}{l} pre(v), \\ pre(u), \quad u \text{ is reachable by a single back edge from some vertex in subtree rooted at } v \end{array} \right]$$



low(v) = pre( )

So $low(v) = \min \left[\begin{matrix} pre(v), \\ low(w) \end{matrix} , \; w \in children \cap v , \; pre(w), \; (u \; \underset{v}{\rightarrow} \; w \text{ single back edge}) \right]$

don't count tree edge !!

v is an articulation point

$\Leftrightarrow$ $\exists$ a child w of v, with

$low(w) \geq pre(v)$ DFS

$\Leftrightarrow$ $\max_{w \in children \cap v} [low(w)] \geq pre(v)$ DFS

$maxlow(v) = \max_{w \in children \cap v} [low(w)]$

So v is an articulation point

$\Leftrightarrow$ $maxlow(v) \geq pre(v)$ DFS