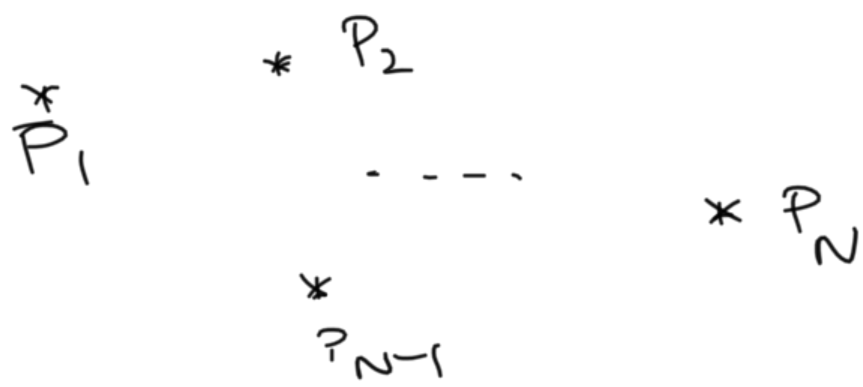
Closest pair of points
Divide and conquer

Naive :



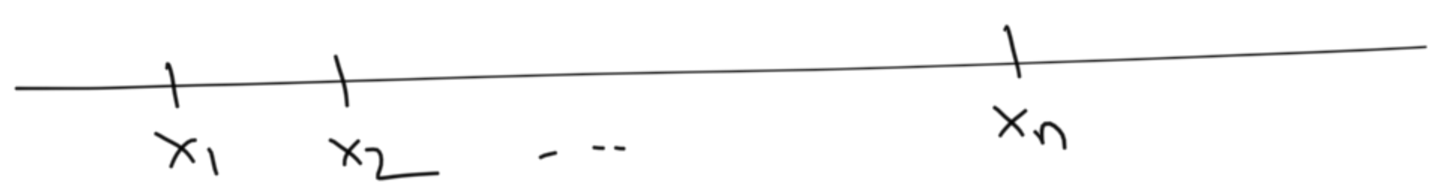
Find $\min [(P_i - P_j) \mid \forall \, i < j]$

$O(n^2)$

Divide and conquer

" Assume no 2 x coordinates same
no 2 y coordinates same" ****

Ⓐ 1-d simpler version : $\{ x_1, \dots \, x_n \}$



sort $O(n \log n)$

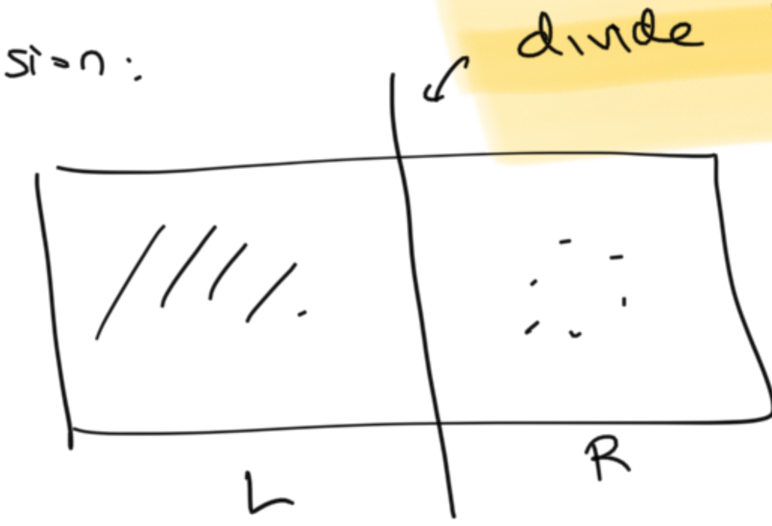
$\min\limits_i [x_i - x_{i+1}]$ $\quad O(n)$

$O(n \log n)$

Ⓑ 2-d version : ****

divide by a vertical point

Divide:



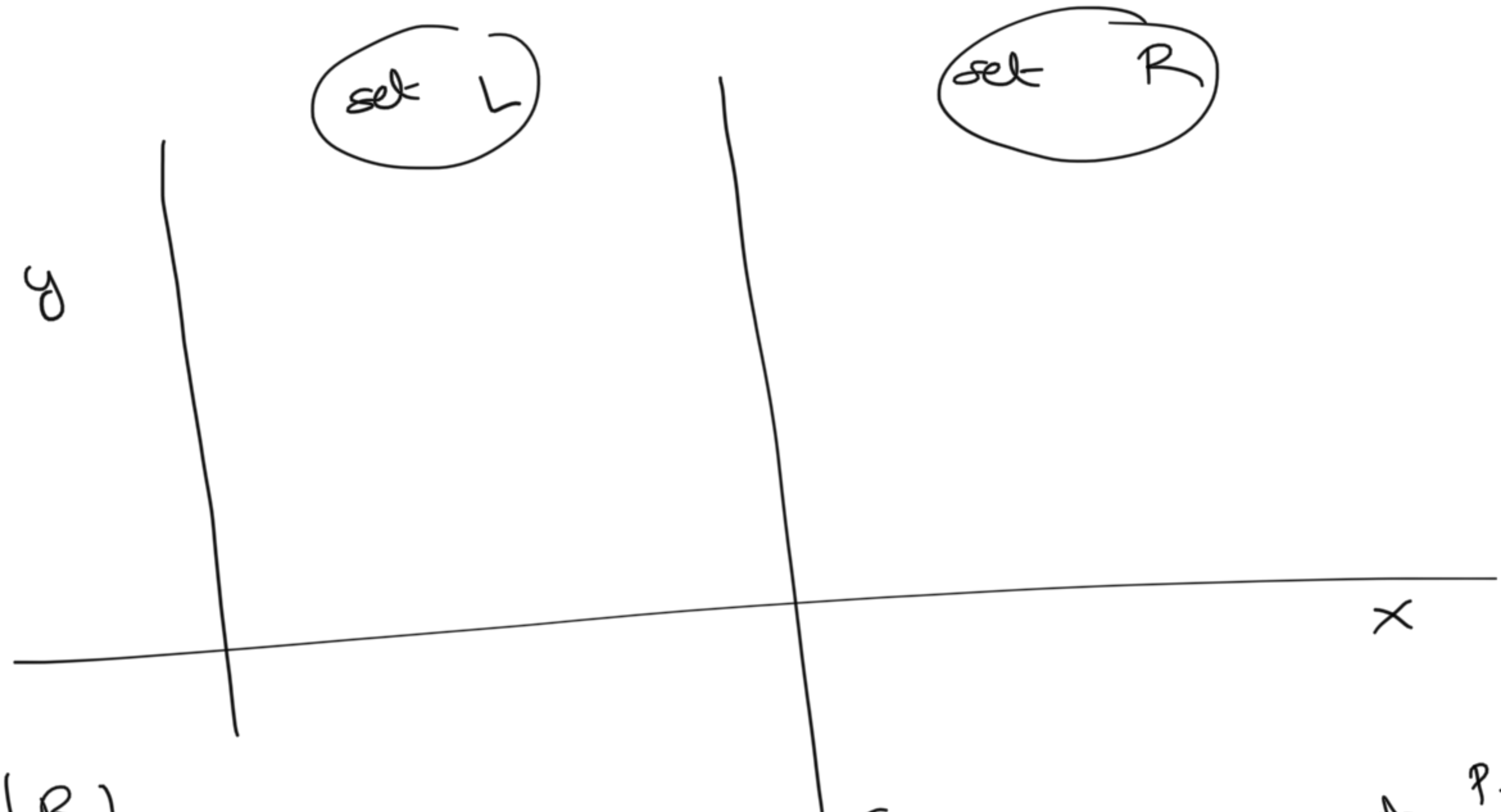
n points in L $\approx$ # n points in R

Recursively solve sub problems : closest pairs (L), closest pairs (R)

combine : what about
min $\{ d(l, r) \mid l \in L, r \in R \}$??

P = given list of points ($|P| = n$)

P_x = list sorted by x coordinate
P_y = list sorted by y coordinate $\Big\}$ $O(n \log n)$

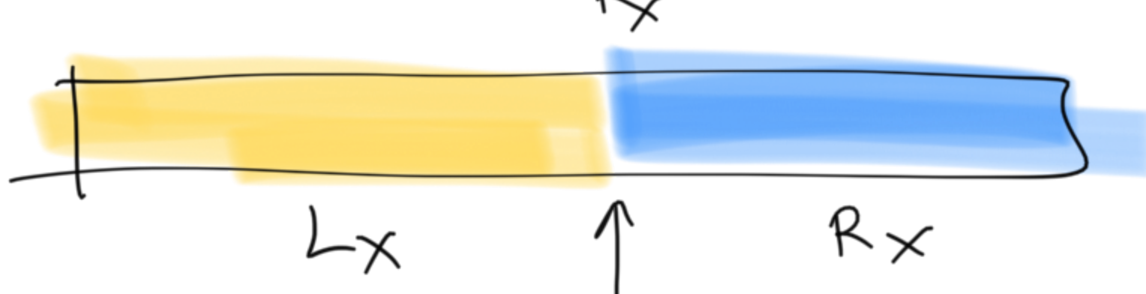


$|L| = |R|$

x = median of P_x
x coordinate

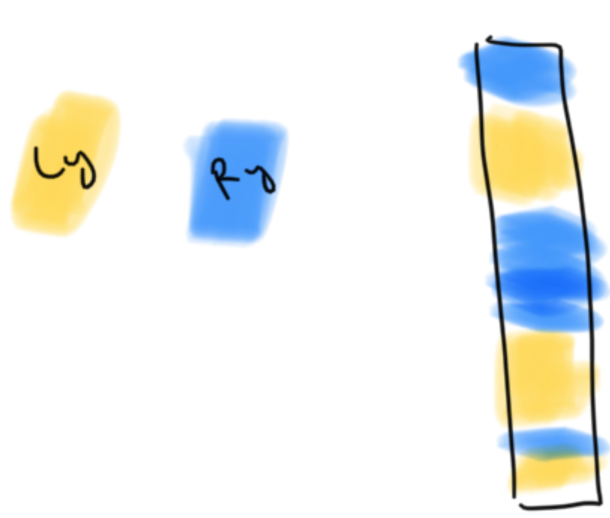
what is L_x, L_y ?
R_x, R_y ?
(Don't re compute!!)

**** no 2 points have same x coordinate



x = median of P_x coordinate

??



P_y

Scan P_y. if x coordinate $<$ median of P_x x coordinate

lands in L_y, else lands in R_y

P_x, P_y $\quad O(n \log n)$

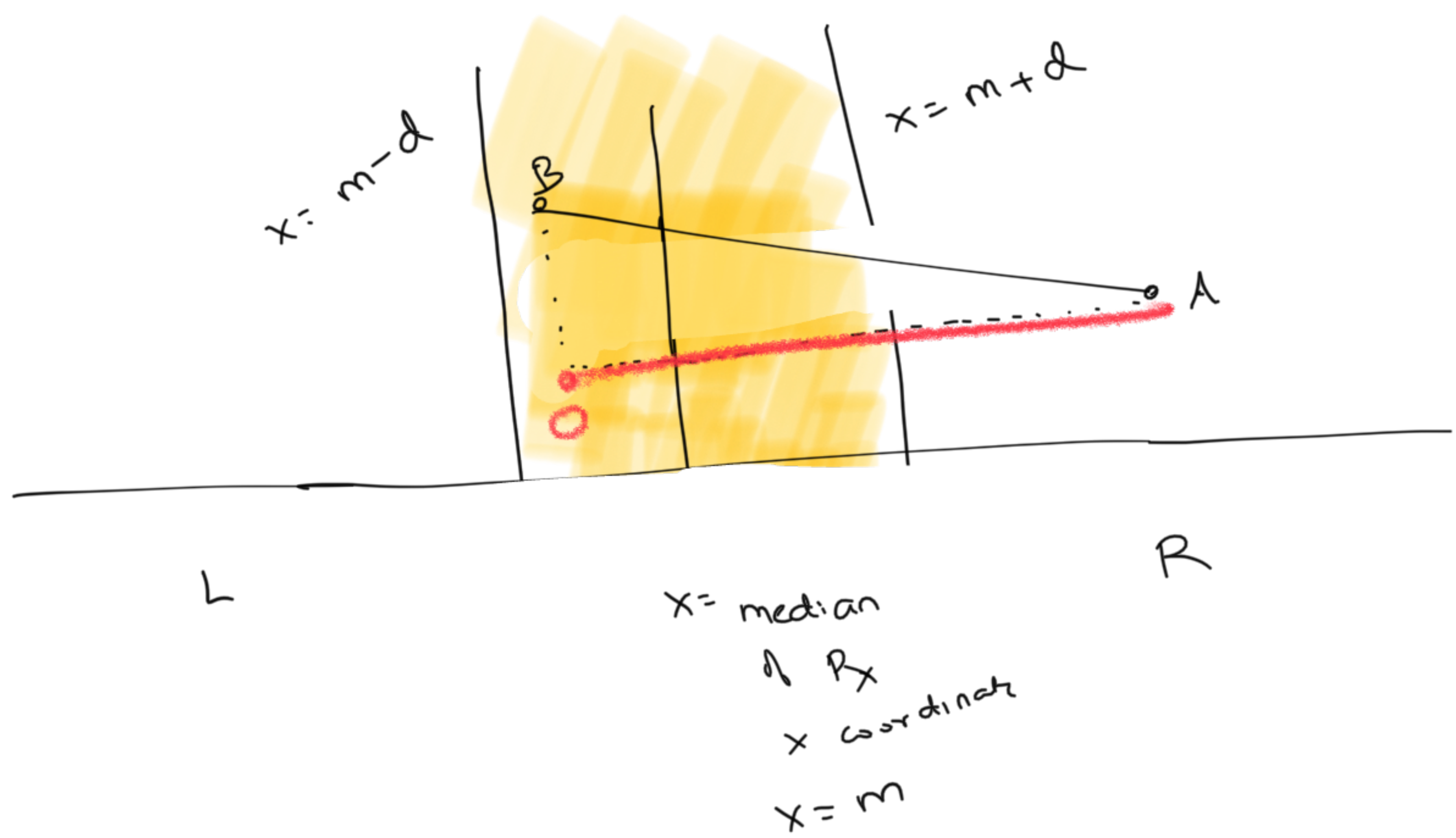
Finding L_x, L_y, R_x, R_y $\quad O(n)$

$d_L =$ ClosestPairs (L_x, L_y) ✓ $\quad T(n/2)$

$d_R =$ ClosestPairs (R_x, R_y) ✓ $\quad T(n/2)$

$d = \min(d_L, d_R)$

- Finding $\min\{ d(l,r) \mid l \in L, r \in R \}$??
- Finding $\min\{ d(l,r) \mid l \in L, r \in R, d(l,r) < d \}$??



$$A - B = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

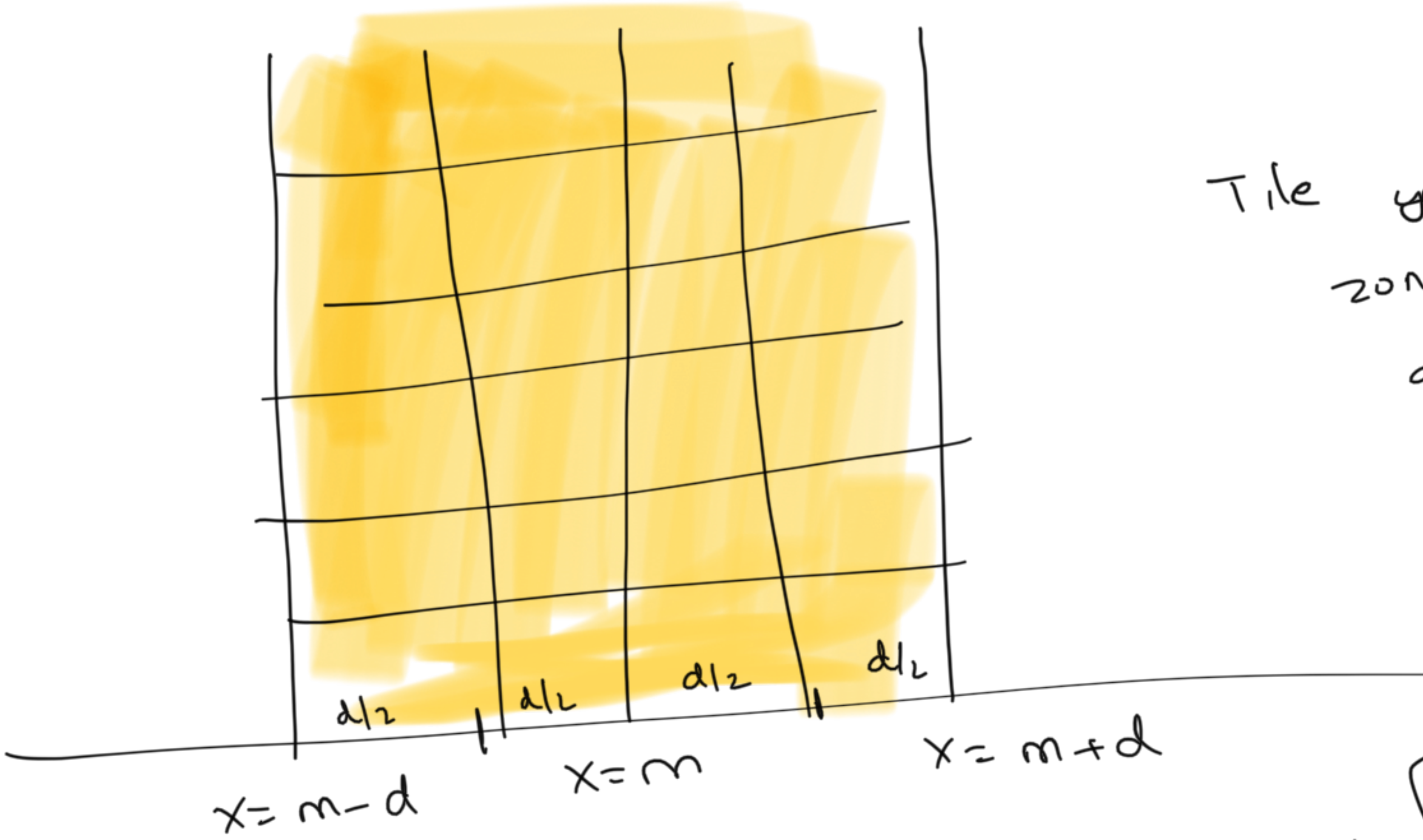
$$\geq A_x - B_x$$

$$= |AO| > d$$

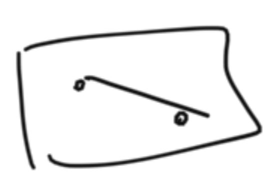
so no need to consider !!

Only need to find $\min \left\{ d(l, r) \;\middle|\; \begin{array}{l} l \in L \\ r \in R \\ l, r \text{ lie in yellow zone} \end{array} \right\}$

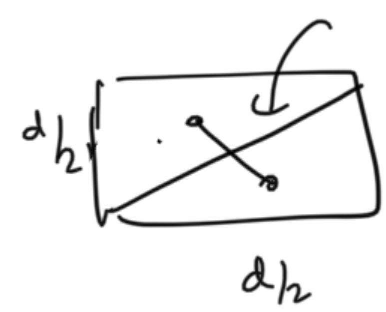
Tile yellow zone with $d/2 \times d/2$ squares



$< \sqrt{\frac{d^2}{2}} = d/\sqrt{2}$



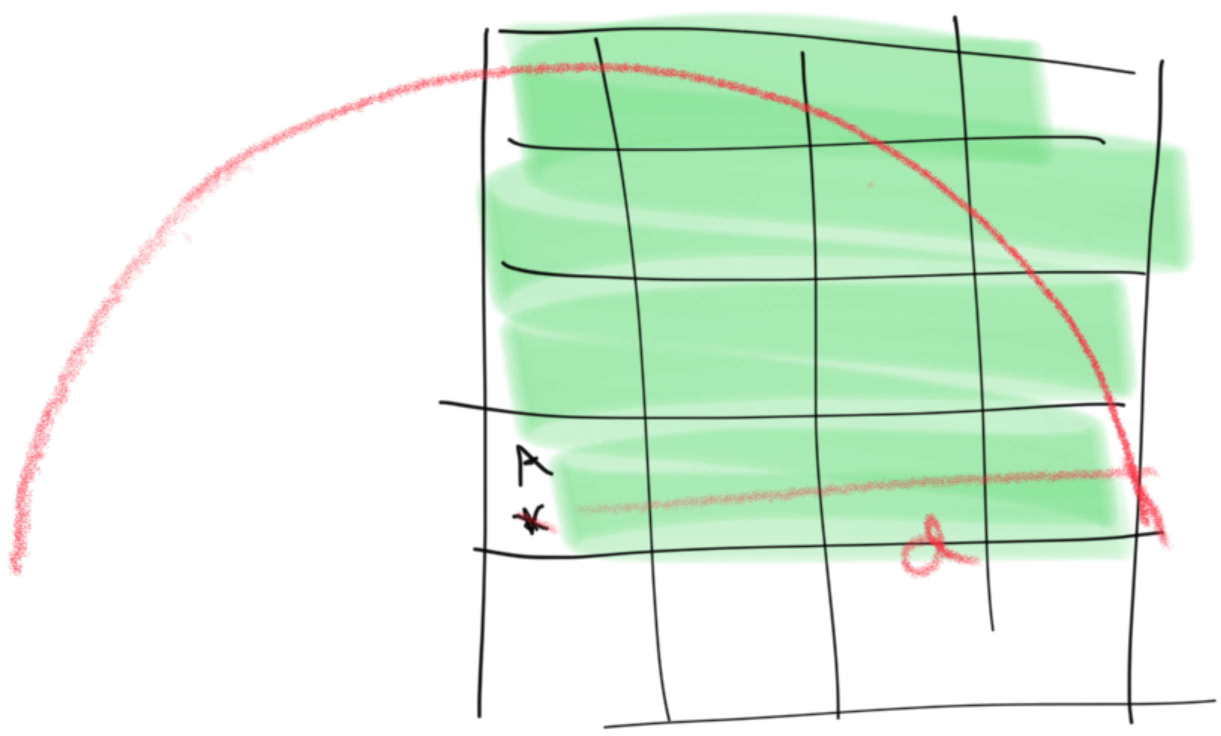
2 points in same box (lies in L say) $\Rightarrow$



$< d$

$\rightarrow\leftarrow$

so each box has atmost 1 point



check A against 15 other points

S_y = (sorted y) in yellow zone

$O(n)$

L_y merge R_y

$\leftarrow A$

S_y

compare A with 15 pts over it in S_y ---

$\forall A \in S_y$ // ~ min $= d_s$

$O(n)$

$$\boxed{\text{ans} = \min(d, d_s)}$$

$$O(n\log n) + \underbrace{2T(n/2) + O(n)}_{= O(n\log n)} = O(n\log n)$$