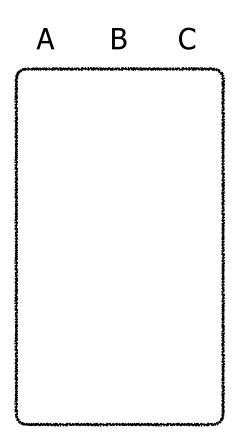
Write-Optimized Indexing

Niv Dayan 31 January, 2023

We will begin at 2:10 pm



We are trying to optimize the following kinds of queries



select * from table where A = "..."

select * from table where B > "..." and B < "..."

Let's reconsider four baselines

Append-Only Table

Sorted Table

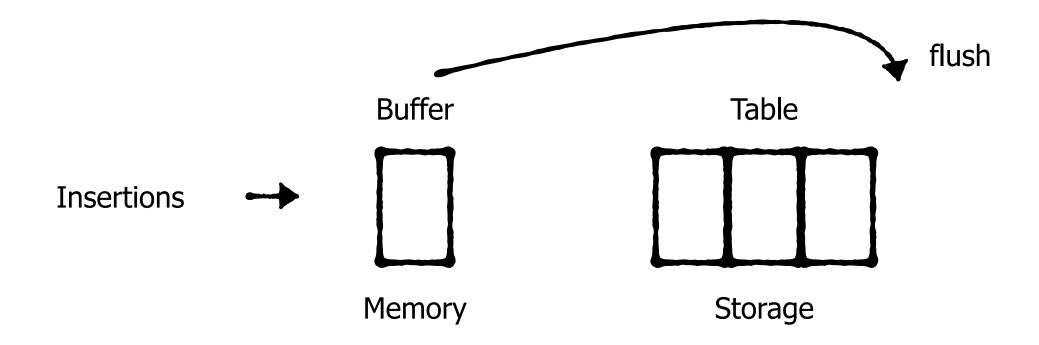
Extendible Hashing

B-tree

Append-Only Table

O(N/B) for any selection query

O(1/B) for insertions



Let's reconsider four baselines

Append-Only Table

Sorted Table

Extendible Hashing

B-tree

Sorted Table

A ...

7 ...

22 ...

32 ...

32 ...

61 ...

-

74 ...

90 ...

97 ...

Binary search: O(log₂ N/B) I/O

Update/insert/delete: O(N/B) read & write I/O

Let's reconsider four baselines

Append-Only Table

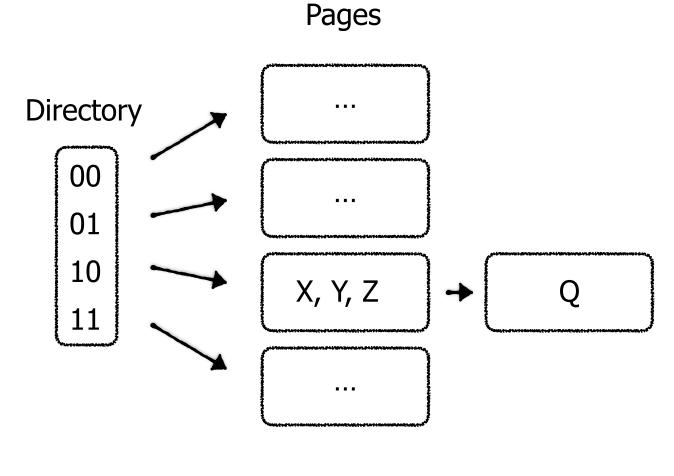
Sorted Table

Extendible Hashing

B-tree

A directory maps pages in storage with a given hash prefix

Handle overflows via chaining

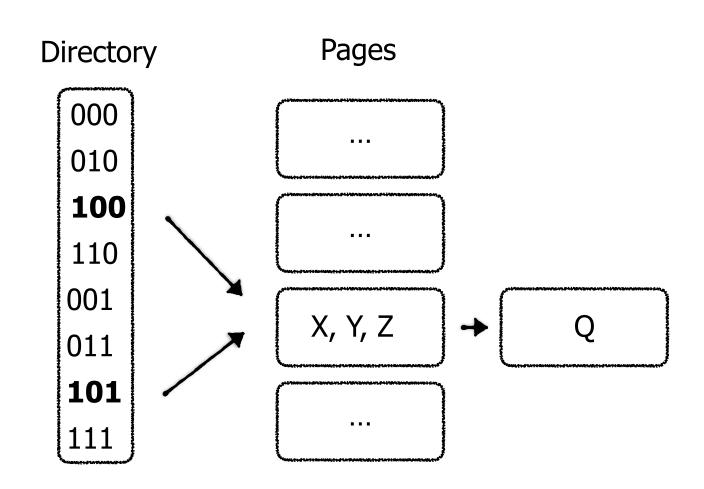


Memory

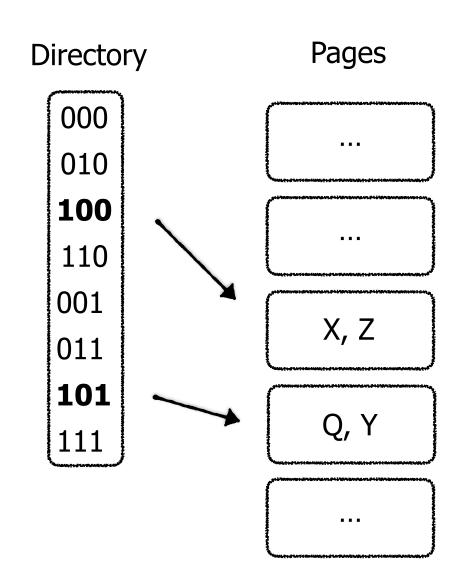
Storage

When we reach capacity, double directory size.

New directory slots still point to previous pages



Split one overflowing bucket at a time by rehashing entries.

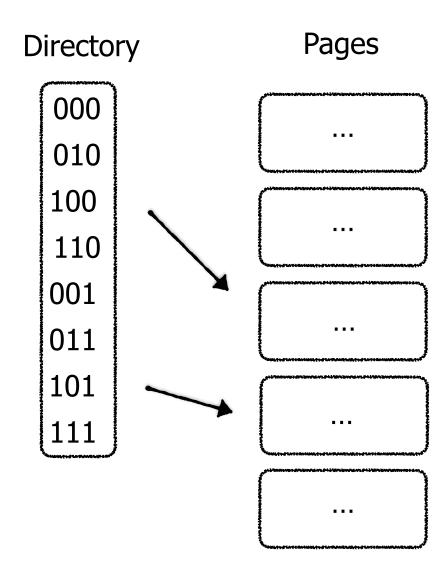


Query cost: O(1) read I/O

Insertion cost:

O(1) read I/O

O(1) write I/O



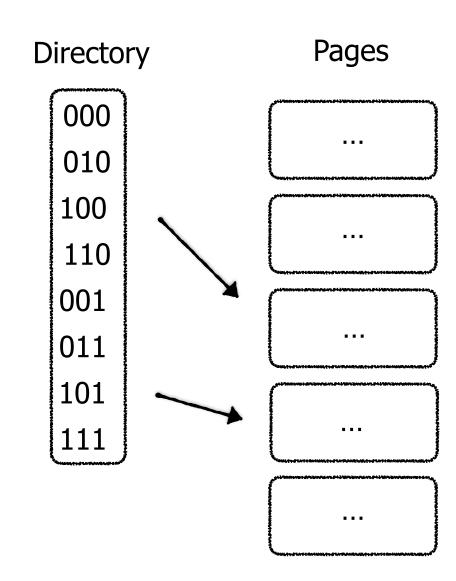
Query cost: O(1) read I/O

Insertion cost:

O(1) read I/O

O(1) * **GC** write I/O

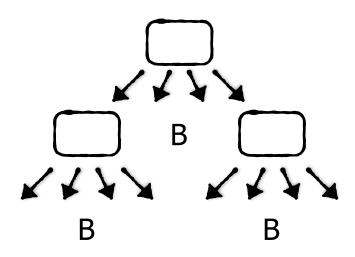
Random writes to storage entail SSD garbage-collection



Let's reconsider four baselines

Append-Only Table Sorted Table Extendible Hashing B-Tree

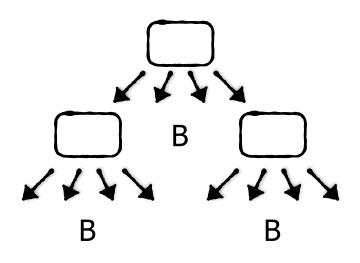
B-Tree



Each node has B children. This allows pruning by a factor of B in each level

Can issue random writes to storage leading to SSD garbage-collection

B-Tree



Each node has B children. This allows pruning by a factor of B in each level

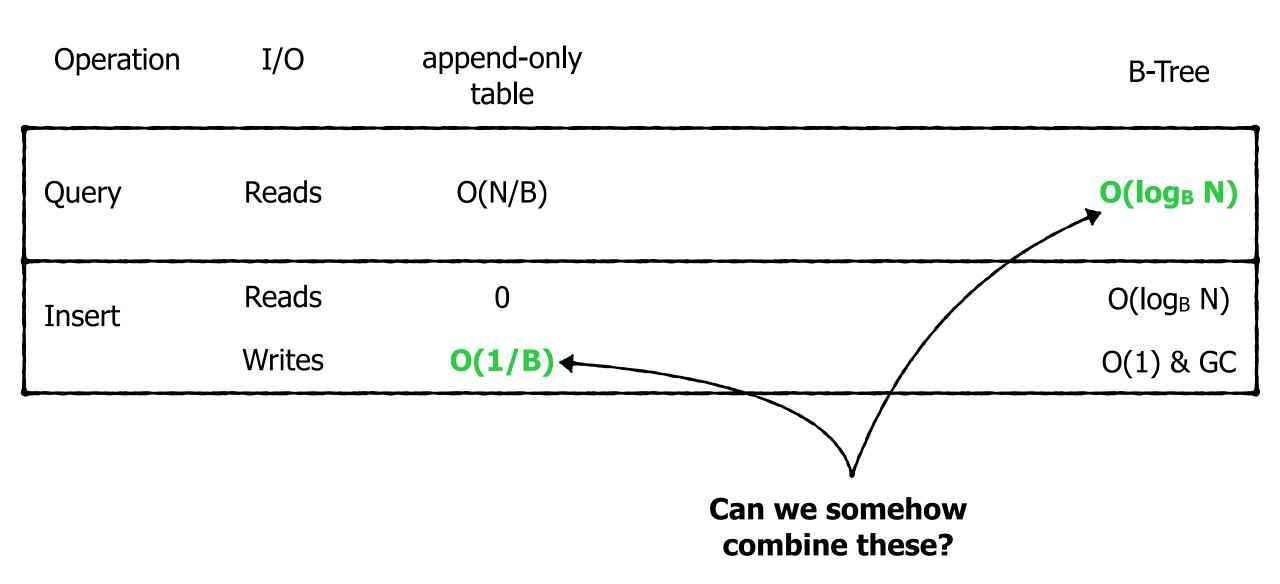
Can issue random writes to storage leading to SSD garbage-collection

Query: O(log_B N) read I/O

Update/insert/query: O(log_B N) read I/O

& O(1) * GC write I/O

Operation	I/O	append-only table	Sorted File	Extendible Hashing	B-Tree
Query	Reads	O(N/B)	O(log ₂ N/B)	O(1)	O(log _B N)
Insert	Reads	0	O(N/B)	O(1)	O(log _B N)
	Writes	O(1/B)	O(N/B)	O(1) & GC	O(1) & GC



The Log-Structured Merge-Tree





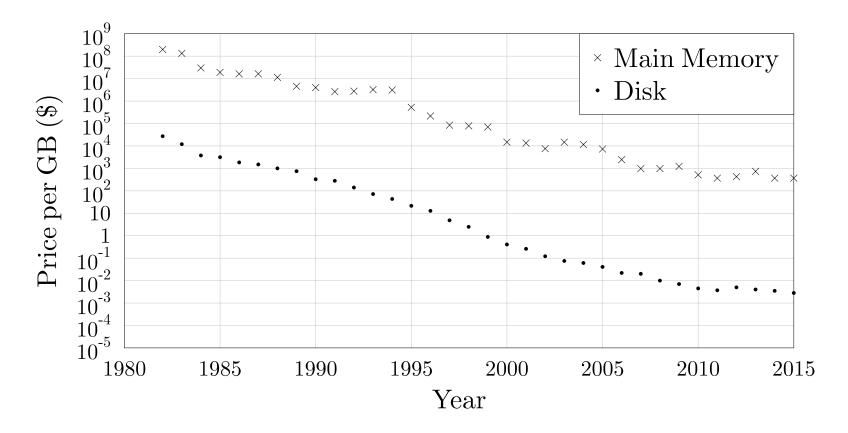
B-tree	LSM-tree is	Google's BigTable	LSM-tree is	
is invented	invented	adopts LSM-tree	widely used	
1970	1996	2005	Today	



B-tree is invented	LSM-tree is invented	Google's BigTable adopts LSM-tree	LSM-tree is widely used
1970	1996	2005	Today

Why was it not invented and used sooner?

The declining costs of storage allow us to store more data cheaply. Hence, application workloads are becoming more write-intensive. This drives a need to optimize for data ingestion.



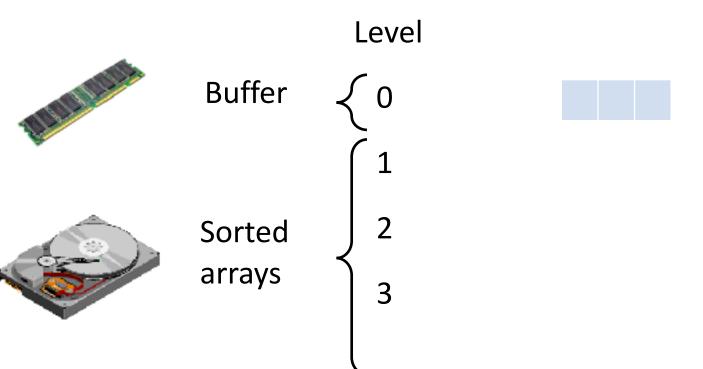
Leveled LSM-tree

Basic LSM-tree

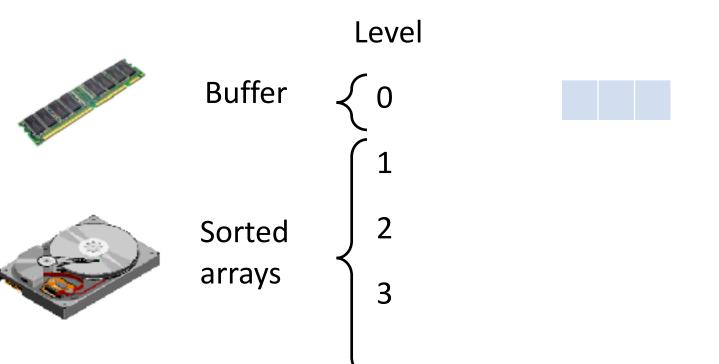
Tiered LSM-tree



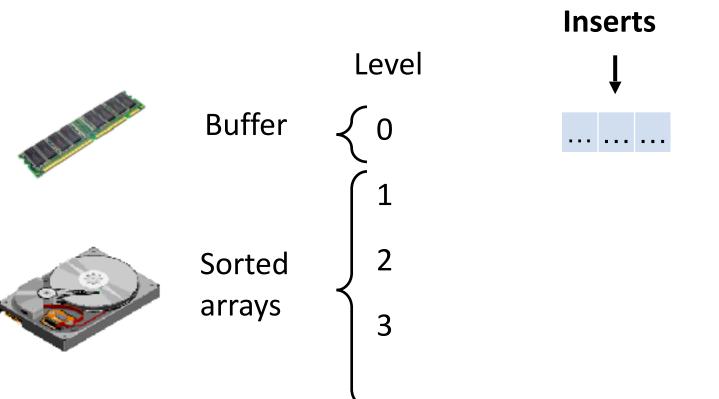




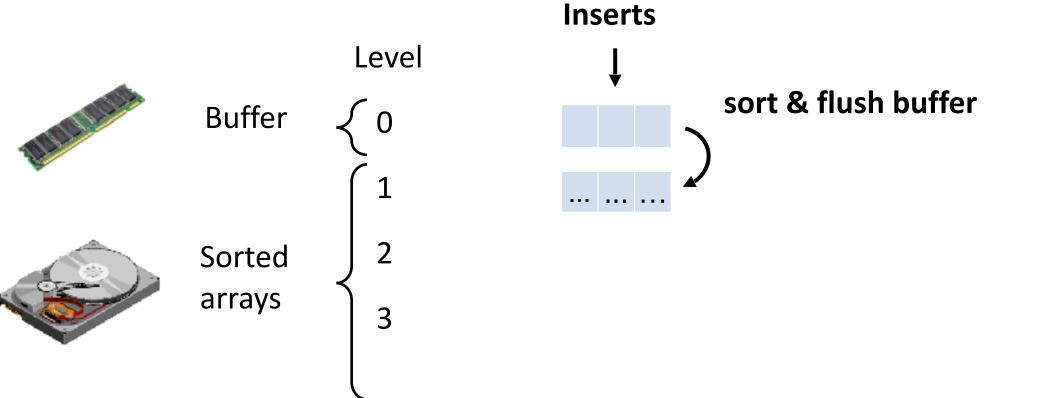
Design principle #1:



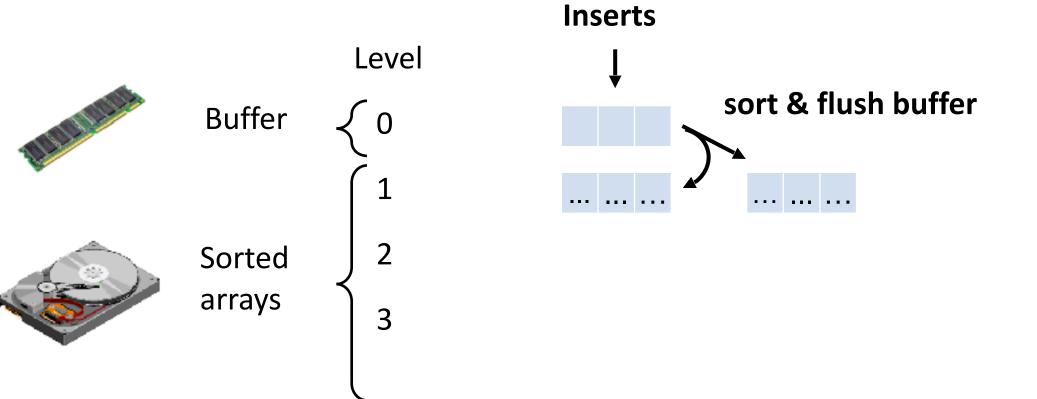
Design principle #1:



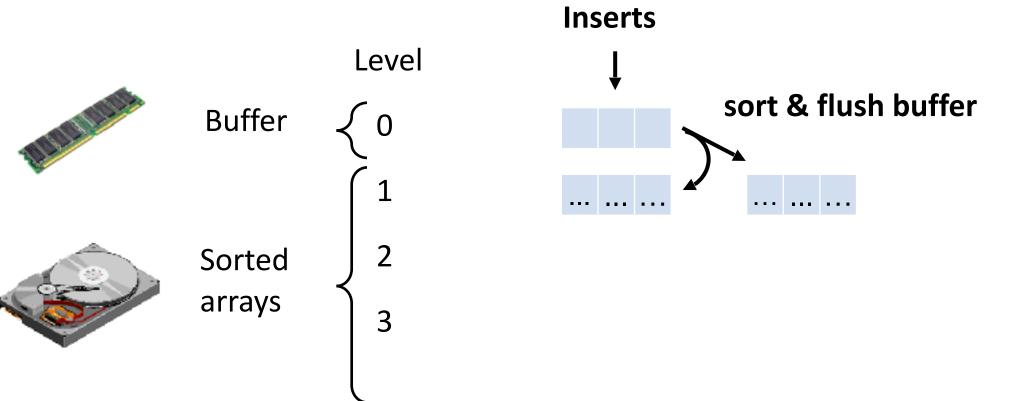
Design principle #1:



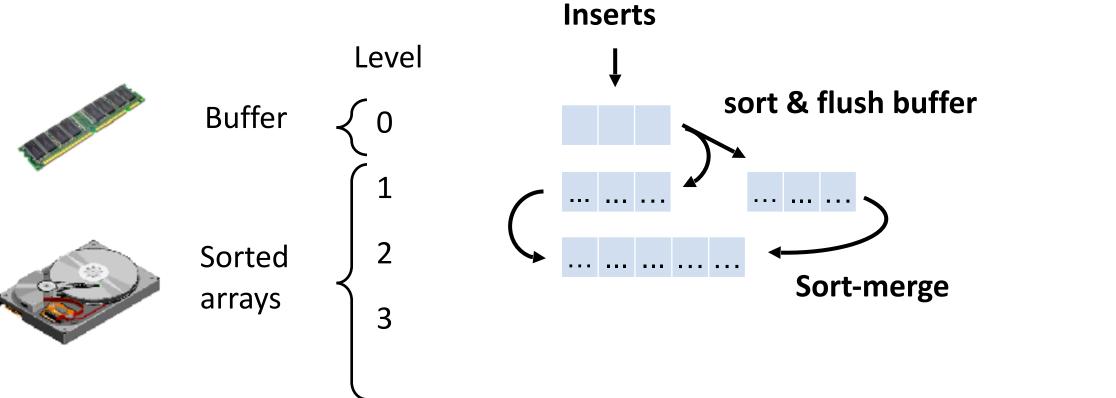
Design principle #1:



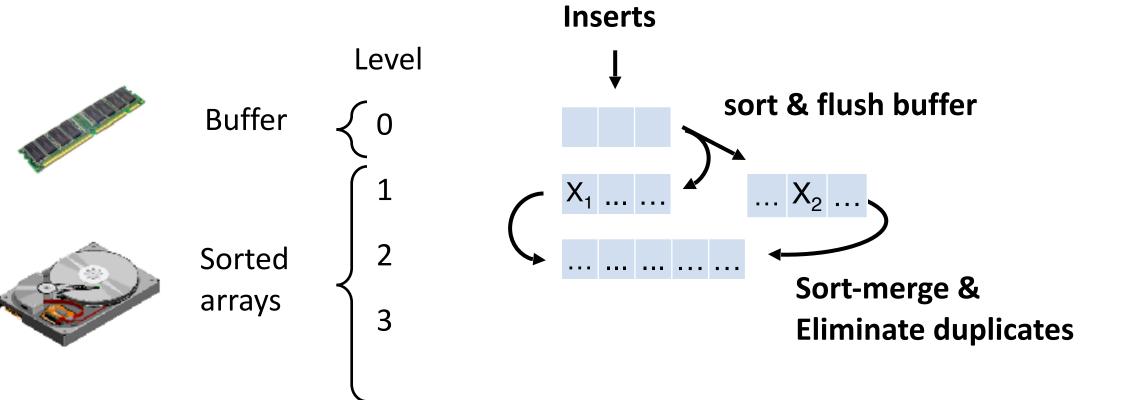
Design principle #1: optimize for insertions by buffering



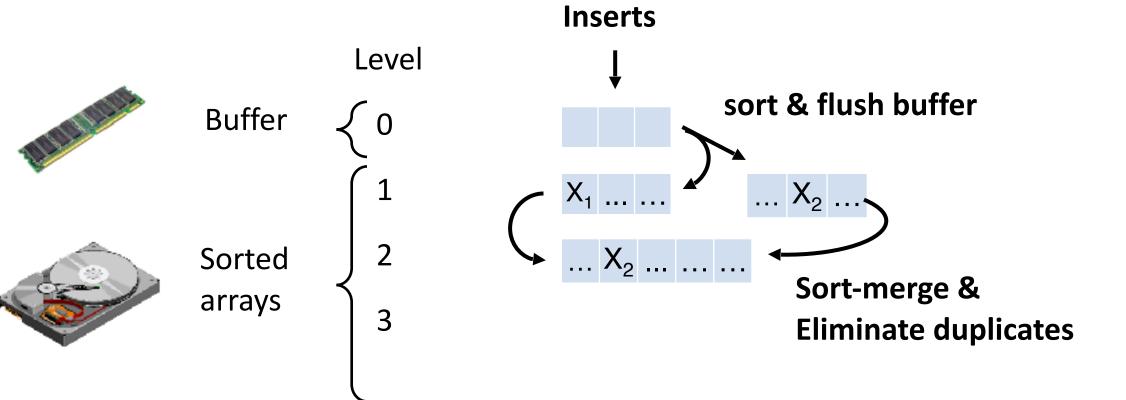
Design principle #1: optimize for insertions by buffering



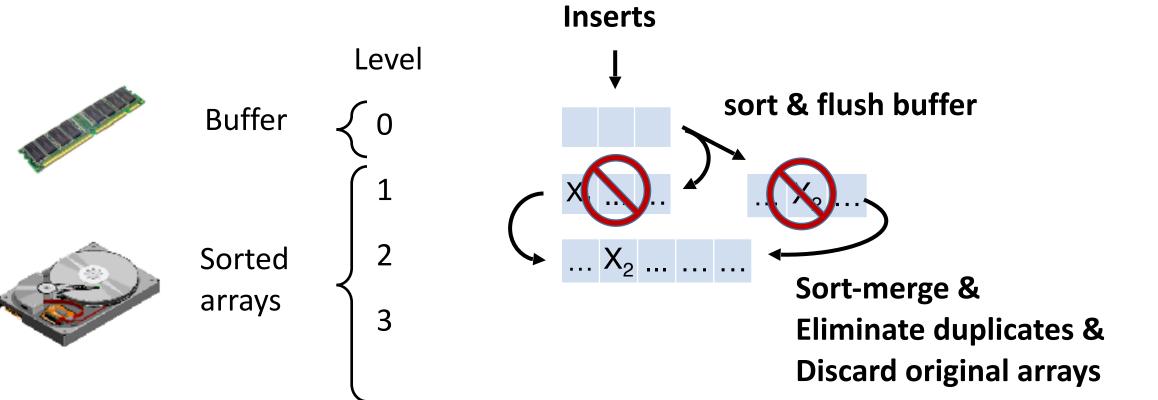
Design principle #1: optimize for insertions by buffering

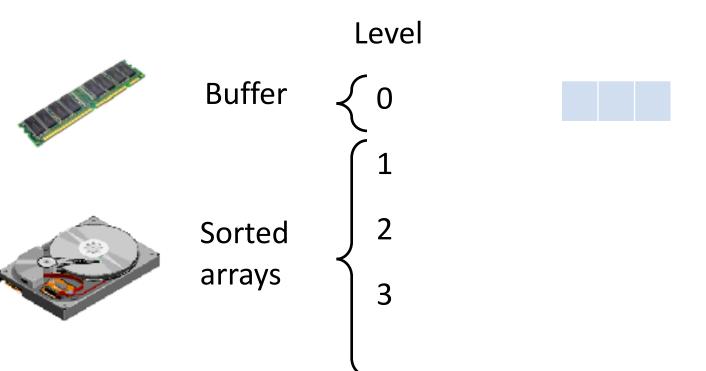


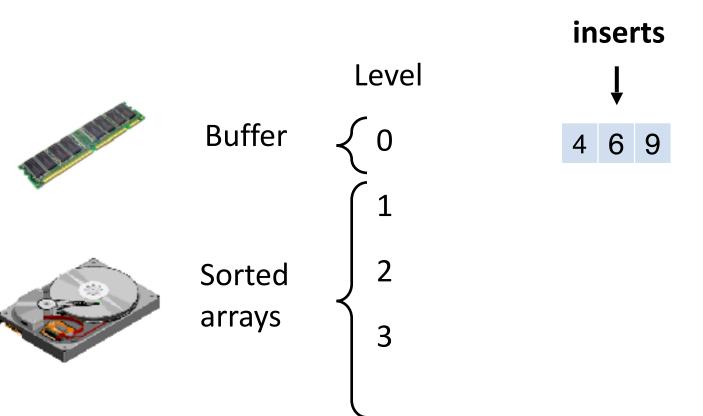
Design principle #1: optimize for insertions by buffering

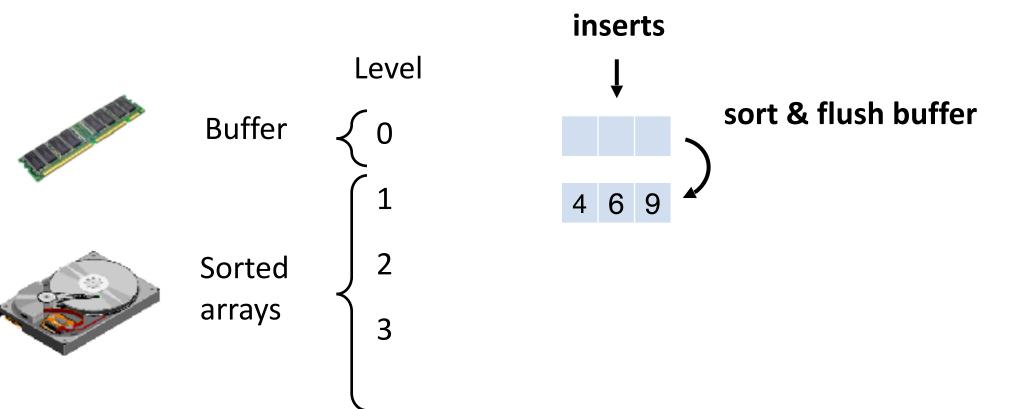


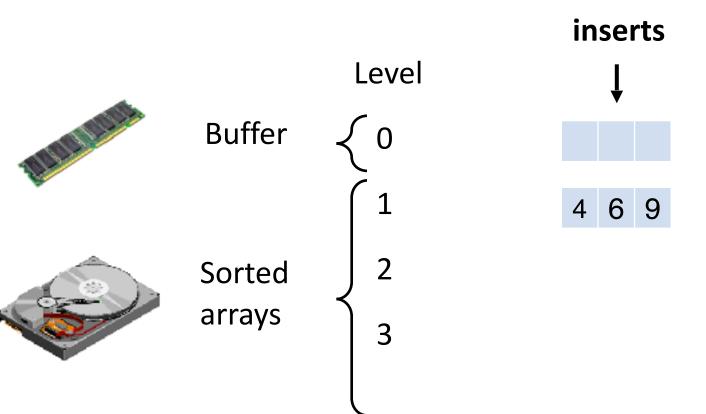
Design principle #1: optimize for insertions by buffering

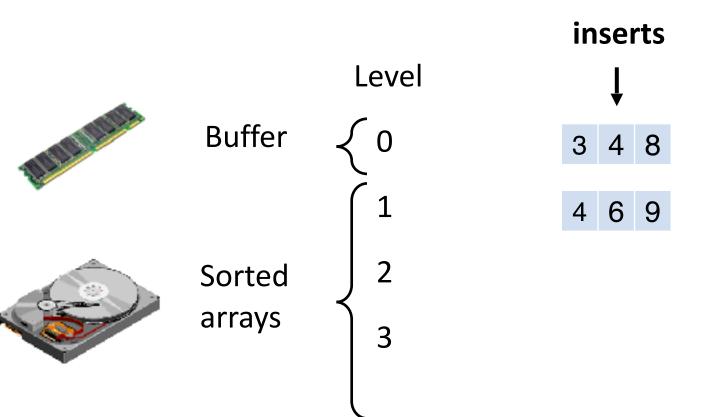


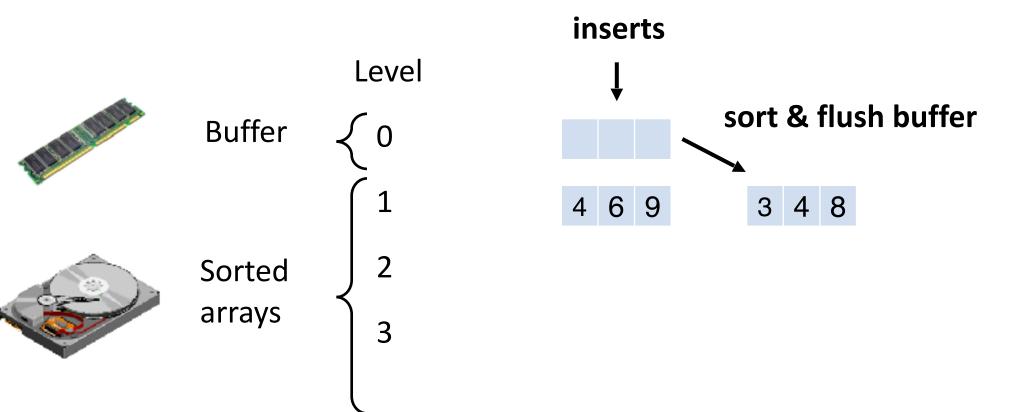


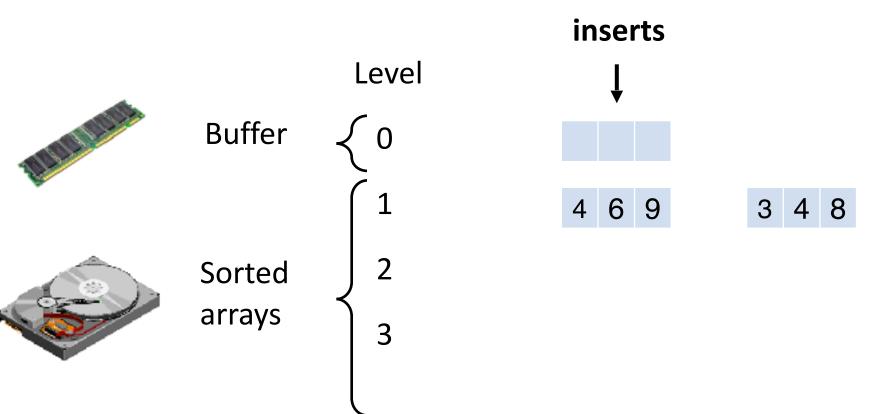


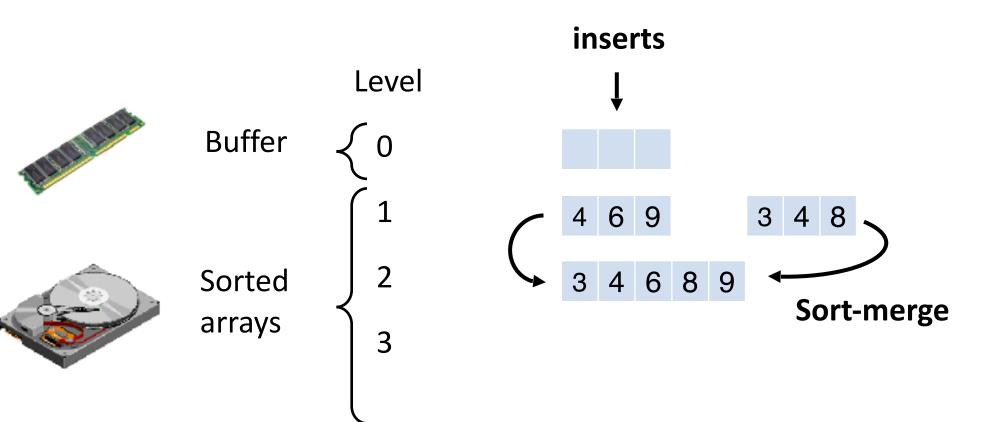


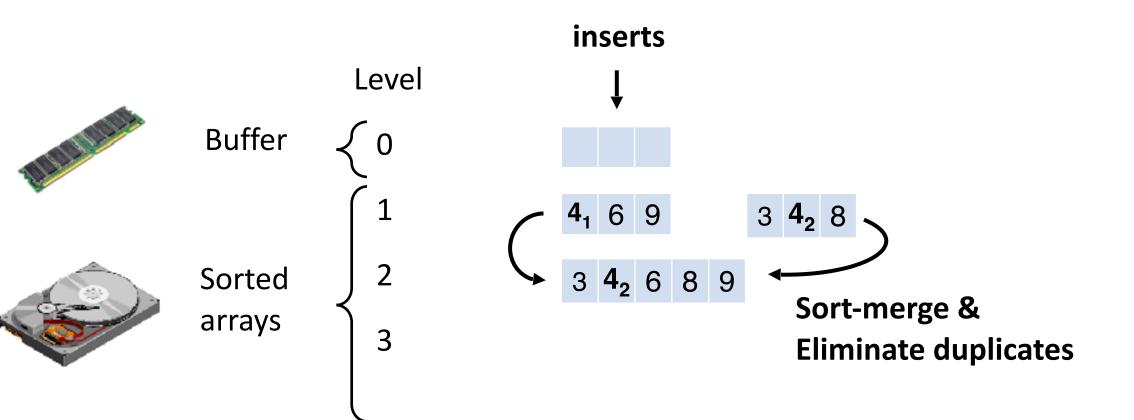


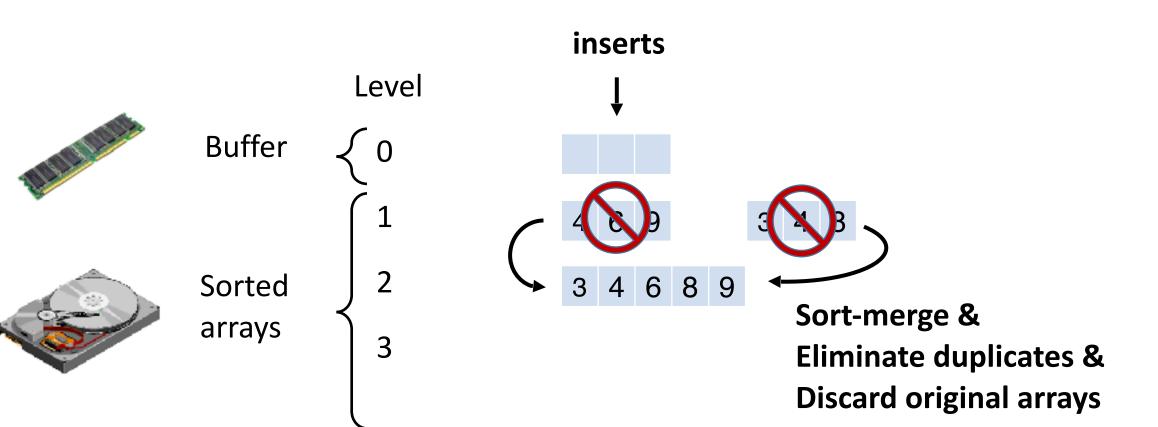


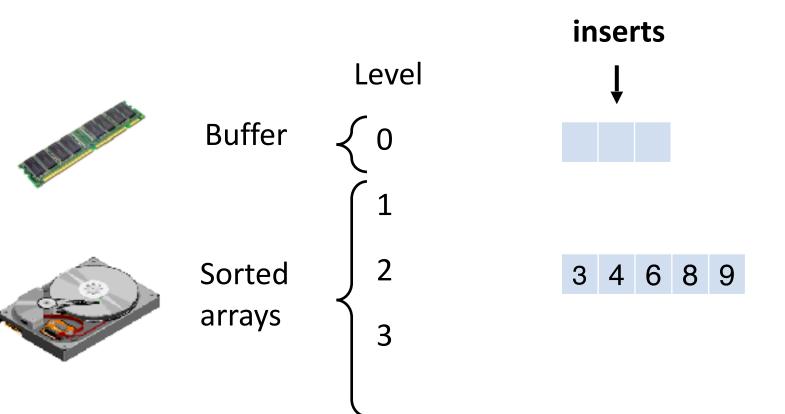


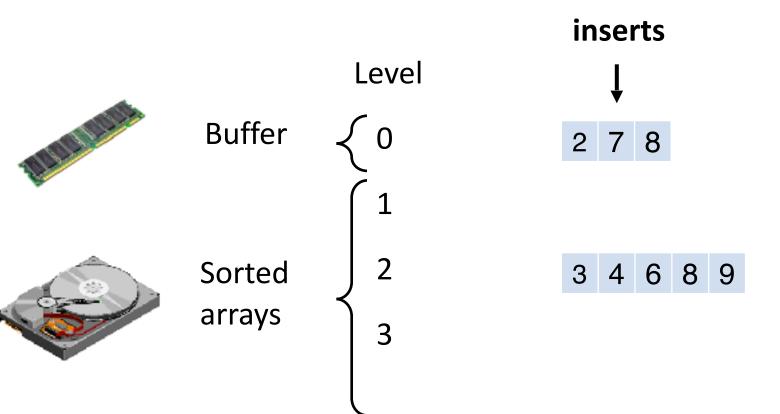


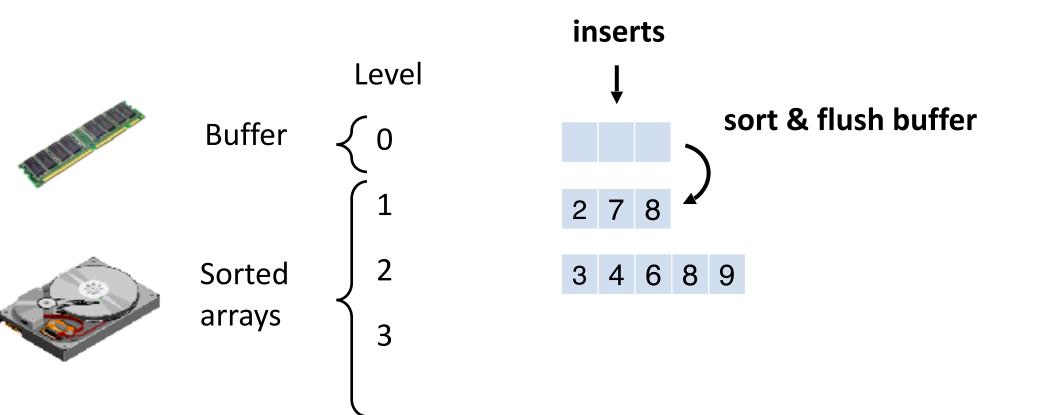


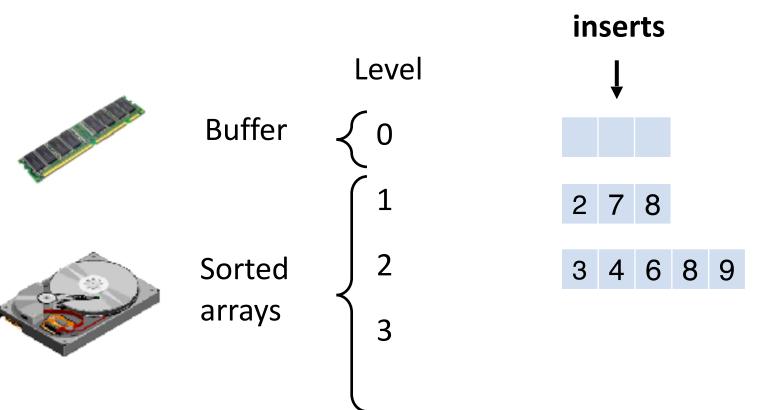






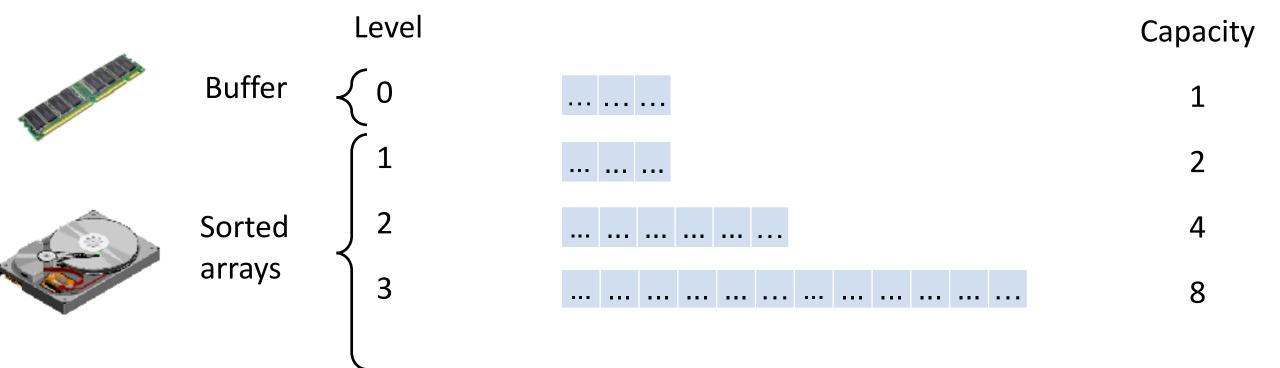






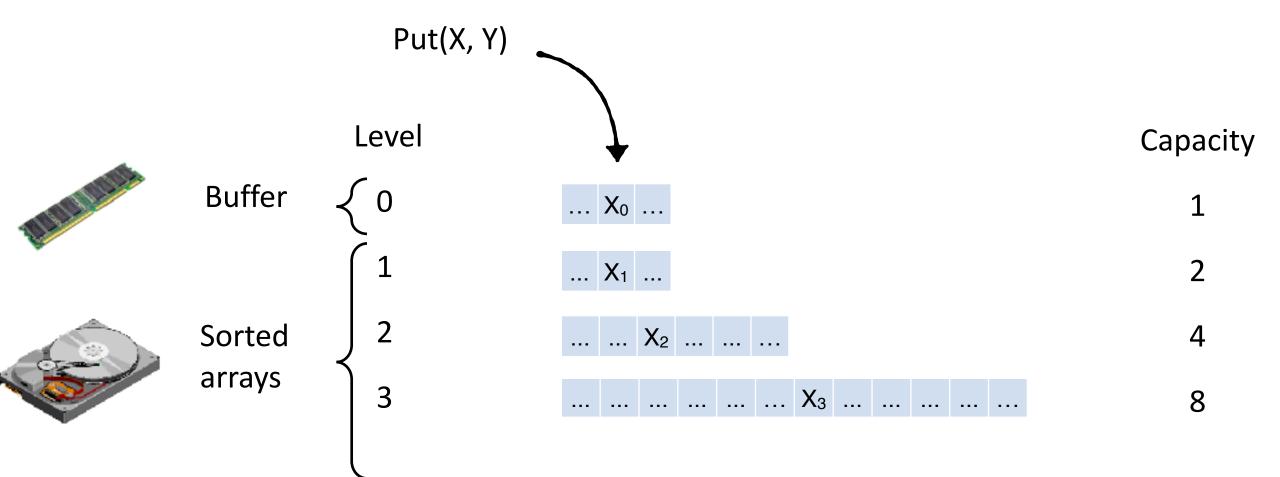
Basic LSM-tree

Levels have exponentially increasing capacities.



Basic LSM-tree - Updates

Can be made out-of-place through an insertion into the buffer



Basic LSM-tree - Updates

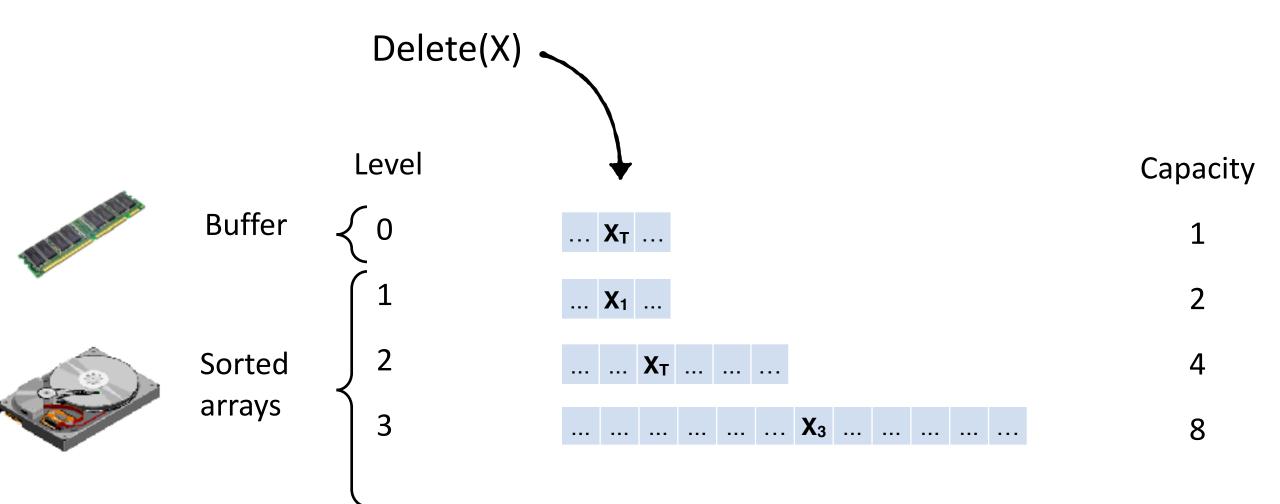
Can be made out-of-place through an insertion into the buffer

Only the most recent version counts and should be returned to users during queries.

	Level		Capacity
Buffer	€ 0	\ldots X_0 \ldots	1
		X ₁	2
Sorted arrays	2	X ₂	4
	3	X ₃	8

Basic LSM-tree - Deletes

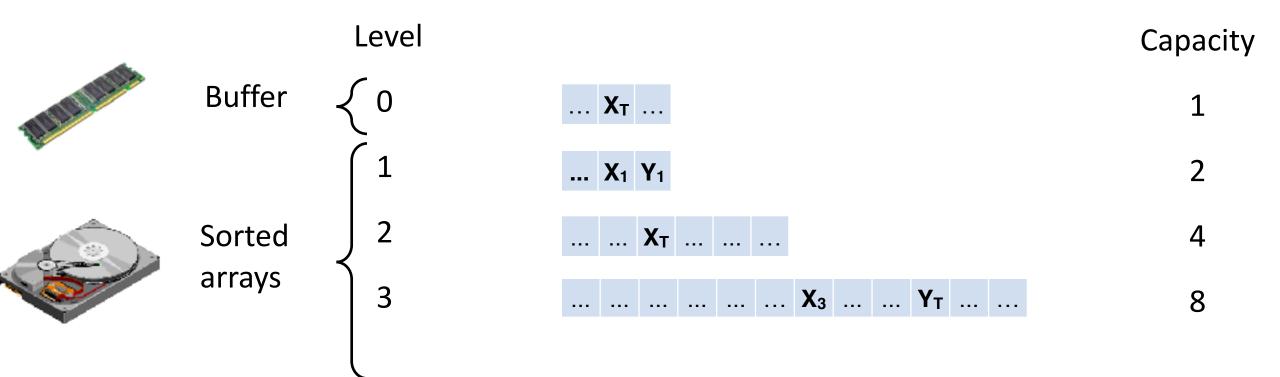
Delete an entry by inserting a tombstone



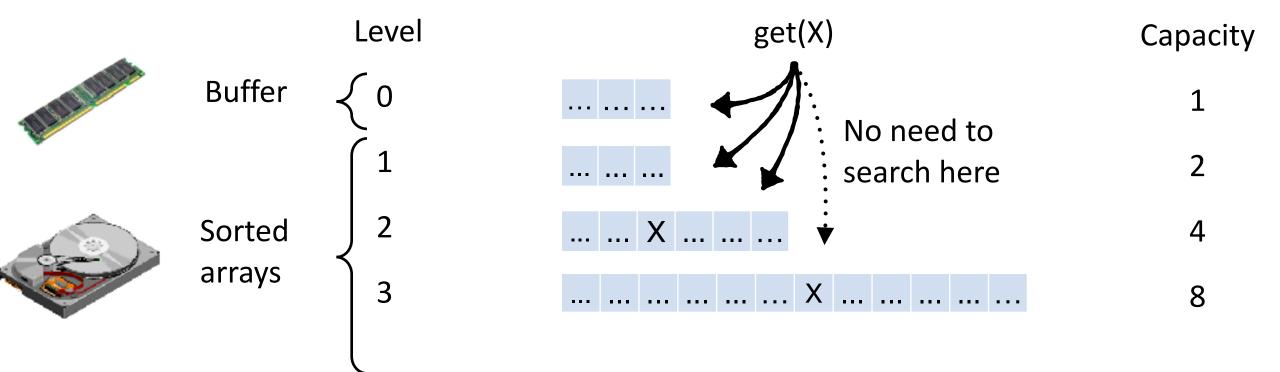
Basic LSM-tree - Deletes

Delete an entry by inserting a tombstone

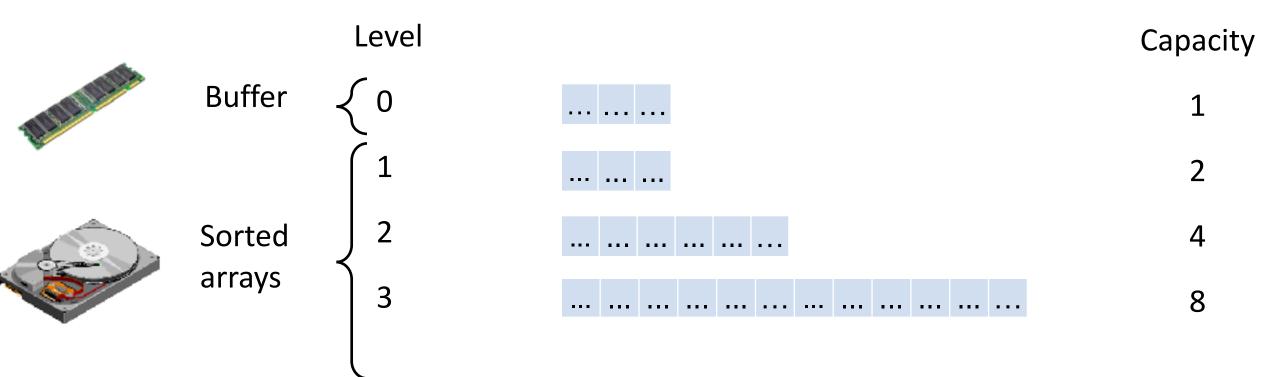
If the most recent version of an entry is a tombstone, it's considered deleted. (X is deleted by Y isn't in this example)



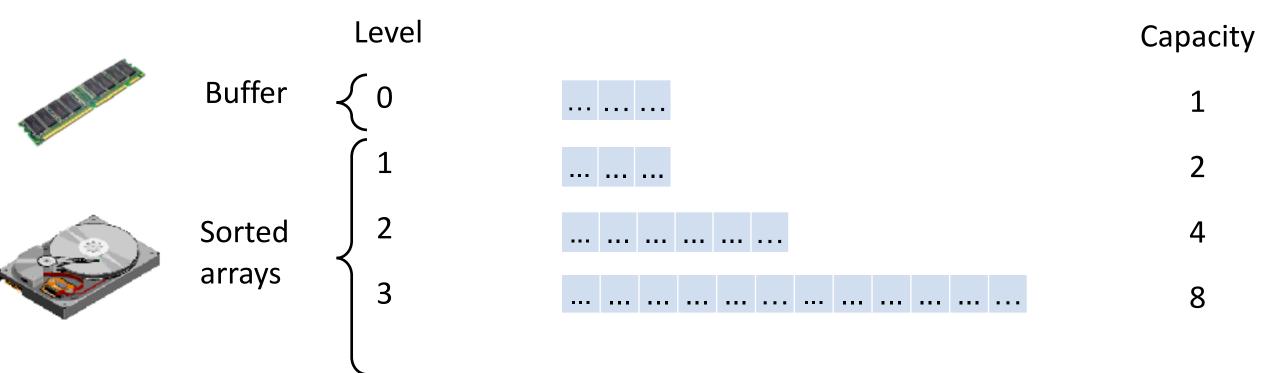
A get query searches the LSM-tree from smaller to larger levels. They stop when they find the first matching entry, as entries at larger levels are older and thus superseded.



How many levels to search?



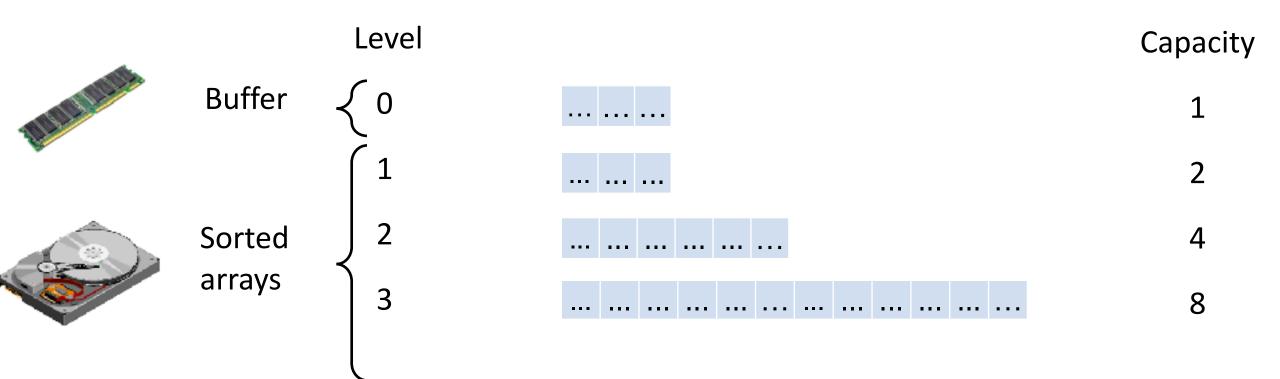
How many levels to search? O(log₂ N/B)



How many levels to search?

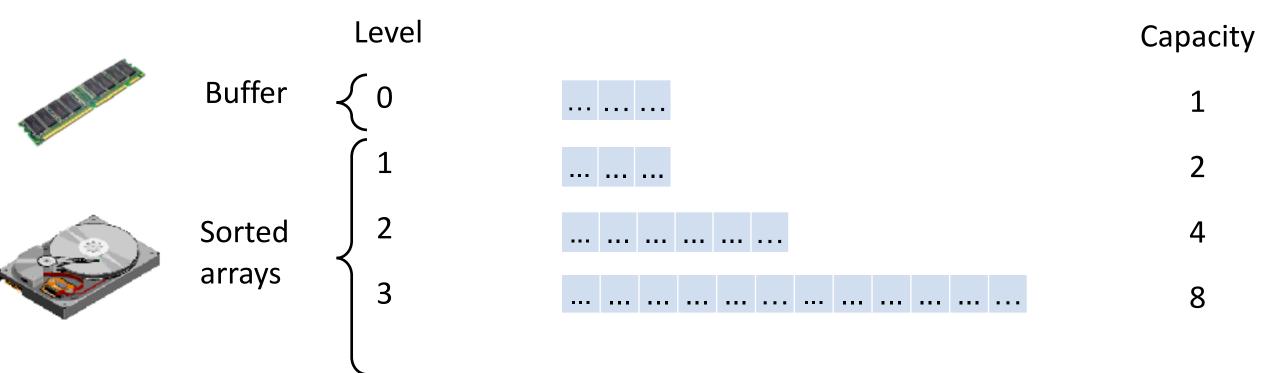
 $O(log_2 N/B)$

Cost per searching each level?



How many levels to search? $O(log_2 N/B)$

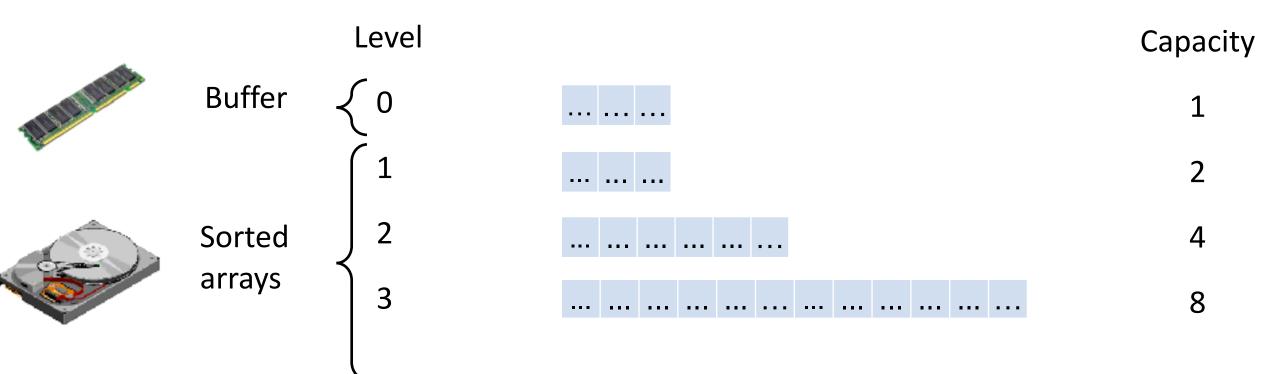
Cost per searching each level? O(log₂ N/B) w. binary search



How many levels to search? O(log₂ N/B)

Cost per searching each level? O(log₂ N/B)

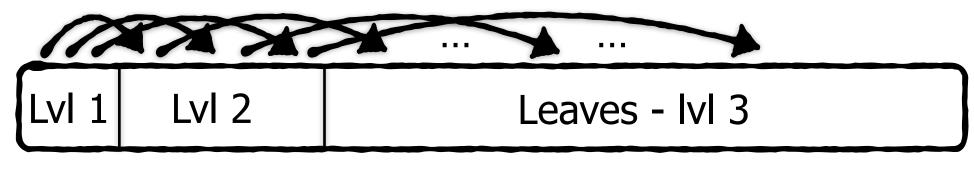
Total search cost: $O(log_2(N/B)^2)$



We can do slightly better by structuring each file (SST) as a static B-tree. How would this impact search cost?

Total search cost:

 $O(log_2(N/B)^2)$

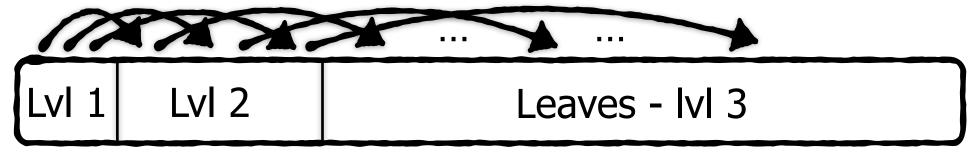


SST static B-tree structure

We can do slightly better by structuring each file (SST) as a static B-tree. How would this impact search cost?

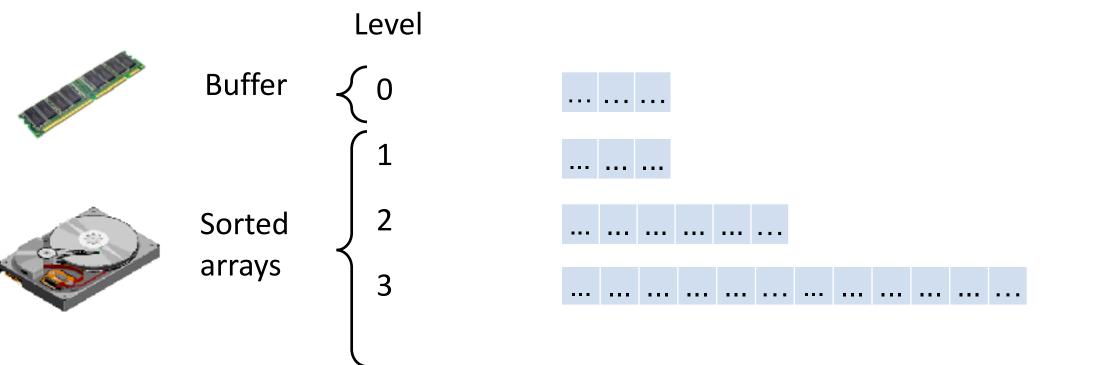
Total search cost:

 $O(log_2(N/B) * log_B N)$



SST static B-tree structure

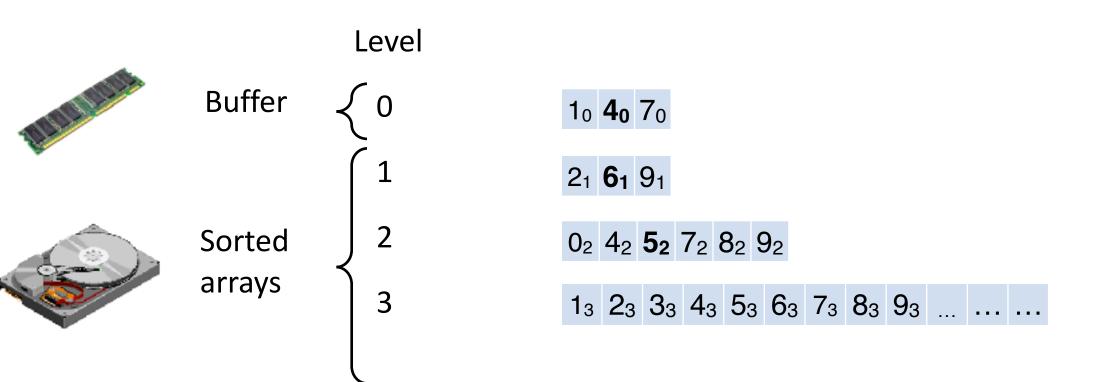
Return most recent version of each entry in the range across entire tree.



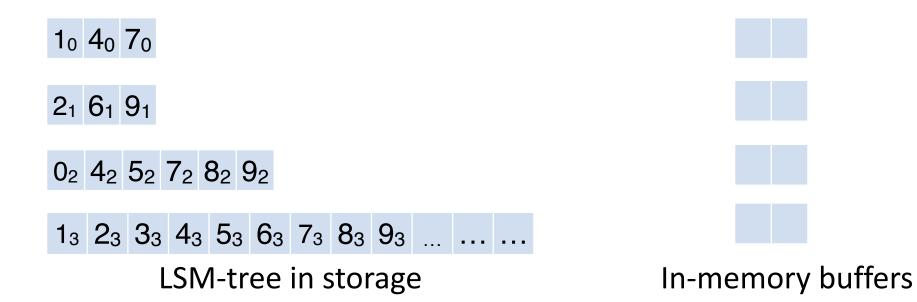
Return most recent version of each entry in the range across entire tree.

e.g., Scan(4, 6)

Expected output: 40 52 61

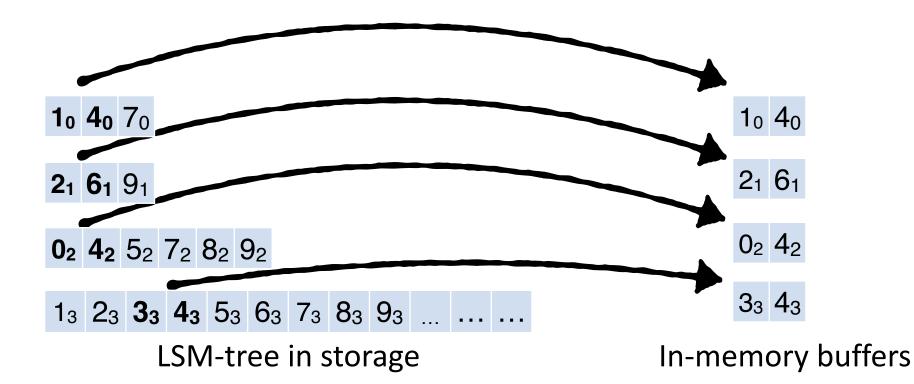


1. Allocate an in-memory buffer (>1 page) for each level



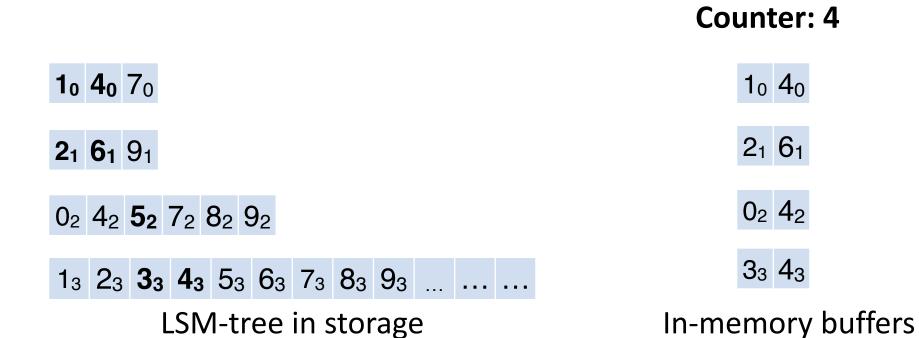
Output

- 1. Allocate an in-memory buffer (>1 page) for each level
- 2. Search for start of key range at each level



Output

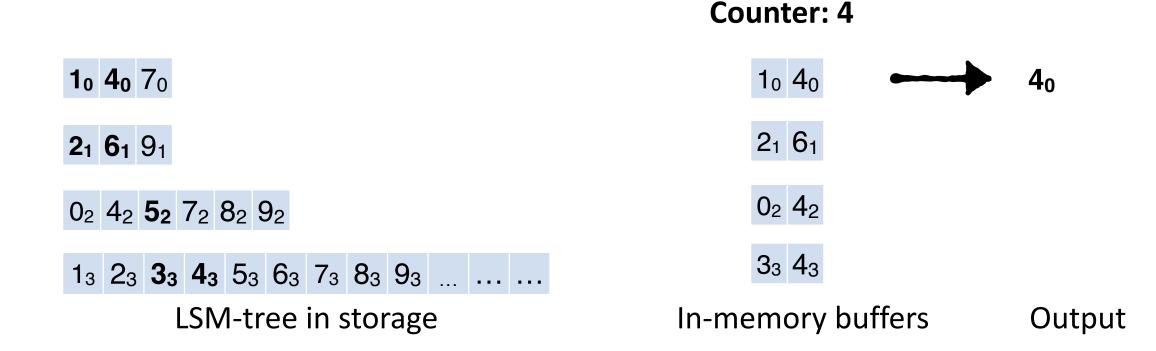
- 1. Allocate an in-memory buffer (>1 page) for each level
- 2. Search for start of key range at each level
- 3. Initialize counter to smallest key in range



Output

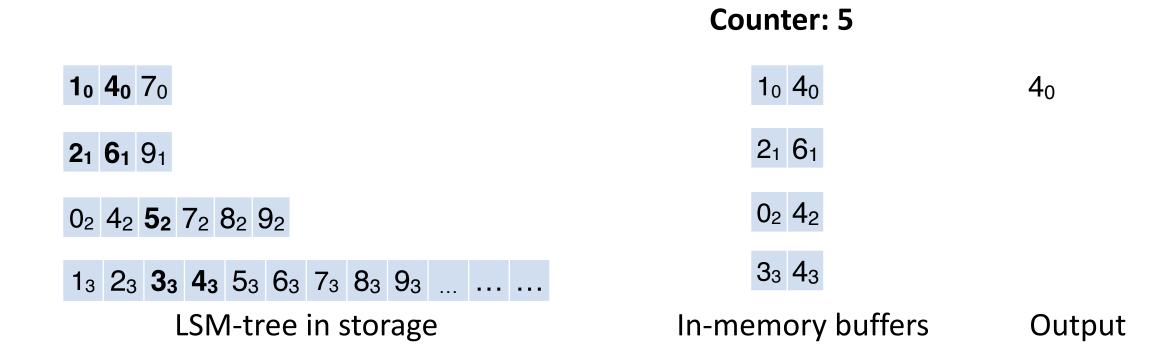
Loop until counter passes end of range

4. Bring youngest matching entry to output



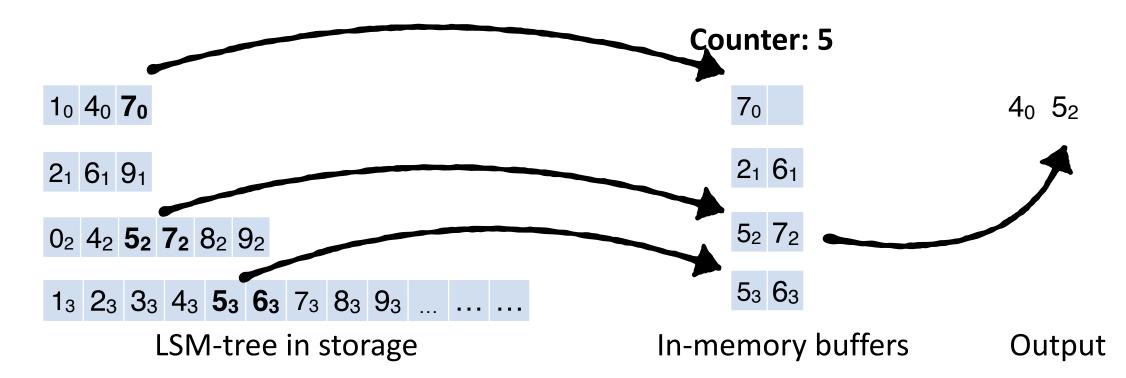
Loop until counter passes end of range

- 4. Bring youngest matching entry to output
- 5. Increment counter



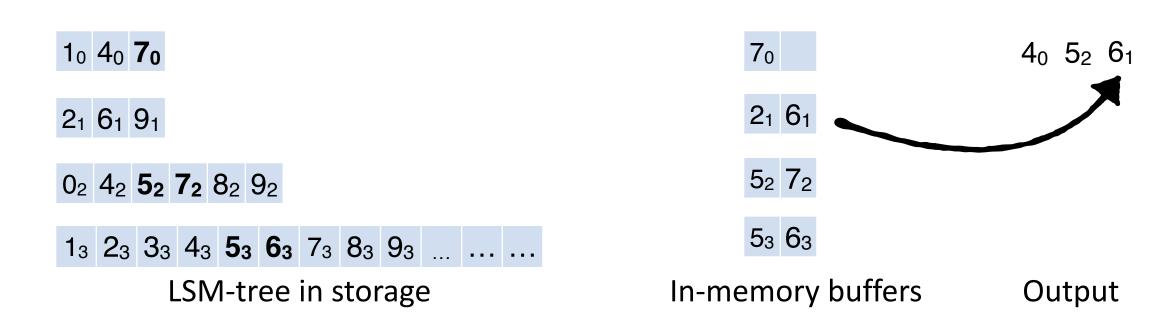
Loop until counter passes end of range

- 4. Bring youngest matching entry to output
- 5. Increment counter
- 6. Bring more pages to buffers if needed



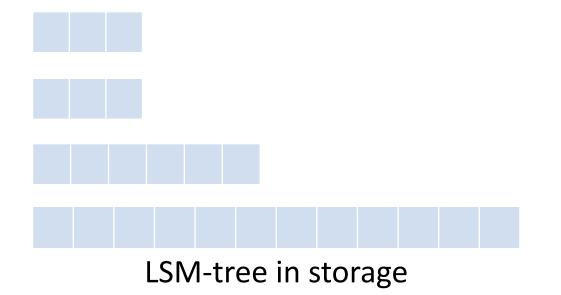
Loop until counter passes end of range

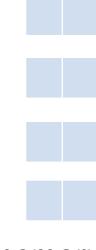
- 4. Bring youngest matching entry to output
- 5. Increment counter
- 6. Bring more pages to buffers if needed



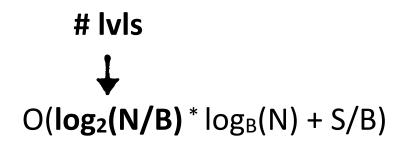
Counter: 6

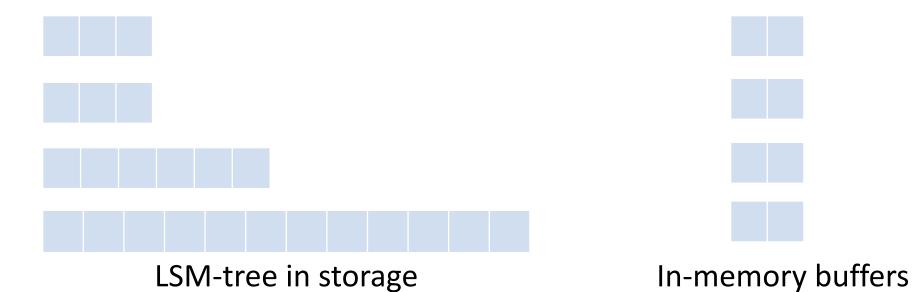
$$O(log_2(N/B) * log_B(N) + S/B)$$

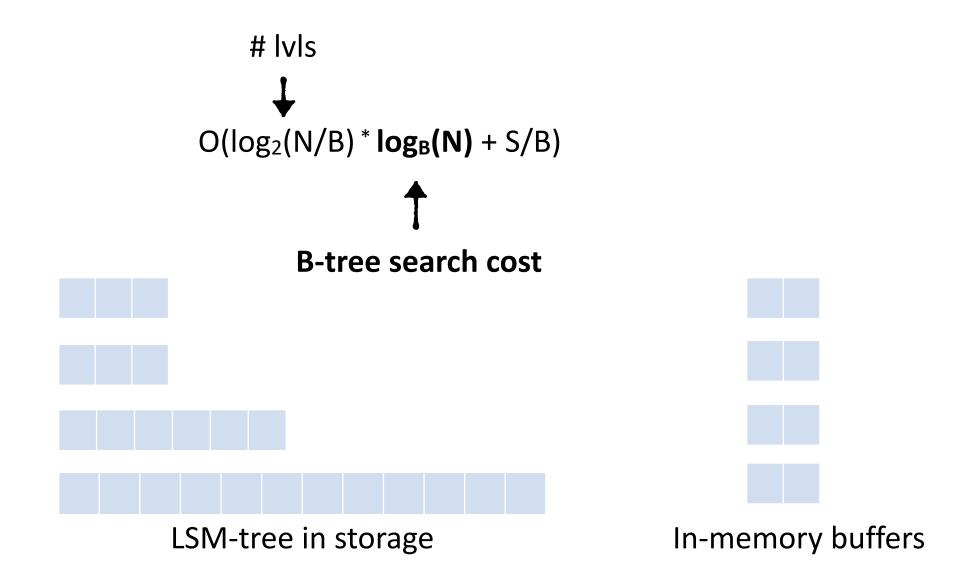


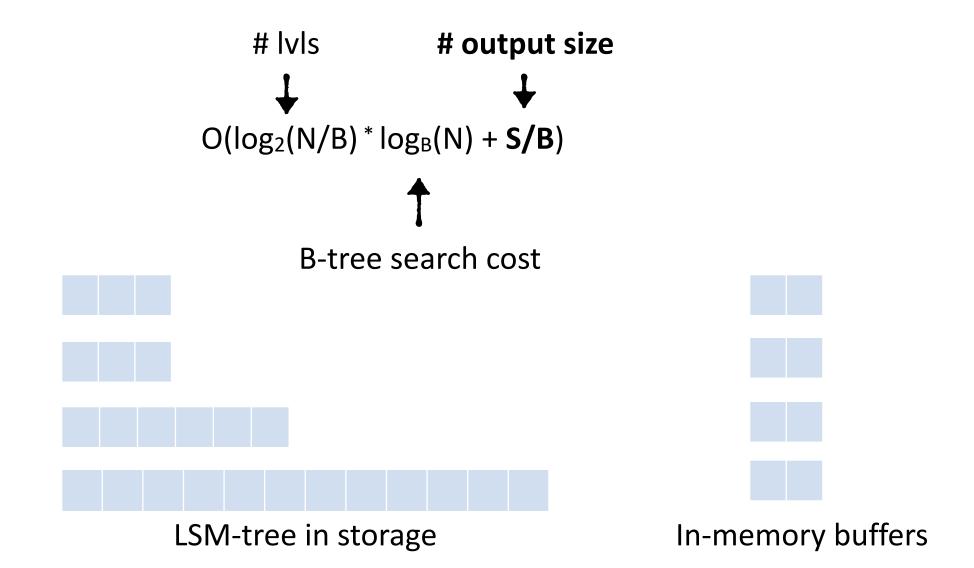


In-memory buffers

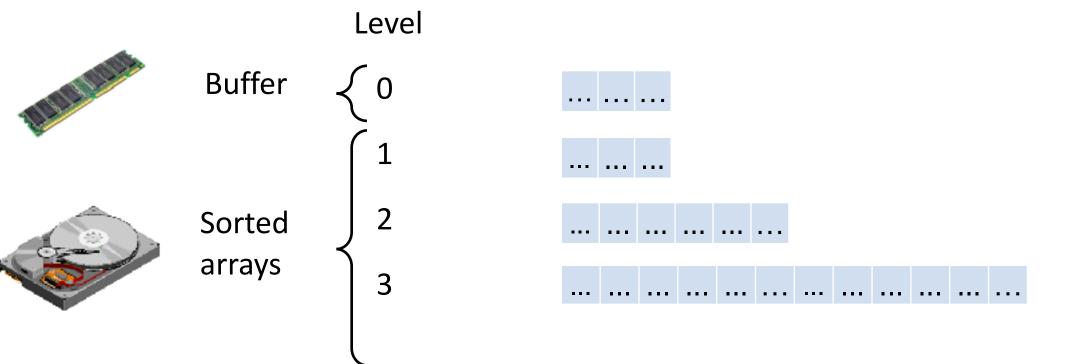




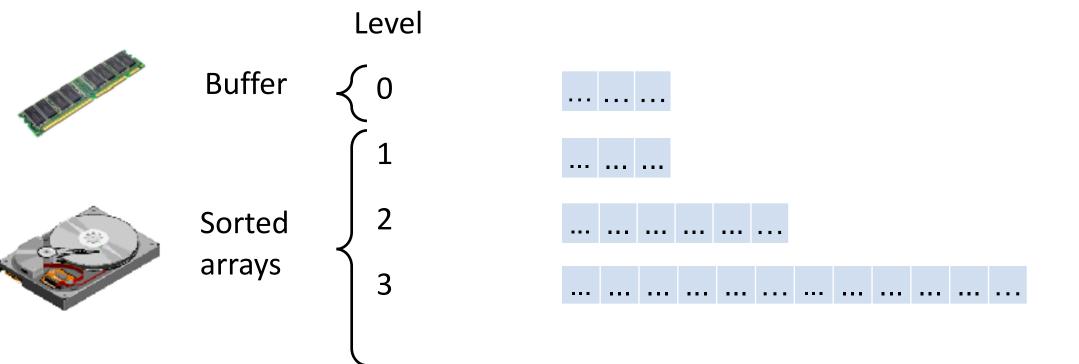




How many times is each entry copied?

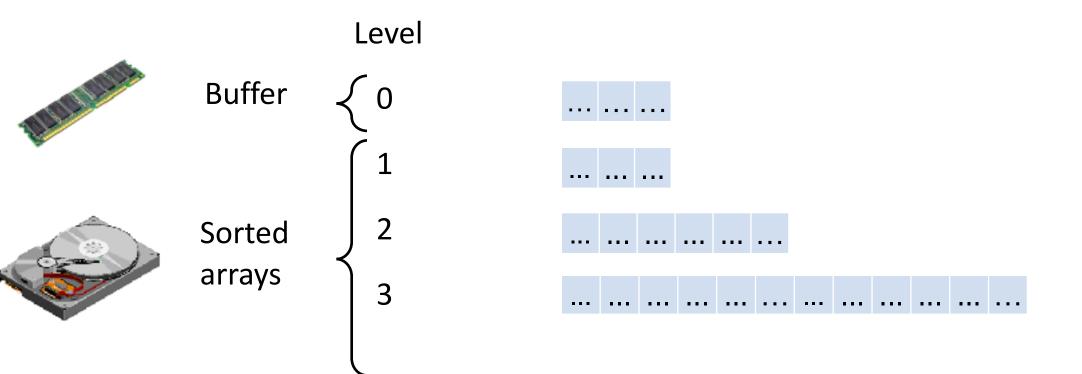


How many times is each entry copied? O(log₂ N/B)



How many times is each entry copied? $O(log_2 N/B)$

Price of each copy?

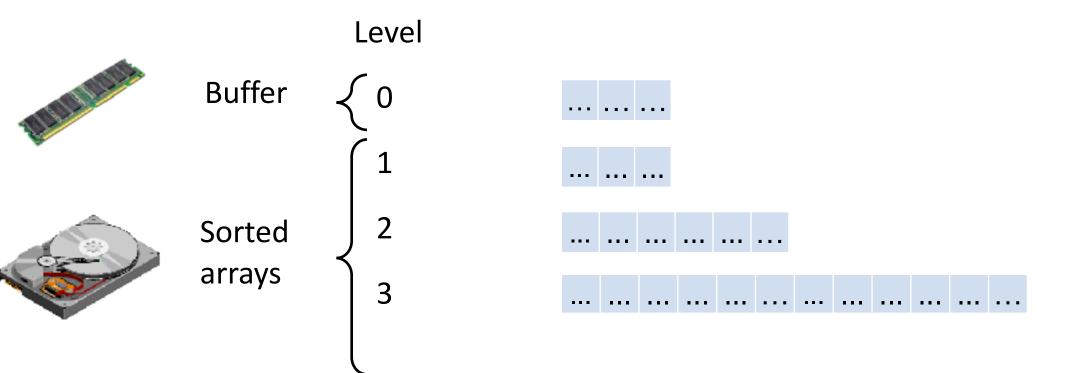


How many times is each entry copied?

 $O(log_2 N/B)$

Price of each copy?

O(1/B) reads & writes



How many times is each entry copied?

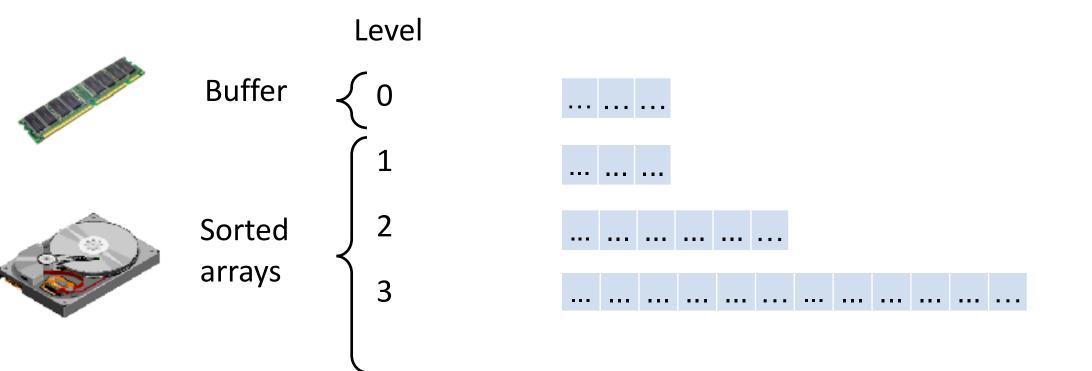
 $O(log_2 N/B)$

Price of each copy?

O(1/B) reads & writes

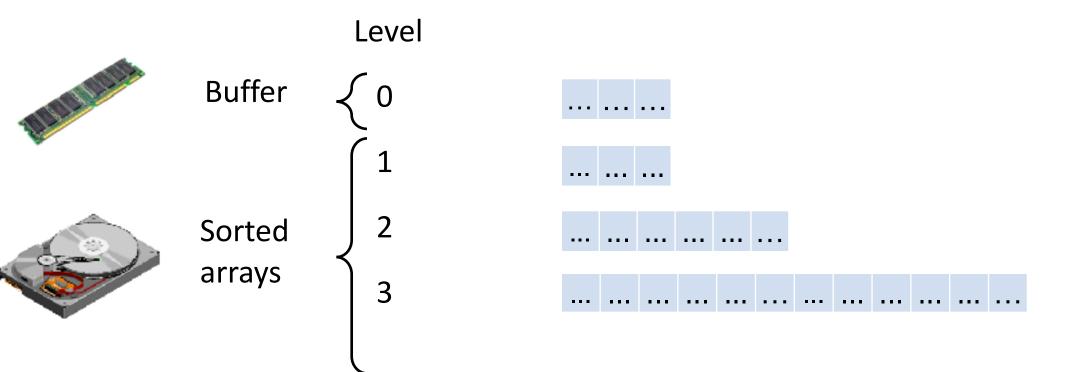
Total cost:

O((log₂ N/B)/B) read & write I/Os



Total cost: O((log₂ N/B)/B) read & write I/Os

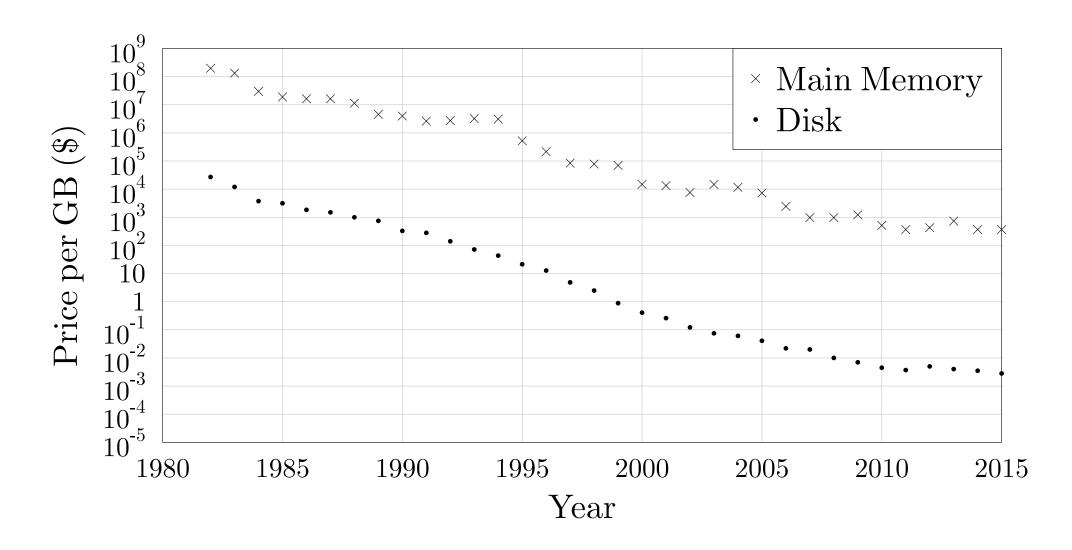
As all writes are large & sequential (rather than random), there is less SSD garbage-collection



Operation	I/O	append-only table	Basic LSM-tree table	B-Tree
Query	Reads	O(N/B)	O(log ₂ (N/B) * log _B N)	O(log _B N)
Insert	Reads	0	O((log ₂ N/B)/B)	O(log _B N)
	Writes	O(1/B)	O((log ₂ N/B)/B)	O(1) & GC

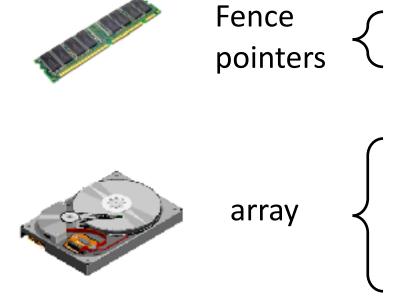
Break:)

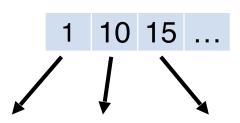
Declining Main Memory Cost



Declining Main Memory Cost

It's viable to pin the internal nodes of B-trees in memory





Block 1	Block 2	Block 3	
1	10	15	
3	11	16	•••
6	13	18	•••

Operation	I/O	append-only table	Basic LSM-tree table	B-Tree
Query	Reads	O(N/B)	$O(log_2(N/B) * N)$	O(10364)
Insert	Reads	0	$O((log_2 N/B)/B)$	
	Writes	O(1/B)	$O((log_2 N/B)/B)$	O(1) & GC

Operation	I/O	append-only table	Basic LSM-tree table	B-Tree
Query	Reads	O(N/B)	O(log ₂ N/B)	O(1)
Insert	Reads	0	$O((log_2 N/B)/B)$	O(log _B N)
	Writes	O(1/B)	O((log ₂ N/B)/B)	O(1) & GC
Memory		O(B)	O(N/B)	O(N/B)

Basic LSM-tree

Tiered LSM-tree

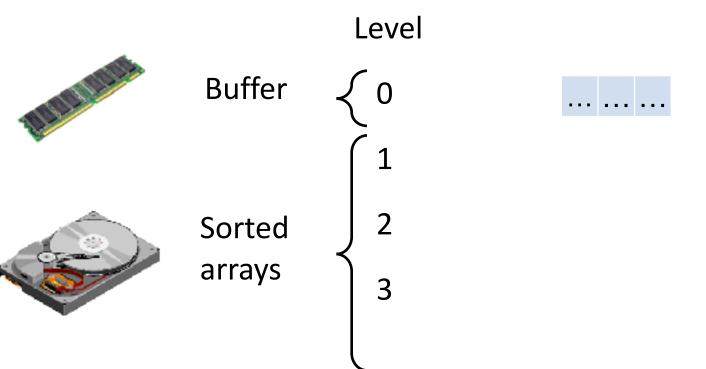




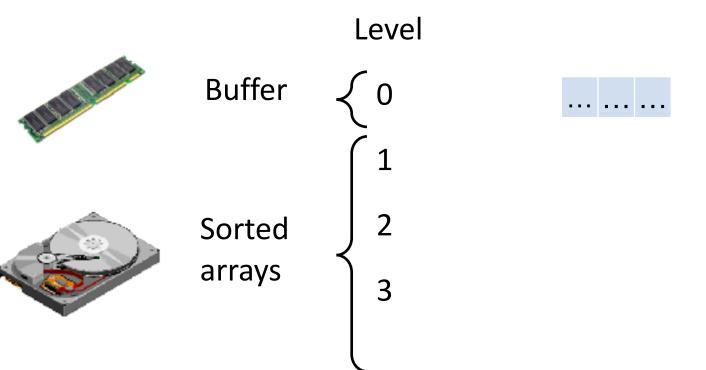




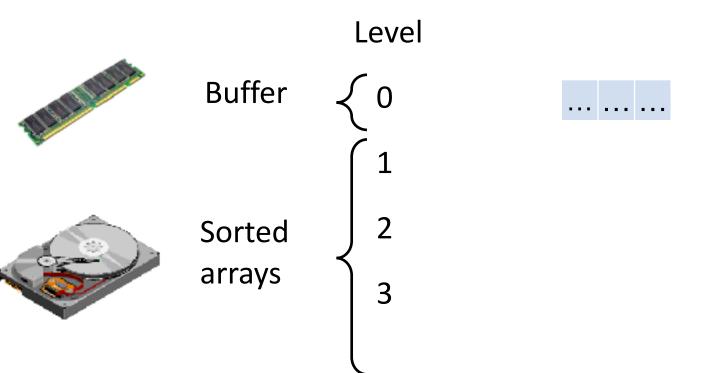
Lookup cost depends on number of levels



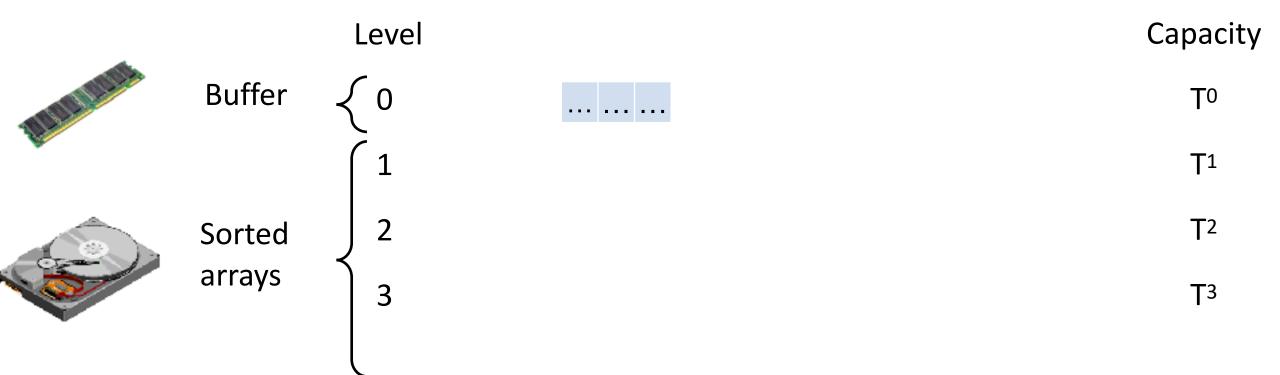
Lookup cost depends on number of levels How to reduce it?



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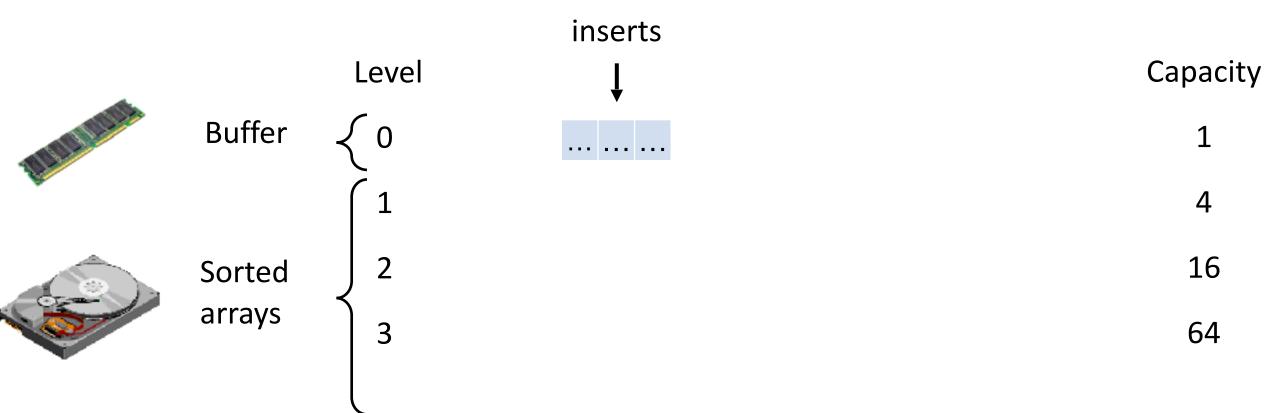
Lookup cost depends on number of levels How to reduce it?

E.g. size ratio of 4



Lookup cost depends on number of levels How to reduce it?

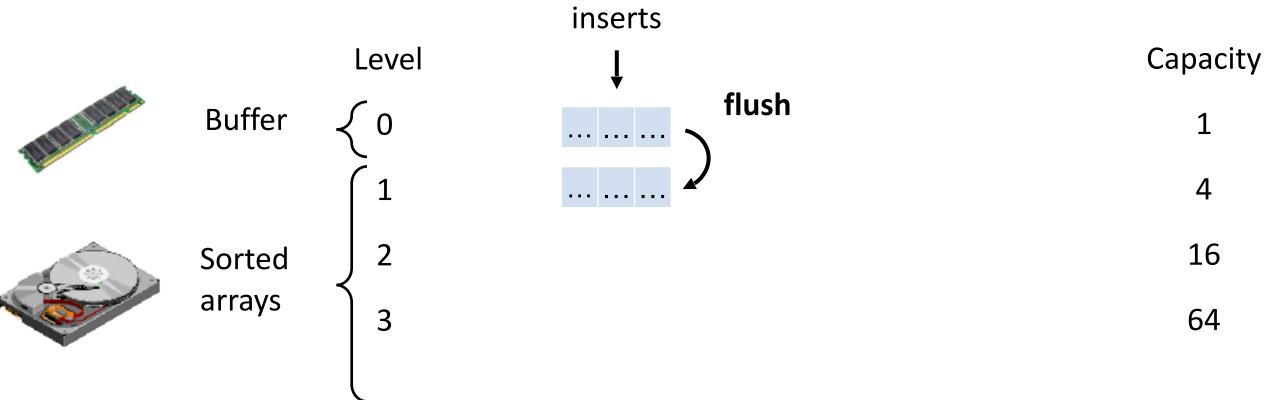
E.g. size ratio of 4



Lookup cost depends on number of levels

How to reduce it?

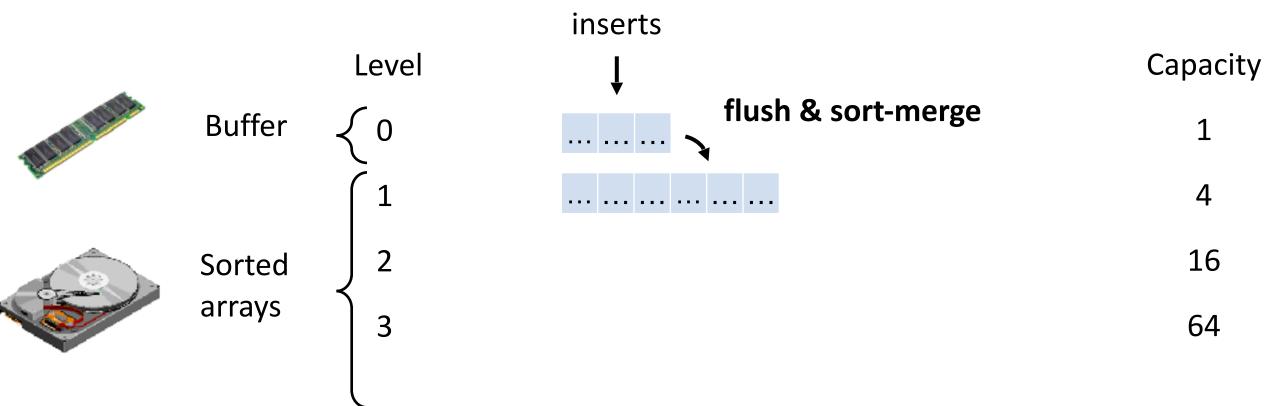
E.g. size ratio of 4



Lookup cost depends on number of levels

How to reduce it?

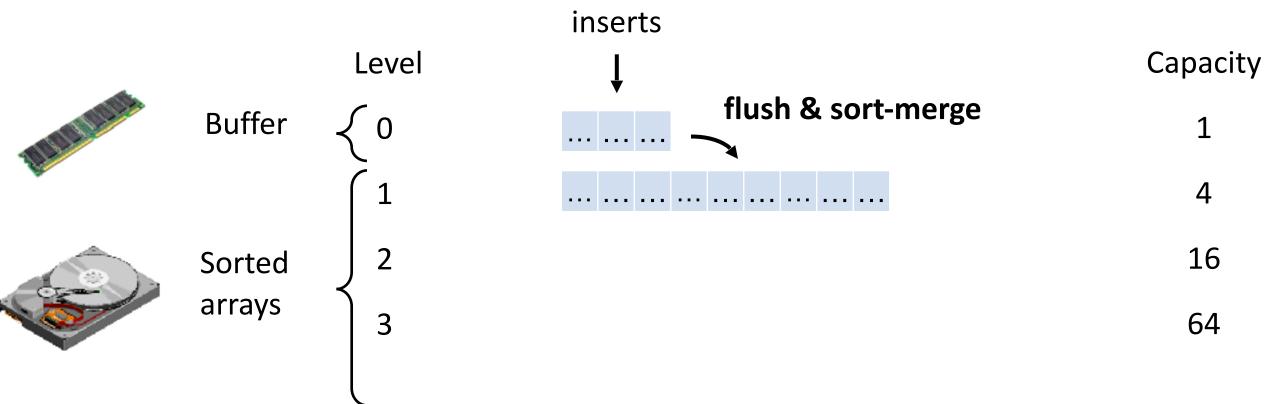
E.g. size ratio of 4



Lookup cost depends on number of levels

How to reduce it?

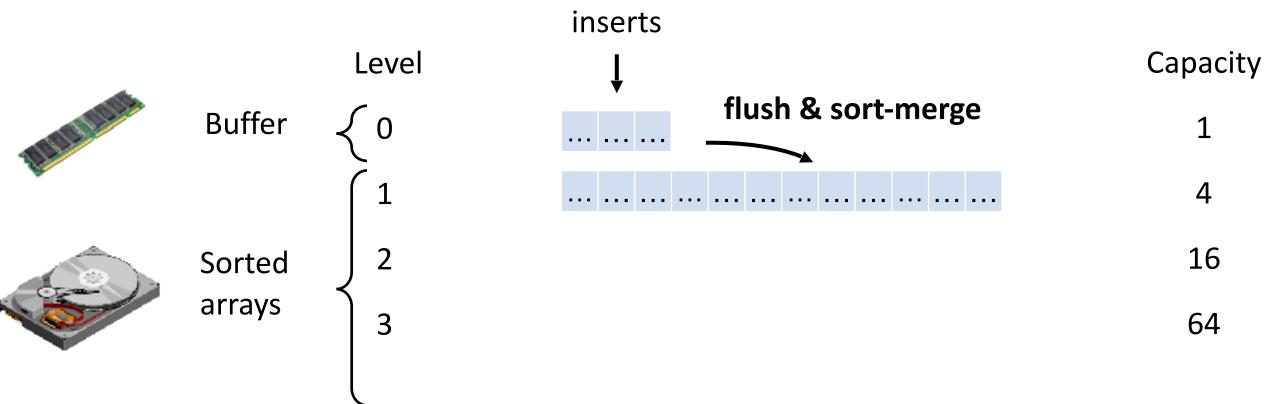
E.g. size ratio of 4



Lookup cost depends on number of levels

How to reduce it?

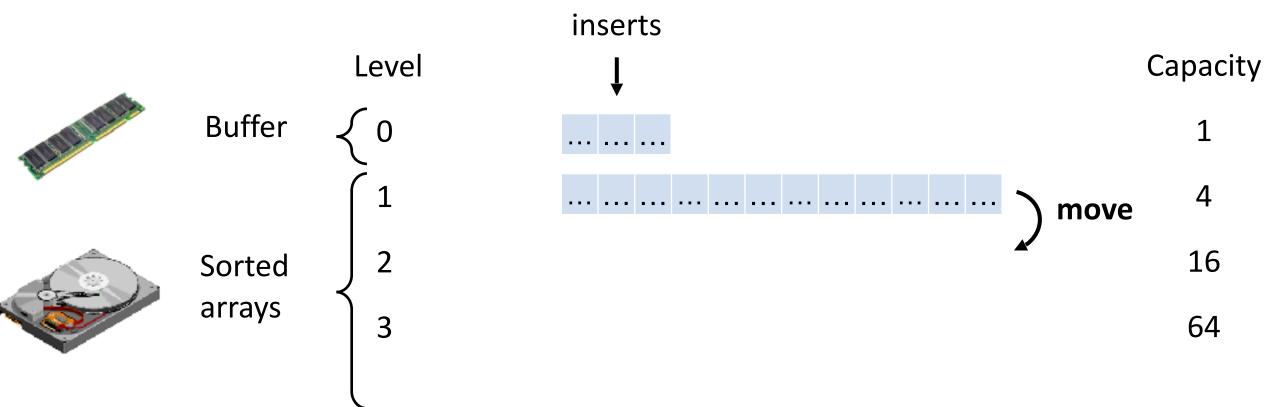
E.g. size ratio of 4



Lookup cost depends on number of levels

How to reduce it?

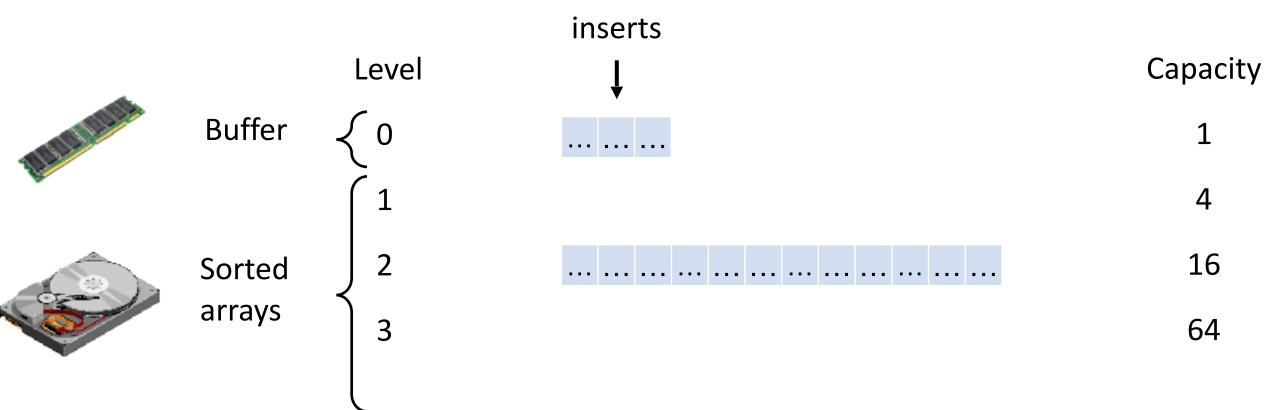
E.g. size ratio of 4



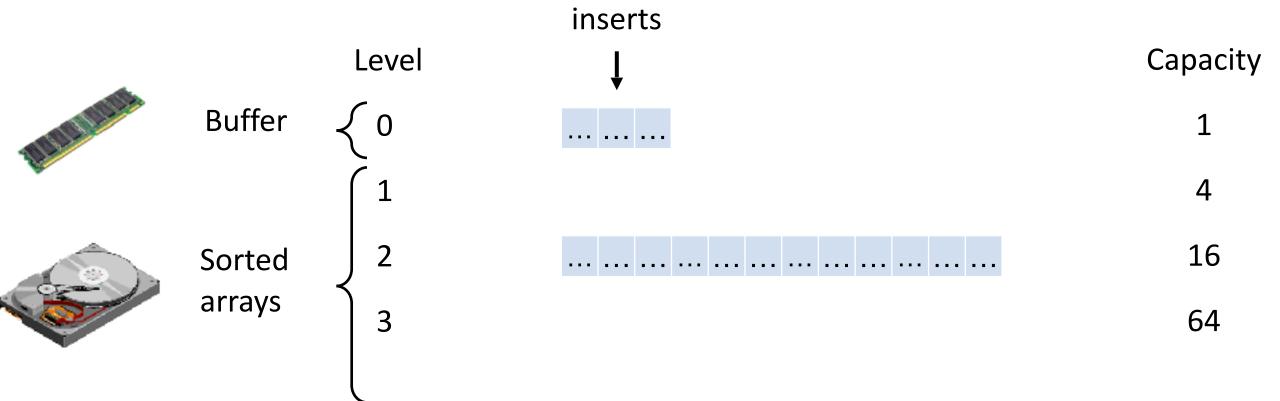
Lookup cost depends on number of levels

How to reduce it?

E.g. size ratio of 4

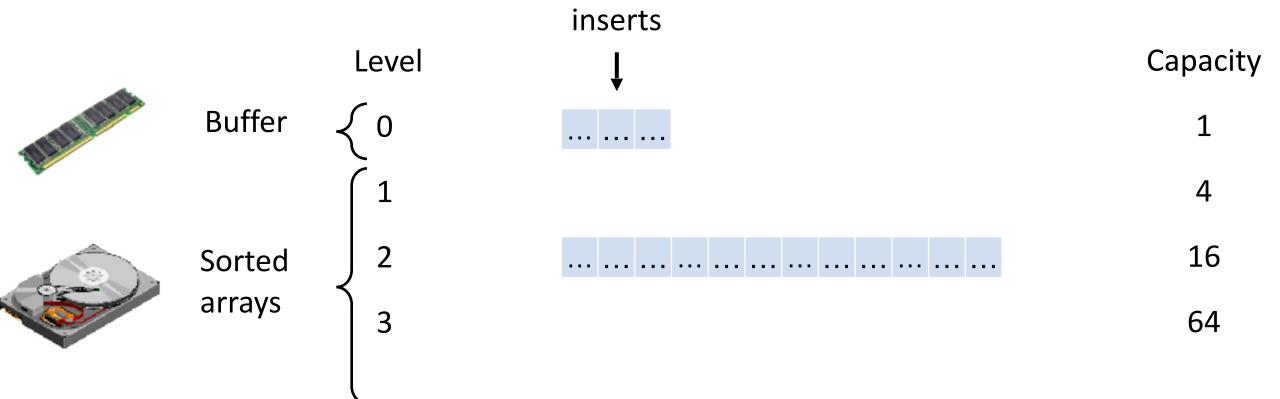


Lookup cost?



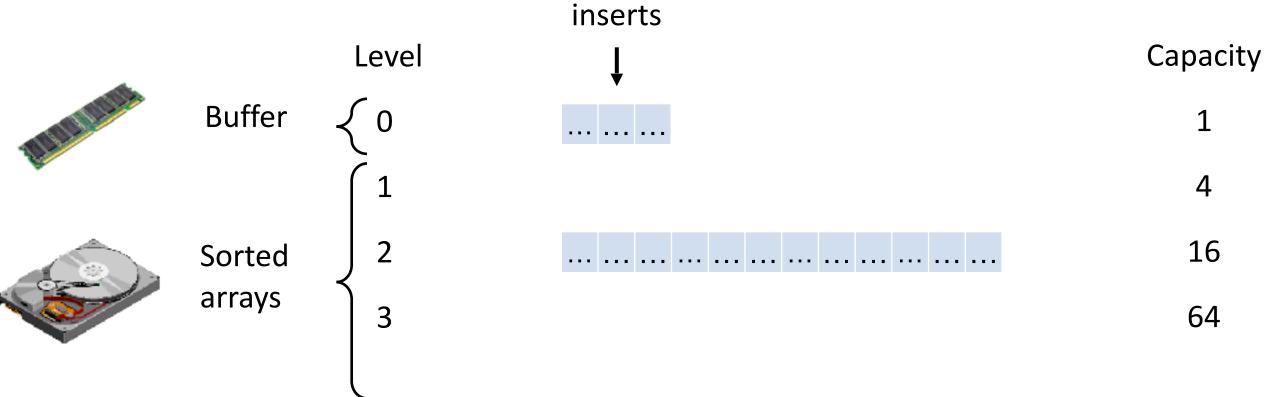
Lookup cost?

 $O(log_T(N/B))$



Lookup cost?
O(log_T(N/B))

Insertion cost?

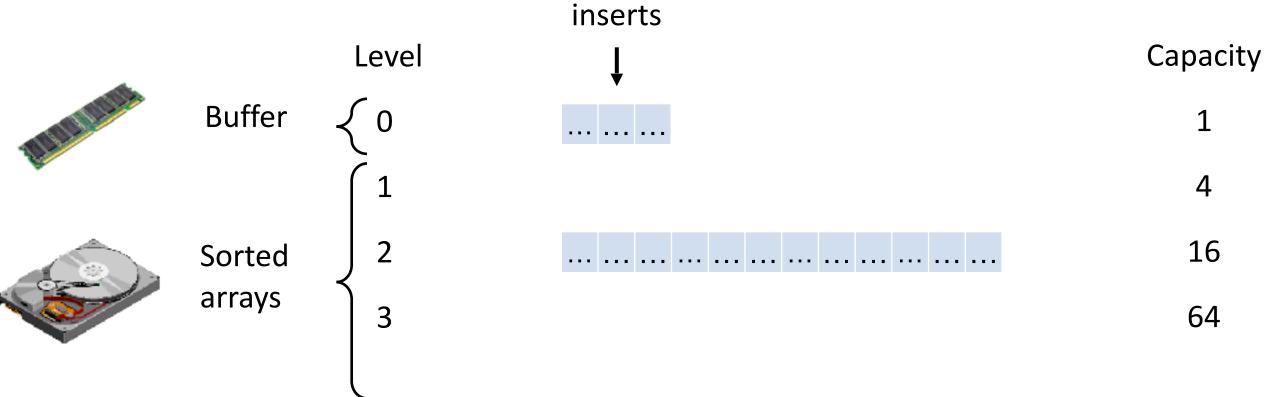


Lookup cost?

 $O(log_T(N/B))$

Insertion cost?

 $O(T/B \bullet log_T(N/B))$



Lookup cost?

 $O(log_T(N/B))$

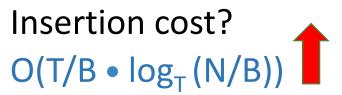
Insertion cost?

 $O(T/B \bullet log_T(N/B))$

What happens as we increase the size ratio T?



Lookup cost?
O(log_T(N/B))



What happens as we increase the size ratio T?

Lookup cost?
O(log_T(N/B))

Insertion cost?
O(T/B • log_T (N/B))

What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?



Lookup cost?
O(log_T(N/B))

Insertion cost?
O(T/B • log_T (N/B))

What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

Lookup cost becomes:

O(1)

Insert cost becomes:

 $O(N/B^2)$



Lookup cost? $O(log_T(N/B))$

Insertion cost?
O(T/B • log_T (N/B))

What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

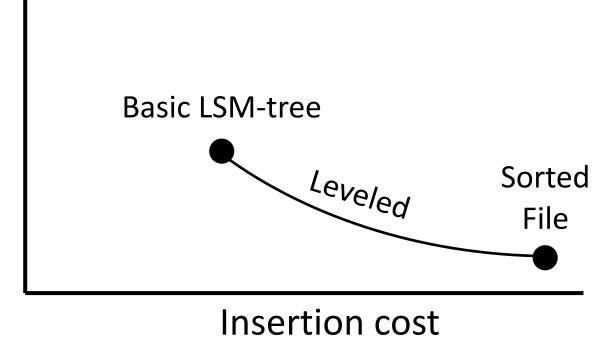
Lookup cost becomes:

O(1)

Insert cost becomes:

 $O(N/B^2)$

The LSM-tree becomes a sorted file!



Leveled LSM-tree

Basic LSM-tree

Tiered LSM-tree









Reduce the number of levels by increasing the size ratio.

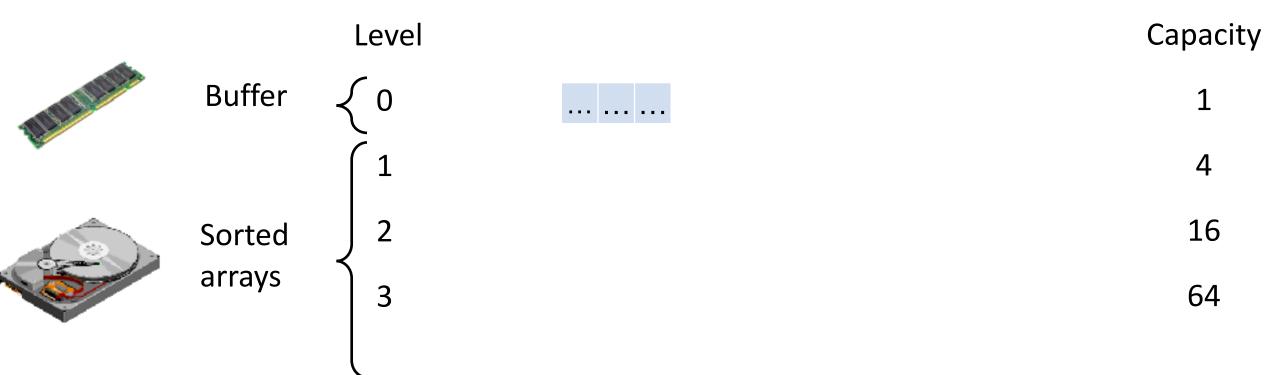


Reduce the number of levels by increasing the size ratio. Do not merge within a level.



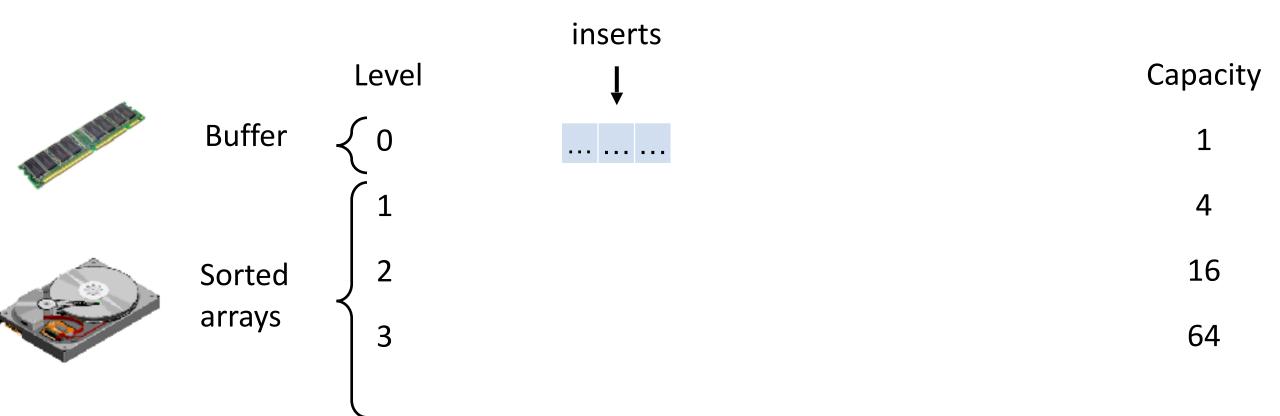
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



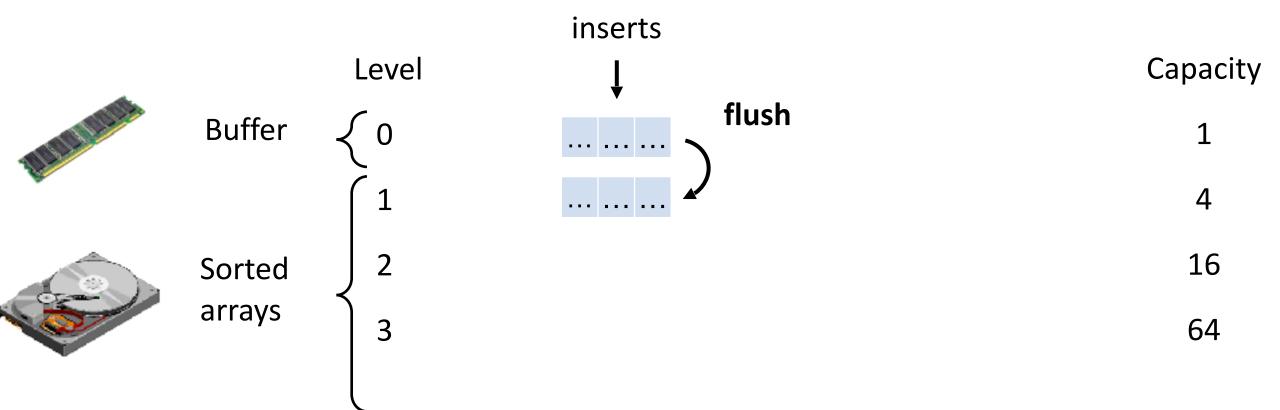
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



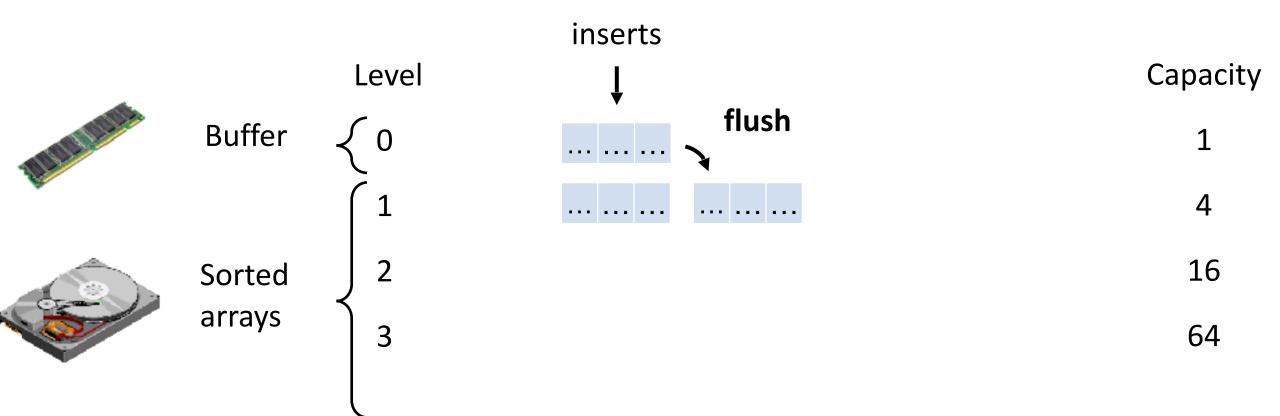
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



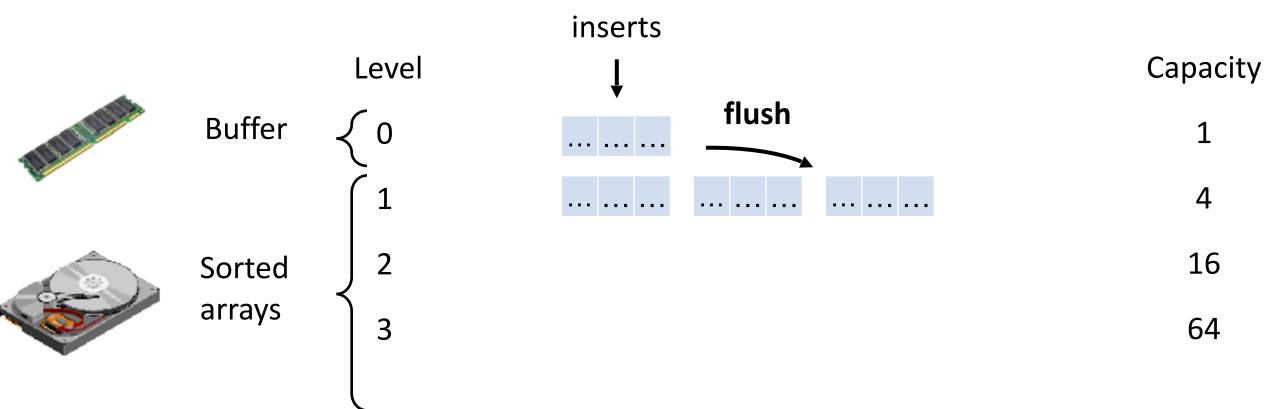
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



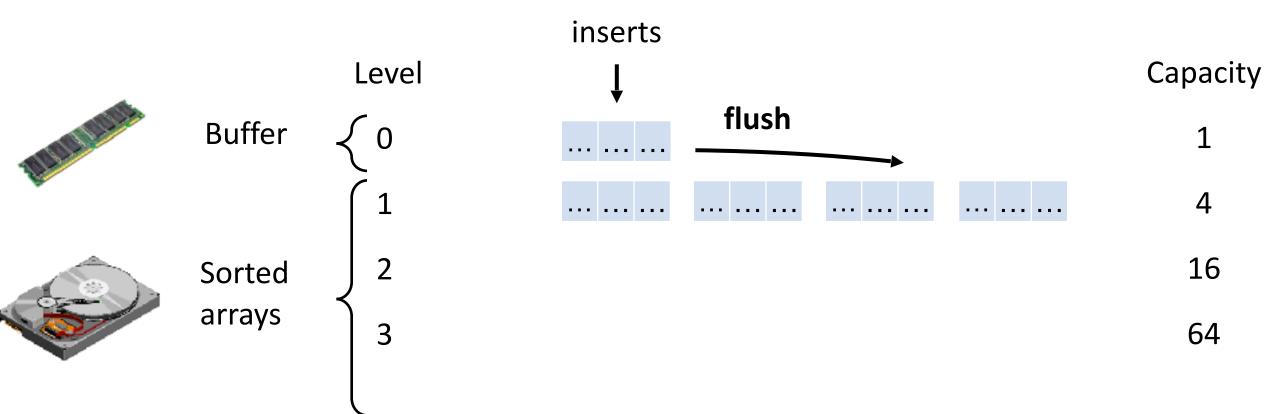
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



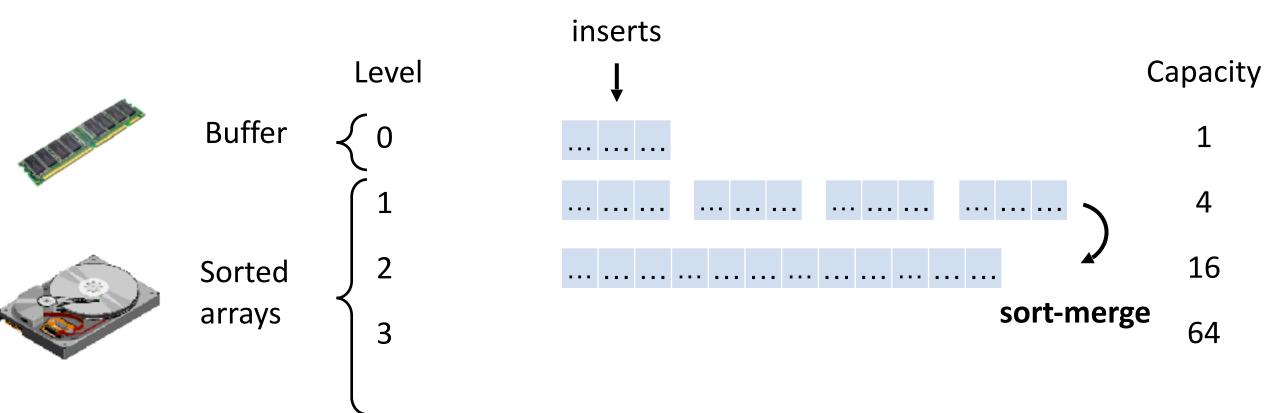
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.



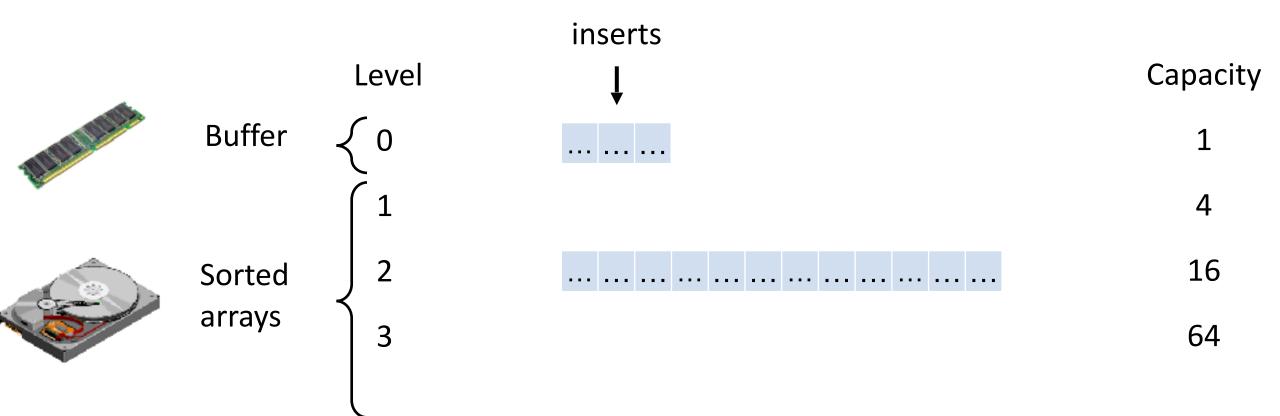
Reduce the number of levels by increasing the size ratio.

Do not merge within a level.

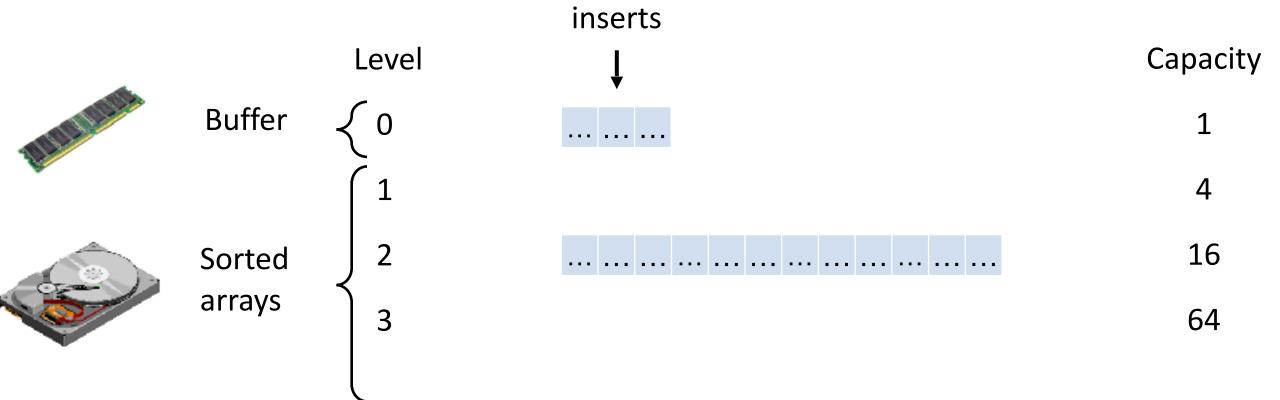


Reduce the number of levels by increasing the size ratio.

Do not merge within a level.

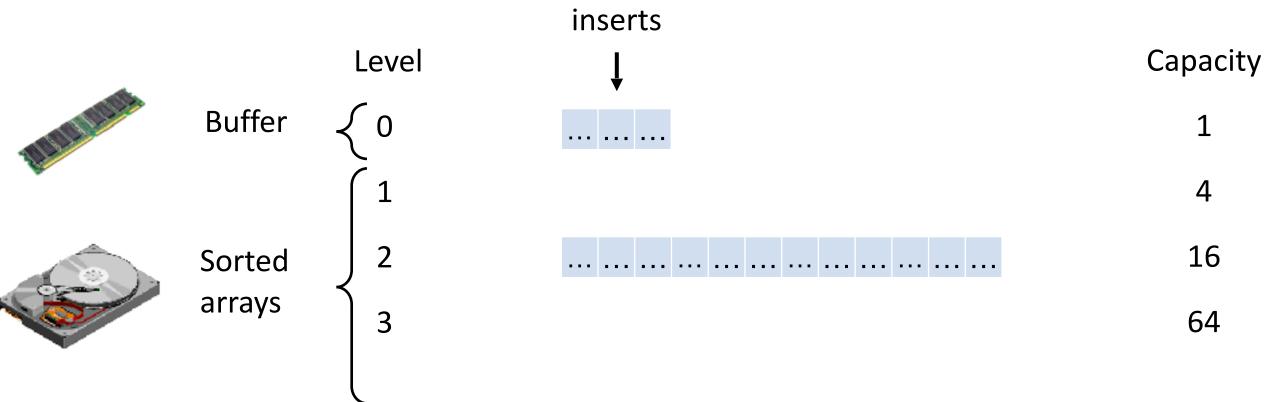


Lookup cost?



Lookup cost?

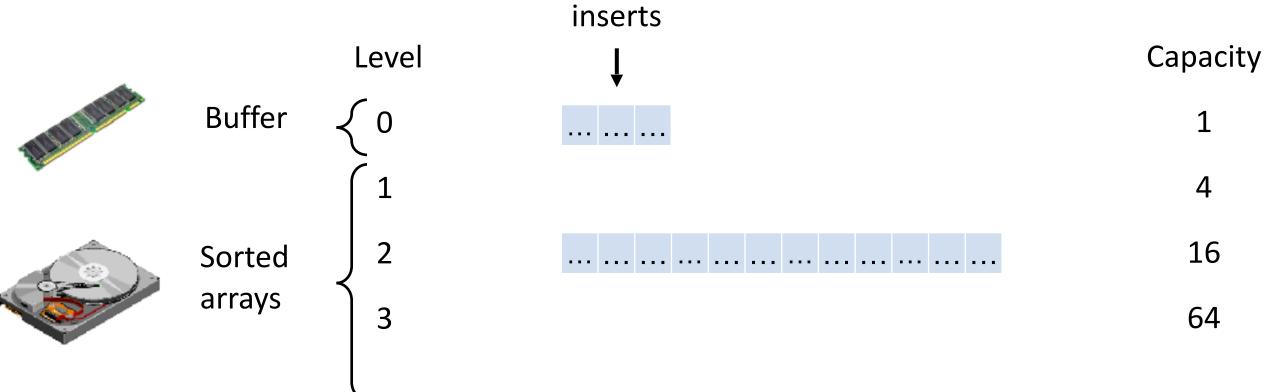
 $O(T \bullet log_T(N/B))$



Lookup cost?

 $O(T \bullet log_T(N/B))$

Insertion cost?

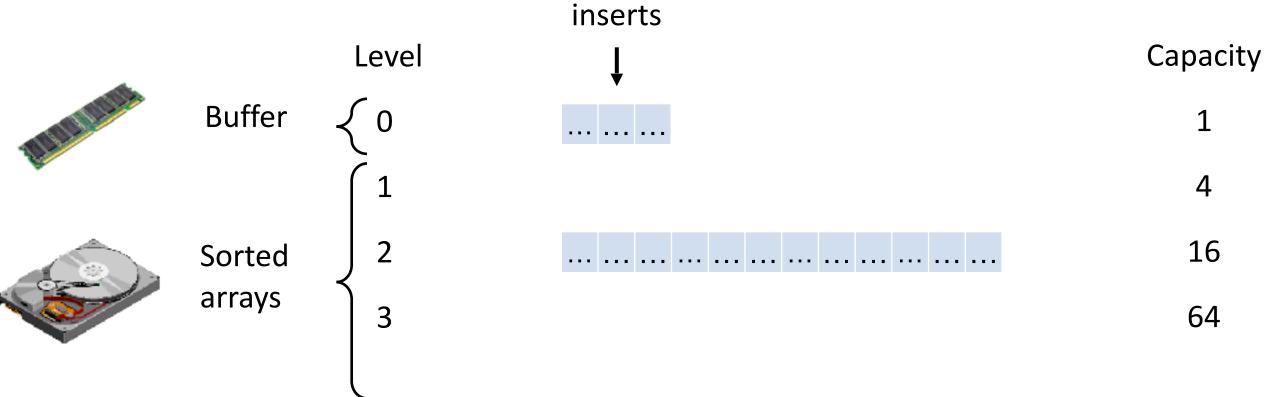


Lookup cost?

 $O(T \bullet log_T(N/B))$

Insertion cost?

 $O(1/B \bullet log_T(N/B))$



Lookup cost?

 $O(T \bullet log_T(N/B))$

Insertion cost?

 $O(1/B \bullet log_T(N/B))$

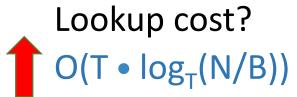
What happens as we increase the size ratio T?

Lookup cost?

Insertion cost?

 $O(1/B \bullet log_T(N/B))$

What happens as we increase the size ratio T?



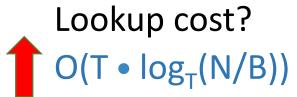
Insertion cost?

 $O(1/B \bullet log_T(N/B))$



What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?



Insertion cost?

 $O(1/B \bullet log_T(N/B))$



What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

Lookup cost becomes:

O(N/B)

Insert cost becomes:

O(1/B)



Insertion cost?

 $O(1/B \bullet log_T(N/B))$



What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

Lookup cost becomes:

O(N/B)

Insert cost becomes:

O(1/B)

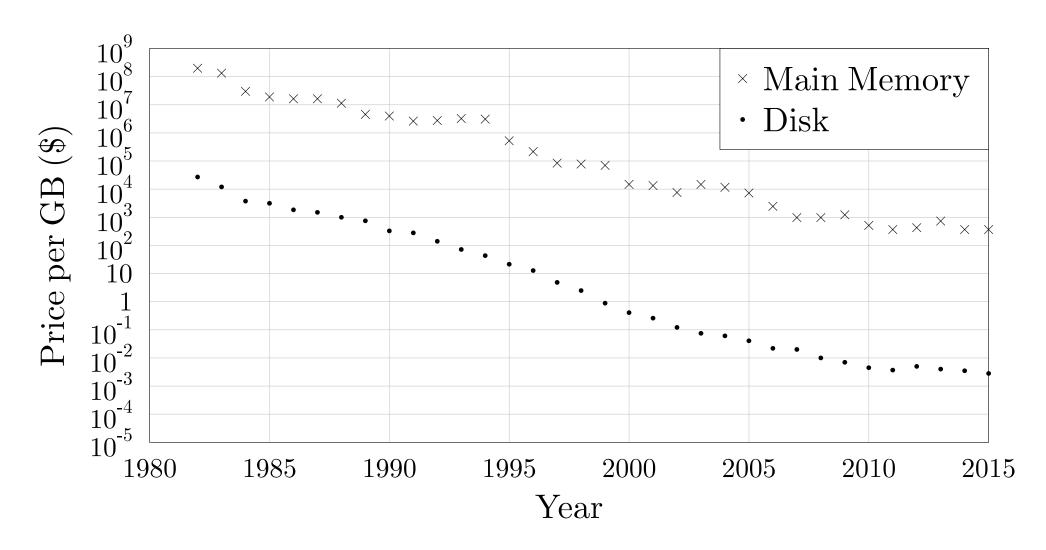
The tiered LSM-tree becomes an append-only file!

Insertion cost

Operation		Unordered File	Leveled LSM-tree	Tiered LSM-tree	B-Tree
Query	Reads	O(N/B)	O(L)	O(L * T)	O(1)
Insert	Reads	0	O((L * T)/B)	O(L/B)	O(log _B N)
	Writes	O(1/B)	O((L * T)/B)	O(L/B)	O(1) & GC
Memory		O(B)	O(N/B)	O(N/B)	O(N/B)

We let $L = log_T(N/B)$

Declining Main Memory Cost

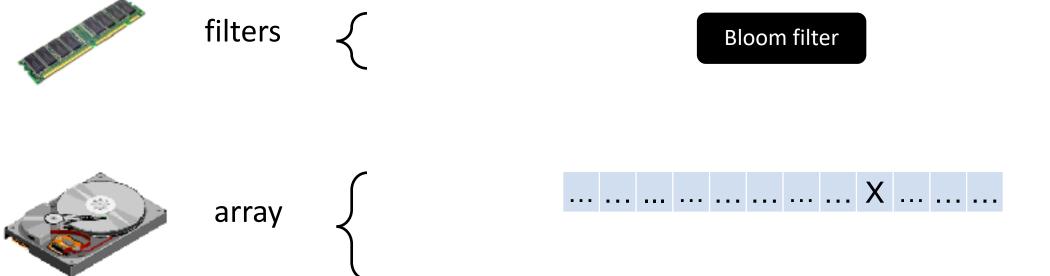


Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.

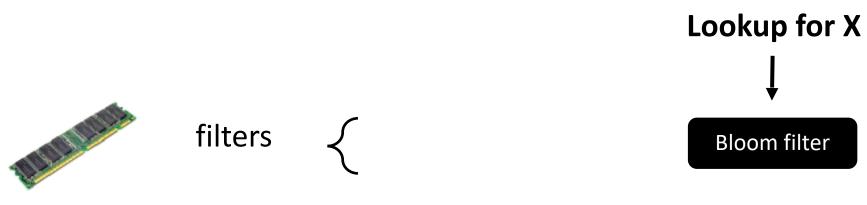


Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

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Answers set-membership queries

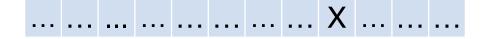
Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.





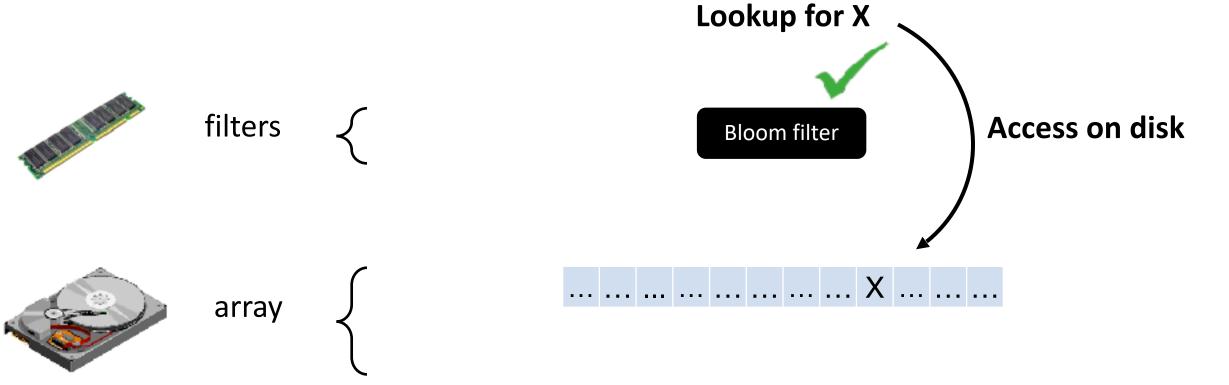


Answers set-membership queries

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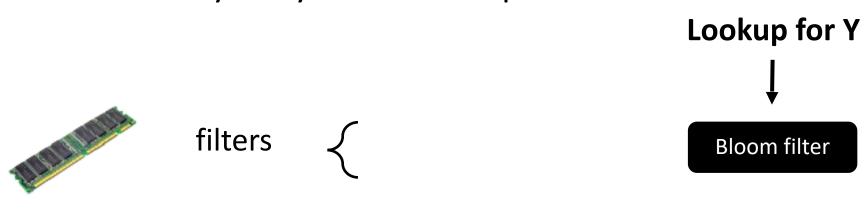


Answers set-membership queries

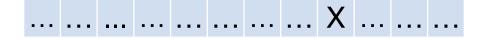
Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.





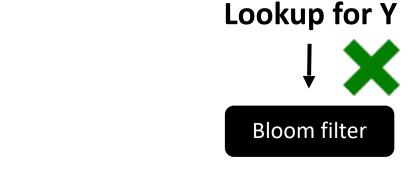


Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.





filters







Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.

Lookup for Y



filters



Bloom filter





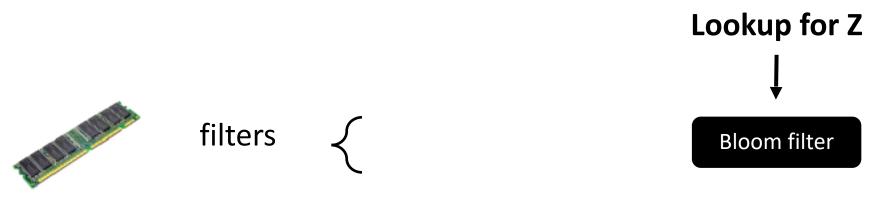


Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.







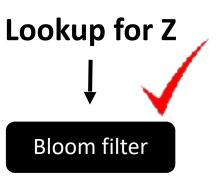
Answers set-membership queries

Smaller than data it represents

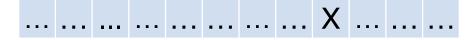
Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.







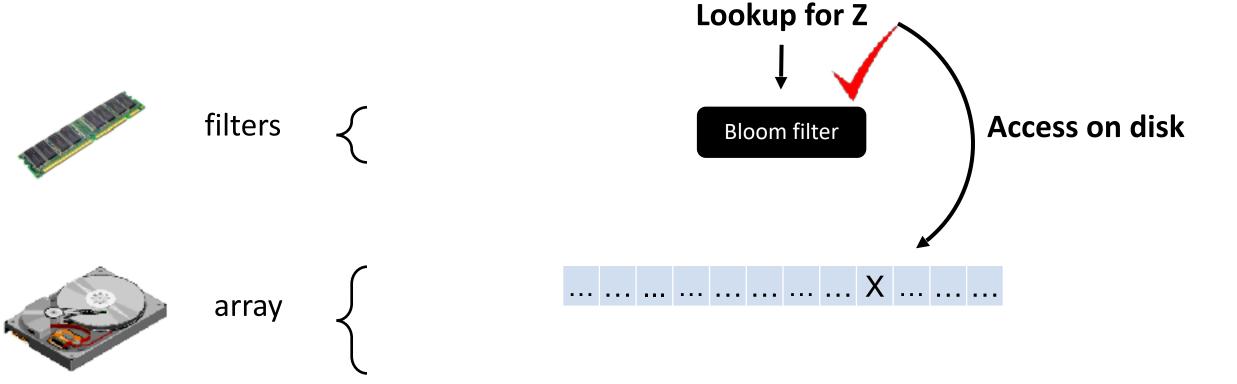


Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

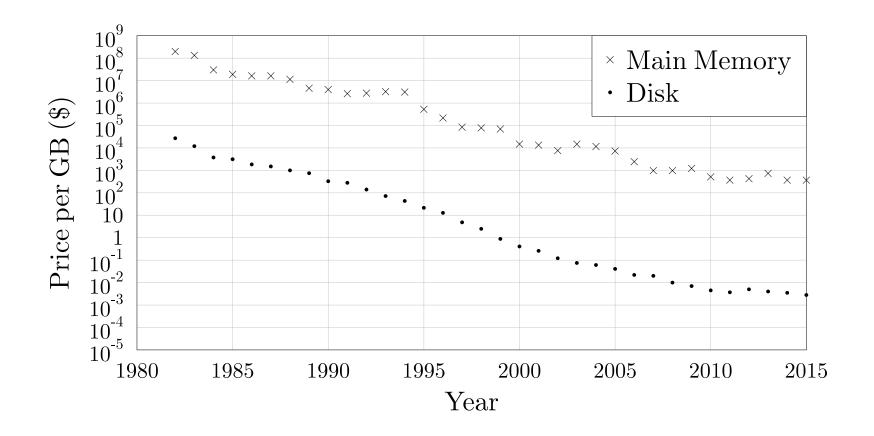
Subtlety: may return false positives.



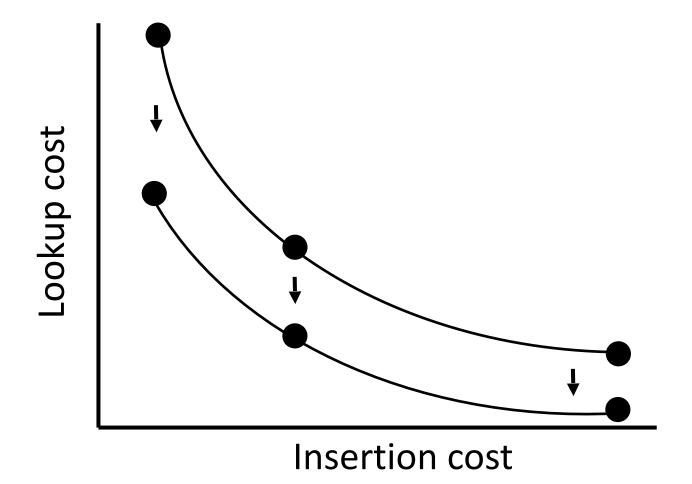
The more main memory, the less false positives



cheaper lookups



The more main memory, the less false positives _____ cheaper lookups



Write-optimized

Write-optimized

Highly tunable

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Highly tunable

Backbone of many modern systems

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Highly tunable

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Trade-off between lookup and insert cost (tiering/leveling, size ratio)

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Trade-off between lookup and insert cost (tiering/leveling, size ratio)

Trade main memory for lookup cost (fence pointers, Bloom filters)

Write-optimized

Highly tunable

Backbone of many modern systems

Trade-off between lookup and insert cost (tiering/leveling, size ratio)

Trade main memory for lookup cost (fence pointers, Bloom filters)

The short-term future

Today: office hours after class in my office, 5230 in Bahen.

Thursday: Indexing exercises in tutorial this week.

Next week: research lecture. Important for your project.