Write-Optimized Indexing

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31 January, 2023

We will begin at 2:10 pm

We are trying to optimize the following kinds of queries

A   B   C

select * from table where A = "…"

select * from table where B > "…" and B < "…"

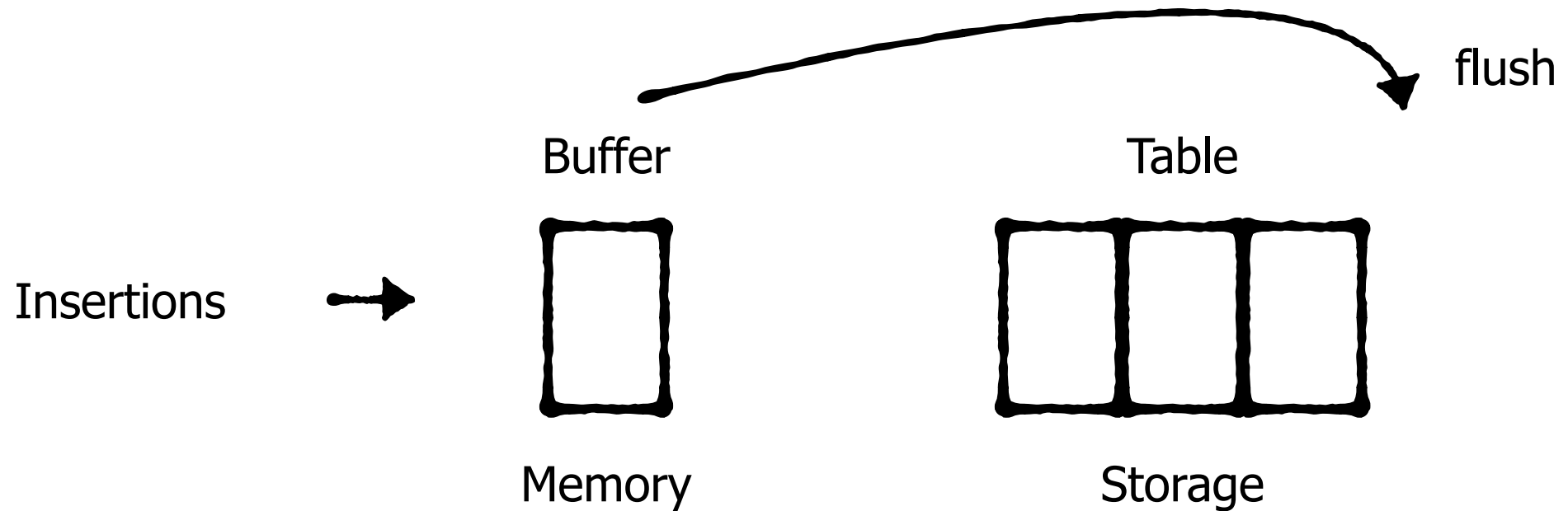# Let's reconsider four baselines

Append-Only Table   Sorted Table   Extendible Hashing   B-tree

Append-Only Table

O(N/B) for any selection query

O(1/B) for insertions

flush

Buffer

Table

Insertions →

Memory

Storage

Let's reconsider four baselines

Append-Only Table   Sorted Table   Extendible Hashing   B-tree

Sorted Table

A	...
2	...
7	...
22	...
32	...
32	...
61	...
74	...
90	...
97	...

Binary search: $O(\log_2 N/B)$ I/O

Update/insert/delete: $O(N/B)$ read & write I/O

Let's reconsider four baselines

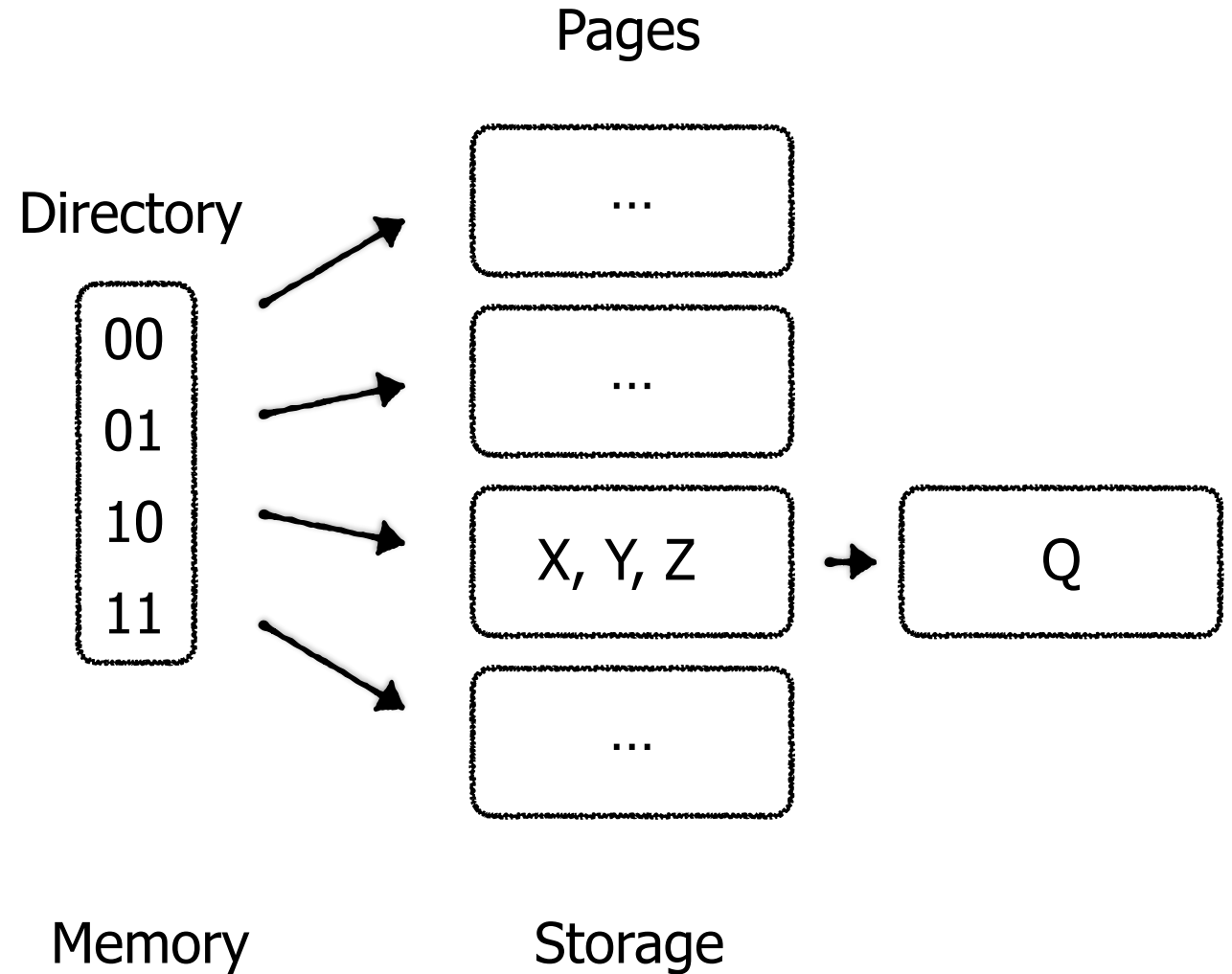Append-Only Table   Sorted Table   Extendible Hashing   B-tree

Extendible Hashing
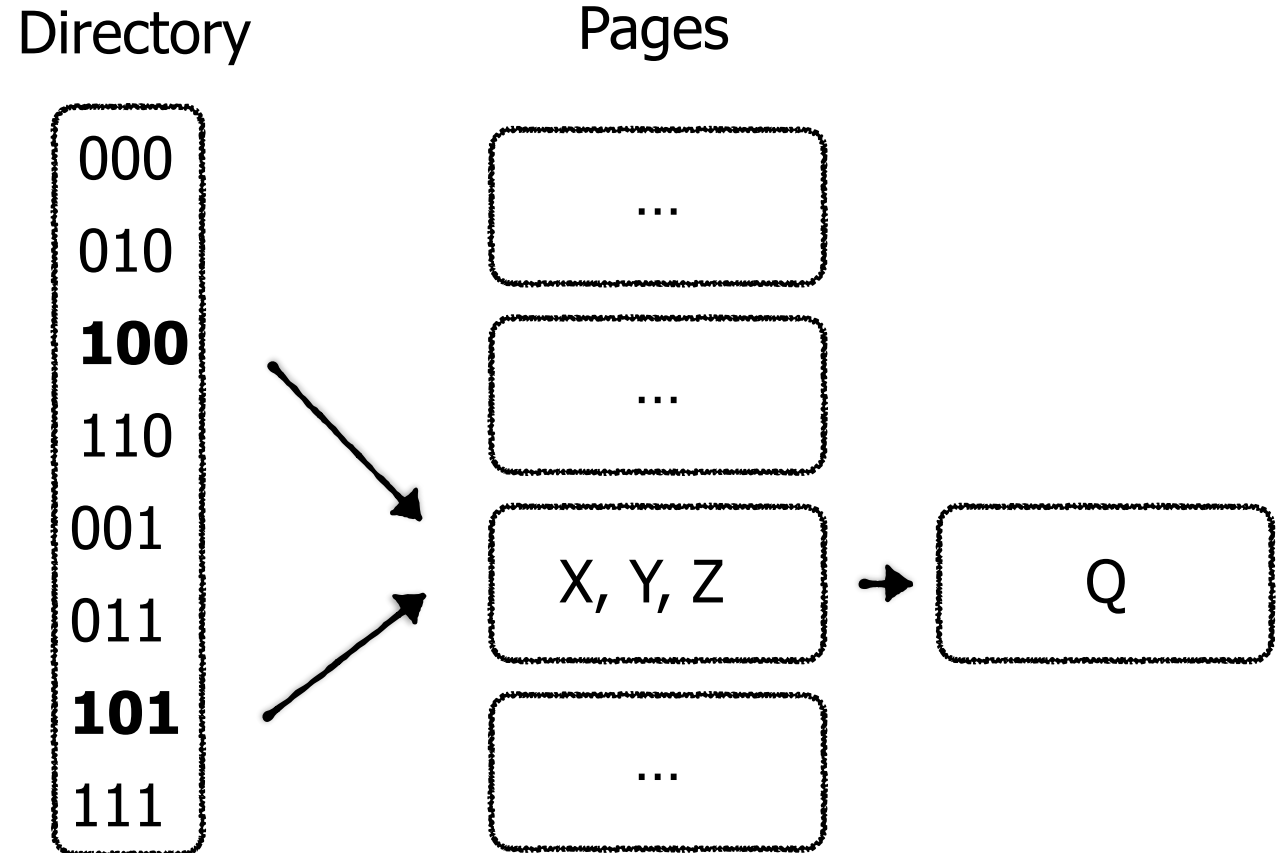
A directory maps pages in storage with a given hash prefix

Handle overflows via chaining

Pages

Directory

00
01
10
11

X, Y, Z → Q

...

...

...

Memory   Storage

Extendible Hashing

When we reach capacity, double directory size.

New directory slots still point to previous pages

Directory

000
010
100
110
001
011
101
111

Pages

...

...

X, Y, Z → Q

...

Extendible Hashing

Directory   Pages

Split one overflowing bucket at a
time by rehashing entries.

```
000
010
100
110
001
011
101
111
```

```
...
```

```
...
```

```
X, Z
```

```
Q, Y
```

```
...
```

Extendible Hashing

Query cost: O(1) read I/O

Insertion cost:
    O(1) read I/O
    O(1) write I/O

Directory

000
010
100
110
001
011
101
111

Pages

...

...

...

...

...
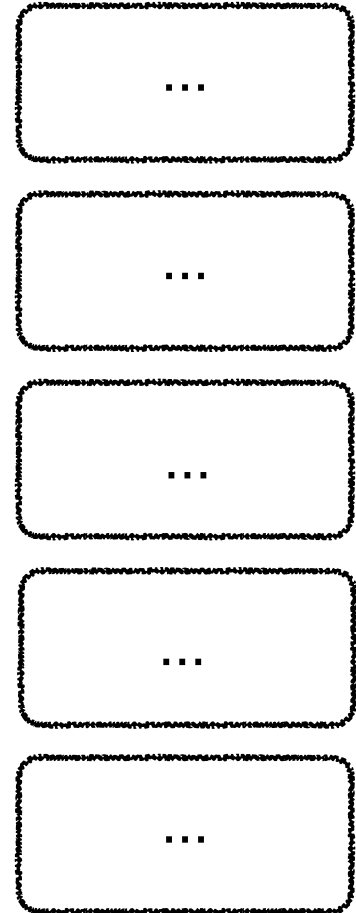
Extendible Hashing

Query cost: O(1) read I/O

Insertion cost:
    O(1) read I/O
    O(1) * **GC** write I/O

Random writes to storage entail SSD garbage-collection

Directory

000
010
100
110
001
011
101
111

Pages

...

...

...

...

...
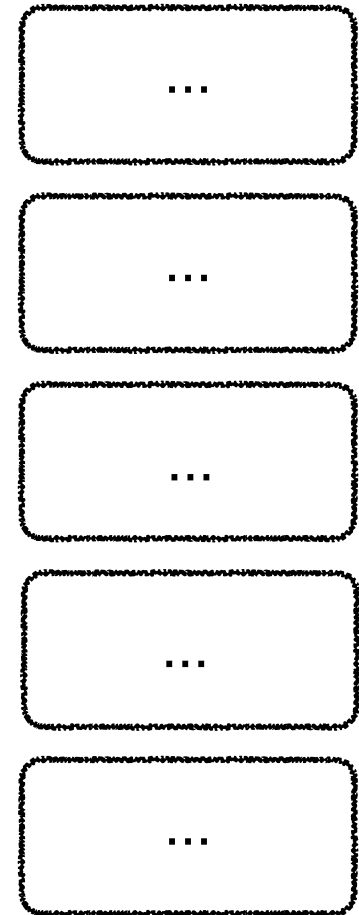
Let's reconsider four baselines
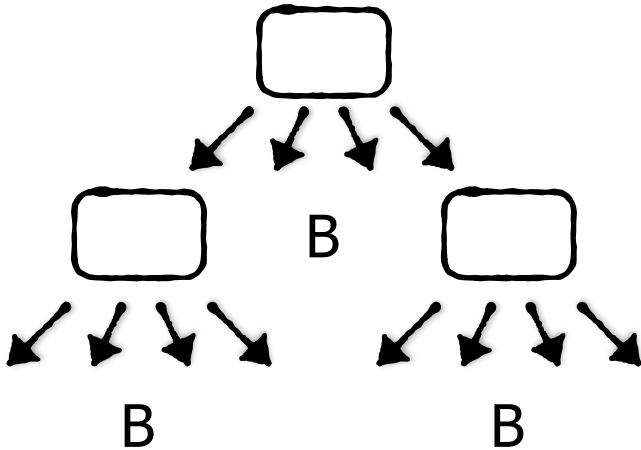
Append-Only Table   Sorted Table   Extendible Hashing   B-Tree
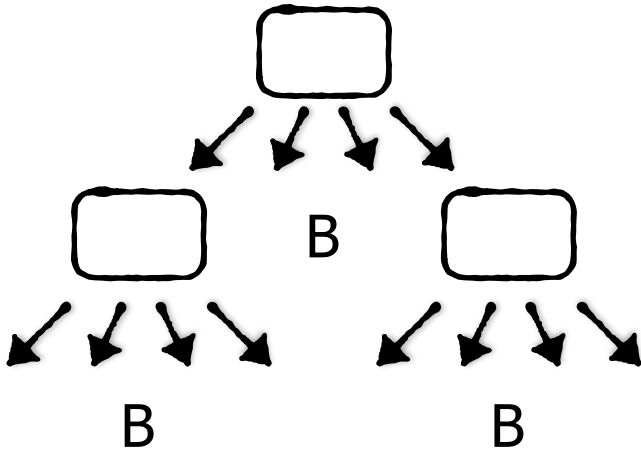
B-Tree

Each node has B children. This allows pruning by a factor of B in each level

Can issue random writes to storage leading to SSD garbage-collection

B-Tree

Each node has B children. This allows pruning by a factor of B in each level

Can issue random writes to storage leading to SSD garbage-collection

Query: $O(\log_B N)$ read I/O

Update/insert/query: $O(\log_B N)$ read I/O

& $O(1)$ * GC write I/O

Operation	I/O	append-only table	Sorted File	Extendible Hashing	B-Tree
Query	Reads	$O(N/B)$	$O(\log_2 N/B)$	$O(1)$	$O(\log_B N)$
Insert	Reads	0	$O(N/B)$	$O(1)$	$O(\log_B N)$
	Writes	$O(1/B)$	$O(N/B)$	$O(1)$ & GC	$O(1)$ & GC

Operation	I/O	append-only table	B-Tree
Query	Reads	$O(N/B)$	**$O(\log_B N)$**
Insert	Reads	0	$O(\log_B N)$
	Writes	**$O(1/B)$**	$O(1)$ & GC

Can we somehow combine these?

The Log-Structured Merge-Tree

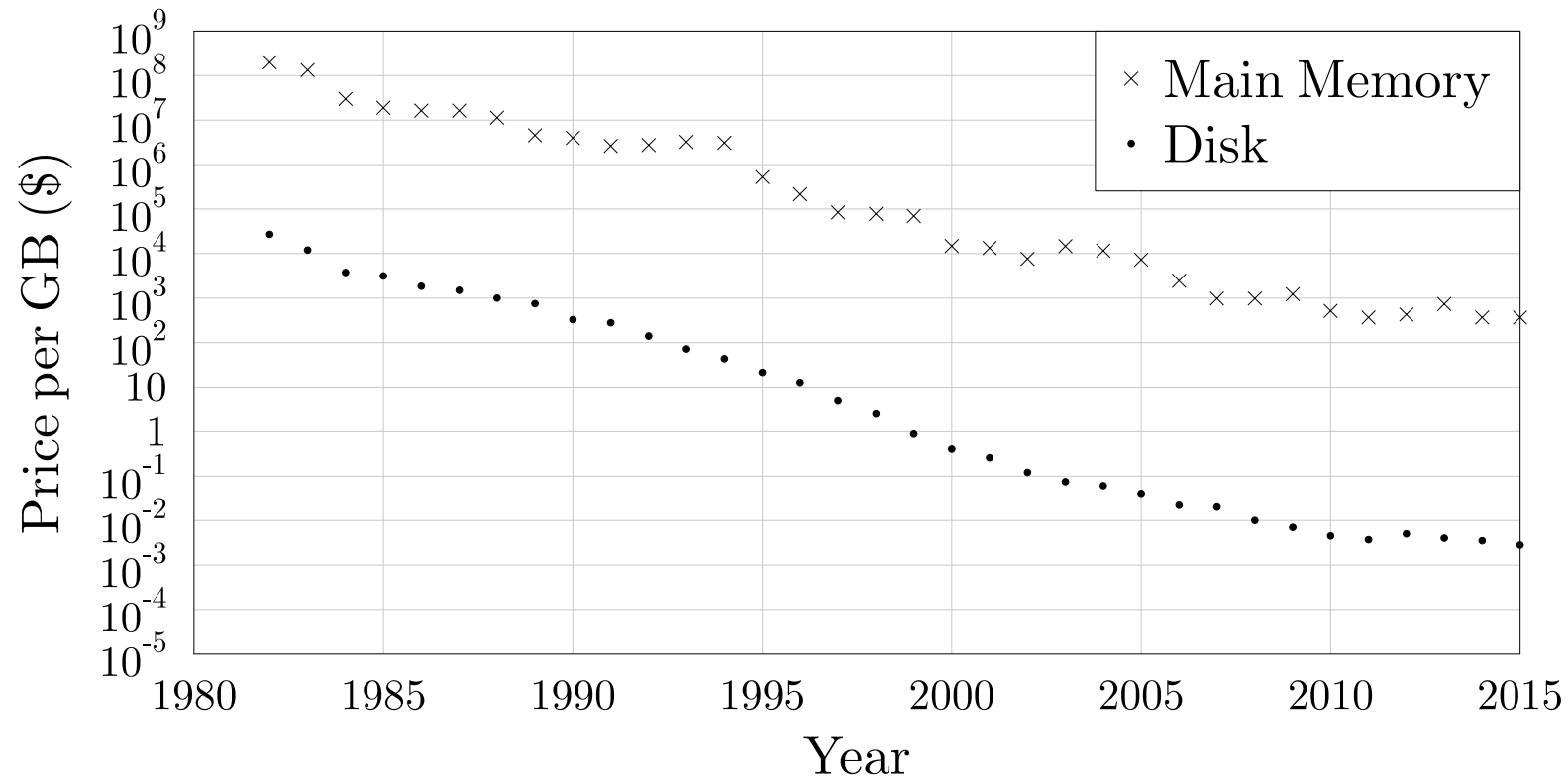| B-tree is invented	LSM-tree is invented	Google's BigTable adopts LSM-tree	LSM-tree is widely used
1970	1996	2005	Today

B-tree
is invented

LSM-tree is
invented

Google's BigTable
adopts LSM-tree

LSM-tree is
widely used

1970

1996

2005
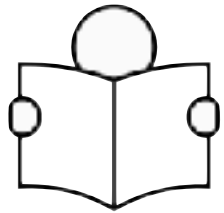
Today

Why was it not invented and used sooner?

The declining costs of storage allow us to store more data cheaply. Hence, application workloads are becoming more write-intensive. This drives a need to optimize for data ingestion.
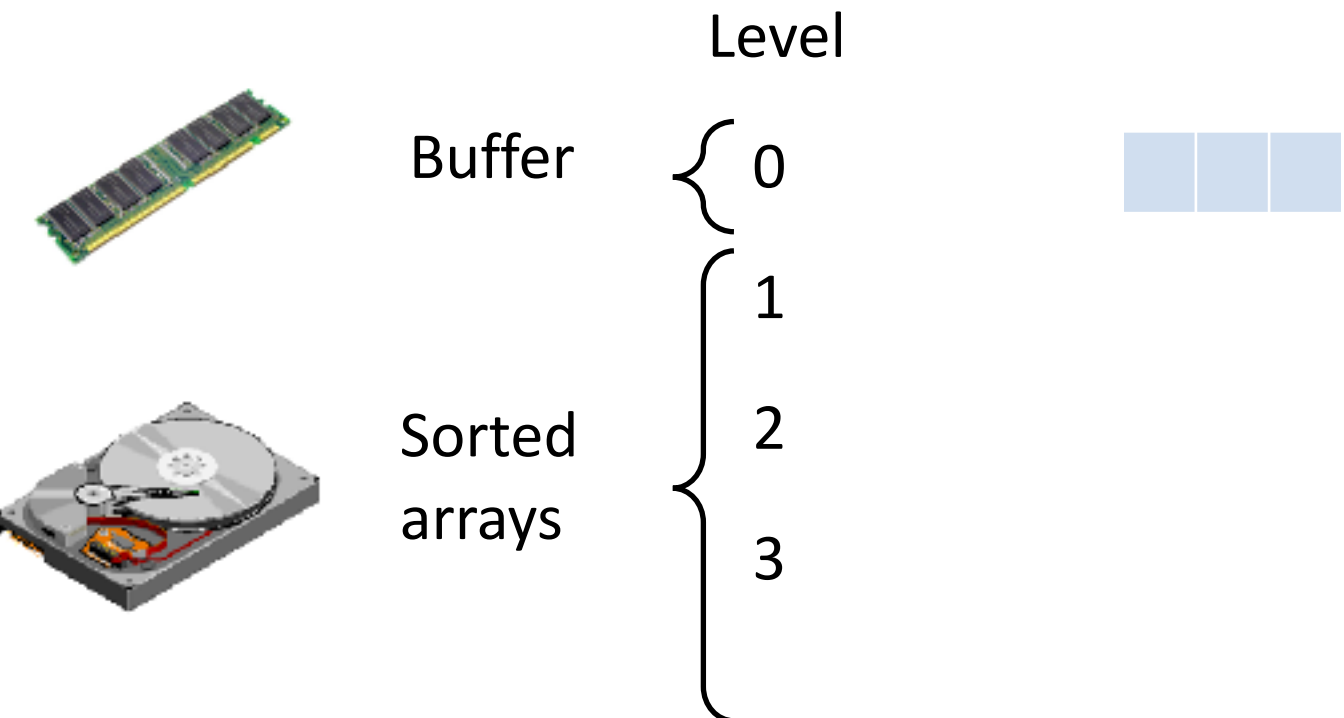
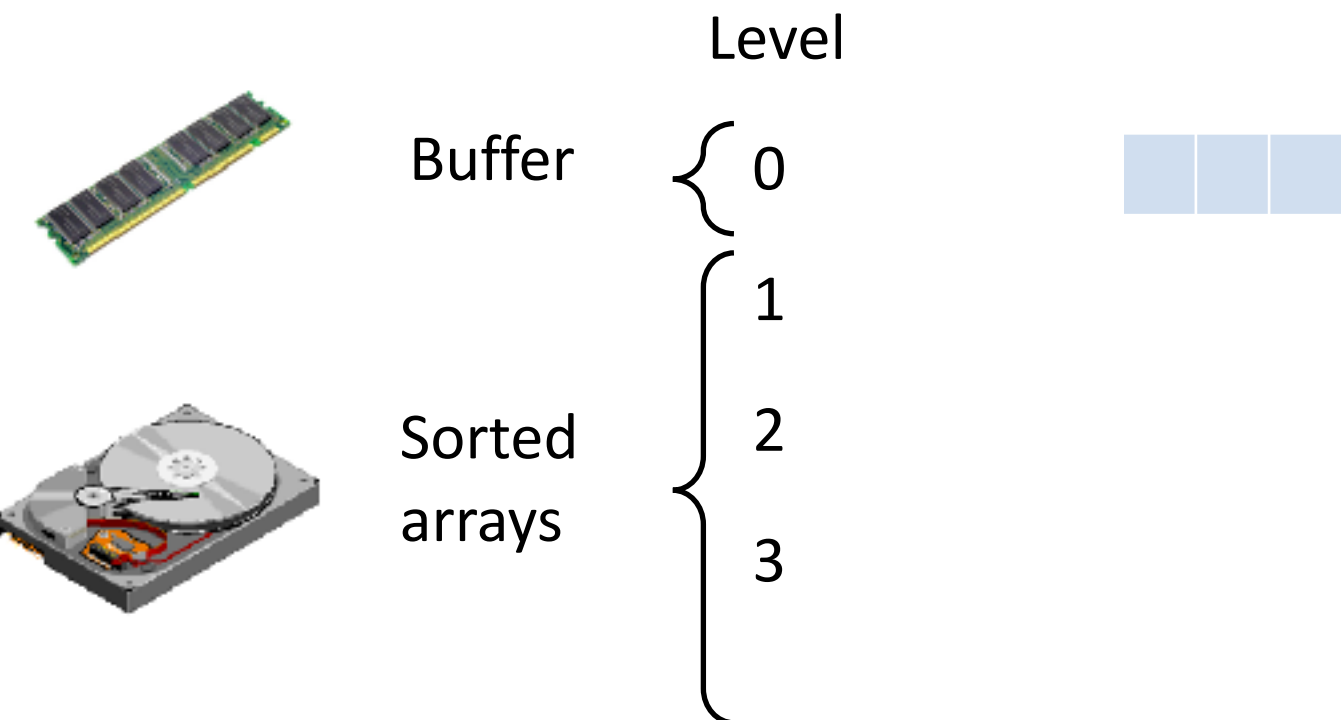Leveled LSM-tree   Basic LSM-tree   Tiered LSM-tree
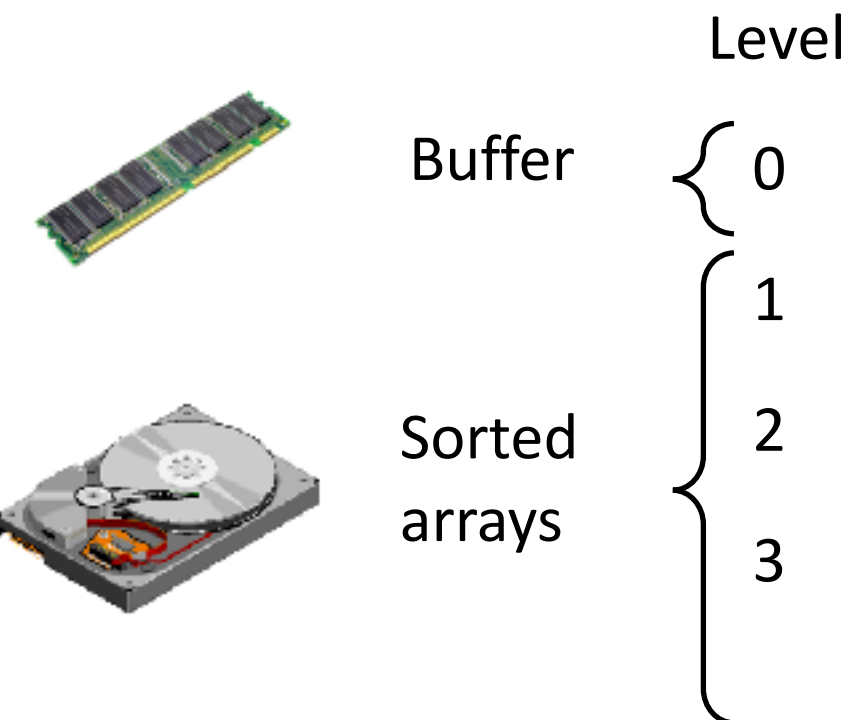
Basic LSM-tree

Level

Buffer   0

Sorted
arrays

1

2

3

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Level

Buffer   0

Sorted   1
arrays   2
            3

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Inserts

Level

Buffer   0   … … …

Sorted   1
arrays   2

          3

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Inserts

Level

Buffer   0

Sorted
arrays   1

              2

              3

sort & flush buffer

… … …

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Inserts

Level

Buffer   0

sort & flush buffer

Sorted
arrays   1

2

3

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Design principle #2:   optimize for lookups by sort-merging SSTs

Inserts

Level

Buffer   0

sort & flush buffer

Sorted
arrays   1

          2

          3

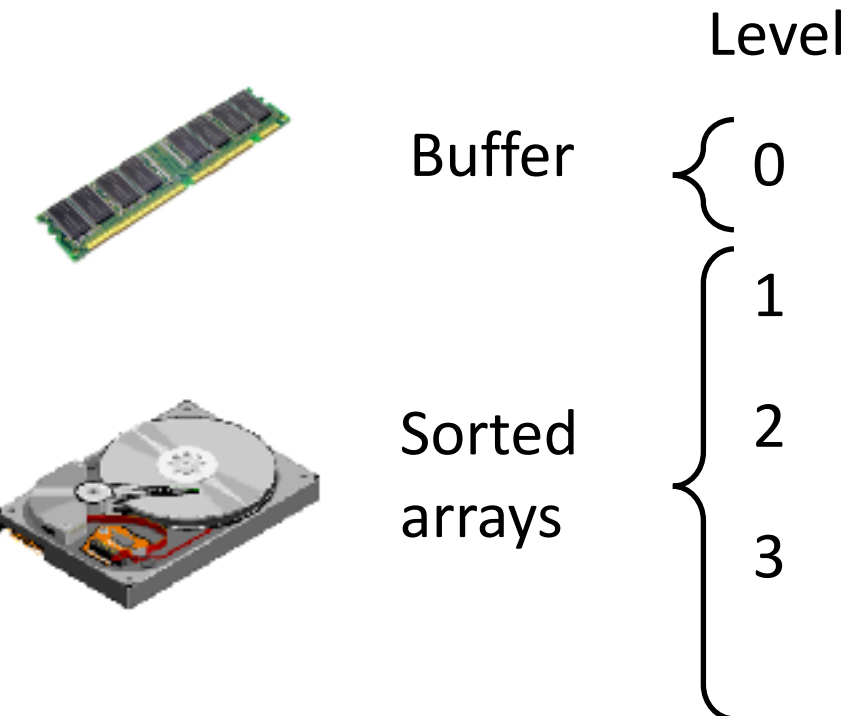# Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Design principle #2:   optimize for lookups by sort-merging SSTs

Inserts

Level

Buffer   0

Sorted
arrays

1

2

3

sort & flush buffer

Sort-merge

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Design principle #2:   optimize for lookups by sort-merging SSTs

Inserts

Level

Buffer   0

Sorted arrays   1

2

3

sort & flush buffer

X_1 ... ... X_2 ...

... ...

Sort-merge & Eliminate duplicates

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Design principle #2:   optimize for lookups by sort-merging SSTs
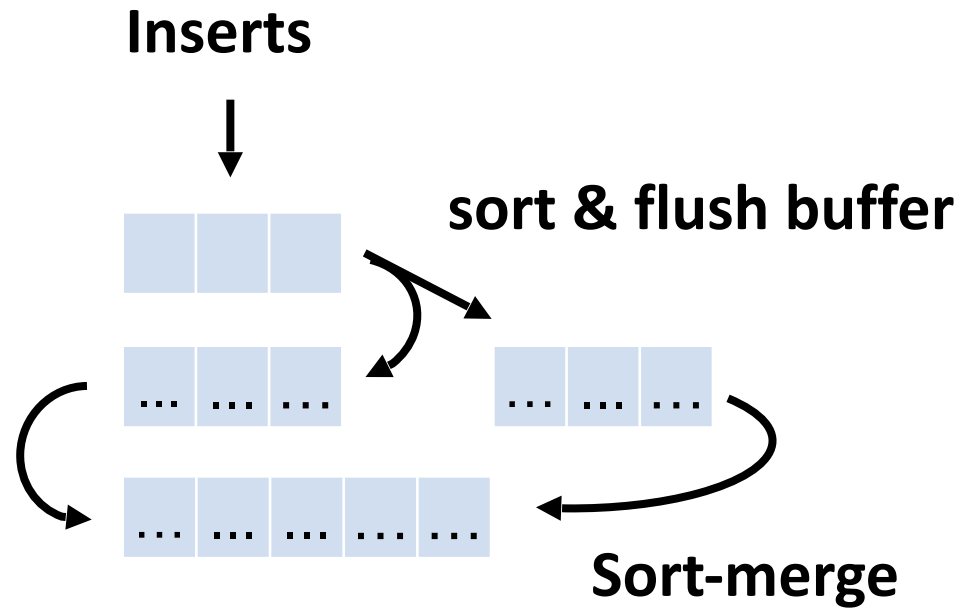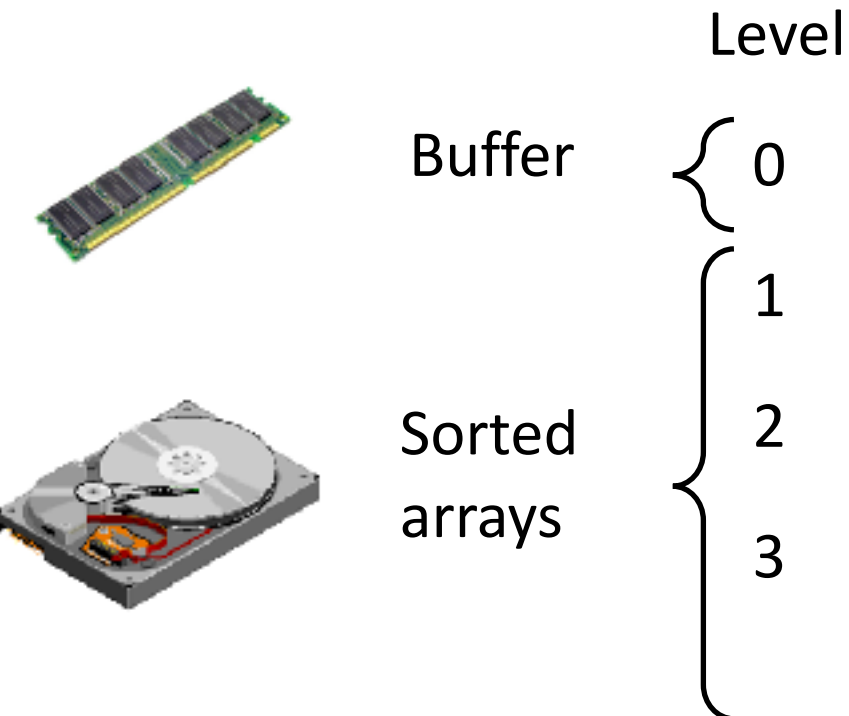
Inserts

Level

Buffer   0   **sort & flush buffer**

1   X_1 ... ... X_2 ...

Sorted arrays   2   ... X_2 ... ...
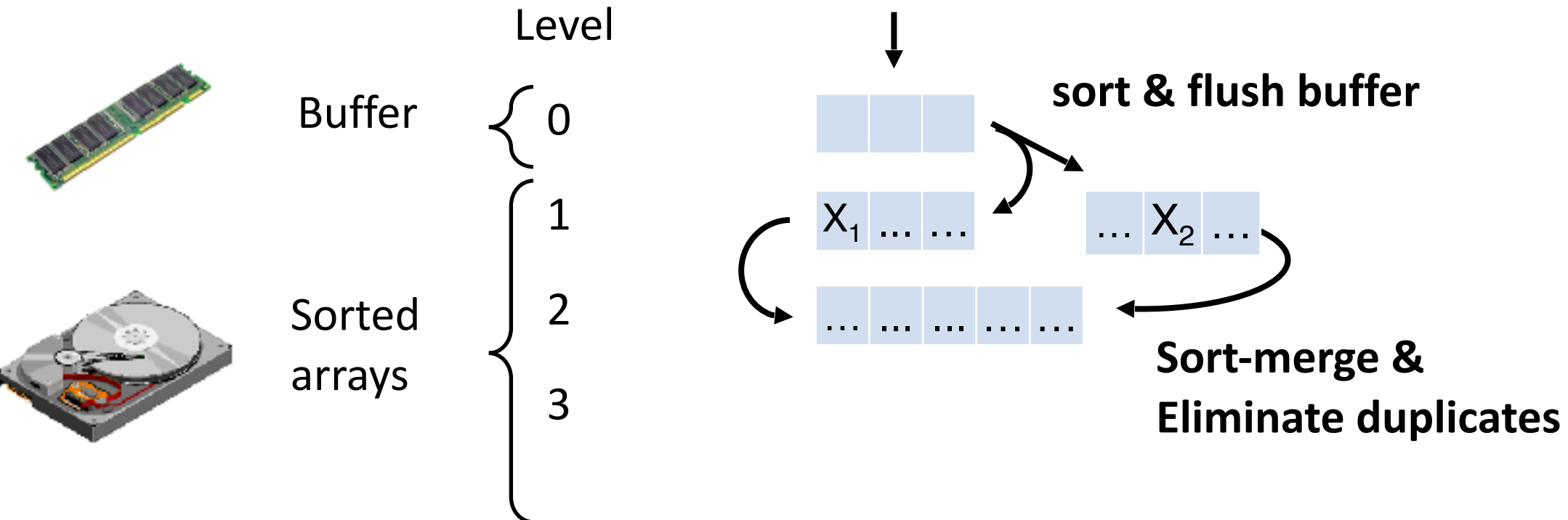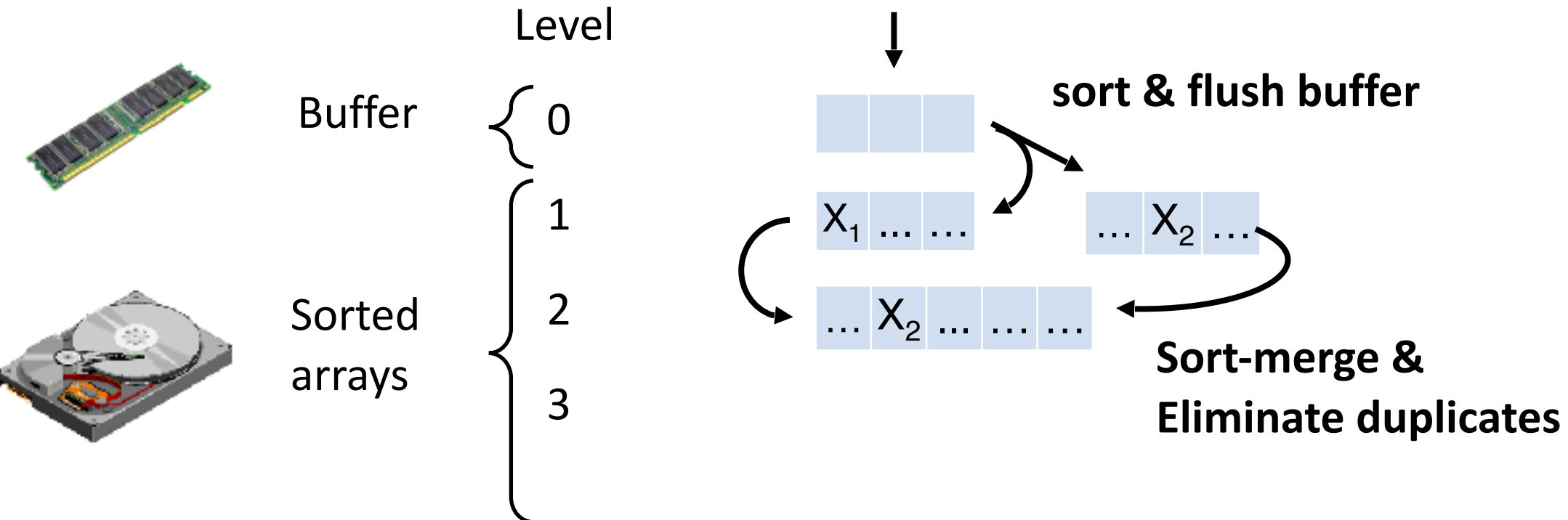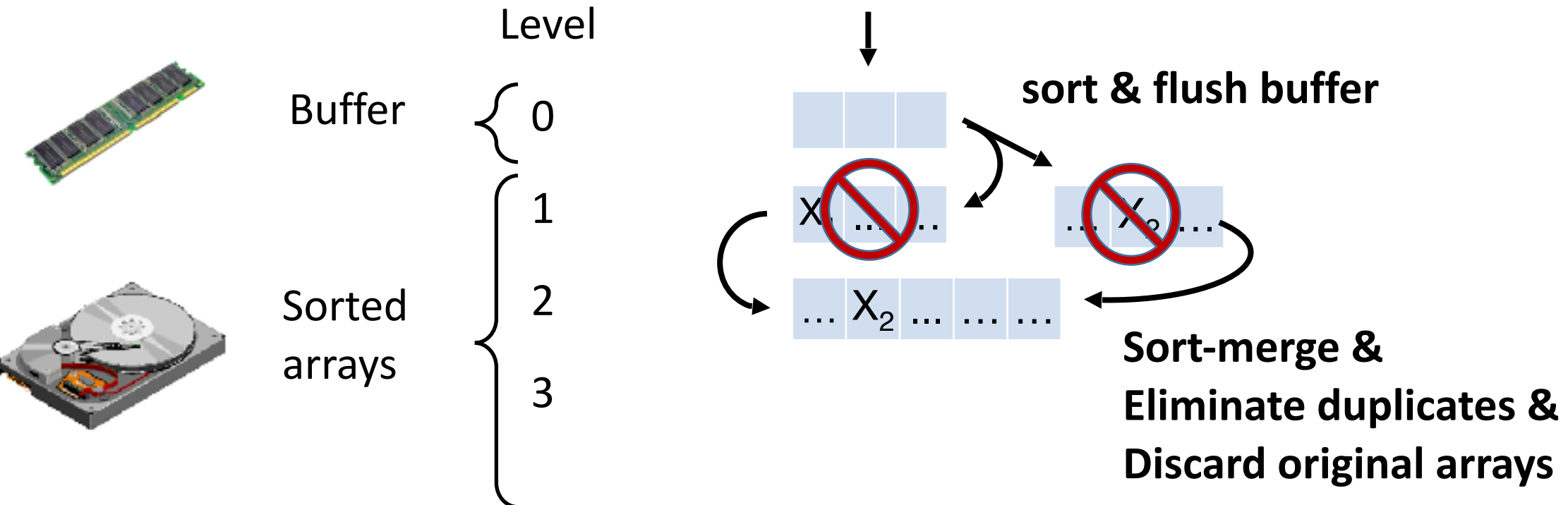
3   **Sort-merge & Eliminate duplicates**

Basic LSM-tree

Design principle #1:   optimize for insertions by buffering

Design principle #2:   optimize for lookups by sort-merging SSTs

Inserts

Level

Buffer   0

sort & flush buffer

Sorted arrays

1

X_1 ... ...

... X_2 ...

2

... X_2 ... ...

**Sort-merge &
Eliminate duplicates &
Discard original arrays**

3

Basic LSM-tree – Example

Level

Buffer   0

Sorted
arrays   1

  2

  3

Basic LSM-tree – Example

inserts

Level

Buffer   0

| 4 | 6 | 9 |

Sorted arrays

1

2

3

Basic LSM-tree – Example

inserts

Level

Buffer

0

sort & flush buffer

1

| 4 | 6 | 9 |

Sorted
arrays

2

3

Basic LSM-tree – Example

inserts

Level

Buffer 0

Sorted
arrays

1 | 4 | 6 | 9 |

2

3

Basic LSM-tree – Example

inserts

Level

Buffer ⎰ 0

| 3 | 4 | 8 |

Sorted arrays ⎰ 1

| 4 | 6 | 9 |

2

3

Basic LSM-tree – Example

Level

inserts

Buffer 0

sort & flush buffer

Sorted arrays 1   4 6 9   3 4 8

2

3

Basic LSM-tree – Example

inserts

Level

Buffer   0

Sorted arrays   1 | 4 | 6 | 9 | | 3 | 4 | 8 |

2

3

Basic LSM-tree – Example

inserts

Level

Buffer

0

Sorted arrays

1

| 4 | 6 | 9 | | 3 | 4 | 8 |

2

| 3 | 4 | 6 | 8 | 9 |

Sort-merge

3

Basic LSM-tree – Example

inserts

Level

Buffer   0

Sorted
arrays

1   4_1 6 9   3 4_2 8

2   3 4_2 6 8 9

3

**Sort-merge &
Eliminate duplicates**

Basic LSM-tree – Example

inserts

Level

Buffer 0

Sorted arrays

1

2

3

4 6 9

3 4 8

3 4 6 8 9

**Sort-merge &
Eliminate duplicates &
Discard original arrays**

Basic LSM-tree – Example

inserts

Level

Buffer
0

Sorted arrays
1

2 | 3 | 4 | 6 | 8 | 9 |

3

Basic LSM-tree – Example

inserts

Level

Buffer 0 | 2 | 7 | 8 |

1

Sorted 2 | 3 | 4 | 6 | 8 | 9 |
arrays

3

Basic LSM-tree – Example

inserts

Level

Buffer   0

sort & flush buffer

| | | |

1

| 2 | 7 | 8 |

Sorted arrays   2

| 3 | 4 | 6 | 8 | 9 |

3

Basic LSM-tree – Example

inserts

Level

Buffer 0

Sorted arrays

1 | 2 | 7 | 8 |

2 | 3 | 4 | 6 | 8 | 9 |

3

Basic LSM-tree

Levels have exponentially increasing capacities.
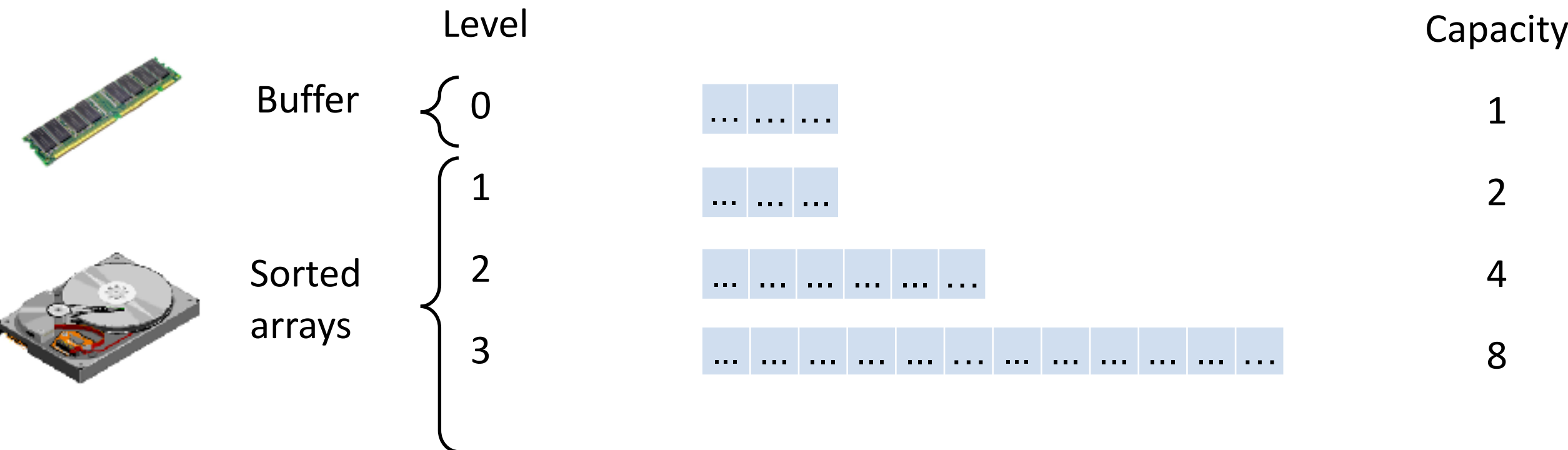
Basic LSM-tree - Updates

Can be made out-of-place through an insertion into the buffer

Put(X, Y)

Level

Capacity

Buffer 0

... X_0 ...

1

Sorted arrays 1

... X_1 ...

2

2

... ... X_2 ... ...

4

3

... ... X_3 ... ...

8
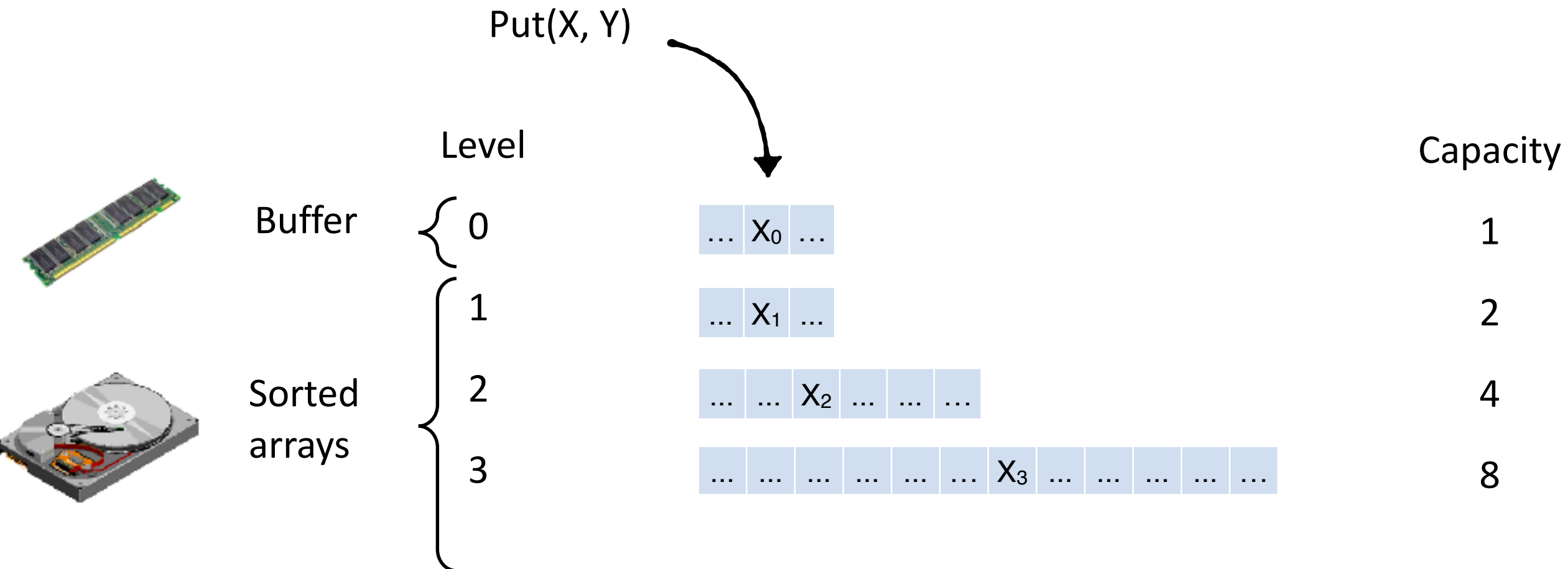
Basic LSM-tree - Updates

Can be made out-of-place through an insertion into the buffer

Only the most recent version counts and should be returned to users during queries.
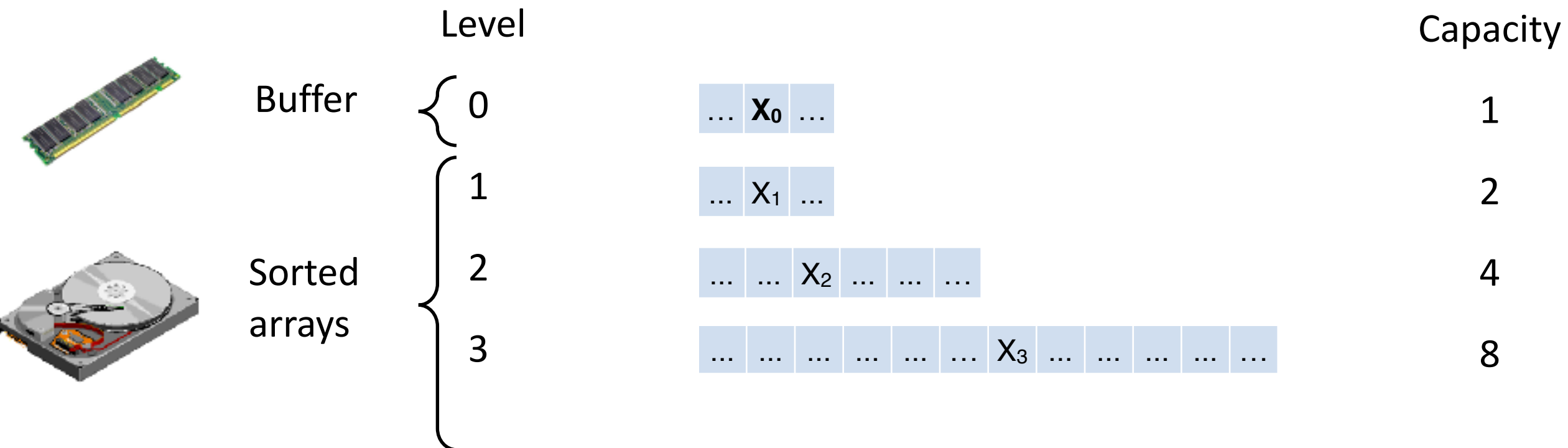
	Level		Capacity
Buffer	0	... X_0 ...	1
Sorted arrays	1	... X_1 ...	2
	2	... ... X_2	4
	3	... ... X_3	8

Basic LSM-tree - Deletes

Delete an entry by inserting a tombstone

Delete(X)

Level

Capacity

Buffer 0   ... X_T ...   1

Sorted arrays

1   ... X_1 ...   2

2   ... ... X_T   4

3   ... ... X_3   8
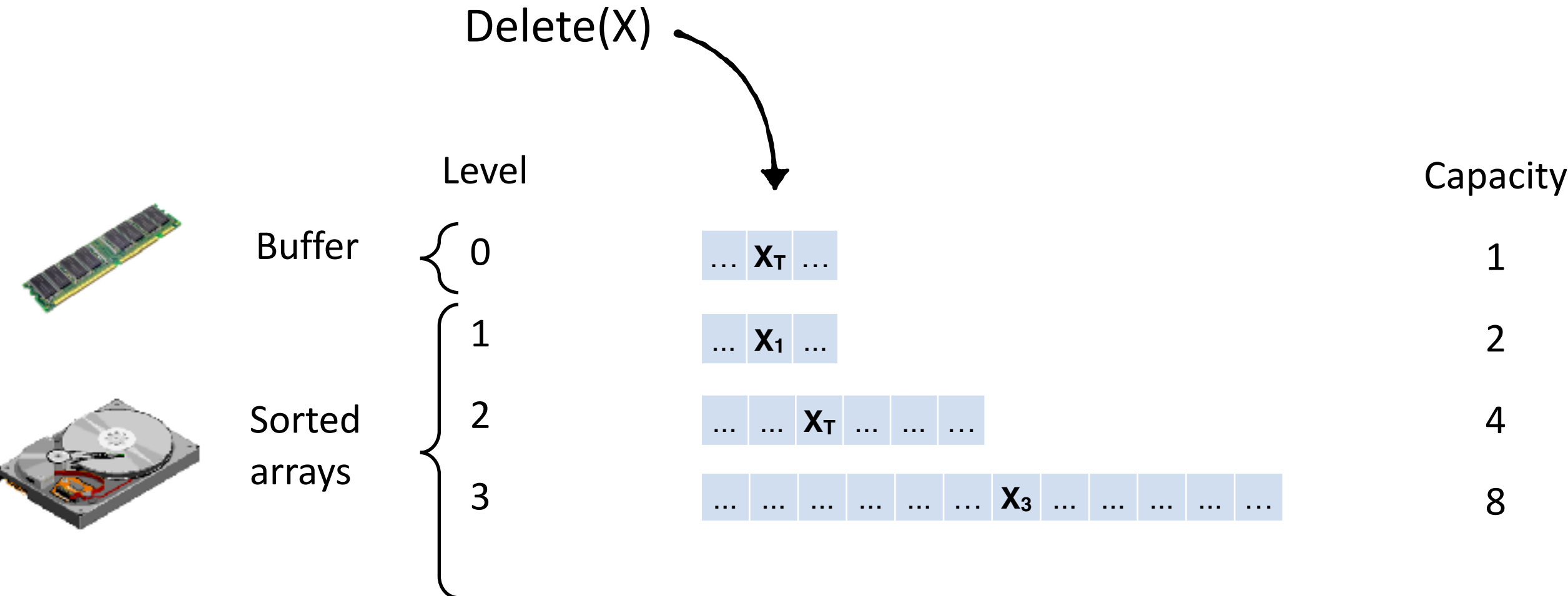
Basic LSM-tree - Deletes

Delete an entry by inserting a tombstone

If the most recent version of an entry is a tombstone, it's considered deleted. (X is deleted by Y isn't in this example)

	Level		Capacity
Buffer	0	… X_T …	1
Sorted arrays	1	… X_1 Y_1	2
	2	… … X_T … … …	4
	3	… … … … … … X_3 … … Y_T … …	8
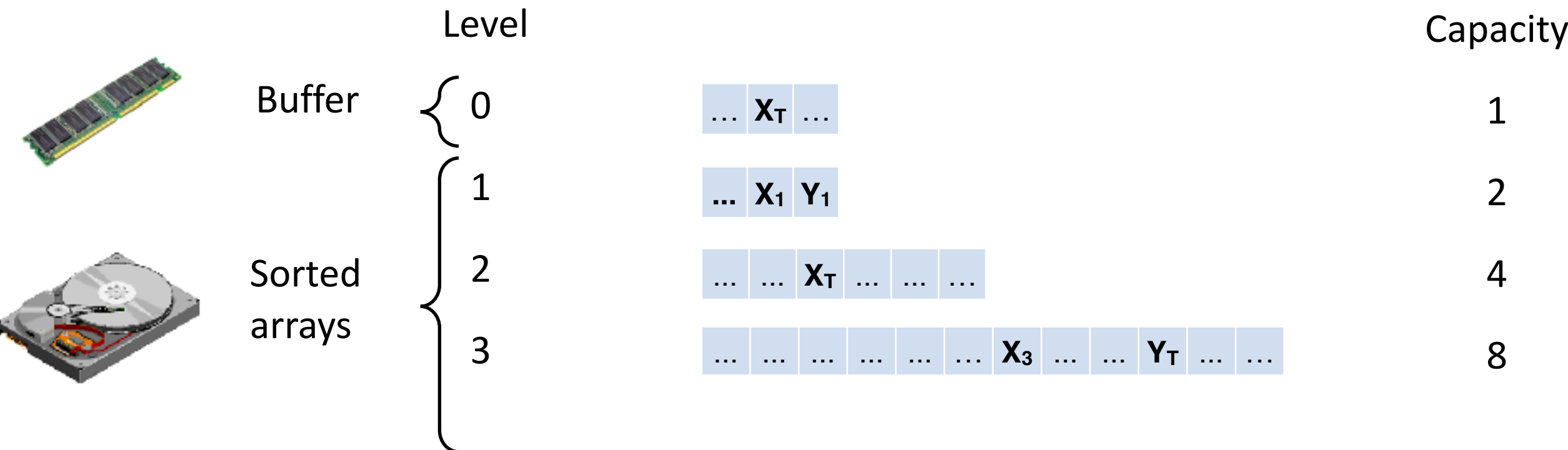
Basic LSM-tree – Get Queries

A get query searches the LSM-tree from smaller to larger levels. They stop when they find the first matching entry, as entries at larger levels are older and thus superseded.
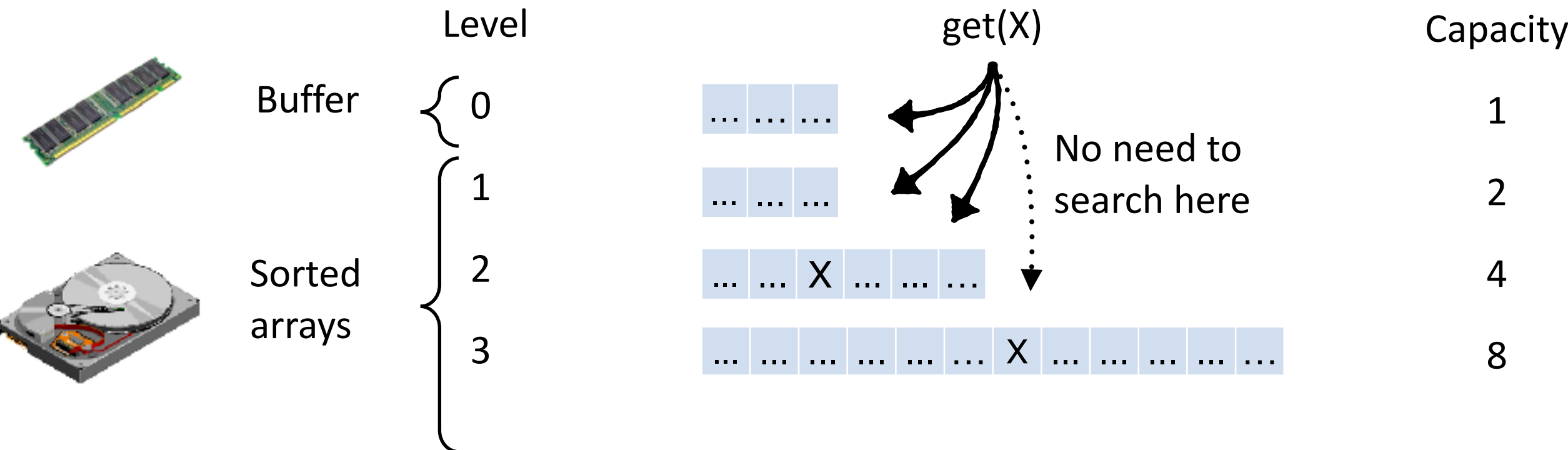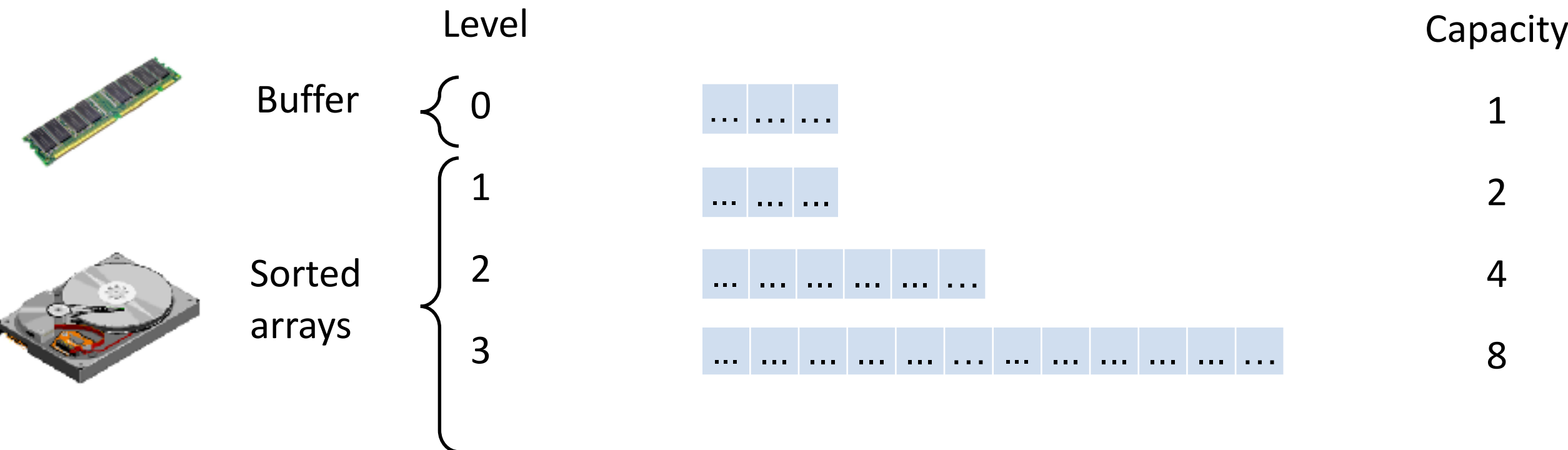
Level   get(X)   Capacity

Buffer   0   … … …   1

        1   … … …   No need to   2

Sorted   2   … … X … … …   search here   4

arrays   3   … … … … … … X … … … … …   8

Basic LSM-tree – Get Queries Cost

How many levels to search?

Level

Capacity

Buffer { 0   1

Sorted arrays {
1   2

2   4

3   8

Basic LSM-tree – Get Queries Cost

How many levels to search?   **$O(\log_2 N/B)$**

	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1	... ...	2
	2	... ...	4
	3	... ...	8

Basic LSM-tree – Get Queries Cost

How many levels to search? $O(\log_2 N/B)$

Cost per searching each level?

	Level		Capacity
Buffer	0	… … …	1
Sorted arrays	1	… … …	2
	2	… … … … … …	4
	3	… … … … … … … … … … … …	8
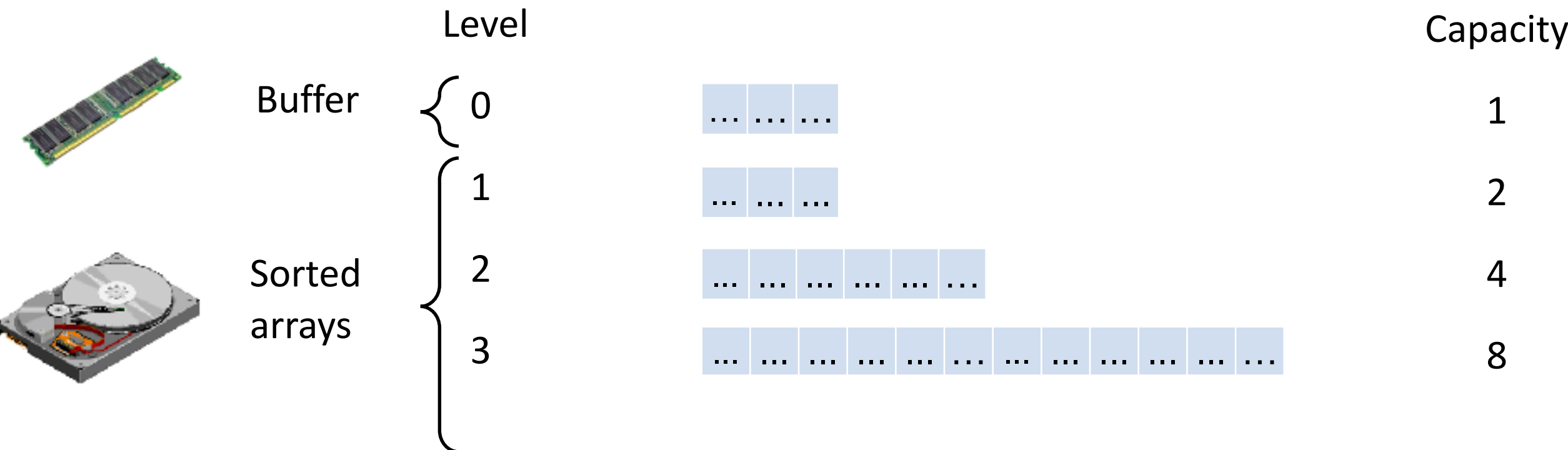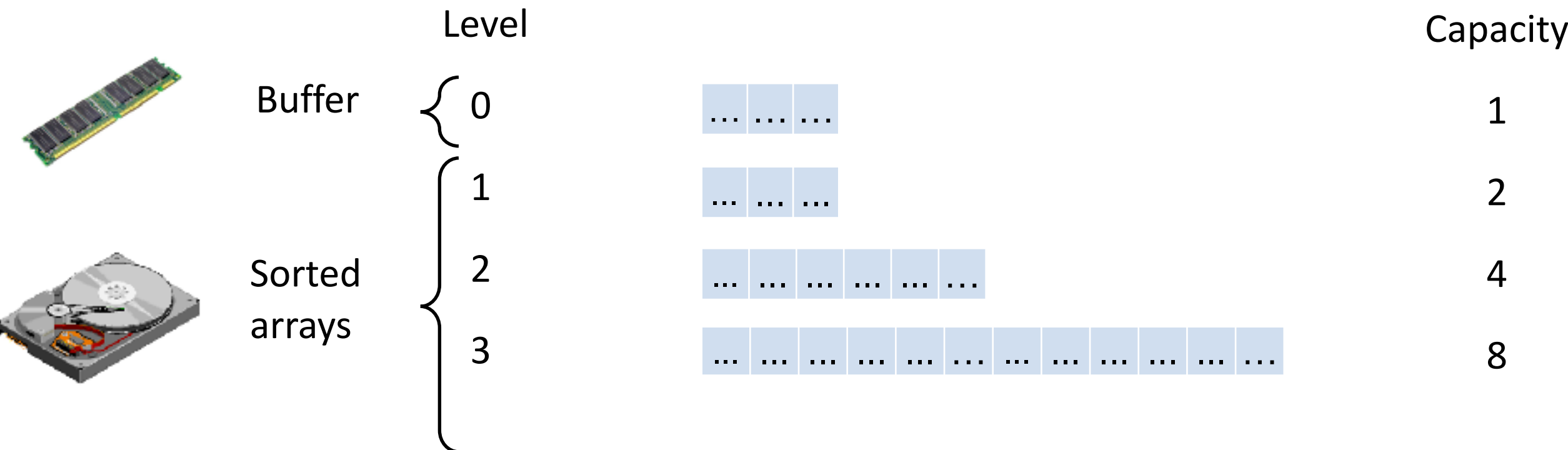
Basic LSM-tree – Get Queries Cost

How many levels to search? $O(\log_2 N/B)$

Cost per searching each level? $O(\log_2 N/B)$ **w. binary search**
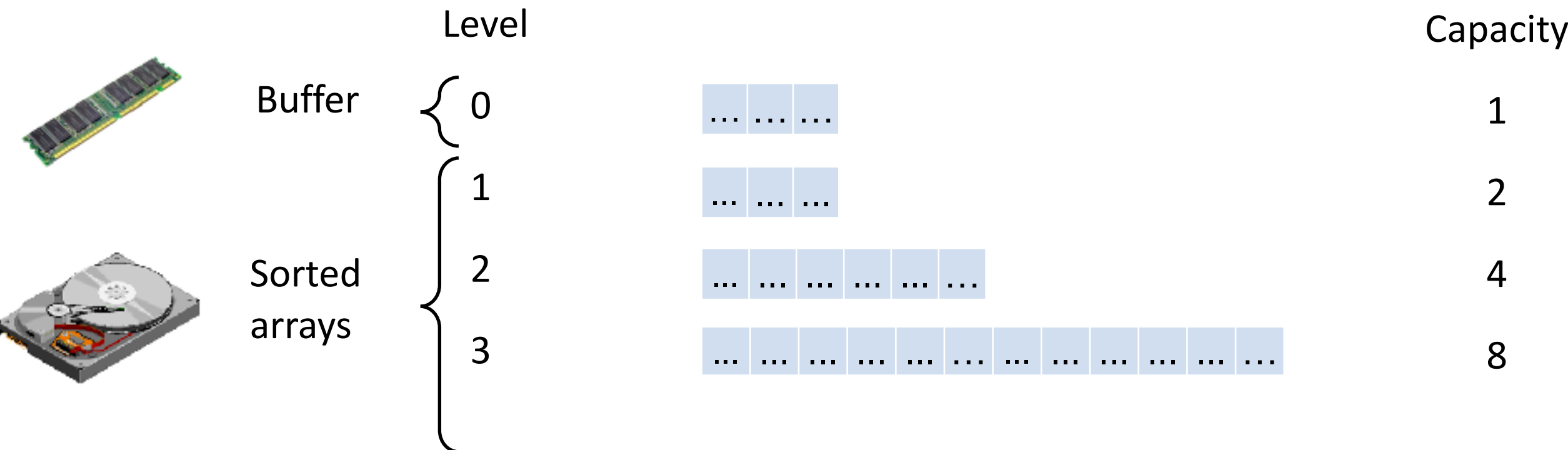
	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1	... ...	2
	2	... ...	4
	3	... ...	8

Basic LSM-tree – Get Queries Cost

How many levels to search? $O(\log_2 N/B)$
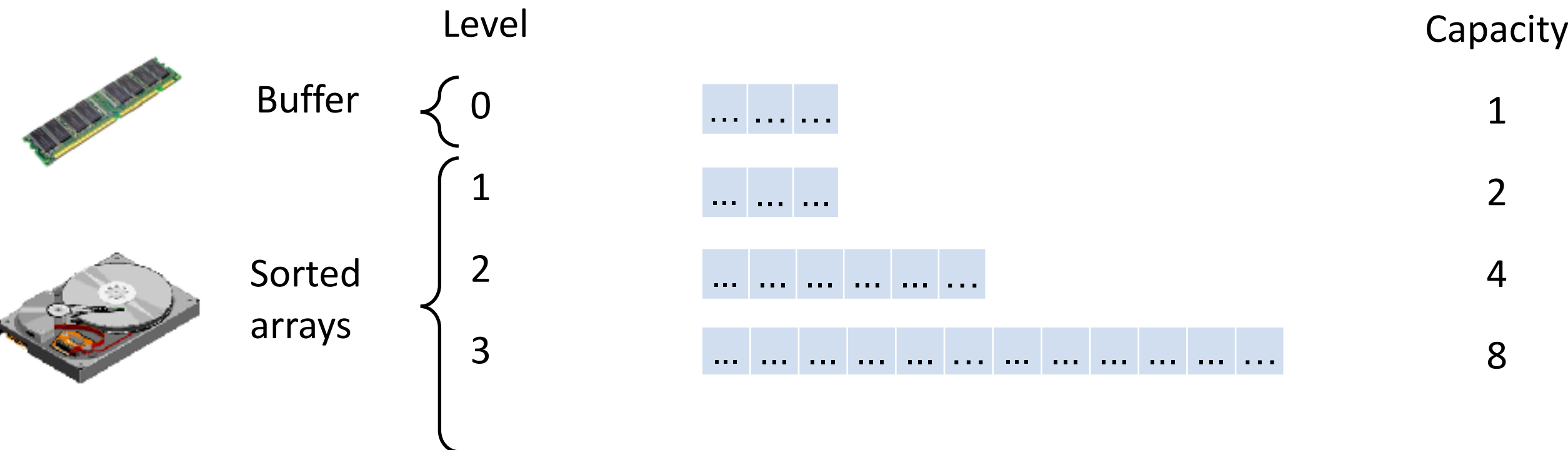
Cost per searching each level? $O(\log_2 N/B)$

Total search cost: **$O(\log_2(N/B)^2)$**

	Level		Capacity
Buffer	0	… … …	1
Sorted arrays	1	… … …	2
	2	… … … … … …	4
	3	… … … … … … … … … … … …	8

Basic LSM-tree – Get Queries Cost

We can do slightly better by structuring each file (SST) as a static B-tree. How would this impact search cost?

Total search cost: $O(\log_2(N/B)^2)$

Lvl 1	Lvl 2	Leaves - lvl 3

SST static B-tree structure

Basic LSM-tree – Get Queries Cost

We can do slightly better by structuring each file (SST) as a static B-tree.
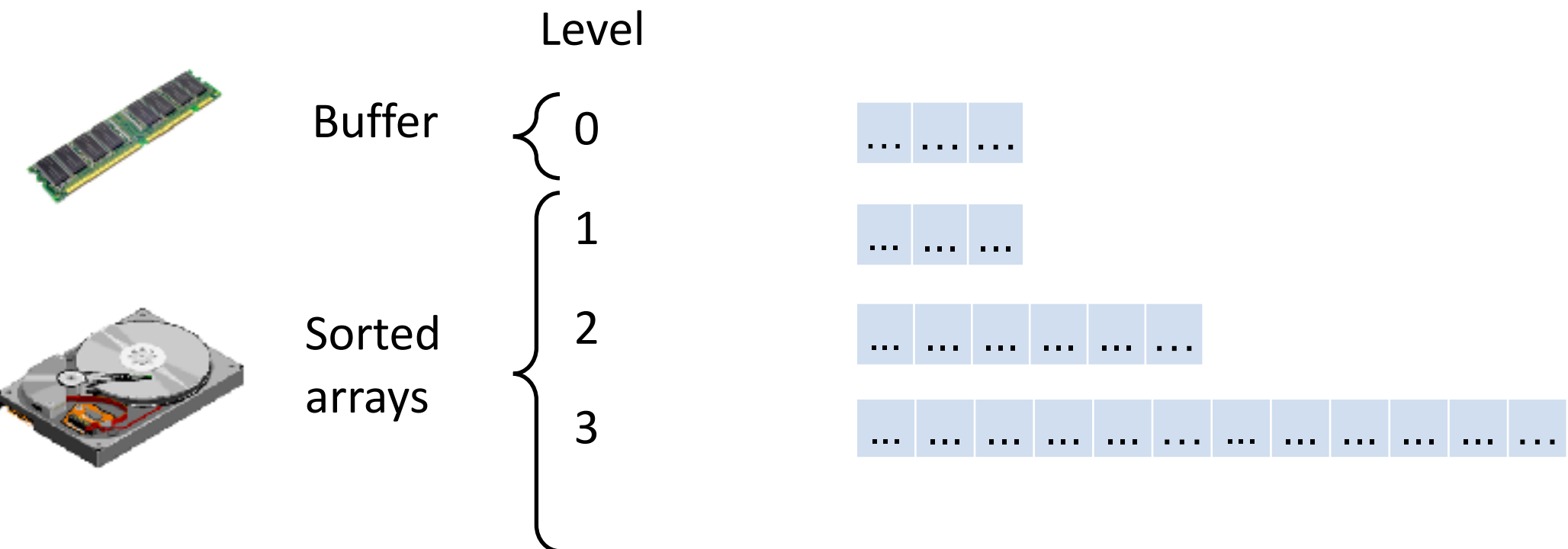How would this impact search cost?

Total search cost: $O(\log_2(N/B) * \mathbf{\log_B N})$

SST static B-tree structure

Basic LSM-tree – Scan Queries

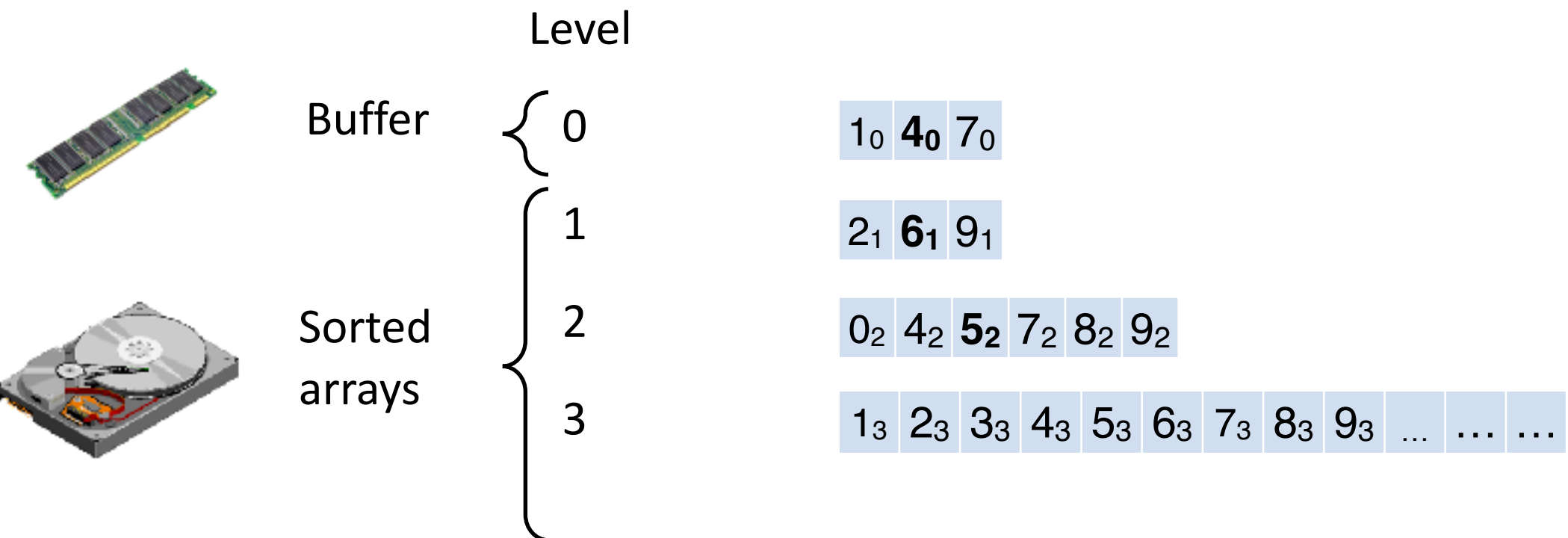Return most recent version of each entry in the range across entire tree.

Basic LSM-tree – Scan Queries

Return most recent version of each entry in the range across entire tree.

e.g., Scan(4, 6)

Expected output: 4_0 5_2 6_1

Level

Buffer   0 1_0 **4_0** 7_0

         1   2_1 **6_1** 9_1

Sorted   2 0_2 4_2 **5_2** 7_2 8_2 9_2
arrays
         3   1_3 2_3 3_3 4_3 5_3 6_3 7_3 8_3 9_3 ... ...

Basic LSM-tree – Scan Queries

1. Allocate an in-memory buffer (>1 page) for each level

1_0 4_0 7_0

2_1 6_1 9_1

0_2 4_2 5_2 7_2 8_2 9_2

1_3 2_3 3_3 4_3 5_3 6_3 7_3 8_3 9_3 ... ...

LSM-tree in storage   In-memory buffers   Output

Basic LSM-tree – Scan Queries

1. Allocate an in-memory buffer (>1 page) for each level
2. Search for start of key range at each level

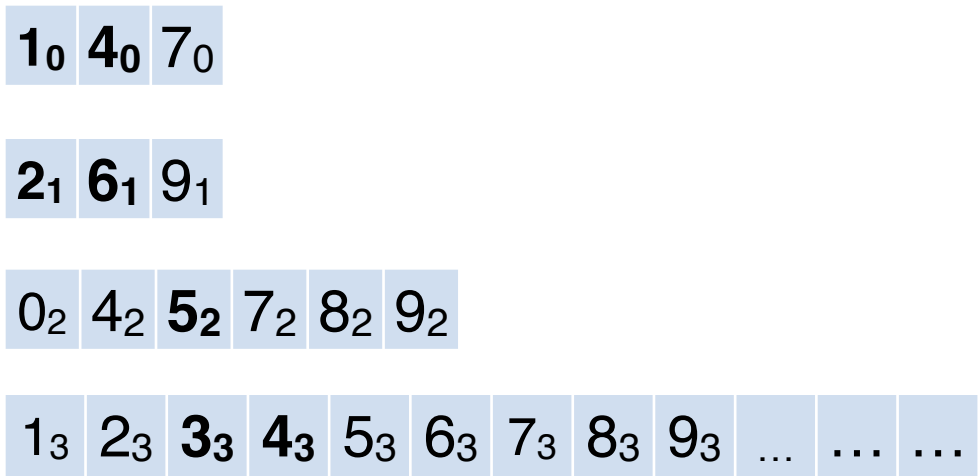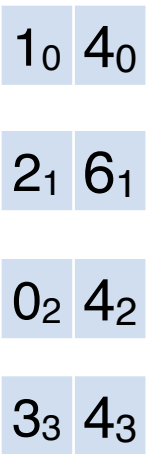LSM-tree in storage   In-memory buffers   Output
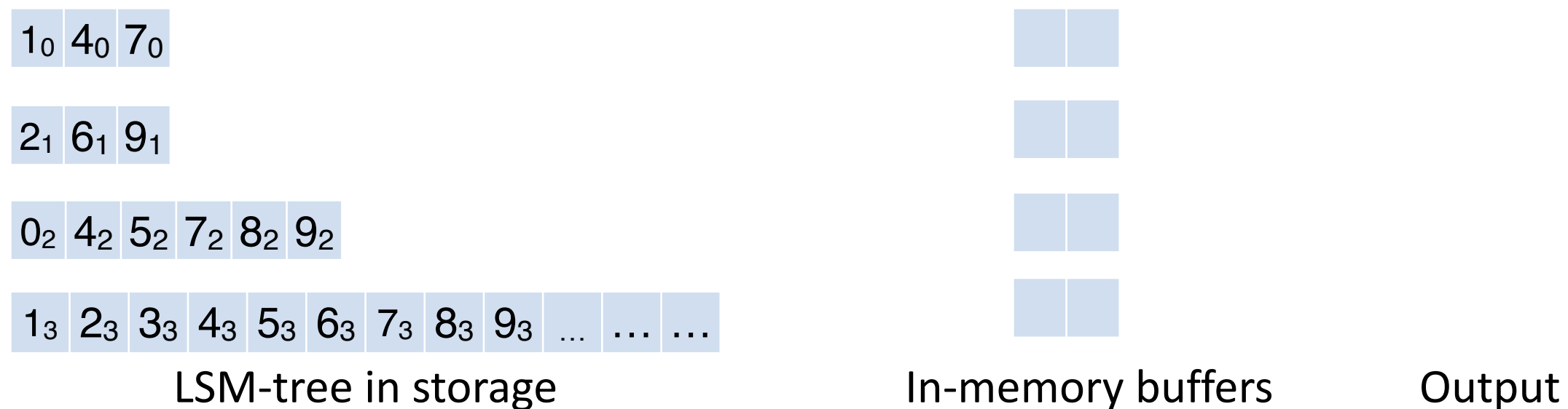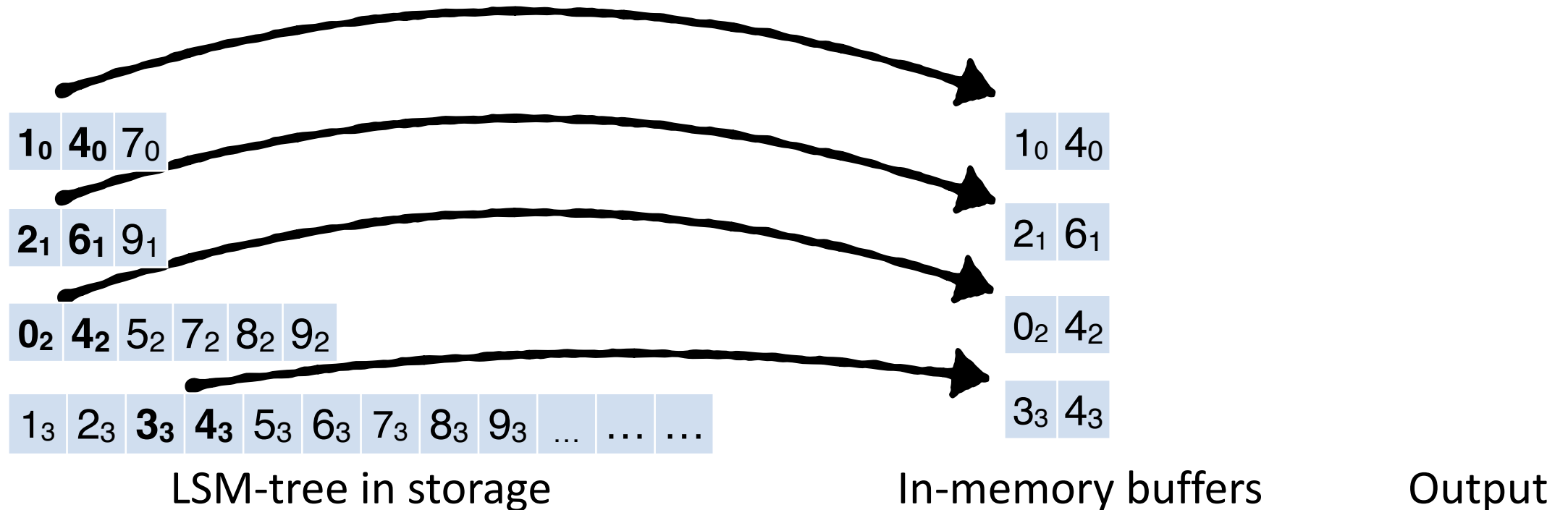
Basic LSM-tree – Scan Queries

1. Allocate an in-memory buffer (>1 page) for each level
2. Search for start of key range at each level
3. Initialize counter to smallest key in range

Counter: 4

1_0 $\mathbf{4_0}$ 7_0 1_0 4_0

$\mathbf{2_1}$ $\mathbf{6_1}$ 9_1 2_1 6_1

0_2 4_2 $\mathbf{5_2}$ 7_2 8_2 9_2 0_2 4_2

1_3 2_3 $\mathbf{3_3}$ $\mathbf{4_3}$ 5_3 6_3 7_3 8_3 9_3 ... ... 3_3 4_3

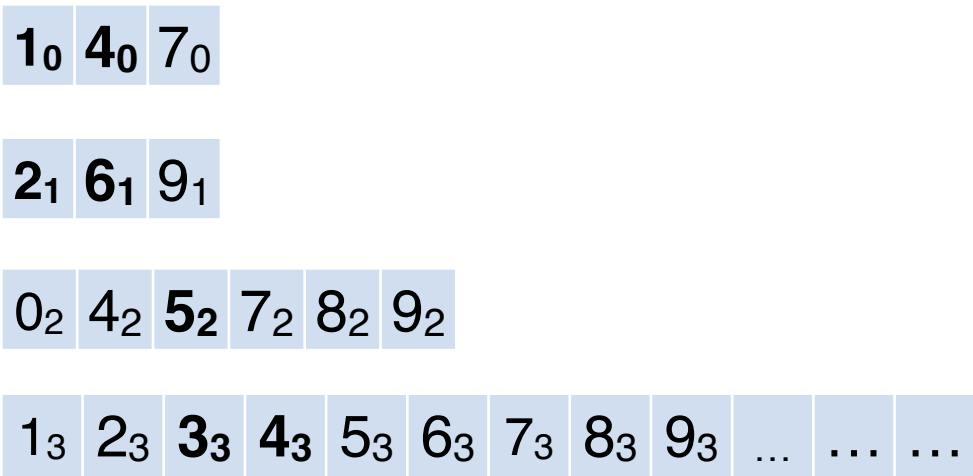LSM-tree in storage   In-memory buffers   Output

Basic LSM-tree – Scan Queries

Loop until counter passes end of range
    4. Bring youngest matching entry to output

Counter: 4

1_0 4_0 7_0

2_1 6_1 9_1

0_2 4_2 5_2 7_2 8_2 9_2

1_3 2_3 3_3 4_3 5_3 6_3 7_3 8_3 9_3 ... ...

LSM-tree in storage

1_0 4_0 → 4_0

2_1 6_1

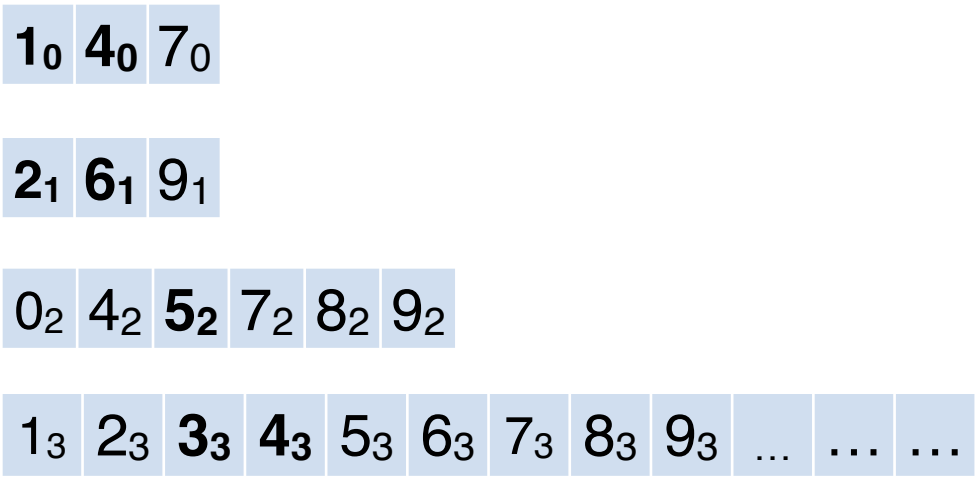0_2 4_2

3_3 4_3

In-memory buffers   Output

Basic LSM-tree – Scan Queries

Loop until counter passes end of range

    4. Bring youngest matching entry to output

    5. Increment counter

Counter: 5

1_0 4_0 7_0

2_1 6_1 9_1

0_2 4_2 5_2 7_2 8_2 9_2

1_3 2_3 3_3 4_3 5_3 6_3 7_3 8_3 9_3 ... ...

LSM-tree in storage

1_0 4_0

2_1 6_1

0_2 4_2

3_3 4_3
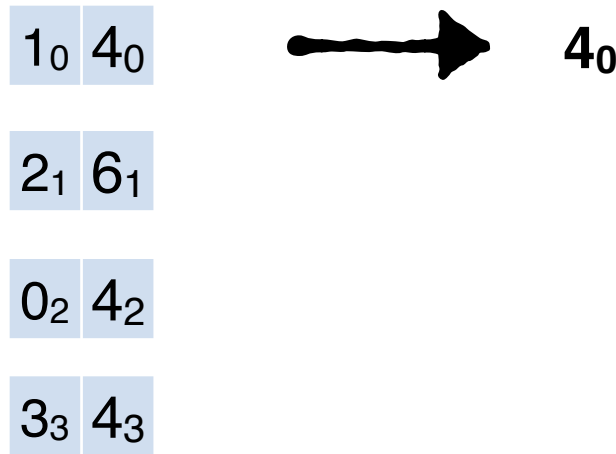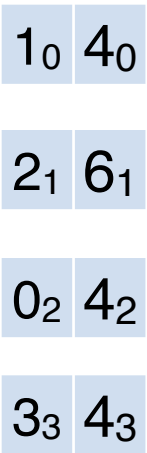
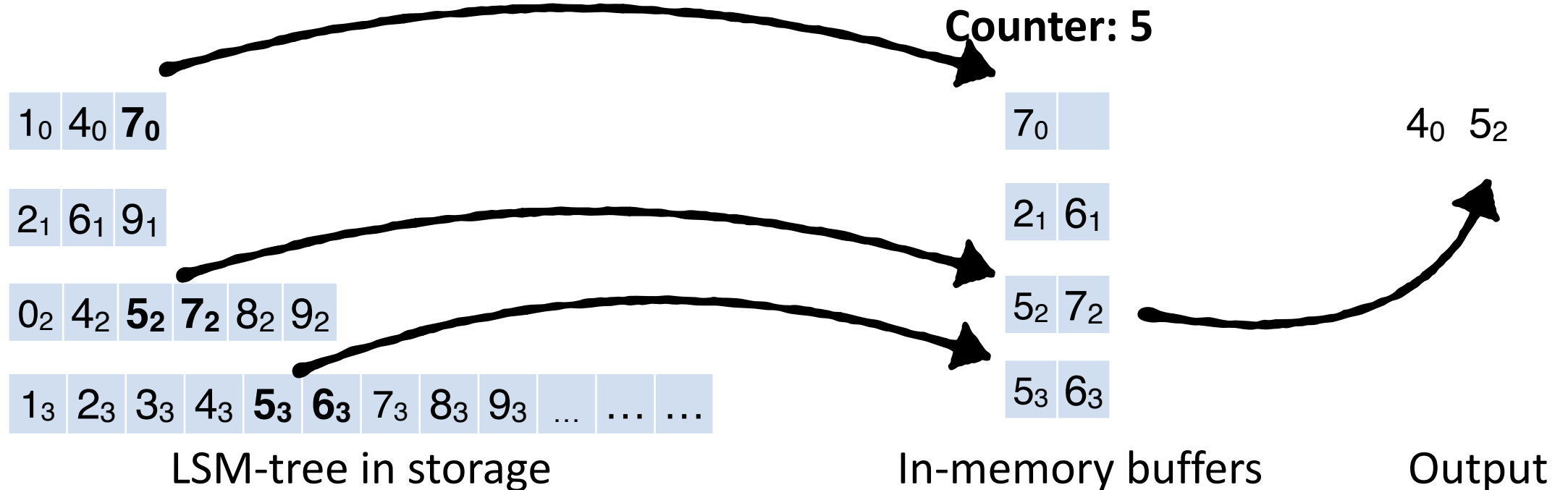In-memory buffers

4_0

Output

Basic LSM-tree – Scan Queries

Loop until counter passes end of range

    4. Bring youngest matching entry to output

    5. Increment counter

    6. Bring more pages to buffers if needed

Counter: 5

1_0 4_0 **7_0**

2_1 6_1 9_1

0_2 4_2 **5_2** **7_2** 8_2 9_2

1_3 2_3 3_3 4_3 **5_3** **6_3** 7_3 8_3 9_3 ... ...

LSM-tree in storage

7_0

2_1 6_1

5_2 7_2

5_3 6_3

In-memory buffers

4_0 5_2
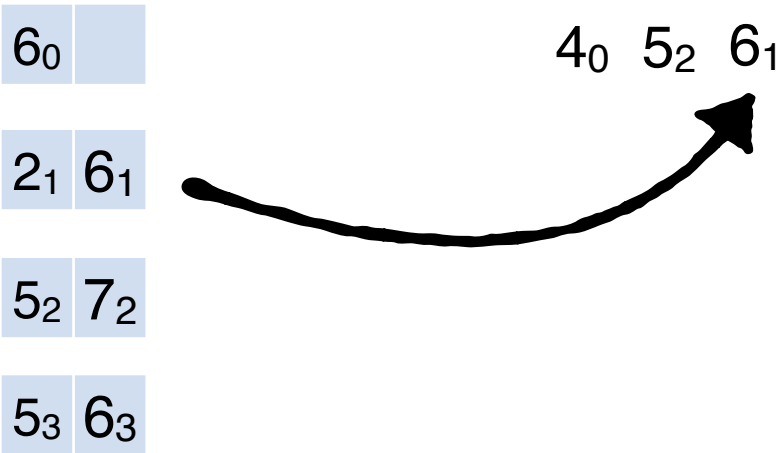
Output

Basic LSM-tree – Scan Queries

Loop until counter passes end of range

    4. Bring youngest matching entry to output

    5. Increment counter

    6. Bring more pages to buffers if needed

Counter: 6

1_0 4_0 **7_0**

2_1 6_1 9_1

0_2 4_2 **5_2** **7_2** 8_2 9_2

1_3 2_3 3_3 4_3 **5_3** **6_3** 7_3 8_3 9_3 ... ...

LSM-tree in storage

6_0

2_1 6_1
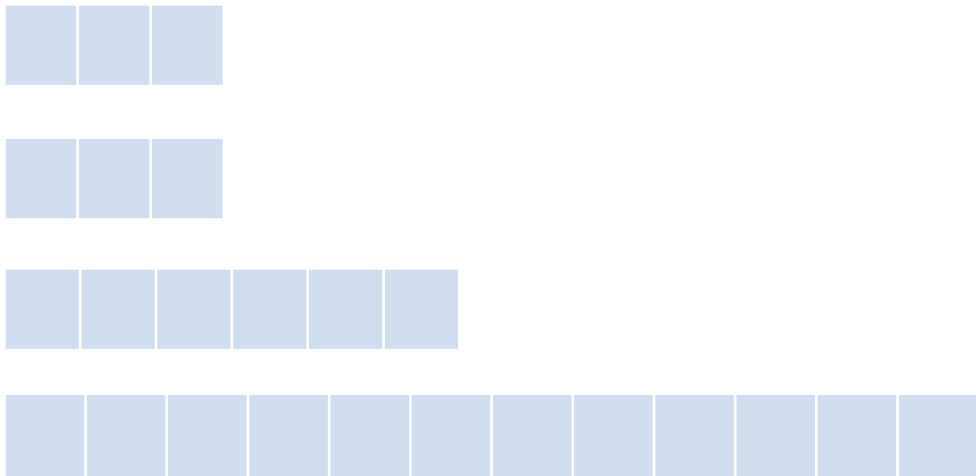
5_2 7_2

5_3 6_3

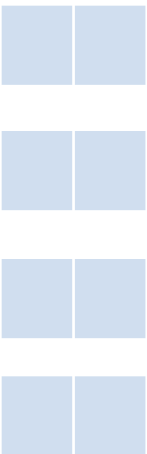In-memory buffers

4_0 5_2 6_1
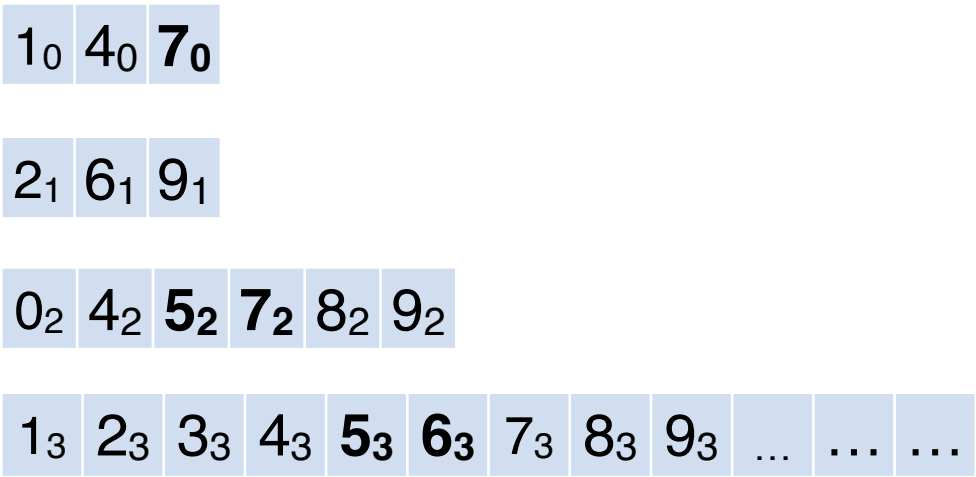
Output
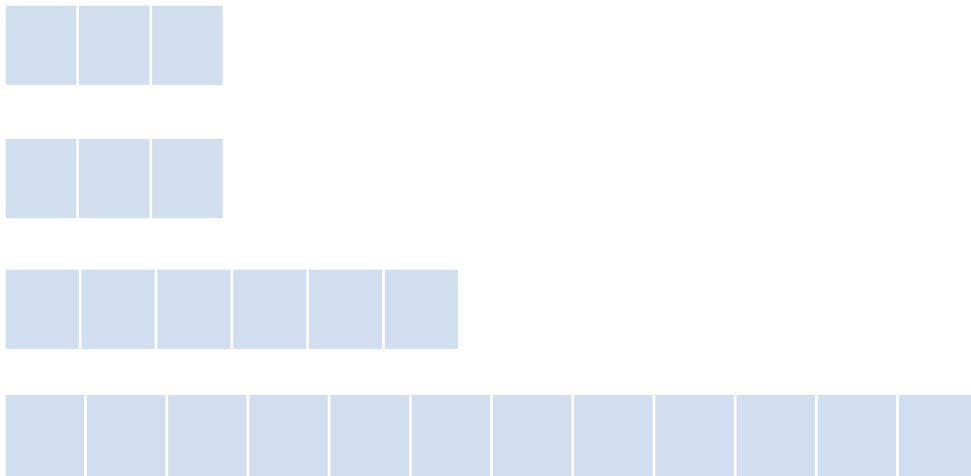
Basic LSM-tree – Scan Queries I/O Cost

$$O(\log_2(N/B) * \log_B(N) + S/B)$$

LSM-tree in storage

In-memory buffers

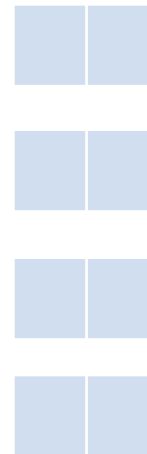# Basic LSM-tree – Scan Queries I/O Cost

 # lvls

$\downarrow$

$O(\mathbf{\log_2(N/B)} * \log_B(N) + S/B)$

LSM-tree in storage
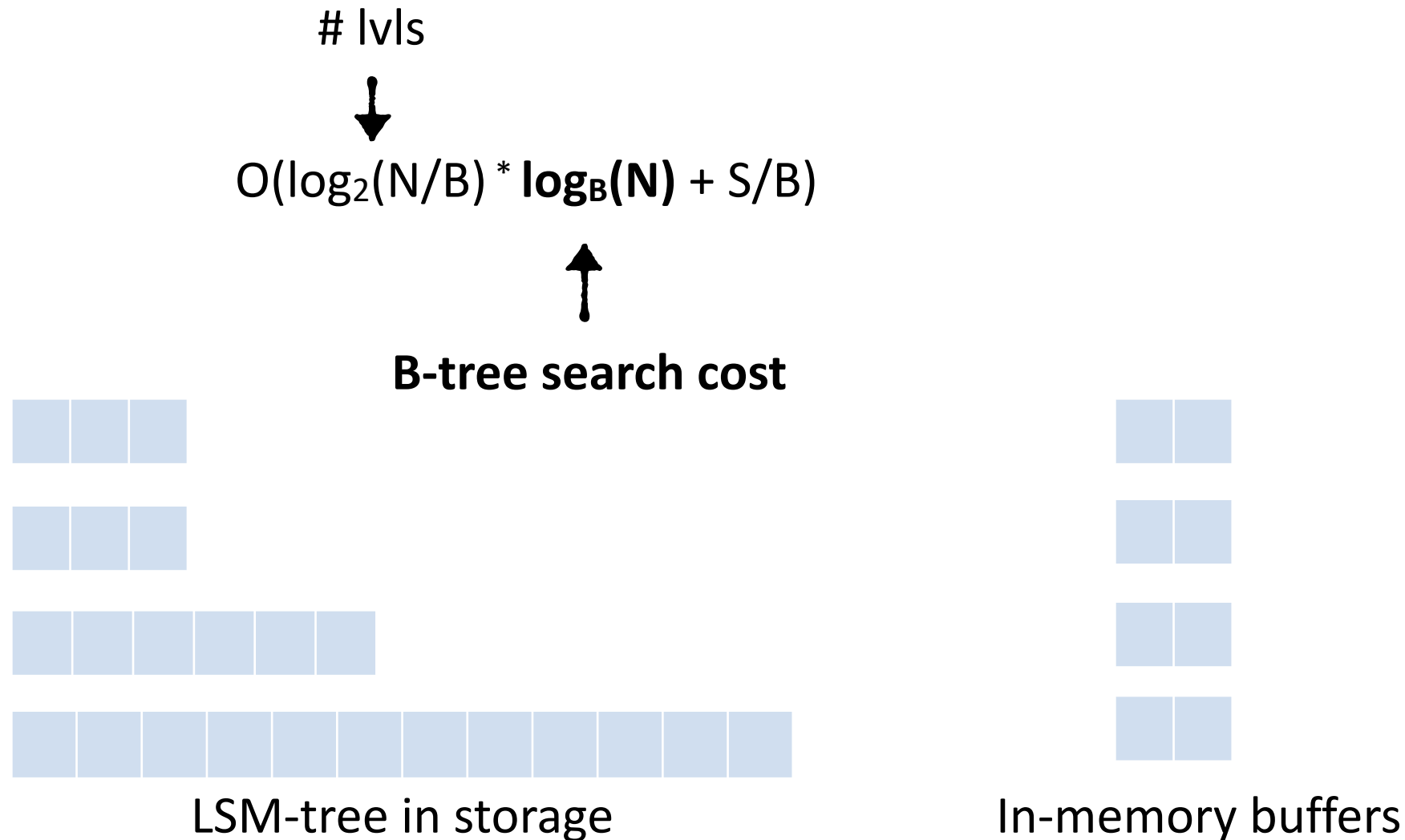
In-memory buffers

Basic LSM-tree – Scan Queries I/O Cost

\# lvls

$\downarrow$

$O(\log_2(N/B) * \mathbf{\log_B(N)} + S/B)$

$\uparrow$

B-tree search cost

LSM-tree in storage

In-memory buffers

Basic LSM-tree – Scan Queries I/O Cost

lvls   **# output size**

$$O(\log_2(N/B) * \log_B(N) + S/B)$$

B-tree search cost

LSM-tree in storage   In-memory buffers

Basic LSM-tree – Insertion/Update/Delete cost

How many times is each entry copied?

Level

Buffer 0

Sorted arrays 1

2

3

Basic LSM-tree – Insertion/Update/Delete cost

How many times is each entry copied?   **$O(\log_2 N/B)$**

Level

Buffer $\left\{ \begin{array}{c} \\ \end{array} \right.$ 0   … … …

Sorted arrays $\left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right.$

1   … … …

2   … … … … … …

3   … … … … … … … … … … … …

Basic LSM-tree – Insertion/Update/Delete cost

How many times is each entry copied? $O(\log_2 N/B)$

Price of each copy?

Level

Buffer   0   ... ...

Sorted
arrays

1   ... ...

2   ... ...

3   ... ...

Basic LSM-tree – Insertion/Update/Delete cost

How many times is each entry copied? $O(\log_2 N/B)$

Price of each copy? $O(1/B)$ reads & writes

Level

Buffer   0   … … …

1   … … …

Sorted arrays

2   … … … … … …

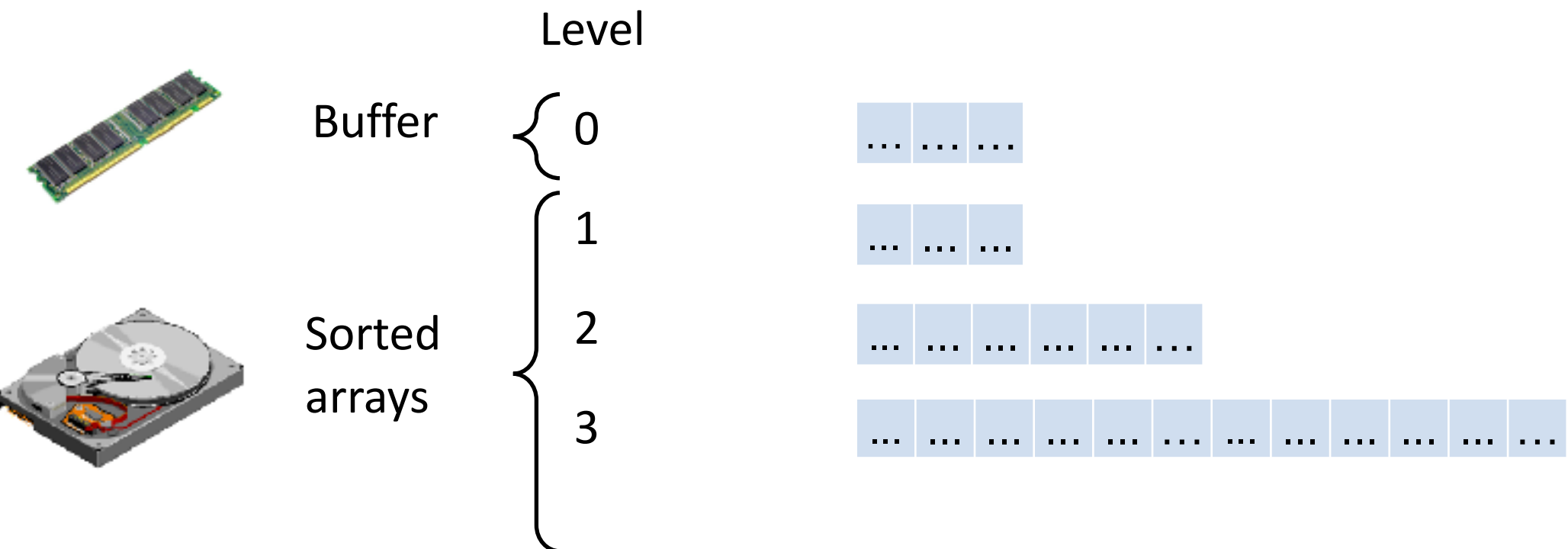3   … … … … … … … … … … … …
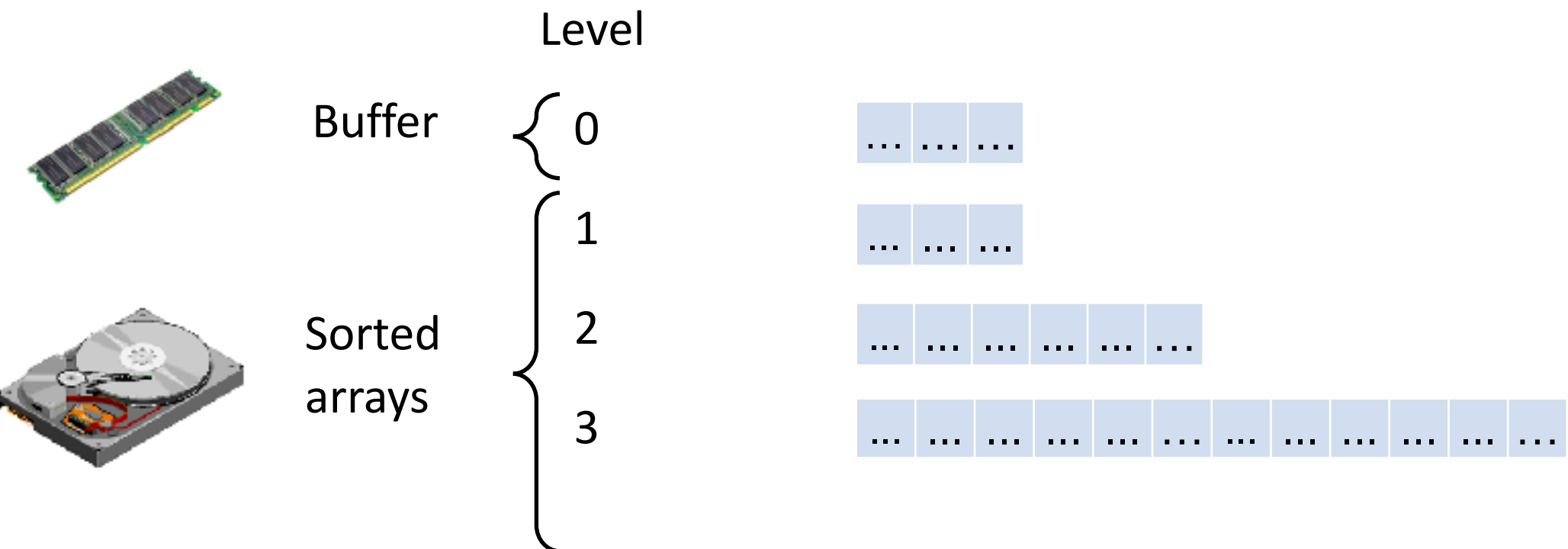
Basic LSM-tree – Insertion/Update/Delete cost

How many times is each entry copied? $O(\log_2 N/B)$

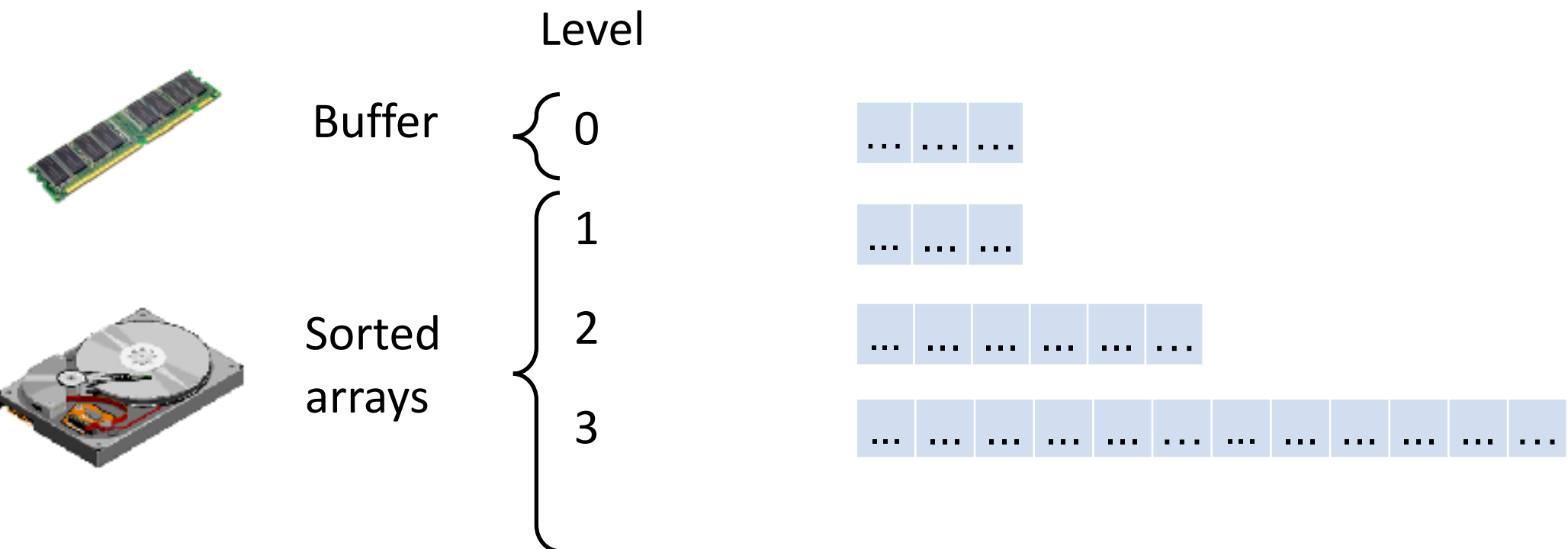**Price of each copy?** $O(1/B)$ reads & writes

Total cost: $O((\log_2 N/B)/B)$ read & write I/Os

Level

Buffer   0   … … …

Sorted arrays

1   … … …

2   … … … … … …

3   … … … … … … … … … … … …
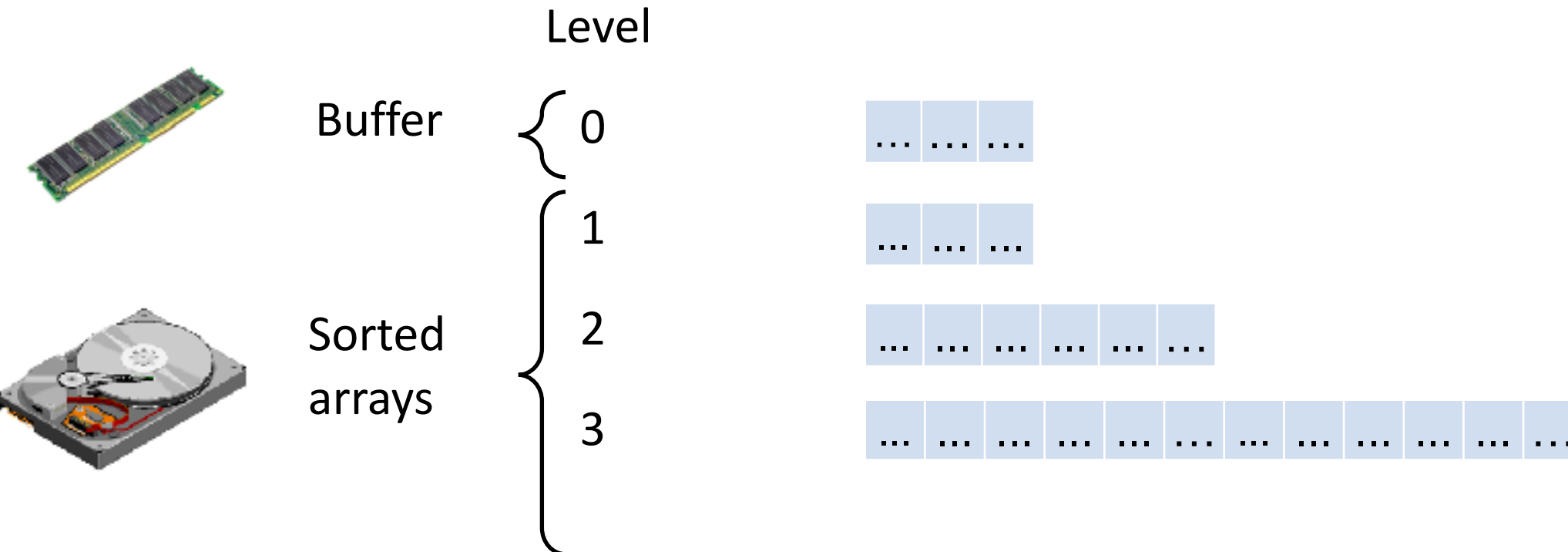
Basic LSM-tree – Insertion/Update/Delete cost

Total cost: $O((\log_2 N/B)/B)$ read & write I/Os

As all writes are large & sequential (rather than random), there is less SSD garbage-collection

Level

Buffer { 0   ... ...

Sorted arrays {

1   ... ...

2   ... ...

3   ... ...

Operation	I/O	append-only table	Basic LSM-tree table	B-Tree
Query	Reads	$O(N/B)$	$O(\log_2(N/B) * \log_B N)$	$O(\log_B N)$
Insert	Reads	0	$O((\log_2 N/B)/B)$	$O(\log_B N)$
	Writes	$O(1/B)$	$O((\log_2 N/B)/B)$	$O(1)$ & GC

Break :)
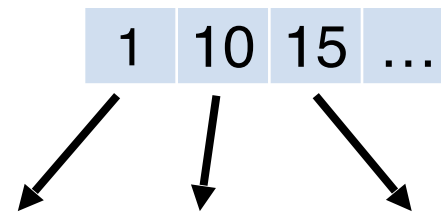
Declining Main Memory Cost
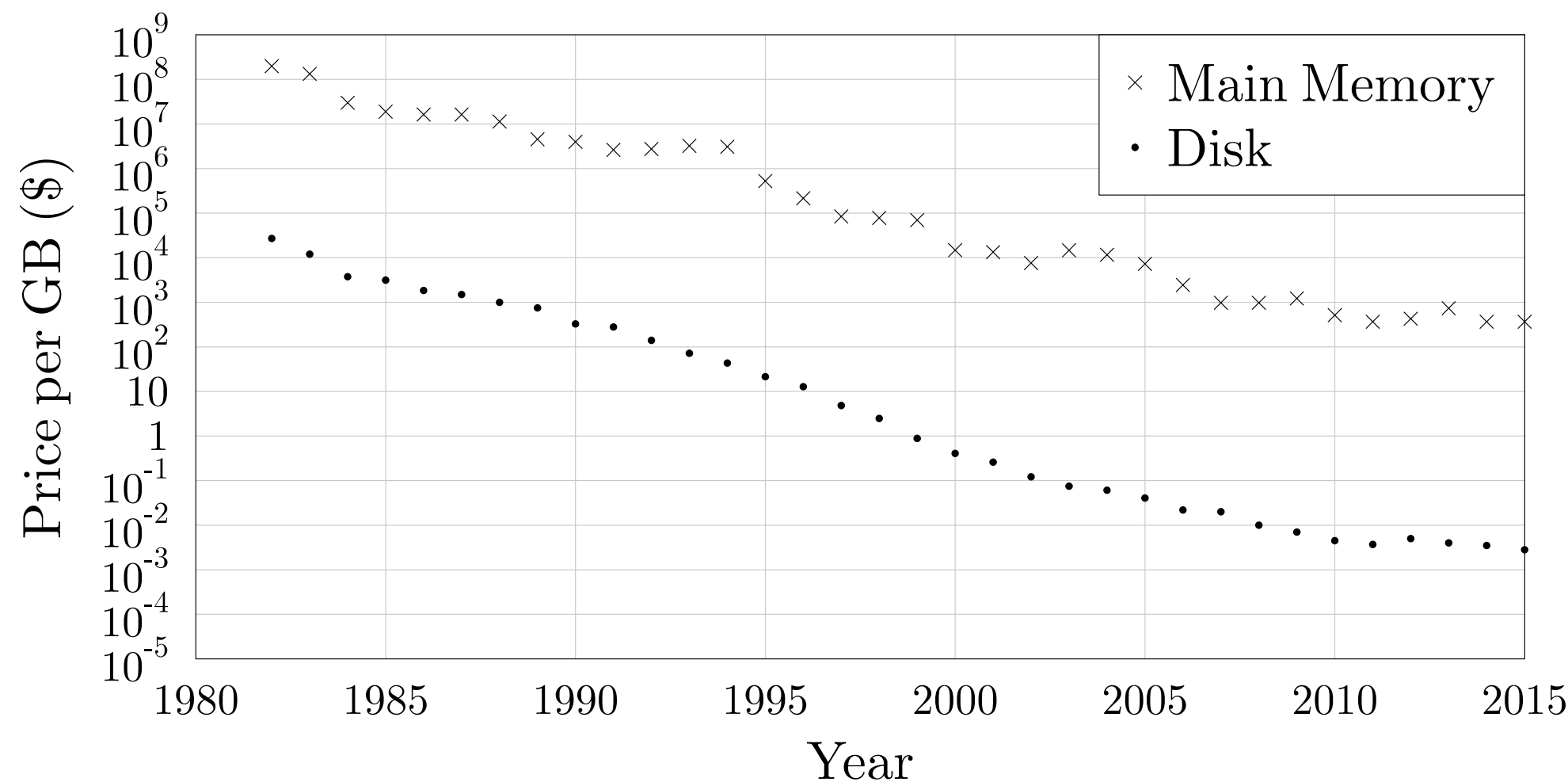
Declining Main Memory Cost

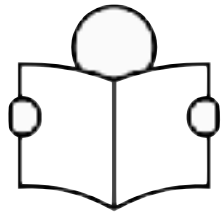It's viable to pin the internal nodes of B-trees in memory

Fence pointers

array

| 1 | 10 | 15 | ... |

Block 1	Block 2	Block 3	...
1	10	15	...
3	11	16	...
6	13	18	...

Operation	I/O	append-only table	**Basic LSM-tree table**	**B-Tree**
Query	Reads	$O(N/B)$	$O(\log_2(N/B) * ~~\log_B N~~)$	$O(~~\log_B N~~)$
Insert	Reads	0	$O((\log_2 N/B)/B)$	$O(\log_B N)$
	Writes	$O(1/B)$	$O((\log_2 N/B)/B)$	$O(1)$ & GC

Operation	I/O	append-only table	Basic LSM-tree table	B-Tree
Query	Reads	$O(N/B)$	$O(\log_2 N)$	$O(1)$
Insert	Reads	0	$O((\log_2 N/B)/B)$	$O(\log_B N)$
	Writes	$O(1/B)$	$O((\log_2 N/B)/B)$	$O(1)$ & GC
Memory		$O(B)$	$O(N/B)$	$O(N/B)$

Leveled LSM-tree   Basic LSM-tree   Tiered LSM-tree
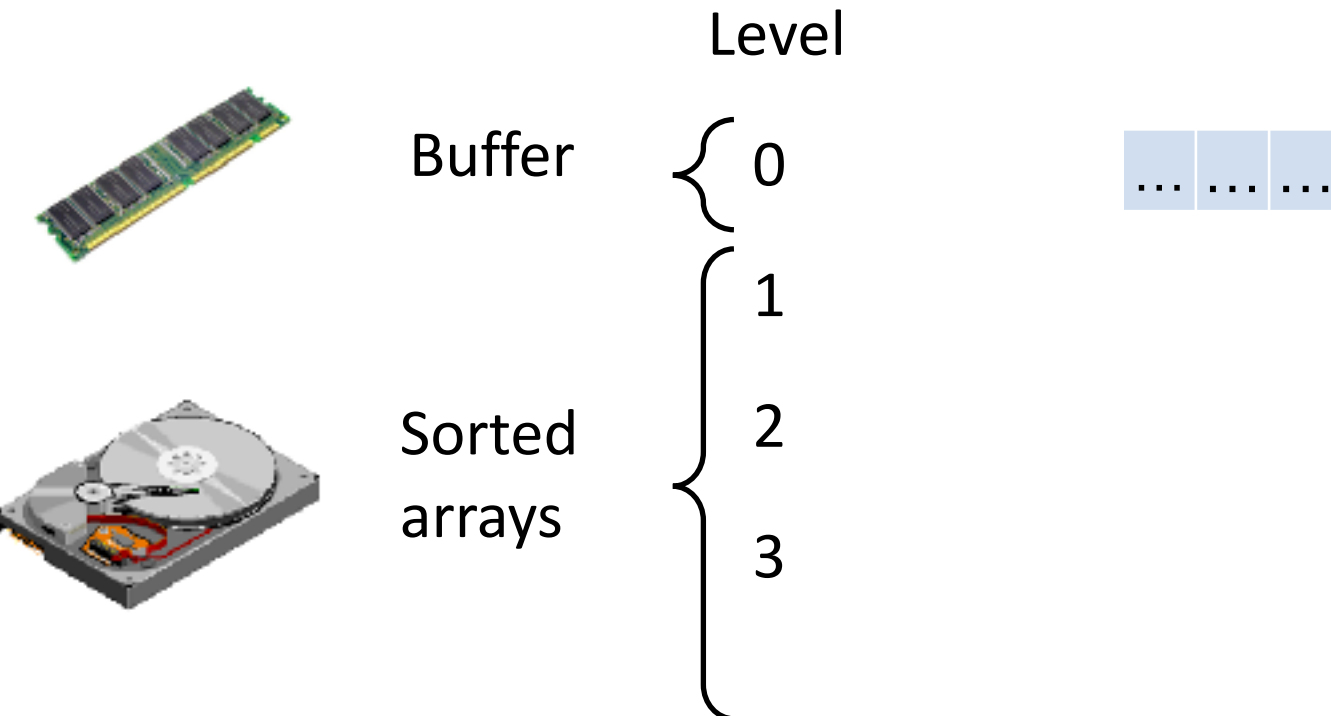
Leveled LSM-tree

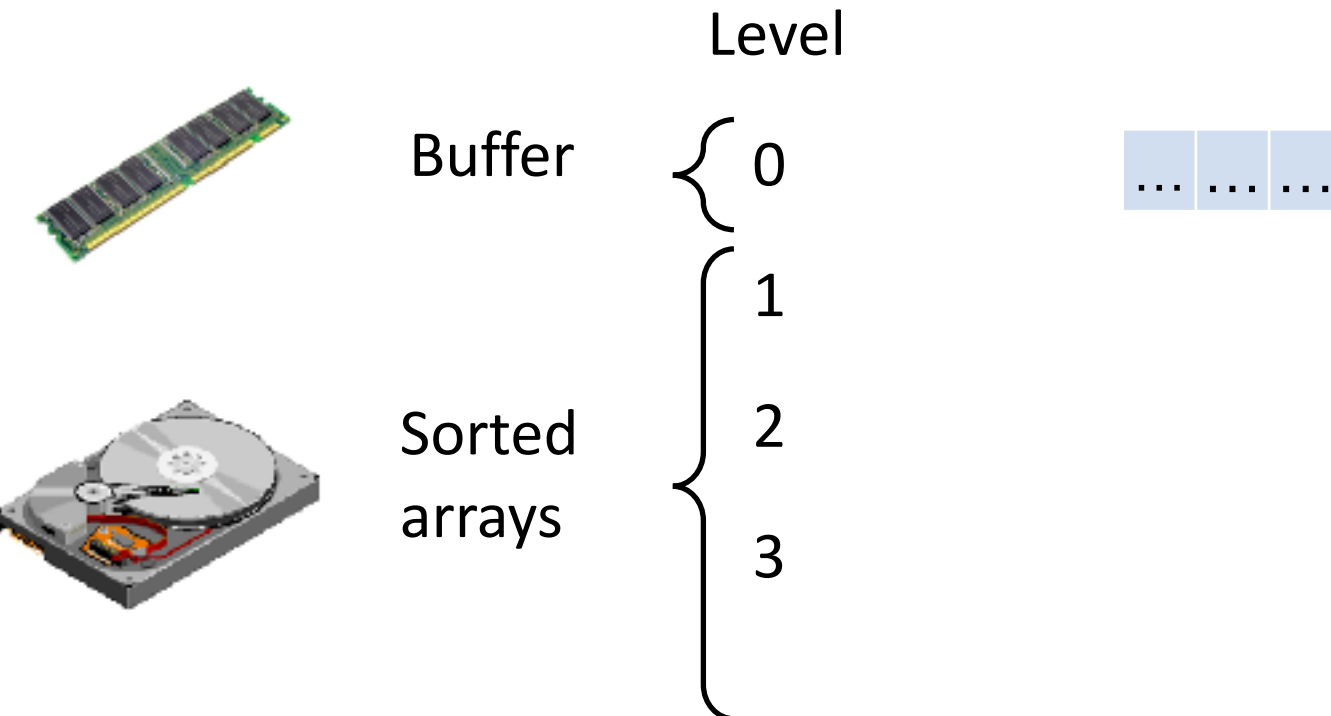⬇ Lookup cost ⬆ Update cost

Leveled LSM-tree

Lookup cost depends on number of levels

Level

Buffer $\quad$ 0 $\qquad$ … … …

Sorted
arrays $\quad$ 1

2

3

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?

Level

Buffer { 0   ... ...

Sorted { 1
arrays   2
         3

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T

Level

Buffer   0   ... ...

          1

Sorted   2
arrays
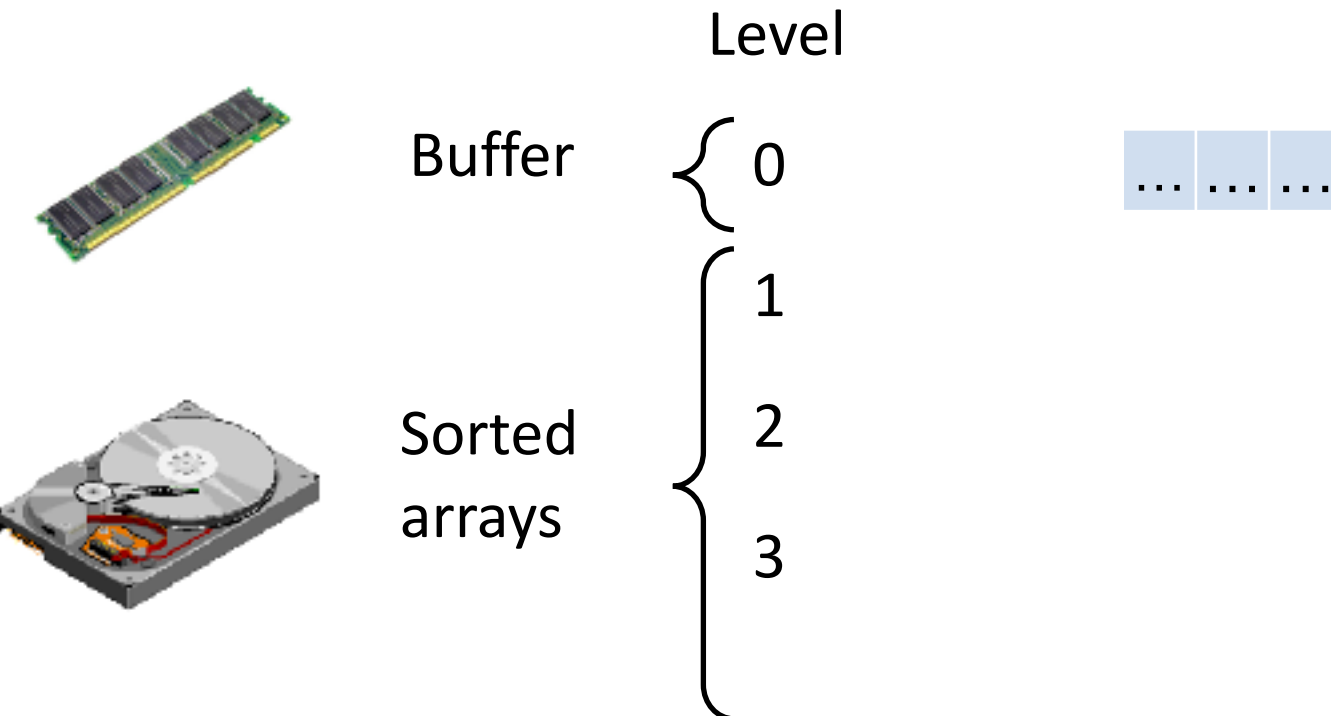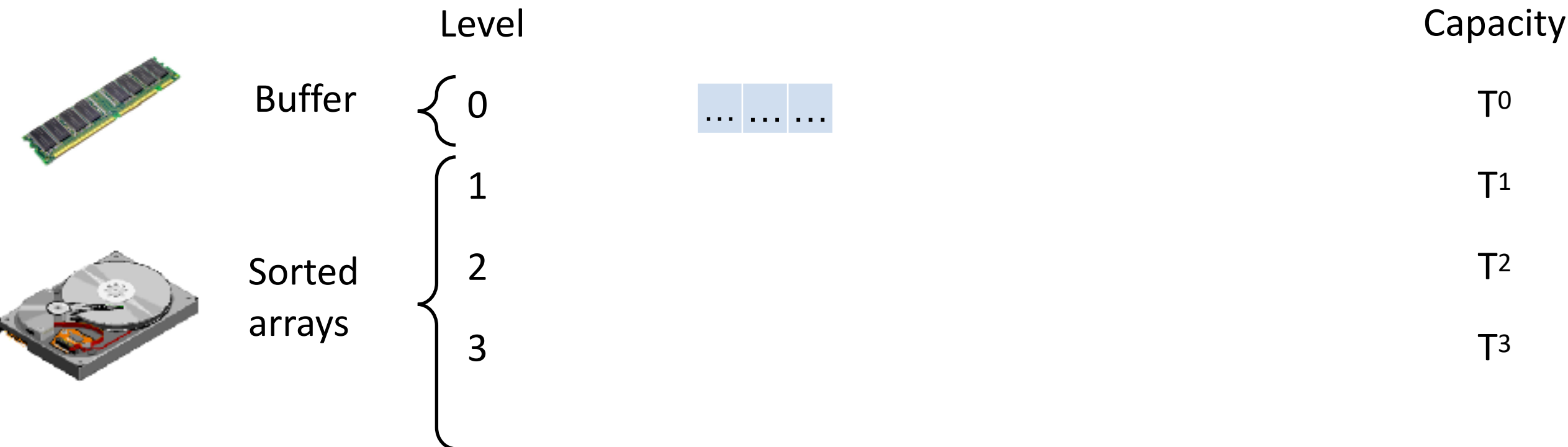   3

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T

Level   Capacity

Buffer   0   ... ... T^0

                1 T^1

Sorted   2 T^2
arrays
                3 T^3

Leveled LSM-tree

Lookup cost depends on number of levels
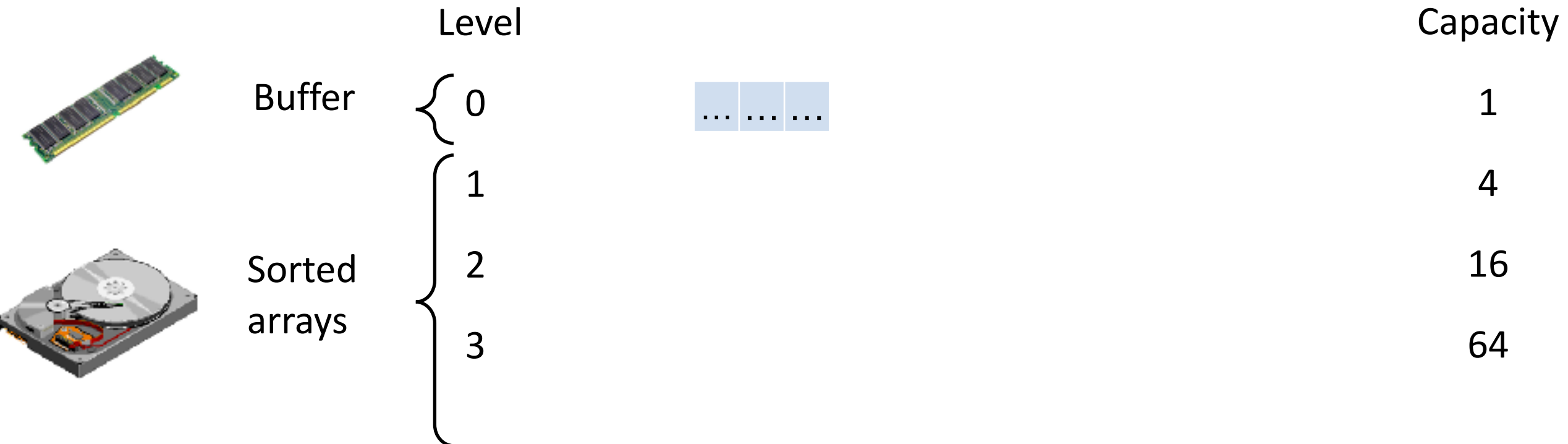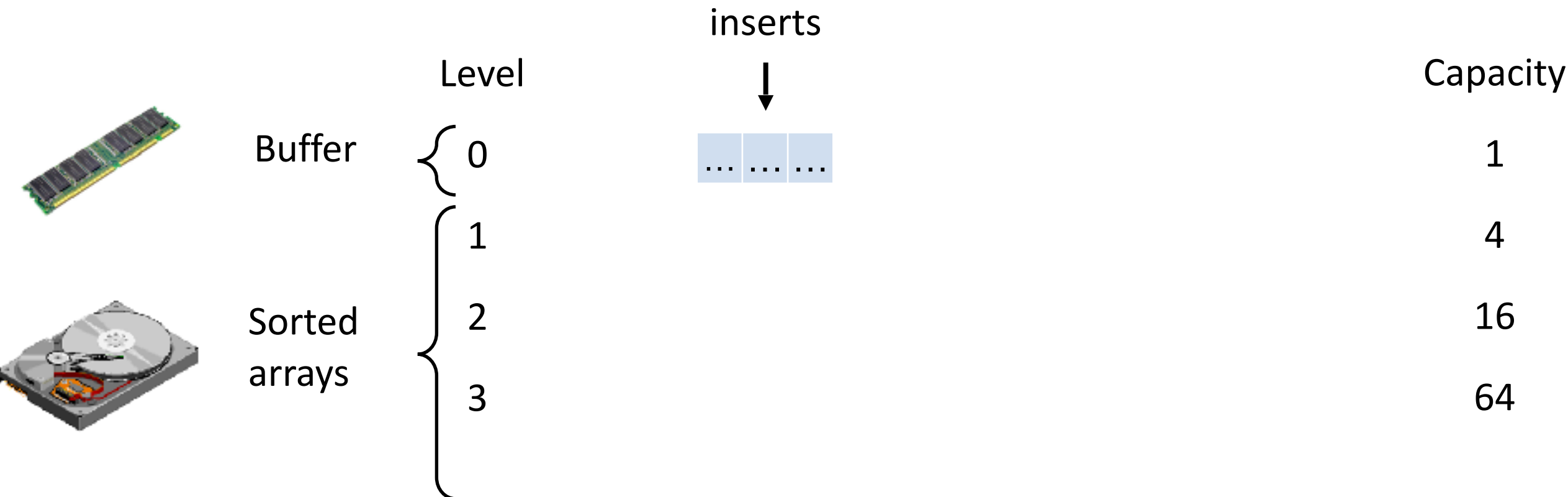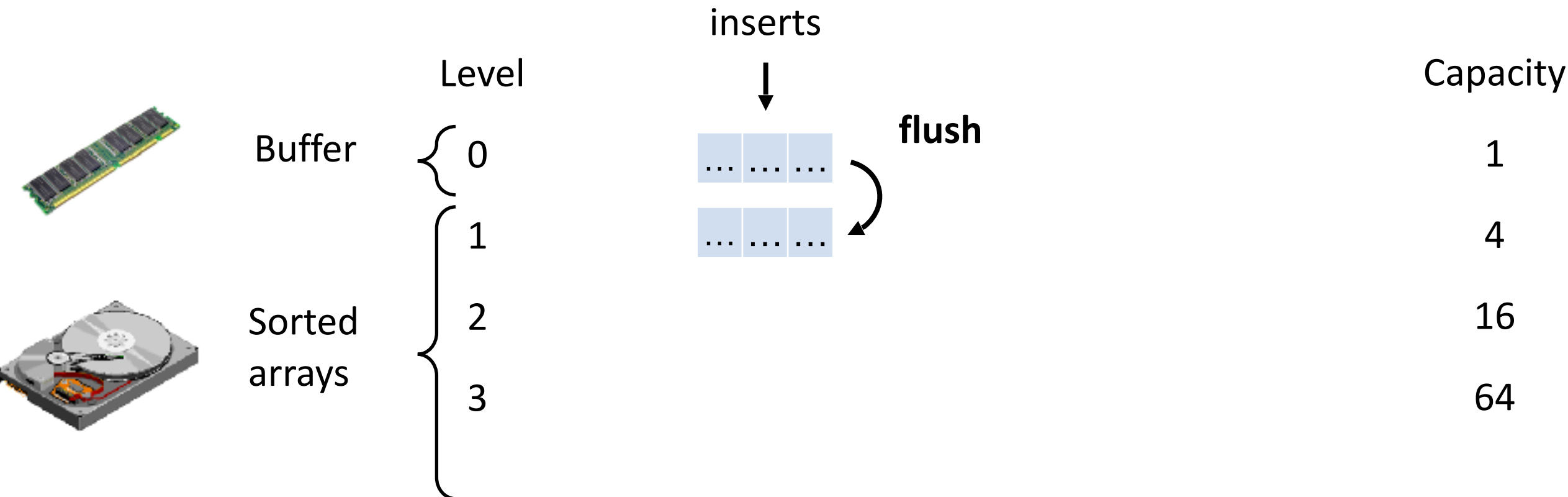How to reduce it?   Increase size ratio T
E.g. size ratio of 4

	Level		Capacity
Buffer	0	… … …	1
Sorted arrays	1		4
	2		16
	3		64

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

	Level		Capacity
Buffer	0	… … …	1
Sorted arrays	1		4
	2		16
	3		64

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1	... ...	4
	2		16
	3		64

flush

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

Level

Capacity

flush & sort-merge

Buffer   0   … … …   1

          1   … … … … … …   4

Sorted
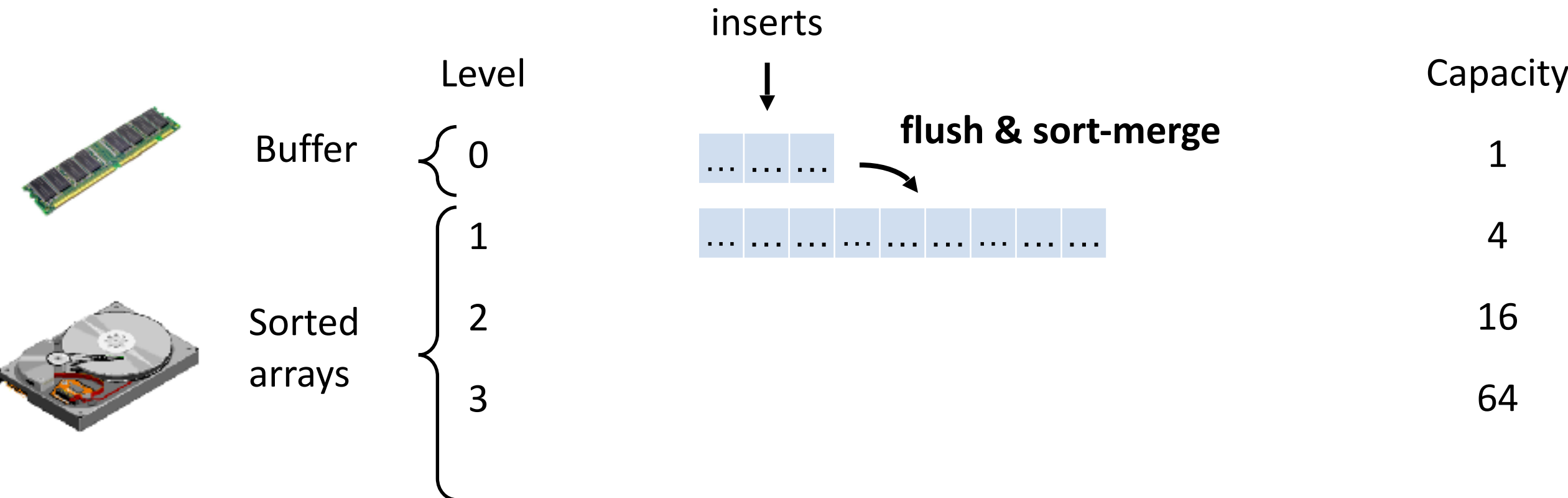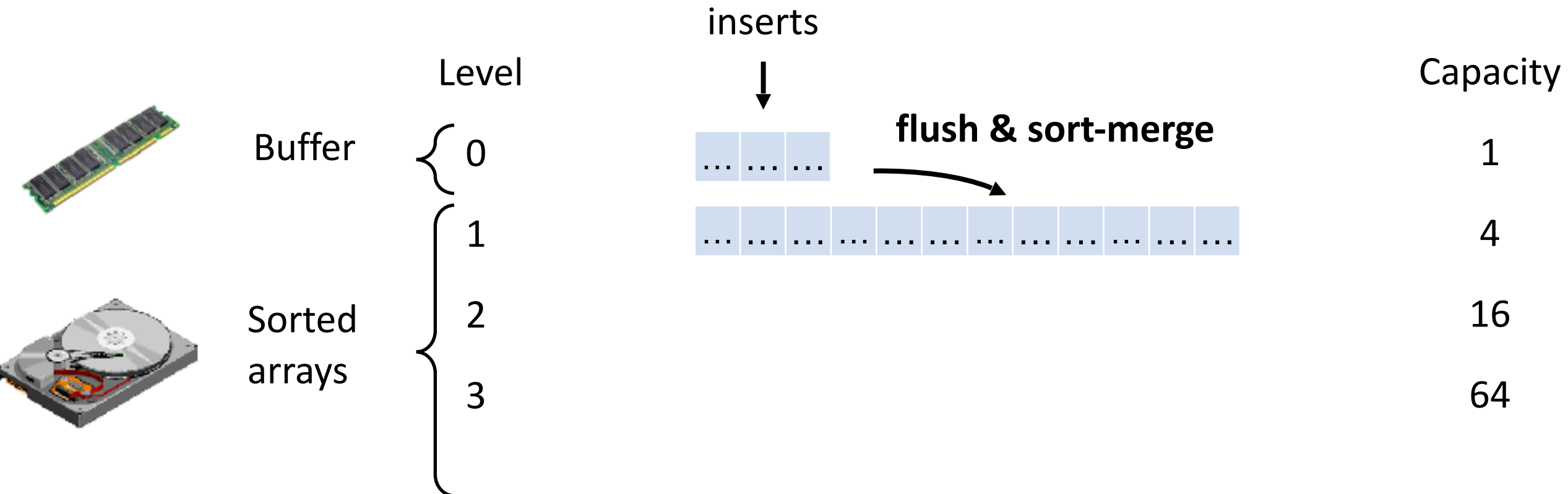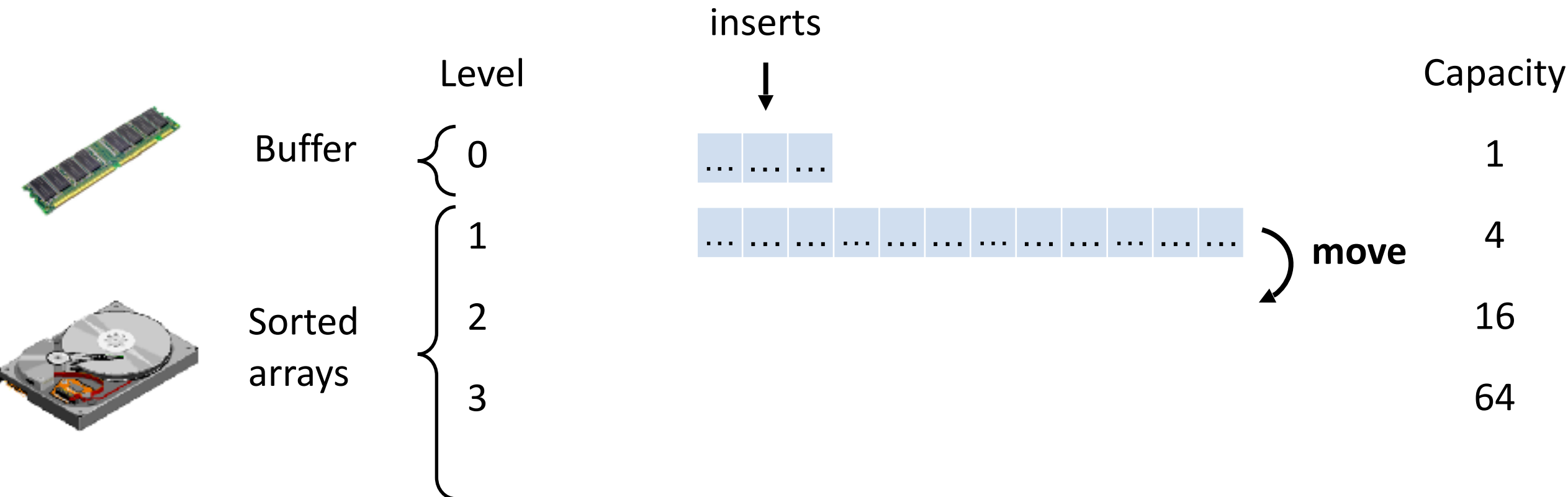arrays   2   16

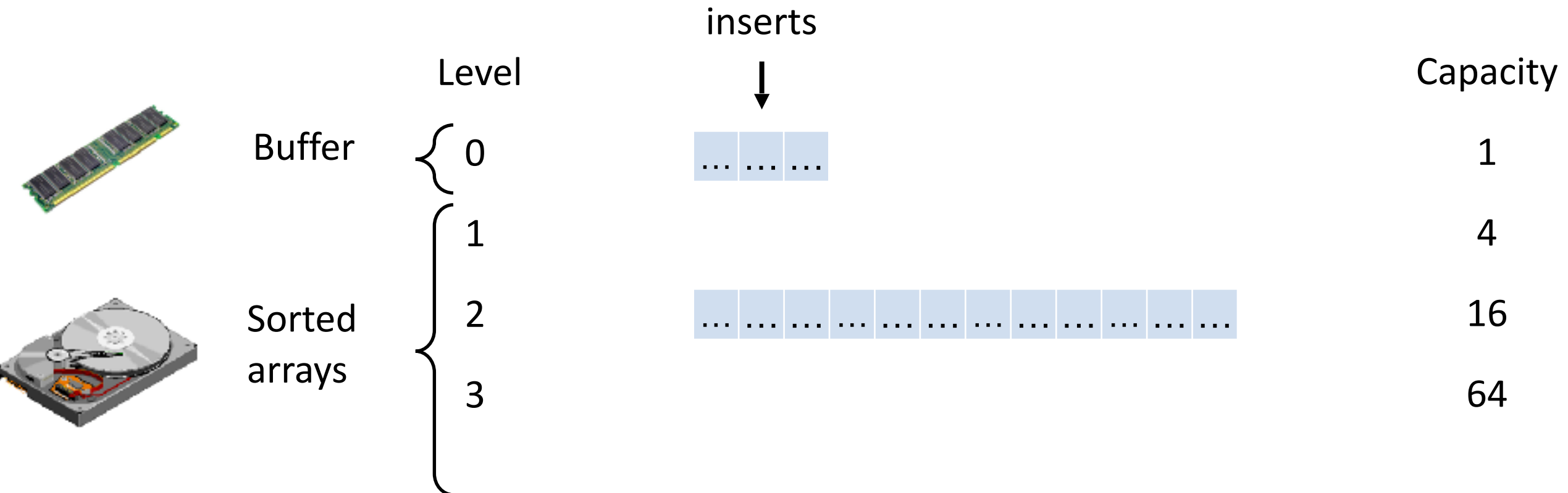   3   64

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

	Level			Capacity
Buffer	0	… … …	**flush & sort-merge**	1
Sorted arrays	1	… … … … … … … … …		4
	2			16
	3			64

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

	Level			Capacity
Buffer	0	... ...	**flush & sort-merge**	1
Sorted arrays	1	... ...		4
	2			16
	3			64

Leveled LSM-tree

Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

Level   Capacity

Buffer   0   1

         1   ... ...) **move**   4

Sorted
arrays   2   16

         3   64

Leveled LSM-tree

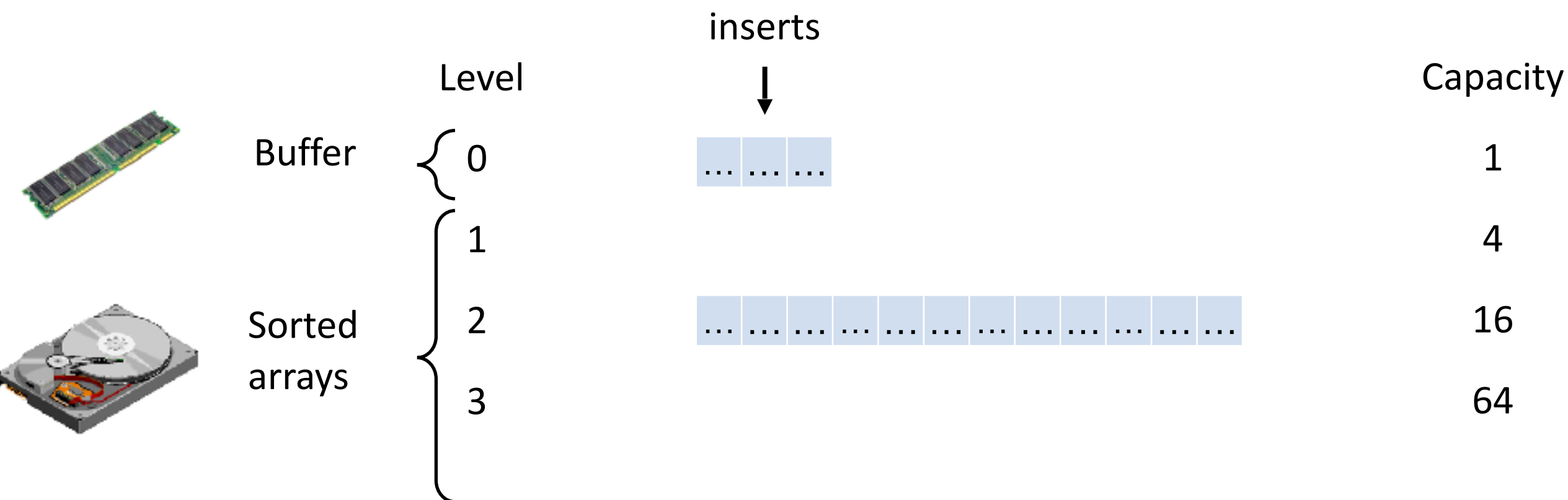Lookup cost depends on number of levels
How to reduce it?   Increase size ratio T
E.g. size ratio of 4

inserts

	Level		Capacity
Buffer	0	... ...	1
	1		4
Sorted arrays	2	... ...	16
	3		64

Leveled LSM-tree

Lookup cost?

inserts

Level

Capacity

Buffer   0   … … …   1

        1   4

Sorted   2   … … … … … … … … … … … …   16

arrays   3   64

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

inserts

	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1		4
	2	... ...	16
	3		64

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?

inserts

	Level		Capacity
Buffer	0	... ...	1
	1		4
Sorted arrays	2	... ...	16
	3		64

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?
$O(T/B \bullet \log_T(N/B))$

inserts

	Level		Capacity
Buffer	0	... ...	1
	1		4
Sorted arrays	2	... ...	16
	3		64

Leveled LSM-tree

Lookup cost?

$O(\log_T(N/B))$

Insertion cost?

$O(T/B \bullet \log_T (N/B))$

What happens as we increase the size ratio T?

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?
$O(T/B \bullet \log_T(N/B))$

What happens as we increase the size ratio T?

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?
$O(T/B \bullet \log_T(N/B))$

What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?
$O(T/B \bullet \log_T (N/B))$
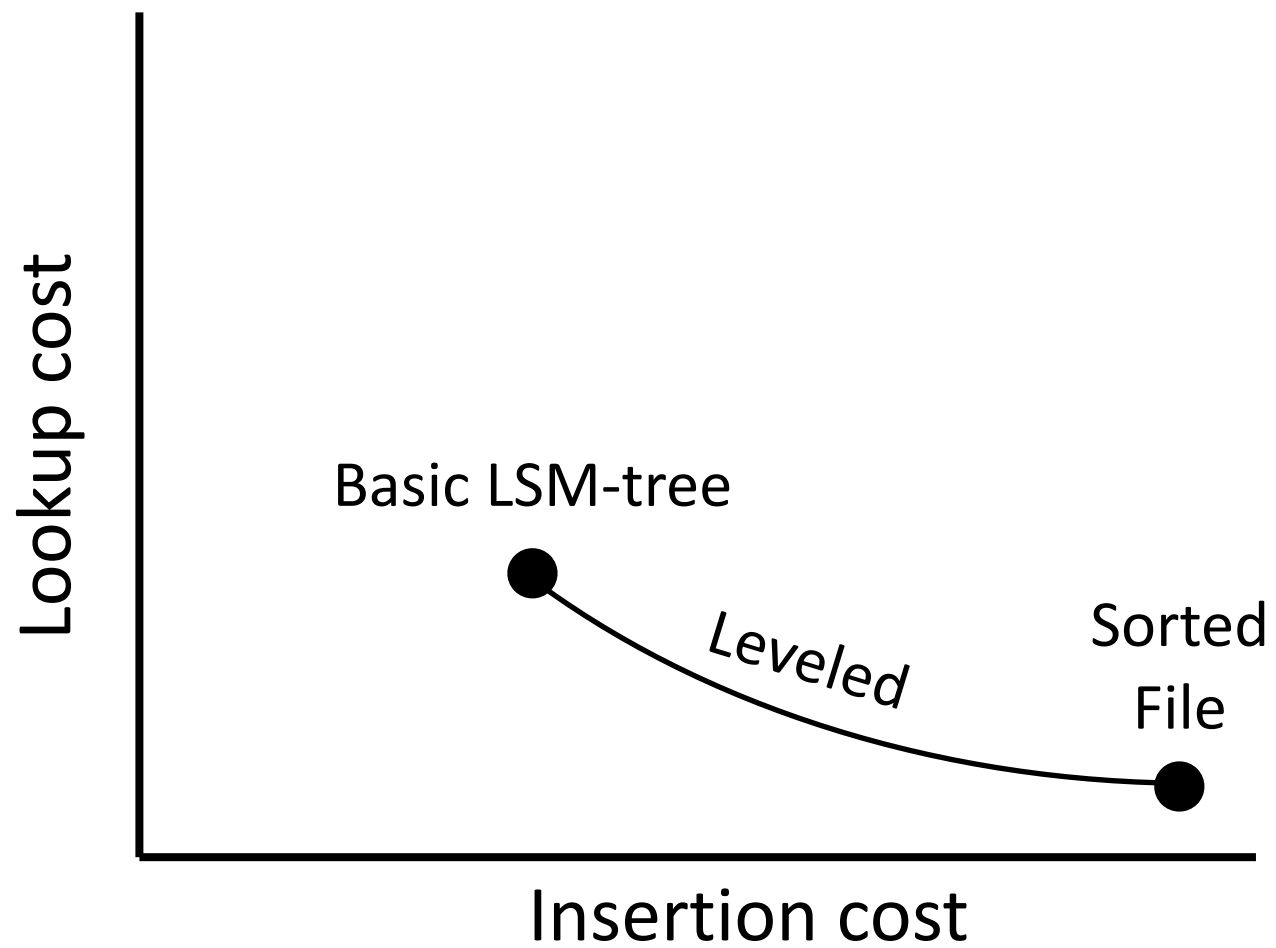
What happens as we increase the size ratio T?

What happens when size ratio T is set to be N/B?

Lookup cost becomes:
$O(1)$

Insert cost becomes:
$O(N/B^2)$

Leveled LSM-tree

Lookup cost?
$O(\log_T(N/B))$

Insertion cost?
$O(T/B \cdot \log_T(N/B))$

What happens as we increase the size ratio T?
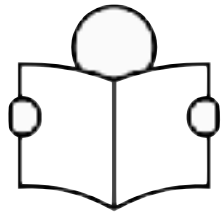
What happens when size ratio T is set to be N/B?

Lookup cost becomes:
$O(1)$

Insert cost becomes:
$O(N/B^2)$

The LSM-tree becomes a sorted file!

Leveled LSM-tree
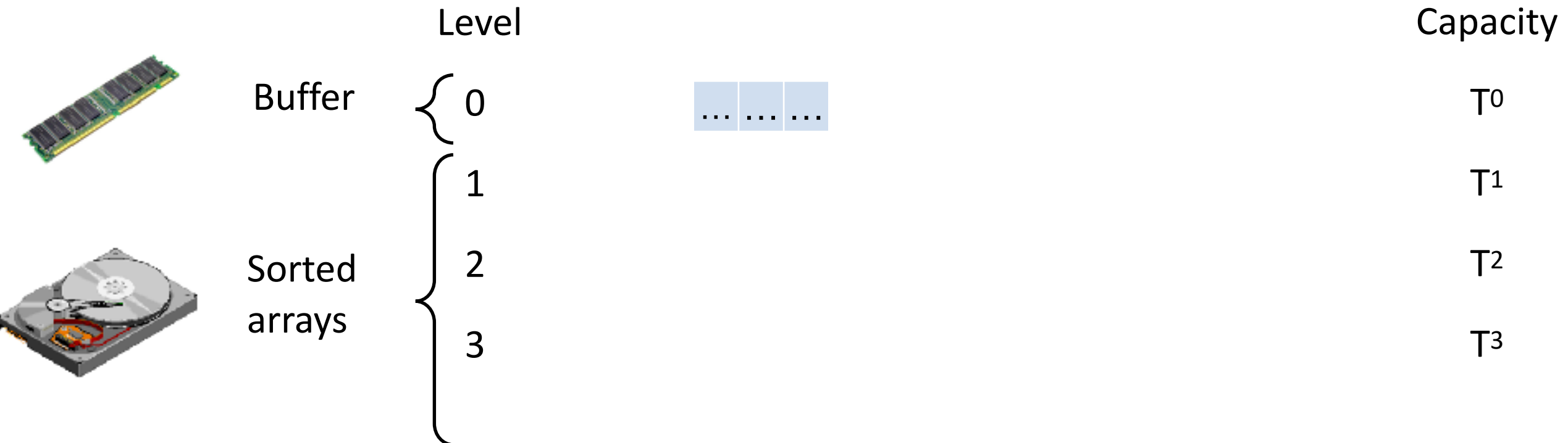
Basic LSM-tree

Tiered LSM-tree

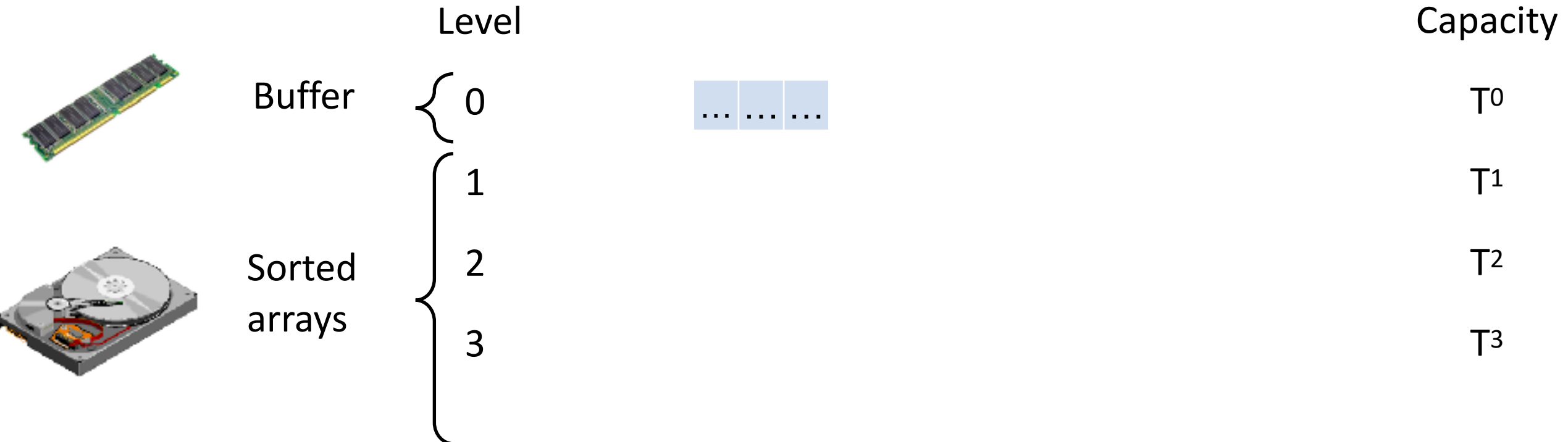# Tiered LSM-tree

↑ Lookup cost   ↓ Insertion cost

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.

Level

Capacity

Buffer 0 ... ... T^0

1 T^1

Sorted 2 T^2
arrays

3 T^3

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.

Level   Capacity

Buffer   0   ... ... T^0

          1 T^1

Sorted
          2 T^2
arrays
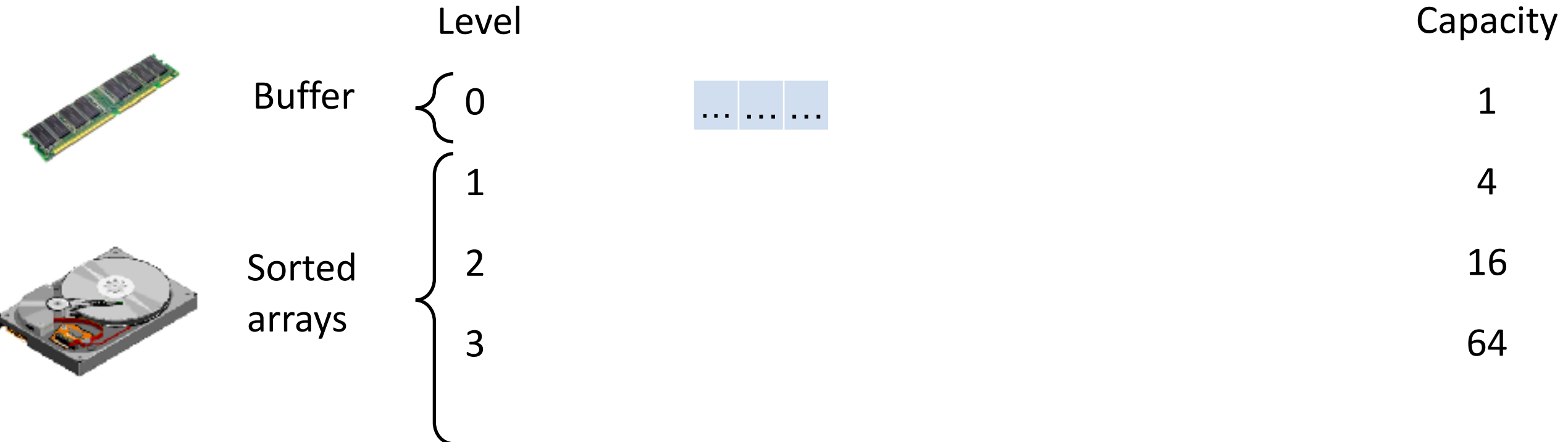
          3 T^3

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

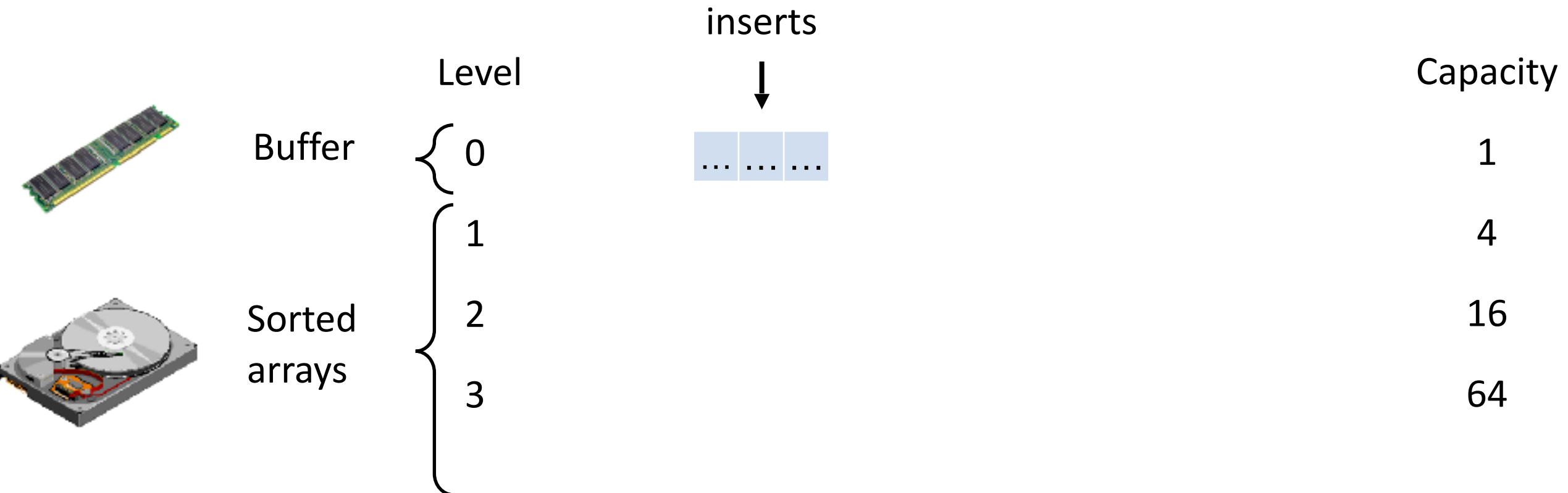|	Level		Capacity
Buffer	0	… … …	1
	1		4
Sorted arrays	2		16
	3		64

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level | Capacity

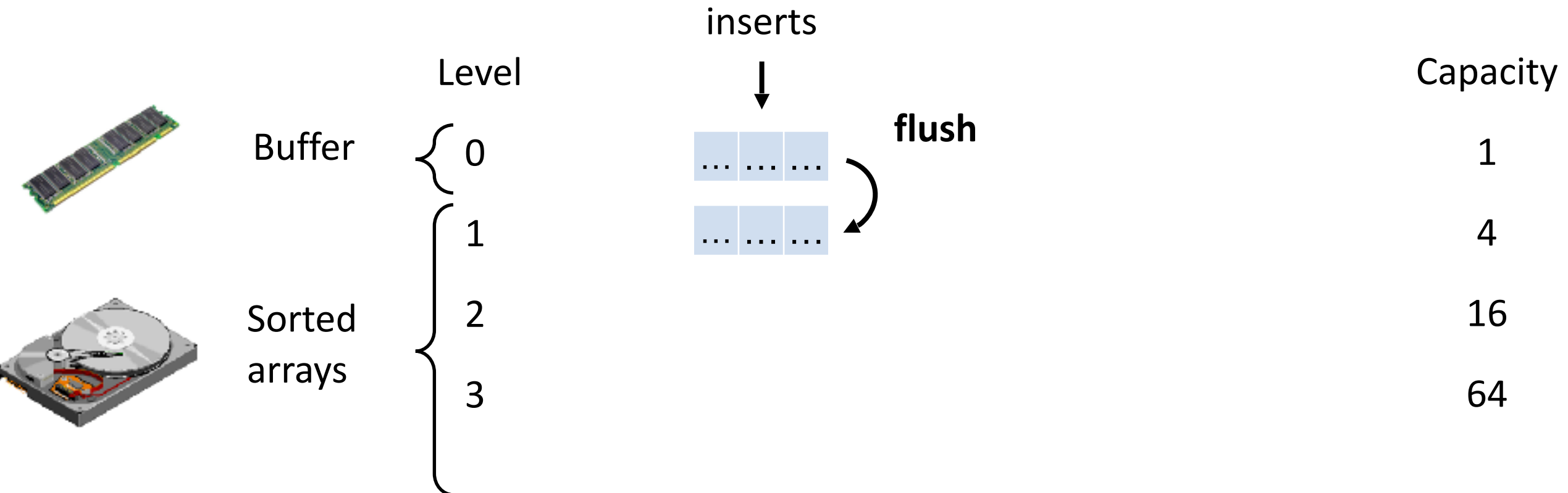Buffer   0   … … …   1

Sorted   1   4
arrays   2   16
          3   64

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level

Capacity

Buffer   0   ... ...   **flush**   1

     1   4

Sorted   2   16

arrays   3   64

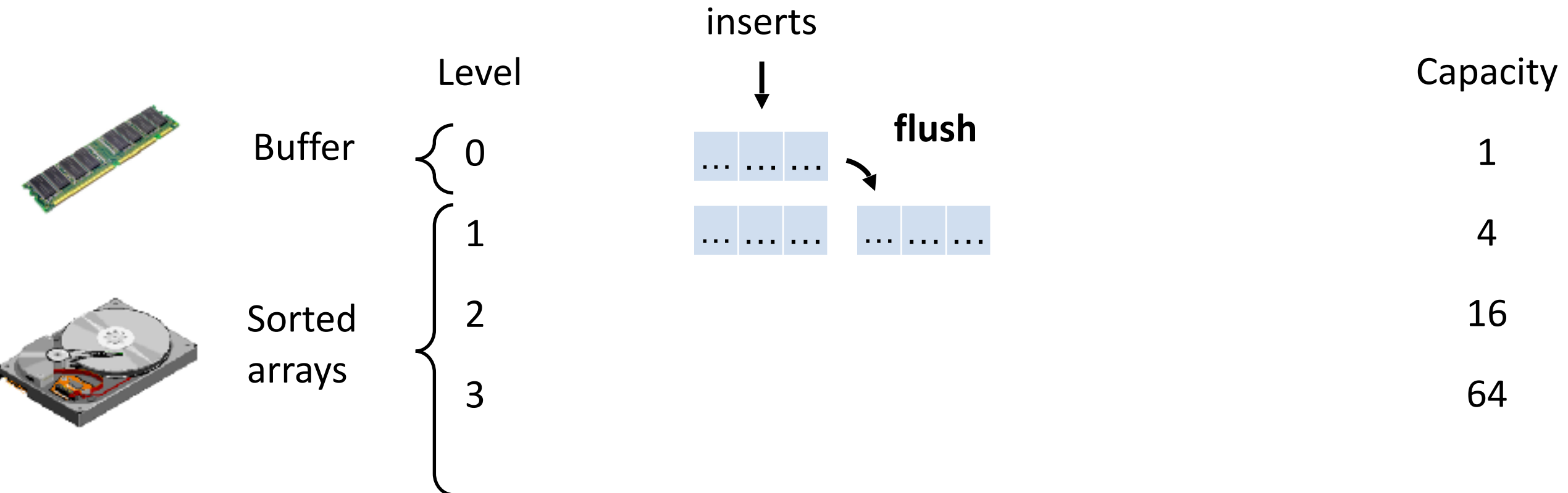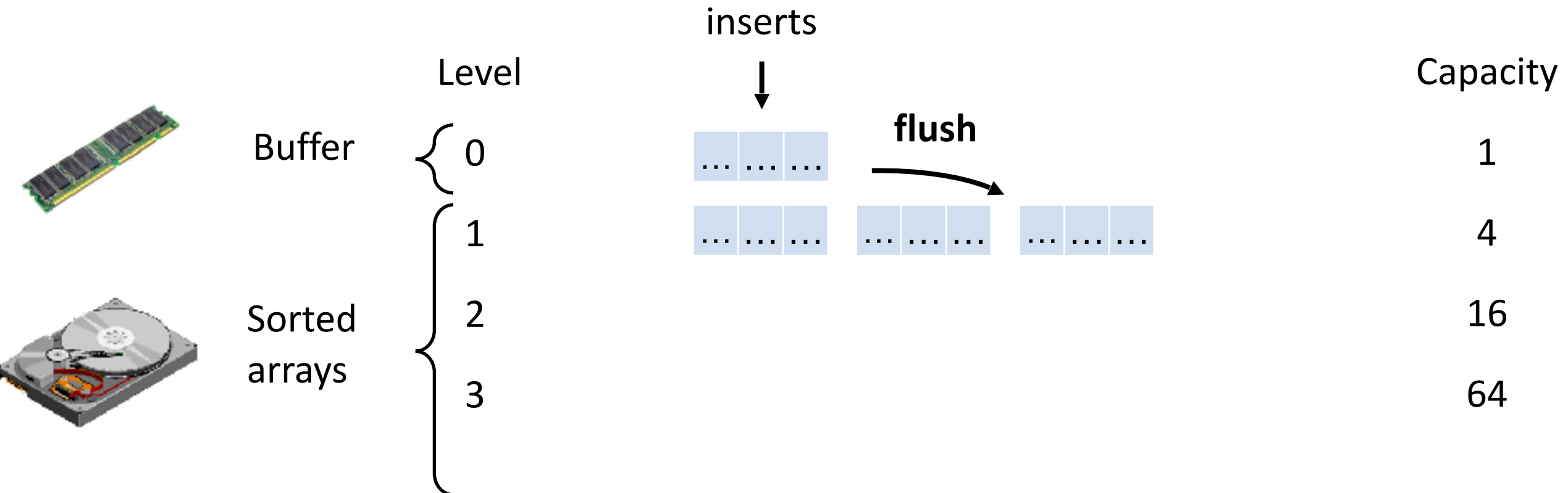# Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level   Capacity

Buffer   0   ... ...   **flush**   1

            1   4

Sorted   2   16
arrays
            3   64

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
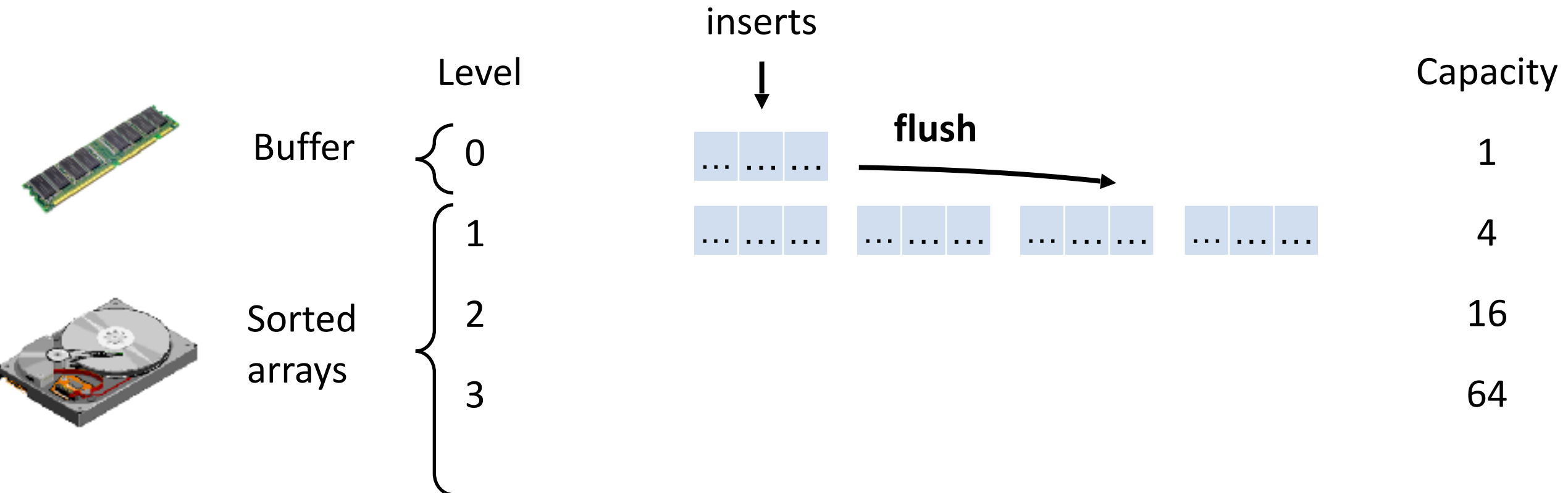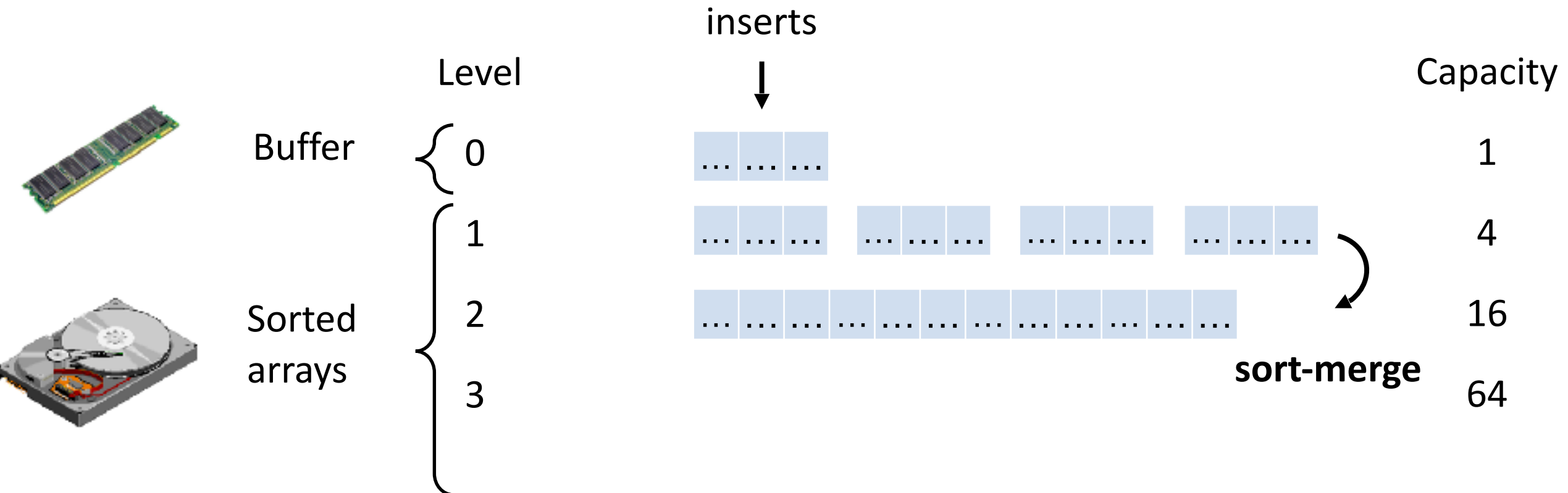Do not merge within a level.
E.g. size ratio of 4

inserts

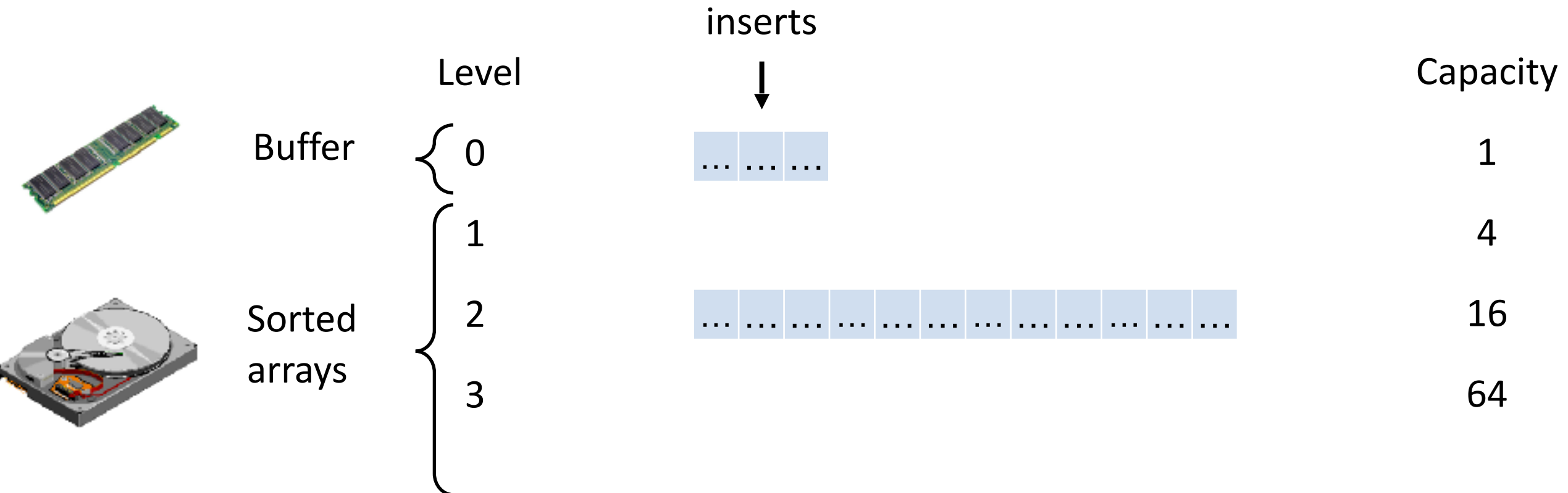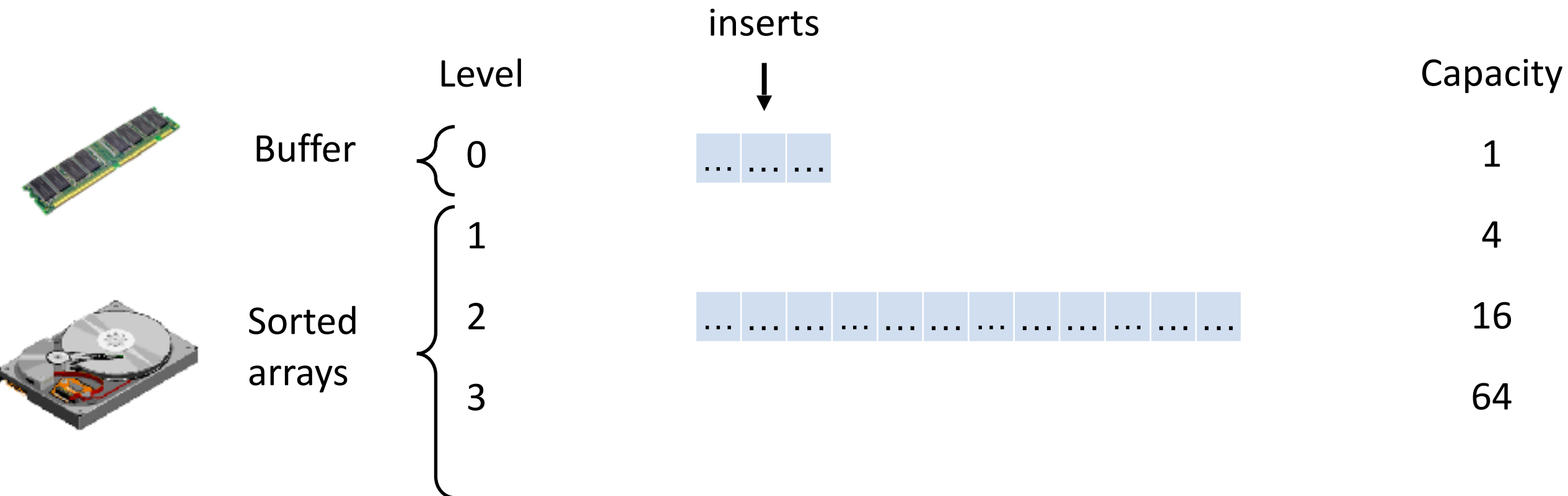| Level		Capacity
Buffer	0	1
Sorted arrays	1	4
	2	16
	3	64

flush

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level		Capacity
Buffer	0	1
Sorted arrays	1	4
	2	16
	3	64

flush

... ...

... ...

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level

Capacity

Buffer 0 1

Sorted 1 4

2 16

arrays 3 **sort-merge** 64

Tiered LSM-tree

Reduce the number of levels by increasing the size ratio.
Do not merge within a level.
E.g. size ratio of 4

inserts

Level		Capacity
Buffer 0	... ...	1
1		4
Sorted arrays 2	... ...	16
3		64

Tiered LSM-tree

Lookup cost?

inserts

Level

Capacity

Buffer $\left\{ \begin{array}{c} 0 \end{array} \right.$ 1

Sorted arrays $\left\{ \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \end{array} \right.$

4

... ... 16

64

Tiered LSM-tree

Lookup cost?

$O(T \bullet \log_T(N/B))$

inserts

	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1		4
	2	... ...	16
	3		64

Tiered LSM-tree

Lookup cost?
$O(T \bullet \log_T(N/B))$

Insertion cost?

inserts

	Level		Capacity
Buffer	0	... ...	1
Sorted arrays	1		4
	2	... ...	16
	3		64

Tiered LSM-tree

Lookup cost?
$O(T \bullet \log_T(N/B))$

Insertion cost?
$O(1/B \bullet \log_T(N/B))$

inserts
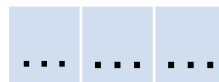
Level

Capacity

Buffer   0   1

1   4

Sorted arrays   2   16

3   64

Tiered LSM-tree

Lookup cost?

$O(T \cdot \log_T(N/B))$

Insertion cost?

$O(1/B \cdot \log_T(N/B))$

What happens as we increase the size ratio T?

Tiered LSM-tree

Lookup cost?

$O(T \bullet \log_T(N/B))$

Insertion cost?

$O(1/B \bullet \log_T (N/B))$

What happens as we increase the size ratio T?

Tiered LSM-tree

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The tiered LSM-tree becomes an append-only file!
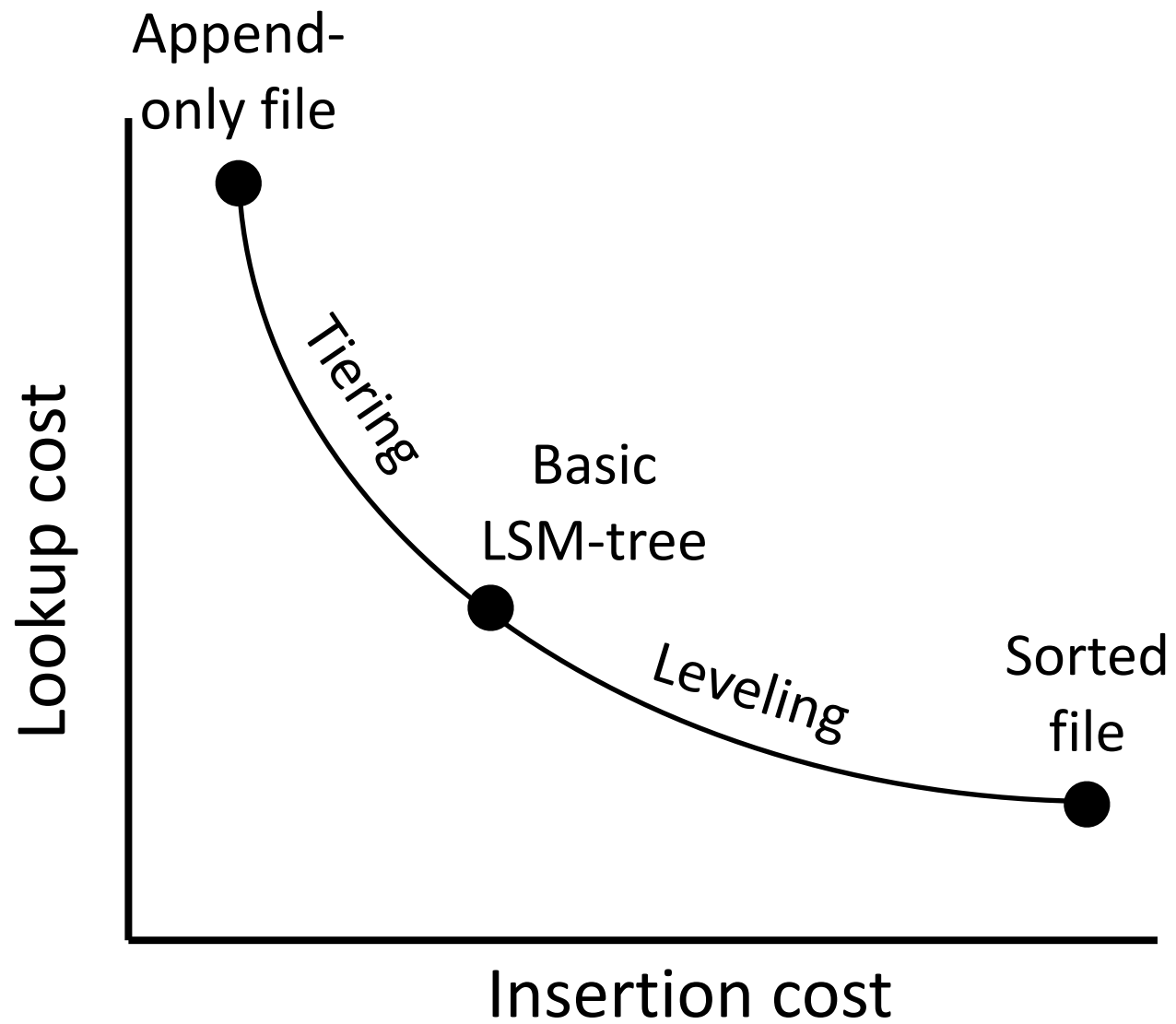
Operation		Unordered File	Leveled LSM-tree	Tiered LSM-tree	B-Tree
Query	Reads	$O(N/B)$	$O(L)$	$O(L * T)$	$O(1)$
Insert	Reads	0	$O((L * T)/B)$	$O(L/B)$	$O(\log_B N)$
	Writes	$O(1/B)$	$O((L * T)/B)$	$O(L/B)$	$O(1)$ & GC
Memory		$O(B)$	$O(N/B)$	$O(N/B)$	$O(N/B)$

We let $L = \log_T(N/B)$

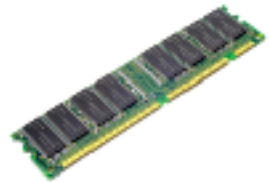# Bloom filters

Declining Main Memory Cost
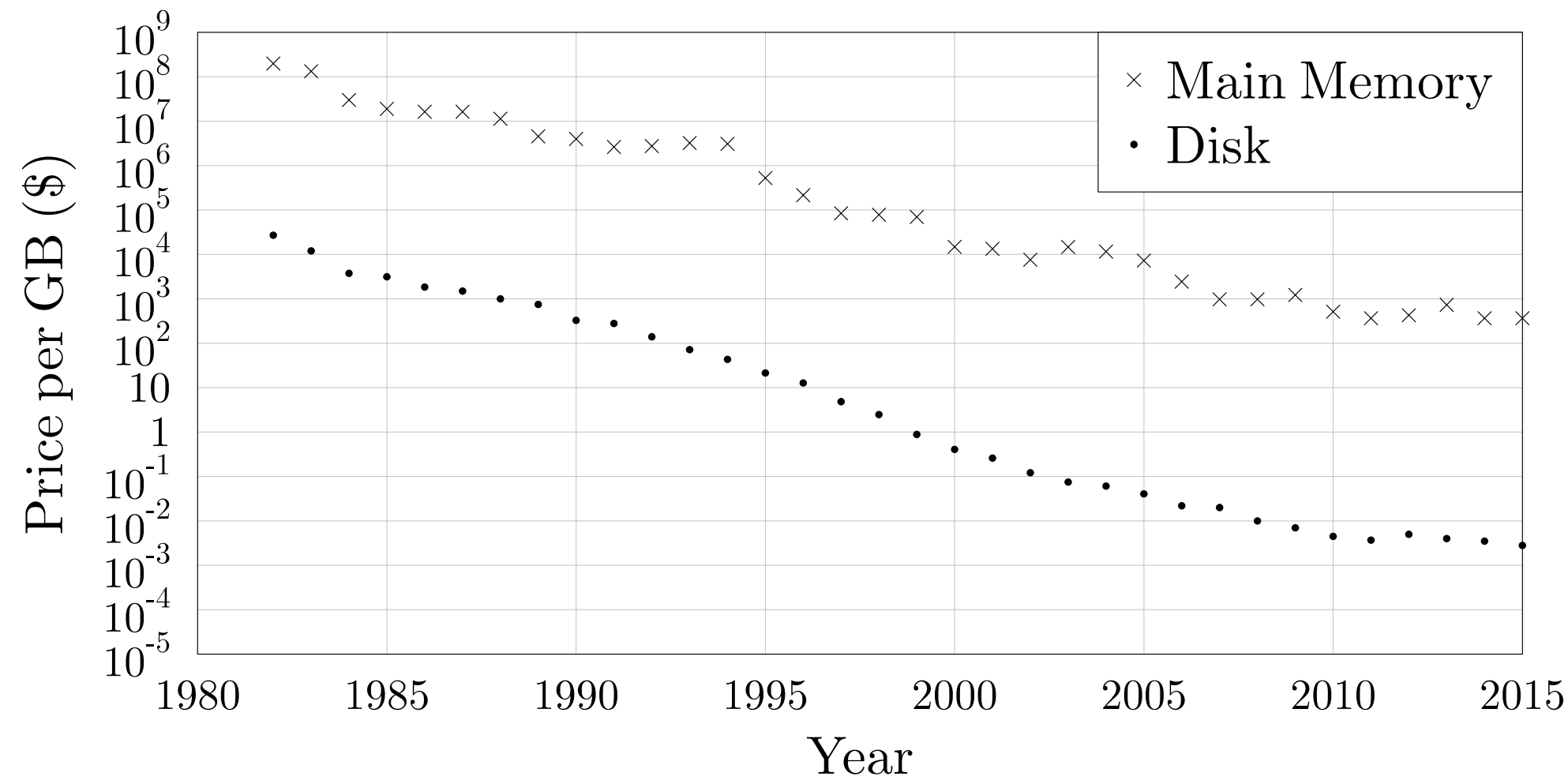
Bloom Filters
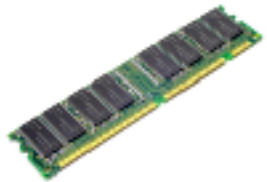
Answers set-membership queries
Smaller than data it represents
Purpose: avoid accessing storage if entry is not present
Subtlety: may return false positives.

filters {

Bloom filter

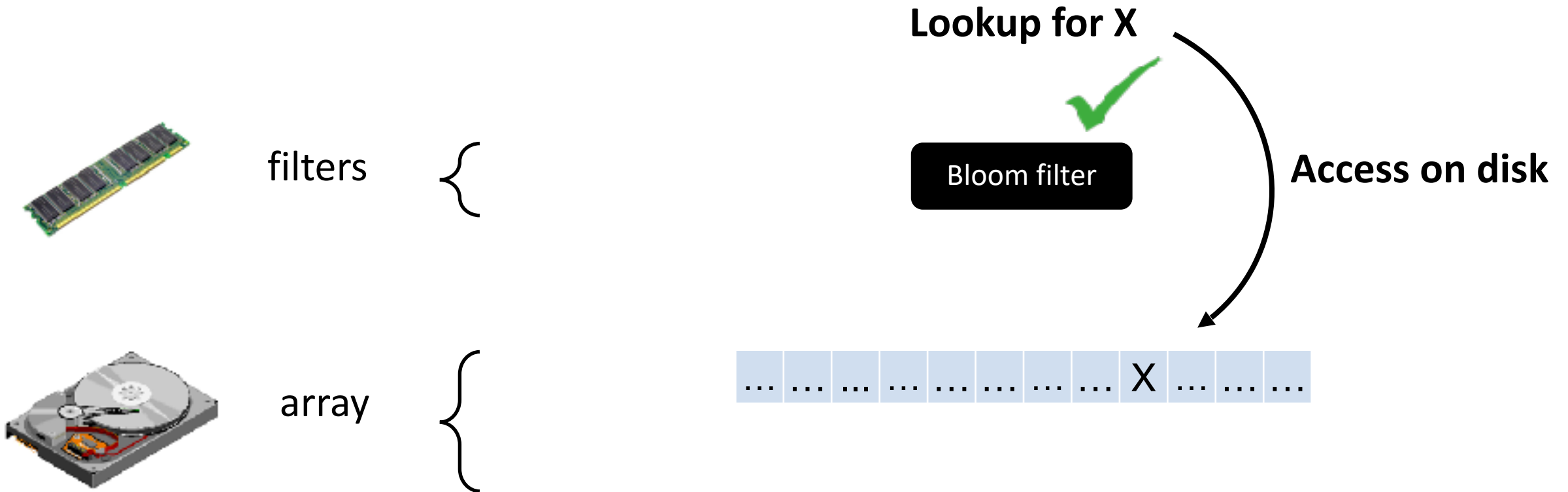array {

... ... X ... ...

Bloom Filters

Answers set-membership queries
Smaller than data it represents
Purpose: avoid accessing storage if entry is not present
Subtlety: may return false positives.

Lookup for X

Bloom filter
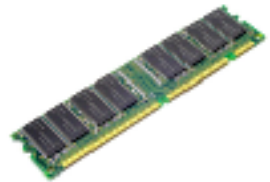
filters

array

… … … … … … … … X … … …

Bloom Filters

Answers set-membership queries

Smaller than data it represents

Purpose: avoid accessing storage if entry is not present

Subtlety: may return false positives.

Lookup for X

Bloom filter

filters {

array {

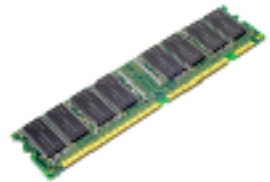... ... ... X ... ...

Bloom Filters

Answers set-membership queries
Smaller than data it represents
Purpose: avoid accessing storage if entry is not present
Subtlety: may return false positives.

Lookup for X

✔

Bloom filter

Access on disk

filters

array

… … … … … … … … X … … …

Bloom Filters

Answers set-membership queries
Smaller than data it represents
Purpose: avoid accessing storage if entry is not present
Subtlety: may return false positives.

Lookup for Y

filters {

array {

Bloom filter

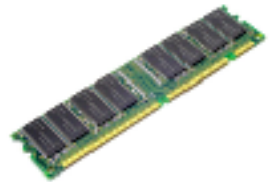… … … … … … … … X … … …

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Lookup for Y

Bloom filter
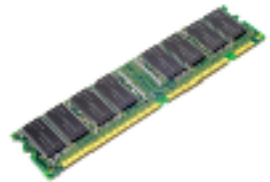
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… … … … … … … … X … … …

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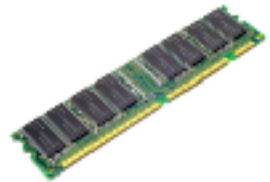
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Bloom Filters

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Lookup for Z

filters

array

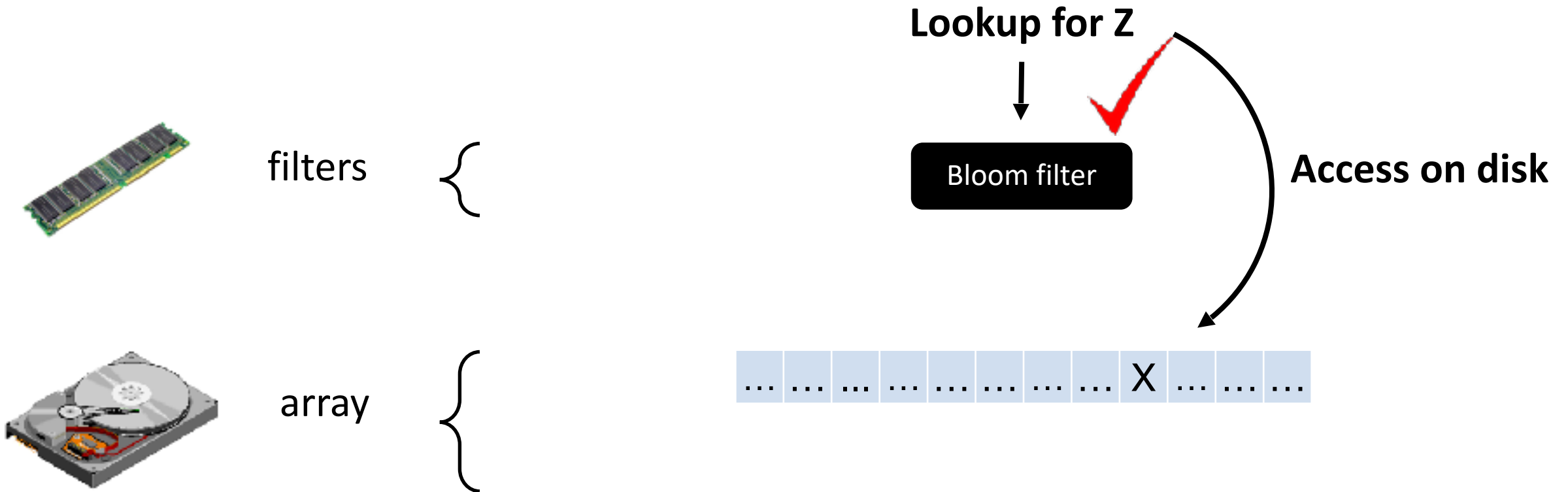Bloom filter

… … … … … … … … X … … …

Bloom Filters

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Subtlety: may return false positives.

Lookup for Z

Bloom filter

filters

array

... ... X ... ...

Bloom Filters

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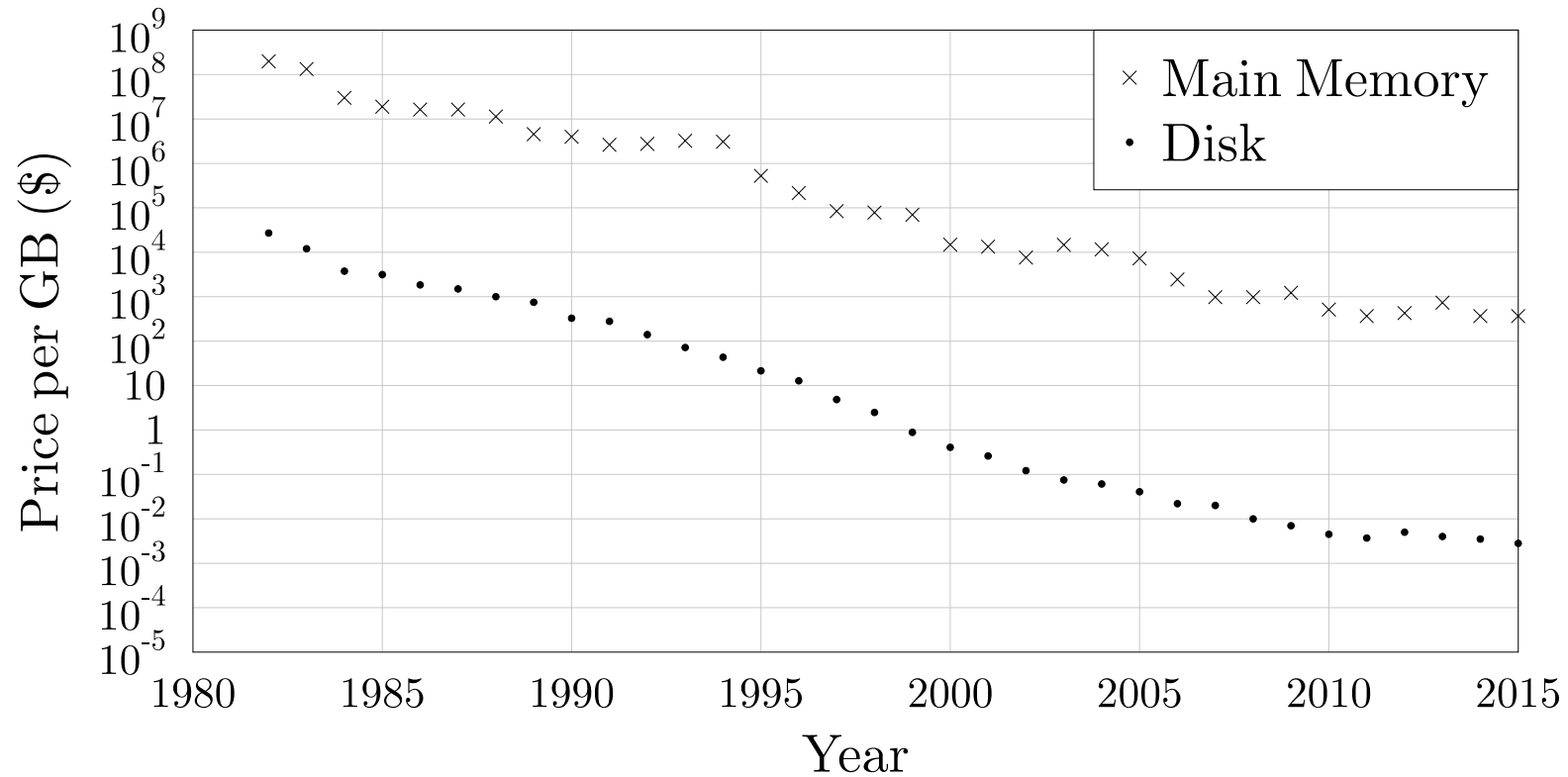Smaller than data it represents
Purpose: avoid accessing storage if entry is not present
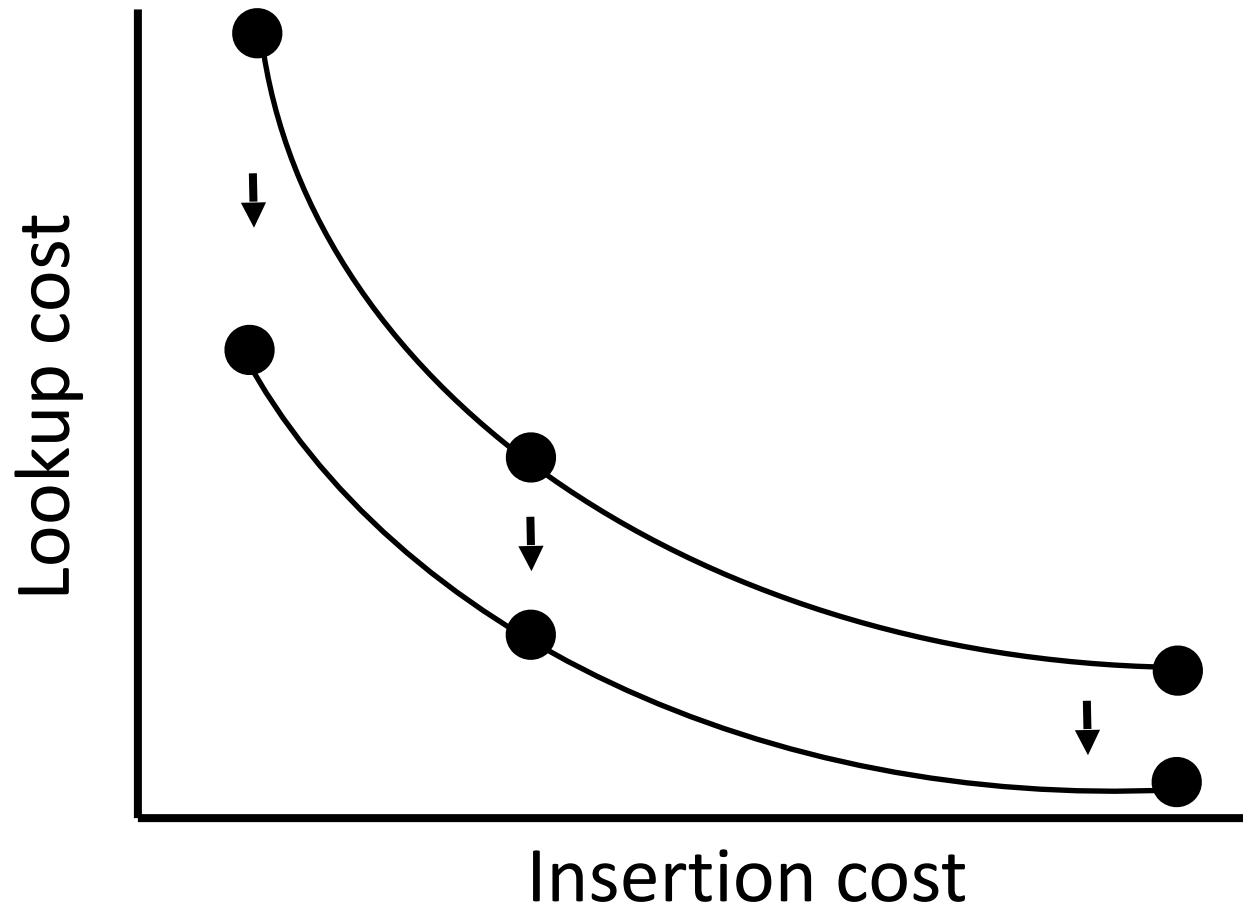Subtlety: may return false positives.

Bloom Filters

The more main memory, the less false positives $\Longrightarrow$ cheaper lookups

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Write-optimized

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The short-term future

Today: office hours after class in my office, 5230 in Bahen.

Thursday: Indexing exercises in tutorial this week.

Next week: research lecture. Important for your project.