Report on Paper - A High-Accuracy Rotation Estimation Algorithm Based on 1D Phase-Only Correlation

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***Abstract*—This paper proposes a high-accuracy rotation estimation algorithm using 1D Phase-Only Correlation (POC). In general, the rotation angle between two images is estimated as follows: (i) convert the image rotation into the image shift by polar mappings of the amplitude spectrum of images, and (ii) estimate the translational displacement between the polar mappings to obtain the rotation angle. The problem of rotation estimation between two images is replaced to 1D displacement estimation between pairs of horizontal lines at the same vertical position in two polar mappings. The proposed algorithm employs 1D POC instead of 2D matching for estimating a rotation angle. The use of 1D POC to estimate the rotation angle makes it possible to reduce the computational cost significantly without sacrificing the estimation accuracy.**

I. INTRODUCTION

High-accuracy image matching is an important fundamental task in many fields,such as image sensing, image/video processing, computer vision, industrial vision, etc. Over the years, various techniques for image matching have been developed. Typical examples include methods using image correlation functions, Fourier-transform-based methods, image-feature-based methods, and others. Among many methods, image matching techniques using Phase-Only Correlation (POC) have high-accuracy and robust performance.

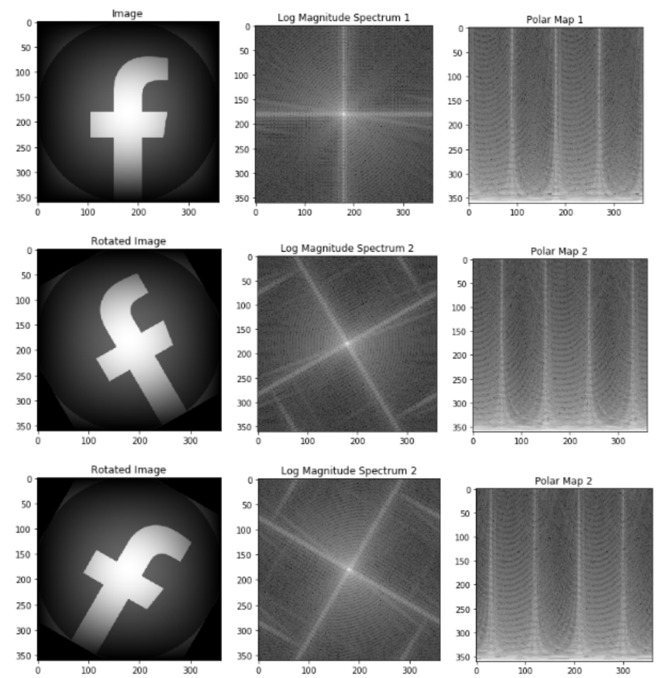

Fig. 1. Images and their spectrum

II. BASIC PRINCIPLE OF IMAGE ROTATION ESTIMATION

The problem considered here is to estimate the rotation angle θ between two images that are translated and rotated each other. For estimating the image rotation, we employ the polar mapping of the amplitude spectrum to convert the image rotation into the image translation. Note that the amplitude spectra are not affected by the image shifts, and are rotated with respect to each other at the origin of the spatial frequencies by the same angle as their spatial domain counterparts. The use of polar mapping, we can easily estimate the rotation angle θ The procedure for 2D POC is mentioned below, in our algorithm we use 1D POC.

Input: Images $f(n_1, n_2)$ and $g(n_1, n_2)$

Output: relative rotation angle θ between $f(n_1, n_2)$ and $g(n_1, n_2)$

Step 1: Calculate 2D DFTs of the discrete images $f(n_1, n_2)$ and $g(n_1, n_2)$ to obtain $F(k_1, k_2)$ and $G(k_1, k_2)$, where we assume that the index ranges are $n_1 = -M, , M$, $n_2 = -M, , M$, $k_1 = -M, , M$ and $k_2 = -M, , M$. The image size is $N * N, (N = 2M + 1)$. In order to reduce the effect of discontinuity at images border, we also apply 2D Hanning window to the input images $f(n_1, n_2)$ and $g(n_1, n_2)$.

Step 2: Calculate the amplitude spectra $|F(k_1, k_2)|$ and $|G(k_1, k_2)|$. For natural images, most energy is concentrated in low-frequency domain. Hence, we had better to use $log|F(k_1, k_2)| + 1$ and $log|G(k_1, k_2)| + 1$ instead of $|F(k_1, k_2)|$ and $|G(k_1, k_2)|$.

Step 3: Calculate the polar mappings of the amplitude spectrum $F_P(l_1, l_2)$ and $G_P(l_1, l_2)$ of $log|F(k_1, k_2)| + 1$ and $log|G(k_1, k_2)| + 1$(Fig. 1 (d)), where the index ranges of the transformed image are $l_1 = -M, , M$ and $l_2 = -M, , M$.

Step 4: Estimate the image displacement between $F_P(l_1, l_2)$ and $G_P(l_1, l_2)$ using 2D POC function to obtain the rotation angle θ, where the displacement in the horizontal direction (l_2 direction) corresponds to the image rotation.

III. Rotation Estimation Algorithm Using 1D Phase-Only Correlation

The problem of rotation estimation is replaced to 1D displacement estimation between row lines in two polar mappings. The use of 1D POC function makes it possible to reduce the computational cost significantly for estimating the rotation angle. In this section, we first define the 1D POC function and then introduce the details of the proposed rotation estimation algorithm.

A. 1-Dimensional Phase-Only Correlation (1D POC)

Consider two 1D image signals, $f(n)$ and $g(n)$, where we assume that the index range is $n = -M, , M$ for mathematical simplicity, and hence the signal length is $N = 2M + 1$. The discussion could be easily generalized to non-negative index ranges with power-of-two signal length. Let $F(k)$ and $G(k)$ denote the Discrete Fourier Transforms (DFTs) of the two signals. $F(k)$ and $G(k)$ are given by :

$$F(k) = \sum_{n=-M}^{M} f(n) W_N^{kn} = A_F(k) e^{j\theta_F(k)} \tag{1}$$

$$G(k) = \sum_{n=-M}^{M} g(n) W_N^{kn} = A_G(k) e^{j\theta_G(k)} \tag{2}$$

The cross-phase spectrum (or normalized cross spectrum) $R(k)$ is defined as

$$R(k) = \frac{F(k)\overline{G(k)}}{|F(k)\overline{G(k)}|} = e^{j\theta(k)} \tag{3}$$

The POC function $r(n)$ is the Inverse Discrete Fourier Transform (IDFT) of $R(k)$ and is given by:

$$r(n) = \frac{1}{N} \sum_{k=-M}^{M} R(k) W_N^{-kn} \tag{4}$$

If there is a similarity between two signals, the POC function gives a distinct sharp peak. (When $f(n) = g(n)$, the POC function becomes the Kronecker delta function.) If not, the peak drops significantly. The height of the peak can be used as a good similarity measure for signal matching, and the location of the peak shows the translation displacement between the two signals.

Now consider $f_c(t)$ as a signal defined in continuous space with a real number index t. Let represent a displacement of $f_c(t)$. So, the displaced signal can be represented as $f_c(t\delta)$. Assume that $f(n)$ and $g(n)$ are spatially sampled signals of $f_c(t)$ and $f_c(t\delta)$, and are defined as

$$f(n) = f_c(t)|_{t=nT} \tag{5}$$

$$g(n) = f_c(t-\delta)|_{t=nT} \tag{6}$$

where T is the sampling interval and the index range is given by $n = -M, , M$. For simplicity, we assume $T = 1$. The cross-phase spectrum $R(k)$ and the POC function $r(n)$ between $f(n)$ and $g(n)$ will be given by

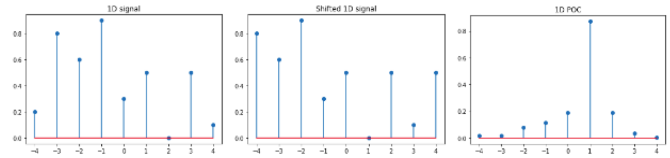

Fig. 2. (a) Signal, (b) Shifted signal, (c)1D phase correlation

$$R(k) = \frac{F(k)\overline{G(k)}}{|F(k)\overline{G(k)}|} \simeq e^{j\frac{2\pi}{N}k\delta} \tag{7}$$

$$r(n) = \frac{1}{N} \sum_{k=-M}^{M} R(k) W_N^{-kn} \simeq \frac{\alpha}{N} \frac{\sin\{\pi(n+\delta)\}}{\sin\left\{\frac{\pi}{N}(n+\delta)\right\}} \tag{8}$$

For the signal matching task, we evaluate the similarity between the two signals by the peak value α, and estimate the displacement by the peak position δ.

*a) **Function fitting for high-accuracy estimation of peak location**:* We fit a function to get the true estimate of the peaks, since we are using discrete samples, the highest peak doesn't always correspond to best fit value, this value can be estimated by getting the peak value of the function that we use to fit the data (equation 8).

*b) **Spectral weighting technique to reduce aliasing and noise effects**:* Natural images tend to have most reliable information in their lower frequency components. We improve the accuracy by adding a low pass type weighing function, in our case we used a Hanning window for this purpose.

B. High-Accuracy Image Rotation Estimation Algorithm

To get the total rotation angle θ we perform 1D phase correlation for every pairs of row lines in polar mappings, but care must be taken to select the line which lie in the most effective frequency bands.

*a) **Effective line extraction**:* We compute the Polar mappings of the amplitude spectrum and extract the displacement δ_{l_1}, set a threshold δ_{th} and consider the lines above this threshold only (effective lines), as described in detail below.

Input: Registered image $f(n_1, n_2)$.
Output: Indices l of effective lines.
Step 1: Generate the rotated image $f(n_1, n_2)$ by rotating the registered image $f(n_1, n_2)$ by θ degree, where $\theta = 40$ in this paper.
Step 2: Calculate the polar mappings of the amplitude spectrum $F_P(l_1, l_2)$ and $F'_P(l_1, l_2)$ of $f(n_1, n_2)$ and $f'(n_1, n_2)$, respectively.
Step 3: Extract 1D image signals $u_{l_1}(l_2)$ and $u'_{l_1}(l_2)$ in the horizontal direction (l_2 direction) from $F_P(l_1, l_2)$ and $F'_P(l_1, l_2)$, respectively, as shown in Fig. 3. Next, we calculate the 1D POC functions $r_{l_1}(l_2)$ between $u_{l_1}(l_2)$ and $u'_{l_1}(l_2)$, and obtain the displacement δ_{l_1} and the correlation peak value α_{l1} . The number of 1D POC functions is N.
Step 4: Select indices l by comparing between δ_{l_1} and θ as follows: $l = l_1 : |\delta_{l_1} N\theta/\pi| < \delta_{th}, -M \leq l_1 \leq M$,

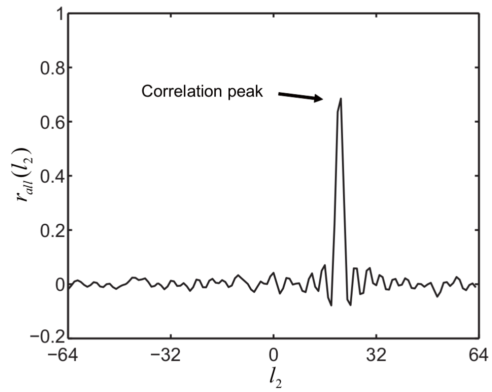

Fig. 3. Averaged 1D POC function

where $\delta_{th} = 1$ in this paper. Note that $N\theta/\pi$ indicates the displacement between two polar mappings which is equivalent to the rotation angle θ.
Step 5: Update the indices l by extracting upper half of the correlation peak value α_i.

*b) **Rotation estimation using 1D POC function**:* Average all the 1 D POC functions and we obtain the displacement δ and rotation angle using this δ as described below. An example of the 1D POC averaged function is shown in Fig 3.
Input: Registered image $f(n_1, n_2)$, input image $g(n_1, n_2)$ and indices l of effective lines.
Output: Rotation angle θ
Step 1: Calculate the polar mappings of the amplitude spectrum $F_P(l_1, l_2)$ and $G_P(l_1, l_2)$ of $f(n_1, n_2)$ and $g(n_1, n_2)$, respectively.
Step 2: Extract 1D image signals $u_{l_1}(l_2)$ and $v_{l_1}(l_2)$ in the horizontal direction (l_2 direction) from $F_P(l_1, l_2)$ and $G_P(l_1, l_2)$, respectively, where $l_1 \epsilon l$. Next, we calculate the 1D POC functions $r_{l_1}(l_2)$ between $u_{l_1}(l_2)$ and $v_{l_1}(l_2)$, where the number of 1D POC functions equals to the number of effective lines.
Step 3: Calculate the average of all 1D POC functions $r_{l_1}(l_2)$, which denotes $r_{all}(l_2)$. Next, we obtain the displacement δ from the peak location of the averaged 1D POC function $r_{all}(l_2)$. Then, we calculate the rotation angle θ as $\theta = \delta\pi/N$.

IV. Experiments and Discussions

	Polar map generation	*Displ. estimation*	*Total*
2D-POC	2N	4N	6N
1D-POC	2N	N/2 + 1	2.5N + 1

Table : Computational cost for estimating a rotation angle between two images

We have been able to successfully implement the High-Accuracy Image Rotation Estimation Algorithm, using effective line extraction and rotation estimation using 1D phase correlation.

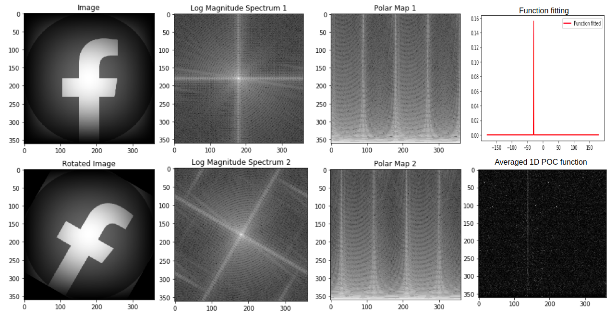

Fig. 4. Images, their transformations and rotation angle -30 degree

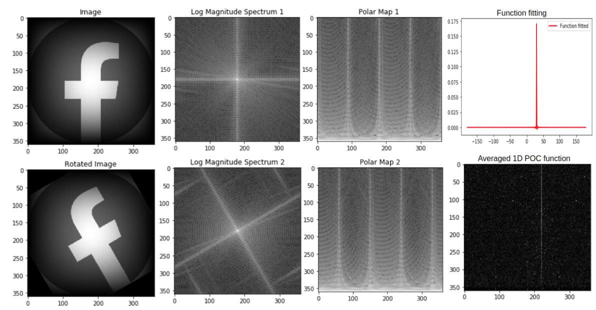

Fig. 5. Images, their transformations and rotation angle +30 degree

NOTE: This algorithm fails when there absolute symmetry between the image and the rotated image. Roughly speaking, the amount of symmetry in the image inversely proportional to the reliability of this method.

References

[1] Nagashima, Sei Ito, Koichi Aoki, Takafumi Ishii, Hideaki Kobayashi, Koji. (2009). High-Accuracy Estimation of Image Rotation Using 1D Phase-Only Correlation. IEICE Transactions. 92-A. 235-243. 10.1587/transfun.E92.A.235.
[2] M.H.G. Peeters,Implementation of the phase correlation algorithm.