We aim to choose stocks to create an **aggressive portfolio** catered towards investors of our age group who can afford to take on more risk. Overall, we have opted for high risk stocks with potential for large abnormal returns. Through our weighting function using the **efficient frontier**, covariances between the stocks will be taken into consideration. Therefore, our portfolio will **consist of both shorts and longs** of a collection of stocks. Despite only choosing stocks we believe have potential for large returns based from a sentimental or speculative point of view, we will let the statistics do the talking when it comes to weighting and which stocks to long or short. Using this approach will further hedge our portfolio against potentially large losses if our sentiment, especially regarding the current COVID-19 situation is wrong.

The first stock we have chosen is **TPG Telecom LTD (TPM)**. This stock has had a log return of 8.68% over the last 1 year period with a standard deviation of 2.41% on the daily returns and a market cap of $6.74 billion AUD which is significantly less than its competitors like Telstra and Optus (owned by Singtel). Although the average return hasn't been significant, we believe TPM has a lot of potential with their recent partnership with Vodafone for an aggressive rollout of the 5G network. In an age evolving faster and faster towards technological dependence, the reliance on a fast network for ubiquitous computing will become increasingly necessary. This is further exacerbated by the current global pandemic restrictions placing even further strain on our network systems.

To diversify our portfolio, we have decided to include **A2 Milk Company LTD (A2M)**. A2M has by far outperformed the market in the past 12 months, yielding a log return of 18.56% over the last 1 year period with a standard deviation of 2.44% in daily returns. It is clear investor confidence in this company is at a high as well, with the current PE ratio around 43.4. We have chosen A2M as it is a consumer staple, expanding internationally and becoming particularly popular amongst Asian and American shoppers who recognise the brand.

Appen (APX) is a global leader in the development of high-quality training data for machine learning and artificial intelligence. Despite posting a log return of -19% over the past 1 year period and a standard deviation of 3.44% in the daily returns, we are confident prices will correct once the panic sell off ends and as business activity that depends on APX's technology increases again. Furthermore, with a taste of working remotely being embraced by white collar employees, we are confident this will act as a catalyst for more dependence on APX's technology which can facilitate the implementation of more automation and less in person business.

Altium (ALU) is a software company that provides PC-based electronic design software. It has been doing well due to its potential as its plans to accelerate its new cloud platform for its rapidly growing consumer. At the end of the first half of the financial market, its growth is 19% and profit growth has been at 23% with a healthy cash flow. The recent release of 5G has also aided in its potential by bringing in new opportunities to its cloud based software that is very likely to evolve in the next few years.

The final stock we have chosen is **After Pay Ltd (APT)**, which is a fintech company revolutionizing the concept of use now, pay later. With an overall log return of around -55.6% (simple returns of -42.7%) over the past one year period and a standard deviation of 5.66% in the daily returns, this is definitely one of our most risky stocks. However, it has actually been a very steady stock, consistently outperforming the market prior to the recent crash. After understanding the recent crash was due to investors fearing consumers defaulting on their loans, we have confidence the price will rebound substantially once the stimulus packages take into effect and the economy recovers.

In order to compute the daily portfolio value and the daily return, we had to assign the weightings for each stock. The stock weightings were chosen using the efficient frontier, the explanation on the calculation of weights is provided in **Appendix B2-1.** Using this yields us the following weights for each stock:
A2M: buy 47.78%,
APT: buy 9.57%,
APX: short 26.16%,
ALU: borrow and buy 116.74%,
TPM: short 47.92%.

We will be using these weightings throughout the holding period (3 April 2020 to 11 May 2020). Daily value and return of our portfolio are computed based on two-month historical closing prices for each stock starting from the day when the assignment was released to the day before the due date (3 April 2020 to 11 May 2020). The value of the portfolio based on the daily closing price of the stocks in the portfolio and the daily return on the portfolio are documented and provided in **Appendix B2-2** for reference.

The overall return of our portfolio till date is 30.4%. The overall return of the portfolio is computed by taking the average of daily log returns throughout the holding period as shown in **Appendix B2-2.**

In order to test for persistence in portfolio daily returns, we constructed a correlogram of residuals (as shown in Appendix B1a). By looking at ACF, we can see that there are some spikes, but they are all within the 95% confidence band. As such, it appears to have no serial correlation. To confirm, we also looked at the p-values for each lag.

Since the p-values for all 36 lags are greater than 0.05 (level of significance), we do not reject the null hypothesis and conclude that there is insufficient evidence to prove that there is serial autocorrelation (refer to Equation b). Therefore, we can conclude that there are no serial autocorrelations present in daily log returns of the portfolio as such the daily log returns are not persistent. Furthermore, as shown in Appendix B1b, it can be seen that for the daily return series, the mean of returns is centered around 0 and there are some signs of volatility clustering, with variance being non-constant over time. This indicates that there may be ARCH effects present in the returns. Given that returns keep going to the mean frequently, it indicates that it is a stationary series**.**

Since there are no autocorrelations observed in daily log returns and the fact that it is a stationary series, therefore a constant mean equation would be sufficient to model the mean component of the returns.

For the daily log return series, the constant mean model equation is (refer to Appendix B1c),

$$\widehat{r_t} = 0.000847 + \widehat{u_t}$$

As shown in Appendix B1b, there were some signs of volatility clustering observed which indicates that there may be ARCH effects present in the returns. By looking at the correlogram of squared residuals after fitting the constant mean model (as shown in Appendix B2a), we could observe that for the first two lags, the bars are within the 95% confidence band and their -values are slightly greater than 0.05, which therefore indicates there are no serial correlations in the squared residuals (refer to Equation b). But if we were to compare it at 10% level of significance, the p-value for the first lags are less than 0.10 indicating that there are serial correlations present in the squared residuals. Starting from the third lag, large spikes are observed and these bars exceed the 95% confidence bands, which indicates that there are serial correlations observed in the squared residuals of this model. Furthemore, since p-value = 0 < 0.05, we are rejecting the null that says there are no autocorrelations observed (refer to Equation b) in the squared residuals.

All in all, this model displays strong ARCH effects especially in the later lags and hence we can model the variances and improve our model.

ARCH model

We started off by fitting a constant mean model with ARCH(1) errors (refer to Appendix B2b). To determine if there are any excess autocorrelations that are existing in the 5% level of significance, we look at the correlogram because we want to make sure that by fitting an ARCH(1) model, we get rid of all persistence. Referring to Appendix B2c, in this correlogram, there appears to be no serial correlation as the p-values are greater than 5% level of significance for all lags (refer to Equation b). Even though there are big spikes observed, it is within 95% confidence bands. We want to check for leftover ARCH effects to see if the ARCH(1) model is adequate. We can formally test this by doing an ARCH test up to 5 lags. Please refer to Appendix B2d for detailed ARCH tests on ARCH(1) model for upto 5 lags. Based on the ARCH tests, we can observe that there are some leftover ARCH effects present especially in lags 3 and 5 and as such we can conclude the ARCH(1) model is inadequate.

So, we decided to fit a constant mean model with ARCH(2) errors (refer to Appendix B2e). To determine if there are any excess autocorrelations that are existing in the 5% level of significance, we look at the correlogram in Appendix B2f, and there appears to be no serial correlation as the p-values are greater than 5% level of significance for all lags. Even though there are big spikes observed, it is within the 95% confidence band. Once again, we want to check for leftover ARCH effects to assess if the ARCH(2) model is adequate. We can formally test this by doing an ARCH test up to 5 lags. Please refer to Appendix B2g for detailed ARCH tests on ARCH(2) model for upto 5 lags. Based on the ARCH tests, we can see that there are no leftover ARCH effects present over the 5 lags which suggests ARCH(2) specification is adequate.

One of the underlying assumption made when modelling an ARCH model is that u_t follows a non-normal distribution, that is a t-distribution to account for the excess kurtosis (kurtosis >3) displayed by the portfolio returns (Appendix 2h). From standardised residuals, it could be seen that the model has captured this stylized fact of financial data. We can further conclude that the returns are not normal and be tested with a Jarque-Bera test in Appendix B2h.

Therefore, the ARCH(2) model will be (refer to Appendix B2e)

$$\sigma_t^2 = 0.000475 + 0.234\varepsilon_{t-1}^2 + 0.466\varepsilon_{t-2}^2$$

Since the ARCH(2) model captures the time dependency in the squares of the errors, it means it captures the time variability in the volatility as well. However, we want to assess if adding a lagged variance component would capture the variability in the volatility even better. GARCH model permits a wider range of behaviour, in particular, more persistent volatility, and improves modeling of conditional variance. As such we would be fitting a constant mean model with GARCH(2,1) errors (refer to Appendix B2i).

<u>GARCH model</u>

The underlying assumption we would be making when modelling a GARCH model is that u_t follows a non-normal distribution, that is a t-distribution. To determine if there are any excess autocorrelations that are existing in the 5% level of significance, we look at the correlogram because we want to make sure that by fitting a GARCH(2,1) model, we get rid of all persistence. Referring to Appendix B2j, in this correlogram, there appears to be no serial correlation as the p-values are greater than 5% level of significance for all lags (refer to equation b above).Once again, we want to check for leftover ARCH effects to assess if the GARCH(2,1) model is adequate. We can formally test this by doing an ARCH test up to 5 lags. Please refer to Appendix B2k for detailed ARCH tests on the GARCH(2,1) model for upto 5 lags. Based on the ARCH tests, we can see that there are no leftover ARCH effects present over the 5 lags which suggests GARCH(2,1) specification is adequate.

As mentioned earlier, we have assumed the u_t follows a t-distribution to account for excess kurtosis displayed by the residuals of daily portfolio returns. When we look at the histogram of standardised residuals as shown in Appendix B2l, the distribution is not normal, as it follows t-distribution which is further supported by a formal Jarque Bera test (refer to Appendix B2l for the formal test).

Therefore, the GARCH(2,1) model will be (refer to Appendix B2i)

$$\sigma_t^2 = 0.0000386 + 0.135\varepsilon_{t-1}^2 + 0.00348\varepsilon_{t-2}^2 + 0.846\sigma_{t-1}^2$$

<u>ARCH(2) vs GARCH(2,1)</u>

We observe that ARCH(2) is nested in the GARCH(2,1) model. Because if we can add a restriction to the GARCH(2,1) specification, it will bring us back to ARCH(2) specification therefore we can say it is nested. As such we can conduct a likelihood ratio test to formally decide which is a better model.

$H_0 : \beta_1 = 0$ (if coefficients of GARCH components are 0, then it is just an ARCH model)

$H_1 : \beta_1 \neq 0$

$LR_{stat} = 2 * [LL(\widehat{\theta}_G) - LL(\widehat{\theta}_A)]$, where $LL(\widehat{\theta}_G)$ is likelihood from GARCH(2,1) model and $LL(\widehat{\theta}_A)$ is likelihood from the ARCH(2) model.

$LR_{stat} = 2 * [188.31\text{-}184.90] = 6.82$ and $LR_{crit} = \mathscr{X}^2_{(0.05,1)} = 3.841$

Since $LR_{stat} > LR_{crit}$ (6.82>3.841), we reject H_0 under the 5% level of significance. Therefore, we can conclude that ARCH(2) is inadequate which means GARCH(2,1) would be a better model specification.

Forecasting

Using a constant mean model with GARCH(2,1) error, we could do forecasting for 11 May 2020. The fitted model for daily portfolio returns are (refer to Appendix B2i):

$$\widehat{r_t} = 0.004512 + \widehat{u_t}$$

$$\sigma_t^2 = 0.0000386 + 0.135\varepsilon_{t-1}^2 + 0.00348\varepsilon_{t-2}^2 + 0.846\sigma_{t-1}^2$$

To forecast 11 May 2020 returns, we have

$r_{8may} = -0.000790$, $\widehat{\varepsilon}_{8may} = -0.005302$, $\widehat{\varepsilon}_{7may} = 0.015099$, $\widehat{\sigma}^2_{8may} = 0.000819$

Conditional mean: $E[r_{11may} | I_{8may}] = 0.004512$

Conditional variance:

$\widehat{V}[r_{11may} | I_{8may}] = 0.0000386 + 0.135(-0.005302)^2 + 0.00348(0.015099)^2 + 0.8466(0.000819) = 0.0007365$

Volatility:

$\widehat{\sigma}[r_{11may} | I_{8may}] = \sqrt{0.0007365} = 0.027139$

If we assume that $u_t \sim N(0,1)$, so that $\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$, then the conditional distribution of returns will also be normally distributed. We can form prediction returns for the return on 11 May 2020. A 95% prediction interval for r_{11may} would be:

$$0.004512 \pm 1.96\,(\,0.027139\,) = [-0.00487, 0.0577]$$

The actual return on 11 May 2020 is 0.0191 which falls within the 95% prediction interval. This suggests that the constant mean model with GARCH(2,1) is a good model for forecasting the daily returns of this portfolio.

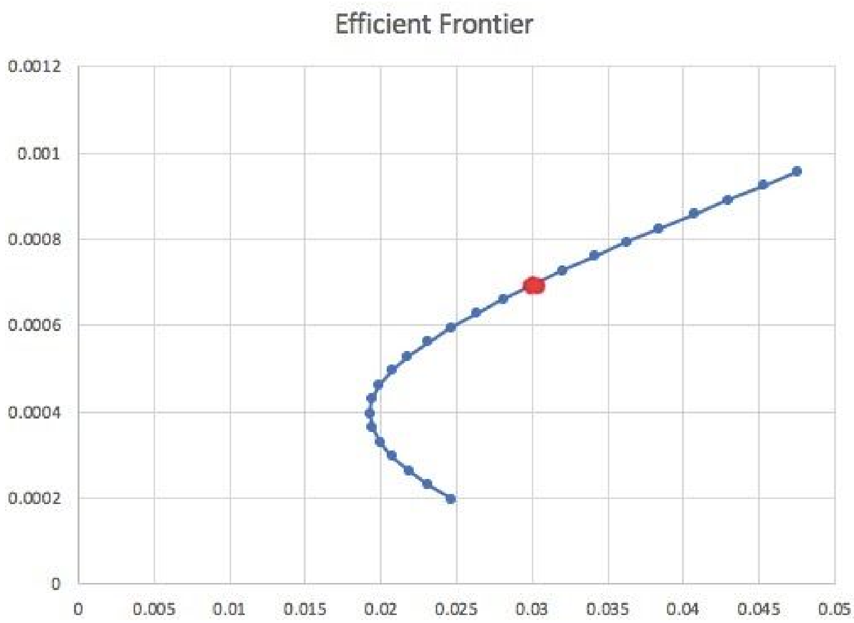
APPENDIX

Appendix B2-1

Weight:

A2M	0.47766161
APT	0.09567618
APX	-0.2615537
ALU	1.16741058
TPM	-0.4791947

The weight of each stock in our portfolio is based on the efficient frontier we created. Our group chooses to have an aggressive portfolio with the intention of taking slightly more risk so as to earn higher returns. Therefore, the red spot shown in the graph below is where we decided the weight of each stock to be. We will be using these weightings throughout the holding period.



Explanation on how we get the efficient frontier are provided as follows

Step 1: In order to derive the weights of each stock, we have gathered one year historical data of each stock and we reported the daily closing prices of each stock. We then calculated the log returns from the daily closing prices for each stock ($r_{t+1} = log(\frac{P_{t+1}}{P_t})$). From there, we calculated the excess returns for each stock (return of each stock at time t - average return of the stock over the 1 year period).

Step 2: Calculated the mean return of each stock.

Step 3: Transposed the excess return table that was created in Step 1

Step 4: Created a variance covariance table, also made up two random constants in our case which is 0.0002 and 0.1. Following that, we created two tables called mean constant where we subtracted the mean return of each stock with each of our random constants.

Step 5: Created two portfolios x and y which are the two efficient portfolios sitting on the efficient frontier. The weights of each stock in portfolio x and y is determined by each single z-value divided by the sum of all z-values.

Date	Adj Close_A2M	Adj Close_APT	Adj Close_APX	Adj Close_ALU	Adj Close_TPM	return_A2M	return_APT	return_APX	return_ALU	return_TPM	STEP 1	excess_return	A2M	APT	APX	ALU	TPM
6/5/19	15.05	27.5	24.215994	31.757648	6.812149												
7/5/19	15.44	28.459999	24.71406	31.925732	6.971032	0.0255836	0.0343136	0.020359	0.0052788	0.0230556			0.02468	0.0332	0.0201	0.005	0.02295
8/5/19	15.24	26.559999	24.24588	31.292952	6.027659	-0.013038	-0.0690933	-0.0191256	-0.0200194	-0.1454046			-0.0139	-0.0702	-0.0194	-0.02	-0.1455
9/5/19	15.11	26.43	23.927118	31.579681	6.007799	-0.0085668	-0.0049066	-0.0132342	0.009121	-0.0033003			-0.0095	-0.006	-0.0135	0.0089	-0.0034
10/5/19	14.99	26.209999	23.767736	31.569794	6.305706	-0.0079735	-0.0083587	-0.0066834	-0.0003131	0.0483965			-0.0089	-0.0094	-0.0069	-6E-04	0.04829
13/5/19	14.83	26.17	23.87731	30.956787	6.256055	-0.0107312	-0.0015273	0.0045996	-0.0196085	-0.0079051			-0.0116	-0.0026	0.0043	-0.02	-0.008
14/5/19	15.02	26	24.046652	30.472315	6.265985	0.0127305	-0.0065172	0.0070671	-0.0157737	0.001586			0.01183	-0.0076	0.0068	-0.016	0.00148
15/5/19	15.4	26.139999	24.076536	31.846634	6.246124	0.0249849	0.0053701	0.001242	0.0441131	-0.0031747			0.02408	0.0043	0.001	0.0439	-0.0033
16/5/19	15.38	25.459999	24.176149	31.975168	6.226264	-0.0012995	-0.0263581	0.0041288	0.0040279	-0.0031846			-0.0022	-0.0274	0.0039	0.0038	-0.0033
17/5/19	15.39	25.6	25.092592	32.242123	6.196473	0.00065	0.0054838	0.0372061	0.0083142	-0.0047962			-0.0003	0.0044	0.0369	0.0081	-0.0049
20/5/19	15.34	25.969999	24.634371	30.749157	6.246124	-0.0032542	0.0143496	-0.01843	-0.0474112	0.0079809			-0.0042	0.0133	-0.0187	-0.048	0.00788
21/5/19	15.25	24.700001	24.026731	30.086716	6.196473	-0.0058843	-0.0501387	-0.0249757	-0.0217788	-0.0079809			-0.0068	-0.0512	-0.0252	-0.022	-0.0081
22/5/19	15.13	24.379999	24.136305	30.20536	6.246124	-0.0079	-0.0130402	0.0045501	0.0039356	0.0079809			-0.0088	-0.0141	0.0043	0.0037	0.00788
23/5/19	15.08	25.08	25.132439	30.551413	6.246124	-0.0033102	0.0283076	0.0404423	0.0113915	0			-0.0042	0.0272	0.0402	0.0112	-0.0001
24/5/19	14.96	23.98	24.992979	29.266077	6.206403	-0.0079894	-0.0448506	-0.0055645	-0.0429819	-0.0063796			-0.0089	-0.0459	-0.0058	-0.043	-0.0065
27/5/19	14.74	23.93	26.138531	30.304234	6.45466	-0.0148151	-0.0020872	0.0448156	0.0348584	0.0392208			-0.0157	-0.0032	0.0446	0.0346	0.03912
28/5/19	14.9	24.93	27.652655	31.065546	6.265985	0.0107963	0.040939	0.0563112	0.0248119	-0.0296666			0.0099	0.0399	0.0561	0.0246	-0.0298
29/5/19	14.92	25.049999	27.662615	31.085321	6.206403	0.0013414	0.0048019	0.0003601	0.0006364	-0.0095543			0.00044	0.0037	0.0001	0.0004	-0.0097
30/5/19	14.58	24.040001	27.772188	30.561298	6.256055	-0.0230519	-0.0411546	0.0039532	-0.0170013	0.0079683			-0.024	-0.0422	0.0037	-0.017	0.00787
31/5/19	14.68	24.15	25.929344	30.581076	6.246124	0.0068353	0.0045652	-0.0686598	0.0006469	-0.0015887			0.00593	0.0035	-0.0689	0.0004	-0.0017
3/6/19	14.26	23.18	25.570736	30.175701	6.216334	-0.0290276	-0.0409945	-0.0139267	-0.0133444	-0.0047808			-0.0299	-0.0421	-0.0142	-0.014	-0.0049
4/6/19	13.03	22.32	24.963095	28.880478	6.216334	-0.090204	-0.0378067	-0.02405	-0.0438711	0			-0.0911	-0.0389	-0.0243	-0.044	-0.0001
5/6/19	13.48	23.9	25.491045	29.493484	6.246124	0.0339527	0.0683953	0.0209287	0.0210035	0.0047808			0.03305	0.0673	0.0207	0.0208	0.00468
6/6/19	13.45	23.450001	26.048882	30.294346	6.226264	-0.002228	-0.0190079	0.0216476	0.0267917	-0.0031846			-0.0031	-0.0201	0.0214	0.0266	-0.0033
7/6/19	13.64	24.17	26.606714	30.966675	6.345427	0.0140275	0.0302417	0.0211887	0.0219505	0.0189579			0.01313	0.0292	0.0209	0.0217	0.01886
11/6/19	13.95	24.17	26.766098	31.886185	6.405008	0.0224729	0	0.0059725	0.0292612	0.0093458			0.02157	-0.0011	0.0057	0.029	0.00924
12/6/19	13.92	25.639999	27.383699	32.390427	6.444729	-0.0021529	0.0590414	0.0228118	0.0156901	0.0061824			-0.0031	0.058	0.0226	0.0155	0.00608
13/6/19	13.87	22.549999	25.4512	31.727989	6.424869	-0.0035984	-0.1284186	-0.073185	-0.0206637	-0.0030863			-0.0045	-0.1295	-0.0734	-0.021	-0.0032
14/6/19	14.04	21.59	25.720156	32.271782	6.47452	0.0121822	-0.0435048	0.0105121	0.016994	0.0076982			0.01128	-0.0446	0.0103	0.0168	0.0076
17/6/19	13.86	20.27	25.799847	32.261898	6.45466	-0.0129034	-0.0630883	0.0030936	-0.0003063	-0.0030721			-0.0138	-0.0642	0.0028	-5E-04	-0.0032

STEP 2		A2M	APT	APX	ALU	TPM
	mean	0.00090033	0.00107552	0.00025656	0.00023699	0.00010187

STEP 3	transpose of excess return																		
		0.02468322	-0.0139383	-0.0094671	-0.0088738	-0.0116315	0.01183016	0.02408453	-0.0021999	-0.0002503	-0.0041545	-0.0067846	-0.0088003	-0.0042105	-0.0088897	-0.0157154	0.00989599	0.00044105	-0.02395
		0.03323803	-0.0701688	-0.0059821	-0.0094343	-0.0026028	-0.0075927	0.00429461	-0.0274336	0.00440828	0.01327411	-0.0512142	-0.0141157	0.02723211	-0.0459261	-0.0031628	0.03986347	0.00372637	-0.04223
		0.02010243	-0.0193822	-0.0134908	-0.00694	0.00434304	0.00681058	0.00098542	0.00387225	0.03694953	-0.0186866	-0.0252322	0.00429357	0.0401857	-0.005821	0.044559	0.05605467	0.00010355	0.003696
		0.00504177	-0.0202564	0.00888402	-0.0005501	-0.0198455	-0.0160107	0.04387614	0.00379092	0.00807718	-0.0476482	-0.0220158	0.00369866	0.01115456	-0.0432189	0.03462139	0.02457493	0.00039937	-0.01723
		0.02295377	-0.1455064	-0.0034021	0.04829461	-0.008007	0.00148413	-0.0032766	-0.0032865	-0.0048981	0.00787898	-0.0080827	0.00787898	-0.0001019	-0.0064815	0.03911898	-0.0297684	-0.0096562	0.007866

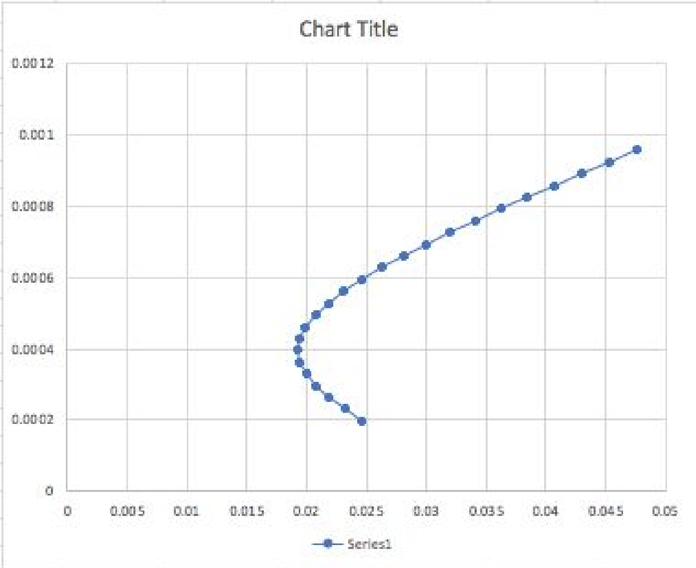
STEP 4	var-co table	A2M	APT	APX	ALU	TPM		mean
	A2M	0.00061911	0.00039679	0.00029325	0.00026209	0.00019355		0.00090033
	APT	0.00039679	0.00436544	0.00087327	0.00063284	0.00074933		0.00107552
	APX	0.00029325	0.00087327	0.0012862	0.00066184	0.00030334		0.00025656
	ALU	0.00026209	0.00063284	0.00066184	0.00070025	0.00030357		0.00023699
	TPM	0.00019355	0.00074933	0.00030334	0.00030357	0.00064948		0.00010187
		set two constant						
	constant	0.0002		0.1				
		mean-constant						
		0.00070033		-0.0990997				
		0.00087552		-0.0989245				
		5.6563E-05		-0.0997434				
		3.6987E-05		-0.099763				
		-9.813E-05		-0.0998981				

STEP 5		z	x(weight)	z	y(weight)
	A2M	1.35294386	0.48668668	-106.53393	0.39643605
	APT	0.25882798	0.11291846	15.9905311	-0.0595043
	APX	-0.1518811	-0.292641	-4.8995794	0.01823241
	ALU	-0.2570154	1.26990443	-65.829483	0.24496589
	TPM	-0.6618218	-0.5768686	-107.45671	0.39986991
			1		1

Step 6: We assume the two portfolios we created as two single stocks and computed their portfolio expected return, variance, covariance and correlation.

Step 7 and Step 8: We created a data table to get all the possible proportions of portfolio x and outline them all in the table. We can get a series of data representing how different weights of x in the portfolio affect the overall portfolio risk and return. Using that we created an efficient frontier where the x axis represents risk of the portfolio (i.e. standard deviation) and the y axis represents the return of the portfolio (i.e mean).

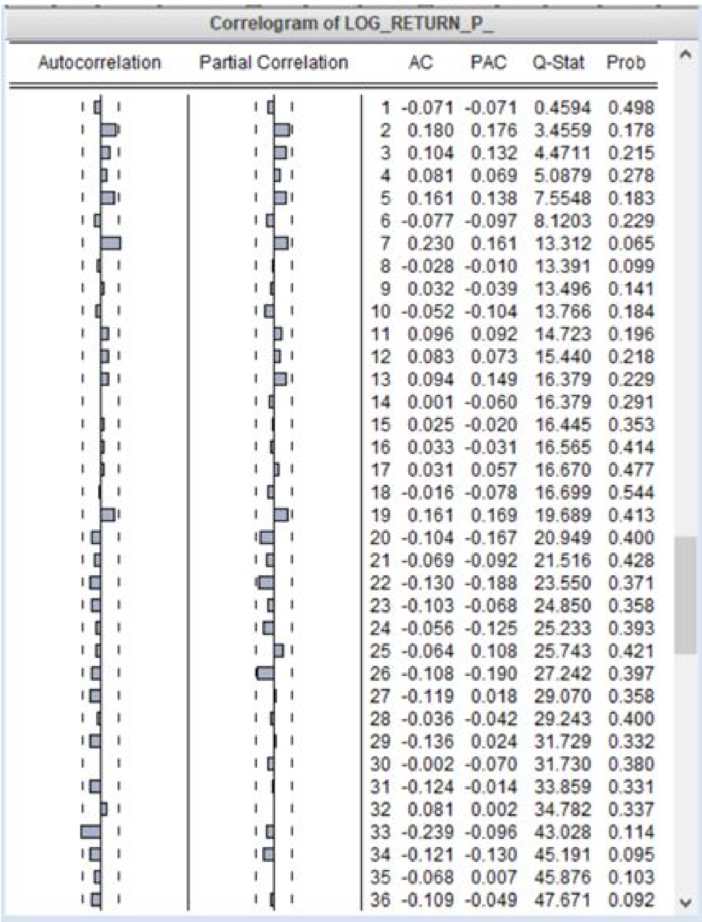
STEP 6	portfolio	x	y
	mean	0.0007267	0.0003964
	Var	0.0010251	0.0003706
	stdev	0.0320174	0.0192522
	cov(x,y)	0.0003694	
	corr(x,y)	0.5993104	
STEP 7	proportion x	0.1	
	$E(r_p)$	0.0004294	
	σ_p	0.0194157	
STEP 8	prop of x	σ_p	$E(r_p)$
		0.0194157	0.0004294
	-0.6	0.0246701	0.0001982
	-0.5	0.023154	0.0002312
	-0.4	0.0218343	0.0002643
	-0.3	0.0207487	0.0002973
	-0.2	0.0199353	0.0003303
	-0.1	0.0194284	0.0003634
	0	0.0192522	0.0003964
	0.1	0.0194157	0.0004294
	0.2	0.0199106	0.0004625
	0.3	0.0207131	0.0004955
	0.4	0.0217892	0.0005285
	0.5	0.0231008	0.0005616
	0.6	0.0246102	0.0005946
	0.7	0.0262834	0.0006276
	0.8	0.0280911	0.0006607
	0.9	0.030009	0.0006937
	1	0.0320174	0.0007267
	1.1	0.0341001	0.0007598
	1.2	0.0362445	0.0007928
	1.3	0.0384402	0.0008258
	1.4	0.0406789	0.0008589
	1.5	0.0429539	0.0008919
	1.6	0.0452597	0.0009249
	1.7	0.0475918	0.000958



Appendix B2-2

	Adj Close_A2M	Adj Close_APT	Adj Close_APX	Adj Close_ALU	Adj Close_TPM		weight		Date	portfolio value	log return(p)	overall return
/4/20	16.620001	19.549999	19.959999	27.030001	6.991151		A2M	0.47766161	3/4/20	32.79358153	#VALUE!	0.30405928
/4/20	16.879999	20.120001	21.25	29.879999	7.27		APT	0.09567618	6/4/20	35.82839857	0.08850802	
/4/20	17.049999	21.059999	21.219999	30.450001	7.369589		APX	-0.2615537	7/4/20	36.62508718	0.02199261	
/4/20	16.82	19.879999	21.66	30.23	7.27		ALU	1.16741058	8/4/20	36.07813475	-0.0150465	
/4/20	16.92	22	22.76	30.93	7.3		TPM	-0.4791947	9/4/20	36.84383706	0.02100136	
/4/20	17.48	28.4	24.110001	31.34	7.76				14/4/20	37.62876621	0.02108046	
/4/20	18.1	27.879999	23.85	31.719999	7.77				15/4/20	38.38199182	0.01981957	
/4/20	19	27.280001	23.809999	31.549999	7.65				16/4/20	38.62398773	0.00628514	
/4/20	19	29	24.719999	32.349998	7.81				17/4/20	39.40779298	0.02009006	
/4/20	18.73	29	24.83	32.110001	7.59				20/4/20	39.07530099	-0.008473	
/4/20	18.309999	27.32	23.18	30.700001	7.47				21/4/20	37.55696462	-0.0396317	
/4/20	18.620001	26.57	23.440001	30.860001	7.32				22/4/20	37.82394423	0.00708351	
/4/20	18.59	27.15	23.01	30.84	7.39				23/4/20	37.92068141	0.0025543	
/4/20	18.65	27.01	22.799999	31.299999	7.37				24/4/20	38.53746457	0.01613423	
/4/20	18.77	27.75	23.83	33.209999	7.64				27/4/20	40.49655544	0.04958605	
/4/20	18.93	28.309999	24.1	33.029999	7.5				28/4/20	40.41289373	-0.002068	
/4/20	18.6	28.15	24.190001	33.360001	7.52				29/4/20	40.59208115	0.00442412	
/4/20	18.219999	31.200001	25.83	34.150002	7.36				30/4/20	41.27236066	0.01662004	
/5/20	17.950001	29.16	25.35	33.029999	7.06				1/5/20	39.91001429	-0.0335658	
/5/20	18.9	36.099998	25.84	33.720001	6.99				4/5/20	41.73868285	0.04480107	
/5/20	18.77	38.18	26.59	35.009998	7.14				5/5/20	43.11350519	0.03240795	
/5/20	18.540001	39.799999	28.940001	35.299999	7.33				6/5/20	42.79149071	-0.007497	
/5/20	18.299999	39.5	29.049999	36.119999	7.2				7/5/20	43.63894981	0.01961083	
/5/20	18.17	39.880001	30	36.349998	7.26				8/5/20	43.60448667	-0.00079	
/5/20	18.059999	42.209999	29.68	36.860001	7.275				11/5/20	44.44676088	0.01913204	

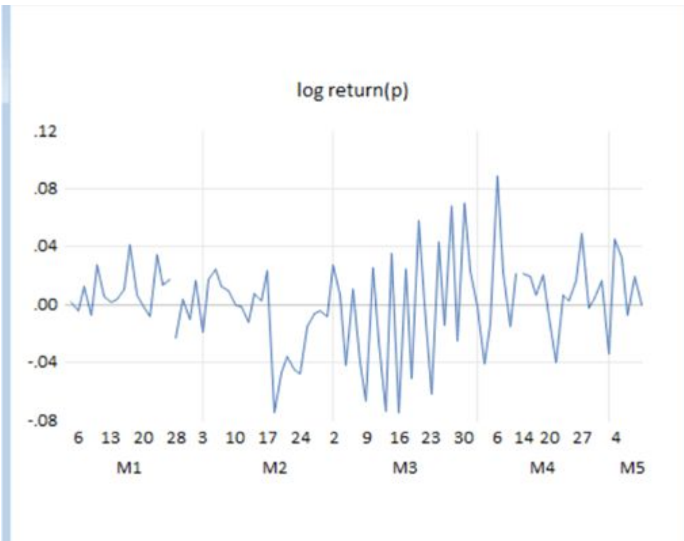
Appendix B1a



Correlogram of LOG_RETURN_P_

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.071	-0.071	0.4594	0.498
		2	0.180	0.176	3.4559	0.178
		3	0.104	0.132	4.4711	0.215
		4	0.081	0.069	5.0879	0.278
		5	0.161	0.138	7.5548	0.183
		6	-0.077	-0.097	8.1203	0.229
		7	0.230	0.161	13.312	0.065
		8	-0.028	-0.010	13.391	0.099
		9	0.032	-0.039	13.496	0.141
		10	-0.052	-0.104	13.766	0.184
		11	0.096	0.092	14.723	0.196
		12	0.083	0.073	15.440	0.218
		13	0.094	0.149	16.379	0.229
		14	0.001	-0.060	16.379	0.291
		15	0.025	-0.020	16.445	0.353
		16	0.033	-0.031	16.565	0.414
		17	0.031	0.057	16.670	0.477
		18	-0.016	-0.078	16.699	0.544
		19	0.161	0.169	19.689	0.413
		20	-0.104	-0.167	20.949	0.400
		21	-0.069	-0.092	21.516	0.428
		22	-0.130	-0.188	23.550	0.371
		23	-0.103	-0.068	24.850	0.358
		24	-0.056	-0.125	25.233	0.393
		25	-0.064	0.108	25.743	0.421
		26	-0.108	-0.190	27.242	0.397
		27	-0.119	0.018	29.070	0.358
		28	-0.036	-0.042	29.243	0.400
		29	-0.136	0.024	31.729	0.332
		30	-0.002	-0.070	31.730	0.380
		31	-0.124	-0.014	33.859	0.331
		32	0.081	0.002	34.782	0.337
		33	-0.239	-0.096	43.028	0.114
		34	-0.121	-0.130	45.191	0.095
		35	-0.068	0.007	45.876	0.103
		36	-0.109	-0.049	47.671	0.092

Appendix B1b



Appendix B1c

Dependent Variable: LOG_RETURN_P_
Method: Least Squares
Date: 05/29/20 Time: 07:34
Sample (adjusted): 1/03/2020 5/08/2020
Included observations: 88 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000847	0.003391	0.249714	0.8034

Root MSE	0.031629	R-squared	0.000000
Mean dependent var	0.000847	Adjusted R-squared	0.000000
S.D. dependent var	0.031811	S.E. of regression	0.031811
Akaike info criterion	-4.046729	Sum squared resid	0.088037
Schwarz criterion	-4.018577	Log likelihood	179.0561
Hannan-Quinn criter.	-4.035387	Durbin-Watson stat	2.142048

Appendix B2a

Correlogram of Residuals Squared

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.042	0.042	0.1634	0.686
		2	0.247	0.246	5.7912	0.055
		3	0.217	0.212	10.197	0.017
		4	0.022	-0.046	10.245	0.036
		5	0.387	0.316	24.539	0.000
		6	-0.057	-0.117	24.848	0.000
		7	0.126	-0.016	26.400	0.000
		8	0.111	0.025	27.626	0.001
		9	-0.052	-0.053	27.892	0.001
		10	0.281	0.156	35.925	0.000
		11	-0.038	0.011	36.073	0.000
		12	0.019	-0.104	36.111	0.000
		13	0.194	0.152	40.074	0.000
		14	0.000	0.060	40.074	0.000
		15	0.175	-0.027	43.391	0.000
		16	-0.024	-0.030	43.456	0.000
		17	0.138	0.146	45.589	0.000
		18	0.084	-0.081	46.384	0.000
		19	-0.021	-0.023	46.434	0.000
		20	-0.001	-0.165	46.434	0.001
		21	-0.073	-0.057	47.059	0.001
		22	-0.103	-0.143	48.321	0.001
		23	-0.045	-0.050	48.563	0.001
		24	-0.060	0.010	49.009	0.002
		25	-0.069	0.034	49.602	0.002
		26	-0.100	-0.063	50.877	0.002
		27	-0.118	-0.096	52.677	0.002
		28	-0.100	-0.089	53.991	0.002
		29	-0.075	0.030	54.738	0.003
		30	-0.071	-0.014	55.428	0.003
		31	-0.149	-0.122	58.529	0.002
		32	-0.098	-0.023	59.892	0.002
		33	-0.034	0.148	60.055	0.003
		34	0.042	0.115	60.319	0.004
		35	-0.117	-0.081	62.376	0.003
		36	-0.106	-0.053	64.093	0.003

Appendix B2b

Dependent Variable: LOG_RETURN_P_
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/31/20 Time: 20:03
Sample (adjusted): 1/03/2020 5/08/2020
Included observations: 88 after adjustments
Convergence achieved after 20 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003626	0.002832	1.280687	0.2003
		Variance Equation		
C	0.000859	0.000577	1.489676	0.1363
RESID(-1)^2	0.427657	0.483538	0.884432	0.3765
T-DIST. DOF	3.706023	2.786848	1.329826	0.1836

Root MSE	0.031751	R-squared	-0.007723
Mean dependent var	0.000847	Adjusted R-squared	-0.007723
S.D. dependent var	0.031811	S.E. of regression	0.031933
Akaike info criterion	-4.025609	Sum squared resid	0.088717
Schwarz criterion	-3.913003	Log likelihood	181.1268
Hannan-Quinn criter.	-3.980243	Durbin-Watson stat	2.125632

Appendix B2c

Correlogram of Standardized Residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.060	-0.060	0.3278	0.567
		2	0.143	0.140	2.2222	0.329
		3	0.078	0.096	2.7828	0.426
		4	0.082	0.075	3.4217	0.490
		5	0.137	0.128	5.2180	0.390
		6	-0.022	-0.034	5.2652	0.510
		7	0.220	0.176	9.9764	0.190
		8	0.034	0.045	10.090	0.259
		9	-0.006	-0.069	10.093	0.343
		10	-0.081	-0.148	10.763	0.376
		11	0.155	0.130	13.248	0.277
		12	0.036	0.039	13.384	0.342
		13	0.112	0.118	14.720	0.325
		14	-0.032	-0.069	14.831	0.390
		15	0.009	-0.047	14.840	0.463
		16	-0.002	-0.043	14.840	0.536
		17	0.020	0.077	14.885	0.604
		18	-0.008	-0.072	14.892	0.669
		19	0.202	0.207	19.589	0.420
		20	-0.102	-0.152	20.795	0.409
		21	-0.024	-0.024	20.863	0.467
		22	-0.138	-0.196	23.131	0.394
		23	-0.138	-0.126	25.455	0.327
		24	-0.025	-0.143	25.531	0.377
		25	-0.053	0.108	25.877	0.414
		26	-0.094	-0.196	26.995	0.410
		27	-0.140	0.002	29.522	0.336
		28	-0.026	-0.012	29.608	0.382
		29	-0.149	0.036	32.591	0.295
		30	0.005	-0.054	32.595	0.340
		31	-0.143	-0.019	35.453	0.266
		32	0.100	-0.004	36.857	0.254
		33	-0.235	-0.103	44.825	0.082
		34	-0.146	-0.126	47.953	0.057
		35	-0.065	0.003	48.576	0.063
		36	-0.091	-0.061	49.847	0.062

Appendix B2d

We want to check for leftover ARCH effects to see if the ARCH(1) model is adequate. We can formally test this by doing an ARCH test up to 5 lags.
A LM test for ARCH effect (5 lags) will be used, desired hypothesis:
Auxiliary equation:

$$\sigma_t^2 = \gamma_0 + \gamma_1 \widehat{\varepsilon}_{t-1}^{\,2} + \gamma_2 \widehat{\varepsilon}_{t-2}^{\,2} + \ldots + \gamma_5 \widehat{\varepsilon}_{t-5}^{\,2} + v_t$$

$$H_0: \gamma_1 = \gamma_2 = \ldots = \gamma_5 = 0 \ \text{[No ARCH]}$$
$$H_1: \text{At least one of } \gamma_i \neq 0 \ \text{[ARCH]}$$

Lag 1

Heteroskedasticity Test: ARCH

F-statistic	1.132376	Prob. F(1,85)	0.2903
Obs*R-squared	1.143783	Prob. Chi-Square(1)	0.2849

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:10
Sample (adjusted): 1/06/2020 5/08/2020
Included observations: 87 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.966357	0.182918	5.283019	0.0000
WGT_RESID^2(-1)	-0.114652	0.107743	-1.064132	0.2903

Root MSE	1.450040	R-squared	0.013147
Mean dependent var	0.866975	Adjusted R-squared	0.001537
S.D. dependent var	1.468129	S.E. of regression	1.467001
Akaike info criterion	3.627037	Sum squared resid	182.9277
Schwarz criterion	3.683724	Log likelihood	-155.7761
Hannan-Quinn criter.	3.649863	F-statistic	1.132376
Durbin-Watson stat	1.956121	Prob(F-statistic)	0.290283

The p-value = 0.2849 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 1.

Lag 2

Heteroskedasticity Test: ARCH

F-statistic	2.157452	Prob. F(2,83)	0.1221
Obs*R-squared	4.249924	Prob. Chi-Square(2)	0.1194

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:12
Sample (adjusted): 1/07/2020 5/08/2020
Included observations: 86 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.795781	0.210891	3.773414	0.0003
WGT_RESID^2(-1)	-0.096595	0.107792	-0.896119	0.3728
WGT_RESID^2(-2)	0.189022	0.107675	1.755486	0.0829

Root MSE	1.428922	R-squared	0.049418
Mean dependent var	0.876220	Adjusted R-squared	0.026512
S.D. dependent var	1.474190	S.E. of regression	1.454517
Akaike info criterion	3.621486	Sum squared resid	175.5965
Schwarz criterion	3.707103	Log likelihood	-152.7239
Hannan-Quinn criter.	3.655943	F-statistic	2.157452
Durbin-Watson stat	2.084237	Prob(F-statistic)	0.122060

The p-value = 0.1194 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 2.

<u>Lag 3</u>

Heteroskedasticity Test: ARCH

F-statistic	2.846131	Prob. F(3,81)	0.0427
Obs*R-squared	8.105611	Prob. Chi-Square(3)	0.0439

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:13
Sample (adjusted): 1/08/2020 5/08/2020
Included observations: 85 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.632712	0.227379	2.782628	0.0067
WGT_RESID^2(-1)	-0.141153	0.108399	-1.302167	0.1965
WGT_RESID^2(-2)	0.206922	0.106955	1.934665	0.0565
WGT_RESID^2(-3)	0.220757	0.108246	2.039408	0.0447

Root MSE	1.399821	R-squared	0.095360
Mean dependent var	0.885360	Adjusted R-squared	0.061855
S.D. dependent var	1.480486	S.E. of regression	1.433968
Akaike info criterion	3.604683	Sum squared resid	166.5573
Schwarz criterion	3.719631	Log likelihood	-149.1990
Hannan-Quinn criter.	3.650918	F-statistic	2.846131
Durbin-Watson stat	1.996129	Prob(F-statistic)	0.042681

The p-value = 0.0439 < 0.05 (level of significance) and therefore we reject the null concluding that there is sufficient evidence to prove that there are leftover ARCH effects present in lag 3.

<u>Lag 4</u>

Heteroskedasticity Test: ARCH

F-statistic	2.049258	Prob. F(4,79)	0.0954
Obs*R-squared	7.896492	Prob. Chi-Square(4)	0.0954

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:14
Sample (adjusted): 1/09/2020 5/08/2020
Included observations: 84 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.649808	0.243266	2.671178	0.0092
WGT_RESID^2(-1)	-0.141521	0.112594	-1.256918	0.2125
WGT_RESID^2(-2)	0.205731	0.110984	1.853708	0.0675
WGT_RESID^2(-3)	0.217350	0.110844	1.960865	0.0534
WGT_RESID^2(-4)	-0.007614	0.112306	-0.067794	0.9461

Root MSE	1.407022	R-squared	0.094006
Mean dependent var	0.894248	Adjusted R-squared	0.048133
S.D. dependent var	1.487095	S.E. of regression	1.450865
Akaike info criterion	3.639875	Sum squared resid	166.2956
Schwarz criterion	3.784566	Log likelihood	-147.8747
Hannan-Quinn criter.	3.698040	F-statistic	2.049258
Durbin-Watson stat	1.993223	Prob(F-statistic)	0.095449

The p-value = 0.0954 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 4.

Lag 5

Heteroskedasticity Test: ARCH

F-statistic	2.817354	Prob. F(5,77)	0.0217
Obs*R-squared	12.83613	Prob. Chi-Square(5)	0.0250

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:15
Sample (adjusted): 1/10/2020 5/08/2020
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.490122	0.250509	1.956505	0.0540
WGT_RESID^2(-1)	-0.140281	0.110231	-1.272607	0.2070
WGT_RESID^2(-2)	0.148056	0.111461	1.328317	0.1880
WGT_RESID^2(-3)	0.162387	0.111114	1.461442	0.1480
WGT_RESID^2(-4)	0.027801	0.111039	0.250367	0.8030
WGT_RESID^2(-5)	0.259195	0.109955	2.357286	0.0210

Root MSE	1.366997	R-squared	0.154652
Mean dependent var	0.897527	Adjusted R-squared	0.099760
S.D. dependent var	1.495830	S.E. of regression	1.419258
Akaike info criterion	3.607689	Sum squared resid	155.1006
Schwarz criterion	3.782545	Log likelihood	-143.7191
Hannan-Quinn criter.	3.677936	F-statistic	2.817354
Durbin-Watson stat	1.979444	Prob(F-statistic)	0.021721

The p-value = 0.0217 < 0.05 (level of significance) and therefore we reject the null concluding that there is sufficient evidence to prove that there are leftover ARCH effects present in lag 5.

Appendix B2e

Dependent Variable: LOG_RETURN_P_
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/31/20 Time: 19:58
Sample (adjusted): 1/03/2020 5/08/2020
Included observations: 88 after adjustments
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.004278	0.002603	1.643532	0.1003
		Variance Equation		
C	0.000475	0.000204	2.325957	0.0200
RESID(-1)^2	0.234458	0.235306	0.996397	0.3191
RESID(-2)^2	0.465957	0.353722	1.317298	0.1877
T-DIST. DOF	5.111721	4.139773	1.234783	0.2169

Root MSE	0.031815	R-squared	-0.011768
Mean dependent var	0.000847	Adjusted R-squared	-0.011768
S.D. dependent var	0.031811	S.E. of regression	0.031997
Akaike info criterion	-4.088742	Sum squared resid	0.089073
Schwarz criterion	-3.947984	Log likelihood	184.9046
Hannan-Quinn criter.	-4.032034	Durbin-Watson stat	2.117133

Appendix B2f

Correlogram of Standardized Residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.059	-0.059	0.3207	0.571
		2	0.059	0.056	0.6408	0.726
		3	0.103	0.111	1.6336	0.652
		4	0.061	0.072	1.9833	0.739
		5	0.051	0.048	2.2275	0.817
		6	0.017	0.005	2.2568	0.895
		7	0.135	0.120	4.0336	0.776
		8	0.005	0.008	4.0362	0.854
		9	-0.006	-0.028	4.0395	0.909
		10	-0.035	-0.071	4.1606	0.940
		11	0.162	0.145	6.8719	0.809
		12	0.006	0.026	6.8758	0.866
		13	0.136	0.138	8.8214	0.786
		14	0.002	-0.024	8.8219	0.842
		15	0.051	0.025	9.0988	0.872
		16	0.019	-0.014	9.1377	0.908
		17	0.036	0.034	9.2825	0.931
		18	-0.036	-0.099	9.4258	0.949
		19	0.171	0.165	12.789	0.849
		20	-0.116	-0.151	14.345	0.813
		21	-0.038	-0.025	14.516	0.846
		22	-0.177	-0.277	18.274	0.690
		23	-0.143	-0.144	20.764	0.595
		24	-0.060	-0.169	21.216	0.626
		25	-0.014	0.095	21.239	0.679
		26	-0.069	-0.139	21.839	0.697
		27	-0.134	-0.033	24.162	0.621
		28	0.052	-0.001	24.522	0.654
		29	-0.116	0.034	26.334	0.608
		30	-0.013	-0.087	26.358	0.657
		31	-0.077	0.013	27.192	0.663
		32	0.146	0.091	30.200	0.558
		33	-0.247	-0.107	38.963	0.219
		34	-0.115	-0.091	40.907	0.193
		35	-0.090	-0.053	42.120	0.190
		36	-0.108	-0.075	43.911	0.171

Appendix B2g

We want to check for leftover ARCH effects to see if the ARCH(2) model is adequate. We can formally test this by doing an ARCH test up to 5 lags.
A LM test for ARCH effect (5 lags) will be used, desired hypothesis:
Auxiliary equation:

$$\sigma_t^2 = \gamma_0 + \gamma_1 \widehat{\varepsilon}_{t-1}^{\,2} + \gamma_2 \widehat{\varepsilon}_{t-2}^{\,2} + \ldots + \gamma_5 \widehat{\varepsilon}_{t-5}^{\,2} + v_t$$

$$H_0: \gamma_1 = \gamma_2 = \ldots = \gamma_5 = 0 \ \text{[No ARCH]}$$
$$H_1: \text{At least one of } \gamma_i \neq 0 \ \text{[ARCH]}$$

<u>Lag 1</u>

Heteroskedasticity Test: ARCH

F-statistic	0.448768	Prob. F(1,85)	0.5047
Obs*R-squared	0.456915	Prob. Chi-Square(1)	0.4991

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:22
Sample (adjusted): 1/06/2020 5/08/2020
Included observations: 87 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.035402	0.210487	4.919073	0.0000
WGT_RESID^2(-1)	-0.072462	0.108169	-0.669902	0.5047

Root MSE	1.685077	R-squared	0.005252
Mean dependent var	0.965465	Adjusted R-squared	-0.006451
S.D. dependent var	1.699313	S.E. of regression	1.704786
Akaike info criterion	3.927476	Sum squared resid	247.0350
Schwarz criterion	3.984164	Log likelihood	-168.8452
Hannan-Quinn criter.	3.950302	F-statistic	0.448768
Durbin-Watson stat	2.008604	Prob(F-statistic)	0.504736

The p-value = 0.4991 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 1.

<u>Lag 2</u>

Heteroskedasticity Test: ARCH

F-statistic	0.397211	Prob. F(2,83)	0.6735
Obs*R-squared	0.815332	Prob. Chi-Square(2)	0.6652

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:23
Sample (adjusted): 1/07/2020 5/08/2020
Included observations: 86 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.113191	0.242660	4.587448	0.0000
WGT_RESID^2(-1)	-0.080545	0.109591	-0.734963	0.4644
WGT_RESID^2(-2)	-0.061115	0.109498	-0.558139	0.5783

Root MSE	1.688880	R-squared	0.009481
Mean dependent var	0.975084	Adjusted R-squared	-0.014387
S.D. dependent var	1.706896	S.E. of regression	1.719131
Akaike info criterion	3.955776	Sum squared resid	245.2991
Schwarz criterion	4.041392	Log likelihood	-167.0984
Hannan-Quinn criter.	3.990232	F-statistic	0.397211
Durbin-Watson stat	1.989504	Prob(F-statistic)	0.673463

The p-value = 0.6652 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 2.

<u>Lag 3</u>

Heteroskedasticity Test: ARCH

F-statistic	0.448464	Prob. F(3,81)	0.7191
Obs*R-squared	1.388765	Prob. Chi-Square(3)	0.7082

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:25
Sample (adjusted): 1/08/2020 5/08/2020
Included observations: 85 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.044652	0.276661	3.775934	0.0003
WGT_RESID^2(-1)	-0.079085	0.110823	-0.713618	0.4775
WGT_RESID^2(-2)	-0.058696	0.110925	-0.529151	0.5981
WGT_RESID^2(-3)	0.077133	0.110737	0.696545	0.4881

Root MSE	1.690559	R-squared	0.016338
Mean dependent var	0.984748	Adjusted R-squared	-0.020093
S.D. dependent var	1.714658	S.E. of regression	1.731799
Akaike info criterion	3.982114	Sum squared resid	242.9293
Schwarz criterion	4.097062	Log likelihood	-165.2398
Hannan-Quinn criter.	4.028349	F-statistic	0.448464
Durbin-Watson stat	2.004782	Prob(F-statistic)	0.719061

The p-value = 0.7082 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 3.

Lag 4

Heteroskedasticity Test: ARCH

F-statistic	0.358941	Prob. F(4,79)	0.8371
Obs*R-squared	1.499384	Prob. Chi-Square(4)	0.8268

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:26
Sample (adjusted): 1/09/2020 5/08/2020
Included observations: 84 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.026582	0.306661	3.347615	0.0013
WGT_RESID^2(-1)	-0.084784	0.112546	-0.753322	0.4535
WGT_RESID^2(-2)	-0.059763	0.112554	-0.530975	0.5969
WGT_RESID^2(-3)	0.076459	0.112565	0.679240	0.4990
WGT_RESID^2(-4)	0.034915	0.112319	0.310852	0.7567

Root MSE	1.697481	R-squared	0.017850
Mean dependent var	0.993317	Adjusted R-squared	-0.031879
S.D. dependent var	1.723124	S.E. of regression	1.750374
Akaike info criterion	4.015215	Sum squared resid	242.0410
Schwarz criterion	4.159906	Log likelihood	-163.6390
Hannan-Quinn criter.	4.073380	F-statistic	0.358941
Durbin-Watson stat	1.999446	Prob(F-statistic)	0.837084

The p-value = 0.8268 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 4.

Heteroskedasticity Test: ARCH

F-statistic	0.330989	Prob. F(5,77)	0.8928
Obs*R-squared	1.746366	Prob. Chi-Square(5)	0.8830

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:26
Sample (adjusted): 1/10/2020 5/08/2020
Included observations: 83 after adjustments

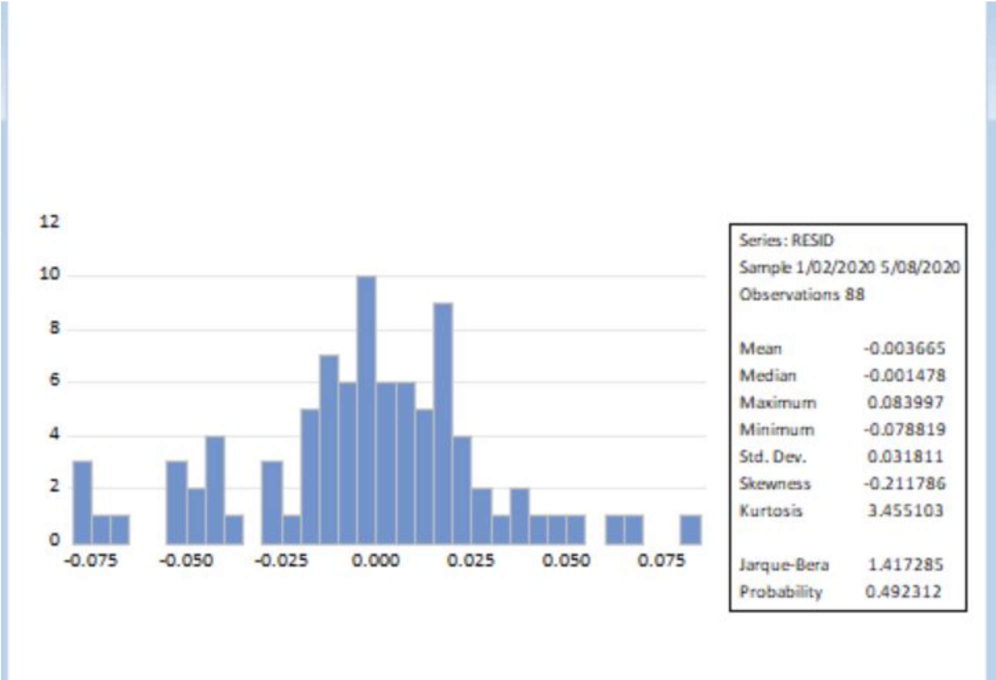
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.970400	0.333892	2.906325	0.0048
WGT_RESID^2(-1)	-0.087050	0.114046	-0.763292	0.4476
WGT_RESID^2(-2)	-0.064468	0.114455	-0.563261	0.5749
WGT_RESID^2(-3)	0.079490	0.114271	0.695629	0.4888
WGT_RESID^2(-4)	0.039455	0.114214	0.345452	0.7307
WGT_RESID^2(-5)	0.057018	0.113836	0.500880	0.6179

Root MSE	1.704900	R-squared	0.021041
Mean dependent var	0.993365	Adjusted R-squared	-0.042528
S.D. dependent var	1.733599	S.E. of regression	1.770078
Akaike info criterion	4.049468	Sum squared resid	241.2547
Schwarz criterion	4.224324	Log likelihood	-162.0529
Hannan-Quinn criter.	4.119715	F-statistic	0.330989
Durbin-Watson stat	1.978493	Prob(F-statistic)	0.892780

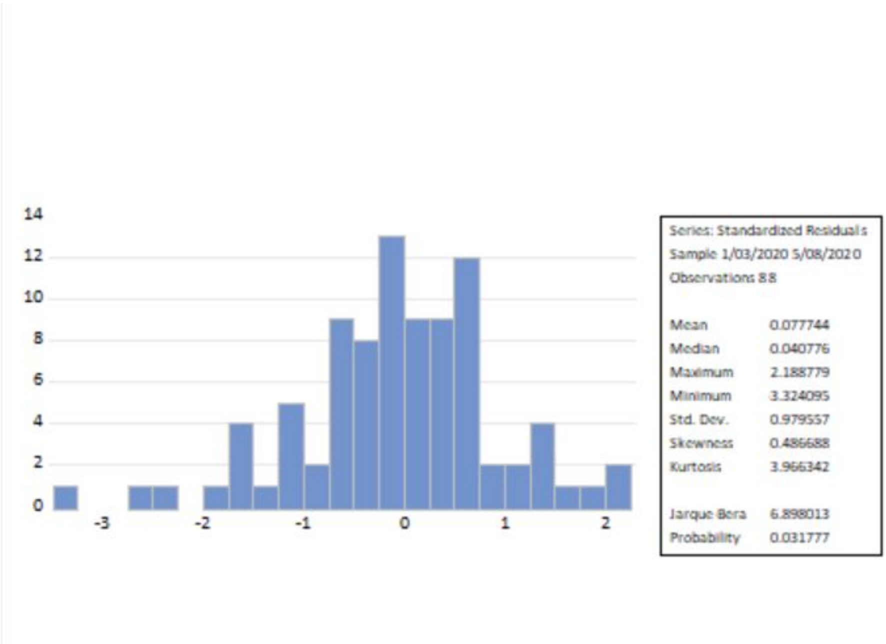
The p-value = 0.8830 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 5.

Appendix B2h

Residuals of Portfolio Returns



Standardised Residuals of ARCH(2)



Jarque-Bera test would be used for normality testing, hypothesis testing:

$$H_0 : Skewness\ (x) = 0\ and\ Kurtosis\ (x) = 3$$
$$H_1 : Skewness\ (x) \neq 0\ or\ Kurtosis\ (x) \neq 3$$

$$JB\ stat = \frac{n}{6}(\widehat{Sk}(x)^2 + \frac{(\widehat{K}(x)-3)^2}{4}) \sim \mathscr{X}^2_{(\alpha,m)} \text{ under } H_0$$

JB stat= 6.90. Under chi square distribution, the critical value at 5% level of significance and 2 degrees of freedom, $\mathscr{X}^2_{(0.05,2)}$ is 5.99. Since 6.90 > 5.99 (JB stat > $\mathscr{X}^2_{(0.05,2)}$), we reject the null hypothesis and conclude that there is sufficient evidence to prove that the distribution of the regression model is not normal.

Appendix B2i

Dependent Variable: LOG_RETURN_P_
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/31/20 Time: 18:43
Sample (adjusted): 1/03/2020 5/08/2020
Included observations: 88 after adjustments
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.004512	0.002744	1.643939	0.1002
Variance Equation				
C	3.86E-05	3.49E-05	1.105753	0.2688
RESID(-1)^2	0.134628	0.197346	0.682196	0.4951
RESID(-2)^2	0.003481	0.210535	0.016534	0.9868
GARCH(-1)	0.845603	0.117775	7.179796	0.0000
T-DIST. DOF	6.142670	4.866835	1.262149	0.2069

Root MSE	0.031841	R-squared	-0.013424
Mean dependent var	0.000847	Adjusted R-squared	-0.013424
S.D. dependent var	0.031811	S.E. of regression	0.032024
Akaike info criterion	-4.143402	Sum squared resid	0.089219
Schwarz criterion	-3.974493	Log likelihood	188.3097
Hannan-Quinn criter.	-4.075353	Durbin-Watson stat	2.113673

Appendix B2j

Correlogram of Standardized Residuals

	AC	PAC	Q-Stat	Prob*
1	-0.031	-0.031	0.0850	0.771
2	0.138	0.137	1.8303	0.400
3	0.076	0.085	2.3668	0.500
4	0.094	0.082	3.1977	0.525
5	0.137	0.126	4.9867	0.418
6	0.026	0.010	5.0538	0.537
7	0.105	0.063	6.1222	0.526
8	0.004	-0.020	6.1238	0.633
9	-0.013	-0.062	6.1400	0.726
10	-0.046	-0.084	6.3504	0.785
11	0.153	0.144	8.7610	0.644
12	-0.020	-0.005	8.8014	0.720
13	0.157	0.148	11.389	0.578
14	0.011	0.026	11.401	0.654
15	0.079	0.050	12.079	0.673
16	0.008	-0.042	12.087	0.738
17	0.076	0.046	12.724	0.754
18	-0.075	-0.165	13.357	0.770
19	0.173	0.156	16.779	0.605
20	-0.104	-0.135	18.029	0.585
21	-0.025	-0.021	18.106	0.642
22	-0.209	-0.286	23.372	0.381
23	-0.132	-0.080	25.493	0.325
24	-0.030	-0.120	25.601	0.374
25	-0.067	0.101	26.170	0.399
26	-0.069	-0.128	26.779	0.421
27	-0.176	-0.025	30.794	0.280
28	0.012	-0.027	30.813	0.325
29	-0.170	-0.010	34.696	0.215
30	0.010	-0.103	34.709	0.253
31	-0.096	0.031	35.983	0.247
32	0.110	0.050	37.683	0.225
33	-0.229	-0.112	45.208	0.076
34	-0.125	-0.140	47.485	0.062
35	-0.083	0.010	48.521	0.064
36	-0.099	-0.080	50.013	0.060

Correlogram of Standardized Residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.031	-0.031	0.0850	0.771
		2	0.138	0.137	1.8303	0.400
		3	0.076	0.085	2.3668	0.500
		4	0.094	0.082	3.1977	0.525
		5	0.137	0.126	4.9867	0.418
		6	0.026	0.010	5.0538	0.537
		7	0.105	0.063	6.1222	0.526
		8	0.004	-0.020	6.1238	0.633
		9	-0.013	-0.062	6.1400	0.726
		10	-0.046	-0.084	6.3504	0.785
		11	0.153	0.144	8.7610	0.644
		12	-0.020	-0.005	8.8014	0.720
		13	0.157	0.148	11.389	0.578
		14	0.011	0.026	11.401	0.654
		15	0.079	0.050	12.079	0.673
		16	0.008	-0.042	12.087	0.738
		17	0.076	0.046	12.724	0.754
		18	-0.075	-0.165	13.357	0.770
		19	0.173	0.156	16.779	0.605
		20	-0.104	-0.135	18.029	0.585
		21	-0.025	-0.021	18.106	0.642
		22	-0.209	-0.286	23.372	0.381
		23	-0.132	-0.080	25.493	0.325
		24	-0.030	-0.120	25.601	0.374
		25	-0.067	0.101	26.170	0.399
		26	-0.069	-0.128	26.779	0.421
		27	-0.176	-0.025	30.794	0.280
		28	0.012	-0.027	30.813	0.325
		29	-0.170	-0.010	34.696	0.215
		30	0.010	-0.103	34.709	0.253
		31	-0.096	0.031	35.983	0.247
		32	0.110	0.050	37.683	0.225
		33	-0.229	-0.112	45.208	0.076
		34	-0.125	-0.140	47.485	0.062
		35	-0.083	0.010	48.521	0.064
		36	-0.099	-0.080	50.013	0.060

Appendix B2k

We want to check for leftover ARCH effects to see if the GARCH(2,1) model is adequate.
We can formally test this by doing an ARCH test up to 5 lags.
A LM test for ARCH effect (5 lags) will be used, desired hypothesis:
Auxiliary equation:

$$\sigma_t^2 = \gamma_0 + \gamma_1 \widehat{\varepsilon}_{t-1}^{\,2} + \gamma_2 \widehat{\varepsilon}_{t-2}^{\,2} + \ldots + \gamma_5 \widehat{\varepsilon}_{t-5}^{\,2} + v_t$$

$$H_0: \gamma_1 = \gamma_2 = \ldots = \gamma_5 = 0 \text{ [No ARCH]}$$
$$H_1: \text{At least one of } \gamma_i \neq 0 \text{ [ARCH]}$$

Lag 1

Heteroskedasticity Test: ARCH

F-statistic	0.046578	Prob. F(1,85)	0.8296
Obs*R-squared	0.047648	Prob. Chi-Square(1)	0.8272

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:31
Sample (adjusted): 1/06/2020 5/08/2020
Included observations: 87 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.964315	0.234382	4.114295	0.0001
WGT_RESID^2(-1)	0.023405	0.108447	0.215820	0.8296

Root MSE	1.921955	R-squared	0.000548
Mean dependent var	0.987436	Adjusted R-squared	-0.011211
S.D. dependent var	1.933627	S.E. of regression	1.944435
Akaike info criterion	4.190540	Sum squared resid	321.3704
Schwarz criterion	4.247228	Log likelihood	-180.2885
Hannan-Quinn criter.	4.213367	F-statistic	0.046578
Durbin-Watson stat	1.997904	Prob(F-statistic)	0.829645

The p-value = 0.8272 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 1.

Lag 2

Heteroskedasticity Test: ARCH

F-statistic	0.057725	Prob. F(2,83)	0.9439
Obs*R-squared	0.119456	Prob. Chi-Square(2)	0.9420

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:33
Sample (adjusted): 1/07/2020 5/08/2020
Included observations: 86 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.000758	0.261584	3.825764	0.0003
WGT_RESID^2(-1)	0.022638	0.109825	0.206124	0.8372
WGT_RESID^2(-2)	-0.030126	0.109772	-0.274437	0.7844

Root MSE	1.931498	R-squared	0.001389
Mean dependent var	0.993346	Adjusted R-squared	-0.022674
S.D. dependent var	1.944177	S.E. of regression	1.966095
Akaike info criterion	4.224236	Sum squared resid	320.8388
Schwarz criterion	4.309853	Log likelihood	-178.6422
Hannan-Quinn criter.	4.258693	F-statistic	0.057725
Durbin-Watson stat	1.996770	Prob(F-statistic)	0.943948

The p-value = 0.9420 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 2.

Lag 3

Heteroskedasticity Test: ARCH

F-statistic	0.047177	Prob. F(3,81)	0.9863
Obs*R-squared	0.148260	Prob. Chi-Square(3)	0.9855

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:33
Sample (adjusted): 1/08/2020 5/08/2020
Included observations: 85 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.993501	0.289632	3.430221	0.0010
WGT_RESID^2(-1)	0.022214	0.111193	0.199781	0.8422
WGT_RESID^2(-2)	-0.032252	0.111205	-0.290027	0.7725
WGT_RESID^2(-3)	0.016985	0.111217	0.152720	0.8790

Root MSE	1.941373	R-squared	0.001744
Mean dependent var	1.000425	Adjusted R-squared	-0.035228
S.D. dependent var	1.954600	S.E. of regression	1.988731
Akaike info criterion	4.258786	Sum squared resid	320.3590
Schwarz criterion	4.373734	Log likelihood	-176.9984
Hannan-Quinn criter.	4.305021	F-statistic	0.047177
Durbin-Watson stat	1.998343	Prob(F-statistic)	0.986321

The p-value = 0.9855 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 3.

Lag 4

Heteroskedasticity Test: ARCH

F-statistic	0.037644	Prob. F(4,79)	0.9972
Obs*R-squared	0.159801	Prob. Chi-Square(4)	0.9970

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:34
Sample (adjusted): 1/09/2020 5/08/2020
Included observations: 84 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.985197	0.316523	3.112561	0.0026
WGT_RESID^2(-1)	0.021436	0.112654	0.190278	0.8496
WGT_RESID^2(-2)	-0.032243	0.112687	-0.286132	0.7755
WGT_RESID^2(-3)	0.015882	0.112774	0.140827	0.8884
WGT_RESID^2(-4)	0.013400	0.112616	0.118992	0.9056

Root MSE	1.952480	R-squared	0.001902
Mean dependent var	1.003887	Adjusted R-squared	-0.048634
S.D. dependent var	1.966077	S.E. of regression	2.013319
Akaike info criterion	4.295125	Sum squared resid	320.2229
Schwarz criterion	4.439817	Log likelihood	-175.3953
Hannan-Quinn criter.	4.353290	F-statistic	0.037644
Durbin-Watson stat	1.989384	Prob(F-statistic)	0.997243

The p-value = 0.9970 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 4.

Lag 5

Heteroskedasticity Test: ARCH

F-statistic	0.031502	Prob. F(5,77)	0.9995
Obs*R-squared	0.169435	Prob. Chi-Square(5)	0.9994

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/31/20 Time: 23:35
Sample (adjusted): 1/10/2020 5/08/2020
Included observations: 83 after adjustments

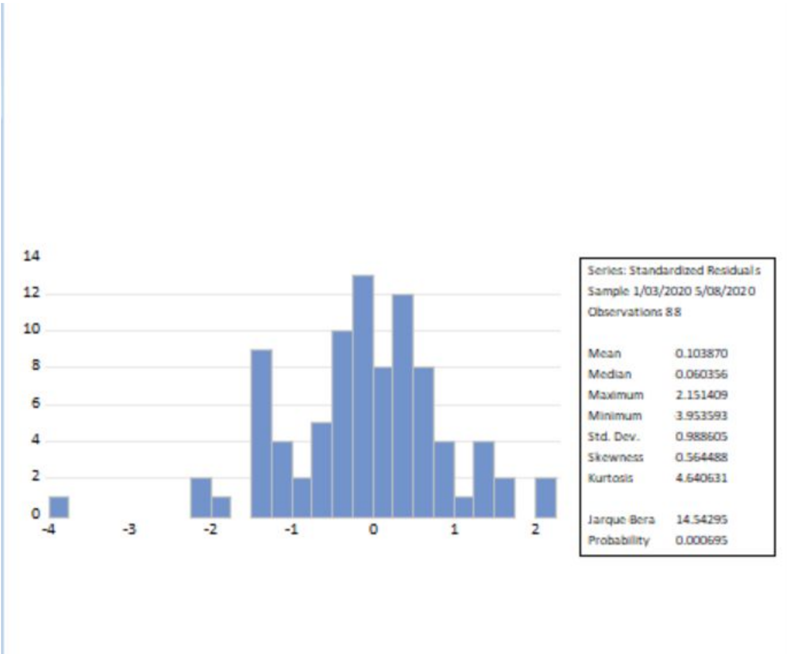
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.966024	0.340801	2.834568	0.0059
WGT_RESID^2(-1)	0.022736	0.113839	0.199722	0.8422
WGT_RESID^2(-2)	-0.029527	0.113933	-0.259162	0.7962
WGT_RESID^2(-3)	0.017806	0.114031	0.156151	0.8763
WGT_RESID^2(-4)	0.017375	0.113953	0.152475	0.8792
WGT_RESID^2(-5)	-0.006392	0.113930	-0.056106	0.9554

Root MSE	1.959040	R-squared	0.002041
Mean dependent var	0.988545	Adjusted R-squared	-0.062761
S.D. dependent var	1.972964	S.E. of regression	2.033935
Akaike info criterion	4.327365	Sum squared resid	318.5407
Schwarz criterion	4.502221	Log likelihood	-173.5856
Hannan-Quinn criter.	4.397612	F-statistic	0.031502
Durbin-Watson stat	1.987100	Prob(F-statistic)	0.999482

The p-value = 0.994 > 0.05 (level of significance) and therefore we do not reject the null concluding that there is insufficient evidence to prove that there are leftover ARCH effects present in lag 5.

Appendix B2l

Standardised Residuals of GARCH(2,1)



Jarque-Bera test would be used for normality testing, hypothesis testing:

$$H_0 : Skewness\ (x) \ = \ 0\ and\ Kurtosis\ (x) \ = 3$$
$$H_1 : Skewness\ (x) \ \neq \ 0\ or\ Kurtosis\ (x) \ \neq 3$$

$$JB\ stat \ = \ \frac{n}{6}(\widehat{Sk}(x)^2 + \frac{(\widehat{K}(x)-3)^2}{4}) \sim \mathscr{X}^2_{(\alpha,m)} \text{ under } H_0$$

JB stat= 14.54. Under chi square distribution, the critical value at 5% level of significance and 2 degrees of freedom, $\mathscr{X}^2_{(0.05,2)}$ is 5.99. Since 14.54 > 5.99 (JB stat > $\mathscr{X}^2_{(0.05,2)}$), we reject the null hypothesis and conclude that there is sufficient evidence to prove that the distribution of the regression model is not normal.

Equation b:

Auxiliary regression for serial correlation : $\widehat{u_t} = \gamma_0 + \rho_1 \widehat{u}_{t-1} + ... + \rho_{36} \widehat{u}_{t-36} + v_t\ where\ i > 1$

Hypotheses: $H_0 : \rho_1 \ = \ \rho_2 \ = \ ... \ = \ \rho_{36} \ = 0\ [No\ autocorrelation]$

$$H_1 : any\ \rho_i \ \neq 0\ [Autocorrelation]$$