Radio Interferometry:

Aperture Synthesis and Antenna Arrays

Dunlap Summer School 2019
Introduction to Astronomical Instrumentation

Introduction

Modern radio telescopes can be divided into two broad classes: single dish and interferometric. The former works much like a reflecting optical telescope, using a large metal mirror to focus light from an aperture into a feed (or cluster of feeds). A single antenna can be used by itself or added to a larger array for use as an *interferometric element*.

Aperture Synthesis is the name given to the coherent joining of an array of antennas into a single large instrument. Developed in the 1960s and cited in the 1974 Nobel Prize, aperture synthesis has become a mainstay of radio astronomy.

You've already spent some time making antennas, understanding the basics of heterodyne receivers, and operating simple radio systems. Today we'll get into things a little deeper, and explore the basics of radio correlation and aperture synthesis.

The goals for this laboratory session are to:

1) Understand radio interferometers!
2) Play with Adding Interferometry.
3) Understand Baselines and Visibilities.
4) Experiment with Radio Arrays.
5) Cross-correlate feeds.
6) Experiment with visibility phases and source localization.

Lab Equipment:

- 2 Antennas from Tue/Wed Antenna Lab
- 2 AirSpy Device2 (with USB cable)
- Laptop running CentOS, with software packages:
  - `dss_js_interferometry` – interactive website
  - `dss_airspy_xcorr` – correlator and visualizer
- 1 rotating tray (AKA 'Lazy Suzan') with tube antenna stand attached
- 360° Protractor Ruler
- Cables:
  - 1 RG58 Coaxial Cable, 3m BNC M-M, carrying 10MHz TTL clock
  - 2 RG174 Coaxial Cable, 36" SMA M-M
  - 2 RG174 Coaxial Cable, 6" SMA M-M
  - 2 RG174 Coaxial Cable, 6" BNC-MCX M-M
  - 1 BNC Tee, FFF
  - 1 SMA Barrel, FF
- 1 RF Power Splitter, SMA FFF, 1-2GHz

Adding Interferometry (60 min)

Goals: Build a phased array!

Let's begin with a simple two-antenna system, and consider a one-dimensional sky, containing a single far-field point source. The layout is shown in Figure 1.

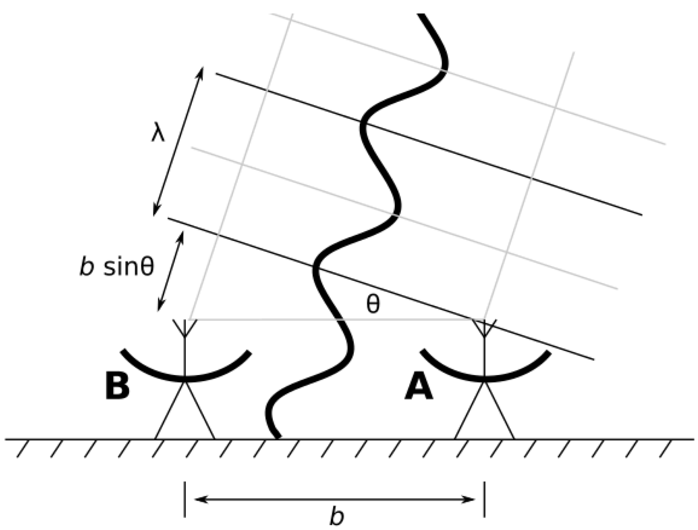


Figure 1: Basic Interferometery Setup

Images are formed by sampling and processing a the incoming wavefront, which will arrive at the two antennas at slightly different times according to its origin on the sky:

$$\Delta t = b/c \cdot \sin(\theta)$$

Equivalently, **B** will sample the wavefront with phase delay proportional to the wavelength being studied:

$$\Delta \phi = 2\pi b/\lambda \cdot \sin(\theta)$$

We can then write the electric fields striking each antenna as:

$$E_B(\omega, t) = Ee^{-i(\omega t+\phi(t)+\Delta\phi)}$$
$$E_A(\omega, t) = Ee^{-i(\omega t+\phi(t))}$$

These are the signals we get to play with to try and figure out what's happening on the sky. As a first option, we can build an **adding interferometer**. In an adding interferometer (also called a phased array), the electric fields are summed and allowed to directly interfere. This is can be done by carrying signals down cables into a combiner, as shown in Figure 2.

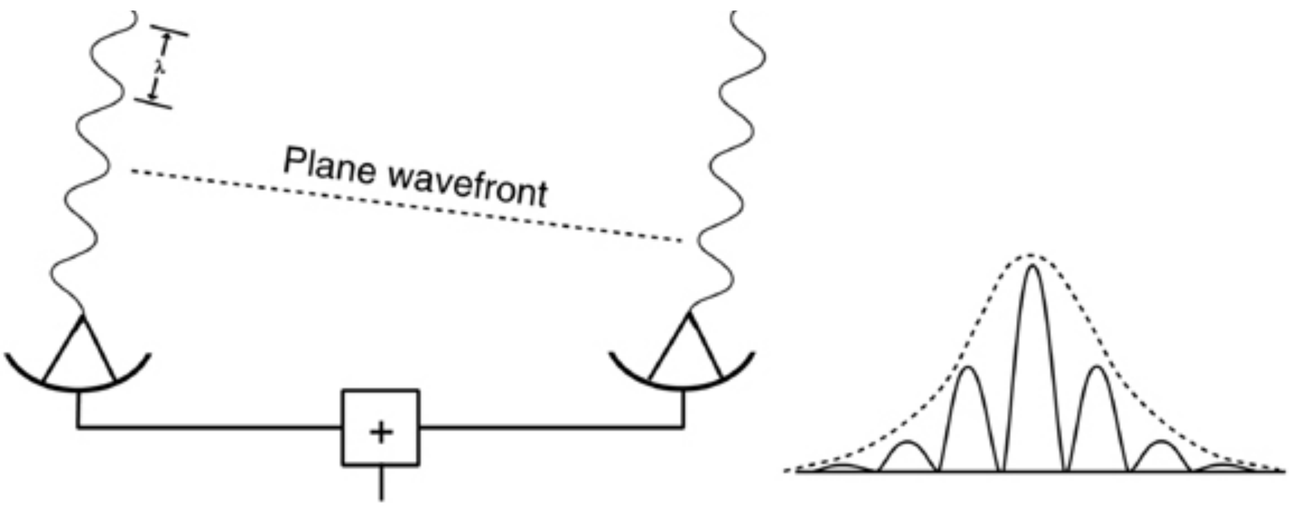


Figure 2: A simple adding interferometer is shown on the left. The individual antenna beams ("primary beams") are shown by the dashed line, while the summed signal is shown in solid lines. The fringe spacing is set by the physical separation of antennas, *b*.

This is essentially a two-slit experiment! By changing the relative lengths of cabling, we change what position on the interference screen is being sampled.

In radio interferometry, these periodic peaks and valleys in the response are called **fringes**. You'll see more of these later on, but in general, a fringe is simply a periodic spatial variation in the response of an instrument.

To start, we're going to build an adding interferometer and measure the summed beam. Take two similar antennas from yesterday, build a little tee to set them on, and connect everything as shown in Figure 3: two antennas connected into the splitter / combiner, with the sum fed into the AirSpy.

If you don't have a pair of very similar antennas, retrieve a pen from the instructors and draw up a pair of simple dipoles tuned to 1420MHz.

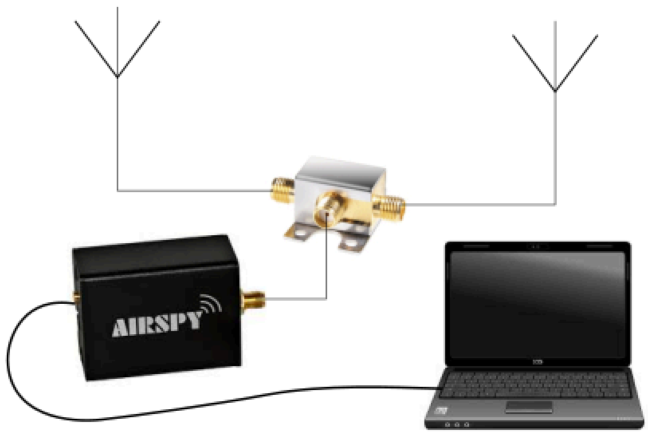


Figure 3: Adding Interferometer setup.

Set-up Tips for Best Results:

1. Make sure the leads for both of your antennas are connected to the same pins (i.e center and left versus center and right) on their SMA connectors. What would happen if they were not?
2. Start with the two antennas relatively close together.
3. For most intuitive results, align your set-up so that 0 degrees corresponds to both antennas pointing directly at the source.

Load up GQRX as in the antenna lab. Transcribe your single antenna maps from earlier – or roughly re-measure – below. Then combine them through the combiner and look at the beam shape.

Rotate your phased array and look for the minima and maxima.

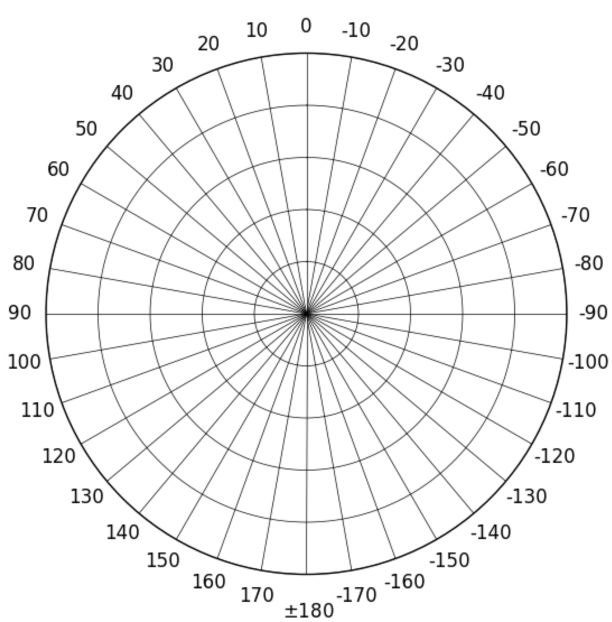
- How much power can you get at maximum?
- How well can you null out the power?
- Connect a single antenna and compare the results.

Consider two feeds that are spaced a distance ***b*** apart and looking directly overhead. Their signals will add coherently at zenith (along bore sight), with a maximal response. At what angle do we expect the first null?

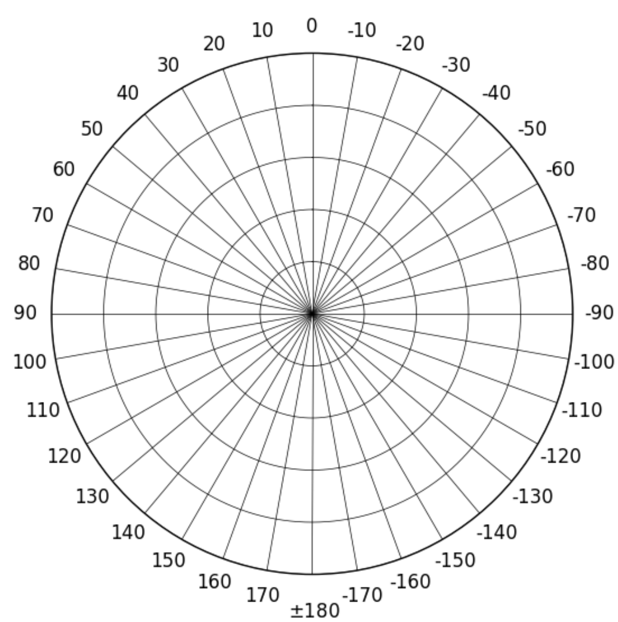
Measure the formed beam pattern!

Measure the power at multiple angles, with antenna spacing ____cm.

Angle (°)	Power (db)	Angle (°)	Power (db)	Angle (°)	Power (db)
0		120		-120	
10		130		-110	
20		140		-100	
30		150		-90	
40		160		-80	
50		170		-70	
60		±180		-60	
70		-170		-50	
80		-160		-40	
90		-150		-30	
100		-140		-20	
110		-130		-10	



Antenna A,B primary beams

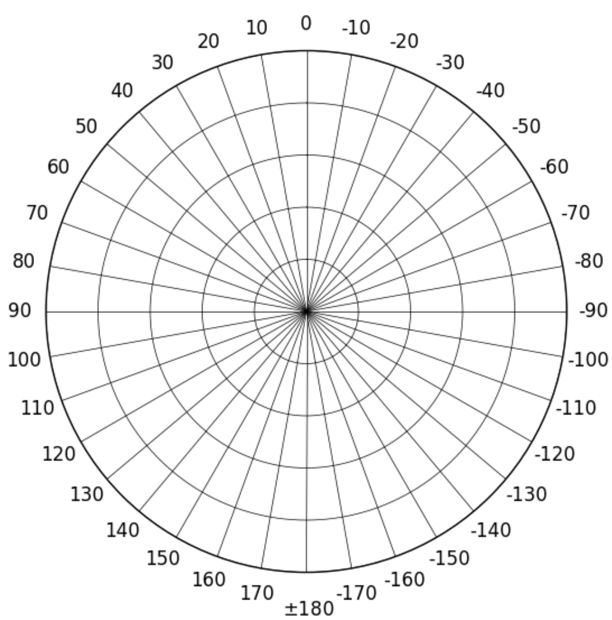


Summed Beam, b = ____ cm

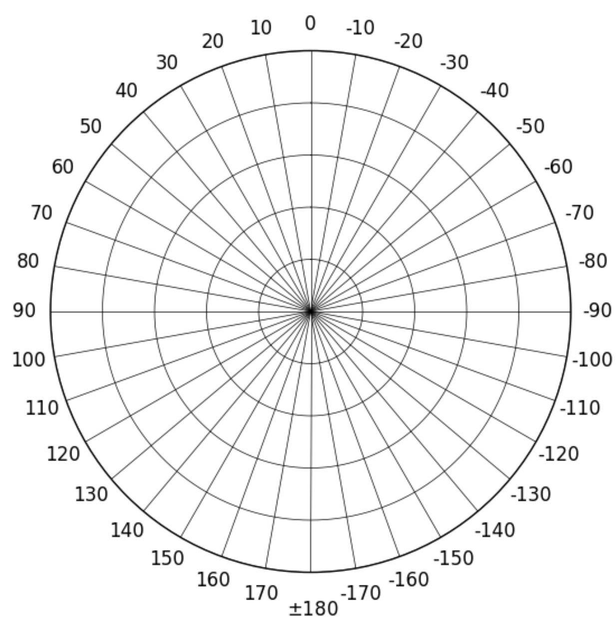
Try at a different (wider / narrower) feed separation.
What happens to the pattern?

Measure the power at multiple angles, with antenna spacing ____cm.

Angle (°)	Power (db)	Angle (°)	Power (db)	Angle (°)	Power (db)
0		120		-120	
10		130		-110	
20		140		-100	
30		150		-90	
40		160		-80	
50		170		-70	
60		±180		-60	
70		-170		-50	
80		-160		-40	
90		-150		-30	
100		-140		-20	
110		-130		-10	



Summed Beam, b = ___ cm

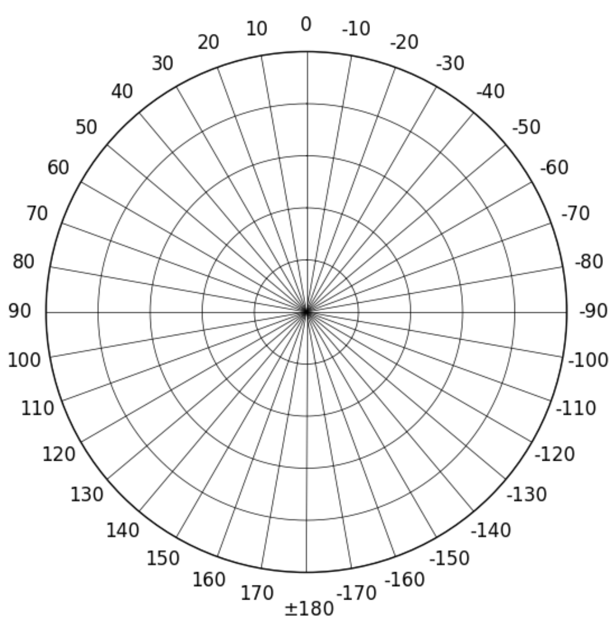


Summed Beam, b = ___ cm

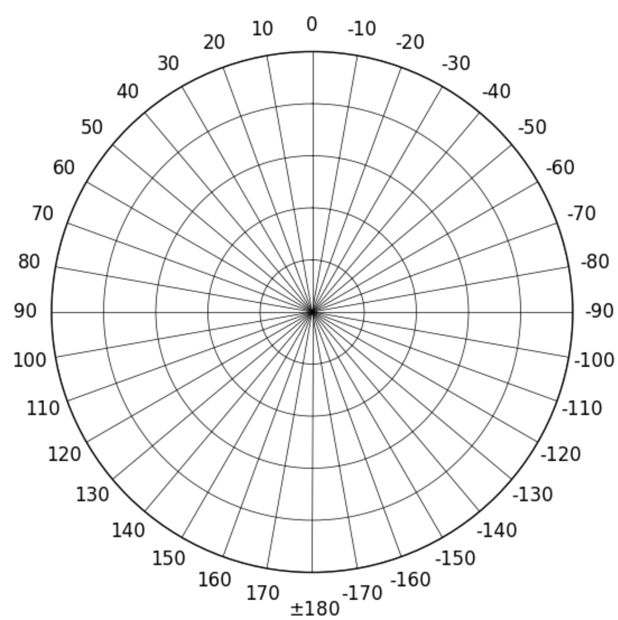
Add an extra short cable to one leg of your interferometer, so that light from one antenna is slightly delayed relative to the other.
What happens to the beam pattern?

Measure the power at multiple angles, with antenna spacing ____cm.

Angle (°)	Power (db)	Angle (°)	Power (db)	Angle (°)	Power (db)
0		120		-120	
10		130		-110	
20		140		-100	
30		150		-90	
40		160		-80	
50		170		-70	
60		±180		-60	
70		-170		-50	
80		-160		-40	
90		-150		-30	
100		-140		-20	
110		-130		-10	



Summed Beam, lag = ___ cm



Summed Beam, lag = ___ cm

Adding interferometers are great, but they're clearly throwing away information: we no longer know about the regions that destructively interfere away. Further, think about what happens if the two antennas aren't perfectly matched. Mismatched amplitudes in their responses will mean things don't full interfere away or add optimally, while phase delays will mean our array is pointing in the wrong direction!

Take a moment to work through the effect. Returning to our earlier example of two feeds spaced a distance ***b*** apart, imagine one of them has slightly longer cable than the order, introducing a $\lambda/2$ lag.

At what angle do their signals now add coherently?

At what angle do we expect the first null?

Compare to your earlier calculations, and you'll see why phased arrays can be dangerous without a careful calibration! Thankfully, there's another option, which makes better use of the available data.

Close GQRX and carry on!

Correlation Interferometry (15 min)

Goals: Correlation basics and the van Cittert-Zernike theorem!

If we generalize to a sky with multiple sources, described by angular coordinates (l,m), we can write the detected fields as

$$E_A(\omega, t) = \int_{l,m} E(l,m)\, e^{-i(\omega t + \phi_{lm}(t))}$$
$$E_B(\omega, t) = \int_{l,m} E(l,m)\, e^{-i(\omega t + \phi_{lm}(t) + \Delta\phi)}$$

where we now have a 2d phase shift given by

$$\Delta\phi = \ 2\pi/\lambda \cdot (u \sin(l) + v \sin(m)) \approx 2\pi/\lambda \cdot (ul + vm)$$

$$u \equiv \Delta x = x_A - x_B \quad ; \quad v \equiv \Delta y = y_A - y_B$$

Consider the time-averaged product,

$$V_{AB} \equiv < E_A E_B^* >$$

This is the definition of a **visibility**, and it describes the correlation between the two antennas. Note that it only depends on $\Delta\phi = b/\lambda \sin(\theta)$, so will be the same for any pair of antennas separated by a distance b.

Plugging in and rearranging, we find,

$$V_{AB} = \frac{1}{\Delta t} \int_t \left[\int_{l,m} E(l,m) e^{-i(\omega t + \phi_{lm}(t))} \cdot \int_{l,m} E(l,m) e^{-i(\omega t + \phi_{lm}(t) + \Delta\phi)} \right]$$

Expanding and collecting terms,

$$V_{AB} = \frac{1}{\Delta t}\int_t \left[\int_{l,m} E^2(l,m)e^{i\Delta\phi} \right.$$
$$\left. + \int_{\substack{l_A \neq l_B \\ m_A \neq m_B}} E(l_A, m_A)E(l_B, m_B)e^{-i\left(\phi_{l_A m_A}(t)\phi_{l_B m_B}(t)+\Delta\phi\right)}\right]$$

Since the phases of light are uncorrelated across sky, the second integral is composed of randomized sinusoids, all of which vanish when integrated over. This leads to the tremendous simplification,

$$V_{AB} = < \int_{l,m} I(l,m)\, e^{i\Delta\phi} >$$
$$\approx < \int_{l,m} E^2(l,m)e^{i(2\pi/\lambda)(ul+vm)} >$$

which the astute reader will recognize as a Fourier Transform. This is a simplified statement of the **Van Cittert-Zernike** theorem, that **Visibilities (V) are the Fourier Transform of the intensity pattern $I(l,m) = E^2(l,m)$ of the sky**.

As we saw in lecture, radio interferometers measure these visibilities, which sample **UV space**, the 2d Fourier conjugate of the sky image. The more parts of UV space we sample, the more information we have about the sky: visibilities near the origin (with short baselines) sample large angular scales on the sky, while visibilities with large baselines sample fine angular structures. You will play with this in the next section.

Calculating visibilities is a computationally intensive task, and the job of a **radio correlator**, which we'll get into in the following sections.

There are a few complications to this formulation. First, the Fourier Transform equality only holds in the flat sky limit, when $\sin(l) \approx l$ and $\sin(m) \approx m$, i.e., within a small field of view. Each antenna's measurement also contains a noise term N_A, which vanishes in all cross-correlations $V_{A\neq B}$ but adds an additional term to auto-correlation visibilities,

$$V_{AB} = < E_A E_B^* > = I e^{-i\Delta\phi} + \delta_{AB} < N_A N_B^* >$$

This is the radio equivalent of read noise, and adds a significant amount of additional power to any measurement made with single antennas. Because of this, interferometers tend to discard the autocorrelation visibilities, and so have no sensitivity to the largest scales on the sky.

JavaScript Interferometer Playpen (15 min)

Goals: Understand 2d Interferometry: UV space and aperture arrays!
Ok, time to play with phony arrays on the computer. On the desktop, open the HTML document `dss_js_interferometry/index.html`

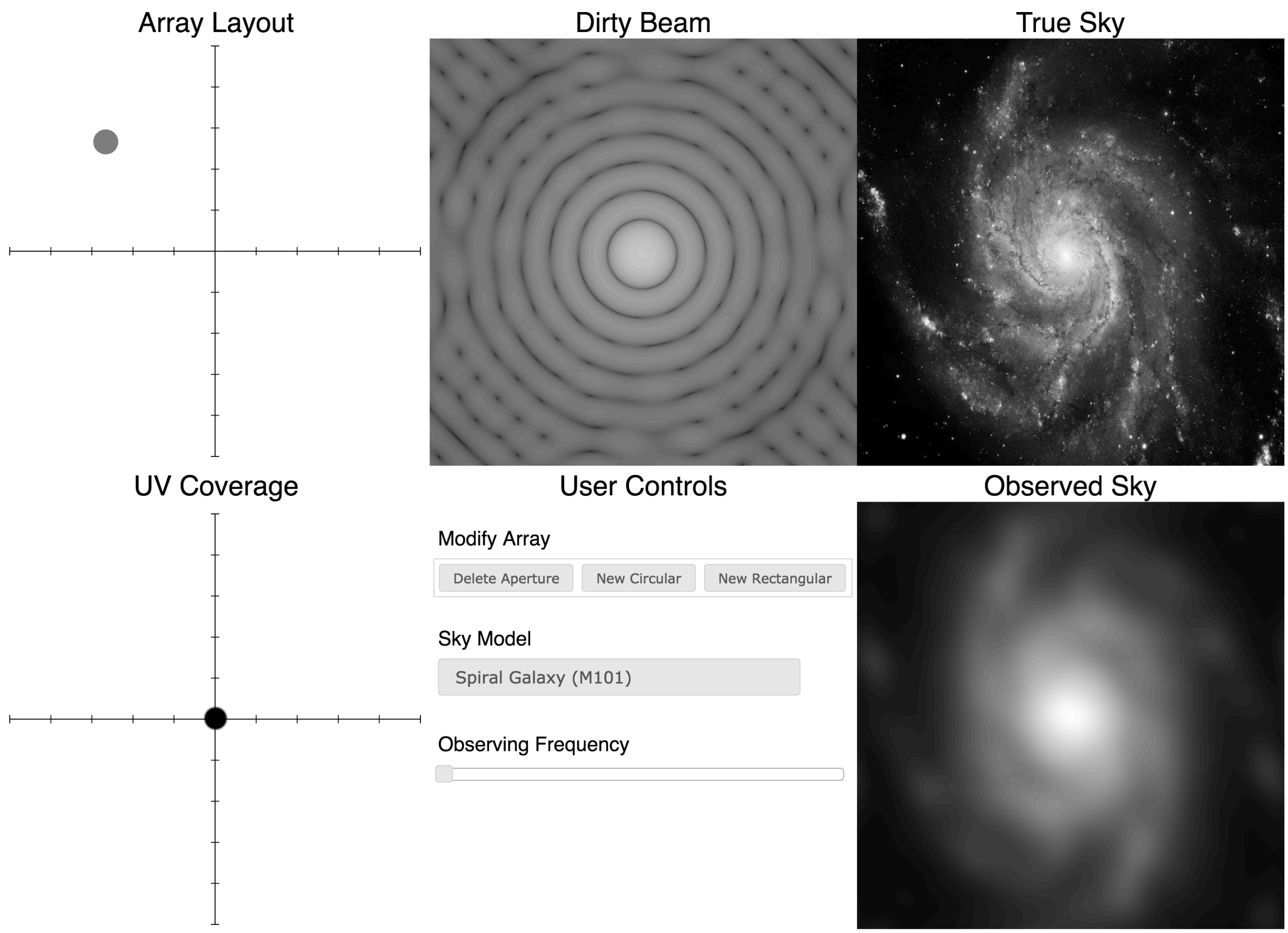


Figure 4: The virtual aperture array interactive webpage.

There are 6 panels in the webpage, you may need to zoom out (ctrl+–) to fit them all:

Array Layout: Top-down view of the layout of your simulated radio array. Each antenna is shown by a grey circle/rectangle, and can be selected (click), moved (click+drag on selected antenna), or resized (click+drag on resize boxes of selected antenna). As you modify things in this panel, the others will automatically update to reflect your changes.

UV Coverage: The UV-plane and which portions are sampled by your array. Darker shades mean more copies of that region are being measured.

Dirty Beam: The on-sky performance of your array. This is the radio equivalent of a point-spread-function, and shows how you will be sampling the sky. Note that the colors here are on a logarithmic scale, to help show sidelobes and structure away from the main beam. This is the Fourier conjugate of the UV coverage.

True Sky: The sky your array will be used to observe. Different skies can be selected from the user interface panel (below).

Observed Sky: How the true sky appears to your array. This is simply the try sky convolved with the dirty beam.

User Controls: Allows you to add antennas to the array (with either circular apertures or rectangular), or delete the selected existing antenna. You can also change the sky model in this pane.

There are several things you should be sure to try here, but feel free to explore freely as well! At a minimum, make sure you:

1) Look at the behavior of a single antenna.
    a. Increase and decrease the size, what happens to the beam pattern and observed sky? Why?
    b. Try moving the aperture. What happens, and why?
    c. Stretch along one axis, leading to an elliptical aperture.
    d. Try a rectangular aperture, explain the shape of the beam & observation.
    e. Look at both the diffuse (galaxy) and point source skies.
2) Add a second antenna.
    a. Close to and far away from the first, observe the fringes.
    b. Place it at different angles relative to the first, observe the fringes.
    c. Look at the UV plane as you move the source. Explain the symmetry.
3) Add additional antennas.
    a. How does adding a new aperture populate UV space? How many new parts of UV are samples with each new antenna?
    b. Move a single feed and observe how the UV sampling changes.
    c. Make a co-linear array, look at the dirty beam and observed sky.
    d. Extend a few apertures perpendicular to the linear array. What happens to the UV coverage, dirty beam, and observed sky?

Cross Correlation: Initialization (30 min)

Goals: Understand the importance of clocking stability in an interferometer. Use external clocks to synchronize the AirSpy units and measure clocking / phase stability of the system.

Before we can do any useful interferometry, it's important to understand the system we'll be using. The basic data flow is shown in Figure 5: a GUI allows us to interact with a high-speed backend, which queries the AirSpy units and processes the samples into visibilities.

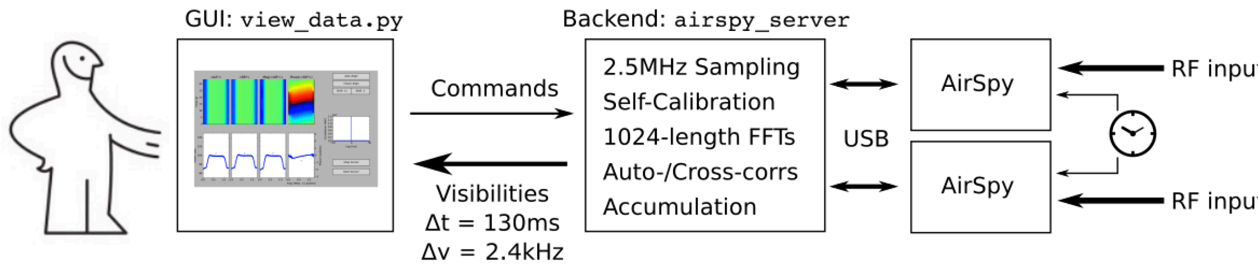


Figure 5: The software chain used in the cross correlation measurements that follow.

Before that will work, though, we need to get all our antennas on the same page, which here means getting the clocks synchronized. While the AirSpy units come with extremely stable on-board clocks, even slight discrepancies can lead to major problems offline. For example, if one AirSpy is generating 10MHz exactly while the other generates 10,000,001Hz, they will slip relative to one another by 1 sample per second. Using an example of correlating every 1024 samples, the signals will completely de-correlate every 17 minutes!

To get around this problem, we will use an external clock that is shared between all units in the lab. Clock distribution is a central part of any radio telescope, but is thankfully a largely solved problem.

A central 10MHz clock is provided for everyone to use, broken out via a clock distribution box. Each station should have a long BNC cable carrying the 10MHz clock to be shared between their receivers. **Connect this BNC to a splitter and to the 6" BNC-MCX patch cables. Connect these MCX cables to the AirSpy external clock input.** See Figure 6 for the layout.

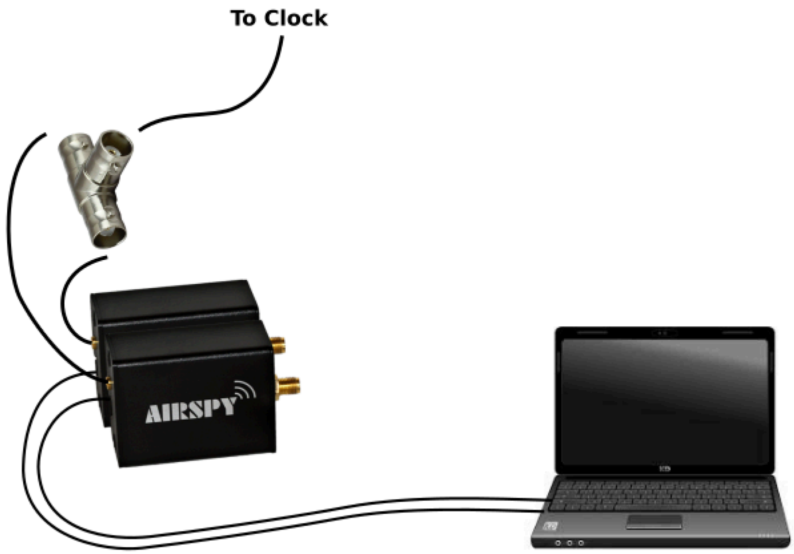


Figure 6: AirSpy with USB and external clock cables connected. Depending on the setup required, in the steps below you will also supply signal input from either from your antenna or from a calibration source.

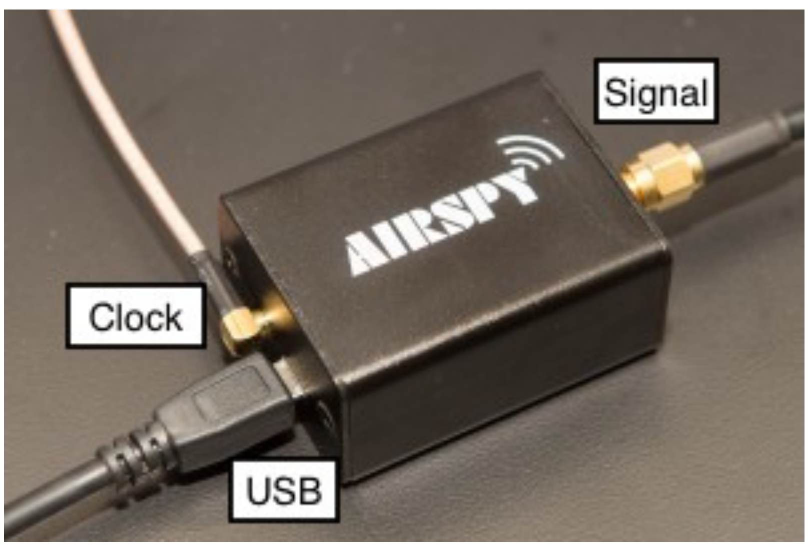


Figure 7: AirSpy with USB, external clock, and signal cables connected.
(The signal cable will be connected in steps below.)

The AirSpy units only look for an external clock on boot, so **power them down (unplug the USB and wait a moment), then bring them back up**. Confirm the external clock is active by opening a terminal and running the commands

```
cd /home/dss/Desktop/dss_airspy_xcorr
python check_clocks.py
```

If it didn't come up, try again. There's no point moving on until this works!

Next, in your terminal, we can **start the viewer**. It is responsible for talking to the AirSpy units, collecting information, aligning the signals, and processing them into a useful form. Bring up the data viewer by running (in a terminal):

```
cd /home/dss/Desktop/dss_airspy_xcorr
python -i view_data.py
```

Once the viewer comes up, click the Start Server button to initialize the correlator backend. Data should begin to flow, and you can see the major pieces of information a radio telescope records.

The top row shows **waterfall** plots, which scroll color-coded spectra across the screen over time. The bottom row shows plots of the most recent spectra.

Left-to-right, the columns show the auto-correlations <AA*>, <BB*> (1st and 2nd), followed by the cross-correlation <AB*>. Because the cross-correlation is a complex value, it is displayed as a Magnitude (3rd column) and Phase (4th column). An example is shown in Figure 8.

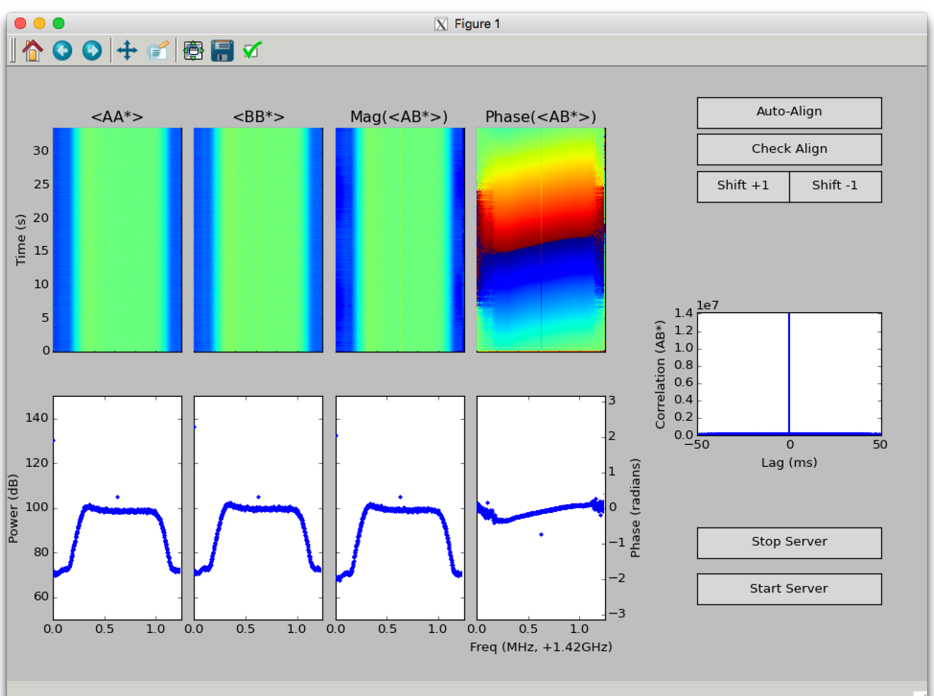


Figure 8: A typical set of plots showing measured data.

The last step before things make sense is to *align the data*. Because the two AirSpy units were initialized independently, their data may be several milliseconds misaligned. To fix this, you will need a calibration signal. **Request the broadband calibration source, and connect it via the SMA combiner/splitter to both AirSpy units' inputs**, as shown in Figure 9. With both units receiving the same signal, we are ready to synchronize. Click the **Check Align** button.

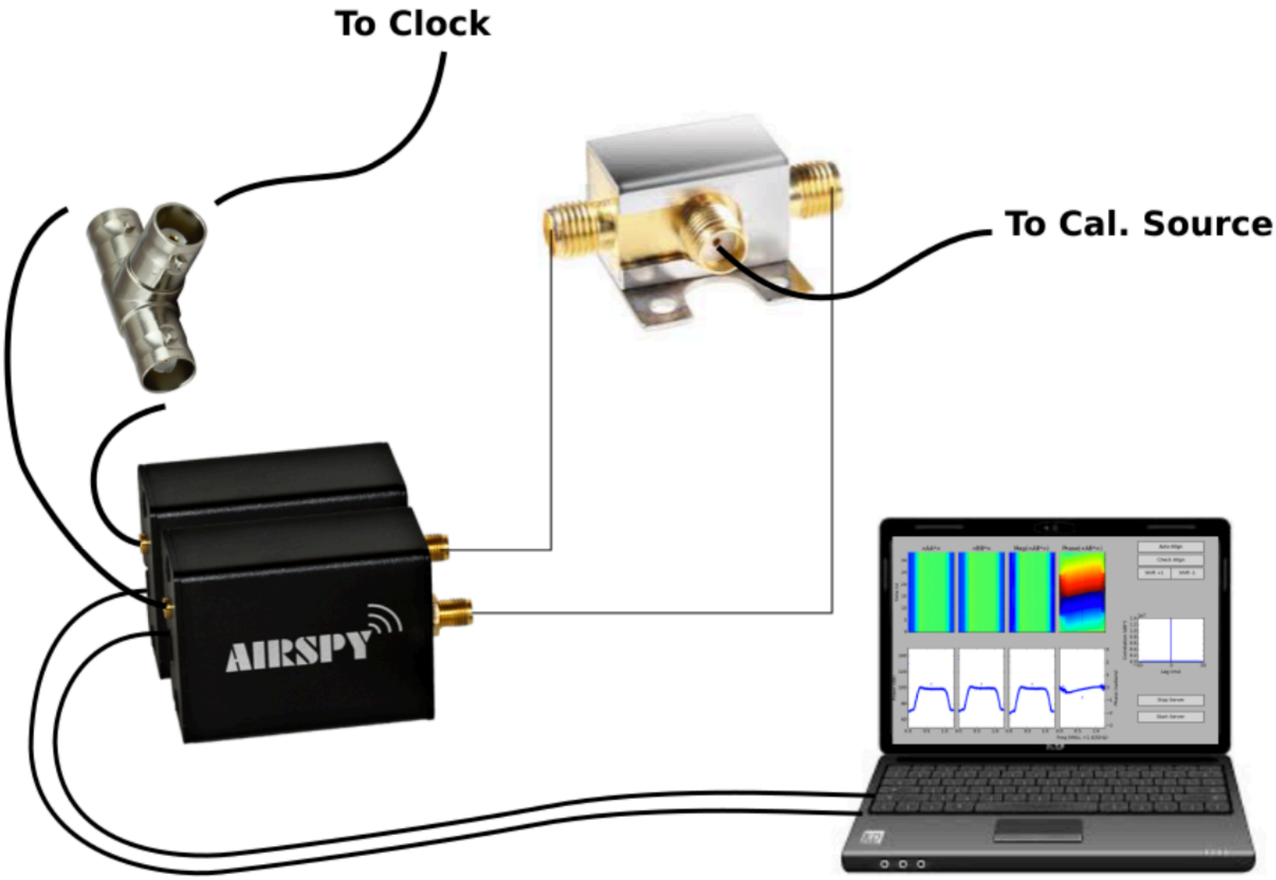


Figure 9: How to attach the calibration source.

This will trigger a lag correlation in the backend software, which will convolve the two signals and display the result in the plot on the right. White noise correlated with itself should yield a delta function: zero correlation at any delay other than perfect alignment, and a strong correlation when aligned. A typical (successful) result is shown in Figure 10.

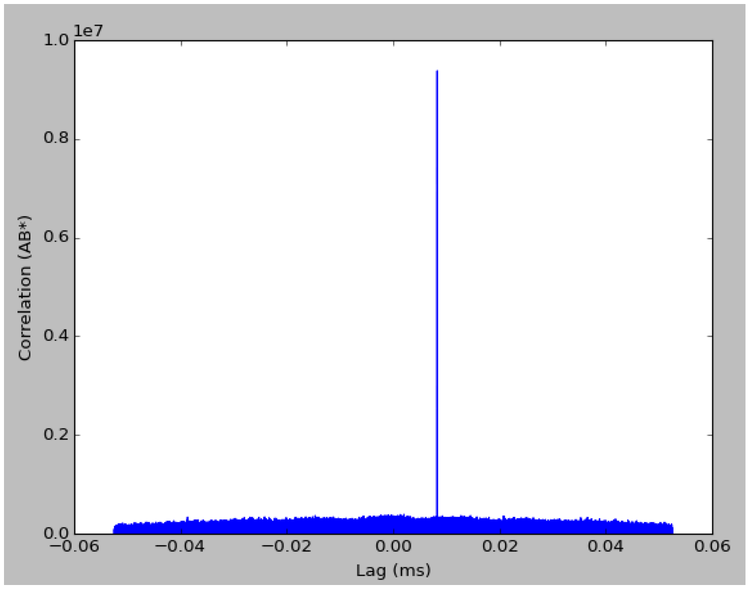


Figure 10: Correlation spectrum for white noise, showing a strong peak at ~0.86ms.

Provided things look good, click **Auto-Align** button, which will shift the time-streams in memory to place that peak at zero time delay, i.e. it will synchronize the two units. If successful, the peak will move to 0 lag, and the phase measured in the cross correlation window will change from apparently random noise to a nice steady value. If you see a large phase shift across the frequency band (i.e. if the lower right plot displays a diagonal, rather than flat, line), the calibration is off by a sample of two. Click the **Shift +1** or **Shift -1** buttons to shift the time-streams by a single sample and flatten the band.

WARNING:
We have noticed with the AirSpy units that external clocks on some units don't lock properly. If the time-streams align, but the phase continues to "wander" as opposed to remaining at a constant value in the right-hand plots, use the **Restart Server** button as a shortcut to Stop, Start, and Auto-Align in sequence. Keep restarting until you get a stable phase. If you're having trouble after a few dozen restarts, ask for help, we may be able to swap units. Don't proceed until the phase is stable over many seconds, this stability is important for later measurements.

That's it, hopefully you now have a calibrated interferometer backend! Still, let's do a couple of sanity checks before proceeding.

- The software is sampling at 2.5MHz, so covers a 1.25MHz band. How flat is the phase across your full band? If the time streams were offset by 1 sample, how would the phase vary? Try it, by adding a ±1 sample shift manually!

This sort of calibration is always required in an interferometer, and is generally referred to as **phasing up** the array.

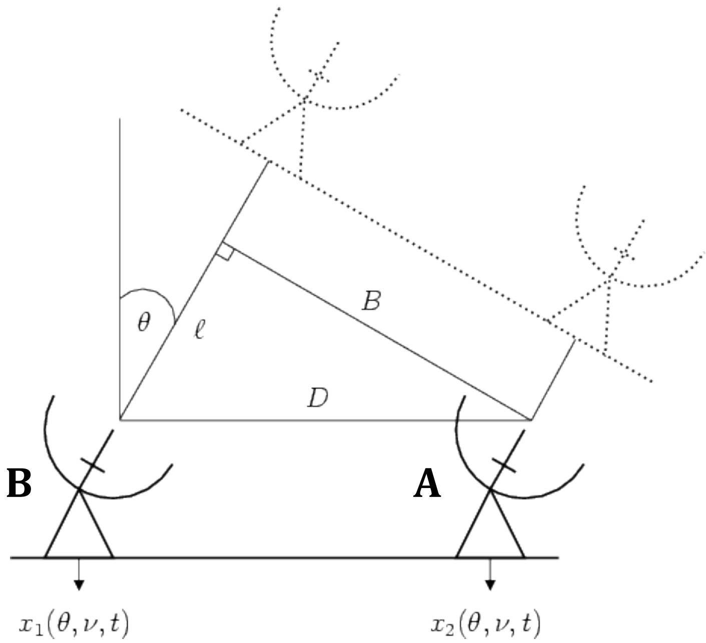


Figure 11: Phasing up an off-zenith array.

Remember that interference patterns between antennas take place within their primary beams. This is one of the reasons the flat sky approximation is usually justified: each antenna sees a small region of sky, and aperture synthesis allows us to resolve structure within that region.

When dishes are pointed off zenith, an additional delay *l* is introduced, as shown in Figure 11. To correct for this, the time-stream from B is advanced: the wavefront from field center arrives simultaneously in both data streams!

Once your antennas are phased up, return the calibration source and disconnect the splitter, then connect your feeds as shown in Figure 12, and proceed with the lab.

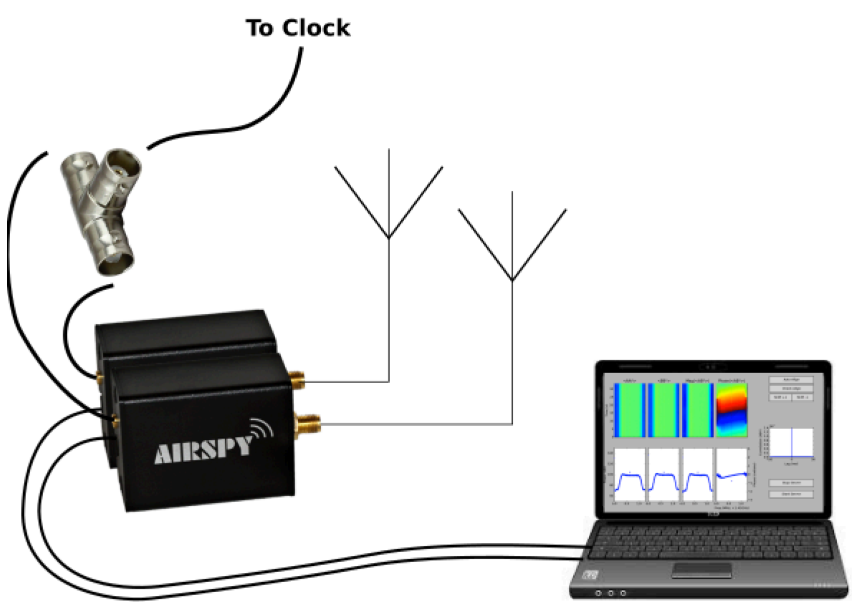


Figure 12: Connect your feeds, and you've got an interferometer!

Cross Correlation: Measurements (60 min)

Goals: Measure a beam in cross correlation. Understand the point of phase.

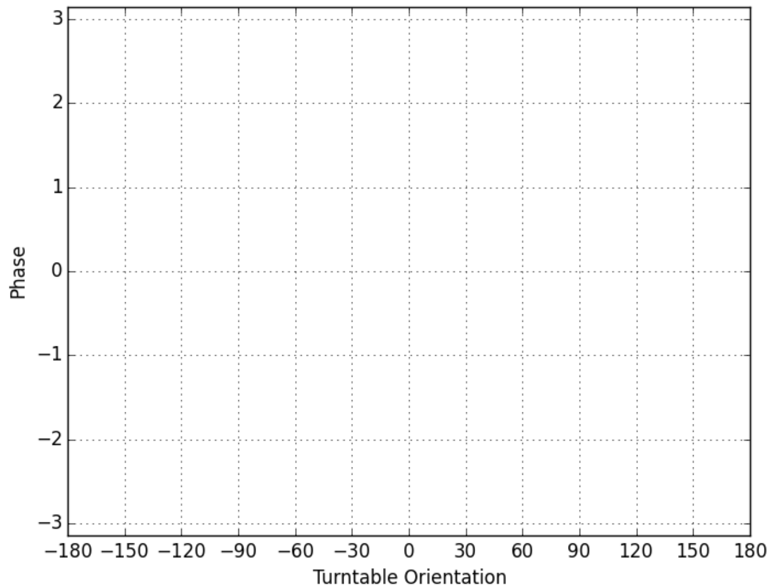
Now that we've got a fully functional interferometer, let's measure some correlations. As always, feel free to play with the setup and explore along the way.

To start, set one feed on top of the rotating table and aim it at the transmitter. Take the other antenna in your hand, and move it toward the transmitter, keeping an eye on the phase while you do so. How far do you move it before observing a full 2π phase shift? What wavelength are you observing?

Next, mount both feeds on a tee set on the rotating plate. Again, arrange so that an angle of 0 corresponds to both antennas pointing towards the transmitter. Try rotating things, watching the phase of the cross correlation. Remember, this is just the difference in times it takes the signal to arrive at each of the feeds. Pick a frequency that's easy to measure and record the phase there as a function of angle.

Measure the phase at multiple angles, with antenna spacing _____cm.

Angle (°)	Phase (rad)	Angle (°)	Phase (rad)	Angle (°)	Phase (rad)
0		120		-120	
10		130		-110	
20		140		-100	
30		150		-90	
40		160		-80	
50		170		-70	
60		±180		-60	
70		-170		-50	
80		-160		-40	
90		-150		-30	
100		-140		-20	
110		-130		-10	

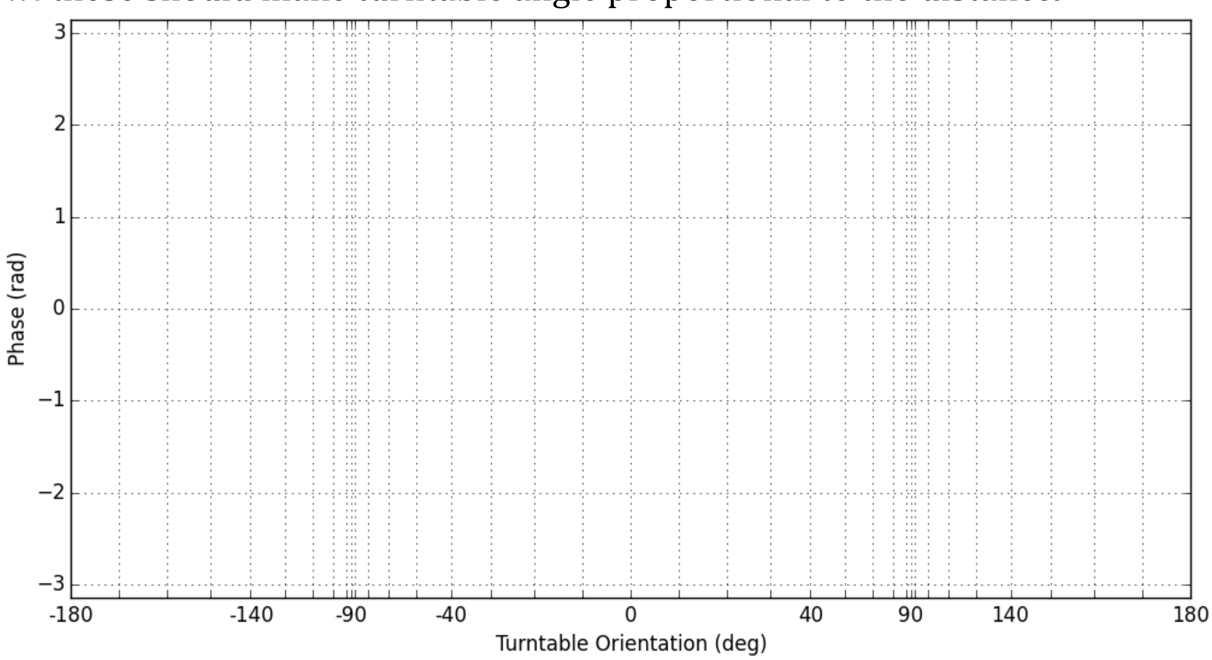


Note: When connecting points on the plot, remember that phase is periodic over 2π: it can 'wrap' from +π (top of plot) to −π (bottom of plot) and vice versa.

Notice how sensitive this phase measurement is compared to your previous measurements of power with the adding interferometer. You have much more resolving power with an interferometer than a total power receiver!

Add a short cable lag to one of the feeds, what is the effect on your measurement?

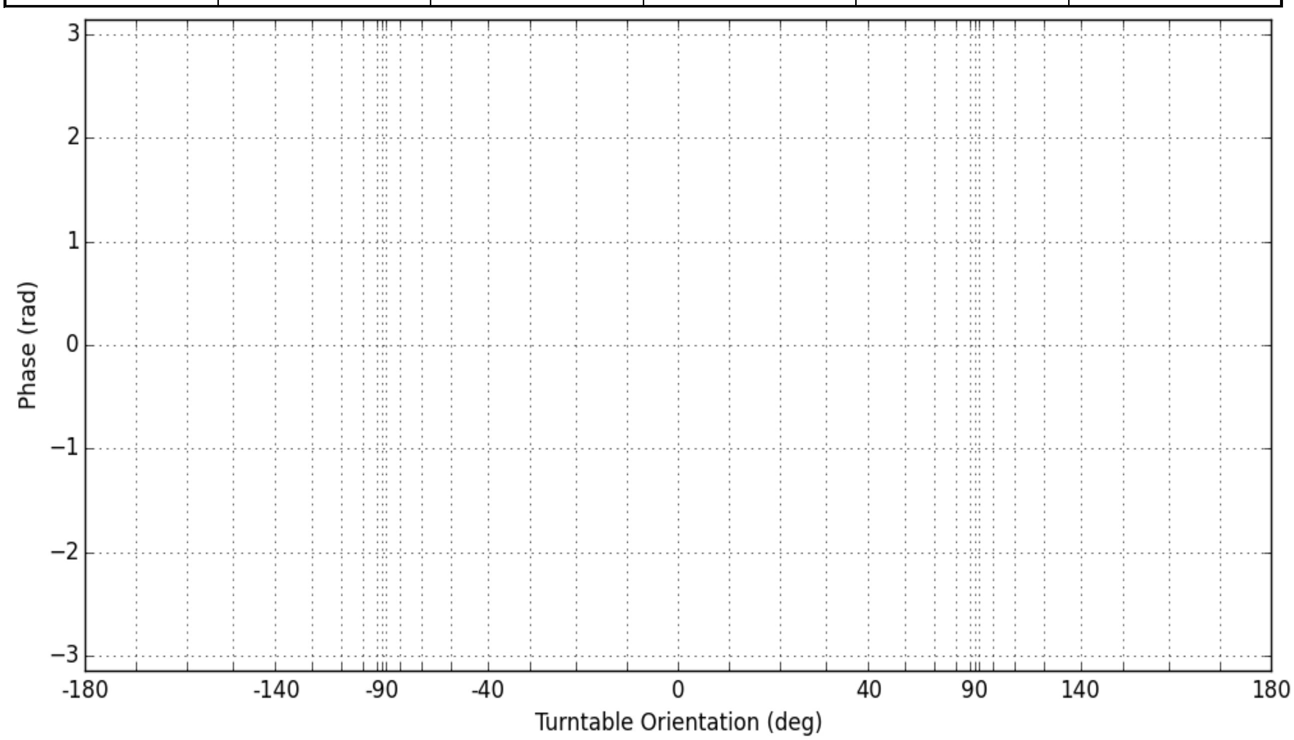
Your curve may be a little messy, because this turntable is well outside the small angle limit: what actually matters is the extra distance the signal has to travel, not the orientation of your mini array. Re-plot your values using the nonlinear axes below: these should make turntable angle proportional to the distance.



Next, try varying the antenna spacing and repeating the measurement. How does the fringe rate vary with spacing? Plot the results below.

Measure the phase at multiple angles, with antenna spacing _____cm. .

Angle (°)	Phase (rad)	Angle (°)	Phase (rad)	Angle (°)	Phase (rad)
0		120		-120	
10		130		-110	
20		140		-100	
30		150		-90	
40		160		-80	
50		170		-70	
60		±180		-60	
70		-170		-50	
80		-160		-40	
90		-150		-30	
100		-140		-20	
110		-130		-10	



Based on the rate at which the phase changes as you rotate your feed assembly, how far apart are your feeds? Measure the distance between your feeds and compare. Are there major discrepancies?

Bonus Activities! (leftover time)

Goals: Play with your fancy new radio interferometer.

You probably noticed a few odd patches in the phase-vs-angle plots. These are mostly caused by a few factors:

1) Non-idealities in your feeds, which may have nulls in odd places.
2) People walking about, changing the boundary conditions, reflecting and blocking signal paths.
3) Static reflections within the room. Can you identify reflected signals and where they're coming from?